

Anticipated Shocks, Optimal Monetary Policy, and Welfare

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1 Introduction

1.1 Topics and Literature Review

Intertemporal general-equilibrium models with Keynesian features such as imperfect competition and imperfectly flexible prices have become the workhorse model for monetary macroeconomics.¹ These type of models combine elements of the real business cycle literature initiated by Kydland and Prescott (1982) with Keynesian features proposed by, among others, Taylor (1980), Rotemberg (1982), Calvo (1983), or Blanchard and Kiyotaki (1987).

These New Keynesian or New Neoclassical Synthesis models describe the rational behavior of utility-maximizing households and profit-maximizing firms which act in monopolistically competitive markets. The optimal price setting behavior of firms and/or the optimal wage setting behavior of labor suppliers are constrained by mechanism that leads to a stickiness of prices and/or wages. The assumption of fairly representative agents makes it possible to aggregate the individual optimality conditions of households and firms. As a consequence, the model finally yields typical macroeconomic variables like aggregate production (output), aggregate absorption, aggregate employment, inflation, and nominal and real interest rates. Market clearing conditions for each individual type of good and each type of labor finally explain the description of these class of models as a general-equilibrium framework.²

The consideration of nominal rigidities leads to a (at least) short-run non-neutrality of monetary policy. The question then naturally arises how the monetary policy should be conducted in order to maximize overall welfare.³

An optimal monetary policy is thereby either the commitment to a policy plan which is derived from the intertemporal optimization of the policy-maker's objective function (policy under commitment), the solution of the optimization of the parameters of a given simple monetary instrument rule (optimal simple rule), or the solution of a sequential optimization problem without committing to future policies (policy under discretion).⁴

It is well-known that the solution of dynamic optimal control problems with forward-looking rational agents is dynamic inconsistent or strategic incoherent (see, for example, Wohltmann and Krömer (1989), Currie and Levine (1993), or McCallum (2005)). In order to overcome the problem of the dynamic inconsistency of optimal policy under commitment, Woodford (1999a) proposes the concept of timeless perspective policy-making.⁵

¹Woodford (2009) reviews the development of the new synthesis in macroeconomics and outlines the main elements of the new macroeconomic workhorse model.

²An introduction to the New Keynesian framework is provided by Walsh (2003b), Galí (2008), or for German readers, Wohltmann and Winkler (2008c).

³In the current literature welfare is defined as the aggregate utility of households. In earlier studies, however, welfare is often represented by an ad-hoc (quadratic) social loss function.

⁴An introduction to the problem of optimal monetary policy-making in the New Keynesian framework can be found in Clarida, Galí, and Gertler (1999), Walsh (2003b), Galí (2008), or for German readers, Wohltmann and Winkler (2008d).

⁵Dennis (2001), McCallum and Nelson (2004), Giannoni and Woodford (2003), and McCallum (2005) extensively discuss the timeless perspective policy approach.

The widely recognized shortcomings of policy-making from a timeless perspective are that the problem of time-inconsistency in the sense of Kydland and Prescott (1977) and Barro and Gordon (1983) is in fact not solved and that the policy generally fails to globally maximize the policy makers' objective function (see, among others, Jensen and McCallum (2002)). Nevertheless, the concept of timeless perspective commitment achieved a lot of attention and is widely considered as a good approximation of the globally optimal policy under commitment.

From a methodological point of view, the New Keynesian literature on optimal monetary policy can be divided into the linear-quadratic approach and the Ramsey approach. The first is widely applied and is based on a second-order approximation of the households' utility function.⁶ Rotemberg and Woodford (1999) and Woodford (2003) show that, under certain circumstances, this leads to a quadratic loss function in inflation and the output gap.⁷ However, the assumptions which must be made to derive this function are restrictive.⁸

Thus, Khan, King, and Wolman (2003) propose to use the Ramsey approach for the analysis of optimal monetary policy since it does not rely on any approximation of the utility function. Instead, the optimal policy is derived by maximizing the utility of households subject to the non-linearized model equations.⁹

This thesis now adds to the literature on the optimal design of monetary policy by demonstrating the possible inferiority of Ramsey monetary policy under timeless perspective commitment to simple (non-optimized) monetary policy rules. More precisely, we show that the Ramsey policy under timeless perspective commitment is identical to the globally optimal policy under commitment only if the government optimally subsidizes the cost of production. This fiscal policy instrument leads to the efficiency of the model without nominal rigidities as well as to the efficiency of the steady state. However, if we abandon the assumption that optimal subsidies are in place, there exist monetary instrument rules which are superior to the optimal policy under timeless perspective commitment.

In particular, we do find non-optimized but welfare-enhancing instrument rules in the presence of cost-push shocks (wage mark-up shocks, price mark-up shocks) and technology shocks (Chapter 4) as well as in the presence of oil price shocks (Chapter 5).

A similar result is reported by Blake (2001) and Jensen and McCallum (2002) who show within the linear-quadratic framework that simple policy rules deliver superior welfare results compared to the optimal policy under timeless perspective commitment. However, these studies apply the canonical New Keynesian model with a quadratic and ad-hoc loss function which does not rely explicitly on the utility of households. We are able to show the inferiority of policy-making from a timeless perspective for the Ramsey-type monetary policy with a utility-based welfare criteria.

Hence, our findings strongly support and generalize the results of Blake (2001) and Jensen and McCallum (2002) and call into question the appropriateness of the proposed Ramsey

⁶See, among others, Woodford (2003), Giannoni and Woodford (2004), or Galí (2008).

⁷The output gap is thereby defined as the deviation of actual output from the natural level of output which, in turn, is defined as the output that would prevail in the absence of nominal rigidities.

⁸Note that a lot of studies use a quadratic objective function which does not rely explicitly on the utility of households. See, among others, Clarida, Galí, and Gertler (1999), Woodford (1999b), Svensson (1999, 2000), Steinsson (2003), Walsh (2003a), Wollmershäuser (2006), Adolfson (2007), or Wohltmann and Winkler (2008a).

⁹The Ramsey approach is applied by, among others, Yun (2005), Faia (2008a, 2008b), Faia and Monacelli (2004, 2007, 2008), or Schmitt-Grohé and Uribe (2004b, 2005, 2007b).

policy under timeless perspective commitment as a guideline for the conduct of monetary policy.

After pointing out the contribution to the literature on the optimal design of monetary policy, we now turn to the second novel contribution of this thesis, namely the (welfare) analysis of anticipated shocks under (optimal) monetary policy.

A characteristic of dynamic general equilibrium models of the New Neoclassical Synthesis, established since the real business cycle revolution of Kydland and Prescott (1982), is that unanticipated random disturbances are considered as the main driving force in explaining business cycles.¹⁰ As a consequence, the literature on the optimal design of monetary policy usually considers only unanticipated shocks (see, among others, Clarida, Galí, and Gertler (1999), Svensson (1999), King, Khan, and Wolman (2003), or Woodford (2003)).

However recently, a number of macroeconometric studies emphasize the role of anticipated shocks as sources of macroeconomic fluctuations. Beaudry and Portier (2006) find that more than one half of business cycle fluctuations are caused by news about future technological opportunities. Davis (2007) and Fujiwara, Hirose, and Shintani (2008) analyze the importance of anticipated shocks in large scale DSGE models closely related to the model of Christiano, Eichenbaum, and Evans (2005) and report that these disturbances are important components of aggregate fluctuations. Schmitt-Grohé and Uribe (2008) conduct a Bayesian estimation of a real business cycle model and find that anticipated shocks are the most important source of aggregate fluctuations. In particular, they report that anticipated shocks explain two thirds of the volatility in consumption, output, investment, and employment.¹¹

In light of these findings, this thesis analyzes optimal monetary policy in the presence of anticipated shocks and systematically investigates the welfare consequences of the anticipation of future shocks.

The welfare effects of anticipated shocks were so far ignored by the literature. To the best of our knowledge, the only exception is the study of Wohltmann and Winkler (2008a) that compares the welfare effects of anticipated and unanticipated energy price shocks under different monetary policy regimes. However, this study applies a stylized New Keynesian model without rigorous micro-foundations.

This thesis, however, conducts an analysis of the welfare effects of anticipated shocks under different (optimal) monetary policy regimes within several fully micro-founded New Keynesian models. In particular, we investigate whether the anticipation of a future cost-push shock enhances welfare compared to an unanticipated shock of equal size. Or, to put it differently, we pose the question: Is the additional information about future cost disturbances a source of additional welfare? This thesis claims: Generally not!

¹⁰An exception is the stream of literature that analyzes anticipated disinflations going back to Ball (1994) who shows that a simple variant of the New Keynesian model predicts a boom in response to an anticipated disinflation.

¹¹Recent theoretical studies on the role of anticipations for business cycle fluctuations include Beaudry and Portier (2004, 2007), Beaudry, Collard, and Portier (2006), Jaimovich and Rebelo (2006, 2008), Den Haan and Kaltenbrunner (2007), or Christiano, Ilut, Motto, and Rostagno (2008). Contributions which do not utilize general equilibrium models are, among others, Brock's (1975) analysis of anticipated inflations and deflations in a monetary model with utility-maximizing agents, Wilson's (1979), Gray and Turnovsky's (1979), Turnovsky's (1986) and Clausen and Wohltmann's (2005) extensions of the celebrated Dornbusch (1976) overshooting model, or Bhandari and Turnovsky's (1984), Wohltmann's (1994b), Wohltmann and Clausen's (2003) and Wohltmann and Winkler's (2005a, 2005b, 2005c) investigations of anticipated shocks in oil-dependent economies. German readers are also referred to Wohltmann (1994a, 2000), Wohltmann and Bulthaupt (1999), or Wohltmann and Clausen (2001, 2002). However, none of these studies considers the welfare effects of the anticipation of future shocks.

More precisely, we find that anticipated cost-push shocks generally entail higher welfare losses than unanticipated cost-push shocks of equal size. We are able to analytically show this result within the canonical purely forward-looking New Keynesian model (Chapter 2). In particular, we find that – for empirically plausible degrees of nominal rigidity – the anticipation of a future cost-push shock leads to a higher welfare loss than an analogous unanticipated shock. A welfare gain from the anticipation of a future cost shock may only occur if prices are sufficiently flexible. These results, which are independent of the monetary policy regime, are consistent with the findings of Schmitt-Grohé and Uribe (2008). They show that the anticipation of future shocks has a stabilizing effect on an economy without nominal rigidities. We point out that precisely the degree of nominal rigidity plays an important role for the evaluation of the welfare effects of anticipations.

The result that the anticipation of a future cost-push shock leads to a higher welfare loss than an unanticipated shock of equal size is then confirmed within a micro-founded and numerically calibrated hybrid New Keynesian model. It includes the features of habit formation in consumption preferences and of a variant of the Calvo (1983) mechanism with partial indexation of non-optimized prices to past inflation (Chapter 3). The results we obtained from our simulations show that the welfare loss of anticipated cost-shocks exceeds the welfare loss of unanticipated cost-shocks of equal magnitude for plausible lengths of the time span between the anticipation and the realization of the shock, henceforth denoted as anticipation period.

Furthermore, we are able to confirm this finding within a non-linearized model for an oil-dependent economy (Chapter 5). We numerically demonstrate that the anticipation of typical cost-push shocks such as wage and price mark-up shocks enhances their welfare-reducing effects. This result holds under the globally optimal Ramsey-type monetary policy as well as under simple (non-optimized) monetary policy rules.

Chapter 5 contains a further novel contribution of this thesis. It adds to the literature on the interplay between monetary policy and oil price shocks by deriving the globally optimal Ramsey-type monetary policy for an oil-dependent economy.¹² Moreover, we compare the optimal monetary policy to simple (non-optimized) monetary policy rules.

Among the numerous empirical contributions that studies the interaction between oil price shocks and monetary policy see, for example, Hamilton (1983), Hamilton and Herrera (2004), Bernanke, Gertler, and Watson (1997, 2004), or Barsky and Kilian (2002). Theoretical contributions are, among others, Leduc and Sill (2004), Medina and Soto (2005), Carlstrom and Fuerst (2006), and Blanchard and Galí (2007).

Leduc and Sill (2004), Medina and Soto (2005), and Carlstrom and Fuerst (2006) seek to answer the question whether the recessionary consequences of oil price shocks are caused by the oil price hike or by the monetary response to the rise in oil prices. However, these studies concentrate on the recessionary consequences and do not, as we do, investigate the welfare effects of oil price shocks under different monetary policy regimes, in particular optimal

¹²Thus, this thesis is also related to the strand of literature that deals in general with the macroeconomic effects of oil price shocks. Contributions which utilize models without rigorous micro-foundations include, among others, Buiter (1978), Bhandari (1981), Bhandari and Turnovsky (1984), Bruno and Sachs (1985), Wohltmann (1994b), Wohltmann and Clausen (2003), and Wohltmann and Winkler (2005a, 2005b, 2005c, 2006, 2008a). German readers are also referred to Jarchow (1992) or Wohltmann (1987, 1993). In contrast, Kim and Loungani (1992), Rotemberg and Woodford (1996), and Finn (2000) analyze the effects of oil price shocks in micro-founded dynamic general equilibrium models of closed economies. Backus and Crucini (2000) consider an open-economy real business cycle model to study the effects of oil on the economy. The latter studies are based on the assumption of completely flexible prices. Hence, there is no role for monetary policy.

monetary policy of Ramsey-type.

The only studies, we are aware of, that analyze optimal monetary policy in the presence of oil shocks are De Fiori, Lombardo, and Stebunovs (2006), Montoro (2007), and Wohltmann and Winkler (2008a). De Fiori, Lombardo, and Stebunovs (2006) consider a three-country framework and search for optimal parameters of a simple monetary policy rule. Montoro (2007) considers a closed and oil-dependent economy with staggered price setting and investigates optimal monetary policy under commitment by conducting a second-order approximation of the utility function according to Benigno and Woodford (2005). Hence, this study is based on the linear-quadratic approach. This approach is also applied by Wohltmann and Winkler (2008a) who consider a stylized small-open economy framework with a loss function that reflects, according to Svensson (2000), the policy objective of flexible domestic inflation targeting. They derive the optimal policy under commitment and search for optimal simple policy rules. This thesis, for the first time, derives the globally optimal *Ramsey monetary policy* under commitment for an oil-dependent economy.

Our results contribute to the ongoing discussion whether the monetary policy amplifies or dampens the recessionary effects of oil price shocks by pointing out that the welfare-maximizing policy in fact calls for a sharp and prolonged output slump. In contrast, simple rules in the spirit of Taylor (1993) lead to a dampening of the output drop that is welfare-reducing.

1.2 Outline

The thesis is organized in two methodologically different parts. The first one, which consists of Chapter 2 and Chapter 3, is devoted to analyze linear dynamic systems with overall welfare measured by quadratic loss functions. The second part of the thesis, which comprises Chapter 4 and Chapter 5, deals with nonlinear dynamic systems. Thereby, we apply the Ramsey approach to the analysis of optimal monetary policy. In contrast to existing literature, the models of this part are solved without any approximation. Note that a log-linear approximation of the intrinsically nonlinear New Keynesian models leads to at least qualitatively inaccurate dynamic adjustment paths and welfare results. The reason is that variables which measure the distortions due to nominal rigidities (quadratic wage and price adjustment costs or measures of price and wage dispersion) can be ignored up to a first-order approximation of the respective system.

In more detail, the remainder of this thesis is organized as follows. Chapter 2 compares the welfare effects of unanticipated and anticipated cost-push shocks in the canonical New Keynesian model with a monetary authority which minimizes a quadratic loss function that weights the volatility of inflation and the output gap. It presents an analytical solution to dynamics and welfare in the case of optimal monetary policy under both timeless perspective commitment and discretion. Furthermore, a solution of welfare as a function of the time span between the anticipation and the realization of the shock is derived. This allows to discover the dependency of welfare on the length of the anticipation period. Moreover, a systematic investigation of the role of nominal rigidities for the welfare impacts of anticipations is conducted.

In order to analyze the (welfare) effects of anticipated shocks in more elaborate models, Chapter 3 presents a general solution method for linear dynamic rational expectations models with anticipated shocks and optimal policy. This method extends the work of Söderlind

(1999) who uses the generalized Schur decomposition method, advocated by Klein (2000), to solve linear rational expectations models with optimal policy. However, Söderlind (1999) only considers stochastic models with white noise shocks which are, by definition, unpredictable. In the case of anticipated shocks, the occurrence of all future shocks is known exactly at the time when the solution of the model is computed. The method presented in this thesis also contains unanticipated shocks as a limiting case.¹³

As an economic example, Chapter 3 presents a calibrated New Keynesian model for a closed and cashless economy with internal habit formation in consumption preferences, a variant of Calvo price staggering with partial indexation to past inflation, a time-varying wage mark-up which represents a typical cost-push shock and a monetary authority which seeks to minimize a quadratic loss function. The model is numerically simulated and the effects of mark-up shocks for different lengths of the anticipation period are compared.

Chapter 4 turns to the nonlinear Ramsey approach and studies optimal monetary policy in a standard New Keynesian model for a closed and cashless economy with price and wage rigidities resulting from quadratic adjustment costs. In particular, it derives the Ramsey-type optimal monetary policy in the presence of cost-push shocks such as wage and price mark-up shocks for the case of an inefficient and efficient steady state. Subsequently, welfare implications of Ramsey policy and simple monetary policy rules are discussed.

Chapter 5 analyzes the optimal monetary policy in an oil-dependent economy with staggered price and wage setting. This chapter picks up the two main topics of this thesis, namely the analysis of the welfare effects of anticipated shocks and the welfare analysis of Ramsey monetary policy under timeless perspective commitment in comparison to simple monetary policy rules. Furthermore, it raises the problem of optimal policy-making in the presence of energy price shocks.

More precisely, Chapter 5 compares the optimal Ramsey-type monetary policy under timeless perspective commitment to simple monetary policy rules for both the case of unanticipated and anticipated energy price shock. Thereby, we again study the performance of different monetary policy regimes in the case of an efficient and inefficient steady state. Moreover, we investigate the welfare effects of the anticipation of future shocks and analyze whether the findings reported in Chapter 2 and 3 are robust when we do not consider a linear-quadratic approach and instead utilize a non-linearized framework. Finally, we discuss what will be the best or optimal monetary policy in response to an increase in oil prices.

In the last chapter, our main results are summarized and an outlook for future research is given.

¹³Matlab codes that compute impulse response functions and the minimal value of the loss function for the policy regimes commitment, discretion and (optimal) simple rules can be downloaded from the authors' webpage at: http://www.wiso.uni-kiel.de/vwlinstitute/Wohltmann/REAS_solution.zip.

Part I: Anticipated Shocks and Optimal
Monetary Policy in Linear Dynamic
Systems

2 Welfare Effects of Anticipated Shocks in the Canonical New Keynesian Model

2.1 Introduction

Does the *anticipation* of future shocks have a stabilizing and thus welfare-enhancing effect on the economy compared to *unanticipated* shocks? In this chapter¹, we seek to answer this question by comparing the welfare effects of unanticipated and anticipated cost-push shocks in the canonical New Keynesian model with a monetary authority which minimizes a quadratic loss function that weights the volatility of inflation and the output gap. In particular, we analytically solve for dynamics and welfare in the case of optimal monetary policy under timeless perspective commitment and discretion. We distinguish the usual case of unanticipated cost-push shocks and the case of future cost-push shocks that are known in advance. We derive a solution of welfare as a function of the time span between the anticipation and the realization of the shock which enables us to discover the dependency of welfare on the length of the anticipation period. Furthermore, we systematically investigate the role of nominal rigidities for the welfare impacts of anticipations.

To the best of our knowledge, Wohltmann and Winkler (2008a) is the only study that compares the welfare effects of anticipated and unanticipated shocks. They analyze energy price shocks under different monetary policy regimes including optimal monetary policy. However, this study relies on numerical simulations and do not, as we do in this chapter, investigate the role of nominal rigidities.

The main results of this chapter are the following. For empirically plausible degrees of nominal rigidity, the anticipation of a future cost-push shock leads to a higher welfare loss than an analogous unanticipated shock. A welfare gain from the anticipation of a future cost shock may only occur if prices are sufficiently flexible. This result is consistent with the findings of Schmitt-Grohé and Uribe (2008) who show that the anticipation of future shocks has a stabilizing effect on an economy without nominal rigidities. We point out that precisely the degree of nominal rigidity plays an important role for the evaluation of the welfare effects of anticipations.

Our results are driven by two opposing effects. On the one hand, we obtain the well-known result that the anticipation of a future shock dampens its impact effect. On the other hand, we show that the anticipation of future cost-push shocks enhances the persistence of output and inflation and thus enhances the welfare loss. This persistence effect, in turn, is amplified by the degree of price stickiness.

Nevertheless, at a first glance, our findings seem to be puzzling since it suggests that the information about the occurrence of future shocks is in general welfare-reducing. But then the

¹For a different version of this chapter see Wohltmann and Winkler (2008b).

question arises, why rational agents do not ignore the knowledge about future disturbances. In the remainder of this chapter, we will seek to shed more light on this question.

This chapter is organized as follows. Section 2 presents the canonical New Keynesian model and its solution under the policy regimes timeless perspective commitment and discretion. In Section 3, we report and discuss our main findings. Furthermore, we provide analytical proofs and, for the sake of illustration, numerical simulations. Section 4 concludes.

2.2 The Framework

We consider the canonical New Keynesian model. It consists of an optimizing IS-type relationship of the form

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1}) \quad (\sigma > 0) \quad (2.1)$$

and a price adjustment equation of Calvo-Rotemberg type, often referred to as New Keynesian Phillips Curve (NKPC)

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + k_t \quad (0 < \beta < 1, \kappa > 0) \quad (2.2)$$

x_t denotes the output gap, π_t is inflation, and i_t is the nominal interest rate. E_t is the expectations operator conditional on information up to date t . β is the discount factor and $1/\sigma$ denotes the intertemporal elasticity of substitution. It is well-known that under the assumptions of Calvo (1983) price setting, a constant returns to scale production function with labor as single input, and perfect labor markets, the slope parameter κ is given by $\kappa = (\eta + \sigma) \frac{(1-\omega)(1-\beta\omega)}{\omega}$, where η is the inverse of the labor supply elasticity.² Obviously, κ is negatively correlated with the degree of price rigidity ω . According to the Calvo price adjustment mechanism, a fraction $1 - \omega$ of firms can adjust their price in period t . Simultaneously, ω is the probability that a single price which is reoptimized in period t , also holds in the next period $t + 1$. The Calvo parameter ω is therefore a measure of the degree of price rigidity on the goods markets.

In the NKPC, k_t represents a temporary cost-push shock that is assumed to be autoregressive of order one with AR parameter $\varphi \in [0, 1)$ and a one-unit cost shock ε_t

$$k_t = \varphi k_{t-1} + \varepsilon_t \quad (t \geq T > 0) \quad (2.3)$$

Since we consider anticipated cost-push shocks, the one-unit cost shock ε_t is not white noise, but known to the public before the shock actually occurs.³ Assume that at time $t = 0$ the public anticipates the cost-push shock to take place at some future time $T > 0$. Then,

$$\varepsilon_t = \begin{cases} 1 & \text{for } t = T > 0 \\ 0 & \text{for } t \neq T \end{cases} \quad (2.4)$$

The adjustment dynamics induced by anticipated shocks involve two phases, the time span between the anticipation and the realization of the shock ($0 \leq t < T$) and the time span after the implementation of the shock ($T \leq t \leq \infty$). The lead time T up to the realization

²See, for example, Walsh (2003b) for a derivation of the NKPC under Calvo pricing.

³Schmitt-Grohé and Uribe (2007c) study the impacts of anticipated cost shocks on the pass-through to prices.

of the shock is equal to the length of the anticipation phase $0 \leq t < T$. An implication of our definition of anticipated shocks is that rational expectations are equivalent to perfect foresight so that we can omit the expectations operator.

The policy maker's objective at the time of anticipation $t = 0$ is to minimize the intertemporal loss function

$$V = E_0 \sum_{t=0}^{\infty} \beta^t (\alpha_1 \pi_t^2 + \alpha_2 x_t^2) \quad (\alpha_1 > \alpha_2 > 0, 0 < \beta \leq 1) \quad (2.5)$$

which reflects the objective of flexible inflation targeting (see, for example, Svensson (1999)). Rotemberg and Woodford (1999) and Woodford (2003) show that, under certain conditions, a quadratic loss function in inflation and the output gap is the correct approximation to the representative agent's utility function.

The first-order conditions of the policy problem under timeless perspective precommitment monetary policy as well as under discretion are well known and need not to be derived here (see, for example, Walsh (2003b)). Under the optimal timeless perspective precommitment policy, inflation satisfies the targeting rule

$$\pi_t = -\frac{\alpha_2}{\alpha_1 \kappa} (x_t - x_{t-1}) \quad (2.6)$$

while the output gap is described by the second-order difference equation

$$\left(1 + \beta + \frac{\alpha_1 \kappa^2}{\alpha_2}\right) x_t - x_{t-1} - \beta E_t x_{t+1} = -\frac{\alpha_1 \kappa}{\alpha_2} k_t \quad (2.7)$$

where the expectational operator can be omitted in the case of anticipated shocks.

To solve the difference equation for x_t , write equation (2.7) as

$$\begin{pmatrix} x_{t+1} \\ w_{t+1} \end{pmatrix} = C \begin{pmatrix} x_t \\ w_t \end{pmatrix} + \begin{pmatrix} \frac{\alpha_1 \kappa}{\alpha_2 \beta} \\ 0 \end{pmatrix} k_t \quad (2.8)$$

where $w_t = x_{t-1}$ and

$$C = \begin{pmatrix} \frac{1}{\beta} \left(1 + \beta + \frac{\alpha_1 \kappa^2}{\alpha_2}\right) & -\frac{1}{\beta} \\ 1 & 0 \end{pmatrix} \quad (2.9)$$

The auxiliary variable w_t is backward-looking (with the initial value $w_0 = 0$, while the output gap x_t is forward-looking. The system matrix C has two real eigenvalues r_1 and r_2 with $r_1 > 1 > r_2 > 0$ so that the Blanchard and Kahn (1980) saddlepath stability condition is satisfied.

The solution for the output gap over the anticipation phase is given by

$$x_t = -\frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} \frac{\alpha_1 \kappa}{\alpha_2 \beta} r_1^{-T} (r_1^{t+1} - r_2^{t+1}) \quad \text{for } t < T \quad (2.10)$$

with the initial values

$$x_0 = -\frac{1}{r_1 - \varphi} \frac{\alpha_1 \kappa}{\alpha_2 \beta} r_1^{-T}, \quad x_{-1} = 0 \quad (2.11)$$

while the solution for $t \geq T$ is defined by

$$x_t = \frac{\alpha_1 \kappa}{\alpha_2 \beta} \frac{1}{(r_1 - \varphi)(r_2 - \varphi)} \left[\varphi^{t+1-T} - \frac{(r_1 - \varphi)r_2^{-T} - (r_2 - \varphi)r_1^{-T}}{r_1 - r_2} r_2^{t+1} \right] \text{ for } t \geq T \quad (2.12)$$

In the limiting case of unanticipated shocks ($T = 0$), the term in brackets in equation (2.12) simplifies to $\varphi^{t+1} - r_2^{t+1}$. Note that the solution formula (2.10) also holds in the shock period $t = T$.

Using (2.6), the solution time path of the inflation rate follows

$$\pi_t = \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} r_1^{-T} [(r_1 - 1)r_1^t - (r_2 - 1)r_2^t] \text{ for } t \leq T \quad (2.13)$$

with the initial value

$$\pi_0 = \frac{1}{\beta} \frac{1}{r_1 - \varphi} r_1^{-T} \quad (2.14)$$

and

$$\pi_t = \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_2 - \varphi} \left[(1 - \varphi)\varphi^{t-T} - \frac{(r_1 - \varphi)r_2^{-T} - (r_2 - \varphi)r_1^{-T}}{r_1 - r_2} (1 - r_2)r_2^t \right] \text{ for } t \geq T \quad (2.15)$$

In the limiting case $T = 0$, the term in brackets simplifies to $(1 - \varphi)\varphi^t - (1 - r_2)r_2^t$.

To determine the welfare loss under the optimal precommitment policy, write the loss function V as $V_1 + V_2$, where

$$V_1 = E_0 \sum_{t=0}^{T-1} \beta^t (\alpha_1 \pi_t^2 + \alpha_2 x_t^2) \quad (2.16)$$

is the loss in the anticipation period and

$$V_2 = E_0 \sum_{t=T}^{\infty} \beta^t (\alpha_1 \pi_t^2 + \alpha_2 x_t^2) \quad (2.17)$$

is the loss caused by the realization of the shock.

By inserting the solution for x_t and π_t , the loss V_1 can be rewritten as

$$V_1 = \alpha_1 \lambda^2 r_1^{-2T} (r_1^T - r_2^T) \left(\frac{r_1 - 1}{r_2^T} + \frac{1 - r_2}{r_1^T} \right) \quad (2.18)$$

where

$$\lambda = \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} \quad (2.19)$$

Accordingly, the loss V_2 can be rewritten as

$$V_2 = \frac{\alpha_1 \beta^T}{\beta^2 (r_1 - \varphi)^2} \left\{ \frac{(r_2^T - r_1^T)^2 (1 - r_2)}{(r_1 - r_2)^2 r_1^{2T}} + \frac{r_1}{r_1 r_2 - \varphi^2} \right\} \quad (2.20)$$

The total loss V is then simply given by $V = V_1 + V_2$.

Under the policy regime discretion (D), the central bank is unable to make a commitment to future policies. Now private expectations are given for the central bank and the reduced form of the first-order conditions reads as

$$\pi_t = -\frac{\alpha_2}{\alpha_1 \kappa} x_t \quad (2.21)$$

$$E_t x_{t+1} = \frac{1}{\beta} \left[1 + \frac{\alpha_1 \kappa^2}{\alpha_2} \right] x_t + \frac{\alpha_1 \kappa}{\alpha_2 \beta} k_t \quad (2.22)$$

with $E_t x_{t+1} = x_{t+1}$ in the case of anticipated shocks. The difference equation in x_t has the unstable eigenvalue

$$r_D = \frac{1}{\beta} \left[1 + \frac{\alpha_1 \kappa^2}{\alpha_2} \right] = \frac{1}{\alpha_2 \beta} [\alpha_2 + \alpha_1 \kappa^2] > 1 \quad (2.23)$$

and the forward solution

$$x_t = - \sum_{s=0}^{\infty} r_D^{-s} \frac{1}{r_D} \frac{\alpha_1 \kappa}{\alpha_2 \beta} k_{t+s} \quad (2.24)$$

Since

$$k_{t+s} = \begin{cases} \varphi^{t+s-T} & \text{for } t+s \geq T \\ 0 & \text{for } t+s < T \end{cases} \quad (2.25)$$

we obtain for $t \geq T$

$$x_t = - \frac{\alpha_1 \kappa}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} \varphi^{t-T} \quad (2.26)$$

and for $t < T$

$$x_t = - \frac{\alpha_1 \kappa}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} r_D^{t-T} \quad (2.27)$$

Due to $r_D^{t-T} = 1$ for $t = T$, the solution formula (2.27) also holds in the shock period $t = T$. For $t = 0$ we obtain

$$x_0 = - \frac{\alpha_1 \kappa}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} r_D^{-T} \quad (2.28)$$

so that the size of the initial jump of x_t decreases with increasing T .

For the inflation rate π_t we obtain the solution time path

$$\pi_t = \begin{cases} \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} r_D^{t-T} & \text{if } 0 \leq t \leq T \\ \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} \varphi^{t-T} & \text{if } t \geq T \end{cases} \quad (2.29)$$

Note that the limiting case $\varphi = 0$ implies $\pi_t = x_t = 0$ for $t > T$.

It is well-known that the loss under discretion (V_D) is greater than the total loss under the optimal precommitment policy. By inserting the solution time paths for π_t and x_t in the

loss function, we obtain

$$\begin{aligned}
V_D &= V_1^D + V_2^D = \sum_{t=0}^{T-1} \beta^t \left[\frac{\alpha_2^2}{\alpha_1 \kappa^2} + \alpha_2 \right] x_t^2 + \sum_{t=T}^{\infty} \beta^t \left[\frac{\alpha_2^2}{\alpha_1 \kappa^2} + \alpha_2 \right] x_t^2 \\
&= \frac{\alpha_1 \alpha_2 [\alpha_2 + \alpha_1 \kappa^2]}{[\alpha_2 (1 - \beta \varphi) + \alpha_1 \kappa^2]^2} \left(\frac{r_D^{-2T} - \beta^T}{1 - \beta r_D^2} + \frac{\beta^T}{1 - \beta \varphi^2} \right) \\
&= \frac{\alpha_1 \alpha_2 [\alpha_2 + \alpha_1 \kappa^2]}{[\alpha_2 (1 - \beta \varphi) + \alpha_1 \kappa^2]^2} \frac{1}{1 - \beta r_D^2} \left(r_D^{-2T} - \frac{\beta (r_D^2 - \varphi^2)}{1 - \beta \varphi^2} \beta^T \right)
\end{aligned} \tag{2.30}$$

where

$$\frac{1}{1 - \beta r_D^2} = \frac{\alpha_2^2 \beta}{\alpha_2^2 \beta - (\alpha_2 + \alpha_1 \kappa^2)^2} < 0 \tag{2.31}$$

2.3 Main Results

In this section, we compare the welfare loss induced by anticipated shocks ($T > 0$) to the corresponding loss if the same deterministic shock is not anticipated in advance ($T = 0$). In particular, we investigate the properties of the welfare loss V considered as function of the lead time T .

Since the size of the initial jumps of the forward-looking variables x_t and π_t are negatively correlated with the lead time T , we can conjecture that the loss function $V = V(T)$ is a decreasing function in T . In the following, we will demonstrate that this conjecture is false in general. It is only true, if the degree of price flexibility is very high.

Our main results can be summarized in the form of four propositions.

Proposition 1. *Without discounting (i.e. $\beta = 1$) the welfare loss induced by an anticipated cost-push shock is greater than the corresponding loss in the case of an unanticipated shock. This result is independent of the length of the lead time T and the degree of price rigidity ω :*

$$\begin{aligned}
\text{If } \beta = 1, \text{ then } V(0) < V(T) \text{ for all } T > 0 \\
\text{and all } \omega > 0.
\end{aligned} \tag{2.32}$$

A similar result holds with discounting ($\beta < 1$) provided the degree of price rigidity ω is sufficiently high and the time span between anticipation and realization of the shock is not too large.

Proposition 2. *If β is less than unity and the degree of price flexibility $1 - \omega$ low, there exists a positive upper bound T_c^* for the lead time T , positively depending on ω , such that*

$$V(0) < V(T) \text{ for all } 0 < T < T_c^*. \tag{2.33}$$

Proposition 3. *If the degree of price flexibility is very high (i.e. ω very small) then $T_c^* = 0$ so that*

$$V(T) < V(0) \text{ for all } T > 0. \tag{2.34}$$

Only in this case (which seems empirically not very realistic), the welfare loss under anticipated cost-push shocks is always smaller than under unanticipated shocks.

Proposition 4. *The propositions 1, 2, and 3 hold under the optimal monetary policy regimes timeless perspective commitment and discretion. They also hold under (optimal) simple rules of Taylor-type.*

Sketch of Proof of Propositions 1, 2, and 3. Consider the partial loss function V_1 (given by (2.18)) as function of T (the time span between the anticipation and realization of the cost-push shock).

The function $V_1 = V_1(T)$ has the following properties:

$$V_1(0) = 0, \quad \lim_{T \rightarrow \infty} V_1(T) = \begin{cases} 0 & \text{for } \beta < 1 \\ \bar{V}_1 > 0 & \text{for } \beta = 1 \end{cases} \quad (2.35)$$

where

$$\bar{V}_1 = \frac{\alpha_1(r_1 - 1)}{(r_1 - \varphi)^2(r_1 - r_2)^2} \quad (2.36)$$

The derivative of V_1 with respect to T , i. e.

$$\begin{aligned} \frac{dV_1}{dT} = \alpha_1 \lambda^2 & \left\{ 2 \ln r_1 \cdot r_1^{-2T} [r_1 + r_2 - 2] - (r_1 - 1) \ln(r_1 r_2) \cdot (r_1 r_2)^{-T} \right. \\ & \left. - (1 - r_2) \ln \left(\frac{r_2}{r_1^3} \right) \cdot \left(\frac{r_2}{r_1^3} \right)^T \right\} \end{aligned} \quad (2.37)$$

is positive at time $T = 0$:

$$\left. \frac{dV_1}{dT} \right|_{T=0} = \alpha_1 \frac{1}{\beta^2} \frac{1}{(r_1 - \varphi)^2} \frac{1}{r_1 - r_2} [\ln r_1 - \ln r_2] > 0 \quad (2.38)$$

Therefore, $V_1(T)$ starts to rise with increasing T (although the size of the initial jumps of x_t and π_t is decreasing in T). For $\beta < 1$, the limit value $\lim_{T \rightarrow \infty} V_1(T)$ is equal to zero. Therefore, $V_1(T)$ must decrease if T is sufficiently large.

The loss function $V_2 = V_2(T)$ (given by (2.20)) has the following properties:

$$V_2(0) = \frac{\alpha_1}{\beta^2(r_1 - \varphi)^2} \frac{r_1}{r_1 r_2 - \varphi^2} > 0 \quad (2.39)$$

$$\lim_{T \rightarrow \infty} V_2(T) = \begin{cases} 0 & \text{if } \beta < 1 \\ \bar{V}_2 > V_2(0) \Big|_{\beta=1} = \frac{\alpha_1 r_1}{(r_1 - \varphi)^2(1 - \varphi^2)} & \text{if } \beta = 1 \end{cases} \quad (2.40)$$

where

$$\bar{V}_2 = \frac{\alpha_1}{(r_1 - \varphi)^2} \left\{ \frac{1 - r_2}{(r_1 - r_2)^2} + \frac{r_1}{1 - \varphi^2} \right\} \quad (2.41)$$

The first derivative of V_2 with respect to T

$$\begin{aligned} \frac{dV_2}{dT} = \frac{\alpha_1}{\beta^2(r_1 - \varphi)^2} \beta^T & \left\{ \frac{r_1}{r_1 r_2 - \varphi^2} \ln \beta \right. \\ & \left. + \frac{1 - r_2}{(r_1 - r_2)^2} \left[(\ln r_2 - 3 \ln r_1) \left(\frac{r_2}{r_1} \right)^{2T} + 4 \ln r_1 \left(\frac{r_2}{r_1} \right)^T + \ln \beta \right] \right\} \end{aligned} \quad (2.42)$$

implies for $\beta < 1$ and $T = 0$

$$\left. \frac{dV_2}{dT} \right|_{T=0} = \frac{\alpha_1}{\beta^2(r_1 - \varphi)^2} \frac{r_1}{r_1 r_2 - \varphi^2} \ln \beta < 0 \quad (2.43)$$

since $\beta = 1/(r_1 r_2)$. For $\beta < 1$, the derivative dV_2/dT is also negative if T is sufficiently large. In the limiting case $\beta = 1$, the loss function $V_2(T)$ is an increasing function in T with a limit value $\bar{V}_2 > V_2(0)$.

We can now investigate the development of the total loss $V = V_1 + V_2$.

In the limiting case $\beta = 1$, the total loss $V(T)$ is an overall increasing function in T with $V(0) = V_2(0) > 0$ and

$$\lim_{T \rightarrow \infty} V(T) = \frac{\alpha_1}{(r_1 - \varphi)^2} \left\{ \frac{1}{r_1 - r_2} + \frac{r_1}{1 - \varphi^2} \right\} > V_2(0) \Big|_{\beta=1} > 0 \quad (2.44)$$

If $\beta = 1$, we can write $V(T)$ as $V_1(T) + V_2(T)$, where

$$V_1(T) = \frac{\alpha_1}{(r_1 - \varphi)^2 (r_1 - r_2)^2} \left[(r_1 - 1) + (2 - r_1 - r_2) r_1^{-2T} - (1 - r_2) \left(\frac{r_2}{r_1} \right)^T \right] \quad (2.45)$$

$$V_2(T) = \frac{\alpha_1}{(r_1 - \varphi)^2} \left\{ \frac{1 - r_2}{(r_1 - r_2)^2} \left[1 - \left(\frac{r_2}{r_1} \right)^T \right]^2 + \frac{r_1}{1 - \varphi^2} \right\} \quad (2.46)$$

Then

$$\begin{aligned} \frac{dV_1}{dT} &= \frac{\alpha_1}{(r_1 - \varphi)^2 (r_1 - r_2)^2} \left\{ 2[r_1 + r_2 - 2] \ln r_1 \right. \\ &\quad \left. + [3 \ln r_1 - \ln r_2] (1 - r_2) \left(\frac{r_2}{r_1} \right)^T \right\} r_1^{-2T} > 0 \quad \text{for all } T \geq 0 \end{aligned} \quad (2.47)$$

(due to $r_1 + r_2 = \text{tr } C > 2$ and $\ln r_2 < 0$) and

$$\begin{aligned} \frac{dV_2}{dT} &= \frac{\alpha_1}{(r_1 - \varphi)^2 (r_1 - r_2)^2} \left\{ -2 \left(1 - \left(\frac{r_2}{r_1} \right)^T \right) \ln \left(\frac{r_2}{r_1} \right) \right\} \left(\frac{r_2}{r_1} \right)^T \\ &\stackrel{(&=)}{>} 0 \quad \text{if } T \stackrel{(&=)}{>} 0 \end{aligned} \quad (2.48)$$

(because $0 < r_2 < 1 < r_1$). Therefore, $dV/dT > 0$ for all $T \geq 0$ so that V is a monotonically increasing function in T . This result holds independently of the degree of price rigidity ω .

For $\beta < 1$, $V(0) = V_2(0) > 0$ (with $V_2(0)$ defined in (2.39)) and $\lim_{T \rightarrow \infty} V(T) = 0$. For small values of ω , i.e. a high degree of price flexibility, the total loss V is a decreasing function in T implying $V(T) < V(0)$ for all $T > 0$. With high price flexibility, the welfare loss under anticipated shocks is smaller than under unanticipated shocks.

For the derivative dV/dT at time $T = 0$ we obtain

$$\left. \frac{dV}{dT} \right|_{T=0} = \frac{\alpha_1}{\beta^2(r_1 - \varphi)^2} \left\{ \left[\frac{1}{r_1 - r_2} - \frac{r_1}{r_1 r_2 - \varphi^2} \right] \ln r_1 - \left[\frac{1}{r_1 - r_2} + \frac{r_1}{r_1 r_2 - \varphi^2} \right] \ln r_2 \right\} \quad (2.49)$$

Then

$$\left. \frac{dV}{dT} \right|_{T=0} > 0 \Leftrightarrow 2 \left(\frac{1}{\beta} - \varphi^2 \right) \ln r_1 + (r_1^2 - \varphi^2) \ln \beta > 0 \quad (2.50)$$

A rising ω induces a fall in the unstable eigenvalue r_1 since $d\kappa/d\omega < 0$. Since the fall in r_1^2 is stronger than the decrease in $\ln r_1$, and $1/\beta - \varphi^2 > 0$, inequality (2.50) is fulfilled if the degree of price rigidity ω is sufficiently large. In this case, $V(T)$ starts to rise and due to $\lim_{T \rightarrow \infty} V(T) = 0$ its development must be hump-shaped implying the existence of an upper bound $T_c^* > 0$ such that $V(T) > V(0) > 0$ for all $T < T_c^*$.

The value of the upper bound T_c^* is the positive solution of the equation $V(T) = V(0)$, where $V(0) = V_2(0)$ is given by (2.39). This leads to the equation

$$1 - \left(\frac{r_2}{r_1} \right)^T = [(r_1 r_2)^T - 1] \frac{r_1(r_1 - r_2)}{r_1 r_2 - \varphi^2} \quad (2.51)$$

Equation (2.51) can be written as

$$\beta^T r_1^{2T} \left[\beta r_1^2 \left(1 - \frac{1}{\beta^T} \right) + \frac{1}{\beta^T} - \beta \varphi^2 \right] = 1 - \beta \varphi^2 \Leftrightarrow \quad (2.52)$$

$$r_1^{2T} [\beta^{T+1} (r_1^2 - \varphi^2) + (1 - \beta r_1^2)] = 1 - \beta \varphi^2 \quad (2.53)$$

so that T_c^* is also the positive solution of (2.52) and (2.53). The value of T_c^* is dependent on ω and β . A rising ω (a higher degree of price rigidity) decreases the unstable eigenvalue r_1 so that the left-hand side of equation (2.52) is decreased while the right-hand side remains unchanged. Since $\beta^T r_1^{2T} = (r_1/r_2)^T$ is increasing in T , equation (2.52) implies that the solution value T_c^* must increase if ω rises. Conversely, a higher degree of price flexibility induces a fall in T_c^* . For sufficiently small values of ω , the only solution of (2.53) is $T_c^* = 0$ (so that $V(T) < V(0)$ for all $T > 0$). If a positive solution T_c^* of (2.53) exists, then it is also an increasing function in the discount factor β with $T_c^* = \infty$ if $\beta = 1$. \square

Sketch of Proof of Proposition 4. Consider V_D (given by (2.30)) as function in T . Then

$$V_D(0) = \frac{\alpha_1 \alpha_2 [\alpha_2 + \alpha_1 \kappa^2]}{[\alpha_2(1 - \beta \varphi) + \alpha_1 \kappa^2]^2} \frac{1}{1 - \beta \varphi^2} > 0 \quad (2.54)$$

and

$$\lim_{T \rightarrow \infty} V_D(T) = \begin{cases} 0 & \text{if } \beta < 1 \\ \frac{\alpha_1 \alpha_2 [\alpha_2 + \alpha_1 \kappa^2]}{[\alpha_2(1 - \beta \varphi) + \alpha_1 \kappa^2]^2} \left(\frac{1}{r_D^2 - 1} + \frac{1}{1 - \varphi^2} \right) > V_D(0) > 0 & \text{if } \beta = 1 \end{cases} \quad (2.55)$$

The partial loss function

$$V_2^D(T) = \frac{\alpha_1 \alpha_2 [\alpha_2 + \alpha_1 \kappa^2]}{[\alpha_2(1 - \beta \varphi) + \alpha_1 \kappa^2]^2} \frac{\beta^T}{1 - \beta \varphi^2} \quad (2.56)$$

has the properties

$$V_2^D(0) = V_D(0) \quad (2.57)$$

$$\lim_{T \rightarrow \infty} V_2^D(T) = 0 \quad \text{if } \beta < 1 \quad (2.58)$$

$$\frac{dV_2^D}{dT} = (\ln \beta)V_2^D(T) < 0 \quad \text{if } \beta < 1 \quad \text{for all } 0 \leq T < \infty \quad (2.59)$$

For $\beta = 1$, the function $V_2^D(T)$ is constant (independent of T).

The partial loss function $V_1^D(T)$ given by

$$V_1^D(T) = \frac{\alpha_1 \alpha_2 [\alpha_2 + \alpha_1 \kappa^2]}{[\alpha_2(1 - \beta\varphi) + \alpha_1 \kappa^2]^2} \frac{r_D^{-2T} - \beta^T}{1 - \beta r_D^2} \quad (2.60)$$

has similar properties as the corresponding function $V_1(T)$ under the policy regime timeless perspective commitment:

$$V_1^D(0) = 0 \quad (2.61)$$

$$\lim_{T \rightarrow \infty} V_1^D(T) = \begin{cases} 0 & \text{if } \beta < 1 \\ \frac{\alpha_1 \alpha_2 [\alpha_2 + \alpha_1 \kappa^2]}{[\alpha_2(1 - \beta\varphi) + \alpha_1 \kappa^2]^2} \frac{1}{r_D^2 - 1} > 0 & \text{if } \beta = 1 \end{cases} \quad (2.62)$$

The first derivative with respect to T

$$\frac{dV_1^D(T)}{dT} = \frac{\alpha_1 \alpha_2 [\alpha_2 + \alpha_1 \kappa^2]}{[\alpha_2(1 - \beta\varphi) + \alpha_1 \kappa^2]^2} \frac{1}{1 - \beta r_D^2} [-2(\ln r_D)r_D^{-2T} - (\ln \beta)\beta^T] \quad (2.63)$$

is positive at time $T = 0$, since $1 - \beta r_D^2 < 0$ and $-2 \ln r_D - \ln \beta < 0$ due to $r_D > 1 \geq \beta$.

In the case $\beta < 1$, the development of $V_1^D(T)$ is hump-shaped with the maximum value at time T_d^* which is the solution of the equation

$$2(\ln r_D)r_D^{-2T} + (\ln \beta)\beta^T = 0 \quad (2.64)$$

Equation (2.64) is equivalent to

$$-\frac{2 \ln r_D}{\ln \beta} = (\beta r_D^2)^T \quad (2.65)$$

with the solution

$$T_d^* = \frac{\ln \left[-\frac{2 \ln r_D}{\ln \beta} \right]}{\ln(\beta r_D^2)} > 0 \quad (2.66)$$

The total loss function $V_D(T) = V_1^D(T) + V_2^D(T)$ has a similar development as the corresponding function $V(T)$ under timeless perspective commitment. In the limiting case $\beta = 1$, it is overall increasing. For $\beta < 1$, it is hump-shaped, if the degree of price flexibility is not too large, while it is monotonically decreasing in T if the value of ω is small. For small values of ω , the derivative of V_D at time $T = 0$ is negative, while it is positive if ω is sufficiently large. For the sake of brevity, the proof for the case of simple (optimal) Taylor rules is presented in Appendix A. \square

The propositions 1 to 3 follow from two opposing effects on the welfare loss which change in opposite directions with increasing lead time T . On the one hand, the size of the initial jumps of the forward-looking variables x_t and π_t taking place at the time of anticipation, is inversely related to the time span between anticipation and realization of the cost-push

shock. The longer the lead time T , the smaller is the response of output and inflation on impact so that the contribution of this *anticipation effect* to the welfare loss V *decreases* with increasing T . On the other hand, the *persistence effect* of the cost-push shock on the target variables x_t and π_t is *increasing* in T . Thereby, persistence is measured as the total variation of a variable over time, i.e. its intertemporal deviation from the respective initial steady state. For example, the persistence of the price level p_t is given by $\sum_{t=0}^{\infty} |p_t - \bar{p}_0|$ where the initial steady state can be normalized to zero. In Appendix A, we derive the persistence of p_t , x_t , and π_t under the optimal monetary policy regimes commitment and discretion and show that persistence is smaller in the case of unanticipated shocks than in the case of anticipated shocks.

For the sake of illustration, we numerically simulated our solutions by using a standard calibration. The time unit is one quarter. The discount rate is equal to $\beta = 0.99$ implying an annual steady state real interest rate of approximately 4 percent. The inverse of the intertemporal elasticity of substitution, σ , is set to $\sigma = 2$. We set $\eta = 1$ implying a quadratic disutility of labor. The Calvo parameter ω is either set to 0.25 implying an average duration of price contracts of four months or to 0.75 implying an average duration of price contracts of one year. The weights in the loss function are set to $\alpha_1 = 1$ and $\alpha_2 = 0.5$ reflecting the objective of flexible inflation targeting. Finally, we assume the cost-push shock to be persistent and choose φ equal to 0.5.

Figure 2.1 depicts impulse response functions of inflation, output gap, and price level in the case of low ($\omega = 0.25$, left column) and high ($\omega = 0.75$, right column) price rigidity under the optimal monetary policy with timeless perspective commitment. Solid lines with triangles denote responses to a cost-push shock that unexpectedly emerges in period $t = 0$, solid lines with circles denote responses to a cost-push shock whose realization in period $T = 2$ is anticipated in period $t = 0$.

We first consider the empirically plausible case of *high price rigidity*. In the case of an unanticipated cost-shock, both the price level and inflation rise whereas output falls in response to the realization of the increase in the costs of production.⁴ Subsequently, all variables converge in a hump-shaped fashion to their respective steady state values.

Anticipated cost shocks have two effects, namely the anticipation effect which reflects the change in x_t , π_t , and p_t in response to the anticipation of a future change in costs, and the realization effect which occurs when the anticipated change in costs actually takes place. Under the optimal monetary policy with commitment, output starts to decline and prices begin to increase in response to the anticipation of a future rise in the costs of production. Both variables respond in a hump-shaped fashion peaking at the date of realization. The increase in prices causes inflation to jump at the time of anticipation, peaking at the date of realization and then returning in a hump-shaped fashion to its initial steady state level.

In the case of *low price rigidity*, an unanticipated cost shock causes an immediate rise in prices and an immediate drop in output. Subsequently, both variables converge monotonically to their initial steady state levels. After the initial jump, inflation falls sharply and converges from below to its pre-shock level. The announcement of a future rise in costs has negligible anticipation effects when prices are highly flexible. The reason is that the price setting problem of firms becomes more of an atemporal (static) nature when the Calvo parameter ω decreases. In this case firms know that, with a high probability, they will be

⁴We could think about this cost-push shock as an exogenous rise in wage mark-ups (see Chapter 3 or, for example, Galí (2008)).

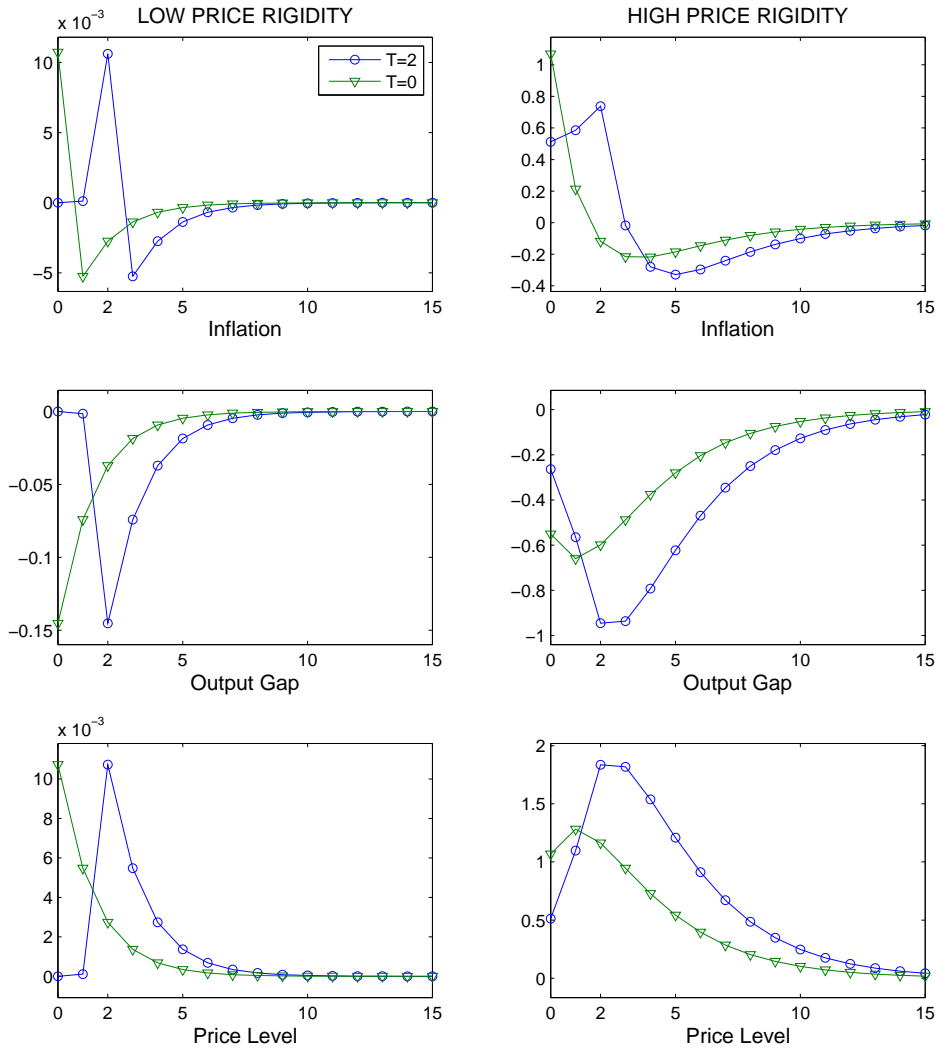


Figure 2.1: Impulse response functions under optimal policy with timeless perspective commitment.

Notes: Solid lines with triangles denote responses to an unanticipated cost-push shock, solid lines with circles denote responses to an anticipated cost-push shock. In the case of low price rigidity, the Calvo parameter ω is set to 0.25; in the case of high price rigidity, ω is set to 0.75.

able to raise their price when the anticipated shock actually materializes in period T . Thus, output and prices change only slightly in response to an announcement or anticipation of future cost-push shocks.

Regardless of the degree of price rigidity, Figure 2.1 illustrates that the initial jumps of inflation, output gap and price level are greater in the case of unanticipated ($T = 0$) than in the case of anticipated shocks ($T = 2$). On the other hand, anticipated shocks amplify the

persistence of p_t , x_t , and π_t compared to unanticipated shocks.⁵

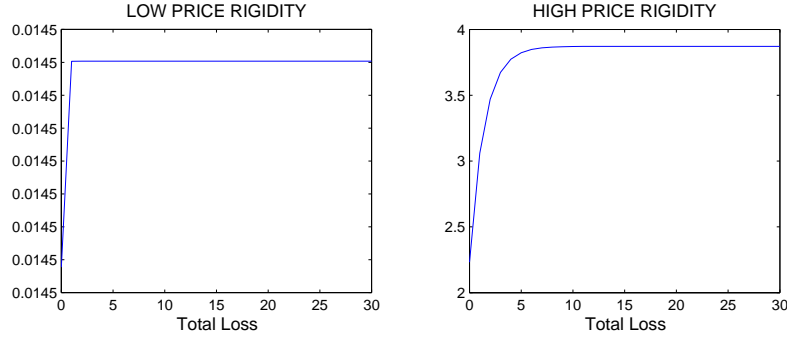


Figure 2.2: Welfare loss for different lengths of the anticipation period under optimal timeless perspective commitment policy in the case $\beta = 1$.

Figure 2.2 illustrates the welfare loss $V = V(T)$ in the case $\beta = 1$. Without time discounting in the intertemporal loss function, the persistence effect always dominates the anticipation effect so that proposition 1 holds. In Figure 2.2, the total loss $V = V(T)$ is overall increasing in T if $\beta = 1$.

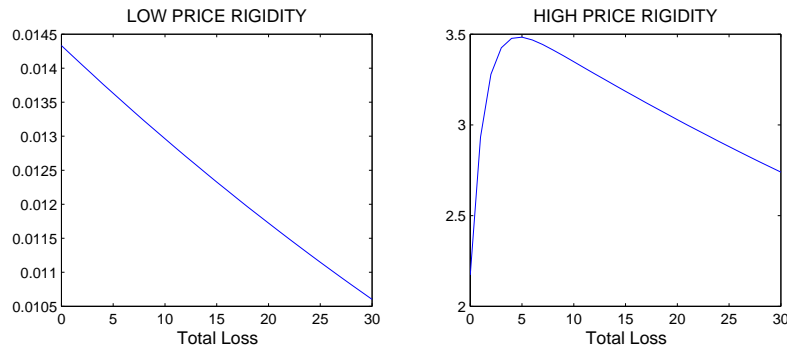


Figure 2.3: Welfare loss for different lengths of the anticipation period under optimal timeless perspective commitment policy in the case $\beta = 0.99$.

If future deviations of the state variables from their initial steady state levels are discounted, the contribution of the initial jumps of output and inflation for the determination of the total loss becomes more important. The same holds for increasing degree of price flexibility $1 - \omega$, since the persistence of prices, output and inflation is a decreasing function of $1 - \omega$. If the degree of price flexibility is high, the value of the total loss is almost completely determined by the size of the initial jumps of x_t and π_t which in turn is inversely proportional to the lead time T . With a sufficiently high degree of price flexibility, the total loss under unanticipated cost-push shocks is greater than the loss under anticipated shocks so that proposition 3 holds. This result is also illustrated in Figure 2.3, where $V(T)$ is a monotonically decreasing function in the lead time T if the degree of price rigidity ω is very small.

⁵This result also holds in the special case $\varphi = 0$, i.e. if the shock exhibits no serial correlation. It is well-known that even in this case the optimal precommitment policy introduces inertia in the impulse response functions.

From an empirical point of view, the parameter ω is not that small so that the development of the impulse response functions displays inertia or strong serial correlation. Then, if the time span between the anticipation and the implementation of the cost-push shock is not too long, the persistence effect dominates and the value of the total loss $V(T)$ is greater than $V(0)$. This is illustrated in Figure 2.3, where the development of the loss function $V(T)$ is hump-shaped and monotonically increasing for small values of T .

Propositions 1 to 3 are independent of the chosen optimal monetary policy regime. They hold under timeless perspective commitment as well as under discretion (see Figure 2.4 and 2.5 for a numerical visualization). They also hold under simple monetary policy rules (such as Taylor-type rules or money growth peg).

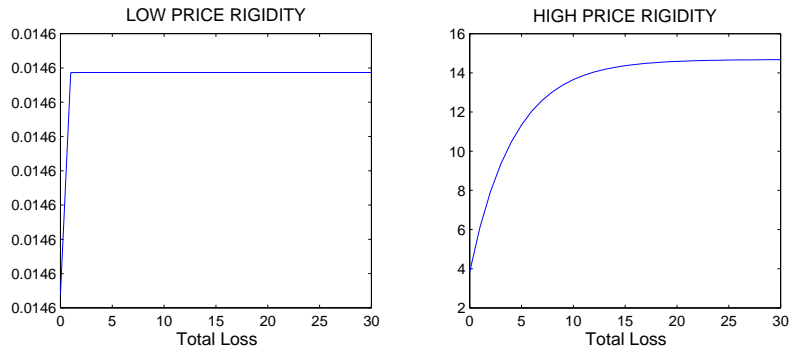


Figure 2.4: Welfare loss for different lengths of the anticipation period under optimal discretionary policy in the case $\beta = 1$.

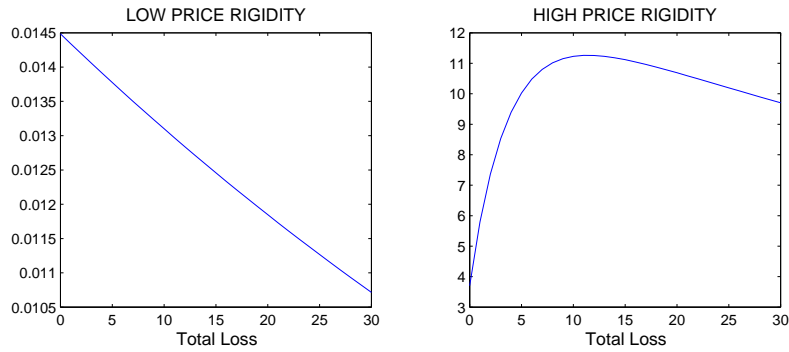


Figure 2.5: Welfare loss for different lengths of the anticipation period under optimal discretionary policy in the case $\beta = 0.99$.

In order to check whether the welfare-reducing effects of anticipations hold for empirically plausible degrees of nominal rigidity, we compute the critical anticipation values T_c^* (commitment) and T_d^* (discretion). Table 1 depicts the values of T_c^* and T_d^* for a persistent ($\varphi = 0.5$) and a one-off cost-push shock ($\varphi = 0$).

Table 1 shows that the anticipation of cost-push shocks dampens the welfare loss induced by such shocks only for empirically unrealistic degrees of nominal rigidity. For the widely applied values of $\omega = 0.75$ or $\omega = 0.66$, the anticipation period or lead time T must be extremely large to obtain a welfare gain from anticipation. Under commitment and a value

$\omega = 0.75$, the loss under an anticipated shock is smaller than the loss under an unanticipated shock of same size when the shock is anticipated to take place in $T_c^* = 54$ (for $\varphi = 0.5$) or $T_c^* = 66$ (for $\varphi = 0$) quarters. Even larger values are obtained under optimal discretionary policy. A Calvo parameter of 0.5 represents the lower bound in the range of values that are reported in the literature. In this case and under the monetary policy regime commitment, the anticipation of future cost shocks has a welfare-enhancing effect if the lead time is larger or equal to two quarters for persistent and three quarters for one-off shocks, respectively. Under discretionary monetary policy, these critical values are three and four quarters.

Our simulations illustrate that for a wide range of empirically realistic degrees of nominal rigidities (i.e., $\omega \geq 0.5$) in conjunction with a plausible length of the anticipation period, the welfare loss of anticipated cost shocks exceeds the welfare loss of unanticipated cost shocks.

Table 2.1: Values of the critical lead time T_c^* and T_d^*

Monetary policy	Degree of price rigidity ω							
	0.75	0.66	0.60	0.55	0.50	0.45	0.40	0.25
<i>With $\varphi = 0.5$</i>								
Commitment	53.09	19.82	9.00	4.23	1.82	0.69	0.16	0
Discretion	125.90	40.41	15.61	6.37	2.42	0	0	0
<i>With $\varphi = 0$</i>								
Commitment	65.78	25.57	11.79	5.59	2.41	0.95	0.28	0
Discretion	146.99	50.77	20.25	8.38	3.20	0	0	0

Note: For an anticipation period $0 < T < T_i^*$ it is true that $V|_T > V|_{T=0}$, for $T > T_i^*$ it is true that $V|_T < V|_{T=0}$ where $i = c, d$.

2.4 Conclusion

In this chapter we investigated the welfare effects resulting from the anticipation of future shocks. In particular, we analyzed the welfare loss for different lengths of the time span between the anticipation and the realization of cost-push shocks. This includes the widely applied case of unanticipated cost-push shocks. Our analysis was based on the canonical New Keynesian model with optimal monetary policy.

We emphasized the role of nominal rigidities for the welfare effects of anticipations. We have shown that for empirically plausible degrees of nominal rigidity, anticipated cost shocks entail higher welfare losses than unexpected cost shocks. The anticipation of a future cost-push shock dampens the volatility of output and inflation only if prices are highly flexible. These results hold independently of the monetary policy regime (timeless perspective commitment, discretion, (optimal) simple rules).

Our results imply that the knowledge about the realization of future cost shocks is in general welfare-reducing. The question remains why rational agents do not simply ignore this information. However, this would be inconsistent with the profit-maximizing behavior of individual firms and the utility-maximizing behavior of individual households on which our model is based. The firm's optimality condition in fact calls for an increase in prices in response to the anticipation of a future rise in costs. By simply ignoring this information, the firm would make a loss.

Hence, our results reveal a contradiction between the optimal behavior of individuals and the optimum from a social point of view.

3 Anticipated Shocks and Optimal Policy: A General Solution Method and a New Keynesian Example

3.1 Introduction

In Chapter 2 we investigated whether the anticipation of future cost-push shocks has a stabilizing effect on the economy and thus reduces the welfare loss compared to unanticipated shocks. In order to provide analytical results which do not rely on calibrations, we considered the baseline New Keynesian model with purely forward-looking IS and Phillips curves. This has enabled us to derive an analytical solution of welfare as a function of the time span between the anticipation and the realization of the shock. We found that – for empirically plausible degrees of nominal rigidity – the anticipation of a future cost-push shock leads to a higher welfare loss than an analogous unanticipated shock.

In order to conduct an analysis of the (welfare) effects of anticipated shocks in more elaborate models, this chapter¹ presents a general solution method for linear dynamic rational expectations models with anticipated shocks and optimal policy. Our method extends the work of Söderlind (1999) who uses the generalized Schur decomposition method, advocated by Klein (2000), to solve linear rational expectations models with optimal policy. However, Söderlind (1999) only considers stochastic models with white noise shocks which are, by definition, unpredictable. In the case of anticipated shocks, the occurrence of all future shocks is known exactly at the time when the solution of the model is computed. Our method also contains unanticipated shocks as a limiting case.

As an economic example, we lay out a calibrated New Keynesian model for a closed and cashless economy with internal habit formation in consumption preferences, a variant of Calvo price staggering with partial indexation to past inflation, and a time-varying wage mark-up which represents a typical cost-push shock. We compare the effects of mark-up shocks under optimal monetary policy for different lengths of the anticipation period. Our results confirm the finding reported in Chapter 2, that anticipated cost-push shocks entail higher welfare losses than unexpected cost shocks.

The chapter is organized as follows. Section 2 discusses optimal policies in RE models with *anticipated* temporary shocks. We first determine the optimal unrestricted policy under precommitment and calculate the minimum value of the intertemporal loss function. We then consider (optimal) simple rules and demonstrate how the Schur decomposition can be used to solve the model under these conditions. Section 3 derives the hybrid New Keynesian model, presents the welfare-theoretic loss function and discusses the effects of anticipated and unanticipated cost-push shocks. Finally, Section 4 provides concluding remarks. In Appendix B, we present a short discussion of the well known stochastic case with white noise shocks.

¹For a different version of this chapter see Wohltmann and Winkler (2009).

3.2 The Model

In this chapter we discuss the following linear expectational difference equations

$$A \begin{pmatrix} w_{t+1} \\ \mathbb{E}_t v_{t+1} \end{pmatrix} = B \begin{pmatrix} w_t \\ v_t \end{pmatrix} + C u_t + D \nu_{t+1} \quad (3.1)$$

where w_t is an $n_1 \times 1$ vector of predetermined variables, assuming w_0 given, v_t an $n_2 \times 1$ vector of non-predetermined variables, u_t an $m \times 1$ vector of policy instruments, and ν_{t+1} an $r \times 1$ vector of exogenous shocks. The matrices A and B are $n \times n$ (where $n = n_1 + n_2$), while the matrices C and D are $n \times m$ and $n \times r$ respectively. We allow matrix A to be singular which is the case if static (intratemporal) equations are included among the dynamic relationships. The vector w , composed of backward-looking variables can include exogenous variables following autoregressive processes. $\mathbb{E}_t v_{t+1}$ denotes model consistent (rational) expectations of v_{t+1} formed at time t . We assume that the shocks are anticipated by the public in advance and take the following form

$$\nu_t = \begin{cases} \bar{\nu} & \text{for } t = \tau > 0 \\ 0 & \text{for } t \neq \tau \end{cases} \quad (3.2)$$

where $\bar{\nu} = (\bar{\nu}_1, \dots, \bar{\nu}_r)'$ is a constant non-zero $r \times 1$ vector. It is assumed that at time $t = 0$ the public anticipates a shock of the form outlined in (3.2) to take place at some future date $\tau > 0$. Note that τ also defines the lengths of the anticipation period. Since the shocks are anticipated by the public we have $\mathbb{E}_t \nu_{t+1} = \nu_{t+1}$. For notational convenience, we define the $n \times 1$ vector $k_t = (w_t', v_t')'$ and the $n_3 \times 1$ target vector $s_t = \tilde{A}k_t + \tilde{B}u_t$, where the matrices \tilde{A} and \tilde{B} are $n_3 \times n$ and $n_3 \times m$ respectively. Assume that the policy maker's welfare loss at time t is given by

$$J_t = \frac{1}{2} \mathbb{E}_t \sum_{i=0}^{\infty} \lambda^i \{s'_{t+i} W_1 s_{t+i} + u'_{t+i} W_2 u_{t+i}\} \quad (3.3)$$

where W_1 and W_2 are symmetric and non-negative definite matrices and λ is a discount factor with $0 < \lambda \leq 1$. We can rewrite J_t as

$$J_t = \frac{1}{2} \mathbb{E}_t \sum_{i=0}^{\infty} \lambda^i \{k'_{t+i} \tilde{W} k_{t+i} + 2k'_{t+i} P u_{t+i} + u'_{t+i} R u_{t+i}\} \quad (3.4)$$

where $\tilde{W} = \tilde{A}' W_1 \tilde{A}$ and $R = W_2 + \tilde{B}' W_1 \tilde{B}$ are symmetric and non-negative definite and $P = \tilde{A}' W_1 \tilde{B}$.

3.2.1 Optimal Policy with Precommitment

In the following, the policy maker's optimal policy rule at time $t = 0$ is developed. It is assumed that the policy maker is able to commit to such a rule. From the Lagrangian

$$\mathcal{L}_0 = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \lambda^t \{k'_t \tilde{W} k_t + 2k'_t P u_t + u'_t R u_t + 2\rho'_{t+1} [B k_t + C u_t + D \nu_{t+1} - A k_{t+1}]\} \quad (3.5)$$

with the $n \times 1$ multiplier ρ_{t+1} , we get the first-order conditions with respect to ρ_{t+1} , k_t , and u_t :

$$\begin{pmatrix} A & 0_{n \times m} & 0_{n \times n} \\ 0_{n \times n} & 0_{n \times m} & \lambda B' \\ 0_{m \times n} & 0_{m \times m} & -C' \end{pmatrix} \begin{pmatrix} k_{t+1} \\ u_{t+1} \\ \rho_{t+1} \end{pmatrix} = \begin{pmatrix} B & C & 0_{n \times n} \\ -\lambda \tilde{W} & -\lambda P & A' \\ P' & R & 0_{m \times n} \end{pmatrix} \begin{pmatrix} k_t \\ u_t \\ \rho_t \end{pmatrix} + \begin{pmatrix} D \\ 0_{n \times r} \\ 0_{m \times r} \end{pmatrix} \nu_{t+1} \quad (3.6)$$

To solve the system of equations (3.6), expand the state and costate vector k_t and ρ_t as $(w'_t, v'_t)'$ and $(p'_{wt}, p'_{vt})'$ respectively and re-order the rows of the $(2n+m) \times 1$ vector $(k'_t, u'_t, \rho'_t)'$ by placing the predetermined vector p_{vt} after w_t . Since v_t is forward-looking with arbitrarily chosen initial value v_0 , the corresponding Lagrange multiplier p_{vt} is predetermined with initial value $p_{v0} = 0$. Re-order the columns of the $(2n+m) \times (2n+m)$ matrices in (3.6) according to the re-ordering of $(k'_t, u'_t, \rho'_t)'$ and write the result as

$$F \begin{pmatrix} \tilde{w}_{t+1} \\ \tilde{v}_{t+1} \end{pmatrix} = G \begin{pmatrix} \tilde{w}_t \\ \tilde{v}_t \end{pmatrix} + \begin{pmatrix} D \\ 0_{n \times r} \\ 0_{m \times r} \end{pmatrix} \nu_{t+1} \quad (3.7)$$

where $\tilde{w}_t = (w'_t, p'_{vt})'$ and $\tilde{v}_t = (v'_t, u'_t, p'_{wt})'$. The $n \times 1$ vector \tilde{w}_t contains the 'backward-looking' variables of (3.6) while the $(n+m) \times 1$ vector \tilde{v}_t contains the 'forward-looking' variables.

Equation (3.6) implies that the $(2n+m) \times (2n+m)$ matrix F is singular. To solve equation (3.7) we apply the generalized Schur decomposition method (Söderlind, 1999; Klein, 2000). The decomposition of the square matrices F and G is given by

$$F = \bar{Q}' S \bar{Z}', \quad G = \bar{Q}' T \bar{Z}' \quad (3.8)$$

or equivalently

$$Q F Z = S, \quad Q G Z = T \quad (3.9)$$

where Q, Z, S , and T are square matrices of complex numbers, S and T are upper triangular and Q and Z are unitary, i.e.

$$Q \cdot \bar{Q}' = \bar{Q}' \cdot Q = I_{(2n+m) \times (2n+m)} = Z \cdot \bar{Z}' = \bar{Z}' \cdot Z \quad (3.10)$$

where the non-singular matrix \bar{Q}' is the transpose of \bar{Q} , which denotes the complex conjugate of Q . \bar{Z}' is the transpose of the complex conjugate of Z . The matrices S and T can be arranged in such a way that the block with the stable generalized eigenvalues (the i th diagonal element of T divided by the i th diagonal element of S) comes first. Premultiply both sides of equation (3.7) with Q and define auxiliary variables \tilde{z}_t and \tilde{x}_t so that

$$\begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} = \bar{Z}' \begin{pmatrix} \tilde{w}_t \\ \tilde{v}_t \end{pmatrix} \quad (3.11)$$

Partitioning the triangular matrices S and T in order to conform with \tilde{z} and \tilde{x} and set

$$Q \begin{pmatrix} D \\ 0_{n \times r} \\ 0_{m \times r} \end{pmatrix} = \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \quad (3.12)$$

where Q_1 is $n \times r$ and Q_2 is $(n+m) \times r$. We then obtain the equivalent system

$$\begin{pmatrix} S_{11} & S_{12} \\ 0_{(n+m) \times n} & S_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_{t+1} \\ \tilde{x}_{t+1} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ 0_{(n+m) \times n} & T_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} + \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \nu_{t+1} \quad (3.13)$$

where the $n \times n$ matrix S_{11} and the $(n+m) \times (n+m)$ matrix T_{22} are invertible while S_{22} is singular. The square matrix T_{11} may also be singular. The lower block of equation (3.13) contains the unstable generalized eigenvalues and must be solved forward. Since

$$\tilde{x}_{t+s} = M_2 \tilde{x}_{t+s+1} - T_{22}^{-1} Q_2 \nu_{t+s+1} \quad (s = 0, 1, 2, \dots) \quad (3.14)$$

where $M_2 = T_{22}^{-1}S_{22}$, the unique stable solution for \tilde{x}_t is given by

$$\tilde{x}_t = -\sum_{s=0}^{\infty} M_2^s T_{22}^{-1} Q_2 E_t \nu_{t+s+1} = \begin{cases} -M_2^{\tau-1-t} T_{22}^{-1} Q_2 \bar{\nu} & \text{for } 0 \leq t < \tau \\ 0 & \text{for } t \geq \tau \end{cases} \quad (3.15)$$

The upper block of (3.13) contains the stable generalized eigenvalues and can be solved backward. Since

$$\tilde{z}_{t+1} = M_1 \tilde{z}_t + S_{11}^{-1}(T_{12} \tilde{x}_t - S_{12} \tilde{x}_{t+1}) + S_{11}^{-1} Q_1 \nu_{t+1} \quad (3.16)$$

where $M_1 = S_{11}^{-1} T_{11}$ (which in general is not invertible), the general solution is given by

$$\begin{aligned} \tilde{z}_t &= M_1^t K + \sum_{s=0}^{t-1} M_1^{t-s-1} S_{11}^{-1} (T_{12} \tilde{x}_s - S_{12} \tilde{x}_{s+1} + Q_1 \nu_{s+1}) \\ &= \begin{cases} M_1^t K + \sum_{s=0}^{t-1} M_1^{t-s-1} S_{11}^{-1} (T_{12} \tilde{x}_s - S_{12} \tilde{x}_{s+1}) & \text{for } 0 \leq t < \tau \\ M_1^t K + \sum_{s=0}^{\tau-1} M_1^{t-s-1} S_{11}^{-1} (T_{12} \tilde{x}_s - S_{12} \tilde{x}_{s+1}) + M_1^{t-\tau} S_{11}^{-1} Q_1 \bar{\nu} & \text{for } t \geq \tau \end{cases} \end{aligned} \quad (3.17)$$

where \tilde{x}_s is defined in (3.15).

The solution for $t \geq \tau$ can be rewritten as

$$\tilde{z}_t = M_1^{t-\tau} \tilde{K} \quad \text{for } t \geq \tau \quad (3.18)$$

where

$$\tilde{K} = M_1^\tau K + S_{11}^{-1} Q_1 \bar{\nu} + \sum_{s=0}^{\tau-1} M_1^{\tau-s-1} S_{11}^{-1} (T_{12} \tilde{x}_s - S_{12} \tilde{x}_{s+1}) \quad (3.19)$$

Since

$$\tilde{x}_s = \begin{cases} -M_2^{\tau-1-s} T_{22}^{-1} Q_2 \bar{\nu} & \text{for } 0 \leq s < \tau \\ 0 & \text{for } s \geq \tau \end{cases} \quad (3.20)$$

we can write \tilde{K} as

$$\tilde{K} = M_1^\tau K + S_{11}^{-1} Q_1 \bar{\nu} + [-\tilde{W}_1 + M_1 \tilde{W}_2] T_{22}^{-1} Q_2 \bar{\nu} \quad (3.21)$$

where

$$\tilde{W}_1 = \sum_{s=0}^{\tau-1} M_1^{\tau-s-1} S_{11}^{-1} T_{12} M_2^{\tau-s-1} = \sum_{k=0}^{\tau-1} M_1^k S_{11}^{-1} T_{12} M_2^k \quad (3.22)$$

and

$$\tilde{W}_2 = \sum_{s=0}^{\tau-2} M_1^{\tau-s-2} S_{11}^{-1} S_{12} M_2^{\tau-s-2} = \sum_{k=0}^{\tau-2} M_1^k S_{11}^{-1} S_{12} M_2^k \quad (3.23)$$

\tilde{W}_1 as well as \tilde{W}_2 is a finite geometric sum of matrices and can be written as

$$\tilde{W}_1 = S_{11}^{-1} T_{12} - M_1^\tau S_{11}^{-1} T_{12} M_2^\tau + M_1 \tilde{W}_1 M_2 \quad (3.24)$$

and

$$\tilde{W}_2 = S_{11}^{-1}S_{12} - M_1^{\tau-1}S_{11}^{-1}S_{12}M_2^{\tau-1} + M_1\tilde{W}_2M_2 \quad (3.25)$$

To solve for \tilde{W}_1 and \tilde{W}_2 respectively, we use the matrix identities (Rudebusch and Svensson 1999; Klein, 2000) $\text{vec}(A+B) = \text{vec}(A) + \text{vec}(B)$ and $\text{vec}(ABC) = [C' \otimes A] \text{vec}(B)$ where $\text{vec}(A)$ denotes the vector of stacked column vectors of the matrix A and \otimes denotes the Kronecker product of matrices.

We then obtain from (3.24) and (3.25)

$$\text{vec } \tilde{W}_1 - [M_2' \otimes M_1] \text{vec } \tilde{W}_1 = \text{vec} [S_{11}^{-1}T_{12} - M_1^{\tau}S_{11}^{-1}T_{12}M_2^{\tau}] \quad (3.26)$$

and

$$\text{vec } \tilde{W}_2 - [M_2' \otimes M_1] \text{vec } \tilde{W}_2 = \text{vec} [S_{11}^{-1}S_{12} - M_1^{\tau}S_{11}^{-1}S_{12}M_2^{\tau}] \quad (3.27)$$

with the solution

$$\text{vec } \tilde{W}_1 = [I - M_2' \otimes M_1]^{-1} \cdot \text{vec} [S_{11}^{-1}T_{12} - M_1^{\tau}S_{11}^{-1}T_{12}M_2^{\tau}] \quad (3.28)$$

$$\text{vec } \tilde{W}_2 = [I - M_2' \otimes M_1]^{-1} \cdot \text{vec} [S_{11}^{-1}S_{12} - M_1^{\tau}S_{11}^{-1}S_{12}M_2^{\tau-1}] \quad (3.29)$$

According to (3.17) and (3.20), the solution of \tilde{z}_t for the anticipation period $0 < t < \tau$ can be rewritten as

$$\tilde{z}_t = M_1^t K + [-W_{1t}^* + W_{2t}^*] T_{22}^{-1} Q_2 \bar{v} \quad \text{for } 0 \leq t < \tau \quad (3.30)$$

with

$$W_{1t}^* = \sum_{s=0}^{t-1} M_1^{t-s-1} S_{11}^{-1} T_{12} M_2^{\tau-s-1} = \sum_{k=0}^{t-1} M_1^k S_{11}^{-1} T_{12} M_2^{\tau-t+k} \quad (3.31)$$

and

$$W_{2t}^* = \sum_{s=1}^t M_1^{t-s} S_{11}^{-1} S_{12} M_2^{\tau-s-1} = \sum_{k=0}^{t-1} M_1^k S_{11}^{-1} S_{12} M_2^{\tau-1-t+k} \quad (3.32)$$

W_{1t}^* satisfies the matrix equation²

$$W_{1t}^* = S_{11}^{-1} T_{12} M_2^{\tau-t} - M_1^t S_{11}^{-1} T_{12} M_2^{\tau} + M_1 W_{1t}^* M_2 \quad (0 \leq t < \tau) \quad (3.33)$$

with the solution

$$\text{vec } W_{1t}^* = [I - M_2' \otimes M_1]^{-1} \cdot \text{vec} (S_{11}^{-1} T_{12} M_2^{\tau-t} - M_1^t S_{11}^{-1} T_{12} M_2^{\tau}) \quad (3.34)$$

The matrix W_{2t}^* satisfies the equation³

$$W_{2t}^* = S_{11}^{-1} S_{12} M_2^{\tau-1-t} - M_1^t S_{11}^{-1} S_{12} M_2^{\tau-1} + M_1 W_{2t}^* M_2 \quad (0 \leq t < \tau) \quad (3.35)$$

²Note that equation (3.33) is also well-defined for $t = \tau$. In this case it is equivalent to (3.24) implying $W_{1\tau}^* = \tilde{W}_1$.

³For $t = \tau - 1$ equation (3.35) is equivalent to (3.25) so that $W_{2\tau-1}^* = \tilde{W}_2$. Then, according to (3.21), $\tilde{K} = \tilde{z}_\tau = M_1^{\tau} K + S_{11}^{-1} Q_1 \bar{v} + [-W_{1\tau}^* + M_1 W_{2\tau-1}^*] T_{22}^{-1} Q_2 \bar{v}$.

The definition of W_{1t}^* implies that W_{1t}^* also satisfies the dynamic equation

$$W_{1t+1}^* = S_{11}^{-1} T_{12} M_2^{\tau-(t+1)} + M_1 W_{1t}^* \quad (0 \leq t \leq \tau - 1)$$

with the initial value $W_{10}^* = 0$. Analogical, W_{2t}^* satisfies the matrix difference equation

$$W_{2t+1}^* = S_{11}^{-1} S_{12} M_2^{\tau-t-2} + M_1 W_{2t}^* \quad (W_{20}^* = 0)$$

which only holds for $0 \leq t < \tau - 1$ since $M_2^{\tau-(\tau-1)-2} = M_2^{-1}$ generally does not exist.

with the solution

$$\text{vec } W_{2t}^* = [I - M_2' \otimes M_1]^{-1} \cdot \text{vec } (S_{11}^{-1} S_{12} M_2^{\tau-1-t} - M_1^t S_{11}^{-1} S_{12} M_2^{\tau-1}) \quad (3.36)$$

The constant K can be determined using the initial value of the predetermined vector \tilde{w} . By premultiplying equation (3.11) with Z and by partitioning the matrix Z to conform with the dimension of \tilde{z} and \tilde{x} , we obtain

$$\begin{pmatrix} \tilde{w}_t \\ \tilde{v}_t \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} \quad (3.37)$$

and therefore

$$\tilde{w}_0 = Z_{11} \tilde{z}_0 + Z_{12} \tilde{x}_0 \quad (3.38)$$

with $\tilde{w}_0 = (w_0', 0'_{n_2 \times 1})'$, $\tilde{z}_0 = K$, and

$$\tilde{x}_0 = -M_2^{\tau-1} T_{22}^{-1} Q_2 \bar{v} \quad (3.39)$$

where it is assumed that $\tau > 0$.⁴ Equation (3.38) implies

$$K = Z_{11}^{-1} \tilde{w}_0 - Z_{11}^{-1} Z_{12} \tilde{x}_0 \quad (3.40)$$

provided the inverse Z_{11}^{-1} exists. A necessary condition is that the dynamic system (3.7) has the saddle path property, i.e., that the number of backward-looking variables ($n_1 + n_2 = n$) coincides with the number of stable generalized eigenvalues (Söderlind, 1999; Klein, 2000).

In the case $\tau > 0$ we can assume $w_0 = 0$ so that according to (3.39) the constant K can be written as

$$K = Z_{11}^{-1} Z_{12} M_2^{\tau-1} T_{22}^{-1} Q_2 \bar{v} \quad (3.41)$$

The solution to the state vector $(\tilde{z}_t, \tilde{x}_t)'$ for $0 \leq t < \tau$ now reads as follows

$$\begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} = \Xi_t T_{22}^{-1} Q_2 \bar{v} \quad \text{for } 0 \leq t < \tau \quad (3.42)$$

where

$$\Xi_t = \begin{pmatrix} \phi_t^* \\ -M_2^{\tau-1-t} \end{pmatrix} \quad (0 \leq t < \tau) \quad (3.43)$$

and⁵

$$\phi_t^* = M_1^t Z_{11}^{-1} Z_{12} M_2^{\tau-1} - W_{1t}^* + W_{2t}^* \quad (3.44)$$

⁴In the special case $\tau = 0$ (unanticipated shocks) we have $\tilde{x}_0 = 0$ and $\tilde{z}_t = (S_{11}^{-1} T_{11})^t K + (S_{11}^{-1} T_{11})^t S_{11}^{-1} Q_1 \bar{v}$ implying $\tilde{z}_0 = K + S_{11}^{-1} Q_1 \bar{v}$ and $K = Z_{11}^{-1} \tilde{w}_0 - S_{11}^{-1} Q_1 \bar{v}$ with $w_0 \neq 0$. By contrast, the initial value w_0 can be normalized to zero if $\tau > 0$.

⁵ ϕ_t^* satisfies the dynamic equation

$$\phi_{t+1}^* = M_1 \phi_t^* + S_{11}^{-1} [-T_{12} M_2 + S_{12}] M_2^{\tau-t-2}$$

where the time index t must be restricted to $0 \leq t < \tau - 1$.

If Z_{11} is invertible, equation (3.37) implies

$$\tilde{v}_t = Z_{21}\tilde{z}_t + Z_{22}\tilde{x}_t = Z_{21}(Z_{11}^{-1}\tilde{w}_t - Z_{11}^{-1}Z_{12}\tilde{x}_t) + Z_{22}\tilde{x}_t = N\tilde{w}_t + \hat{Z}\tilde{x}_t \quad (3.45)$$

where $N = Z_{21}Z_{11}^{-1}$ and $\hat{Z} = Z_{22} - Z_{21}Z_{11}^{-1}Z_{12}$. Write equation (3.45) as

$$\begin{pmatrix} v_t \\ u_t \\ p_{wt} \end{pmatrix} = \begin{pmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \\ N_{31} & N_{32} \end{pmatrix} \begin{pmatrix} w_t \\ p_{vt} \end{pmatrix} + \begin{pmatrix} \hat{Z}_1 \\ \hat{Z}_2 \\ \hat{Z}_3 \end{pmatrix} \tilde{x}_t \quad (3.46)$$

and assume the $n_2 \times n_2$ matrix N_{12} is invertible. The optimal policy rule under commitment can then be written as

$$\begin{aligned} u_t &= N_{21}w_t + N_{22}p_{vt} + \hat{Z}_2\tilde{x}_t \\ &= N_{21}w_t + N_{22}N_{12}^{-1}(v_t - N_{11}w_t - \hat{Z}_1\tilde{x}_t) + \hat{Z}_2\tilde{x}_t \\ &= N_{22}N_{12}^{-1}v_t + (N_{21} - N_{22}N_{12}^{-1}N_{11})w_t + (\hat{Z}_2 - N_{22}N_{12}^{-1}\hat{Z}_1)\tilde{x}_t \end{aligned} \quad (3.47)$$

where \tilde{x}_t is given by (3.15). For $t < \tau$, u_t depends on the auxiliary variable \tilde{x}_t , while for $t \geq \tau$, u_t is only a linear function of the predetermined state variables w_t and p_{vt} , where p_{vt} can be substituted with the original state variables v_t and w_t .

Minimum Value of the Loss Function. To determine the minimum value of the loss function J_t at time $t = 0$, we express J_t as function of \tilde{w} and \tilde{v} . The loss function (3.4) can be written as

$$J_t = \frac{1}{2} \sum_{i=0}^{\infty} \lambda^i(k'_{t+i}, u'_{t+i}) H \begin{pmatrix} k_{t+i} \\ u_{t+i} \end{pmatrix} = \frac{1}{2} \sum_{i=0}^{\infty} \lambda^i(w'_{t+i}, v'_{t+i}, u'_{t+i}) H \begin{pmatrix} w_{t+i} \\ v_{t+i} \\ u_{t+i} \end{pmatrix} \quad (3.48)$$

where the $(n+m) \times (n+m)$ matrix H is given by

$$H = \begin{pmatrix} \tilde{W} & P \\ P' & R \end{pmatrix} \quad (3.49)$$

with $H = H'$. Define the $n_1 \times n$ matrix \tilde{D}_1 and the $(n_2+m) \times (n+m)$ matrix \tilde{D}_2 by $\tilde{D}_1 = (I_{n_1 \times n_1}, 0_{n_1 \times n_2})$ and $\tilde{D}_2 = (I_{(n_2+m) \times (n_2+m)}, 0_{(n_2+m) \times n_1})$, respectively. Then $w = \tilde{D}_1(w', p'_w)'$, $(v', u')' = \tilde{D}_2(v', u', p'_w)'$, $(w', v', u')' = \tilde{D}(\tilde{w}', \tilde{v}')'$ with

$$\begin{aligned} \tilde{D} &= \begin{pmatrix} \tilde{D}_1 & 0_{n_1 \times (n+m)} \\ 0_{(n_2+m) \times n} & \tilde{D}_2 \end{pmatrix} \\ &= \begin{pmatrix} I_{n_1 \times n_1} & 0_{n_1 \times n_2} & 0_{n_1 \times (n_2+m)} & 0_{n_1 \times n_1} \\ 0_{(n_2+m) \times n_1} & 0_{(n_2+m) \times n_2} & I_{(n_2+m) \times (n_2+m)} & 0_{(n_2+m) \times n_1} \end{pmatrix} \end{aligned} \quad (3.50)$$

which is a $(n+m) \times (2n+m)$ matrix. The loss function J_t can now be rewritten as

$$J_t = \frac{1}{2} \sum_{i=0}^{\infty} \lambda^i(\tilde{w}'_{t+i}, \tilde{v}'_{t+i}) \tilde{D}' H \tilde{D} \begin{pmatrix} \tilde{w}_{t+i} \\ \tilde{v}_{t+i} \end{pmatrix} = J_t^{(1)} + J_t^{(2)} \quad (3.51)$$

where

$$J_t^{(1)} = \frac{1}{2} \sum_{i=0}^{\tau-1} \lambda^i(\tilde{w}'_{t+i}, \tilde{v}'_{t+i}) \tilde{D}' H \tilde{D} \begin{pmatrix} \tilde{w}_{t+i} \\ \tilde{v}_{t+i} \end{pmatrix} \quad (3.52)$$

and

$$J_t^{(2)} = \frac{1}{2} \sum_{i=\tau}^{\infty} \lambda^i (\tilde{w}'_{t+i}, \tilde{v}'_{t+i}) \tilde{D}' H \tilde{D} \begin{pmatrix} \tilde{w}_{t+i} \\ \tilde{v}_{t+i} \end{pmatrix} \quad (3.53)$$

First, we calculate $J_t^{(2)}$. For $t \geq \tau$, we have $\tilde{v}_t = N \tilde{w}_t$ and $\tilde{w}_t = Z_{11} \tilde{z}_t$, where $N = Z_{21} Z_{11}^{-1}$. We then obtain $(\tilde{w}'_t, \tilde{v}'_t)' = \tilde{N} \tilde{w}'_t = \tilde{N} Z_{11} \tilde{z}'_t$, where $\tilde{N} = (I_{n \times n}, N)'$ is a $(2n+m) \times n$ matrix. $J_t^{(2)}$ can then be rewritten as

$$J_t^{(2)} = \frac{1}{2} \sum_{i=\tau}^{\infty} \lambda^i Z'_{11} \tilde{z}'_{t+i} \tilde{N}' \tilde{D}' H \tilde{D} \tilde{N} Z_{11} \tilde{z}_{t+i} = \frac{1}{2} \sum_{i=\tau}^{\infty} \lambda^i Z'_{11} \tilde{z}'_{t+i} H^* Z_{11} \tilde{z}_{t+i} \quad (3.54)$$

with $H^* = \tilde{N}' \tilde{D}' H \tilde{D} \tilde{N}$ is a symmetric $n \times n$ matrix. Inserting (3.18) in (3.54) we obtain

$$\begin{aligned} J_t^{(2)} &= \frac{1}{2} (M_1^t \tilde{K})' \lambda^\tau \left(\sum_{i=\tau}^{\infty} \lambda^{i-\tau} (Z_{11} M_1^{i-\tau})' H^* (Z_{11} M_1^{i-\tau}) \right) M_1^t \tilde{K} \\ &= \frac{1}{2} \lambda^\tau \varphi_t' V^* \varphi_t = \frac{1}{2} \lambda^\tau \text{trace}(V^* \varphi_t \varphi_t') \end{aligned} \quad (3.55)$$

where $\varphi_t = M_1^t \tilde{K}$ and V^* is the convergent geometric sum of matrices

$$V^* = \sum_{i=\tau}^{\infty} \lambda^{i-\tau} (Z_{11} M_1^{i-\tau})' H^* (Z_{11} M_1^{i-\tau}) \quad (3.56)$$

which is of dimension $n \times n$ and satisfies the matrix equation

$$V^* = Z'_{11} H^* Z_{11} + \lambda M_1' V^* M_1 \quad (3.57)$$

with the solution

$$\text{vec}(V^*) = [I - \lambda M_1' \otimes M_1]^{-1} \text{vec}(Z'_{11} H^* Z_{11}) \quad (3.58)$$

For $t = 0$ we obtain from (3.55)

$$J_0^{(2)} = \frac{1}{2} \lambda^\tau \text{trace}(V^* \varphi_0 \varphi_0') = \frac{1}{2} \lambda^\tau \text{trace}(V^* \tilde{K} \tilde{K}') \quad (3.59)$$

with \tilde{K} given by (3.21).

The next step is the calculation of the finite sum $J_t^{(1)}$ as defined in (3.52). Because $(\tilde{w}'_t, \tilde{v}'_t)' = Z(\tilde{z}'_t, \tilde{x}'_t)'$, we can write $J_0^{(1)}$ as

$$J_0^{(1)} = \frac{1}{2} \sum_{i=0}^{\tau-1} \lambda^i (\tilde{z}'_i, \tilde{x}'_i) Z' \tilde{D}' H \tilde{D} Z \begin{pmatrix} \tilde{z}_i \\ \tilde{x}_i \end{pmatrix} = \frac{1}{2} \sum_{t=0}^{\tau-1} \lambda^t (\tilde{z}'_t, \tilde{x}'_t) \tilde{H} \begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} \quad (3.60)$$

where $\tilde{H} = Z' \tilde{D}' H \tilde{D} Z$.

Inserting the solution formulas for \tilde{z}_t and \tilde{x}_t in (3.60), we obtain the expression

$$J_0^{(1)} = \frac{1}{2} (T_{22}^{-1} Q_2 \bar{\nu})' \left[\sum_{t=0}^{\tau-1} \lambda^t \Xi_t' \tilde{H} \Xi_t \right] (T_{22}^{-1} Q_2 \bar{\nu}) = \frac{1}{2} \mu' W^* \mu = \frac{1}{2} \text{trace}(W^* \mu \mu') \quad (3.61)$$

where $\mu = T_{22}^{-1}Q_2\bar{\nu}$ and $W^* = \sum_{t=0}^{\tau-1} \lambda^t \Xi_t' \tilde{H} \Xi_t$.

Ξ_t satisfies the matrix difference equation

$$\begin{aligned} \Xi_{t+1} &= \begin{pmatrix} \phi_{t+1}^* \\ -M_2^{\tau-1-(t+1)} \end{pmatrix} = \begin{pmatrix} M_1 \phi_t^* \\ 0 \end{pmatrix} + \begin{pmatrix} S_{11}^{-1}[-T_{12}M_2 + S_{12}] \\ -I \end{pmatrix} M_2^{\tau-1-(t+1)} \\ &= \begin{pmatrix} M_1 \phi_t^* \\ -M_2^{\tau-1-t} \end{pmatrix} + \begin{pmatrix} S_{11}^{-1}[-T_{12}M_2 + S_{12}] \\ M_2 - I \end{pmatrix} M_2^{\tau-1-(t+1)} \\ &= \tilde{M} \Xi_t + \Omega M_2^{\tau-t-2} \quad (0 \leq t < \tau - 1) \end{aligned} \quad (3.62)$$

with

$$\tilde{M} = \begin{pmatrix} M_1 & 0 \\ 0 & I \end{pmatrix}, \quad \Omega = \begin{pmatrix} S_{11}^{-1}[-T_{12}M_2 + S_{12}] \\ M_2 - I \end{pmatrix} \quad (3.63)$$

and the initial value

$$\Xi_0 = \begin{pmatrix} \phi_0^* \\ -M_2^{\tau-1} \end{pmatrix} = \begin{pmatrix} Z_{11}^{-1}Z_{12} \\ -I \end{pmatrix} M_2^{\tau-1} \quad (3.64)$$

Note that the dynamic equation (3.62) is not defined for $t = \tau - 1$, since $M_2 = T_{22}^{-1}S_{22}$ is generally not invertible. The solution time path for Ξ_t ($0 \leq t < \tau - 1$) can be obtained by either solving equation (3.62) backward or – if possible – by solving equation (3.62) forward.

Solving (3.62) backward in time yields

$$\Xi_t = \tilde{M}^t \Xi_0 + \sum_{s=0}^{t-1} \tilde{M}^{t-s-1} \Omega M_2^{\tau-s-2} \quad (3.65)$$

To obtain the forward solution assume that $M_1 = S_{11}^{-1}T_{11}$ is invertible. Then \tilde{M}^{-1} exists and equation (3.62) can be written as

$$\Xi_t = \tilde{M}^{-1} \Xi_{t+1} - \tilde{M}^{-1} \Omega M_2^{\tau-t-2} \quad (3.66)$$

Given

$$\Xi_{\tau-1} = \begin{pmatrix} \phi_{\tau-1}^* \\ -I \end{pmatrix} \quad (3.67)$$

we obtain recursively for $t = \tau - n$:

$$\begin{aligned} \Xi_{\tau-n} &= \left(\tilde{M}^{-1}\right)^{n-1} \Xi_{\tau-1} - \left(\tilde{M}^{-1}\right)^{n-1} \Omega - \left(\tilde{M}^{-1}\right)^{n-2} \Omega M_2 \\ &\quad - \left(\tilde{M}^{-1}\right)^{n-3} \Omega M_2^2 - \dots - \left(\tilde{M}^{-1}\right) \Omega M_2^{n-2} \\ &= \left(\tilde{M}^{-1}\right)^{n-1} \Xi_{\tau-1} - \sum_{k=1}^{n-1} \left(\tilde{M}^{-1}\right)^{n-k} \Omega M_2^{k-1} \end{aligned}$$

With $t = \tau - n$ we then get the forward solution

$$\Xi_t = \left(\tilde{M}^{-1}\right)^{\tau-t-1} \Xi_{\tau-1} - \sum_{k=1}^{\tau-t-1} \left(\tilde{M}^{-1}\right)^{\tau-t-k} \Omega M_2^{k-1} \quad (0 \leq t < \tau - 2) \quad (3.68)$$

The total loss under the optimal unrestricted policy under commitment is now given by

$$J_0 = J_0^{(1)} + J_0^{(2)} = \frac{1}{2} \text{trace}(W^* \mu \mu') + \frac{1}{2} \lambda^\tau \text{trace}(V^* \tilde{K} \tilde{K}') \quad (3.69)$$

Obviously, the value of J_0 depends on the size of the lead time τ . In New Keynesian models we often have a hump-shaped pattern for the function $J_0 = J_0(\tau)$ where J_0 is increasing in τ for small values of τ (see Section 3).

In the limiting case of unanticipated shocks ($\tau = 0$), the total loss is given by

$$J_0 = J_0^{(2)} = \frac{1}{2} \tilde{K}' V^* \tilde{K} \quad (3.70)$$

where

$$\tilde{K} = K \Big|_{\tau=0} + S_{11}^{-1} Q_1 \bar{v} = Z_{11}^{-1} \tilde{w}_0 - S_{11}^{-1} Q_1 \bar{v} + S_{11}^{-1} Q_1 \bar{v} = Z_{11}^{-1} \tilde{w}_0 \quad (3.71)$$

Then

$$J_0 = \frac{1}{2} \tilde{w}_0' Z_{11}^{-1'} V^* Z_{11}^{-1} \tilde{w}_0 = \frac{1}{2} \tilde{w}_0' V \tilde{w}_0 = \frac{1}{2} \text{trace}(V \tilde{w}_0 \tilde{w}_0') \quad (3.72)$$

where

$$\tilde{w}_0 \tilde{w}_0' = \begin{pmatrix} w_0 \\ p_{v0} \end{pmatrix} (w_0', p_{v0}') = \begin{pmatrix} w_0 w_0' & 0_{n_1 \times n_2} \\ 0_{n_2 \times n_2} & 0_{n_2 \times n_2} \end{pmatrix} \quad (3.73)$$

and $V = Z_{11}^{-1'} V^* Z_{11}^{-1}$ satisfies the matrix equation

$$\begin{aligned} V &= Z_{11}^{-1'} V^* Z_{11}^{-1} = H^* + \lambda Z_{11}^{-1'} M_1' V^* M_1 Z_{11}^{-1} \\ &= H^* + \lambda Z_{11}^{-1'} M_1' Z_{11}' Z_{11}^{-1} V^* Z_{11}^{-1} Z_{11} M_1 Z_{11}^{-1} = H^* + \lambda \Gamma' V \Gamma \end{aligned} \quad (3.74)$$

with $\Gamma = Z_{11} M_1 Z_{11}^{-1}$.

3.2.2 (Optimal) Simple Rules

The policy maker could alternatively commit to a suboptimal simple rule of the form

$$u_t = \Lambda k_t + \Psi E_t k_{t+1} \quad (3.75)$$

where the constant matrices Λ and Ψ are $m \times n$. Assuming rational expectations and exogenous shocks of the form (3.2) which are anticipated in $t = 0$, we obtain the dynamic system

$$\begin{pmatrix} A & 0_{n \times m} \\ \Psi & 0_{m \times m} \end{pmatrix} \begin{pmatrix} k_{t+1} \\ u_{t+1} \end{pmatrix} = \begin{pmatrix} B & C \\ -\Lambda & I_{m \times m} \end{pmatrix} \begin{pmatrix} k_t \\ u_t \end{pmatrix} + \begin{pmatrix} D \\ 0_{m \times r} \end{pmatrix} \nu_{t+1} \quad (3.76)$$

The generalized Schur decomposition yields the system of equations

$$F \begin{pmatrix} \tilde{w}_{t+1} \\ \tilde{v}_{t+1} \end{pmatrix} = G \begin{pmatrix} \tilde{w}_t \\ \tilde{v}_t \end{pmatrix} + \begin{pmatrix} D \\ 0_{m \times r} \end{pmatrix} \nu_{t+1} \quad (3.77)$$

where $\tilde{w} = w$ is an $n_1 \times 1$ vector, $\tilde{v} = (v', u)'$ is an $(n_2 + m) \times 1$ vector and where the square matrices F and G are $(n + m) \times (n + m)$ with the decomposition $QFZ = S$ and $QGZ = T$, where Q , Z , S , and T are $(n + m) \times (n + m)$ matrices. Since

$$\begin{pmatrix} \tilde{w} \\ \tilde{v} \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} \tilde{z} \\ \tilde{x} \end{pmatrix} \quad (3.78)$$

the matrices Z_{11} , Z_{12} , Z_{21} , and Z_{22} are now $n_1 \times n_1$, $n_1 \times (n_2 + m)$, $(n_2 + m) \times n_1$, and $(n_2 + m) \times (n_2 + m)$ respectively. The auxiliary variables \tilde{z} and \tilde{x} satisfy the system of equations

$$\begin{pmatrix} S_{11} & S_{12} \\ 0_{(n_2+m) \times n_1} & S_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_{t+1} \\ \tilde{x}_{t+1} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ 0_{(n_2+m) \times n_1} & T_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} + \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \nu_{t+1} \quad (3.79)$$

where S_{11} and T_{11} are $n_1 \times n_1$ matrices, S_{22} and T_{22} are $(n_2 + m) \times (n_2 + m)$ and S_{12} and T_{12} are $n_1 \times (n_2 + m)$. The matrices Q_1 and Q_2 are $n_1 \times r$ and $(n_2 + m) \times r$ respectively with

$$\begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} = Q \begin{pmatrix} D \\ 0_{m \times r} \end{pmatrix} \quad (3.80)$$

The solution of (3.79) is given by (3.15) and (3.17). For $t \geq \tau$, we obtain $\tilde{v}_t = N\tilde{w}_t = Nw_t$, where $N = Z_{21}Z_{11}^{-1}$ is now an $(n_2 + m) \times n_1$ matrix.

The loss function (3.51) simplifies to

$$J_t = \frac{1}{2} \sum_{i=0}^{\infty} \lambda^i (w'_{t+i}, \tilde{v}'_{t+i}) H \begin{pmatrix} w_{t+i} \\ \tilde{v}_{t+i} \end{pmatrix} \quad (3.81)$$

since $\tilde{D}_1 = I_{n_1 \times n_1}$, $\tilde{D}_2 = I_{(n_2+m) \times (n_2+m)}$ and therefore $\tilde{D} = I_{(n_2+m) \times (n_2+m)}$ (cf. (3.50)). J_t can be partitioned using (3.51). $J_t^{(2)}$ can be written as (3.54) with $H^* = \tilde{N}'H\tilde{N}$ and $\tilde{N} = (I_{n_1 \times n_1}, N)'$. The value of the loss function J_0 for given matrices Λ and Ψ is given by $J_0 = J_0^{(1)} + J_0^{(2)}$, where $J_0^{(1)}$ and $J_0^{(2)}$ are defined in (3.59) and (3.61) respectively.

The minimization of J_0 with respect to the coefficients of the matrices Λ and Ψ yields an optimal simple rule of the form (3.75).

3.3 Example: A Hybrid New Keynesian Model

The model is a standard New Keynesian model for a closed and cashless economy with the additional features of internal habit formation in consumption preferences and a variant of the Calvo (1983) mechanism with partial indexation of non-optimized prices to past inflation.⁶ The economy consists of final goods producers, labor bundlers, households, and intermediate goods producers.

Final goods producers use a continuum of intermediate goods $Y_t(i)$ to produce the homogeneous final good Y_t in a perfectly competitive market. A final goods producer maximizes his profits $P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$, subject to the following CES production function

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{1}{1+\lambda_p}} di \right)^{1+\lambda_p} \quad (3.82)$$

where P_t is the price of the final good, $P_t(i)$ is the price of the intermediate good i , and $(1 + \lambda_p)$ is the mark-up in the intermediate goods market.

The first-order condition for profit maximization yields the demand function for intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\frac{(1+\lambda_p)}{\lambda_p}} Y_t \quad (3.83)$$

⁶Similar models are applied by Smets and Wouters (2003), Giannoni and Woodford (2004), or Casares (2006).

and the equation for marginal costs

$$P_t = \left(\int_0^1 P_t(i)^{-\frac{1}{\lambda_p}} di \right)^{-\lambda_p} \quad (3.84)$$

Analogously to final goods producers, labor bundlers buy differentiated labor types $N_t(j)$, aggregate them to N_t and sell it to the intermediate goods producers under perfectly competitive conditions. A bundler maximizes his profits $W_t N_t - \int_0^1 W_t(j) N_t(j) dj$, subject to the following CES aggregation function

$$N_t = \left(\int_0^1 N_t(j)^{\frac{1}{1+\lambda_{w,t}}} dj \right)^{1+\lambda_{w,t}} \quad (3.85)$$

W_t is the price of the labor bundle N_t , $W_t(j)$ denotes the price of labor type j and $(1 + \lambda_{w,t})$ is the time-varying wage mark-up.

The first-order condition for profit maximization yields the demand function for labor type j

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\frac{(1+\lambda_{w,t})}{\lambda_{w,t}}} N_t \quad (3.86)$$

and the wage index equation

$$W_t = \left(\int_0^1 W_t(j)^{-\frac{1}{\lambda_{w,t}}} dj \right)^{-\lambda_{w,t}} \quad (3.87)$$

The economy is made up by a continuum of households, indexed by $j \in [0, 1]$. Each household j is a monopolistic supplier of labor type $N_t(j)$. The household determines the amount of the final good $C_t(j)$ for consumption, its one-period nominal bond holdings $B_t(j)$, and chooses the wage for its labor type $W_t(j)$ in order to maximize its lifetime utility

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left(\frac{1}{1-\sigma} (C_t(j) - hC_{t-1}(j))^{1-\sigma} - \frac{1}{1+\eta} N_t(j)^{1+\eta} \right) \quad (3.88)$$

where β is the discount factor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, and $\eta > 0$ is the inverse of the labor supply elasticity. $C_{t-1}(j)$ is the consumption of the j th household in period $t-1$ and $N_t(j)$ are the total hours worked. We assume $h \geq 0$ to allow for internal habit formation in consumption. Maximization of (3.88) is subjected to the labor demand function (3.86) and the households' period-by-period budget constraint is given by

$$C_t(j) + \frac{B_t(j)}{P_t} = \frac{W_t(j)}{P_t} N_t(j) + \frac{R_{t-1} B_{t-1}(j)}{P_t} + D_t^r(j) \quad (3.89)$$

where R_t is the one-period gross nominal interest rate on households j th nominal bond holdings $B_t(j)$, and $D_t^r(j)$ are dividends, expressed in real terms.

The first-order conditions for this maximization problem are given by

$$\beta R_t \mathbb{E}_t \pi_{t+1}^{-1} = \mathbb{E}_t \left[\frac{(C_t - hC_{t-1})^{-\sigma} - h\beta(C_{t+1} - hC_t)^{-\sigma}}{(C_{t+1} - hC_t)^{-\sigma} - h\beta(C_{t+2} - hC_{t+1})^{-\sigma}} \right] \quad (3.90)$$

$$\frac{W_t}{P_t} = (1 + \lambda_{w,t}) \mathbb{E}_t \left[\frac{N_t^\eta}{(C_t - hC_{t-1})^{-\sigma} - h\beta(C_{t+1} - hC_t)^{-\sigma}} \right] \quad (3.91)$$

where $\pi_t = P_t/P_{t-1}$ is the gross rate of price inflation. We make use of the fact that all households are faced with the same optimization problem and hence, choose the same amount of consumption $C_t(j) = C_t$, the same nominal wage $W_t(j) = W_t$, and supply the same amount of labor $N_t(j) = N_t$.

Each intermediate goods producer is a monopolistic supplier of the intermediate good $i \in [0, 1]$. Firm i uses the amount $N_t(i)$ of homogenous labor and the constant returns to scale technology $Y_t(i) = N_t(i)$, to produce its intermediate good $Y_t(i)$. Real marginal costs are the same for all firms and is given by $MC_t(i) = W_t/P_t$.

The price-setting decision for profit-maximization is constrained by a standard Calvo mechanism. In each period, the intermediate goods producer faces the constant probability $1 - \theta$ of being allowed to re-optimize his price $P_t(i)$. We follow Smets and Wouters (2003) by assuming that a firm which cannot re-optimize his price, resets the price according to $P_t(i) = P_{t-1}(i)\pi_{t-1}^\gamma$, where γ is the degree of price indexation. The firm chooses $P_t(i)$ in order to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Delta_{t,t+k} \left(\frac{P_t(i)\Pi_{t,t+k-1}}{P_{t+k}} Y_{t+k}(i) - MC_{t+k} Y_{t+k}(i) \right) \quad (3.92)$$

subject to the sequence of demand functions

$$Y_{t+k}(i) = \left(\frac{P_t(i)\Pi_{t,t+k-1}}{P_{t+k}} \right)^{-\frac{(1+\lambda_p)}{\lambda_p}} Y_{t+k} \quad \text{for } k = 0, 1, 2, \dots \quad (3.93)$$

where $\Delta_{t,t+k}$ denotes the stochastic discount factor for real payoffs and

$$\Pi_{t,t+k-1} = \pi_t^\gamma \pi_{t+1}^\gamma \dots \pi_{t+k-1}^\gamma = (P_{t+k-1}/P_{t-1})^\gamma \quad (3.94)$$

The first-order condition for the price-setting problem yields

$$P_t^*(i) = (1 + \lambda_p) \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Delta_{t,t+k} MC_{t+k} (P_{t+k}/\Pi_{t,t+k-1})^{(1+\lambda_p)/\lambda_p} Y_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Delta_{t,t+k} (P_{t+k}/\Pi_{t,t+k-1})^{-1/\lambda_p} Y_{t+k}} \quad (3.95)$$

Dividing equation (3.95) by P_t yields

$$\frac{P_t^*(i)}{P_t} = \mu_p \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Delta_{t,t+k} MC_{t+k} \left(\frac{P_{t+k}}{P_t} \right)^{\frac{1+\lambda_p}{\lambda_p}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{-\frac{\gamma(1+\lambda_p)}{\lambda_p}} Y_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \theta^k \Delta_{t,t+k} \left(\frac{P_{t+k}}{P_t} \right)^{\frac{1}{\lambda_p}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{-\frac{\gamma}{\lambda_p}} Y_{t+k}} \quad (3.96)$$

where $\mu_p = 1 + \lambda_p$.

Since all firms which are allowed to re-optimize their price will choose the same price $P_t^*(i) = P_t^*$, the price index (3.84) can be rewritten as

$$1 = \theta \left(\frac{\pi_{t-1}^\gamma}{\pi_t} \right)^{-\lambda_p} + (1 - \theta) \left(\frac{P_t^*}{P_t} \right)^{-\lambda_p} \quad (3.97)$$

Log-linearizing equation (3.97) yields

$$\hat{P}_t^* - \hat{P}_t = \frac{\theta}{1-\theta}(\hat{\pi}_t - \gamma\hat{\pi}_{t-1}) \quad (3.98)$$

Note that we use the convention that a hat above a variable denotes the percentage deviation from its steady state value.

By combining the latter equation with the log-linearized price-setting condition (3.96), we finally obtain

$$\hat{\pi}_t = \frac{\gamma}{1+\beta\gamma}\hat{\pi}_{t-1} + \frac{\beta}{1+\beta\gamma}\text{E}_t\hat{\pi}_{t+1} + \Theta\hat{M}C_t \quad (3.99)$$

where $\Theta = \frac{(1-\beta\theta)(1-\theta)}{\theta(1+\beta\gamma)}$. By log-linearizing the optimality condition (3.91), using the log-linearized overall resource constraint $\hat{Y}_t = \hat{C}_t$ and using the fact that $\widehat{W}_t/P_t = \widehat{M}C_t$ and $\hat{Y}_t = \hat{N}_t$, we obtain

$$\widehat{M}C_t = \hat{\lambda}_{w,t} + (\eta + \delta_1)\hat{Y}_t - \delta_2\hat{Y}_{t-1} - \beta\delta_2\text{E}_t\hat{Y}_{t+1} \quad (3.100)$$

where $\delta_1 = \frac{\sigma(1+\beta h^2)}{(1-h)(1-\beta h)}$, $\delta_2 = \frac{h\sigma}{(1-h)(1-\beta h)}$. The log-linearized mark-up $\hat{\lambda}_{w,t}$ is described by the AR(1) process

$$\hat{\lambda}_{w,t} = \rho\hat{\lambda}_{w,t-1} + e_t \quad (3.101)$$

By inserting the latter equation into equation (3.99), we obtain a hybrid Phillips curve that follows

$$\hat{\pi}_t = \omega_1\text{E}_t\hat{\pi}_{t+1} + \omega_2\hat{\pi}_{t-1} + \omega_3\hat{Y}_t - \omega_4\hat{Y}_{t-1} - \beta\omega_4\text{E}_t\hat{Y}_{t+1} + \Theta\hat{\lambda}_{w,t} \quad (3.102)$$

where $\omega_1 = \frac{\beta}{1+\beta\gamma}$, $\omega_2 = \frac{\gamma}{1+\beta\gamma}$, $\omega_3 = \Theta(\eta + \delta_1)$, and $\omega_4 = \Theta\delta_2$.

Note that in our model the level of output in the absence of nominal rigidities (the natural level) Y_t^n is constant. Thus, the linearized output \hat{Y}_t coincides with the linearized output gap $\hat{Y}_t^g = \hat{Y}_t - \hat{Y}_t^n$, where $\hat{Y}_t^n = 0$. Further note that for $\gamma = h = 0$, equation (3.102) collapses into the purely forward-looking New Keynesian Phillips

$$\hat{\pi}_t = \beta\text{E}_t\hat{\pi}_{t+1} + \kappa\hat{Y}_t + k_t \quad (3.103)$$

where $\kappa = \frac{(1-\beta\theta)(1-\theta)}{\theta}(\sigma + \eta)$ and $k_t = \frac{(1-\beta\theta)(1-\theta)}{\theta}\hat{\lambda}_{w,t}$ is a cost-push shock.

By log-linearizing the optimality condition (3.90) and using $\hat{Y}_t = \hat{C}_t$, we obtain

$$\hat{Y}_t = \kappa_1\hat{Y}_{t-1} + \kappa_2\text{E}_t\hat{Y}_{t+1} - \kappa_3\text{E}_t\hat{Y}_{t+2} - \kappa_4(\hat{R}_t - \text{E}_t\hat{\pi}_{t+1}) \quad (3.104)$$

where $\kappa_1 = \frac{h}{1+h+\beta h^2}$, $\kappa_2 = \frac{1+\beta h+\beta h^2}{1+h+\beta h^2}$, $\kappa_3 = \frac{\beta h}{1+h+\beta h^2}$, and $\kappa_4 = \frac{(1-h)(1-\beta h)}{\sigma(1+h+\beta h^2)}$. Note that for $h = 0$, we obtain the purely forward-looking New Keynesian IS curve

$$\hat{Y}_t = \text{E}_t\hat{Y}_{t+1} - \frac{1}{\sigma}(\hat{R}_t - \text{E}_t\hat{\pi}_{t+1}) \quad (3.105)$$

Following Woodford (2003, Ch. 6) and Giannoni and Woodford (2004), a second-order approximation to the households' utility yields a loss function of the form

$$J_0 = \text{E}_0 \sum_{t=0}^{\infty} \beta^t \left((\hat{\pi}_t - \gamma\hat{\pi}_{t-1})^2 + \alpha_Y(\hat{Y}_t - \delta\hat{Y}_{t-1})^2 \right) \quad (3.106)$$

where $\alpha_y = \frac{\Theta h \sigma \lambda_p}{(1+\lambda_p)\delta(1-\beta h)(1-h)}$ and δ is the smaller root of the quadratic equation

$$\frac{h\sigma}{(1-\beta h)(1-h)}(1+\beta\delta^2) = \left(\eta + \frac{\sigma}{(1-\beta h)(1-h)}(1+\beta h^2) \right) \delta \quad (3.107)$$

We follow Giannoni and Woodford (2004) and Casares (2006) by assuming that the monetary authority is concerned about the volatility of the nominal interest rate. Therefore, we augment the welfare-theoretic loss function by the additional term $\alpha_R \hat{R}_t^2$, where α_R measures the weight on interest rate stabilization.

The monetary authority then seeks to minimize the loss function

$$J_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left((\hat{\pi}_t - \gamma \hat{\pi}_{t-1})^2 + \alpha_Y (\hat{Y}_t - \delta \hat{Y}_{t-1})^2 + \alpha_R \hat{R}_t^2 \right) \quad (3.108)$$

subject to the model equations (3.101), (3.102), and (3.104). Note that in our model, the discount factor for the policy-maker, λ , is equal to the household's discount factor β .

In order to solve the model by using the methods outlined in Section 2, we define the policy objective parameters $\hat{Y}_t^o = \hat{Y}_t - \delta \hat{Y}_{t-1}$ and $\hat{\pi}_t^o = \hat{\pi}_t - \gamma \hat{\pi}_{t-1}$. Furthermore, we define the auxiliary variables $\tilde{\pi}_t = \hat{\pi}_{t-1}$, $\tilde{Y}_t = \hat{Y}_{t-1}$, and $s_t = E_t \hat{\pi}_{t+1}$. If we add the definition of the real interest rate $\hat{r}_t = \hat{R}_t - E_t \hat{\pi}_{t+1}$, we finally obtain a 3×1 vector w_t of predetermined variables given by $w_t = (\hat{\lambda}_{w,t}, \tilde{\pi}_t, \tilde{Y}_t)'$, a 6×1 vector v_t of non-predetermined variables given by $v_t = (\hat{\pi}_t, \hat{Y}_t, s_t, \hat{r}_t, \hat{\pi}_t^o, \hat{y}_t^o)'$, the vector of policy instruments u_t which is simply the scalar $u_t = \hat{R}_t$, and the 1×1 shock vector $\nu_t = e_t$. The 9×9 matrices A and B are given by

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta}{1+\beta\gamma} & -\beta\omega_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \kappa_4 & \kappa_2 & -\kappa_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B = \begin{pmatrix} \rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ -\Theta & -\frac{\gamma}{1+\beta\gamma} & \omega_4 & 1 & -\omega_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\kappa_1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \gamma & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & \delta & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

while the 9×1 matrices C and D are

$$C = (0 \ 0 \ 0 \ 0 \ \kappa_4 \ 0 \ 1 \ 0 \ 0 \ 0)'$$

$$D = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)'$$

Finally, the matrices \tilde{W} , P , and R are given by $P = \mathbf{0}_{9 \times 9}$, $R = \alpha_R$, and

$$\tilde{W} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_Y \end{pmatrix}$$

We complete the description of the model by presenting the calibration. The time unit is one quarter. The discount rate is equal to $\beta = 0.99$, implying a quarterly steady state real interest rate of approximately one percent. The intertemporal elasticity of substitution, σ , is assumed to $\sigma = 2$. We follow Casares (2006) and set the habit formation parameter to $h = 0.85$ implying that the weight on lagged output in the IS equation is $1/3$. The calibrated $\eta = 3$ implies a labor supply elasticity with respect to the real wage of $1/3$. λ_p is set to $8/7$ which implies a steady state mark-up in the goods market of approximately 14 percent. We assume the linearized wage mark-up $\hat{\lambda}_{w,t}$ to be persistent and choose ρ equal to 0.8. The Calvo parameter θ is set to 0.75 implying an average duration of price contracts of one year. The price indexation parameter γ is set to 0.45 which is roughly equal to the value reported by Smets and Wouters (2003). This implies that the weight on lagged inflation in the Phillips curve equation is 0.31.

The parameter values chosen for our model imply a weight on output in the policy-makers' objective function of approximately $\alpha_Y = 0.69$. Following Casares (2006), we set $\alpha_R = 0.0088$ implying a small preference for interest rate smoothing.

For the analysis concerning anticipated and unanticipated shocks, we assume that the economy is in a deterministic steady state until period $t = 0$. In the case of an unanticipated shock, the mark-up $\hat{\lambda}_{w,t}$ jumps by one percent in period $t = 0$ and begins to fall thereafter. In the case of an anticipated shock, the agents anticipate in period $t = 0$ that a one percent increase in the mark-up will take place at some future date $\tau > 0$. They also know that the mark-up will subsequently decline according to the autoregressive process (3.101), where now $e_t = 1$ for $t = \tau$ and $e_t = 0$ for $t \neq \tau$. Note that τ also defines the lengths of the anticipation period or the time interval between $t = 0$ and $t = \tau$. In order to obtain impulse response functions and welfare results, we simulate dynamic adjustment paths and the welfare loss function by using the methods outlined in Section 2.⁷

Figure 3.1 depicts the impulse response functions of inflation, output, nominal, and real interest rates under the unrestricted optimal monetary policy. The solid lines with circles represent the responses to an unforeseen cost-push shock that emerged in period $t = 0$. The solid lines with squares, triangles, and stars represent responses to a cost-push shock whose realization in period $\tau = 1$, $\tau = 2$, or $\tau = 3$ is anticipated in period $t = 0$.

An *unanticipated* rise in the wage mark-up puts upward pressure on the prices of intermediate goods and hence on inflation. Despite the instantaneous jump in inflation, the real interest rate rises due to the sharp increase in the nominal interest rate. The increase in the real

⁷Matlab codes can be downloaded from the author's webpage at http://www.wiso.uni-kiel.de/vwl/institute/Wohltmann/REAS_solution.zip.

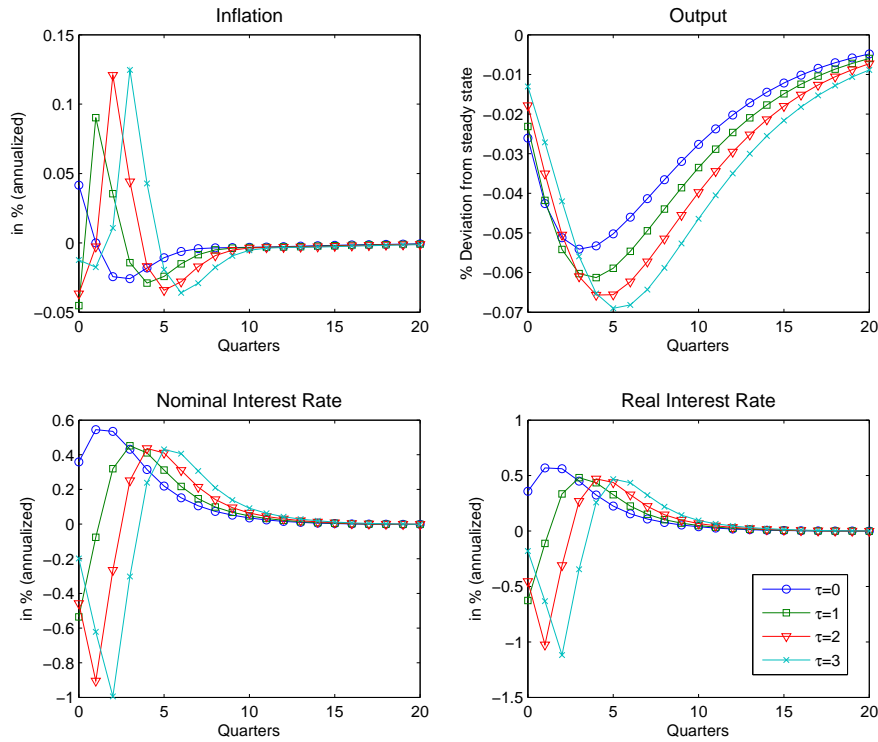


Figure 3.1: Impulse response functions under unrestricted optimal monetary policy.

Notes: Solid lines with circles denote responses to an unanticipated cost-push shock, solid lines with squares, triangles, and stars denote responses to an anticipated cost-push shock taking place in period $\tau = 1$, $\tau = 2$, and $\tau = 3$.

interest rate induces households to postpone consumption which implies an abrupt drop in output. Subsequently, the nominal interest rate continues to rise. This leads – in conjunction with the decline in inflation – to hump-shaped response functions of the real interest rate and output.

In the case of *anticipated* shocks, the optimal policy calls for a decline in nominal and real interest rates in response to the anticipation of a future rise in marginal costs. At the latest with the occurrence of the anticipated shock in period τ , the nominal and real interest rates start to rise and display a hump-shaped development. Inflation declines in response to the anticipation of the future rise in marginal costs. After this initial decline, inflation starts to rise and peaks in the period when the anticipated shock materializes. Output displays a hump-shaped downturn, starting at the point of anticipation, $t = 0$. The drop in output is thereby amplified by the lengths of the anticipation period, τ .

Notably, the anticipation of future shocks leads to an increase in the persistence (or volatility) of inflation, output as well as nominal and real interest rates which increases in lead time τ . Thereby, persistence is measured as the total variation of a variable over time, i.e. by its intertemporal deviation from its initial steady state. The impact or anticipation effect, however, is inversely related to the time span between anticipation and realization of the cost-push shock. It measures the initial jump of a variable taking place at the time of

anticipation.

The opposing effects of anticipations are shown in Figure 3.2 which displays the welfare loss as a function of the time span between the anticipation and the occurrence of the cost-push shock. The welfare function exhibits a hump-shaped pattern implying that for a realistic time

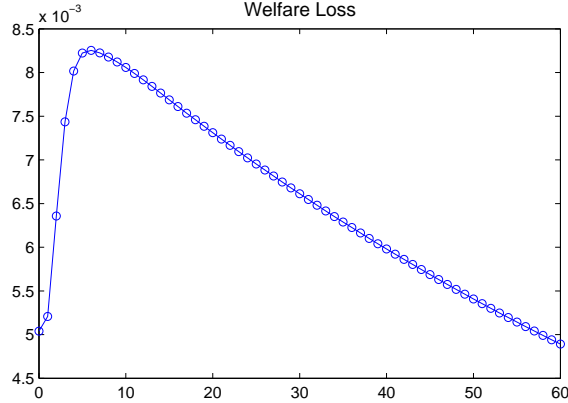


Figure 3.2: Welfare loss for different lengths of the anticipation period under unrestricted optimal monetary policy

span between the anticipation and the realization of cost-push shocks, anticipated shocks entail higher welfare losses than unanticipated shocks of equal size. The rationale is that the anticipation effect is dominated by the persistence effect. A welfare gain from anticipating can only be achieved for very large values of τ . Besides the anticipation effect, this can also be explained by discounting the realization impacts from period τ to period $t = 0$.

The results we obtained from our simulations show that the welfare loss of anticipated cost-shocks exceeds the welfare loss of an unanticipated cost-shock of equal magnitude for plausible lengths of the anticipation period. Hence, our results strongly support the findings reported in Chapter 2, where a similar result within the purely forward-looking canonical New Keynesian model was obtained.

3.4 Conclusion

In this chapter, we presented a method to solve linear dynamic rational expectations models with anticipated shocks and optimal policy by using the generalized Schur decomposition method. Our approach also allows for the evaluation of the widely discussed case of unpredictable shocks and can therefore be seen as a generalization of the methods summarized by Söderlind (1999). We demonstrated our method by means of a calibrated New Keynesian model with internal habit formation in consumption preferences, a variant of Calvo price staggering with partial indexation to past inflation, a time-varying wage mark-up, and a utility-based loss function. We simulated the model economy's responses to unanticipated and anticipated cost-push shocks under the unrestricted optimal monetary policy. It was then shown that anticipated shocks amplify both, the stagflationary effects of cost-push shocks and the overall welfare loss. Hence, our results strongly support the findings reported in Chapter 2, where we have demonstrated the welfare-reducing effects of anticipations within the purely forward-looking canonical New Keynesian model.

Part II: Nonlinear Dynamics – Ramsey
Policy, Simple Rules, and Welfare

4 On the Superiority of Simple Rules to Ramsey Monetary Policy under Timeless Perspective Commitment

4.1 Introduction

This chapter studies optimal monetary policy in a standard New Keynesian model for a closed and cashless economy with price and wage rigidities resulting from quadratic adjustment costs. In particular, we derive the Ramsey-type optimal monetary policy in the presence of cost-push shocks such as wage and price mark-up shocks for the case of an inefficient and efficient steady state and analyze welfare implications of Ramsey policy and simple monetary policy rules. We show that in the case of an efficient steady state, the Ramsey policy is the globally optimal policy under commitment, whereas in the case of an inefficient steady state, there exist monetary policy rules which are superior to the Ramsey policy under timeless perspective commitment.

The design of optimal monetary policy in the presence of cost-push shocks is one of the most discussed topics in monetary macroeconomics. However, a bulk of papers apply the linear-quadratic approach (see, among others, Clarida, Galí, and Gertler (1999), Svensson (1999), or the textbooks of Woodford (2003) and Galí (2008)).

Rotemberg and Woodford (1999) and Woodford (2003) show that, under certain circumstances, a second-order approximation of the households' utility function leads to a quadratic loss function similar to those widely applied in the linear-quadratic literature on optimal monetary policy. However, the assumptions which must be made to derive this utility-based welfare function are restrictive.

Thus, Khan, King, and Wolman (2003) propose to use the Ramsey approach for the analysis of optimal monetary policy since it does not rely on any approximation of the utility function. Instead, the optimal policy is derived by maximizing the utility of households subject to the non-linearized model equations. In this chapter, we follow Khan, King, and Wolman (2003), Yun (2005), Faia (2008a, 2008b), Faia and Monacelli (2004, 2007, 2008), or Schmitt-Grohé and Uribe (2004b, 2005, 2007b) and apply the Ramsey approach for the derivation of optimal policy.

Since the seminal works of Kydland and Prescott (1977) and Barro and Gordon (1983), it is well-known that the solution of optimal policy problems in models with forward-looking rational agents is time-inconsistent. In dynamic optimal control problems with forward-looking rational agents the assimilable problem of dynamic inconsistent or strategic incoherent solutions arises (see, for example, Wohltmann and Krömer (1989), Currie and Levine (1993), or McCallum (2005)). In order to overcome the problem of dynamic inconsistency, Woodford (1999a) proposes the concept of timeless perspective policy-making. Dennis (2001), McCallum and Nelson (2004), Giannoni and Woodford (2003), and McCallum (2005) extensively

discuss the concept of a timeless perspective within the linear-quadratic framework. Faia and Monacelli (2004) provide a detailed discussion of the timeless perspective approach within the framework of Ramsey-type optimal policy.

The widely recognized shortcomings of policy-making from a timeless perspective are that the problem of time-inconsistency in the sense of Kydland and Prescott (1977) and Barro and Gordon (1983) is in fact not solved and that the policy generally fails to globally maximize the policy makers' objective function (see, among others, Jensen and McCallum (2002)). Nevertheless, the concept of timeless perspective commitment achieved a lot of attention and is widely considered as a good approximation of the globally optimal policy under commitment.¹

However, Blake (2001) and Jensen and McCallum (2002) show within the linear-quadratic framework that alternative simple instrument rules deliver superior welfare results. These studies apply the canonical New Keynesian model with a quadratic welfare function which does not rely explicitly on the utility of households.

In this chapter, we demonstrate the non-optimality of timeless perspective commitment by using the Ramsey monetary policy approach with a utility-based welfare function. In particular, we show that even simple instrument rules such as the interest rate rule originally proposed by Taylor (1993) or an interest rate peg may be superior to the Ramsey monetary policy under timeless perspective commitment. Hence, our findings strongly support the results of Blake (2001) and Jensen and McCallum (2002) and call into question the appropriateness of the proposed Ramsey policy under timeless perspective commitment as a guideline for the conduct of monetary policy.

The remainder of this chapter is organized as follows. Section 2 presents the model. Section 3 describes the conduct of monetary policy and derives the optimal Ramsey-type monetary policy. Section 4 presents the calibration and discusses the solution method. In Section 5, the results of our simulations are reported. Section 6 provides concluding remarks.

4.2 The Model

The model is a standard New Keynesian model for a closed and cashless economy which consists of households, intermediate goods producers, final goods producers, and labor bundlers.

We model price rigidity by assuming that each intermediate goods producer faces a quadratic cost of adjusting nominal prices as in Hairault and Portier (1993). Accordingly, we follow Kim (2000) and introduce nominal wage rigidity by assuming that each household faces a quadratic cost of adjusting nominal wages.²

In the following, we present the model in detail.

4.2.1 Final Goods Producers

Final goods producers use the continuum of intermediate goods $Y_t(i)$ to produce the homogeneous final good Y_t in a perfectly competitive market. A final goods producer maximizes his

¹Most studies that analyze optimal policy under commitment apply in fact the timeless perspective. However, they simply denote this as commitment.

²Alternatively, we could introduce nominal rigidities by using the well-known Calvo (1983) mechanism.

profits

$$P_t Y_t - \int_0^1 P_t(i) Y_t(i) di \quad (4.1)$$

subject to the following CES production function

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon_{p,t}-1}{\varepsilon_{p,t}}} di \right)^{\frac{\varepsilon_{p,t}}{\varepsilon_{p,t}-1}} \quad (4.2)$$

where P_t is the price of the final good, $P_t(i)$ is the price of intermediate good i , and $\varepsilon_{p,t} > 1$ denotes the (possibly time-varying) elasticity of substitution between different intermediate goods. $\varepsilon_{p,t}$ follows an autoregressive process given by

$$(\varepsilon_{p,t}/\varepsilon_p) = (\varepsilon_{p,t-1}/\varepsilon_p)^{\rho_p} \exp(e_{p,t}) \quad (4.3)$$

where ε_p denotes the steady state value.³

The first-order condition for profit maximization is given by

$$P_t \left(\int_0^1 Y_t(i)^{\frac{\varepsilon_{p,t}-1}{\varepsilon_{p,t}}} di \right)^{\frac{1}{\varepsilon_{p,t}-1}} Y_t(i)^{-\frac{1}{\varepsilon_{p,t}}} = P_t(i) \quad (4.4)$$

By rearranging (4.4), we obtain the demand function for intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_{p,t}} Y_t \quad (4.5)$$

To obtain the price of final goods, we substitute the demand function (4.5) into the CES production function (4.2) which yields

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon_{p,t}} di \right)^{\frac{1}{1-\varepsilon_{p,t}}} \quad (4.6)$$

4.2.2 Labor Bundlers

Analogously to final goods producers, labor bundlers buy the differentiated labor types $N_t(j)$, aggregate them to N_t and sell it to the intermediate goods producers under perfectly competitive conditions. A bundler maximizes his profits

$$W_t N_t - \int_0^1 W_t(j) N_t(j) dj \quad (4.7)$$

subject to the following CES aggregation function

$$N_t = \left(\int_0^1 N_t(j)^{\frac{\varepsilon_{w,t}-1}{\varepsilon_{w,t}}} dj \right)^{\frac{\varepsilon_{w,t}}{\varepsilon_{w,t}-1}} \quad (4.8)$$

³Note that equation (4.3) is the nonlinear counterpart of the usual expression $\hat{\varepsilon}_{p,t} = \rho_p \hat{\varepsilon}_{p,t-1} + e_{p,t}$, where $\hat{\varepsilon}_{p,t} = \log(\varepsilon_{p,t}) - \log(\varepsilon_p) \approx (\varepsilon_{p,t} - \varepsilon_p)/\varepsilon_p$ denotes log-deviation from the steady state.

where W_t is the price of the labor bundle N_t , $W_t(j)$ is the price of labor type j , and $\varepsilon_{w,t} > 1$ denotes the (possibly time-varying) elasticity of substitution between different labor types. $\varepsilon_{w,t}$ follows an autoregressive process given by

$$(\varepsilon_{w,t}/\varepsilon_w) = (\varepsilon_{w,t-1}/\varepsilon_w)^{\rho_w} \exp(e_{w,t}) \quad (4.9)$$

where ε_w denotes the steady state value.

The first-order condition for profit maximization is given by

$$W_t \left(\int_0^1 N_t(j)^{\frac{\varepsilon_{w,t}-1}{\varepsilon_{w,t}}} dj \right)^{\frac{1}{\varepsilon_{w,t}-1}} N_t(j)^{-\frac{1}{\varepsilon_{w,t}}} = W_t(j) \quad (4.10)$$

By rearranging (4.10) and using (4.8), we obtain the demand function for labor type j

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_{w,t}} N_t \quad (4.11)$$

The demand function (4.11) can be substituted into the CES aggregation function (4.8) to obtain the wage index

$$W_t = \left(\int_0^1 W_t(j)^{1-\varepsilon_{w,t}} dj \right)^{\frac{1}{1-\varepsilon_{w,t}}} \quad (4.12)$$

4.2.3 Households

The economy is made up by a continuum of differentiated households, indexed by $j \in [0, 1]$. Each household seeks to maximize its discounted sum of per-period utility $U(C_t(j), N_t(j))$

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t(j), N_t(j)) \quad (4.13)$$

where $\beta \in (0, 1)$ is the discount factor and E_0 is the expectation operator conditional on information available in period $t = 0$. $C_t(j)$ is household j 'th consumption of final goods and $N_t(j)$ are its hours worked. Maximization of (4.13) is subjected to the demand function (4.11) for household j 'th differentiated labor type and the households' period-by-period budget constraint

$$C_t(j) + \frac{B_t(j)}{P_t} = \frac{W_t(j)}{P_t} N_t(j) + \frac{R_{t-1} B_{t-1}(j)}{P_t} - AC_t^w(j) - T_t^r(j) + \Pi_t^r(j) \quad (4.14)$$

where R_t denotes the gross nominal interest rate on one-period nominal bond holdings $B_t(j)$. $T_t^r(j)$ are time-varying lump-sum taxes and $\Pi_t^r(j)$ are dividends, both expressed in real terms. Following Kim (2000), we assume that the household is faced with a quadratic cost of adjusting its nominal wage given by

$$AC_t^w(j) = \frac{\theta_w}{2} \left(\frac{W_t(j)}{W_{t-1}(j)} - \pi^w \right)^2 \quad (4.15)$$

where $\theta_w \geq 0$ measures the degree of nominal wage rigidity and π^w is the steady state gross rate of wage inflation.

By inserting the constraint (4.14) into the intertemporal utility function (4.13), we obtain the maximization problem

$$\text{Max}_{B_t(j)} \text{E}_0 \sum_{t=0}^{\infty} \beta^t U \left(-\frac{B_t(j)}{P_t} + \frac{R_{t-1}B_{t-1}(j)}{P_t} + t.i.o., N_t(j) \right) \quad (4.16)$$

where $t.i.o. = \frac{W_t(j)}{P_t}N_t(j) - T_t^r(j) + \Pi_t^r(j) - AC_t^w(j)$ comprises all terms independent of this optimization problem. Differentiating (4.16) with respect to $B_t(j)$ gives the familiar consumption Euler equation

$$U_{C,t}(j) = \beta U_{C,t+1}(j) R_t \frac{P_t}{P_{t+1}} \quad (4.17)$$

where $U_{C,t}(j)$ denotes the marginal utility of consumption.

The household is a monopolistic supplier of labor type $N_t(j)$ and chooses its nominal wage $W_t(j)$ in order to maximize utility. By inserting the constraints (4.11) and (4.14) into the intertemporal utility function (4.13), we obtain the wage setting problem

$$\begin{aligned} \text{Max}_{W_t(j)} \text{E}_0 \sum_{t=0}^{\infty} \beta^t U & \left[\frac{W_t(j)}{P_t} \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_{w,t}} N_t - \frac{\theta_w}{2} \left(\frac{W_t(j)}{W_{t-1}(j)} - \pi^w \right)^2 \right. \\ & \left. + t.i.w., \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_{w,t}} N_t \right] \end{aligned} \quad (4.18)$$

where $t.i.w. = -(B_t(j) - R_{t-1}B_{t-1}(j))/P_t - T_t^r(j) + \Pi_t^r(j)$ comprises all terms independent of the wage-setting problem. Differentiating (4.18) with respect to $W_t(j)$ yields the first-order condition

$$\begin{aligned} 0 = U_{C,t}(j) (1 - \varepsilon_{w,t}) \frac{N_t(j)}{P_t} - U_{C,t}(j) \theta_w \left(\frac{W_t(j)}{W_{t-1}(j)} - \pi^w \right) \frac{1}{W_{t-1}(j)} \\ - \varepsilon_{w,t} U_{N,t}(j) \frac{N_t(j)}{W_t(j)} + \beta \text{E}_t \left\{ U_{C,t+1}(j) \theta_w \left(\frac{W_{t+1}(j)}{W_t(j)} - \pi^w \right) \frac{W_{t+1}(j)}{W_t(j)^2} \right\} \end{aligned} \quad (4.19)$$

Note that in the case of perfectly flexible wages, i.e., $\theta_w = 0$, the optimality condition (4.19) collapses into the static wage setting rule

$$\frac{W_t(j)}{P_t} = \mu_{w,t} MRS_t(j) \quad (4.20)$$

where $\mu_{w,t} = \frac{\varepsilon_{w,t}}{\varepsilon_{w,t} - 1}$, and $MRS_t(j) = -\frac{U_{N,t}(j)}{U_{C,t}(j)}$ denotes the marginal rate of substitution between consumption and labor. The monopolistic structure of the labor market drives a wedge (of the amount of the mark-up $\mu_{w,t} > 1$) between the marginal rate of substitution and the real wage.

4.2.4 Intermediate Goods Producers

Each intermediate goods producer is a monopolistic supplier of the intermediate good $i \in [0, 1]$. Firm i uses the amount $N_t(i)$ of homogenous labor and the decreasing returns to scale technology

$$A_t N_t(i)^{1-\alpha} \quad (4.21)$$

to produce the intermediate good $Y_t(i)$. A_t is a technology shifter which may be described by the autoregressive process

$$A_t/A = (A_{t-1}/A)^{\rho_a} \exp(e_{a,t}) \quad (4.22)$$

The firm takes the real price of labor W_t/P_t as given. Note that we allow for the existence of wage subsidies τW_t , financed by lump-sum taxes. The real marginal costs $MC_t(i)$ can be obtained from the minimization of total real costs $TC_t(i) = \frac{W_t}{P_t}(1 - \tau)N_t(i)$ subject to the production function (4.21) which yields

$$MC_t(i) = \frac{1 - \tau}{1 - \alpha} \frac{W_t}{P_t} Y_t(i)^{\frac{\alpha}{1-\alpha}} A_t^{-\frac{1}{1-\alpha}} \quad (4.23)$$

Following Hairault and Portier (1993), the firm faces a quadratic cost of price adjustment given by

$$AC_t^p(i) = \frac{\theta_p}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi^p \right)^2 \quad (4.24)$$

where $\theta_p \geq 0$ measures the degree of nominal price rigidity, and π^p denotes the steady state gross rate of price inflation.⁴

The firm chooses $P_t(i)$ in order to maximize his real profits

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \Delta_{0,t} \left(\frac{P_t(i)}{P_t} Y_t(i) - MC_t(i) Y_t(i) - AC_t^p(i) \right) \quad (4.25)$$

subject to the demand function (4.5), where $\Delta_{0,t} = \beta^t U_{C,t}/U_{C,0}$ denotes the stochastic discount factor for real payoffs. Plugging equations (4.5) and (4.24) into the intertemporal profit function (4.25) yields the following maximization problem

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \Delta_{0,t} \left[\left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_{p,t}} Y_t - MC_t(i) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_{p,t}} Y_t - \frac{\theta_p}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi^p \right)^2 \right] \quad (4.26)$$

By differentiating (4.26) with respect to $P_t(i)$, we obtain the first-order condition

$$0 = (1 - \varepsilon_{p,t}) \frac{Y_t(i)}{P_t} + \varepsilon_{p,t} MC_t(i) \frac{Y_t(i)}{P_t(i)} - \theta_p \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi^p \right) \frac{1}{P_{t-1}(i)} \quad (4.27)$$

$$+ \mathbb{E}_t \left\{ \frac{\Delta_{0,t+1}}{\Delta_{0,t}} \theta_p \left(\frac{P_{t+1}(i)}{P_t(i)} - \pi^p \right) \frac{P_{t+1}(i)}{P_{t-1}(i)^2} \right\}$$

Note that in the case of perfectly flexible prices, i.e., $\theta_p = 0$, the optimality condition (4.27) will read as

$$P_t(i) = \mu_{p,t} P_t MC_t(i) \quad (4.28)$$

where $\mu_{p,t} = \frac{\varepsilon_{p,t}}{\varepsilon_{p,t} - 1}$. The monopolistic supplier of good i will set its price $P_t(i)$ as a mark-up $\mu_{p,t}$ over nominal marginal costs given by $P_t MC_t(i)$.

⁴Note that this approach is observably equivalent to the pricing assumption introduced by Rotemberg (1982) when the steady state gross rate of price inflation is equal to one.

4.2.5 The Government

The government collects lump-sum taxes from households to finance wage subsidies. The government budget constraint is given by

$$P_t T_t^r = \tau W_t N_t \quad (4.29)$$

where T_t^r denotes real time-varying lump-sum taxes and $N_t = \int_0^1 N_t(i) di$ is the aggregated labor demand.

4.2.6 Aggregation, Market Clearing, and Functional Forms

The market clearing condition for labor type j and the market clearing condition for intermediate good i are given by

$$N_t(j) \equiv N_t^s(j) = N_t^d(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_{w,t}} N_t \quad (4.30)$$

$$Y_t(i) \equiv Y_t^s(i) = Y_t^d(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_{p,t}} Y_t \quad (4.31)$$

We assume a symmetric equilibrium in which all intermediate goods producers and households are identical and are faced with the same price and wage setting problems, respectively. Thus, each intermediate goods producer and each household will choose the same price and wage, i.e., $W_t(j) = W_t$ and $P_t(i) = P_t$. Then, the market clearing conditions (4.30) and (4.31) imply $N_t(j) = N_t$ and $Y_t(i) = Y_t$. According to equations (4.15) and (4.24), each household is faced with identical real costs of wage adjustment, i.e. $AC_t^w(j) = AC_t^w$, and each intermediate goods producer is faced with identical costs of price adjustment, i.e. $AC_t^p(i) = AC_t^p$. Furthermore, each household chooses the same amount of consumption, holds the same amount of bonds, pays the same amount of taxes, and receives the same amount of dividends, i.e., $C_t(j) = C_t$, $B_t(j) = B_t$, $T_t^r(j) = T_t^r$, and $\Pi_t^r(j) = \Pi_t^r$.

In what follows, we assume the following functional form for the per-period utility function:

$$U(C_t, N_t) = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\eta} N_t^{1+\eta} \quad (4.32)$$

where $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, and $1/\eta$ with $\eta > 0$ measures the labor supply elasticity. The marginal utility of consumption $U_{C,t}$ and the marginal disutility of labor $U_{N,t}$ are then given by $U_{C,t} = C_t^{-\sigma}$ and $U_{N,t} = -N_t^\eta$, respectively.

Now, the consumption Euler equation (4.17) and the optimal wage setting condition (4.19) can be rewritten as

$$C_t^{-\sigma} = C_{t+1}^{-\sigma} R_t (\pi_{t+1}^p)^{-1} \quad (4.33)$$

and

$$(\pi_t^w - \pi^w) \pi_t^w = \beta \mathbf{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1}^w - \pi^w) \pi_{t+1}^w \right\} + \frac{N_t (\varepsilon_{w,t} - 1)}{\theta_w} \left(\mu_{w,t} C_t^\sigma N_t^\eta - \frac{W_t}{P_t} \right) \quad (4.34)$$

where $\pi_t^p = P_t/P_{t-1}$ and $\pi_t^w = W_t/W_{t-1}$ denote the gross rate of price and wage inflation, respectively. Equation (4.34) represents a nonlinear wage Phillips curve.⁵

Note that wage and price inflation are connected by the identity

$$\frac{W_t}{P_t} = \frac{W_{t-1}}{P_{t-1}} \frac{\pi_t^w}{\pi_t^p} \quad (4.35)$$

Analogously to the wage setting equation, we can rewrite the optimal price setting condition (4.27) to obtain

$$(\pi_t^p - \pi^p)\pi_t^p = \beta \mathbf{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1}^p - \pi^p)\pi_{t+1}^p \right\} + \frac{Y_t \varepsilon_{p,t}}{\theta_p} \left(MC_t - \frac{1}{\mu_{p,t}} \right) \quad (4.36)$$

where marginal costs are given by

$$MC_t = \frac{1 - \tau}{1 - \alpha} \frac{W_t}{P_t} Y_t^{1-\alpha} A_t^{-\frac{1}{\alpha}} \quad (4.37)$$

and

$$Y_t = A_t N_t^{1-\alpha} \quad (4.38)$$

Note that equation (4.36) represents a nonlinear New Keynesian Phillips curve.⁶

Inserting the bond market clearing condition, $B_t = 0$, and the definition of the real cost of wage adjustment, $AC_t^w = \frac{\theta_w}{2} (\pi_t^w - \pi^w)^2$, into the aggregate budget constraint of households yields

$$C_t = \frac{W_t}{P_t} N_t - T_t^r + \Pi_t^r - \frac{\theta_w}{2} (\pi_t^w - \pi^w)^2 \quad (4.39)$$

Combining (4.39) with the government budget constraint (4.29) and the aggregated profits of intermediate goods producers

$$\Pi_t^r = Y_t - \frac{W_t}{P_t} (1 - \tau) N_t - AC_t^p \quad (4.40)$$

yields the overall resource constraint

$$Y_t = C_t + \frac{\theta_p}{2} (\pi_t^p - \pi^p)^2 + \frac{\theta_w}{2} (\pi_t^w - \pi^w)^2 \quad (4.41)$$

where we make use of the definition of the real cost of price adjustment, $AC_t^p = \frac{\theta_p}{2} (\pi_t^p - \pi^p)^2$. Equation (4.41) shows that the quadratic price and wage adjustment costs limit the consumption possibilities of the economy.⁷ Note that the functional form assumed for AC_t^p and AC_t^w ensures that the real costs of price and wage adjustment are zero in the steady state.⁸

⁵By log-linearizing equation (4.34) around the steady state, we obtain a standard linear wage Phillips curve given by $\widehat{\pi}_t^w = \beta \mathbf{E}_t \widehat{\pi}_{t+1}^w + \lambda_w (\widehat{MRS}_t - \widehat{W}_t/\widehat{P}_t)$, where $\lambda_w = N^{1+\eta} C^\sigma \epsilon_w / \theta_w$. Note that variables without time subscript refer to steady state values and a hat above a variable denotes the percentage deviation from its steady state value.

⁶By log-linearizing equation (4.36) around the steady state, we obtain a standard linear New Keynesian Phillips curve given by $\widehat{\pi}_t^p = \beta \mathbf{E}_t \widehat{\pi}_{t+1}^p + \lambda_p \widehat{MC}_t$, where $\lambda_p = Y(\varepsilon_p - 1)/\theta_p$.

⁷By log-linearizing equation (4.41) around the steady state, we obtain $\widehat{Y}_t = \widehat{C}_t$. Thus, the distortion due to quadratic wage and price adjustment costs vanishes in the linearized system.

⁸Furthermore, it rules out any long-run effects of trend inflation, i.e. $\pi^p = \pi^w > 1$, by leading to vertical long-run Phillips curves. A discussion of the effects of trend inflation under the Rotemberg (1982) pricing assumption is provided by Ascari and Rossi (2008).

4.2.7 The Steady State

We calculate the steady state of the model by omitting the time indices t . For simplicity, we normalize steady state technology to unity, i.e., $A = 1$. The equations describing wage and price setting now read as

$$MC = 1/\mu_p = (\varepsilon_p - 1)/\varepsilon_p \quad (4.42)$$

and

$$W/P = \mu_w C^\sigma N^\eta \quad (4.43)$$

where $\mu_w = \varepsilon_w/(\varepsilon_w - 1)$. Inserting the labor demand function $N = Y^{1/(1-\alpha)}$ and the resource constraint $Y = C$ into equation (4.43) yields

$$W/P = \mu_w Y^{\frac{\sigma(1-\alpha)+\eta}{1-\alpha}} \quad (4.44)$$

Marginal costs given by equation (4.37) can be rewritten as

$$MC = \frac{1-\tau}{1-\alpha} \mu_w Y^{\frac{\eta+\sigma+\alpha(1-\sigma)}{1-\alpha}} \quad (4.45)$$

By inserting (4.42) into the latter equation, we obtain

$$Y = \left(\frac{(1-\alpha)}{\mu_p \mu_w (1-\tau)} \right)^{\frac{1-\alpha}{\eta+\sigma+\alpha(1-\sigma)}} \quad (4.46)$$

The steady state level of output Y differs from the efficient level of output due to the existence of constant wage and price mark-ups $\mu_p > 1$ and $\mu_w > 1$. Efficiency of the steady state can be achieved by choosing the subsidy rate τ such that $\mu_p \mu_w (1-\tau) = 1$. It then follows that the optimal subsidy rate is given by

$$\tau = 1 - 1/(\mu_p \mu_w) \quad (4.47)$$

The gross real interest rate can be obtained by using the steady state Euler equation $C^{-\sigma} = \beta C^{-\sigma} R(\pi^p)^{-1}$. Hence, we have $R/\pi^p = 1/\beta$. The gross nominal interest rate is then given by $R = \pi^p/\beta$.

The steady state amount of hours worked can easily be computed by inserting equation (4.46) into the labor demand function $N = Y^{1/(1-\alpha)}$.

4.2.8 The Welfare Measure

We follow Schmitt-Grohé and Uribe (2004a) or Faia and Monacelli (2007) and measure absolute welfare by the expectation of the discounted sum of aggregate lifetime utility conditional on the set of information at time equal to zero

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right) \quad (4.48)$$

We also compute a relative welfare measure V_0^r which is defined as the percentage deviation of absolute welfare under a monetary policy rule from absolute welfare under the Ramsey policy, discussed below.⁹

⁹Let $V_0^{a,k}$ denote absolute welfare under the monetary policy regime $k \in \{\text{rule}, \text{Ramsey}\}$, we formally obtain $V_0^r = \frac{(V_0^{a,\text{rule}} - V_0^{a,\text{Ramsey}})}{V_0^{a,\text{Ramsey}}} \cdot 100$. For $\sigma > 1$ and $\eta > 0$, $V_0^{a,k} \in \mathbb{R}_0^-$ holds. Then, the Ramsey policy is superior if $V_0^r > 0$, whereas the opposite is true for $V_0^r < 0$.

4.3 Monetary Policy

We consider two different monetary policy regimes. First, we derive the optimal monetary policy under commitment. In doing so, we distinguish between the policy under time-zero commitment and the policy under timeless-perspective commitment. Second, we assume that the monetary authority commits itself to a simple time-invariant instrument rule. In doing so, we apply an interest rate peg and two types of the interest rate rule originally proposed by Taylor (1993).

For the determination of optimal monetary policy under commitment, we use the Ramsey approach described in Khan, King, and Wolman (2003). In this approach, the monetary authority seeks to maximize the utility of households subject to the constraints that describe the competitive economy, given by

$$0 = A_t N_t^{1-\alpha} - C_t - \frac{\theta_p}{2} (\pi_t^p - \pi^p)^2 - \frac{\theta_w}{2} (\pi_t^w - \pi^w)^2 \quad (4.49)$$

$$0 = (\pi_t^p - \pi^p) \pi_t^p - \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1}^p - \pi^p) \pi_{t+1}^p \right\} \\ - A_t N_t^{1-\alpha} \frac{\varepsilon_{p,t}}{\theta_p} \left(\frac{1-\tau}{1-\alpha} \frac{W_t}{P_t} N_t^\alpha \frac{1}{A_t} - \frac{1}{\mu_{p,t}} \right) \quad (4.50)$$

$$0 = (\pi_t^w - \pi^w) \pi_t^w - \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1}^w - \pi^w) \pi_{t+1}^w \right\} \\ - \frac{N_t (\varepsilon_{w,t} - 1)}{\theta_w} \left(\mu_{w,t} C_t^\sigma N_t^\eta - \frac{W_t}{P_t} \right) \quad (4.51)$$

$$0 = \frac{W_t}{P_t} - \frac{W_{t-1}}{P_{t-1}} \frac{\pi_t^w}{\pi_t^p} \quad (4.52)$$

where we have used the production function (4.38) to substitute for Y_t and equation (4.37) to substitute for the real marginal costs MC_t . Note that the consumption Euler equation (4.33) is not a binding restriction for the Ramsey problem. We assume that the monetary authority is able to commit to the policy plan determined at time $t = 0$.

The optimal Ramsey monetary policy can be obtained by solving the following optimization problem. Let $\lambda_{1,t}$, $\lambda_{2,t}$, $\lambda_{3,t}$, $\lambda_{4,t}$ represent the Lagrange multipliers on the constraints (4.49), (4.50), (4.51) and (4.52), respectively. For given exogenous processes (4.3), (4.9), and (4.22), the monetary authority optimizes the Lagrangian

$$\mathcal{L}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right) \\ + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_{1,t+1} \left(A_t N_t^{1-\alpha} - C_t - \frac{\theta_p}{2} (\pi_t^p - \pi^p)^2 - \frac{\theta_w}{2} (\pi_t^w - \pi^w)^2 \right) \\ + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_{2,t+1} \left((\pi_t^p - \pi^p) \pi_t^p - \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1}^p - \pi^p) \pi_{t+1}^p \right. \\ \left. - A_t N_t^{1-\alpha} \frac{\varepsilon_{p,t}}{\theta_p} \left(\frac{1-\tau}{1-\alpha} \frac{W_t}{P_t} N_t^\alpha \frac{1}{A_t} - \frac{1}{\mu_{p,t}} \right) \right)$$

$$\begin{aligned}
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_{3,t+1} \left((\pi_t^w - \pi^w) \pi_t^w - \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} (\pi_{t+1}^w - \pi^w) \pi_{t+1}^w \right. \\
& \quad \left. - \frac{N_t(\varepsilon_{w,t} - 1)}{\theta_w} \left(\mu_{w,t} C_t^\sigma N_t^\eta - \frac{W_t}{P_t} \right) \right) \\
& + \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \lambda_{4,t+1} \left(\frac{W_t}{P_t} - \frac{W_{t-1}}{P_{t-1}} \frac{\pi_t^w}{\pi_t^p} \right)
\end{aligned}$$

with respect to the set of control variables $\Omega_t = \{N_t, C_t, \pi_t^p, \pi_t^w, W_t/P_t\}$ and the set of costate variables $\Lambda_{t+1} = \{\lambda_{1,t+1}, \lambda_{2,t+1}, \lambda_{3,t+1}, \lambda_{4,t+1}\}$.

The first-order conditions with respect to N_t , C_t , π_t^p , π_t^w , and W_t/P_t are then given by

$$\begin{aligned}
0 = & -N_t^\eta + \tilde{\lambda}_{1,t} A_t (1 - \alpha) N_t^{-\alpha} - \tilde{\lambda}_{2,t} \frac{\varepsilon_{p,t}}{\theta_p} \frac{\alpha(1 - \tau)}{1 - \alpha} \frac{W_t}{P_t} \\
& - \tilde{\lambda}_{2,t} \frac{\varepsilon_{p,t}}{\theta_p} A_t N_t^{-\alpha} (1 - \alpha) \left(\frac{1 - \tau}{1 - \alpha} \frac{W_t}{P_t} \frac{N_t^\alpha}{A_t} - \frac{1}{\mu_{p,t}} \right) \\
& - \tilde{\lambda}_{3,t} \frac{(\varepsilon_{w,t} - 1)}{\theta_w} \left(\mu_{t,w} C_t^\sigma N_t^\eta (1 + \eta) - \frac{W_t}{P_t} \right)
\end{aligned} \tag{4.53}$$

$$0 = C_t^{-\sigma} - \tilde{\lambda}_{1,t} - \tilde{\lambda}_{2,t} \sigma \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{C_t} (\pi_{t+1}^p - \pi^p) \pi_{t+1}^p \right\} \tag{4.54}$$

$$\begin{aligned}
& - \tilde{\lambda}_{3,t} \left[\sigma \beta \mathbb{E}_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{C_t} (\pi_{t+1}^w - \pi^w) \pi_{t+1}^w \right\} + \mu_{w,t} \frac{(\varepsilon_{w,t} - 1) \sigma}{\theta_w} N_t^{1+\eta} C_t^{\sigma-1} \right] \\
& + \tilde{\lambda}_{2,t-1} \sigma \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{1}{C_t} (\pi_t^p - \pi^p) \pi_t^p + \tilde{\lambda}_{3,t-1} \sigma \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} \frac{1}{C_t} (\pi_t^w - \pi^w) \pi_t^w
\end{aligned}$$

$$\begin{aligned}
0 = & -\tilde{\lambda}_{1,t} \theta_p (\pi_t^p - \pi^p) + \tilde{\lambda}_{2,t} (2\pi_t^p - \pi^p) \\
& - \tilde{\lambda}_{2,t-1} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} (2\pi_t^p - \pi^p) + \tilde{\lambda}_{4,t} \left(\frac{W_{t-1}}{P_{t-1}} \right) \frac{\pi_t^w}{(\pi_t^p)^2}
\end{aligned} \tag{4.55}$$

$$\begin{aligned}
0 = & -\tilde{\lambda}_{1,t} \theta_w (\pi_t^w - \pi^w) + \tilde{\lambda}_{3,t} (2\pi_t^w - \pi^w) \\
& - \tilde{\lambda}_{3,t-1} \left(\frac{C_t}{C_{t-1}} \right)^{-\sigma} (2\pi_t^w - \pi^w) - \tilde{\lambda}_{4,t} \left(\frac{W_{t-1}}{P_{t-1}} \right) \frac{1}{\pi_t^p}
\end{aligned} \tag{4.56}$$

$$0 = -\tilde{\lambda}_{2,t} N_t \frac{\varepsilon_{p,t}(1 - \tau)}{\theta_p(1 - \alpha)} + \tilde{\lambda}_{3,t} \frac{N_t(\varepsilon_{w,t} - 1)}{\theta_w} + \tilde{\lambda}_{4,t} - \beta \mathbb{E}_t \left\{ \tilde{\lambda}_{4,t+1} \frac{\pi_{t+1}^w}{\pi_{t+1}^p} \right\} \tag{4.57}$$

where we have redefined the costate variables as $\lambda_{i,t+1} = \tilde{\lambda}_{i,t}$ ($i = 1, \dots, 4$). Note that the costate variables $\lambda_{2,t} = \tilde{\lambda}_{2,t-1}$ and $\lambda_{3,t} = \tilde{\lambda}_{3,t-1}$ are predetermined, whereas $\tilde{\lambda}_{1,t}$ and $\tilde{\lambda}_{4,t}$ are non-predetermined (jump) variables.

To obtain a global optimum under *time zero commitment*, the initial values of the predetermined costate variables must be

$$\lambda_{2,0} = \tilde{\lambda}_{2,-1} = 0 = \lambda_{3,0} = \tilde{\lambda}_{3,-1} \tag{4.58}$$

However, in the spirit of the *timeless perspective commitment* policy proposed by Woodford (1999a), the literature usually neglects the optimality condition (4.58).¹⁰ Instead, the initial

¹⁰See, among others, Khan, King, and Wolman (2003), Faia (2008a, 2008b), Faia and Monacelli (2008).

values of the predetermined costate variables are set equal to their respective steady state values

$$\begin{aligned}\lambda_{2,0} &= \tilde{\lambda}_{2,-1} = \lambda_2 \\ \lambda_{3,0} &= \tilde{\lambda}_{3,-1} = \lambda_3\end{aligned}\tag{4.59}$$

The Ramsey policy under time zero commitment and the Ramsey policy under timeless perspective commitment obviously coincide when the steady state values of the predetermined costate variables λ_2 and λ_3 are equal to zero. Besides this exception, the Ramsey policy under timeless perspective commitment fails to maximize the policy makers' objective function V_0^a . McCallum and Nelson (2004) point out that the timeless perspective policy can be considered as "policy-making according to a rule".¹¹ As a consequence, other monetary policy rules (in addition to time zero commitment) may exist which are superior to the Ramsey policy under timeless perspective commitment.

After having discussed the policy regime *Ramsey monetary policy*, we now turn to the alternative regime *commitment to a time-invariant simple rule*. For this purpose, we consider a interest rate rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^\phi \left(\left(\frac{\pi_t^p}{\pi^p}\right)^{\delta_\pi} \left(\frac{Y_t}{Y}\right)^{\delta_y}\right)^{1-\phi}\tag{4.60}$$

where δ_π is the constant weight on price inflation, and δ_y denotes the constant weight on output. A positive value of the constant parameter ϕ indicates a preference for interest rate smoothing.¹²

4.4 Calibration and Computation

We employ a standard calibration. The time unit is one quarter. The discount rate is equal to $\beta = 0.99$, implying an annual steady state real interest rate of approximately 4%. The inverse of the intertemporal elasticity of substitution in consumption, σ , is set to $\sigma = 2$. The calibrated $\eta = 3$ implies a labor supply elasticity with respect to the real wage of 1/3. The steady state values of the elasticity of substitution between different intermediate goods, ε_p , and the elasticity of substitution between different labor types, ε_w , are both set to 8. This implies steady state mark-ups of approximately 14 percent as well as an optimal subsidy rate given by $\tau = 0.2344$. The degree of decreasing returns to scale $(1 - \alpha)$ is set to 0.7. The parameters which measure the degree of nominal price and wage rigidity are set to $\theta_p = 342.48$ and $\theta_w = 1546.68$, respectively. These values imply slopes of the price and wage Phillips curves which are equal to the slopes under Calvo (1983) price and wage setting with standard Calvo parameters equal to 0.75. We set ρ_p , ρ_w , and ρ_a either equal to zero implying one-off shocks or to 0.95 implying that the exogenous processes are highly persistent.

Equation (4.60) comprises several well-known monetary policy rules. The rule originally proposed by Taylor (1993) can be obtained by setting $\phi = 0$, $\delta_\pi = 1.5$, and $\delta_y = 0.125$. To consider a Taylor rule with smoothing, we set $\phi = 0.8$, $\delta_\pi = 1.5$, and $\delta_y = 0.125$. Finally, we

¹¹See McCallum and Nelson (2004), p. 45.

¹²Note that equation (4.60) is the nonlinear counterpart of the linear rule usually considered in the literature. By log-linearizing, we obtain $\hat{R}_t = \phi \hat{R}_{t-1} + (1 - \phi)(\delta_\pi \hat{\pi}_t^p + \delta_y \hat{Y}_t)$ where a hat above a variable denotes the percentage deviation from its steady state value.

follow Collard and Dellas (2005) and set $\phi = 0.999$, $\delta_\pi = 1.001$ and $\delta_y = 0$ to consider an interest rate peg.

In what follows, we assume deterministic shock terms $e_{p,t}$, $e_{w,t}$, and $e_{a,t}$ in the exogenous processes. Thus, we depart from the usual assumption of white noise shocks. In order to simulate a 10 percent decrease in the elasticities $\varepsilon_{p,t}$ and $\varepsilon_{w,t}$ occurring at time $t = 0$, we set $e_{p,t} = e_{w,t} = \log(7.2/\varepsilon_l)$ for $t = 0$, where $\varepsilon_l = 8$ with $l \in \{p, w\}$ and $e_{p,t} = e_{w,t} = 0$ for $t \neq 0$. Note that this implies jumps in the price and wage mark-ups $\mu_{p,t} = \varepsilon_{p,t}/(\varepsilon_{p,t} - 1)$ and $\mu_{w,t} = \varepsilon_{w,t}/(\varepsilon_{w,t} - 1)$ of approximately 1.6 percent. To analyze a 10 percent increase in technology A_t , we assume $e_{a,0} = \log(1.1/A)$, where the steady state value of technology is normalized to $A = 1$.

The overall deterministic model under timeless perspective commitment is described by the equations (4.37), (4.38), (4.49), (4.50), (4.51), (4.52), and the optimality conditions (4.53) to (4.57). For given exogenous processes (4.3), (4.9), and (4.22) this set of equations determines the optimal paths of the non-predetermined (or jump) variables C_t , Y_t , N_t , MC_t , π_t^p , π_t^w , the predetermined variable W_t/P_t , the predetermined costate variables $\lambda_{2,t} = \tilde{\lambda}_{2,t-1}$, $\lambda_{3,t} = \tilde{\lambda}_{3,t-1}$ and the non-predetermined costate variables $\tilde{\lambda}_{1,t}$, $\tilde{\lambda}_{4,t}$. By dropping the time indices we can solve for the steady state of this system, which we denote as the Ramsey steady state. To analyze our model under a simple instrument rule, we replace the set of optimality conditions (4.53) to (4.57) by equation (4.60).

Our deterministic approach allows us to solve the model without any approximation. For the deterministic but nonlinear simulations, we use the software DYNARE developed by Juillard (1996).

4.5 Simulation Results

At first, we briefly discuss the properties of the Ramsey steady state and then turn to the welfare implications of wage and price mark-up shocks.

The numerical solution for the steady state values of the costate variables suggests the following: If we assume the existence of an optimal subsidy as defined in (4.47) that guarantees steady state efficiency, the steady state values of the predetermined costate variables λ_2 and λ_3 will be zero. In this case, the Ramsey policy under timeless perspective commitment corresponds to the policy under time zero commitment and is therefore the globally optimal policy under commitment. Otherwise, if the government does not subsidize the costs of employment, the initial steady state is distorted and the numerical solution for the steady state values yields $\lambda_2 \neq 0$ and $\lambda_3 \neq 0$. Thus, the optimality condition (4.58) is violated and the policy under timeless commitment is not the globally optimal policy under commitment.

Table 4.1 reports the absolute and relative welfare results for persistent and one-off wage and price mark-up shocks where both the case of an efficient (with optimal subsidies) and the case of an inefficient steady state (without optimal subsidies) are considered. Note that our definition of relative welfare implies that the Ramsey policy is superior if $V_0^r > 0$, whereas the opposite is true for $V_0^r < 0$. Or, to put it differently, V_0^r measures the welfare costs or gains of committing to a simple rule instead of committing to the timeless perspective Ramsey policy.

The key results to note are as follows. First, and as already mentioned, the existence of optimal subsidies ensures that Ramsey policy under timeless perspective commitment is the

Table 4.1: Welfare effects

	Persistent shock		One-off shock	
	V_0^a	V_0^r	V_0^a	V_0^r
Wage mark-up shock				
<i>With optimal subsidies</i>				
Ramsey	-123.9107	–	-123.9106	–
Interest rate peg	-123.9111	$2.7176 \cdot 10^{-4}$	-123.9106	$3.0191 \cdot 10^{-7}$
Taylor rule	-123.9122	0.0012	-123.9106	$3.5262 \cdot 10^{-7}$
Taylor rule with smoothing	-123.9118	$8.8664 \cdot 10^{-4}$	-123.9106	$3.2783 \cdot 10^{-7}$
<i>Without optimal subsidies</i>				
Ramsey	-124.4438	–	-124.4371	–
Interest rate peg	-124.4435	$-2.7136 \cdot 10^{-4}$	-124.4377	$4.3549 \cdot 10^{-4}$
Taylor rule	-124.4431	$-5.8738 \cdot 10^{-4}$	-124.4377	$4.2061 \cdot 10^{-4}$
Taylor rule with smoothing	-124.4430	$-6.5441 \cdot 10^{-4}$	-124.4377	$4.2527 \cdot 10^{-4}$
Price mark-up shock				
<i>With optimal subsidies</i>				
Ramsey	-123.9117	–	-123.9106	–
Interest rate peg	-123.9121	$3.0186 \cdot 10^{-4}$	-123.9106	$4.9620 \cdot 10^{-8}$
Taylor rule	-123.9147	0.0024	-123.9106	$5.9649 \cdot 10^{-7}$
Taylor rule with smoothing	-123.9134	0.0014	-123.9106	$3.3600 \cdot 10^{-7}$
<i>Without optimal subsidies</i>				
Ramsey	-124.4469	–	-124.4377	–
Interest rate peg	-124.4497	0.0022	-124.4378	$8.2695 \cdot 10^{-5}$
Taylor rule	-124.4456	-0.0011	-124.4376	$-6.8903 \cdot 10^{-5}$
Taylor rule with smoothing	-124.4466	$-2.9624 \cdot 10^{-4}$	-124.4377	$-1.7009 \cdot 10^{-5}$

Notes: Relative welfare V_0^r is the percentage of the welfare under a policy regime relative to the welfare under the Ramsey policy. The definition of relative welfare implies that the Ramsey policy is superior when $V_0^r > 0$, whereas the opposite is true for $V_0^r < 0$.

globally optimal policy under commitment. An obvious implication of the previous result is that no superior rule exists within the class of time-invariant rules which the monetary authority commits itself to.

Second, if the initial steady state is distorted, the Ramsey monetary policy under timeless perspective commitment is generally *not* superior to simple monetary policy rules. In the case of a persistent wage mark-up shock, the monetary authority could achieve a welfare gain by switching to all the rules under consideration. In the case of a persistent price mark-up shock, a Taylor rule performs 0.0011 percent better than the Ramsey policy. A small welfare gain could also be achieved by following a simple Taylor rule with smoothing. By considering one-off shocks, we obtain mixed results. In the case of wage mark-up shocks, the Ramsey policy performs better than the simple rules under consideration, whereas in the case of price mark-up shocks, the Ramsey policy is again inferior to a Taylor rule with and without smoothing.¹³

¹³However, also in the case of a one-off wage mark-up shock a time-invariant instrument rule exists that performs better than the Ramsey monetary policy under timeless perspective commitment. This superior rule is an augmented Taylor rule with a positive weight on the elasticity of substitution between labor types

Note that we have also found the non-optimality of Ramsey policy under timeless perspective commitment in the case of technology shocks. For a persistent as well as for a one-off 10 percent increase in A_t , a more complicated rule with a high weight on price and wage inflation, a significant degree of interest rate smoothing and a negative weight on output would perform better than the Ramsey policy under timeless perspective commitment.

The rationale for these results has already been mentioned: Ramsey policy from a timeless perspective implies that the monetary authority ignores the optimality condition (4.58) and instead sets the initial values of the predetermined costate variables equal to their respective values in the by now distorted steady state. As a consequence, the policy under timeless perspective commitment fails to maximize the policy makers' objective function.

Nevertheless, this policy is widely considered as a good approximation of the globally optimal policy under commitment. Or, according to McCallum and Nelson (2004), the policy under timeless perspective commitment should be optimal within the class of time-invariant rules which the monetary authority commits itself to. Our results reveal that this is, however, *not* generally true.

This is particularly notable, since we do not even search for optimal simple rules by maximizing the policy makers' objective function with respect to the parameters of a given simple rule.¹⁴ If we did, we would improve the welfare results of simple rules which in turn would amplify the relative welfare loss of the Ramsey policy under timeless perspective commitment.

In this chapter, however, our purpose is not to find the optimal monetary policy rule or to investigate the structure of optimal simple rules but solely to demonstrate a possible inferiority of the Ramsey policy under timeless perspective commitment to widely-discussed simple monetary policy rules within a standard New Keynesian model under a reasonable calibration.

The existence of such superior rules challenges the appropriateness of the proposed Ramsey approach under timeless perspective commitment as a guideline for the conduct of monetary policy.

4.6 Conclusion

In this chapter, the Ramsey monetary policy in the presence of technology and cost-push shocks was derived. In particular, we compared the optimal policy to simple monetary policy rules. We have demonstrated that Ramsey policy under timeless perspective commitment may be inferior to simple ad-hoc monetary policy rules when the initial steady state is distorted. Thus, our work sounds a cautionary note about proposed Ramsey policies under timeless perspective commitment as a guideline for monetary policy.

To find a time-invariant rule which is utility-maximizing for different shocks that hit the economy and under different calibrations as well as to discover the structure of such an "optimal rule" within a non-linearized framework would be a fruitful area for future research.

leading to a sharp decline in the nominal interest rate in response to a rise in the wage mark-up.

¹⁴Among the numerous studies which solve such a constrained optimization problem see, for example, Schmitt-Grohé and Uribe (2007a).

5 Ramsey Monetary Policy, Oil Price Shocks, and Welfare

5.1 Introduction

Blanchard and Galí (2007) seek to answer the question, why the actual hike in oil prices has smaller impacts on inflation and economic activity than the oil price shocks of the seventies. They identify three main reasons, a smaller oil share in production, a higher flexibility of labor markets, and a better monetary policy. However, they do not ask what will be the best or optimal monetary policy in response to an increase in oil prices.

In this chapter, we seek to shed light on this question by deriving the optimal Ramsey-type monetary policy in an oil-dependent economy with staggered price and wage setting. We derive the optimal Ramsey-type monetary policy for the case of an efficient and inefficient steady state and compare the optimal policy to simple monetary policy rules. Furthermore, we investigate the usual case of an unanticipated shock, but also allow for the case that the occurrence of the energy price hike is known in advance. In order to check the robustness of our results, we also compare the welfare effects of anticipated and unanticipated cost-push shocks.

This chapter is related to different strands of literature. The first strand deals with the macroeconomic effects of oil price shocks. Contributions which utilize models without rigorous micro-foundations include, among others, Buiters (1978), Bhandari (1981), Bhandari and Turnovsky (1984), Bruno and Sachs (1985), Wohltmann (1994b), Wohltmann and Clausen (2003), and Wohltmann and Winkler (2005a, 2005b, 2005c, 2006, 2008a).¹

In contrast, Kim and Loungani (1992), Rotemberg and Woodford (1996), and Finn (2000) analyze the effects of oil price shocks in micro-founded dynamic general equilibrium models of closed economies. Backus and Crucini (2000) consider an open-economy real business cycle model to study the effects of oil on the economy. The latter studies are based on the assumption of completely flexible prices. Hence, there is no role for monetary policy.

A second strand of the literature studies the interaction between oil price shocks and monetary policy. Among the numerous empirical studies see, for example, Hamilton (1983), Hamilton and Herrera (2004), Bernanke, Gertler, and Watson (1997, 2004), or Barsky and Kilian (2002). Theoretical contributions are, among others, Leduc and Sill (2004), Medina and Soto (2005), Carlstrom and Fuerst (2006), and Blanchard and Galí (2007).

Leduc and Sill (2004), Medina and Soto (2005) and Calstrom and Fuerst (2006) seek to answer the question whether the recessionary consequences of oil price shocks are caused by the oil price hike or by the monetary response to the rise in oil prices. However, these studies concentrate on the recessionary consequences and do not, as we do, investigate the welfare effects of oil price shocks under different monetary policy regimes, in particular the optimal Ramsey monetary policy.

¹German readers are also referred to Jarchow (1992) or Wohltmann (1987, 1993).

Before reviewing the small stream of literature on optimal monetary policy and oil shocks, we highlight our methodological relation and contribution to the vast literature on the optimal design of monetary policy.

In this chapter, we again use the Ramsey approach under timeless perspective commitment for the derivation of optimal policy. Recall that the optimal policy under timeless perspective commitment is the globally optimal policy under commitment if the steady state of the system is efficient. Efficiency can be achieved by a fiscal policy that optimally subsidizes the cost of production. If we abandon the assumption that optimal subsidies are in place, the Ramsey policy under timeless perspective commitment is not the globally optimal policy under commitment. In Chapter 4, we have demonstrated that in this case, the Ramsey monetary policy under timeless perspective commitment may be inferior to simple (non-optimized) monetary policy rules.

In this chapter, we are able to show the possible inferiority of Ramsey policy under timeless perspective commitment in the presence of energy price shocks. In particular, we show that even simple instrument rules such as the interest rate rule originally proposed by Taylor (1993) or an interest rate peg may be superior to the Ramsey monetary policy under timeless perspective commitment. Hence, our findings strongly support the results reported in Chapter 4.

After having discussed our methodological contribution, we now relate this chapter to the combination of two strands of literature, namely the interaction of oil shocks and the optimal design of monetary policy.

The only studies, we are aware of, that analyze optimal monetary policy in the presence of oil shocks are De Fiori, Lombardo, and Stebunovs (2006), Montoro (2007), and Wohltmann and Winkler (2008a). De Fiori, Lombardo, and Stebunovs (2006) consider a three-country framework and search for optimal parameters of a simple monetary policy rule. Montoro (2007) considers a closed and oil-dependent economy with staggered price setting and investigates optimal monetary policy under commitment by conducting a second-order approximation of the utility function according to Benigno and Woodford (2005). Hence, this chapter is based on the linear-quadratic approach. This approach is also used by Wohltmann and Winkler (2008a) who consider a stylized small-open economy framework with a loss function that reflects, according to Svensson (2000), the policy objective of flexible domestic inflation targeting. They derive the optimal policy under commitment and search for optimal simple policy rules.

This chapter contributes to the literature by deriving the globally optimal Ramsey monetary policy under commitment for an oil-dependent economy with staggered price and wage setting. Thereby, we do neither approximate the utility-based policy makers' objective function nor the competitive equilibrium conditions of the model economy. In this perspective, we follow the suggestion of Ascari and Merkl (2007) and take into account the effects of non-linearities. Our results suggest that the question whether monetary policy amplifies or dampens the recessionary effects of energy price shocks is negligible from a welfare point of view since the welfare-maximizing policy in fact calls for a significant recession.

In Chapter 2 and Chapter 3, we have shown that anticipated cost-push shocks may lead to higher welfare losses than unanticipated cost-push shocks. Wohltmann and Winkler (2008a) report the same result for energy price shocks.

In this chapter, we compare the welfare effects of anticipated and unanticipated energy price and cost-push shocks under different monetary policy regimes. Our simulations suggest that

the anticipation of a future energy price shock is welfare-enhancing, whereas the anticipation of cost-push shocks induces higher welfare losses than unanticipated cost-push shocks.

Our mixed results stand in contrast to the findings of Wohltmann and Winkler (2008a) but confirm our previous findings reported in Chapter 2 and Chapter 3. In this respect, we suggest that the welfare effects of anticipation depend on the nature of the shock that is known in advance.

The remainder of this chapter is organized as follows. Section 2 presents a micro-founded model for an oil-dependent economy. First, we lay out the baseline model with staggered price and wage setting as well as monopolistic competition on product and labor markets. Second, we derive the real business cycle core of the model which is the limiting case of the baseline model with perfectly flexible prices and wages. Then, we solve for the steady state of the model which is independent of the degree of price and wage flexibility. Thereby, it is shown how wage and energy price subsidies can eliminate the monopolistic distortions and, thus, lead to the efficiency of both the steady state level of output and the output level without nominal rigidities (henceforth denoted as natural level). At the end of Section 2, an absolute and a relative measure of overall welfare is presented.

Section 3 describes two different monetary policy regimes. First, we derive the optimal Ramsey-type monetary policy under commitment. In doing so, we distinguish between the policy under time-zero commitment and the policy under timeless perspective commitment. We show under which conditions these commitment policies are identical. Second, we assume that the monetary authority commits itself to a simple (and time-invariant) instrument rule. We apply an interest rate peg and several types of the interest rate rule originally proposed by Taylor (1993).

Section 4 presents the calibration and discusses the solution method.

In Section 5, the results of our simulations are reported. First, we analyze the steady state effects of permanent changes in the real price of energy. Second, we briefly discuss the properties of the Ramsey steady state which is defined as the long-run equilibrium of the model under the optimal Ramsey-type monetary policy. Then, we turn to dynamics and analyze the effects of energy price shocks. Initially, we consider the real business cycle core of the model in order to obtain the natural responses to the energy price shock as a benchmark for our subsequent analysis. Thereafter, we consider the baseline model with staggered price and wage setting. We present impulse response functions for the model under the globally optimal policy under commitment and under the simple interest rate rule that was introduced by Taylor (1993). Thereby, we investigate whether the optimal policy amplifies or dampens the recessionary and inflationary effects of a persistent increase in the real price of energy. Furthermore, we discuss the welfare effects of different monetary policy regimes. In particular, we compare simple ad-hoc interest rate rules to the optimal Ramsey monetary policy under commitment. In doing so, we emphasize the role of the efficiency of the initial steady state for the welfare-ranking of monetary policies. Finally, we compare the welfare effects of anticipated and unanticipated shocks. Thereby, we do not restrict our analysis to the case of energy price shocks but extend it to wage and price mark-up shocks.

Section 6 provides a summary of our main findings and some suggestions for future research.

5.2 A New Keynesian Model for an Oil-Dependent Economy

The model is a standard New Keynesian model for a closed and cashless economy which uses energy, e.g. oil, as an input in production. We follow Calstrom and Fuerst (2006) and assume that energy is bought from exogenous sources at an exogenously given real energy price. The economy consists of households, intermediate goods producers, final goods producers, and labor bundlers.

Households consume homogenous final goods, hold bonds and monopolistically supply differentiated labor. Intermediate goods firms monopolistically supply differentiated intermediate goods produced with homogenous labor and energy. Final goods producers buy intermediate goods, aggregate them to a homogenous final good and provide the final good under perfectly competitive conditions. Similarly, labor bundlers buy differentiated labor types, aggregate them to a homogenous labor bundle and sell it to intermediate goods producers in a perfectly competitive market.

In addition, we follow Erceg, Henderson, and Levin (2000) and assume that both the price setting decision of intermediate goods producers and the wage setting decision of households are constrained by a usual Calvo (1983) mechanism.² In the following, we present the model in detail.

5.2.1 Final Goods Producers

Final goods producers use the continuum of intermediate goods $Y_t(i)$ to produce the homogenous final good \tilde{Y}_t in a perfectly competitive market. A final goods producer maximizes his profits

$$P_t \tilde{Y}_t - \int_0^1 P_t(i) Y_t(i) di \quad (5.1)$$

subject to the following CES production function

$$\tilde{Y}_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \quad (5.2)$$

where P_t is the price of the final good, $P_t(i)$ is the price of intermediate good i , and $\varepsilon_p > 1$ denotes the elasticity of substitution between different intermediate goods.

The first-order condition for profit maximization is given by

$$P_t \left(\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right)^{\frac{1}{\varepsilon_p - 1}} Y_t(i)^{-\frac{1}{\varepsilon_p}} = P_t(i) \quad (5.3)$$

By rearranging (5.3), we obtain the demand function for intermediate good i

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} \tilde{Y}_t \quad (5.4)$$

²Merkel and Snower (2009) show that wage and price staggering à la Calvo are in general complementary in generating monetary persistence.

The demand function (5.4) can be substituted into the CES production function (5.2) which yields

$$P_t = \left(\int_0^1 P_t(i)^{1-\varepsilon_p} di \right)^{\frac{1}{1-\varepsilon_p}} \quad (5.5)$$

The RHS of equation (5.5) represents the marginal costs of a final goods producer. Thus, equation (5.5) states that the price equals marginal costs which, obviously, must hold under perfect competition.

5.2.2 Labor Bundlers

Analogously to final goods producers, labor bundlers buy the differentiated labor types $N_t(j)$, aggregate them to \tilde{N}_t and sell it to the intermediate goods producers under perfectly competitive conditions. A bundler maximizes his profits

$$W_t \tilde{N}_t - \int_0^1 W_t(j) N_t(j) dj \quad (5.6)$$

subject to the following CES aggregation function

$$\tilde{N}_t = \left(\int_0^1 N_t(j)^{\frac{\varepsilon_w-1}{\varepsilon_w}} dj \right)^{\frac{\varepsilon_w}{\varepsilon_w-1}} \quad (5.7)$$

where W_t is the price of the labor bundle \tilde{N}_t , $W_t(j)$ is the price of labor type j , and $\varepsilon_w > 1$ denotes the elasticity of substitution between different labor types.

The first-order condition for profit maximization is given by

$$W_t \left(\int_0^1 N_t(j)^{\frac{\varepsilon_w-1}{\varepsilon_w}} dj \right)^{\frac{1}{\varepsilon_w-1}} N_t(j)^{-\frac{1}{\varepsilon_w}} = W_t(j) \quad (5.8)$$

By rearranging (5.8) and using (5.7), we obtain the demand function for labor type j

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} \tilde{N}_t \quad (5.9)$$

The demand function (5.9) can be substituted into the CES aggregation function (5.7) to obtain

$$W_t = \left(\int_0^1 W_t(j)^{1-\varepsilon_w} dj \right)^{\frac{1}{1-\varepsilon_w}} \quad (5.10)$$

The RHS of (5.10) represents the marginal costs of a labor bundler, such that equation (5.10) is the 'price equals marginal costs' condition in a perfectly competitive market.

5.2.3 Households

The economy is made up by a continuum of differentiated households indexed by $j \in [0, 1]$. Each household seeks to maximize its discounted sum of per-period utility $U(C_t(j), N_t(j))$

$$E_t \sum_{k=0}^{\infty} \beta^k U(C_{t+k}(j), N_{t+k}(j)) \quad (5.11)$$

where $\beta \in (0, 1)$ is the discount factor and E_t is the expectation operator conditional on information available in period t . $C_t(j)$ denotes household j th consumption of final goods and $N_t(j)$ are its total hours worked. Maximization of (5.11) is subjected to the demand function (5.9) for household j th differentiated labor type and the households' period-by-period budget constraint

$$C_t(j) + \frac{B_t(j)}{P_t} = \frac{W_t(j)}{P_t} N_t(j) + \frac{R_{t-1} B_{t-1}(j)}{P_t} - T_t^r(j) + \Pi_t^r(j) \quad (5.12)$$

where R_t is the gross nominal interest rate on its one-period nominal bond holdings $B_t(j)$. $T_t^r(j)$ are lump-sum taxes and $\Pi_t^r(j)$ are dividends, both expressed in real terms and assumed to be equal for all households j ($T_t^r(j) = T_t^r$, $\Pi_t^r(j) = \Pi_t^r$).

Furthermore, we assume the existence of complete financial markets, which ensures the household against variations in its wage income $W_t(j)N_t(j)$. As a consequence, each household chooses the same level of consumption ($C_t(j) = C_t$).

By inserting the constraint (5.12) into the intertemporal utility function (5.11), we obtain the maximization problem

$$\text{Max}_{B_t(j)} E_t \sum_{k=0}^{\infty} \beta^k U \left(-\frac{B_{t+k}(j)}{P_{t+k}} + \frac{R_{t+k-1} B_{t+k-1}(j)}{P_{t+k}} + t.i.o., N_{t+k}(j) \right) \quad (5.13)$$

where $t.i.o. = \frac{W_{t+k}(j)}{P_{t+k}} N_{t+k}(j) - T_{t+k}^r(j) + \Pi_{t+k}^r(j)$ comprises all terms independent of this optimization problem. Differentiating (5.13) with respect to $B_t(j)$ gives the familiar consumption Euler equation

$$U_{C,t} = \beta U_{C,t+1} R_t \frac{P_t}{P_{t+1}} \quad (5.14)$$

where $U_{C,t}$ denotes the marginal utility of consumption where we can drop the index j due to the assumption of complete financial markets.

In what follows, we take the following functional form for the per-period utility function:

$$U(C_t, N_t(j)) = \frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\eta} N_t(j)^{1+\eta} \quad (5.15)$$

where $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution in consumption, and $1/\eta$ with $\eta > 0$ measures the labor supply elasticity.³ The marginal utility of consumption $U_{C,t}$ and the marginal disutility of labor $U_{N,t}(j)$ are then given by

$$U_{C,t} = C_t^{-\sigma} \quad (5.16)$$

and

$$U_{N,t}(j) = -N_t(j)^\eta = - \left[\left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} \tilde{N}_t \right]^\eta \quad (5.17)$$

where in the latter equation we make use of (5.9). The consumption Euler equation (5.14) now reads as follows:

$$C_t^{-\sigma} = C_{t+1}^{-\sigma} R_t (\pi_{t+1}^p)^{-1} \quad (5.18)$$

³Note that in the limiting case $\sigma = 1$, we obtain $U(C_t, N_t(j)) = \log C_t - N_t(j)^{1+\eta}/(1+\eta)$.

where $\pi_t^p = P_t/P_{t-1}$ denotes the gross rate of price inflation.

Following Erceg, Henderson, and Levin (2000), the household is a monopolistic supplier of labor type $N_t(j)$. In each period the household faces the constant probability $(1-\theta_w) \in (0, 1]$ of being allowed to choose its nominal wage. A household j which is able to reset its nominal wage in period t will choose $W_t(j)$ in order to maximize its utility

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k U(C_{t+k}, N_{t+k}(j)) \quad (5.19)$$

subject to the sequence of budget constraints

$$C_{t+k} = \frac{W_t(j)}{P_{t+k}} N_{t+k}(j) - \frac{B_{t+k}(j)}{P_{t+k}} + \frac{R_{t+k-1} B_{t+k-1}(j)}{P_{t+k}} - T_{t+k}^r + \Pi_{t+k}^r \quad (5.20)$$

and labor demand functions

$$N_{t+k}(j) = \left(\frac{W_t(j)}{W_{t+k}} \right)^{-\varepsilon_w} \tilde{N}_{t+k} \quad \text{for } k = 0, 1, 2, \dots \quad (5.21)$$

Combining the constraints (5.20) and (5.21) and plugging them into the intertemporal utility function (5.19) yields the following maximization problem of a household which is able to set its wage $W_t(j)$ in period t

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k U \left(\frac{W_t(j)}{P_{t+k}} \left(\frac{W_t(j)}{W_{t+k}} \right)^{-\varepsilon_w} \tilde{N}_{t+k} + t.i.w., \left(\frac{W_t(j)}{W_{t+k}} \right)^{-\varepsilon_w} \tilde{N}_{t+k} \right) \quad (5.22)$$

where $t.i.w. = -(B_{t+k}(j) - R_{t+k-1} B_{t+k-1}(j))/P_{t+k} - T_{t+k}^r + \Pi_{t+k}^r$ comprises all terms independent of the wage-setting problem. Differentiating (5.22) with respect to $W_t(j)$ yields the first-order condition

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t \left[(1 - \varepsilon_w) \frac{U_{C,t+k}}{P_{t+k}} N_{t+k}(j) - \varepsilon_w \frac{U_{N,t+k}(j)}{W_t(j)} N_{t+k}(j) \right] = 0 \quad (5.23)$$

By inserting (5.16) and (5.17) into the optimality condition (5.23) and rearranging terms, the following optimal wage setting rule can be obtained

$$W_t(j)^{1+\varepsilon_w\eta} = \mu_w \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k W_{t+k}^{\varepsilon_w(1+\eta)} \tilde{N}_{t+k}^{1+\eta}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k C_{t+k}^{-\sigma} P_{t+k}^{-1} W_{t+k}^{\varepsilon_w} \tilde{N}_{t+k}} \quad (5.24)$$

where $\mu_w = \frac{\varepsilon_w}{\varepsilon_w - 1}$.

Note that in the case of perfectly flexible wages, i.e., $\theta_w = 0$, all households choose the same nominal wage $W_t(j) = W_t$ and supply the same amount of labor $N_t(j) = N_t$. Then, the optimality condition (5.23) collapses into the static wage setting rule

$$\frac{W_t}{P_t} = \mu_w \left(-\frac{U_{N,t}}{U_{C,t}} \right) \quad (5.25)$$

The monopolistic structure of the labor market drives a wedge (of the amount of the markup $\mu_w > 1$) between the marginal rate of substitution $-U_{N,t}/U_{C,t}$ and the real wage W_t/P_t . Note that in the limiting case of perfect substitutability between the different types of labor

($\varepsilon_w \rightarrow \infty$), the mark-up μ_w vanishes and the market outcome under perfect competition is restored.

Dividing equation (5.24) by $W_t^{1+\varepsilon_w\eta}$ gives

$$\left(\frac{W_t(j)}{W_t}\right)^{1+\varepsilon_w\eta} = \frac{\Psi_t^w}{\Phi_t^w} \quad (5.26)$$

where

$$\Psi_t^w = \mu_w \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{W_{t+k}}{W_t}\right)^{\varepsilon_w(1+\eta)} \tilde{N}_{t+k}^{1+\eta} \quad (5.27)$$

$$\Phi_t^w = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k C_{t+k}^{-\sigma} \frac{W_{t+k}}{P_{t+k}} \left(\frac{W_{t+k}}{W_t}\right)^{\varepsilon_w-1} \tilde{N}_{t+k} \quad (5.28)$$

Note that $\frac{W_{t+k}}{W_t} = \frac{W_{t+1}}{W_t} \frac{W_{t+2}}{W_{t+1}} \dots \frac{W_{t+k}}{W_{t+k-1}} = \prod_{i=1}^k \pi_{t+i}^w$ is the gross wage inflation rate between period t and $t+k$, and $\pi_t^w = W_t/W_{t-1}$.

According to Schmitt-Grohé and Uribe (2004a), equation (5.27) can be expressed in recursive form as

$$\Psi_t^w = \mu_w \tilde{N}_t^{1+\eta} + \mu_w \mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^{k+1} \left(\frac{W_{t+k+1}}{W_t}\right)^{\varepsilon_w(1+\eta)} \tilde{N}_{t+k+1}^{1+\eta} \quad (5.29)$$

Note that

$$\Psi_{t+1}^w = \mu_w \mathbb{E}_{t+1} \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(\frac{W_{t+k+1}}{W_t}\right)^{\varepsilon_w(1+\eta)} \left(\frac{W_t}{W_{t+1}}\right)^{\varepsilon_w(1+\eta)} \tilde{N}_{t+k+1}^{1+\eta} \quad (5.30)$$

By combining (5.30) and (5.29), we obtain

$$\Psi_t^w = \mu_w \tilde{N}_t^{1+\eta} + \beta\theta_w \mathbb{E}_t \left((\pi_{t+1}^w)^{\varepsilon_w(1+\eta)} \Psi_{t+1}^w \right) \quad (5.31)$$

Rewriting (5.28) in the same way yields

$$\Phi_t^w = C_t^{-\sigma} \frac{W_t}{P_t} \tilde{N}_t + \beta\theta_w \mathbb{E}_t \left((\pi_{t+1}^w)^{\varepsilon_w-1} \Phi_{t+1}^w \right) \quad (5.32)$$

5.2.4 Intermediate Goods Producers

Each intermediate goods producer is a monopolistic supplier of the intermediate good $i \in [0, 1]$. Firm i uses the amount $\tilde{N}_t(i)$ of homogenous labor, the amount $En_t(i)$ of energy from exogenous sources and the constant returns to scale technology

$$\left((1-a)\tilde{N}_t(i)^{1-\rho} + aEn_t(i)^{1-\rho} \right)^{\frac{1}{1-\rho}} \quad (5.33)$$

to produce his intermediate good $Y_t(i)$.⁴ $\rho > 0$ denotes the inverse of the constant elasticity of substitution between energy and labor, $a \in (0, 1)$ represents a share parameter.

⁴This way of modeling energy usage is widely applied in the literature. See, among others, Kamps and Pierdzioch (2002), Medina and Soto (2005), Calstrom and Fuerst (2006), Castillo, Montoro, and Tuesta (2007), or Montoro (2007). For an early reference, see Bhandari and Turnovsky (1984).

The firm takes the real price of labor W_t/P_t and the real price of energy P_t^e as given.⁵ The supply of energy from exogenous sources is assumed to be perfectly elastic. The real price of energy P_t^e is described by the following autoregressive process

$$\frac{P_t^e}{P^e} = \left(\frac{P_{t-1}^e}{P^e} \right)^{\phi_e} \exp(e_t) \quad (5.34)$$

where P^e denotes the steady state value of the real energy price.⁶

We allow for the existence of wage and energy price subsidies $\tau_w W_t/P_t$ and $\tau_e P_t^e$, both financed by lump-sum taxes. The subsidy rates $\tau_w \in [0, 1)$ and $\tau_e \in [0, 1)$ are assumed to be constant.

Factor demands can be obtained from the minimization of total real costs $TC_t(i) = \frac{W_t}{P_t}(1 - \tau_w)\tilde{N}_t(i) + P_t^e(1 - \tau_e)En_t(i)$ subject to the production function (5.33). The solution of the cost minimization problem reads as follows

$$\tilde{N}_t(i) = Y_t(i) \left(\frac{(1-a)MC_t(i)}{(W_t/P_t)(1-\tau_w)} \right)^{\frac{1}{\rho}} \quad (5.35)$$

$$En_t(i) = Y_t(i) \left(\frac{aMC_t(i)}{P_t^e(1-\tau_e)} \right)^{\frac{1}{\rho}} \quad (5.36)$$

where $MC_t(i)$ denotes real marginal costs, which can be expressed as

$$MC_t(i) = \left((1-a)^{\frac{1}{\rho}} ((W_t/P_t)(1-\tau_w))^{\frac{\rho-1}{\rho}} + a^{\frac{1}{\rho}} (P_t^e(1-\tau_e))^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \quad (5.37)$$

Due to our assumption of constant returns to scale and competitive factor markets, real marginal costs do not depend on the quantity produced by firm i , and are thus the same for all firms, i.e., $MC_t(i) = MC_t$.

The profit-maximizing price-setting decision is constrained by a Calvo mechanism. In each period the intermediate goods producer faces the constant probability $(1 - \theta_p) \in (0, 1]$ of being allowed to choose his price $P_t(i)$ in order to maximize his real profits

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k \Delta_{t,t+k} \left(\frac{P_t(i)}{P_{t+k}} Y_{t+k}(i) - MC_{t+k} Y_{t+k}(i) \right) \quad (5.38)$$

subject to the sequence of demand functions

$$Y_{t+k}(i) = \left(\frac{P_t(i)}{P_{t+k}} \right)^{-\varepsilon_p} \tilde{Y}_{t+k} \quad \text{for } k = 0, 1, 2, \dots \quad (5.39)$$

where $\Delta_{t,t+k}$ denotes the stochastic discount factor for real payoffs. Plugging the constraint (5.39) into the intertemporal profit function (5.38) yields the following maximization problem of a firm which is able to re-optimize his price $P_t(i)$ in period t

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k \Delta_{t,t+k} \left(\frac{P_t(i)}{P_{t+k}} \left(\frac{P_t(i)}{P_{t+k}} \right)^{-\varepsilon_p} \tilde{Y}_{t+k} - MC_{t+k} \left(\frac{P_t(i)}{P_{t+k}} \right)^{-\varepsilon_p} \tilde{Y}_{t+k} \right) \quad (5.40)$$

⁵By considering an exogenously given real price of energy, we implicitly assume that the exogenous supplier of energy fully indexes the nominal energy price, $P_t^{e,n}$, to the price index of the energy consumer, P_t .

⁶Note that equation (5.34) is the nonlinear counterpart of the usual expression $\hat{P}_t^e = \phi_e \hat{P}_{t-1}^e + e_t$, where $\hat{P}_t^e = \log(P_t^e) - \log(P^e) \approx (P_t^e - P^e)/P^e$ denotes the log-deviation of the real price of energy from its steady state value.

By differentiating (5.40) with respect to $P_t(i)$, we obtain the first-order condition

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k \Delta_{t,t+k} \left[(1 - \varepsilon_p) \frac{1}{P_{t+k}} Y_{t+k}(i) + \varepsilon_p \frac{1}{P_t(i)} MC_{t+k} Y_{t+k}(i) \right] = 0 \quad (5.41)$$

Inserting (5.39) into the optimality condition (5.41) and rearranging terms yields the following optimal price setting rule

$$P_t(i) = \mu_p \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k \Delta_{t,t+k} MC_{t+k} P_{t+k}^{\varepsilon_p} \tilde{Y}_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k \Delta_{t,t+k} P_{t+k}^{\varepsilon_p - 1} \tilde{Y}_{t+k}} \quad (5.42)$$

where $\mu_p = \frac{\varepsilon_p}{\varepsilon_p - 1}$.

Note that in the case of perfectly flexible prices, i.e., $\theta_p = 0$, all intermediate goods producers choose the same optimal price $P_t(i) = P_t$ and produce the same amount $Y_t(i) = Y_t$. The optimality condition (5.41) will then read as

$$MC_t = \frac{1}{\mu_p} \quad (5.43)$$

The monopolistic supplier of good i will set his price P_t as a constant mark-up μ_p over nominal marginal costs given by $MC_t^n = P_t MC_t$. In the limiting case of perfect substitutability between different types of intermediate goods ($\varepsilon_p \rightarrow \infty$) the mark-up μ_p vanishes and the market outcome under perfect competition ($MC_t = 1$ respectively $P_t = MC_t^n$) is restored.

Dividing equation (5.42) by P_t and using the definition of the stochastic discount factor for real payoffs⁷

$$\Delta_{t,t+k} = \beta^k \frac{U_{C,t+k}}{U_{C,t}} = \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma}, \quad (5.44)$$

the price setting equation (5.42) can be rewritten as follows

$$\frac{P_t(i)}{P_t} = \frac{\Psi_t^p}{\Phi_t^p} \quad (5.45)$$

where

$$\Psi_t^p = \mu_p \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_p)^k C_{t+k}^{-\sigma} \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon_p} MC_{t+k} \tilde{Y}_{t+k} \quad (5.46)$$

$$\Phi_t^p = \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_p)^k C_{t+k}^{-\sigma} \left(\frac{P_{t+k}}{P_t} \right)^{\varepsilon_p - 1} \tilde{Y}_{t+k} \quad (5.47)$$

Note that $\frac{P_{t+k}}{P_t} = \frac{P_{t+1}}{P_t} \frac{P_{t+2}}{P_{t+1}} \dots \frac{P_{t+k}}{P_{t+k-1}} = \prod_{i=1}^k \pi_{t+i}^p$ is the gross price inflation rate between period t and $t+k$, and $\pi_t^p = P_t/P_{t-1}$.

⁷According to (5.14), the stochastic discount factor can also be written as a product of the short term real interest rates

$$\Delta_{t,t+k} = \beta^k \frac{U_{C,t+k}}{U_{C,t}} = \beta \underbrace{\frac{U_{C,t+1}}{U_{C,t}}}_{rr_t^{-1}} \cdot \beta \underbrace{\frac{U_{C,t+2}}{U_{C,t+1}}}_{rr_{t+1}^{-1}} \dots \beta \underbrace{\frac{U_{C,t+k}}{U_{C,t+k-1}}}_{rr_{t+k-1}^{-1}} = \prod_{i=0}^{k-1} rr_{t+i}^{-1}$$

where $rr_t = R_t P_t / P_{t+1}$ is the gross one-period real interest rate.

We again follow Schmitt-Grohé and Uribe (2004a) and rewrite (5.46) recursively to obtain

$$\Psi_t^p = \mu_p C_t^{-\sigma} M C_t \tilde{Y}_t + \mu_p E_t \sum_{k=0}^{\infty} (\beta \theta_p)^{k+1} C_{t+k+1}^{-\sigma} \left(\frac{P_{t+k+1}}{P_t} \right)^{\varepsilon_p} M C_{t+k+1} \tilde{Y}_{t+k+1} \quad (5.48)$$

Note that

$$\Psi_{t+1}^p = \mu_p E_{t+1} \sum_{k=0}^{\infty} (\beta \theta_p)^k C_{t+k+1}^{-\sigma} \left(\frac{P_{t+k+1}}{P_t} \right)^{\varepsilon_p} \left(\frac{P_t}{P_{t+1}} \right)^{\varepsilon_p} M C_{t+k+1} \tilde{Y}_{t+k+1} \quad (5.49)$$

and therefore

$$\Psi_t^p = \mu_p C_t^{-\sigma} M C_t \tilde{Y}_t + \beta \theta_p E_t \left((\pi_{t+1}^p)^{\varepsilon_p} \Psi_{t+1}^p \right) \quad (5.50)$$

Rewriting (5.47) in the same way yields

$$\Phi_t^p = C_t^{-\sigma} \tilde{Y}_t + \beta \theta_p E_t \left((\pi_{t+1}^p)^{\varepsilon_p - 1} \Phi_{t+1}^p \right) \quad (5.51)$$

5.2.5 The Government

The government collects lump-sum taxes from households to finance wage and oil price subsidies. The government budget constraint is given by

$$T_t^r = \tau_w \frac{W_t}{P_t} \tilde{N}_t + \tau_e P_t^e E n_t \quad (5.52)$$

where T_t^r are real lump-sum taxes, $\tilde{N}_t = \int_0^1 \tilde{N}_t(i) di$ and $E n_t = \int_0^1 E n_t(i) di$ are the aggregated factor demands for labor and energy, respectively. The subsidy rates τ_w and τ_e are optimally set to eliminate the distortion resulting from monopolistic competition in labor and product markets and to restore the efficiency of the flexible price equilibrium.

5.2.6 Aggregation and Market Clearing

We assume a symmetric equilibrium in which all intermediate goods firms and households are faced with the same price and wage setting problem, respectively. Thus, each firm which is allowed to change his price in period t will optimally set the same price $P_t(i)$. The mass of those firms is given by $1 - \theta_p$. The rest of the firms (θ_p) will leave their prices unchanged. As a consequence, the price index (5.5) evolves according to the following law of motion

$$P_t^{1-\varepsilon_p} = \theta_p P_{t-1}^{1-\varepsilon_p} + (1 - \theta_p) P_t(i)^{1-\varepsilon_p} \quad (5.53)$$

Dividing equation (5.53) by $P_t^{1-\varepsilon_p}$ yields

$$1 = \theta_p (\pi_t^p)^{\varepsilon_p - 1} + (1 - \theta_p) \left(\frac{P_t(i)}{P_t} \right)^{1-\varepsilon_p} \quad (5.54)$$

The wage index (5.10) can be rewritten in the same way to obtain

$$1 = \theta_w (\pi_t^w)^{\varepsilon_w - 1} + (1 - \theta_w) \left(\frac{W_t(j)}{W_t} \right)^{1-\varepsilon_w} \quad (5.55)$$

Note that wage and price inflation are connected by the identity

$$\frac{W_t}{P_t} = \frac{W_{t-1}}{P_{t-1}} \frac{\pi_t^w}{\pi_t^p} \quad (5.56)$$

The Labor Market. The market clearing condition for labor type j is given by

$$N_t(j) \equiv N_t^s(j) = N_t^d(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} \tilde{N}_t \quad (5.57)$$

where $\tilde{N}_t = \int_0^1 \tilde{N}_t(i) di$ is the aggregated labor demand of all intermediate goods producers which is equal to the supply of the labor bundler. By integrating (5.57) over all labor types j , we obtain

$$N_t \equiv \int_0^1 N_t(j) dj = \left(\int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} dj \right) \tilde{N}_t = s_t^w \tilde{N}_t \quad (5.58)$$

$s_t^w = \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} dj \geq 1$ is a measure of wage dispersion.⁸ Note that the inability of the typical household j to change its price $W_t(j)$ in every period drives a wedge between the aggregated amount of labor supplied N_t and the aggregated amount of labor used in production \tilde{N}_t . Note further that in the case of perfectly flexible wages ($\theta_w = 0$), each household sets the same wage $W_t(i) = W_t$ and thus, the inefficiency induced by wage dispersion vanishes ($s_t^w = 1$).

According to Schmitt-Grohé and Uribe (2004a), s_t^w can be rewritten as

$$\begin{aligned} s_t^w &= (1 - \theta_w) \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} + (1 - \theta_w)\theta_w \left(\frac{W_{t-1}(j)}{W_t} \right)^{-\varepsilon_w} \\ &+ (1 - \theta_w)\theta_w^2 \left(\frac{W_{t-2}(j)}{W_t} \right)^{-\varepsilon_w} + \dots = (1 - \theta_w) \sum_{i=0}^{\infty} \theta_w^i \left(\frac{W_{t-i}(j)}{W_t} \right)^{-\varepsilon_w} \end{aligned} \quad (5.59)$$

The infinite sum can be written recursively as

$$\begin{aligned} s_t^w &= (1 - \theta_w) \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} + (1 - \theta_w)\theta_w \sum_{i=0}^{\infty} \theta_w^i \left(\frac{W_{t-i-1}(j)}{W_t} \right)^{-\varepsilon_w} \\ &= (1 - \theta_w) \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} + \theta_w \left(\frac{W_{t-1}}{W_t} \right)^{-\varepsilon_w} s_{t-1}^w \end{aligned} \quad (5.60)$$

where

$$s_{t-1}^w = (1 - \theta_w) \sum_{i=0}^{\infty} \theta_w^i \left(\frac{W_{t-i-1}(j)}{W_{t-1}} \right)^{-\varepsilon_w}$$

By using the definition of the gross rate of wage inflation, $\pi_t^w = W_t/W_{t-1}$, we finally obtain⁹

$$s_t^w = (1 - \theta_w) \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} + \theta_w (\pi_t^w)^{\varepsilon_w} s_{t-1}^w \quad (5.61)$$

⁸Schmitt-Grohé and Uribe (2004a) show that the measure of wage dispersion is bounded below by 1. Their reasoning goes as follows: Define $x_t(j) = (W_t(j)/W_t)^{1-\varepsilon_w}$. By using the definition of the wage index given in equation (5.10), we can obtain $1 = \left(\int_0^1 x_t(j) dj \right)^{\varepsilon_w/(\varepsilon_w-1)}$ with $\varepsilon_w/(\varepsilon_w-1) > 1$. Furthermore, we can rewrite s_t^w to obtain $s_t^w = \int_0^1 x_t(j)^{\varepsilon_w/(\varepsilon_w-1)} dj$. Jensen's inequality then implies that $s_t^w = \int_0^1 x_t(j)^{\varepsilon_w/(\varepsilon_w-1)} dj \geq \left(\int_0^1 x_t(j) dj \right)^{\varepsilon_w/(\varepsilon_w-1)} = 1$.

⁹Note that a log-linearization of s_t^w around a zero inflation steady state, i.e. $\pi^w = 1$, yields $\hat{s}_t^w = \theta_w \hat{s}_{t-1}^w$. Hence, the measure of wage dispersion follows an autoregressive process which is independent of the dynamics of the remaining linearized system.

The Goods Market. The market clearing condition for intermediate good i is given by

$$Y_t(i) \equiv Y_t^s(i) = Y_t^d(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} \tilde{Y}_t \quad (5.62)$$

where \tilde{Y}_t is the supply of the final goods producers which is equal to the aggregate demand for final goods. By integrating (5.62) over all intermediate goods i , we obtain

$$Y_t \equiv \int_0^1 Y_t(i) di = \left(\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} di \right) \tilde{Y}_t = s_t^p \tilde{Y}_t \quad (5.63)$$

where $Y_t = \int_0^1 Y_t(i) di$ denotes the aggregate production of intermediate goods, and $s_t^p = \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} di \geq 1$ is a measure of price dispersion.¹⁰ The inability of the typical intermediate goods producer to change his price $P_t(i)$ in every period drives a wedge between aggregate production Y_t and aggregate demand \tilde{Y}_t . Note that in the case of perfectly flexible prices ($\theta_p = 0$), each firm charges the same price $P_t(i) = P_t$ and thus, the inefficiency induced by price dispersion vanishes ($s_t^p = 1$).

We can rewrite s_t^p in the same way as s_t^w to obtain

$$s_t^p = (1 - \theta_p) \sum_{j=0}^{\infty} \theta_p^j \left(\frac{P_{t-j}(i)}{P_t} \right)^{-\varepsilon_p} \quad (5.64)$$

or, equivalently,

$$s_t^p = (1 - \theta_p) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} + (1 - \theta_p) \theta_p \left(\frac{P_{t-1}}{P_t} \right)^{-\varepsilon_p} \sum_{j=0}^{\infty} \theta_p^j \left(\frac{P_{t-j-1}(i)}{P_{t-1}} \right)^{-\varepsilon_p} \quad (5.65)$$

Note that

$$s_{t-1}^p = (1 - \theta_p) \sum_{j=0}^{\infty} \theta_p^j \left(\frac{P_{t-j-1}(i)}{P_{t-1}} \right)^{-\varepsilon_p} \quad (5.66)$$

Thus, we finally end up with

$$s_t^p = (1 - \theta_p) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} + \theta_p (\pi_t^p)^{\varepsilon_p} s_{t-1}^p \quad (5.67)$$

where $\pi_t^p = P_t/P_{t-1}$.¹¹

The Aggregate Production Function and the Overall Resource Constraint. The aggregate production function reads as follows

$$Y_t = \left((1 - a) \tilde{N}_t^{1-\rho} + a E n_t^{1-\rho} \right)^{\frac{1}{1-\rho}} \quad (5.68)$$

¹⁰The proof that s_t^p is bounded below by 1 is similar to the proof of $s_t^w \geq 1$, given in footnote 8.

¹¹Note that a log-linearization of s_t^p around a zero inflation steady state, i.e. $\pi^p = 1$, yields $\hat{s}_t^p = \theta_p \hat{s}_{t-1}^p$. As the measure of wage dispersion, the measure of price dispersion follows an autoregressive process which is independent of the dynamics of the remaining linearized system.

where the aggregate factor inputs, $\tilde{N}_t = \int_0^1 \tilde{N}_t(i) di$ and $En_t = \int_0^1 En_t(i) di$, are given by

$$\tilde{N}_t = Y_t \left(\frac{(1-a)MC_t}{(W_t/P_t)(1-\tau_w)} \right)^{\frac{1}{\rho}} \quad (5.69)$$

$$En_t = Y_t \left(\frac{aMC_t}{P_t^e(1-\tau_e)} \right)^{\frac{1}{\rho}} \quad (5.70)$$

Integrating the budget constraint (5.12) over all households $j \in [0, 1]$ yields

$$\int_0^1 C_t(j) dj = \int_0^1 \left(\frac{W_t(j)}{P_t} N_t(j) - \frac{B_t(j) - R_{t-1}B_{t-1}(j)}{P_t} - T_t^r(j) + \Pi_t^r(j) \right) dj \quad (5.71)$$

By using $C_t(j) = C_t$, $T_t^r(j) = T_t^r$, $\Pi_t^r(j) = \Pi_t^r$, the bond market clearing condition $B_t = \int_0^1 B_t(j) dj = 0$, the labor demand function (5.9), and the definition of the wage index (5.10), we obtain¹²

$$C_t = \frac{W_t}{P_t} \tilde{N}_t - T_t^r + \Pi_t^r \quad (5.72)$$

The aggregated real profits of all intermediate goods producers are given by

$$\Pi_t^r = \int_0^1 \left(\frac{P_t(i)}{P_t} Y_t(i) - P_t^e(1-\tau_e)En_t(i) - w_t(1-\tau_w)\tilde{N}_t(i) \right) di \quad (5.73)$$

By inserting the demand function (5.4) and using the definition of the price index (5.5), we obtain

$$\Pi_t^r = \tilde{Y}_t - P_t^e(1-\tau_e)En_t - \frac{W_t}{P_t}(1-\tau_w)\tilde{N}_t \quad (5.74)$$

Combining the aggregated budget constraint of households (5.72) with the government budget constraint (5.52) and the aggregated profits of intermediate goods producers (5.74) yields aggregate absorption

$$\tilde{Y}_t = C_t + P_t^e En_t \quad (5.75)$$

Equation (5.75) shows that the usage of oil from exogenous sources limits the consumption possibilities of the economy.¹³

The overall resource constraint of the oil-dependent economy can be obtained by combining (5.75) and (5.63):

$$Y_t = s_t^p(C_t + P_t^e En_t) \quad (5.76)$$

The overall baseline model for an oil-dependent economy with staggered price and wage setting is summarized in Table 5.1.

¹²Since we assume a mass of households normalized on the unit interval, per capita consumption $C_t(j) = C_t$ is equal to aggregated consumption C_t . The same is true for all per capita terms which are assumed to be equal for all households j .

¹³Alternatively, one can assume that energy is owned by the government which transfers the resulting profits to the households. In this case, we have $\tilde{Y}_t = C_t$. This assumption is used by, for example, Castillo, Montoro, and Tuesta (2007) or Montoro (2007).

Table 5.1: The baseline model at a glance

Euler equation	$C_t^{-\sigma} = \beta C_{t+1}^{-\sigma} R_t (\pi_{t+1}^p)^{-1}$
Production function	$Y_t = \left((1-a) \left(\frac{N_t}{s_t^w} \right)^{1-\rho} + a E n_t^{1-\rho} \right)^{1/(1-\rho)}$
Factor inputs	$E n_t = Y_t \left(\frac{a M C_t}{P_t^e (1-\tau_e)} \right)^{1/\rho}$ $N_t = s_t^w Y_t \left(\frac{(1-a) M C_t}{(W_t/P_t)(1-\tau_w)} \right)^{1/\rho}$
Price Phillips curve	$1 = \theta_p (\pi_t^p)^{\varepsilon_p - 1} + (1 - \theta_p) \left(\frac{P_t(i)}{P_t} \right)^{1-\varepsilon_p}$ $\frac{P_t(i)}{P_t} = \frac{\Psi_t^p}{\Phi_t^p}$ $\Psi_t^p = \mu_p C_t^{-\sigma} M C_t \frac{Y_t}{s_t^w} + \beta \theta_p E_t \left((\pi_{t+1}^p)^{\varepsilon_p} \Psi_{t+1}^p \right)$ $\Phi_t^p = C_t^{-\sigma} \frac{Y_t}{s_t^w} + \beta \theta_p E_t \left((\pi_{t+1}^p)^{\varepsilon_p - 1} \Phi_{t+1}^p \right)$
Wage Phillips curve	$1 = \theta_w (\pi_t^w)^{\varepsilon_w - 1} + (1 - \theta_w) \left(\frac{W_t(j)}{W_t} \right)^{1-\varepsilon_w}$ $\left(\frac{W_t(j)}{W_t} \right)^{1+\varepsilon_w \eta} = \frac{\Psi_t^w}{\Phi_t^w}$ $\Psi_t^w = \mu_w \left(\frac{N_t}{s_t^w} \right)^{1+\eta} + \beta \theta_w E_t \left((\pi_{t+1}^w)^{\varepsilon_w (1+\eta)} \Psi_{t+1}^w \right)$ $\Phi_t^w = C_t^{-\sigma} \frac{W_t}{P_t} \frac{N_t}{s_t^w} + \beta \theta_w E_t \left((\pi_{t+1}^w)^{\varepsilon_w - 1} \Phi_{t+1}^w \right)$
Price dispersion	$s_t^p = (1 - \theta_p) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} + \theta_p (\pi_t^p)^{\varepsilon_p} s_{t-1}^p$
Wage dispersion	$s_t^w = (1 - \theta_w) \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} + \theta_w (\pi_t^w)^{\varepsilon_w} s_{t-1}^w$
Resource constraint	$Y_t = s_t^p (C_t + P_t^e E n_t)$
Real wage	$\frac{W_t}{P_t} = \frac{W_{t-1}}{P_{t-1}} \frac{\pi_t^w}{\pi_t^p}$
Real energy price	$\frac{P_t^e}{P^e} = \left(\frac{P_{t-1}^e}{P^e} \right)^{\phi_e} \exp(e_t)$

5.2.7 The Real Business Cycle Core

The real business cycle core of the oil-dependent economy with monopolistic competition on labor and product markets can be obtained by assuming that prices and wages are perfectly flexible. In this case, the heterogeneity of individual prices, wages, and quantities vanishes and we obtain

$$\frac{P_t(i)}{P_t} = \frac{W_t(j)}{W_t} = s_t^p = s_t^w = 1 \quad (5.77)$$

and $N_t(i) = N_t$, $E n_t(i) = E n_t$, $Y_t(i) = Y_t$, $N_t(j) = N_t$, where we make use of market clearing conditions as well as of equations $\tilde{N}_t = N_t$ and $\tilde{Y}_t = Y_t$ which obviously follow from (5.77). By inserting equations (5.16) and (5.17) into the wage setting rule (5.25), we obtain

$$\frac{W_t}{P_t} = \mu_w C_t^\sigma N_t^\eta \quad (5.78)$$

Inserting equation (5.78) and the price setting equation (5.43) into the cost minimization condition (5.35) yields

$$N_t = Y_t \left(\frac{(1-a)}{\mu_p \mu_w (1-\tau_w)} \right)^{\frac{1}{\rho}} (N_t^\eta C_t^\sigma)^{-\frac{1}{\rho}} \quad (5.79)$$

By inserting the overall resource constraint (5.76), we obtain

$$N_t = Y_t^{\frac{1}{\rho}} \left(\frac{(1-a)}{\mu_p \mu_w (1-\tau_w)} \right)^{\frac{1}{\rho}} (N_t^\eta (Y_t - P_t^e E n_t)^\sigma)^{-\frac{1}{\rho}} \quad (5.80)$$

The natural counterpart of the cost minimization condition (5.36) reads as follows

$$E n_t = Y_t \left(\frac{a}{\mu_p (1-\tau_e) P_t^e} \right)^{\frac{1}{\rho}} \quad (5.81)$$

Substituting the latter equation into (5.80) finally yields

$$N_t = Y_t^{\frac{\rho-\sigma}{\rho+\eta}} \left(\frac{(1-a)}{\mu_p \mu_w (1-\tau_w)} \right)^{\frac{1}{\rho+\eta}} \left(1 - (P_t^e)^{\frac{\rho-1}{\rho}} \left(\frac{a}{\mu_p (1-\tau_e)} \right)^{\frac{1}{\rho}} \right)^{-\frac{\sigma}{\rho+\eta}} \quad (5.82)$$

The natural level of output Y_t^n can be obtained by inserting equation (5.81) and (5.82) into the production function (5.33). After some manipulations, we end up with

$$Y_t^n = \left[\frac{1 - a^{\frac{1}{\rho}} \left(\frac{1}{\mu_p (1-\tau_e)} \right)^{\frac{1-\rho}{\rho}} (P_t^e)^{\frac{\rho-1}{\rho}}}{(1-a)^{\frac{1+\eta}{\rho+\eta}} \left(\frac{1}{\mu_p \mu_w (1-\tau_w)} \right)^{\frac{1-\rho}{\rho+\eta}} \left(1 - \left(\frac{a}{\mu_p (1-\tau_e)} \right)^{\frac{1}{\rho}} (P_t^e)^{\frac{\rho-1}{\rho}} \right)^{\frac{(\rho-1)\sigma}{\rho+\eta}}} \right]^{\frac{\rho+\eta}{(\rho-1)(\sigma+\eta)}} \quad (5.83)$$

Two points are worth mentioning here: First, the natural level of output depends on the exogenously given real price of energy. Thus, a persistent change in this price leads to a persistent change in the natural level of output. This feature distinguishes energy price shocks from usual cost-push shocks like wage or price mark-up shocks where the natural level of output is defined as the level of output under perfectly flexible prices and wages and constant price and wage mark-ups.

Second, the natural level of output Y_t^n differs from the efficient level of output due to the existence of constant wage and price mark-ups $\mu_p > 1$ and $\mu_w > 1$.¹⁴ Efficiency of the natural level of output can be achieved by choosing the subsidy rates τ_e and τ_w such that

$$\mu_p \mu_w (1-\tau_w) = 1, \quad \mu_p (1-\tau_e) = 1 \quad (5.84)$$

It then follows that the optimal subsidy rates are given by

$$\tau_w = 1 - \frac{1}{\mu_p \mu_w}, \quad \tau_e = 1 - \frac{1}{\mu_p} \quad (5.85)$$

The natural level of energy and labor can be computed by inserting equation (5.83) into (5.81) and (5.82), respectively. Similarly, natural consumption can be obtained by combining (5.81) with the overall resource constraint (5.76). Finally, we can solve the consumption Euler equation (5.18) for the natural real interest rate.

¹⁴Note that an efficient market outcome without subsidies requires that $\varepsilon_p \rightarrow \infty$, $\varepsilon_w \rightarrow \infty$, and thus $\mu_w = \mu_p = 1$.

5.2.8 The Steady State

We calculate the steady state of the model by omitting the time indices t . We follow the widely used assumption of a zero inflation steady state which implies a steady state gross rate of price inflation $\pi^p = 1$. According to the identity (5.56), the steady state gross rate of wage inflation is then given by $\pi^w = 1$.

In the steady state, the equations describing wage and price setting read as follows

$$1 - \theta_p(\pi_p)^{\varepsilon_p - 1} = (1 - \theta_p) \left(\frac{P(i)}{P} \right)^{1 - \varepsilon_p} \quad (5.86)$$

$$1 - \theta_w(\pi_w)^{\varepsilon_w - 1} = (1 - \theta_w) \left(\frac{W(j)}{W} \right)^{1 - \varepsilon_w} \quad (5.87)$$

$$\frac{P(i)}{P} = \frac{\Psi_p}{\Phi_p} \quad (5.88)$$

$$\left(\frac{W(i)}{W} \right)^{1 + \varepsilon_w \eta} = \frac{\Psi_w}{\Phi_w} \quad (5.89)$$

$$\Psi^w = \frac{\mu_w \tilde{N}^{1 + \eta}}{1 - \beta \theta_w (\pi^w)^{\varepsilon_w (1 + \eta)}} \quad (5.90)$$

$$\Phi^w = \frac{C_t^{-\sigma} w_t \tilde{N}}{1 - \beta \theta_w (\pi^w)^{\varepsilon_w - 1}} \quad (5.91)$$

$$\Psi^p = \frac{\mu_p C^{-\sigma} M C \tilde{Y}}{1 - \beta \theta_p (\pi^p)^{\varepsilon_p}} \quad (5.92)$$

$$\Phi^p = \frac{C_t^{-\sigma} \tilde{Y}}{1 - \beta \theta_w (\pi^p)^{\varepsilon_p - 1}} \quad (5.93)$$

Inserting $\pi^p = 1$ and $\pi^w = 1$ into (5.86) and (5.87), respectively, yields $P(i) = P$ and $W(j) = W$.

The steady state equations for price and wage dispersion are given by

$$s_p = \frac{(1 - \theta_p) \left(\frac{P(i)}{P} \right)^{-\varepsilon_p}}{1 - \theta_p (\pi^p)^{\varepsilon_p}} \quad (5.94)$$

$$s_w = \frac{(1 - \theta_w) \left(\frac{W(j)}{W} \right)^{-\varepsilon_w}}{1 - \theta_w (\pi^w)^{\varepsilon_w}} \quad (5.95)$$

Using $\pi^p = \pi^w = W(j)/W = P(i)/P = 1$, it is obvious that $s_p = s_w = 1$. Then, (5.58) and (5.63) become $N = \tilde{N}$ and $Y = \tilde{Y}$, respectively.

Under zero inflation, the gross real interest rate and the gross nominal interest rate coincide and can be obtained by using the steady state Euler equation

$$C^{-\sigma} = \beta C^{-\sigma} R (\pi^p)^{-1} \quad (5.96)$$

Hence, we have $R = rr = 1/\beta$.

Inserting (5.92) and (5.93) into (5.88) yields

$$MC = \frac{1}{\mu_p} \quad (5.97)$$

Similarly, we can combine (5.90), (5.91), and (5.89) to obtain

$$\frac{W}{P} = \mu_w C^\sigma N^\eta \quad (5.98)$$

By inserting the wage setting condition (5.98) and the price setting condition (5.97) into (5.69) and (5.70), we obtain

$$En = Y \left(\frac{a}{\mu_p(1-\tau_e)P^e} \right)^{\frac{1}{\rho}} \quad (5.99)$$

$$N = Y \left(\frac{(1-a)}{\mu_p\mu_w(1-\tau_w)} \right)^{\frac{1}{\rho}} (N^\eta C^\sigma)^{-\frac{1}{\rho}} \quad (5.100)$$

We can combine the overall resource constraint (5.76) and equation (5.100), to obtain

$$N = Y^{\frac{1}{\rho}} \left(\frac{(1-a)}{\mu_p\mu_w(1-\tau_w)} \right)^{\frac{1}{\rho}} (N^\eta (Y - P^e En)^\sigma)^{-\frac{1}{\rho}} \quad (5.101)$$

Now, we plug equation (5.99) into (5.101) and after some manipulations we end up with

$$N = Y^{\frac{\rho-\sigma}{\rho+\eta}} \left(\frac{(1-a)}{\mu_p\mu_w(1-\tau_w)} \right)^{\frac{1}{\rho+\eta}} \left(1 - (P^e)^{\frac{\rho-1}{\rho}} \left(\frac{a}{\mu_p(1-\tau_e)} \right)^{\frac{1}{\rho}} \right)^{-\frac{\sigma}{\rho+\eta}} \quad (5.102)$$

The steady state output Y can be obtained by inserting equations (5.102) and (5.99) into the aggregate production function (5.68) which yields

$$Y = \left[\frac{1 - a^{\frac{1}{\rho}} \left(\frac{1}{\mu_p(1-\tau_e)} \right)^{\frac{1-\rho}{\rho}} (P^e)^{\frac{\rho-1}{\rho}}}{(1-a)^{\frac{1+\eta}{\rho+\eta}} \left(\frac{1}{\mu_p\mu_w(1-\tau_w)} \right)^{\frac{1-\rho}{\rho+\eta}} \left(1 - \left(\frac{a}{\mu_p(1-\tau_e)} \right)^{\frac{1}{\rho}} (P^e)^{\frac{\rho-1}{\rho}} \right)^{\frac{(\rho-1)\sigma}{\rho+\eta}}} \right]^{\frac{\rho+\eta}{(\rho-1)(\sigma+\eta)}} \quad (5.103)$$

Three points are worth mentioning here: First, the solution of the steady state is exactly equal to the solution of the real business cycle core besides the fact that in the model with perfectly flexible prices and wages, the real price of energy is described by the autoregressive process (5.34). Second, the steady state level of output depends ambiguously on the steady real price of energy. Third, optimal subsidies as introduced in (5.85) eliminate the inefficiencies caused by monopolistic product and labor markets and, thus, make the steady state level of output efficient.

The steady state level of energy and labor can easily be computed by inserting equation (5.103) into (5.99) and (5.102), respectively. Similarly, steady state consumption can be obtained by combining (5.99) with the overall resource constraint (5.76).

Note that the dependency of steady state output on the real price of energy is unambiguous when we take the simplifying assumption that energy is owned by the government which transfers the resulting profits to the households. The resource constraint can then be reduced to $Y = C$. By inserting the modified resource constraint into (5.101), we obtain

$$N = Y^{\frac{\rho-\sigma}{\rho+\eta}} \left(\frac{(1-a)}{\mu_p\mu_w(1-\tau_w)} \right)^{\frac{1}{\rho}} \quad (5.104)$$

As shown above, the steady state output Y can be obtained by inserting the steady state equations for labor (5.104) and energy (5.99) into the steady state version of the aggregate production function (5.68) which yields

$$Y = \left[\frac{1 - a^{\frac{1}{\rho}} \left(\frac{1}{\mu_p(1-\tau_e)} \right)^{\frac{1-\rho}{\rho}} (P^e)^{\frac{\rho-1}{\rho}}}{(1-a)^{\frac{1+\eta}{\rho+\eta}} \left(\frac{1}{\mu_p\mu_w(1-\tau_w)} \right)^{\frac{1-\rho}{\rho+\eta}}} \right]^{\frac{\rho+\eta}{(\rho-1)(\sigma+\eta)}} \quad (5.105)$$

In the case of the modified resource constraint $Y = C$, an increase in the steady state value of the real price of energy leads unambiguously to a steady state output decline. However, the employment effect of a rising real energy price is ambiguous.¹⁵ On the one hand, firms will employ more labor in production due to the substitution effect while on the other hand, overall production will decline which leads to a decreasing demand for both energy and labor. Equation (5.104) indicates that for $\rho > \sigma$, the overall production effect dominates and the amount of hours worked decreases with increasing real energy prices. For $\sigma > \rho$, the substitution effect dominates the overall production effect and employment rises.

In our model, a further mechanism is at work. The steady state resource constraint $Y = C + P^e E n$ implies, in isolation, a positive correlation between the real energy price P^e and output Y . The rationale is that a rise in the real energy bill due to the hike in the real energy price must be financed by an increase in production. As a consequence, output depends ambiguously on the real price of energy. This can also be seen by inspecting equation (5.103), where, in contrast to equation (5.105), the real price of energy shows up in the denominator and numerator.¹⁶

5.2.9 The Welfare Measure

We follow Schmitt-Grohé and Uribe (2004a) or Faia and Monacelli (2007) and measure absolute welfare by the expectation of the discounted sum of aggregate lifetime utility conditional on the set of information at time equal to zero, that is

$$E_0 \int_0^1 \sum_{t=0}^{\infty} \beta^t U(C_t(j), N_t(j)) dj = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad (5.106)$$

where we make use of the fact that $C_t = C_t(j)$ and $N_t = \int_0^1 N_t(j) dj$. By inserting (5.15), the equation for the per-period utility function, we finally obtain the following absolute welfare measure

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right) \quad (5.107)$$

The steady state level of welfare is then given by

$$V^a = \frac{1}{1-\beta} \left(\frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\eta}}{1+\eta} \right) \quad (5.108)$$

¹⁵This is a well-known result. See, for example, Bhandari and Turnovsky (1984).

¹⁶Note again that exactly the same considerations hold for the natural model.

We can define a relative measure of welfare as the percentage deviation of absolute welfare from its steady state value, formally

$$V_0^r = \frac{(V_0^a - V^a)}{|V^a|} \cdot 100 \quad (5.109)$$

It is worth noting that in our approach V_0^a is identified as both the policy makers' objective function and the overall measure of welfare. Hence, we depart from the widely applied practice of measuring overall welfare by the unconditional expectations of the policy makers' objective function.¹⁷ Recently, Damjanovic, Damjanovic, and Nolan (2008) propose a method for deriving optimal monetary policy based on the optimization of unconditional expectations of the criterion function and thus, as we do, merge the objective function of the monetary authority with the overall measure of welfare.¹⁸

5.3 Monetary Policy

We consider two different monetary policy regimes. First, we derive the optimal monetary policy under commitment. In doing so, we distinguish between the policy under time-zero commitment and the policy under timeless-perspective commitment. Second, we assume that the monetary authority commits itself to a simple instrument rule. We apply an interest rate peg and several types of the interest rate rule originally proposed by Taylor (1993).

For the determination of optimal monetary policy, we use the Ramsey approach described in Khan, King, and Wolman (2003). In this approach, the monetary authority seeks to maximize welfare, as defined in equation (5.107), given the set of competitive equilibrium conditions and exogenous processes summarized in Table 5.1. We assume that the monetary authority is able to commit to the policy plan determined at time $t = 0$. The maximization problem of a monetary authority is then of the form

$$\begin{aligned} \mathcal{L}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t & \left\{ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} + \lambda_{1,t+1} (C_t^{-\sigma} - \beta C_{t+1}^{-\sigma} R_t (\pi_{t+1}^p)^{-1}) \right. \\ & + \lambda_{2,t+1} \left(s_t^p \tilde{Y}_t - \left((1-a) \tilde{N}_t^{1-\rho} + a E n_t^{1-\rho} \right)^{\frac{1}{1-\rho}} \right) \\ & + \lambda_{3,t+1} \left(\tilde{Y}_t - P_t^e E n_t - C_t \right) + \lambda_{4,t+1} \left(N_t - s_t^w \tilde{N}_t \right) \\ & + \lambda_{5,t+1} \left(\tilde{N}_t - s_t^p \tilde{Y}_t \left(\frac{(1-a) M C_t}{(W_t/P_t)(1-\tau_w)} \right)^{\frac{1}{\rho}} \right) \\ & + \lambda_{6,t+1} \left(E n_t - s_t^p \tilde{Y}_t \left(\frac{a M C_t}{P_t^e (1-\tau_e)} \right)^{\frac{1}{\rho}} \right) \\ & \left. + \lambda_{7,t+1} \left(s_t^p - (1-\theta_p) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} - \theta_p (\pi_t^p)^{\varepsilon_p} s_{t-1}^p \right) \right\} \quad (5.110) \end{aligned}$$

¹⁷See, among others, Amato and Laubach (2004), Jensen and McCallum (2002), McCallum and Nelson (2004), Schmitt-Grohé and Uribe (2007a), Steinsson (2003), Walsh (2003a), Woodford (1999a). In our model, the unconditional expectations of the policy makers' objective function formally reads as $\mathbb{E}(V_0^a)$ where \mathbb{E} denotes the unconditional expectations operator.

¹⁸Blake (2001) and Jensen and McCallum (2002) also suggest to optimize unconditional expectations of the criterion function but are not able to derive an analytical solution method.

$$\begin{aligned}
& + \lambda_{8,t+1} \left(s_t^w - (1 - \theta_w) \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} - \theta_w (\pi_t^w)^{\varepsilon_w} s_{t-1}^w \right) \\
& + \lambda_{9,t+1} \left(1 - \theta_p (\pi_t^p)^{\varepsilon_p - 1} - (1 - \theta_p) \left(\frac{P_t(i)}{P_t} \right)^{1 - \varepsilon_p} \right) \\
& + \lambda_{10,t+1} \left(1 - \theta_w (\pi_t^w)^{\varepsilon_w - 1} - (1 - \theta_w) \left(\frac{W_t(j)}{W_t} \right)^{1 - \varepsilon_w} \right) \\
& + \lambda_{11,t+1} \left(\left(\frac{W_t(j)}{W_t} \right)^{1 + \varepsilon_w \eta} - \frac{\Psi_t^w}{\Phi_t^w} \right) + \lambda_{12,t+1} \left(\frac{P_t(i)}{P_t} - \frac{\Psi_t^p}{\Phi_t^p} \right) \\
& + \lambda_{13,t+1} \left(\Psi_t^w - \tilde{N}_t^{1 + \eta} - \beta \theta_w \mathbb{E}_t (\pi_{t+1}^w)^{\varepsilon_w (1 + \eta)} \Psi_{t+1}^w \right) \\
& + \lambda_{14,t+1} \left(\Phi_t^w - C_t^{-\sigma} (W_t/P_t) \tilde{N}_t - \beta \theta_w \mathbb{E}_t (\pi_{t+1}^w)^{\varepsilon_w - 1} \Phi_{t+1}^w \right) \\
& + \lambda_{15,t+1} \left(\Psi_t^p - C_t^{-\sigma} M C_t \tilde{Y}_t - \beta \theta_p \mathbb{E}_t (\pi_{t+1}^p)^{\varepsilon_p} \Psi_{t+1}^p \right) \\
& + \lambda_{16,t+1} \left(\Phi_t^p - C_t^{-\sigma} \tilde{Y}_t - \beta \theta_p \mathbb{E}_t (\pi_{t+1}^p)^{\varepsilon_p - 1} \Phi_{t+1}^p \right) \\
& + \lambda_{17,t+1} \left(\frac{W_t}{P_t} - \frac{W_{t-1}}{P_{t-1}} \frac{\pi_t^w}{\pi_t^p} \right) \Big\}
\end{aligned}$$

where \mathcal{L}_0 denotes the Lagrangian and where make use of the fact that $Y_t = s_t^p \tilde{Y}_t$ and $N_t = s_t^w \tilde{N}_t$. The partial derivative of \mathcal{L}_0 with respect to the monetary instrument R_t is given by

$$\frac{\partial \mathcal{L}_0}{\partial R_t} = \beta^{t+1} \lambda_{1,t+1} C_{t+1}^{-\sigma} (\pi_{t+1}^p)^{-1} = 0 \quad (5.111)$$

It follows that $\lambda_{1,t} = 0$ for all t , so that the consumption Euler equation is not a binding restriction for the Ramsey problem. The remaining set of first-order conditions is given by

$$\begin{aligned} \frac{\partial \mathcal{L}_0}{\partial C_t} = \beta^t & \left[C_t^{-\sigma} - \lambda_{3,t+1} + \lambda_{14,t+1} \sigma C_t^{-\sigma - 1} \frac{W_t}{P_t} \tilde{N}_t \right. \\ & \left. + \lambda_{15,t+1} \sigma C_t^{-\sigma - 1} M C_t \tilde{Y}_t + \lambda_{16,t+1} \sigma C_t^{-\sigma - 1} \tilde{Y}_t \right] = 0 \end{aligned} \quad (5.112)$$

$$\frac{\partial \mathcal{L}_0}{\partial N_t} = \beta^t [-N_t^\eta + \lambda_{4,t+1}] = 0 \quad (5.113)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_0}{\partial \tilde{Y}_t} = \beta^t & \left[\lambda_{2,t+1} s_t^p + \lambda_{3,t+1} - \lambda_{5,t+1} \frac{\tilde{N}_t}{\tilde{Y}_t} - \lambda_{6,t+1} \frac{E n_t}{\tilde{Y}_t} \right. \\ & \left. - \lambda_{15,t+1} C_t^{-\sigma} M C_t - \lambda_{16,t+1} C_t^{-\sigma} \right] = 0 \end{aligned} \quad (5.114)$$

$$\begin{aligned} \frac{\partial \mathcal{L}_0}{\partial \left(\frac{W_t}{P_t} \right)} = \beta^t & \left[\lambda_{5,t+1} \frac{\tilde{N}_t}{\rho(W_t/P_t)} - \lambda_{14,t+1} C_t^{-\sigma} \tilde{N}_t \right. \\ & \left. + \lambda_{17,t+1} - \beta \lambda_{17,t+2} \frac{\pi_{t+1}^w}{\pi_{t+1}^p} \right] = 0 \end{aligned} \quad (5.115)$$

$$\frac{\partial \mathcal{L}_0}{\partial \tilde{N}_t} = \beta^t \left[-\lambda_{2,t+1}(1-a) \left(\frac{s_t^p \tilde{Y}_t}{\tilde{N}_t} \right)^\rho - \lambda_{4,t+1} s_t^w + \lambda_{5,t+1} \right. \quad (5.116)$$

$$\left. - \lambda_{13,t+1}(1+\eta) \tilde{N}_t^\eta - \lambda_{14,t+1} C_t^{-\sigma} \frac{W_t}{P_t} \right] = 0$$

$$\frac{\partial \mathcal{L}_0}{\partial En_t} = \beta^t \left[-\lambda_{2,t+1} a \left(\frac{s_t^p \tilde{Y}_t}{En_t} \right)^\rho - \lambda_{3,t+1} P_t^e + \lambda_{6,t+1} \right] = 0 \quad (5.117)$$

$$\frac{\partial \mathcal{L}_0}{\partial MC_t} = \beta^t \left[-\lambda_{5,t+1} \frac{\tilde{N}_t}{\rho MC_t} - \lambda_{6,t+1} \frac{En_t}{\rho MC_t} - \lambda_{15,t+1} C_t^{-\sigma} \tilde{Y}_t \right] = 0 \quad (5.118)$$

$$\frac{\partial \mathcal{L}_0}{\partial \left(\frac{P_t(i)}{P_t} \right)} = \beta^t \left[\lambda_{7,t+1}(1-\theta_p) \varepsilon_p \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p-1} \right. \quad (5.119)$$

$$\left. - \lambda_{9,t+1}(1-\theta_p)(1-\varepsilon_p) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} + \lambda_{12,t+1} \right] = 0$$

$$\frac{\partial \mathcal{L}_0}{\partial \left(\frac{W_t(j)}{W_t} \right)} = \beta^t \left[\lambda_{8,t+1}(1-\theta_w) \varepsilon_w \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w-1} \right. \quad (5.120)$$

$$\left. - \lambda_{10,t+1}(1-\theta_w)(1-\varepsilon_w) \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} \right. \\ \left. + \lambda_{11,t+1}(1+\varepsilon_w \eta) \left(\frac{W_t(j)}{W_t} \right)^{\varepsilon_w \eta} \right] = 0$$

$$\frac{\partial \mathcal{L}_0}{\partial s_t^p} = \beta^t \left[\lambda_{2,t+1} \tilde{Y}_t - \lambda_{5,t+1} \frac{\tilde{N}_t}{s_t^p} - \lambda_{6,t+1} \frac{En_t}{s_t^p} \right. \quad (5.121)$$

$$\left. + \lambda_{7,t+1} - \lambda_{7,t+2} \beta \theta_p (\pi_{t+1}^p)^{\varepsilon_p} \right] = 0$$

$$\frac{\partial \mathcal{L}_0}{\partial s_t^w} = \beta^t \left[-\lambda_{4,t+1} \tilde{N}_t + \lambda_{8,t+1} - \lambda_{8,t+2} \beta \theta_w (\pi_{t+1}^w)^{\varepsilon_w} \right] = 0 \quad (5.122)$$

$$\frac{\partial \mathcal{L}_0}{\partial \pi_t^p} = \beta^t \left[-\lambda_{7,t+1} \theta_p \varepsilon_p (\pi_t^p)^{\varepsilon_p-1} s_{t-1}^p - \lambda_{9,t+1} \theta_p (\varepsilon_p - 1) (\pi_t^p)^{\varepsilon_p-2} \right. \quad (5.123)$$

$$\left. - \lambda_{15,t} \theta_p \varepsilon_p (\pi_t^p)^{\varepsilon_p-1} \Psi_t^p - \lambda_{16,t} \theta_p (\varepsilon_p - 1) (\pi_t^p)^{\varepsilon_p-2} \Phi_t^p \right. \\ \left. + \lambda_{17,t+1} \frac{W_t}{P_t} \frac{1}{\pi_t^p} \right] = 0$$

$$\frac{\partial \mathcal{L}_0}{\partial \pi_t^w} = \beta^t \left[-\lambda_{8,t+1} \theta_w \varepsilon_w (\pi_t^w)^{\varepsilon_w-1} s_{t-1}^w - \lambda_{10,t+1} \theta_w (\varepsilon_w - 1) (\pi_t^w)^{\varepsilon_w-2} \right. \quad (5.124)$$

$$\left. - \lambda_{13,t} \theta_w \varepsilon_w (1+\eta) (\pi_t^w)^{\varepsilon_w(1+\eta)-1} \Psi_t^w \right. \\ \left. - \lambda_{14,t} \theta_w (\varepsilon_w - 1) (\pi_t^w)^{\varepsilon_w-2} \Phi_t^w - \lambda_{17,t+1} \frac{W_t}{P_t} \frac{1}{\pi_t^w} \right] = 0$$

$$\frac{\partial \mathcal{L}_0}{\partial \Phi_t^w} = \beta^t \left[\lambda_{11,t+1} \frac{\Psi_t^w}{(\Phi_t^w)^2} + \lambda_{14,t+1} - \lambda_{14,t} \theta_w (\pi_t^w)^{\varepsilon_w-1} \right] = 0 \quad (5.125)$$

$$\frac{\partial \mathcal{L}_0}{\partial \Phi_t^p} = \beta^t \left[\lambda_{12,t+1} \frac{\Psi_t^p}{(\Phi_t^p)^2} + \lambda_{16,t+1} - \lambda_{16,t} \theta_p (\pi_t^p)^{\varepsilon_p-1} \right] = 0 \quad (5.126)$$

$$\frac{\partial \mathcal{L}_0}{\partial \Psi_t^w} = \beta^t \left[-\lambda_{11,t+1} \frac{1}{\Phi_t^w} + \lambda_{13,t+1} - \lambda_{13,t} \theta_w (\pi_t^w)^{\varepsilon_w(1+\eta)} \right] = 0 \quad (5.127)$$

$$\frac{\partial \mathcal{L}_0}{\partial \Psi_t^p} = \beta^t \left[-\lambda_{12,t+1} \frac{1}{\Phi_t^p} + \lambda_{15,t+1} - \lambda_{15,t} \theta_p (\pi_t^p)^{\varepsilon_p} \right] = 0 \quad (5.128)$$

For the sake of comparability with the literature, we redefine the costate variables as $\lambda_{i,t+1} = \tilde{\lambda}_{i,t}$ ($i = 2, \dots, 17$). Then, the set of first-order conditions (5.112) to (5.128) reads as follows

$$0 = C_t^{-\sigma} - \tilde{\lambda}_{3,t} + \sigma C_t^{-\sigma-1} \left(\tilde{\lambda}_{14,t} \frac{W_t}{P_t} \tilde{N}_t + \tilde{\lambda}_{15,t} M C_t \tilde{Y}_t + \tilde{\lambda}_{16,t} \tilde{Y}_t \right) \quad (5.129)$$

$$0 = -N_t^\eta + \tilde{\lambda}_{4,t} \quad (5.130)$$

$$0 = \tilde{\lambda}_{2,t} s_t^p + \tilde{\lambda}_{3,t} - \tilde{\lambda}_{5,t} \frac{\tilde{N}_t}{\tilde{Y}_t} - \tilde{\lambda}_{6,t} \frac{E n_t}{\tilde{Y}_t} - C_t^{-\sigma} \left(\tilde{\lambda}_{15,t} M C_t + \tilde{\lambda}_{16,t} \right) \quad (5.131)$$

$$0 = \tilde{\lambda}_{5,t} \frac{\tilde{N}_t}{\rho(W_t/P_t)} - \tilde{\lambda}_{14,t} C_t^{-\sigma} \tilde{N}_t + \tilde{\lambda}_{17,t} - \beta \text{Et} \left\{ \tilde{\lambda}_{17,t+1} \frac{\pi_{t+1}^w}{\pi_{t+1}^p} \right\} \quad (5.132)$$

$$0 = -\tilde{\lambda}_{2,t} (1-a) \left(\frac{s_t^p \tilde{Y}_t}{\tilde{N}_t} \right)^\rho - \tilde{\lambda}_{4,t} s_t^w + \tilde{\lambda}_{5,t} - \tilde{\lambda}_{13,t} (1+\eta) \tilde{N}_t^\eta - \tilde{\lambda}_{14,t} C_t^{-\sigma} \frac{W_t}{P_t} \quad (5.133)$$

$$0 = -\tilde{\lambda}_{2,t} a \left(\frac{s_t^p \tilde{Y}_t}{E n_t} \right)^\rho - \tilde{\lambda}_{3,t} P_t^e + \tilde{\lambda}_{6,t} \quad (5.134)$$

$$0 = -\tilde{\lambda}_{5,t} \frac{\tilde{N}_t}{\rho M C_t} - \tilde{\lambda}_{6,t} \frac{E n_t}{\rho M C_t} - \tilde{\lambda}_{15,t} C_t^{-\sigma} \tilde{Y}_t \quad (5.135)$$

$$0 = \tilde{\lambda}_{7,t} (1-\theta_p) \varepsilon_p \left(\frac{P_t(i)}{P_t} \right)^{-(1+\varepsilon_p)} + \tilde{\lambda}_{12,t} - \tilde{\lambda}_{9,t} (1-\theta_p) (1-\varepsilon_p) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} \quad (5.136)$$

$$0 = \tilde{\lambda}_{8,t} (1-\theta_w) \varepsilon_w \left(\frac{W_t(j)}{W_t} \right)^{-(1+\varepsilon_w)} + \tilde{\lambda}_{11,t} (1+\varepsilon_w \eta) \left(\frac{W_t(j)}{W_t} \right)^{\varepsilon_w \eta} - \tilde{\lambda}_{10,t} (1-\theta_w) (1-\varepsilon_w) \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} \quad (5.137)$$

$$0 = \tilde{\lambda}_{2,t} \tilde{Y}_t - \tilde{\lambda}_{5,t} \frac{\tilde{N}_t}{s_t^p} - \tilde{\lambda}_{6,t} \frac{E n_t}{s_t^p} + \tilde{\lambda}_{7,t} - \beta \theta_p \text{Et} \left\{ \tilde{\lambda}_{7,t+1} (\pi_{t+1}^p)^{\varepsilon_p} \right\} \quad (5.138)$$

$$0 = -\tilde{\lambda}_{4,t} \tilde{N}_t + \tilde{\lambda}_{8,t} - \beta \theta_w \text{Et} \left\{ \tilde{\lambda}_{8,t+1} (\pi_{t+1}^w)^{\varepsilon_w} \right\} \quad (5.139)$$

$$0 = -\tilde{\lambda}_{7,t} \theta_p \varepsilon_p (\pi_t^p)^{\varepsilon_p-1} s_{t-1}^p - \tilde{\lambda}_{9,t} \theta_p (\varepsilon_p - 1) (\pi_t^p)^{\varepsilon_p-2} + \tilde{\lambda}_{17,t} \frac{W_t}{P_t} \frac{1}{\pi_t^p} \quad (5.140)$$

$$- \tilde{\lambda}_{15,t-1} \theta_p \varepsilon_p (\pi_t^p)^{\varepsilon_p-1} \Psi_t^p - \tilde{\lambda}_{16,t-1} \theta_p (\varepsilon_p - 1) (\pi_t^p)^{\varepsilon_p-2} \Phi_t^p$$

$$0 = -\tilde{\lambda}_{8,t} \theta_w \varepsilon_w (\pi_t^w)^{\varepsilon_w-1} s_{t-1}^w - \tilde{\lambda}_{10,t} \theta_w (\varepsilon_w - 1) (\pi_t^w)^{\varepsilon_w-2} \quad (5.141)$$

$$- \tilde{\lambda}_{13,t-1} \theta_w \varepsilon_w (1+\eta) (\pi_t^w)^{\varepsilon_w(1+\eta)-1} \Psi_t^w - \tilde{\lambda}_{14,t-1} \theta_w (\varepsilon_w - 1) (\pi_t^w)^{\varepsilon_w-2} \Phi_t^w - \tilde{\lambda}_{17,t} \frac{W_t}{P_t} \frac{1}{\pi_t^w}$$

$$0 = \tilde{\lambda}_{11,t} \frac{\Psi_t^w}{(\Phi_t^w)^2} + \tilde{\lambda}_{14,t} - \tilde{\lambda}_{14,t-1} \theta_w (\pi_t^w)^{\varepsilon_w-1} \quad (5.142)$$

$$0 = \tilde{\lambda}_{12,t} \frac{\Psi_t^p}{(\Phi_t^p)^2} + \tilde{\lambda}_{16,t} - \tilde{\lambda}_{16,t-1} \theta_p (\pi_t^p)^{\varepsilon_p-1} \quad (5.143)$$

$$0 = -\tilde{\lambda}_{11,t} \frac{1}{\Phi_t^w} + \tilde{\lambda}_{13,t} - \tilde{\lambda}_{13,t-1} \theta_w (\pi_t^w)^{\varepsilon_w(1+\eta)} \quad (5.144)$$

$$0 = -\tilde{\lambda}_{12,t} \frac{1}{\Phi_t^p} + \tilde{\lambda}_{15,t} - \tilde{\lambda}_{15,t-1} \theta_p (\pi_t^p)^{\varepsilon_p} \quad (5.145)$$

Note that the costate variables $\lambda_{13,t} = \tilde{\lambda}_{13,t-1}$, $\lambda_{14,t} = \tilde{\lambda}_{14,t-1}$, $\lambda_{15,t} = \tilde{\lambda}_{15,t-1}$, and $\lambda_{16,t} =$

$\tilde{\lambda}_{16,t-1}$ are predetermined.

To obtain a global optimum under *time zero commitment*, the initial values of the predetermined costate variables must be

$$\begin{aligned}\lambda_{13,0} &= \tilde{\lambda}_{13,-1} = 0 \\ \lambda_{14,0} &= \tilde{\lambda}_{14,-1} = 0 \\ \lambda_{15,0} &= \tilde{\lambda}_{15,-1} = 0 \\ \lambda_{16,0} &= \tilde{\lambda}_{16,-1} = 0\end{aligned}\tag{5.146}$$

However, in the spirit of the *timeless perspective commitment* policy proposed by Woodford (1999a) and discussed at length by Dennis (2001), Giannoni and Woodford (2003), McCallum and Nelson (2004) and McCallum (2005) within the linear-quadratic framework, the literature usually neglects the optimality condition (5.146).¹⁹ Instead, the initial values of the predetermined costate variables are set equal to their respective steady state values

$$\begin{aligned}\lambda_{13,0} &= \tilde{\lambda}_{13,-1} = \lambda_{13} \\ \lambda_{14,0} &= \tilde{\lambda}_{14,-1} = \lambda_{14} \\ \lambda_{15,0} &= \tilde{\lambda}_{15,-1} = \lambda_{15} \\ \lambda_{16,0} &= \tilde{\lambda}_{16,-1} = \lambda_{16}\end{aligned}\tag{5.147}$$

The policy under time zero commitment and the policy under timeless perspective commitment obviously coincide when the steady state values of the predetermined costate variables λ_{13} to λ_{16} are equal to zero. Besides this exception, the Ramsey policy under timeless perspective commitment fails to maximize our welfare measure V_0^a . McCallum and Nelson (2004) point out that the timeless perspective policy can be considered as "policy-making according to a rule".²⁰ As a consequence, other monetary policy rules (in addition to time zero commitment) may exist which are superior to the Ramsey policy under timeless perspective commitment.²¹

After having discussed the policy regime *timeless perspective commitment*, we now turn to the alternative regime *commitment to a time-invariant simple rule*. For this purpose, we consider a general interest rate rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\phi_R} \left(\left(\frac{\pi_t^p}{\pi^p}\right)^{\delta_\pi} \left(\frac{Y_t}{x}\right)^{\delta_y} \right)^{1-\phi_R}\tag{5.148}$$

where x can either represent the steady state level of output Y or the natural output level Y_t^n . δ_π is the weight on price inflation, and δ_y denotes the weight on output stabilization. A positive value of the parameter ϕ_R indicates a preference for interest rate smoothing.²²

¹⁹See, among others, Khan, King, and Wolman (2003), Faia (2008a, 2008b), Faia and Monacelli (2008). Faia and Monacelli (2004) provide a detailed discussion of the timeless perspective approach within the framework of Ramsey optimal policy.

²⁰See, McCallum and Nelson (2004), p. 45.

²¹This exactly has been shown by Jensen and McCallum (2002) within the linear-quadratic framework. A similar result is reported by Blake (2001). However, he uses a welfare measure based on unconditional expectations. Jensen and McCallum (2002) provide results for both an unconditional and a conditional measure of welfare.

²²Note that equation (5.148) is the nonlinear counterpart of the linear rules usually considered in the literature. By log-linearizing and setting $x = Y$, we obtain for instance $\hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R)(\delta_\pi \hat{\pi}_t^p + \delta_y \hat{Y}_t)$, where $\hat{R}_t = \log(R_t) - \log(R)$, $\hat{Y}_t = \log(Y_t) - \log(Y)$ and $\hat{\pi}_t^p = \log(\pi_t^p) - \log(\pi^p)$ denote log-deviations of the gross nominal interest rate, output, and inflation from their respective steady state values.

The general equation (5.148) nests several well-known monetary policy rules. The rule originally proposed by Taylor (1993) can be obtained by setting $\phi_R = 0$, $x = Y$, $\delta_\pi = 1.5$, and $\delta_y = 0.125$. Setting $\phi_R = 0$, $x = Y_t^n$, $\delta_\pi > 1$, and $\delta_y > 0$ yields a Taylor-type rule with a positive weight on output-gap stabilization, the deviation of actual output from its natural level. Note that by log-linearizing the term Y_t/Y_t^n and taking into account that the steady state values of Y_t^n and Y_t coincide, we obtain the following well-known expression for the output gap

$$\hat{Y}_t^g \equiv \log(Y_t) - \log(Y_t^n) = \log(Y_t/Y) - \log(Y_t^n/Y) = \hat{Y}_t - \hat{Y}_t^n \quad (5.149)$$

Furthermore, we can analyze both rules with a positive weight on the lagged interest rate to allow for interest rate smoothing.²³ Finally, we can study an interest rate peg under which the nominal interest rate is kept constant. To achieve $R_t = R$ without an indeterminacy problem, we follow Collard and Dellas (2005) and set $\phi_R = 0.999$, $\delta_\pi = 1.01$, and $\delta_y = 0$.

5.4 Calibration and Computation

We use a standard calibration which closely follows Calstrom and Fuerst (2006). The time unit is one quarter. The discount rate is equal to $\beta = 0.99$, implying a quarterly steady state real interest rate of approximately one percent.²⁴ The inverse of the intertemporal elasticity in consumption, σ , is set to $\sigma = 2$.²⁵ We set $\eta = 3$ implying a labor supply elasticity with respect to the real wage of $1/3$.²⁶ The elasticity of substitution between different intermediate goods, ε_p , and the elasticity of substitution between different labor types, ε_w , are both set to 8. This implies steady state mark-ups of approximately 14 percent as well as optimal subsidy rates given by $\tau_w = 0.2344$ and $\tau_e = 0.125$.²⁷ We set $\theta_w = \theta_p = 0.75$ implying an average duration of price and wage contracts of one year. We normalize the steady state real price of energy to one. The inverse of the elasticity of substitution between labor and energy, ρ , is given by $\rho = 1.7$. We set $a = 0.02$ implying a share of energy in output and marginal costs of 10%.²⁸ Finally, we assume the real price of energy to be highly persistent and choose ϕ_e equal to 0.95. Table 5.2 summarizes the calibration.

In what follows, we depart from the usual assumption that the shock term e_t in the exogenous process $P_t^e/P^e = (P_{t-1}^e/P^e)^{\phi_e} \exp(e_t)$ is an i.i.d. random variable. Instead, we assume e_t to be a deterministic term and calibrate it in a way to analyze a 10 percent increase in the real price of energy. This assumption enables us to deal with anticipated shocks as well as to solve our model without any approximation.

For the analysis of anticipated and unanticipated shocks, we assume that the economy is in a deterministic steady state until period $t = 0$. In the case of an unanticipated shock,

²³See Clarida, Galí, and Gertler (1998, 2000) for a discussion and estimation of interest rate smoothing.

²⁴Note that $rr = 1/\beta = 1/0.99 = 1.0101$. The annual gross real interest rate is then given by $rr^4 = 1.0410$.

²⁵Note that σ is equal to the Arrow-Pratt measure of relative risk-aversion which is defined as $RRA = -C_t U_{CC,t} / U_{C,t}$. $U_{CC,t}$ denotes the second derivative of the per-period utility function $U(C_t, N_t)$ with respect to C_t . By using the utility function (5.15), we obtain $U_{C,t} = C_t^{-\sigma}$, $U_{CC,t} = -\sigma C_t^{-\sigma-1}$, and thus $RRA = \sigma$.

²⁶Note that, according to (5.98), we have $N^s = \mu_w^{1/\eta} w^{1/\eta} C^{-\sigma/\eta}$. Therefore, the steady state labor supply elasticity is given by $(\partial N^s / \partial w) w / N^s = 1/\eta$.

²⁷Remember that the mark-up is defined as $\mu_i = \varepsilon_i / (\varepsilon_i - 1)$ for $i = w, p$, and the subsidy rates are given by $\tau_e = 1 - 1/\mu_p$ and $\tau_w = 1 - 1/(\mu_w \mu_p)$.

²⁸Note that the steady state share of energy in total output is given by $a(En/Y)^{1-\rho}$, whereas the steady state share of energy in the marginal costs is given by $a^{1/\rho}((1 - \tau_e)P^e/MC)^{(\rho-1)/\rho}$.

Table 5.2: Baseline calibration

Parameter	β	σ	η	ε_p	ε_w	θ_w	θ_p	P^e	a	ρ	ϕ_e
Value	0.99	2	3	8	8	0.75	0.75	1	0.02	1.7	0.95

the real price of energy jumps by 10 percent in period $t = 0$ and begins to fall afterwards. In the case of an anticipated shock, the agents anticipate at time $t = 0$ that a 10 percent increase in the real price of energy will take place at some future date $T > 0$. They also know that the price of energy will subsequently fall according to the autoregressive process (5.34), where now

$$e_t = \begin{cases} \log(1.1/P^e) & \text{for } t = T \\ 0 & \text{for } t \neq T \end{cases} \quad (5.150)$$

In our simulations, we set $T = 2$ implying a length of the anticipation period of two quarters.

The overall deterministic model under timeless perspective commitment is described by equations (5.18), (5.26), (5.31), (5.32), (5.45), (5.50), (5.51), (5.55), (5.54), (5.61), (5.67), (5.68), (5.69), (5.70), (5.76) (which are basically the derivatives of \mathcal{L}_0 with respect to the costate variables or multipliers, i.e. $\lambda_{i,t+1}$ for $i = 1, \dots, 17$) and the optimality conditions (5.129) to (5.145). For given exogenous processes (5.34) and (5.150), this set of equations determines the optimal paths of the non-predetermined (or jump) variables ($C_t, Y_t, \tilde{Y}_t, N_t, \tilde{N}_t, En_t, \pi_t^p, \pi_t^w, P_t(i)/P_t, W_t(j)/W_t, \Psi_t^p, \Phi_t^p, \Psi_t^w, \Psi_t^p$), the predetermined variables ($s_t^p, s_t^w, W_t/P_t$), and the costate variables ($\tilde{\lambda}_{2,t}$ to $\tilde{\lambda}_{17,t}$). By dropping the time indices we can solve for the steady state of this system, which we denote as the Ramsey steady state. To analyze our model under a simple instrument rule, we replace the set of optimality conditions (5.129) to (5.145) by equation (5.148).²⁹

As already mentioned, the deterministic approach allows us to solve the model without any approximation. For the deterministic but nonlinear simulations, we use the software DYNARE developed by Juillard (1996). More precisely, we proceed as follows: We solve for the steady state in which the system stays until the energy price shock emerges or is anticipated. This occurs in a period denoted by $t = 0$. DYNARE now simulates the deterministic adjustment paths from the initial steady state to the final steady state. Since we consider temporary energy price shocks which do not change the steady state, the initial steady state coincides with the final steady state. We then compute the percentage or percentage point deviation of all variables from their respective steady state values.

5.5 Simulation Results

In this section, we present the results of our simulations which are based on the calibration described above. Except where otherwise stated, we assume that optimal subsidies are in place.

First, we analyze the steady state effects of permanent changes in the real price of energy. Second, we briefly discuss the properties of the Ramsey steady state. Then we turn to

²⁹To obtain absolute welfare, we rewrite equation (5.107) recursively as $V_t^a = 1/(1-\sigma)C_t^{1-\sigma} - 1/(1+\eta)N_t^{1-\eta} + \beta V_{t+1}^a$. We add this equation to our model and solve for the dynamics of V_t^a . V_0^a is then simply the value of V_t^a at time $t = 0$.

dynamics and analyze the effects of an unanticipated 10 percent increase in the real price of energy.

Initially, we consider the case of perfectly flexible prices and wages in order to obtain the efficient responses to the energy price shock as a benchmark for our subsequent analysis. Furthermore, this allows us to compute gap variables defined as the percentage deviation of a variable from its natural level.³⁰ In addition, both the steady state analysis and the investigation of the real business cycle core will help to gather intuition about the basic mechanisms that are at work in our model.

Subsequently, we consider our baseline model with staggered price and wage setting. We present impulse response functions and discuss the welfare effects of different monetary policy regimes. In particular, we compare simple ad-hoc interest rate rules to the optimal Ramsey monetary policy under commitment. In doing so, we emphasize the role of the initial steady state for the welfare-ranking of monetary policies. Finally, we compare the welfare effects of anticipated and unanticipated shocks. Thereby, we do not restrict our analysis to the case of energy price shocks but extend it to wage and price mark-up shocks.

5.5.1 Steady State Effects of Energy Price Shocks

The steady state of our model was analytically derived in Section 5.2.8. For the sake of illustration, we simulate the steady state of our model for different steady state values of the real price of energy. Figure 5.1 displays the steady state values of output, employment, energy usage, consumption, the real wage, and overall welfare or utility for different steady state real energy prices.³¹

As already mentioned in Section 5.2.8, steady state output depends ambiguously on the real price of energy. Under our baseline calibration, steady state output declines when the real price of energy rises.³² The same holds true for consumption and energy usage. The reasons for the decrease in energy usage are first the substitution from energy to employment caused by both the rise in real energy prices and the decrease in real wages. Second, the overall decline in output reduces the overall amount of factor inputs used in production. However, employment rises with an increasing real price of energy.

As mentioned above, the effect of rising steady state energy prices on hours worked is basically ambiguous. A positive correlation comes from the substitution effect, whereas the decline in output reduces the amount of labor needed in production. Under our baseline calibration, the substitution effect dominates and employment increases. Both, the increase in employment and the decrease in consumption cause welfare to decline. Equation (5.97) shows that in the steady state, real marginal costs are constant. Basically, real marginal costs depend positively on the real price of energy and the real wage (cf. equation (5.37)). Thus, to obtain the constancy of real marginal costs, the real wage must fall when the real price of energy rises.

Figure 5.2 shows that the steady state amount of lump sum taxes needed to finance the optimal subsidies declines with an increasing real energy price. Furthermore, Figure 5.2 displays the average tax rate (AVT), which is defined as the ratio of aggregate lump-sum

³⁰For instance, the output gap is $\hat{Y}_t^g \equiv \hat{Y}_t - \hat{Y}_t^n$, where \hat{Y}_t and \hat{Y}_t^n denote the percentage deviation of actual output Y_t and natural output Y_t^n from their respective steady state values. Recall that with optimal subsidies as defined in (5.85), the natural levels are also efficient.

³¹In our baseline calibration, we set $P^e = 1$.

³²Numerous simulations show that this result seems to be robust for a wide range of parameter sets.

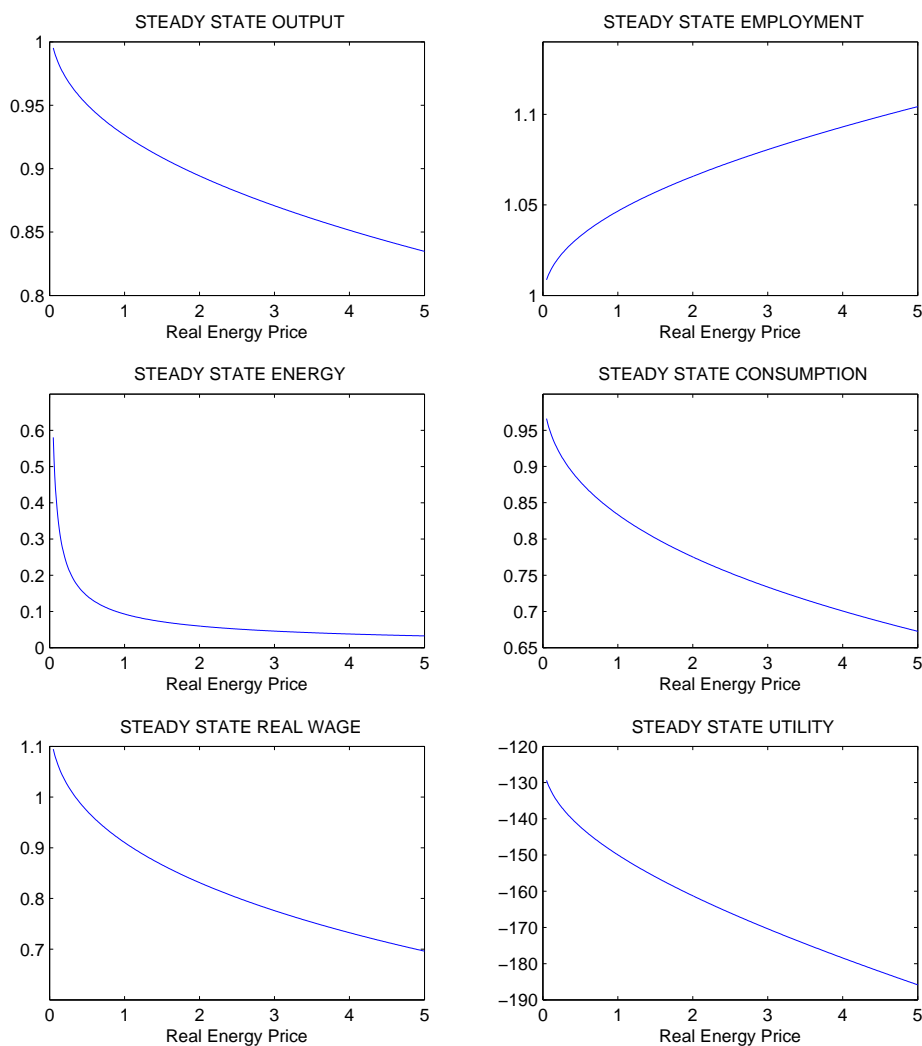


Figure 5.1: Steady state effects of varying the real price of energy

taxes T^r and aggregate income of all households, formally

$$AVT = \frac{T^r}{\Pi^r + wN} \quad (5.151)$$

where the steady state value of distributed real profits is denoted by Π^r . Our simulations illustrate that the average tax rate is a positive function of P^e implying that the tax burden of households increases with a rising energy price.³³

³³Note that our baseline calibration with $P^e = 1$ implies a rather plausible average tax rate of approximately 22 percent.

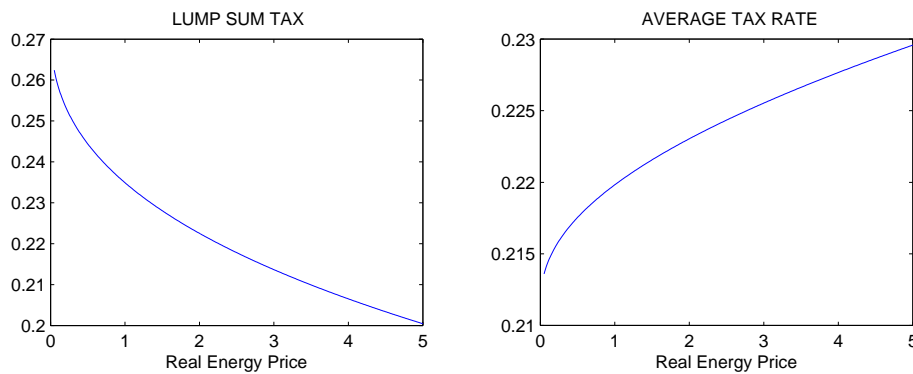


Figure 5.2: Lump-sum tax and tax burden in the steady state

5.5.2 The Ramsey Steady State

In this section, we briefly discuss the properties of the Ramsey steady state. Note that the steady state values of the state variables do not change compared to the above discussed case.

The numerical solution for the costate variables suggests the following: If we assume the existence of optimal subsidies that guarantee steady state efficiency, the steady state values of the predetermined costate variables λ_{13} to λ_{16} will be zero. In this case, the Ramsey policy under timeless perspective commitment corresponds to the policy under time zero commitment and is therefore the globally optimal policy under commitment. Otherwise, if we abandon the assumption that the government optimally subsidizes the costs of employment and energy, the initial steady state will be inefficient which in turn will exclude the global optimality of the timeless perspective approach.³⁴

5.5.3 Energy Price Shocks in the Model with Flexible Prices and Wages

The real business cycle core of the model was derived in Section 5.2.7, where we have already mentioned that the model with flexible prices and wages is rather similar to the steady state of our model. The difference simply consists in the time dependence of the exogenous variable, namely the real price of energy. As a consequence, the steady state effects of an increase in the real price of energy described above carry over to the dynamic responses to a positive energy price shock. Figure 5.3 illustrates the responses to an unanticipated 10 percent increase in the real price of energy.

On impact, natural output drops by approximately 0.4 percent, consumption by approximately 0.8 percent, energy usage by approximately 6 percent, and the real wage by approximately 1 percent. By contrast, the natural real interest rate increases by approximately 0.35 percentage points or 35 basis points and hours worked by approximately 0.2 percent. The increase in the real energy price induces firms to substitute labor for energy. The decline in production reinforces the negative effect on energy usage, whereas the positive effect on

³⁴In this case, the numerical solution of the steady state yields $\lambda_{13} \neq 0$, $\lambda_{14} \neq 0$, $\lambda_{15} \neq 0$, and $\lambda_{16} \neq 0$. Thus, the optimality condition (5.146) is violated.

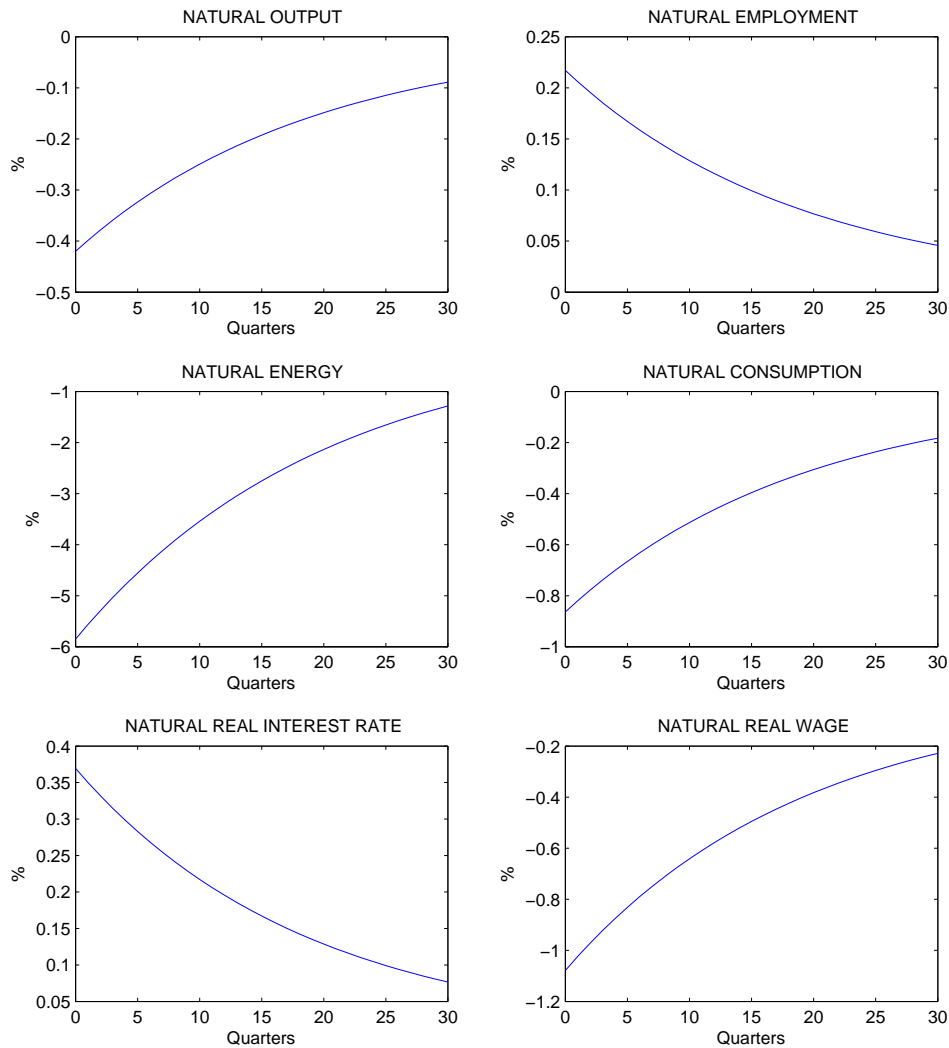


Figure 5.3: Impulse responses of the economy under flexible prices and wages to an unanticipated 10 percent energy price shock.

Note: All variables except for the real interest rate are expressed in percentage deviations from their respective steady state values. The real interest rate is expressed in annualized percentage point deviation from steady state.

employment is dampened.³⁵ The difference between the consumption and output decline is caused by the increase in the real energy bill $P_t^e E n_t$ which in turn follows from the increase in the real energy price.³⁶ The instantaneous and sharp reduction in consumption possibilities puts upward pressure on the real interest rate because of households' efforts to smooth

³⁵As extensively discussed in Section 5.2.8, even a decrease in hours worked may be possible.

³⁶Note that the resource constraint $Y_t = C_t + P_t^e E n_t$ implies a positive effect of an increase in the real energy price on output, holding other things constant.

consumption.³⁷

Following the initial changes, all variables converge monotonically to their unchanged steady state values. The persistence of the endogenous processes is exclusively generated by the persistence of the real energy price. If the energy price exhibits no serial correlation, i.e., $\phi_e = 0$, all variables achieve their steady state values in the period immediately after the occurrence of the shock.

The negative effect on consumption and the positive effect on hours worked cause households' utility and thus welfare to decline. In our baseline scenario, we assume that optimal subsidies are in place. The relative welfare loss due to an unanticipated energy price shock in a model without nominal rigidities and monopolistic distortions is then given by 0.1447 percent. If we do not assume the existence of optimal subsidies, welfare decreases by 0.1456 percent.

5.5.4 Energy Price Shocks and Monetary Policy

After having analyzed the effects of an unanticipated 10 percent increase in the variant of our model without nominal rigidities and hence no role for monetary policy, we now turn to our baseline model with staggered price and wage setting. Figure 5.4 depicts impulse response functions for the model under Ramsey monetary policy and under the original Taylor rule.³⁸ Lines marked with a circle pertain to responses under the Taylor rule. The lines with stars correspond to responses under the Ramsey monetary policy.

The results are qualitatively consistent with the model under perfectly flexible prices and wages. Output, consumption, energy usage, the real energy bill, and the real wage fall while employment and the real interest rate rise. A main qualitative difference is the increase in marginal costs which, by contrast, remains unchanged in the natural model. The rise in real marginal costs puts upward pressure on the prices of intermediate goods and hence on inflation.

Under the Taylor rule, nominal wage inflation rises, whereas it falls under the optimal policy. The opposite responses can be explained by the larger increase in employment and the more moderate decrease in consumption under a Taylor rule. Nevertheless, the shape and magnitude of the response of the real wage is rather similar under both monetary regimes and can be described as hump-shaped.³⁹ The weaker increase in hours worked under the optimal policy causes lump-sum taxes to decline, whereas the tax yield rises under the Taylor rule.

The inability of firms and households to change prices and wages in every period gives rise to a dispersion of prices and wages. Our simulations suggest that both price and wage dispersion respond in a hump-shaped fashion.

Under the Taylor rule, the monetary authority raises the nominal interest rate by around 1.8 percentage points implying an 0.8 percentage point increase in the real interest rate. The optimal monetary policy calls for a more moderate 0.75 percentage point rise in the nominal interest rate. Accordingly, the real interest rate immediately rises by only 0.2 percentage points. Despite this less contractionary effect on consumption and hence output, the optimal

³⁷According to the consumption Euler equation (5.14), the real interest rate depends positively on the change in consumption.

³⁸Note that the original Taylor rule in log-linear form is given by $\hat{R}_t = 1.5\hat{\pi}_t^p + 0.125\hat{Y}_t$, where a hat above a variable denotes the percentage deviation from its steady state value.

³⁹Note that in the case of a simple interest rate rule without smoothing, the real wage and the measures of wage and price dispersion are the only predetermined state variables in our system.

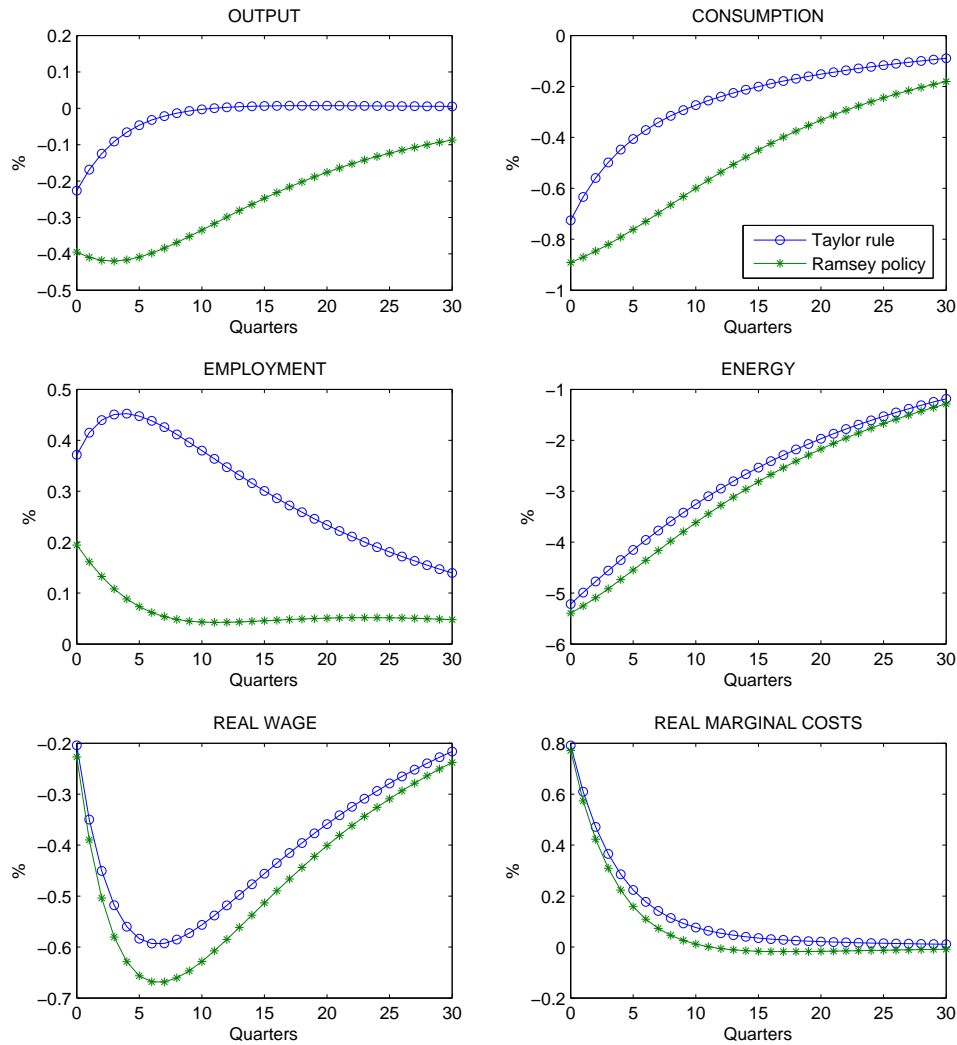


Figure 5.4: Impulse responses to an unanticipated 10 percent energy price shock.

Notes: Solid lines with circles denote responses under the original Taylor rule, solid lines with stars denote responses under Ramsey monetary policy. All real variables except for the real interest rate are expressed in percentage deviations from their respective steady state values. The real interest rate, the nominal interest rate, wage inflation, and price inflation are expressed in annualized percentage point deviation from steady state.

monetary policy implies a larger and more prolonged output and consumption slump. This can be explained by the persistent and hump-shaped development of the real interest rate and the more moderate increase in hours worked.

By looking at the gap variables, two points are of interest. First, the optimal policy obviously seeks to close the gap between the actual and the natural (efficient) levels of output,

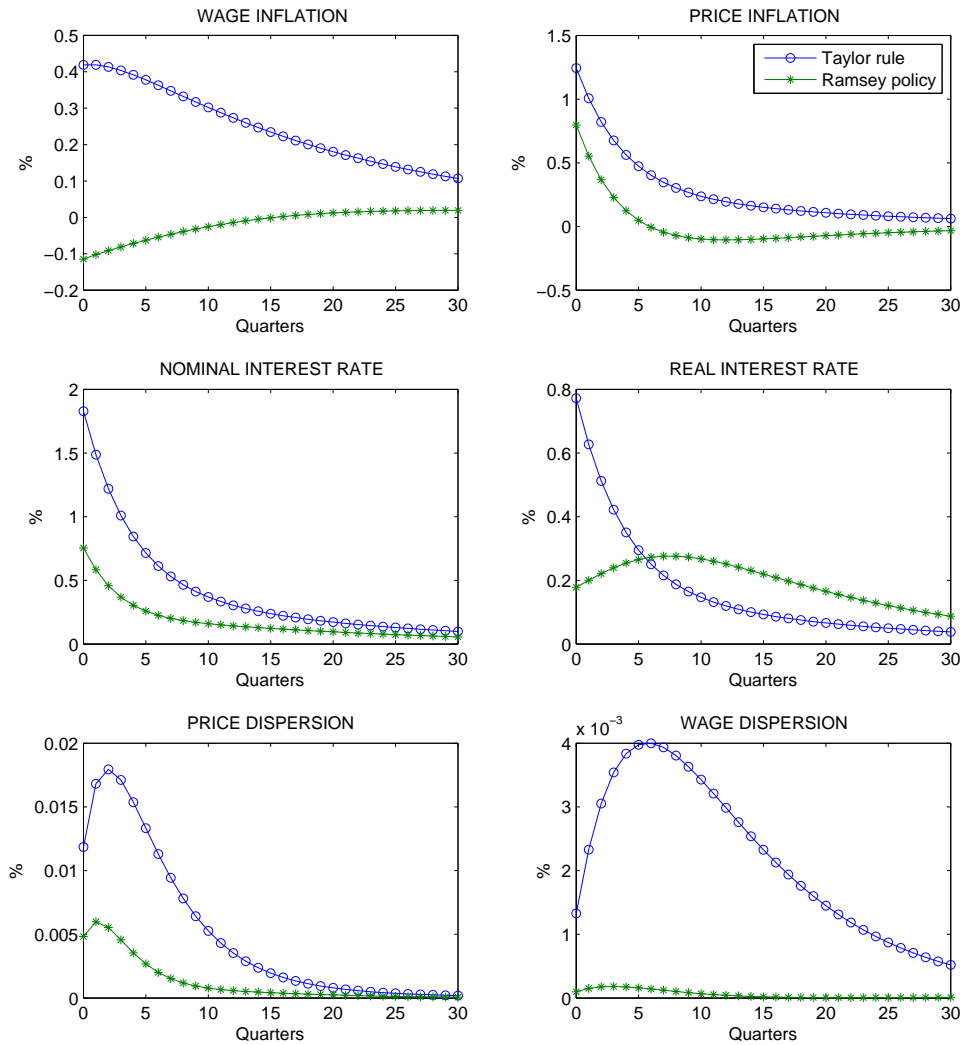


Figure 5.4: — Continued

consumption, employment, and the real interest rate.⁴⁰ Second, if we measure economic activity in terms of the output gap, a Taylor rule in fact leads to a boom, whereas the Ramsey policy entails a moderate decline in the output, consumption, and employment gaps.

With regard to inflation, the optimal policy is able to reduce the change in wages and prices which in turn diminishes the welfare-reducing dispersion of prices and wages. As emphasized by Erceg, Henderson, and Levin (2000), a complete stabilization of the welfare-relevant output gap as well as price and wage inflation is impossible due to the trade-off that arises in a model with staggered price and wage setting.⁴¹

⁴⁰Recall that the natural levels coincides with the efficient levels due to the existence of optimal subsidies.

⁴¹Erceg, Henderson, and Levin (2000) derive the optimal policy with sticky prices and wages by taking the linear-quadratic approach.

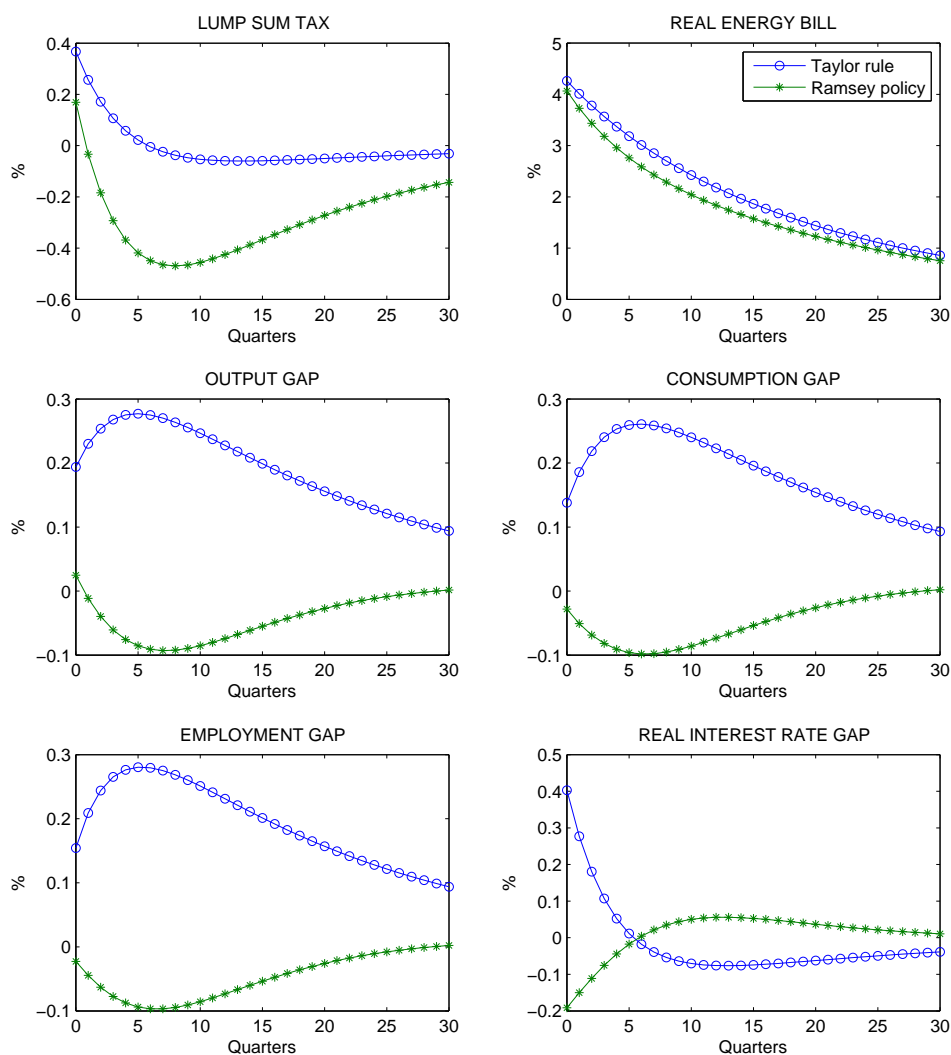


Figure 5.4: — Continued

We conclude by offering a new insight into the question whether monetary policy amplifies or dampens the recessionary effects of energy price shocks. As shown above, the optimal policy calls for a stronger and more prolonged output slump and thus amplifies the recessionary effects compared to a standard Taylor rule.

This result is particularly remarkable since our welfare measure is based on the utility of households. If we instead assume a standard quadratic loss function with a high weight on inflation variability (which Svensson (1999) calls a strict inflation targeting regime), such a result will obviously hold. In our approach, the welfare relevant measure of output is the gap between the actual and the efficient level of output. As already mentioned, a minimization of the welfare-relevant output gap is actually achieved by the optimal policy. However, to emphasize the point again, this policy calls for a significant drop in output.

Table 5.3: Welfare effects of energy price shocks

	Unanticipated shock		Anticipated shock	
	V_0^a	V_0^r	V_0^a	V_0^r
<i>With optimal subsidies</i>				
Ramsey	-150.1563	-0.1451	-150.1518	-0.1421
Interest rate peg	-150.1572	-0.1457	-150.1526	-0.1427
$\hat{R}_t = 1.5\hat{\pi}_t^p + 0.125\hat{Y}_t$	-150.1589	-0.1468	-150.1551	-0.1443
$\hat{R}_t = 1.5\hat{\pi}_t^p + 0.125\hat{Y}_t^g$	-150.1575	-0.1459	-150.1536	-0.1433
$\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2(1.5\hat{\pi}_t^p + 0.125\hat{Y}_t)$	-150.1579	-0.1462	-150.1538	-0.1435
$\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2(1.5\hat{\pi}_t^p + 0.125\hat{Y}_t^g)$	-150.1570	-0.1456	-150.1529	-0.1428
<i>Without optimal subsidies</i>				
Ramsey	-151.1391	-0.1465	-151.1349	-0.1437
Interest rate peg	-151.1273	-0.1387	-151.1262	-0.1380
$\hat{R}_t = 1.5\hat{\pi}_t^p + 0.125\hat{Y}_t$	-151.1199	-0.1338	-151.1198	-0.1337
$\hat{R}_t = 1.5\hat{\pi}_t^p + 0.125\hat{Y}_t^{gap}$	-151.1265	-0.1382	-151.1268	-0.1384
$\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2(1.5\hat{\pi}_t^p + 0.125\hat{Y}_t)$	-151.1233	-0.1361	-151.1226	-0.1356
$\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2(1.5\hat{\pi}_t^p + 0.125\hat{Y}_t^g)$	-151.1301	-0.1406	-151.1295	-0.1402

Notes: Relative welfare is defined as the percentage deviation of absolute welfare from its steady state level. Under our benchmark calibration with and without optimal subsidies the steady state level of welfare is given by -149.9387 and -150.9180 , respectively. \hat{Y}_t^g denotes the output gap.

Table 5.3 shows that the recessionary Ramsey policy is indeed welfare-enhancing compared to the Taylor rule. The Ramsey policy leads to a welfare loss of 0.1451 percent, whereas the standard Taylor rule causes welfare to decline by 0.1468 percent. Additionally, Table 5.3 reports results for three other monetary policy rules, namely an interest rate peg, a Taylor rule with a weight on output-gap stabilization as well as both Taylor rules with a preference for interest rate smoothing. Naturally, all these rules perform worse than the globally optimal Ramsey policy under commitment.

A number of further results are worth noting. First, as emphasized by Woodford (1999b) and shown by Wohltmann and Winkler (2008a) for the case of energy price shocks, history-dependent interest rate rules seem to be welfare-enhancing. This result can be demonstrated by comparing the Taylor rules with and without interest rate smoothing. The original Taylor rule with smoothing causes welfare to decline by 0.1462 percent compared to a 0.1468 percent welfare drop under a rule without smoothing. The same qualitative result holds for the Taylor rule which puts weight on output-gap stabilization. The rule without smoothing leads to a 0.1459 percent decrease in welfare, while the rule with smoothing entails a welfare decline of 0.1456 percent.

Second, a welfare improvement can be achieved by following rules which put weight on output-gap stabilization instead of actual output stabilization. Furthermore, if we follow Leduc and Sill (2004) or Calstrom and Fuerst (2006) and define an interest peg as a form of neutral policy we can conclude that neutral monetary policy dampens the welfare loss compared to active Taylor-type interest rules.⁴²

Finally, all these results hold for anticipated shocks, too. A more detailed welfare analysis of anticipated shocks is provided in the next section.

⁴²This finding is in line with the results of Wohltmann and Winkler (2008a). However, they question the appropriateness of characterizing an interest rate peg as a neutral monetary policy.

Up to now, we have analyzed the welfare effects of energy price shocks and monetary policy under the assumption of optimal employment and energy subsidies. As already mentioned, this additional policy instrument leads to the efficiency of both the natural level and the steady state level of output and ensures the global optimality of the timeless perspective Ramsey approach within the class of commitment policies. In the following, we apply the usual practice of abandoning the assumption that the government pays employment and energy subsidies by setting $\tau_e = \tau_w = 0$.⁴³

Table 5.3 reports the results of simulating the welfare effects, subjected to $\tau_e = \tau_w = 0$. Now, the Ramsey monetary policy performs worse than all other rules under consideration. The optimal policy under timeless perspective commitment entails a 0.1465 percent welfare drop in the case of unanticipated shocks and a 0.1437 percent welfare drop in the case of anticipated shocks, whereas the best policy is now the original Taylor rule which causes welfare to decline by only 0.1338 percent and 0.1337 percent in the case of unanticipated and anticipated shocks, respectively.

The mathematical rationale has already been mentioned: The monetary authority ignores the optimality condition (5.146) and instead sets the initial values of the predetermined costate variables equal to their respective values in the by now inefficient steady state. As a consequence, the policy under timeless perspective commitment fails to maximize our conditional welfare measure. Nevertheless, this policy is widely considered as a good approximation of the globally optimal policy under commitment, or to use the terminology of McCallum and Nelson (2004), the policy under timeless perspective commitment should be optimal within the class of time-invariant rules which the monetary authority commits itself to.⁴⁴ Our results reveal that this is, however, not generally true. This is consistent with the findings of Blake (2001) and Jensen and McCallum (2002) who show the non-optimality of timeless perspective commitment within the linear-quadratic framework.

A rational policy-maker will prefer to use any of the rules discussed above instead of the policy under timeless perspective commitment. This is particularly remarkable, since we do not even search for optimal simple rules by maximizing the policy makers' objective function with respect to the parameters of a given simple rule.⁴⁵ If we did, we would improve the welfare results of simple rules which in turn would amplify the relative welfare loss of the Ramsey policy under timeless perspective commitment. Hence, our results challenge the appropriateness of the proposed Ramsey approach under timeless perspective commitment as a guideline for the conduct of monetary policy.

5.5.5 A Comparison of the Welfare Effects of Anticipated and Unanticipated Shocks

In this section, we compare the welfare effects of anticipated and unanticipated energy price shocks. Recall that we set $T = 2$ implying a length of the anticipation period of two quarters. The results reported in Figure 5.3 suggest that the anticipation of a future energy price hike generally dampens the overall welfare loss. This results seems to be robust to a wide range of variations. Welfare gains through anticipation are achieved i) under perfectly flexible prices and wages as well as under our baseline calibration with a relative high degree of price and

⁴³See, among others, Khan, King, and Wolman (2003), Faia (2008a, 2008b), Faia and Monacelli (2004, 2008).

⁴⁴The simulation of the globally optimal policy under commitment in a model with an inefficient (distorted) steady state remains an open computational problem.

⁴⁵Among the numerous studies which solve such a constrained optimization problem see, for example, Schmitt-Grohé and Uribe (2007a) or Wohltmann and Winkler (2008a).

wage inflexibility, ii) irrespective of the existence of optimal subsidies, and iii) irrespective of the conduct of monetary policy. The only exception is found when we start from an inefficient steady state and use the Taylor rule with a weight on output-gap stabilization. In this exceptional case, an unanticipated energy price shock implies a 0.1382 percent welfare drop, while an anticipated shock causes welfare to decline by 0.1384 percent.

This result stands in contrast to the findings of Wohltmann and Winkler (2008a). However, from a methodological point of view, this study is only loosely related to our work, since they consider a stylized New Keynesian model of a small open economy and apply the linear-quadratic framework with a more traditional loss function which reflects flexible inflation targeting preferences.

The question arises whether the anticipation of future shocks generally leads to welfare gains in the present model. In order to answer this question, we now turn back to cost shocks. Cost-push shocks are widely considered in the literature on optimal monetary policy, because they generate a tradeoff between stabilizing the output gap and inflation.⁴⁶ We can identify two different sources of cost-push shocks in our model, namely the wage mark-up and the price mark-up.⁴⁷ Recall that wage and price mark-ups are defined as $\mu_w = \frac{\varepsilon_w}{\varepsilon_w - 1}$ and $\mu_p = \frac{\varepsilon_p}{\varepsilon_p - 1}$, respectively.

In order to analyze the welfare effects of price and wage mark-up shocks we now assume that both the elasticity of substitution between different intermediate goods, ε_p , and the elasticity of substitution between different labor types, ε_w , follow an autoregressive process given by

$$\frac{\varepsilon_{i,t}}{\varepsilon_i} = \left(\frac{\varepsilon_{i,t-1}}{\varepsilon_i} \right)^{\phi_i} \exp(e_{i,t}) \quad \text{for } i = p, w \quad (5.152)$$

where the steady state values of ε_p and ε_w are set equal to 8. ϕ_p and ϕ_w are set to 0.95 implying a high degree of persistence. The shock terms $e_{p,t}$ and $e_{w,t}$ are again assumed to be deterministic. In order to simulate a 10 percent jump in the mark-ups μ_p and μ_w which ought to occur at time $t = T$, we set $e_{p,t} = e_{w,t} = \log(4.888/8)$ for $t = T$ and $e_{p,t} = e_{w,t} = 0$ for $t \neq T$. We simulate our modified system under the assumption that optimal price and wage subsidies are in place. Hence, we focus on the case of an efficient steady state where, as mentioned above, the Ramsey policy under timeless perspective coincides with the globally optimal policy under commitment.

Table 5.4 reports results for the Ramsey policy, an interest rate peg, and the original Taylor rule with and without smoothing.⁴⁸ Our results indicate that anticipated cost-push shocks lead to higher welfare losses than unanticipated cost-push shocks. This result holds for the optimal policy under commitment, for simple Taylor rules as well as for an interest rate peg. For instance, turn to optimal Ramsey policy. An unanticipated 10 percent increase in the wage mark-up causes welfare to decline by 0.0026 percent, while an anticipated increase in the wage mark-up implies a welfare drop of 0.0029 percent. Accordingly, an unanticipated 10 percent rise in the price mark-up implies a welfare drop of 0.0286 percent. If we instead assume that the price mark-up shock occurs at time $T = 2$ and is anticipated at time $t = 0$, the welfare drops by 0.033 percent.

⁴⁶See, among others, Clarida, Galí, and Gertler (1999) or Woodford (2003).

⁴⁷See, for example, Galí (2008).

⁴⁸Note that, as usually in the literature, the level of output is defined as natural if both prices and wages are perfectly flexible and the mark-ups are constant. It then follows, by definition, that the natural level of output coincides with the steady state level of output if the energy price remains constant at its steady state level.

Table 5.4: Welfare effects of wage and price mark-up shocks

	<i>Unanticipated shock</i>		<i>Anticipated shock</i>	
	V_0^a	V_0^r	V_0^a	V_0^r
<i>Wage mark-up shock</i>				
Ramsey	-149.9426	-0.0026	-149.9431	-0.0029
Interest rate peg	-149.9428	-0.0027	-149.9433	-0.0030
$\hat{R}_t = 1.5\hat{\pi}_t^p + 0.125\hat{Y}_t$	-149.9532	-0.0097	-149.9560	-0.0115
$\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2(1.5\hat{\pi}_t^p + 0.125\hat{Y}_t)$	-149.9487	-0.0067	-149.9505	-0.0078
<i>Price mark-up shock</i>				
Ramsey	-149.9815	-0.0286	-149.9882	-0.0330
Interest rate peg	-149.9869	-0.0321	-149.9923	-0.0357
$\hat{R}_t = 1.5\hat{\pi}_t^p + 0.125\hat{Y}_t$	-150.0473	-0.0724	-150.0933	-0.1031
$\hat{R}_t = 0.8\hat{R}_{t-1} + 0.2(1.5\hat{\pi}_t^p + 0.125\hat{Y}_t)$	-150.0209	-0.0548	-150.0480	-0.0729

Notes: Relative welfare is defined as the percentage deviation of absolute welfare from its steady state level. The table is based on the benchmark calibration with optimal subsidies. The steady state welfare is equal to -149.9387 .

These results confirm the findings reported and discussed in Chapter 2 and Chapter 3, where we apply the linear framework with a quadratic loss function. Here, instead, we explicitly use the utility function of households.

To summarize, our simulations suggest that the welfare effects of the anticipation of future shocks depend on the nature of the shock. Here, our objective is not to provide any explanation, but only to demonstrate this finding and its relation to existing literature.

A further interesting result is that an interest rate peg performs remarkably well, in particular in the case of wage mark-up shocks. For unanticipated as well as anticipated wage mark-up shocks, the welfare loss is only 0.001 percentage points higher than the loss under Ramsey monetary policy. In the case of a 10 percent increase in the price mark-up, an interest rate peg also performs significantly better than the other ad-hoc instrument rules under consideration. Furthermore, our simulations show that a Taylor rule with smoothing performs better than a Taylor rule without smoothing. The rationale for these results is that both rules are associated with a high degree of policy inertia or history-dependence which also characterizes the optimal monetary policy under commitment, as pointed out by Woodford (1999b).

5.6 Conclusion

In this chapter, we laid out a nonlinear model of an oil-dependent economy with staggered price and wage setting. We derived the Ramsey monetary policy in the presence of unanticipated and anticipated energy price shocks. In particular, we compared the optimal policy to simple monetary policy rules.

Our findings contribute to the ongoing discussion whether the monetary policy amplifies or dampens the recessionary effects of oil price shocks by pointing out that the utility-maximizing monetary policy in fact calls for a sharp and prolonged output slump. By contrast, simple rules in the spirit of Taylor (1993) lead to a dampening of the actual output drop that is welfare-reducing. Hence, instead of just looking at the recessionary consequences, we

advocate the consideration of overall welfare in the analysis of the interaction between oil price shocks and monetary policy.

As a methodological contribution, we have shown that Ramsey policy under timeless perspective commitment is inferior to simple ad-hoc monetary policy rules when the initial steady state is distorted. Thus, our work sounds a cautionary note about proposed Ramsey policies as a guideline for monetary policy.

By comparing the welfare effects of unanticipated and anticipated shocks, we obtained mixed results. If the energy price shock is known in advance, its welfare-reducing effects are dampened. However, the opposite is true for typical cost-push shocks such as wage mark-up and price mark-up shocks. We conclude that (i) the welfare effects of the anticipation of future shocks depend on the nature of the shock, and (ii) that the findings reported in Chapter 2 and Chapter 3 with respect to the welfare effects of anticipated cost-push shocks seem to be robust when considering the Ramsey monetary policy approach and taking non-linearities into account.

An important feature of our model is the consideration of imperfect labor markets by assuming monopolistic competition and incomplete nominal wage adjustment. However, an important direction for future research is to incorporate more elaborate labor market rigidities like search and matching frictions (see, for example, Krause and Lubik (2007), Faia (2008a)), efficiency wages (Danthine and Kurmann (2004)), or labor turnover costs (Lechthaler, Merkl, and Snower (2008)). This would enable us to analyze the interaction of oil price shocks, monetary policy, and involuntary unemployment.

A further shortcoming is that we consider a model of an economy that depends on energy bought from exogenous sources, but is otherwise assumed to be closed. Hence, we rule out the potentially important impacts of changes in the nominal exchange rate and in the terms of trade. Taking these effects into account, will also be a fruitful area for future research.

6 Summary and Outlook

This thesis contributes to the literature in four, we think, important dimensions:

The non-optimality of Ramsey monetary policy under timeless perspective commitment

We demonstrate that the Ramsey monetary policy under timeless perspective commitment may be inferior to simple (non-optimized) monetary policy rules. We derive the Ramsey policy in the presence of technology and cost-push shocks for an economy with monopolistic competition on product and labor markets as well as staggered price and wage setting resulting from quadratic adjustment costs. We are able to show that policy-making from a timeless perspective is the globally optimal policy under commitment only if the monopolistic distortions are eliminated by optimal wage subsidies. If the government, however, does not subsidize the cost of production, the optimal policy under timeless perspective commitment may be inferior even to simple monetary policy rules such as an interest rate peg or instrument rules of Taylor-type. This result is confirmed in our analysis of optimal monetary policy-making in the presence of energy price shocks. We base this investigation on a non-linearized model for an oil-dependent economy with, in turn, monopolistic competition on product and labor markets as well as inflexible price and wage adjustment here modeled by the well-known Calvo (1983) mechanism.

Our finding calls into question the appropriateness of the proposed Ramsey policy under timeless perspective commitment as a guideline for the conduct of monetary policy. Then, the question naturally arises what will be the best or optimal monetary policy. To find a time-invariant rule which is utility-maximizing for different shocks that hit the economy and under different calibrations as well as to discover the structure of such an "optimal rule" within a non-linearized framework would be a fruitful area for future research.

The non-optimality of information

We point out that the information about date and magnitude of a future cost-push shock is in general welfare-reducing. We analytically show this result within the canonical linear New Keynesian model with a monetary authority that seeks to minimize a quadratic loss function in inflation and the output gap. We show that for empirically plausible degrees of nominal rigidity, anticipated cost shocks entail higher welfare losses than unexpected cost shocks. The anticipation of a future cost-push shock dampens the volatility of the output gap and inflation only if prices are highly flexible. These results hold independently of the monetary policy regime (timeless perspective commitment, discretion, (optimal) simple rules).

We confirm our finding that the knowledge about future cost-shocks reduces welfare at least for plausible degrees of nominal rigidity (and plausible lengths of the time span between the anticipation and the realization of the shock) within two extensions of the baseline model.

First, the baseline model is extended by considering habit formation in consumption preferences and by assuming a variant of the Calvo mechanism with partial indexation of non-optimized prices to lagged inflation. This leads to hybrid IS and Phillips curves which make

the model empirically more appealing. In addition, the second-order approximation of the households' utility function then leads to a quadratic loss function in quasi-differences in inflation and the output gap.

Second, we take non-linearities into account. We consider an oil-dependent economy with monopolistic competition on product and labor markets as well as staggered price and wage setting à la Calvo (1983), where overall welfare is measured by the aggregate utility of households. As mentioned above, we again find the welfare-reducing effects of the anticipation of typical cost-push shocks such as wage and price mark-up shocks. However, if an increase in the real price of energy is known in advance, its welfare-reducing effects are dampened compared to an unanticipated shock of equal size.

This result suggests that the welfare effects of the anticipation of future shocks depend on the nature of the shock. Thus, an important direction for future research could be to systematically investigating the welfare effects of the anticipation of different shocks under optimal monetary policy. Moreover, a deeper empirical and theoretical analysis of the impacts of anticipated shocks on business cycle fluctuations should be conducted. This should include the quest for welfare-maximizing monetary and fiscal policy in the face of anticipated macroeconomic disturbances.

Optimal monetary policy in the presence of oil price shocks

We derive the optimal Ramsey-type monetary policy in the presence of energy price shocks. We discuss the effects of a persistent increase in the real price of energy both in the limiting case without nominal rigidities and hence no role for monetary policy as well as in our baseline model where sticky prices and wages give a role for monetary policy. Thereby, we compare the response functions under the optimal monetary policy to the responses under the celebrated Taylor rule. Furthermore, we discuss the welfare effects of several monetary policy rules including an interest rate peg and several variants of the rule originally proposed by Taylor (1993).

Our findings contribute to the ongoing discussion whether the monetary policy amplifies or dampens the recessionary effects of oil price shocks by pointing out that the utility-maximizing monetary policy in fact calls for a sharp and prolonged output slump. By contrast, simple rules in the spirit of Taylor (1993) lead to a dampening of the actual output drop that is welfare-reducing. Hence, instead of just looking at the recessionary consequences, we advocate the consideration of overall welfare in the analysis of the interaction between oil price shocks and monetary policy.

An important feature of our model is the consideration of imperfect labor markets by assuming monopolistic competition and incomplete nominal wage adjustment. However, an important direction for future research is to incorporate more elaborate labor market rigidities like search and matching frictions (see, for example, Krause and Lubik (2007), Faia (2008a)), efficiency wages (Danthine and Kurmann (2004)), or labor turnover costs (Lechthaler, Merkl, and Snower (2008)). This would enable us to analyze the interaction of oil price shocks, monetary policy, and involuntary unemployment.

A further shortcoming is that we consider a model of an economy that depends on energy bought from exogenous sources, but is otherwise assumed to be closed. Hence, we rule out the potentially important impacts of changes in the nominal exchange rate and in the terms of trade. Taking these effects into account, will also be a fruitful area for future research.

A general solution method for linear rational expectations models with optimal policy and anticipated shocks

The thesis offers a methodological contribution by extending the methods summarized by Söderlind (1999) who uses the generalized Schur decomposition method to solve linear rational expectations models with optimal policy. In contrast to Söderlind (1999), our approach provides the opportunity to solve linear dynamic rational expectations models with optimal policy and *anticipated shocks*. In particular, we show how to compute response functions and minimal values of the quadratic loss function in the presence of anticipated shocks for the policy regimes commitment, discretion and (optimal) simple rules. In the method proposed, the time span between the anticipation and the realization of shocks is an exogenous variable. The evaluation of unpredictable shocks is then contained as a limiting case.

A limitation of our approach is that the date of occurrence and the magnitude of all future shocks are known exactly at the time when the solution of the model is computed. An important direction for future research would be to allow for uncertainty about timing and magnitude of future shocks.

A Appendix to Chapter 2

Optimal Timeless Perspective Precommitment Policy

The solution time path of the price level p_t under the optimal timeless perspective precommitment policy can be derived from the solution of π_t due to

$$p_t = \sum_{k=0}^t \pi_k \quad (\text{A.1})$$

We then obtain for $t \leq T$:

$$\begin{aligned} p_t &= \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} r_1^{-T} \sum_{k=0}^t [(r_1 - 1)r_1^k - (r_2 - 1)r_2^k] \\ &= \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} r_1^{-T} [r_1^{t+1} - r_2^{t+1}] \end{aligned} \quad (\text{A.2})$$

and for $t \geq T$

$$\begin{aligned} p_t &= \sum_{k=0}^{T-1} \pi_k + \sum_{k=T}^t \pi_k \\ &= \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} r_1^{-T} [r_1^T - r_2^T] + \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_2 - \varphi} \cdot \\ &\quad \cdot \sum_{k=T}^t \left\{ (1 - \varphi)\varphi^{k-T} - \frac{(r_1 - \varphi)r_2^{-T} - (r_2 - \varphi)r_1^{-T}}{r_1 - r_2} (1 - r_2)r_2^k \right\} \\ &= \frac{1}{\beta} \frac{1}{r_2 - \varphi} \frac{1}{r_1 - r_2} r_2^{t+1-T} - \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_1 - r_2} r_1^{-T} r_2^{t+1} - \frac{1}{\beta} \frac{1}{r_1 - \varphi} \frac{1}{r_2 - \varphi} \varphi^{t+1-T} \end{aligned} \quad (\text{A.3})$$

Obviously,

$$\lim_{t \rightarrow \infty} p_t = 0 \quad \text{for all } T \geq 0 \quad (\text{A.4})$$

and

$$p_0 = \frac{1}{\beta} \frac{1}{r_1 - \varphi} r_1^{-T} = \pi_0 > 0 \quad (\text{A.5})$$

so that the size of the initial jump in p is inversely proportional to the lead time T .

Similar results hold for the state variables x_t and π_t . Since

$$\sum_{k=0}^t (x_k - x_{k-1}) = x_t \quad (\text{A.6})$$

equation (2.6) implies

$$p_t = \sum_{k=0}^t \pi_k = -\frac{\alpha_2}{\alpha_1 \kappa} \sum_{k=0}^t (x_k - x_{k-1}) = -\frac{\alpha_2}{\alpha_1 \kappa} x_t \quad (\text{A.7})$$

so that $p_t > 0$ if and only if $x_t < 0$. The optimal policy under timeless perspective implies $p_t > 0$ for all $0 \leq t < \infty$ so that $x_t < 0$ for all $t < \infty$.

In the following, we show that the *persistence* or *total variation* of p_t is positive correlated with T , i.e.

$$\sum_{t=0}^{\infty} p_t \Big|_{T=0} < \sum_{t=0}^{\infty} p_t \Big|_{T>0} \quad \text{for all } T > 0 \quad (\text{A.8})$$

where the infinite sum $\sum_{t=0}^{\infty} p_t \Big|_{T>0}$ is an increasing function in T .

The persistence measure used here is based on the deviation of p_t from its initial steady state level \bar{p}_0 , where the deviation $|p_t - \bar{p}_0|$ is calculated both for $t < T$ and $t \geq T$. Thereafter the differences $|p_t - \bar{p}_0|$ are summed up. Since $\bar{p}_0 = 0$ and $p_t > 0$ for all t we must determine the infinite sum $\sum_{t=0}^{\infty} p_t$.

Inequality (A.8) holds although the initial jump of p_t is a negative function in T . To prove the inequality note that

$$\sum_{t=0}^{\infty} p_t \Big|_{T=0} = \frac{1}{\beta(r_1 - \varphi)(1 - r_2)(1 - \varphi)} \quad (\text{A.9})$$

$$\sum_{t=0}^T p_t \Big|_{T>0} = \frac{1}{\beta(r_1 - \varphi)(r_1 - r_2)} r_1^{-T} \left[r_1 \frac{1 - r_1^{T+1}}{1 - r_1} - r_2 \frac{1 - r_2^{T+1}}{1 - r_2} \right] \quad (\text{A.10})$$

and

$$\begin{aligned} \sum_{t=T+1}^{\infty} p_t \Big|_{T>0} &= \frac{1}{\beta(r_2 - \varphi)(r_1 - r_2)} r_2^{1-T} \frac{r_2^{T+1}}{1 - r_2} - \frac{1}{\beta(r_1 - \varphi)(r_1 - r_2)} r_1^{-T} r_2 \frac{r_2^{T+1}}{1 - r_2} \\ &\quad - \frac{1}{\beta(r_1 - \varphi)(r_2 - \varphi)} \varphi^{1-T} \frac{\varphi^{T+1}}{1 - \varphi} \end{aligned} \quad (\text{A.11})$$

so that

$$\begin{aligned} \sum_{t=0}^{\infty} p_t \Big|_{T>0} &= \frac{1}{\beta(r_1 - \varphi)(r_1 - r_2)} \left[\frac{r_1}{1 - r_1} (r_1^{-T} - r_1) - \frac{r_2 r_1^{-T}}{1 - r_2} \right] \\ &\quad + \frac{1}{\beta(r_2 - \varphi)} \left[\frac{1}{r_1 - r_2} \frac{r_2^2}{1 - r_2} - \frac{1}{r_1 - \varphi} \frac{\varphi^2}{1 - \varphi} \right] \end{aligned} \quad (\text{A.12})$$

After some tedious manipulations, we obtain

$$\sum_{t=0}^{\infty} p_t \Big|_{T>0} > \sum_{t=0}^{\infty} p_t \Big|_{T=0} \Leftrightarrow 1 - r_1^{-T} > 0 \quad (\text{A.13})$$

Since $r_1 > 1$, the last inequality is fulfilled. Note that the total variation of p_t , i.e. $\sum_{t=0}^{\infty} p_t \Big|_{T>0}$ is an increasing function in T . This follows from equation (A.12), since the derivative of

$\frac{r_1}{1-r_1}r_1^{-T} - \frac{r_2}{1-r_2}r_1^{-T}$ with respect to T is positive. An implication of inequality (A.13) is

$$\sum_{t=0}^{\infty} |x_t| \Big|_{T=0} < \sum_{t=0}^{\infty} |x_t| \Big|_{T>0} \quad (\text{A.14})$$

since

$$|x_t| = \frac{\alpha_1 \kappa}{\alpha_2} p_t \quad (\text{A.15})$$

The persistence of output in the case of anticipated cost-push shocks is therefore stronger than in the case of unanticipated shocks.

A similar result can be shown for the inflation rate π_t if the limiting case $\varphi = 0$ is considered. We then obtain for $T = 0$

$$\pi_t = \begin{cases} 1 - (1 - r_2) = r_2 & \text{if } t = 0 \\ -(1 - r_2)r_2^t < 0 & \text{if } t > 0 \end{cases} \quad (\text{A.16})$$

implying

$$\begin{aligned} \sum_{t=0}^{\infty} \pi_t &= \pi_0 + \sum_{t=1}^{\infty} \pi_t = r_2 - (1 - r_2) \sum_{t=1}^{\infty} r_2^t \\ &= r_2 - (1 - r_2) \left[\frac{1}{1 - r_2} - 1 \right] = r_2 - r_2 = 0 \end{aligned} \quad (\text{A.17})$$

and

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{T=\varphi=0} = r_2 + (1 - r_2) \sum_{t=1}^{\infty} r_2^t = 2r_2 \quad (\text{A.18})$$

In the case $T > 0$ and $\varphi = 0$, we obtain

- for $t \leq T$:

$$\pi_t = \frac{r_2}{r_1 - r_2} r_1^{-T} [(r_1 - 1)r_1^t - (r_2 - 1)r_2^t] > 0 \quad (\text{A.19})$$

- for $t > T$:

$$\pi_t = -\frac{r_1 r_2^{-T} - r_2 r_1^{-T}}{r_1 - r_2} (1 - r_2) r_2^t < 0 \quad (\text{A.20})$$

Then

$$\sum_{t=0}^T \pi_t = \frac{r_2}{r_1 - r_2} r_1^{-T} \sum_{t=0}^T [(r_1 - 1)r_1^t - (r_2 - 1)r_2^t] = \frac{r_2}{r_1 - r_2} r_1^{-T} [r_1^{T+1} - r_2^{T+1}] \quad (\text{A.21})$$

and

$$\sum_{t=T+1}^{\infty} \pi_t = -\frac{1 - r_2}{r_1 - r_2} [r_1 r_2^{-T} - r_2 r_1^{-T}] \frac{r_2^{T+1}}{1 - r_2} = -\frac{r_2}{r_1 - r_2} r_1^{-T} [r_1^{T+1} - r_2^{T+1}] \quad (\text{A.22})$$

so that

$$\sum_{t=0}^{\infty} \pi_t = 0 \quad (\text{A.23})$$

and

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{\substack{T>0 \\ \varphi=0}} = 2 \frac{r_2}{r_1 - r_2} r_1^{-T} [r_1^{T+1} - r_2^{T+1}] \quad (\text{A.24})$$

Now

$$\frac{r_2}{r_1 - r_2} r_1^{-T} [r_1^{T+1} - r_2^{T+1}] > r_2 \quad \Leftrightarrow \quad r_2 [r_1^T - r_2^T] > 0 \quad (\text{A.25})$$

Due to $r_1 > 1 > r_2 > 0$ the last inequality is met so that

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{T=\varphi=0} < \sum_{t=0}^{\infty} |\pi_t| \Big|_{\substack{T>0 \\ \varphi=0}} \quad (\text{A.26})$$

The case $\varphi > 0$ is more difficult to analyze since π_t can take both positive and negative values for $t > T > 0$. If $T = 0$, π_t changes sign immediately after the initial jump. Since

$$\pi_t = \frac{1}{\beta(r_1 - \varphi)(r_2 - \varphi)} [(1 - \varphi)\varphi^t - (1 - r_2)r_2^t] \quad (\text{if } T = 0) \quad (\text{A.27})$$

we obtain

$$\pi_0 = \frac{1}{\beta(r_1 - \varphi)} > 0 \quad (\text{A.28})$$

and

$$\begin{aligned} \sum_{t=1}^{\infty} \pi_t \Big|_{T=0} &= \frac{1}{\beta(r_1 - \varphi)(r_2 - \varphi)} \left[(1 - \varphi) \sum_{t=1}^{\infty} \varphi^t - (1 - r_2) \sum_{t=1}^{\infty} r_2^t \right] \\ &= \frac{1}{\beta(r_1 - \varphi)(r_2 - \varphi)} (\varphi - r_2) = -\frac{1}{\beta(r_1 - \varphi)} = -\pi_0 \end{aligned} \quad (\text{A.29})$$

so that

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{T=0} = 2 \frac{1}{\beta(r_1 - \varphi)} \quad (\text{A.30})$$

In the case $T > 0$, π_t is positive for $0 \leq t \leq T$ and due to (2.13) we obtain

$$\begin{aligned} \sum_{t=0}^T \pi_t &= \frac{1}{\beta(r_1 - \varphi)(r_1 - r_2)} r_1^{-T} \left[(r_1 - 1) \frac{1 - r_1^{T+1}}{1 - r_1} - (r_2 - 1) \frac{1 - r_2^{T+1}}{1 - r_2} \right] \\ &= \frac{r_1}{\beta(r_1 - \varphi)(r_1 - r_2)} \left[1 - \left(\frac{r_2}{r_1} \right)^{T+1} \right] > 0 \end{aligned} \quad (\text{A.31})$$

(since $r_1 > 1 > r_2 > 0$). If $t > T$, π_t is negative for sufficiently large values of t . For small values of $t > T$, π_t may be positive. Due to

$$\lim_{t \rightarrow \infty} p_t = 0 \quad \text{and} \quad p_t = \sum_{k=0}^t \pi_k \quad (\text{A.32})$$

we must have

$$\sum_{t=0}^{\infty} \pi_t = 0 \quad (\text{A.33})$$

so that

$$\sum_{t=T+1}^{\infty} \pi_t = -\sum_{t=0}^T \pi_t < 0 \quad (\text{A.34})$$

The last equation also follows from (2.15). With

$$\psi = -\frac{(r_1 - \varphi)r_2^{-T} - (r_2 - \varphi)r_1^{-T}}{r_1 - r_2} \quad (\text{A.35})$$

we obtain

$$\begin{aligned} \sum_{t=T+1}^{\infty} \pi_t &= \frac{1}{\beta(r_1 - \varphi)(r_2 - \varphi)} \left[(1 - \varphi)\varphi^{-T} \sum_{t=T+1}^{\infty} \varphi^t + \psi(1 - r_2) \sum_{t=T+1}^{\infty} r_2^t \right] \\ &= \frac{1}{\beta(r_1 - \varphi)(r_2 - \varphi)} \left[(1 - \varphi)\varphi^{-T} \frac{\varphi^{T+1}}{1 - \varphi} + \psi(1 - r_2) \frac{r_2^{T+1}}{1 - r_2} \right] \\ &= -\frac{r_1}{\beta(r_1 - \varphi)(r_1 - r_2)} \left[1 - \left(\frac{r_2}{r_1} \right)^{T+1} \right] = -\sum_{t=0}^T \pi_t < 0 \end{aligned} \quad (\text{A.36})$$

Therefore,

$$\begin{aligned} \sum_{t=0}^T \pi_t \Big|_{T>0} - \sum_{t=T+1}^{\infty} \pi_t \Big|_{T>0} &= 2 \sum_{t=0}^T \pi_t \Big|_{T>0} > \sum_{t=0}^{\infty} |\pi_t| \Big|_{T=0} = 2\pi_0 \Big|_{T=0} \quad \Leftrightarrow \\ &\sum_{t=0}^T \pi_t \Big|_{T>0} > \pi_0 \Big|_{T=0} \quad \Leftrightarrow \\ &r_1^T > r_2^T \end{aligned} \quad (\text{A.37})$$

The last inequality is met due to $r_1 > 1 > r_2 > 0$. Since

$$-\sum_{t=T+1}^{\infty} \pi_t \Big|_{T>0} \leq \sum_{t=T+1}^{\infty} |\pi_t| \Big|_{T>0} \quad (\text{A.38})$$

the stronger persistence in the case of anticipated shocks follows:

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{T>0} = \sum_{t=0}^T \pi_t + \sum_{t=T+1}^{\infty} |\pi_t| \geq \sum_{t=0}^T \pi_t - \sum_{t=T+1}^{\infty} \pi_t > \sum_{t=0}^{\infty} |\pi_t| \Big|_{T=0} \quad (\text{A.39})$$

Note that for arbitrary $T > 0$

$$\pi_0 \Big|_{T=0} < \sum_{t=0}^T \pi_t \Big|_{T>0} \quad (\text{A.40})$$

but

$$\pi_t \Big|_{T>0} < \pi_0 \Big|_{T=0} \quad \text{for all } 0 \leq t \leq T \quad (\text{A.41})$$

In particular,

$$\pi_T \Big|_{T>0} < \pi_0 \Big|_{T=0} \quad (\text{A.42})$$

since

$$\frac{1}{\beta(r_1 - \varphi)(r_1 - r_2)} \left[(r_1 - 1) - (r_2 - 1) \left(\frac{r_2}{r_1} \right)^T \right] < \frac{1}{\beta(r_1 - \varphi)} \Leftrightarrow \left(\frac{r_2}{r_1} \right)^T < 1 \quad (\text{A.43})$$

Since the last equation holds, the value of the inflation rate at the time of implementation of the cost-push shock is smaller in the case of anticipated compared to unanticipated shocks.¹

Optimal Policy under Discretion

For all $0 \leq \varphi < 1$, the adjustment processes of x_t and π_t in the case of anticipated cost-push shocks show a stronger persistence than in the case $T = 0$. With the abbreviation

$$\tilde{\phi} = \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} > 0 \quad (\text{A.44})$$

we have

$$\sum_{t=0}^{\infty} |\pi_t| \Big|_{T=0} = \tilde{\phi} \sum_{t=0}^{\infty} \varphi^t = \frac{\tilde{\phi}}{1 - \varphi} \quad (\text{A.45})$$

and

$$\begin{aligned} \sum_{t=0}^{\infty} |\pi_t| \Big|_{T>0} &= \sum_{t=0}^{T-1} \pi_t \Big|_{T>0} + \sum_{t=T}^{\infty} \pi_t \Big|_{T>0} \\ &= \tilde{\phi} \frac{1}{1 - \varphi} + \tilde{\phi} \frac{1 - r_D^{-T}}{r_D - 1} > \tilde{\phi} \frac{1}{1 - \varphi} = \sum_{t=0}^{\infty} |\pi_t| \Big|_{T=0} \end{aligned} \quad (\text{A.46})$$

since $r_D > 1$ and $0 < r_D^{-T} < 1$ if $T > 0$. An analogous result holds for x_t .

The policy regime discretion implies

$$\sum_{t=0}^{\infty} \pi_t \Big|_{T=0} = \sum_{t=T}^{\infty} \pi_t \Big|_{T>0} \quad (\text{A.47})$$

and

$$\sum_{t=0}^{\infty} x_t \Big|_{T=0} = \sum_{t=T}^{\infty} x_t \Big|_{T>0} \quad (\text{A.48})$$

¹This result holds under the optimal timeless perspective precommitment policy. Under the policy regime discretion we have (cf. (2.29))

$$\pi_0 \Big|_{T=0} = \pi_T \Big|_{T>0} = \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi}$$

so that the stronger persistence of π_t and x_t in the case $T > 0$ is due to the anticipation effects $\sum_{t=0}^{T-1} \pi_t > 0$ and $\sum_{t=0}^{T-1} x_t < 0$.

The solution time path for the price level p_t results from

$$p_t = \sum_{k=0}^t \pi_k \quad (\text{A.49})$$

For $0 \leq t \leq T$ we obtain

$$p_t = \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} \sum_{k=0}^t r_D^{t-k} = \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} r_D^{-T} \frac{1 - r_D^{t+1}}{1 - r_D} \quad (\text{A.50})$$

and for $t \geq T$

$$p_t = \sum_{k=0}^{T-1} \pi_k + \sum_{k=T}^{\infty} \pi_k = \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} \left[r_D^{-T} \frac{1 - r_D^T}{1 - r_D} - \varphi^{-T} \frac{\varphi^{t+1} - \varphi^T}{1 - \varphi} \right] \quad (\text{A.51})$$

with

$$\lim_{t \rightarrow \infty} p_t = \frac{\alpha_2}{\alpha_2 + \alpha_1 \kappa^2 - \alpha_2 \beta \varphi} \left[\frac{1 - r_D^{-T}}{r_D - 1} + \frac{1}{1 - \varphi} \right] > 0 \quad (\text{A.52})$$

Note that the limit value of p_t is a positive function in T . It is well-known that a temporary cost-push shock yields a permanent rise in the price level under the policy regime discretion. By contrast, under the optimal timeless perspective precommitment policy there is only a temporary rise in the price level.

Total Loss under a Simple Rule

We can also determine the total loss under an ad hoc Taylor rule

$$i_t = \delta_\pi \pi_t + \delta_x x_t \quad (\text{A.53})$$

with exogenously given coefficients δ_π and δ_x . It is well-known that under the condition $\delta_\pi > 1$ and $\delta_x \geq 0$ the baseline New Keynesian model satisfies the Blanchard/Kahn (1980) saddlepath condition. The state equations

$$A \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} = B \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix} k_t \quad (\text{A.54})$$

with

$$A = \begin{pmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{pmatrix}, \quad B = \begin{pmatrix} 1 + \frac{\delta_x}{\sigma} & \frac{\delta_\pi}{\sigma} \\ -\kappa & 1 \end{pmatrix} \quad (\text{A.55})$$

have two unstable eigenvalues belonging to the state matrix $A^{-1}B$. Solving the state equations forward we obtain with

$$v_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}, \quad P = B^{-1}A, \quad q = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \quad (\text{A.56})$$

the solution time paths in the case of anticipated cost-push shocks:

- For $t \geq T$

$$v_t = - \left(\sum_{s=0}^{\infty} \varphi^s P^s \right) B^{-1} q \varphi^{t-T} = -[B - \varphi A]^{-1} q \varphi^{t-T} \quad (\text{A.57})$$

- For $t < T$

$$v_t = - \left(\sum_{s=T-t}^{\infty} \varphi^s P^s \right) B^{-1} q \varphi^{t-T} = -[I_{2 \times 2} - \varphi P]^{-1} P^{T-t} B^{-1} q \quad (\text{A.58})$$

The solution formula for $t < T$ also holds in $t = T$ since

$$v_T = -[B - \varphi A]^{-1} q = -[I_{2 \times 2} - \varphi P]^{-1} B^{-1} q \quad (\text{A.59})$$

The total loss under the simple Taylor rule (V_{STR}) can be written as

$$V_{STR} = \sum_{t=0}^{\infty} \beta^t v_t' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} v_t = V_1^{STR} + V_2^{STR} \quad (\text{A.60})$$

where

$$V_1^{STR} = \sum_{t=0}^{T-1} \beta^t v_t' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} v_t \quad (\text{A.61})$$

and

$$V_2^{STR} = \sum_{t=T}^{\infty} \beta^t v_t' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} v_t \quad (\text{A.62})$$

Define

$$M = (B - \varphi A)^{-1} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \quad (\text{A.63})$$

Then

$$\begin{aligned} V_2^{STR} &= \sum_{t=T}^{\infty} \beta^t q' M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q \varphi^{2(t-T)} = q' M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q \varphi^{-2T} \left(\sum_{t=T}^{\infty} \beta^t \varphi^{2t} \right) \\ &= \frac{(\beta \varphi^2)^T}{1 - \beta \varphi^2} \varphi^{-2T} q' M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q = \frac{\beta^T}{1 - \beta \varphi^2} \text{tr} \left(M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q q' \right) \end{aligned} \quad (\text{A.64})$$

where

$$M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M = \begin{pmatrix} \alpha_2 m_{11}^2 + \alpha_1 m_{21}^2 & \alpha_2 m_{11} m_{12} + \alpha_1 m_{21} m_{22} \\ \alpha_2 m_{11} m_{12} + \alpha_1 m_{21} m_{22} & \alpha_2 m_{12}^2 + \alpha_1 m_{22}^2 \end{pmatrix} \quad (\text{A.65})$$

Since

$$q q' = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad (\text{A.66})$$

we obtain

$$\text{tr} \left(M' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} M q q' \right) = \alpha_2 m_{12}^2 + \alpha_1 m_{22}^2 \quad (\text{A.67})$$

The definition of the matrices A and B implies

$$B - \varphi A = \begin{pmatrix} 1 + \frac{\delta_x}{\sigma} - \varphi & \frac{\delta_\pi}{\sigma} - \frac{\varphi}{\sigma} \\ -\kappa & 1 - \varphi\beta \end{pmatrix} \quad (\text{A.68})$$

$$\Delta = |B - \varphi A| = \left(1 + \frac{\delta}{\sigma} - \varphi\right) (1 - \varphi\beta) + \kappa \left(\frac{\delta_\pi}{\sigma} - \frac{\varphi}{\sigma}\right) = \frac{1}{\sigma} b \quad (\text{A.69})$$

where

$$b = (1 - \varphi)(1 - \varphi\beta)\sigma + \delta_x(1 - \varphi\beta) + \kappa(\delta_\pi - \varphi) > 0 \quad \text{if } \delta_\pi > 1 \text{ and } \delta_x > 0 \quad (\text{A.70})$$

Then

$$M = (B - \varphi A)^{-1} = \frac{1}{b} \begin{pmatrix} \sigma(1 - \varphi\beta) & -(\delta_\pi - \varphi) \\ \sigma\kappa & \sigma(1 - \varphi) + \delta_x \end{pmatrix} \quad (\text{A.71})$$

so that

$$m_{12} = -\frac{1}{b}(\delta_\pi - \varphi), \quad m_{22} = \frac{1}{b}[\sigma(1 - \varphi) + \delta_x] \quad (\text{A.72})$$

and

$$V_2^{STR} = \frac{\beta^T}{1 - \beta\varphi^2} \frac{1}{b^2} [\alpha_2(\delta_\pi - \varphi)^2 + \alpha_1(\sigma(1 - \varphi) + \delta_x)^2] \quad (\text{A.73})$$

The loss function $V_2^{STR} = V_2^{STR}(T)$ has the same properties as the corresponding function under discretion ($V_2^D(T)$).

To calculate the loss V_1^{STR} , set

$$Q = [I_{2 \times 2} - \varphi P]^{-1} \quad (\text{where } P = B^{-1}A) \quad (\text{A.74})$$

and

$$\tilde{q} = B^{-1}q \quad (\text{A.75})$$

Then

$$v_t = -QP^{T-t}\tilde{q} \quad \text{for } t \leq T \quad (\text{A.76})$$

and

$$\begin{aligned} V_1^{STR} &= \tilde{q}' \left(\sum_{t=0}^{T-1} \beta^t (P^{T-t})' Q' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} Q P^{T-t} \right) \tilde{q} \\ &= \tilde{q}' \left(\sum_{k=1}^T \beta^{T-k} (P^k)' Q' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} Q P^k \right) \tilde{q} \\ &= \beta^T \tilde{q}' \tilde{W} \tilde{q} = \beta^T \text{tr}(\tilde{W} \tilde{q} \tilde{q}') \end{aligned} \quad (\text{A.77})$$

where

$$\begin{aligned} \tilde{q} \tilde{q}' &= B^{-1} q q' (B^{-1})' = B^{-1} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} (B^{-1})' \\ &= \frac{1}{(\sigma + \delta_x + \kappa\delta_\pi)^2} \begin{pmatrix} \delta_\pi^2 & -\delta_\pi(\sigma + \delta_x) \\ -\delta_\pi(\sigma + \delta_x) & (\sigma + \delta_x)^2 \end{pmatrix} \end{aligned} \quad (\text{A.78})$$

and

$$\tilde{W} = \sum_{k=1}^T \beta^{-k} (P^k)' Q' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} Q P^k \quad (\text{A.79})$$

\tilde{W} satisfies the following matrix equation. Let

$$\tilde{D} = Q' \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} Q \quad (\text{A.80})$$

The definition of \tilde{W} then implies

$$\begin{aligned} \tilde{W} &= \beta^{-1} P' \tilde{D} P + \sum_{k=2}^T \beta^{-k} (P^k)' \tilde{D} P^k \\ &= \beta^{-1} P' \tilde{D} P + \sum_{k=1}^{T-1} \beta^{-(k+1)} (P^{k+1})' \tilde{D} P^{k+1} \\ &= \beta^{-1} P' \tilde{D} P - \beta^{-(T+1)} (P^{T+1})' \tilde{D} P^{T+1} + \beta^{-1} P' \tilde{W} P \end{aligned} \quad (\text{A.81})$$

or in compact representation

$$\tilde{W} = \tilde{H} + \beta^{-1} P' \tilde{W} P \quad (\text{A.82})$$

where

$$\tilde{H} = \beta^{-1} P' \tilde{D} P - \beta^{-(T+1)} (P^{T+1})' \tilde{D} P^{T+1} \quad (\text{A.83})$$

To solve for \tilde{W} , use the vectorization of a matrix and the Kronecker product of matrices. Since

$$\text{vec}(\beta^{-1} P' \tilde{W} P) = [\beta^{-1} P' \otimes P'] \text{vec} \tilde{W} \quad (\text{A.84})$$

we obtain

$$\text{vec} \tilde{W} - [\beta^{-1} P' \otimes P'] \text{vec} \tilde{W} = \text{vec} \tilde{H} \quad (\text{A.85})$$

with the solution

$$\text{vec} \tilde{W} = [I_{4 \times 4} - \beta^{-1} P' \otimes P']^{-1} \text{vec} \tilde{H} \quad (\text{A.86})$$

where

$$\begin{aligned} \text{vec} \tilde{H} &= \text{vec}(\beta^{-1} P' \tilde{D} P) - \text{vec}(\beta^{-(T+1)} (P^{T+1})' \tilde{D} P^{T+1}) \\ &= \left([\beta^{-1} P' \otimes P'] - [\beta^{-(T+1)} (P^{T+1})' \otimes (P^{T+1})'] \right) \text{vec} \tilde{D} \end{aligned} \quad (\text{A.87})$$

and

$$\begin{aligned} \text{vec} \tilde{D} &= Q' \otimes Q' \text{vec} \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix} \\ &= ([I_{2 \times 2} - \varphi P]^{-1})' \otimes ([I_{2 \times 2} - \varphi P]^{-1})' \begin{pmatrix} \alpha_2 \\ 0 \\ 0 \\ \alpha_1 \end{pmatrix} \end{aligned} \quad (\text{A.88})$$

Note that $\text{vec} \tilde{D}$ equals $\text{vec} \begin{pmatrix} \alpha_2 & 0 \\ 0 & \alpha_1 \end{pmatrix}$ in the special case $\varphi = 0$. The development of V_1^{STR} as function in T is analogous to the loss function $V_1^D(T)$. Therefore, the total loss function $V^{STR}(T) = V_1^{STR}(T) + V_2^{STR}(T)$ has the same properties as the total loss under discretion.

B Appendix to Chapter 3

The Stochastic Case

Assume that ν_{t+1} is an $r \times 1$ vector of independent and identically distributed white noise disturbances with variance-covariance matrix $\Sigma_{\nu\nu} = E(\nu_t \nu_t')$. The i.i.d shocks are, by definition, unpredictable ($\tau = 0$) and occur at time $t = 0$. Since $E_t(\nu_{t+1}) = 0_{r \times 1}$, equation (3.7) implies

$$F \cdot E_t \begin{pmatrix} \tilde{w}_{t+1} \\ \tilde{v}_{t+1} \end{pmatrix} = G \begin{pmatrix} \tilde{w}_t \\ \tilde{v}_t \end{pmatrix} \quad (\text{B.1})$$

The Schur decomposition yields the following system of equations

$$\begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix} E_t \begin{pmatrix} \tilde{z}_{t+1} \\ \tilde{x}_{t+1} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} \quad (\text{B.2})$$

where

$$\begin{pmatrix} \tilde{w}_t \\ \tilde{v}_t \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} \tilde{z}_t \\ \tilde{x}_t \end{pmatrix} \quad (\text{B.3})$$

and $\tilde{x}_t = 0$ for all $t \geq T = 0$. Partitioning the matrices A and B in equation (3.1) to conform with the dimension of w_t and v_t , i.e.

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} \quad (\text{B.4})$$

Equation (3.1) then implies

$$A_{11}w_{t+1} + A_{12}E_t v_{t+1} = B_{11}w_t + B_{12}v_t + C_1u_t + D_1\nu_{t+1} \quad (\text{B.5})$$

and

$$A_{11}E_t w_{t+1} + A_{12}E_t v_{t+1} = B_{11}w_t + B_{12}v_t + C_1u_t \quad (\text{B.6})$$

where

$$C = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}, \quad D = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} \quad (\text{B.7})$$

From (B.5) and (B.6) we get

$$A_{11}(w_{t+1} - E_t w_{t+1}) = D_1\nu_{t+1} \quad (\text{B.8})$$

so that

$$w_{t+1} - E_t w_{t+1} = A_{11}^{-1}D_1\nu_{t+1} \quad (\text{B.9})$$

holds (provided A_{11}^{-1} exists). The corresponding equation for the costate vector p_v is given by (Backus and Driffill, 1986)

$$p_{v,t+1} - E_t p_{v,t+1} = 0_{n_2 \times 1} \quad (\text{B.10})$$

Defining $\tilde{w}_t = (w'_t, p'_{vt})'$ and using equations (B.2) and (B.3) then imply

$$\tilde{w}_{t+1} - E_t \tilde{w}_{t+1} = Z_{11}(\tilde{z}_{t+1} - E_t \tilde{z}_{t+1}) = Z_{11}(\tilde{z}_{t+1} - S_{11}^{-1}T_{11}\tilde{z}_t) = \begin{pmatrix} A_{11}^{-1}D_1\nu_{t+1} \\ 0_{n_2 \times 1} \end{pmatrix} \quad (\text{B.11})$$

and therefore

$$\tilde{z}_{t+1} = (S_{11}^{-1}T_{11})\tilde{z}_t + Z_{11}^{-1} \begin{pmatrix} A_{11}^{-1}D_1\nu_{t+1} \\ 0_{n_2 \times 1} \end{pmatrix} = (S_{11}^{-1}T_{11})\tilde{z}_t + Z_{11}^{-1} \begin{pmatrix} A_{11}^{-1}D_1 \\ 0_{n_2 \times r} \end{pmatrix} \nu_{t+1} \quad (\text{B.12})$$

The solution of the VAR(1) process (B.12) has the general form

$$\tilde{z}_t = (S_{11}^{-1}T_{11})^t K + \sum_{s=0}^{t-1} (S_{11}^{-1}T_{11})^{t-s-1} Z_{11}^{-1} \begin{pmatrix} A_{11}^{-1}D_1 \\ 0_{n_2 \times r} \end{pmatrix} \nu_{s+1} \quad (\text{B.13})$$

where

$$K = \tilde{z}_0 = Z_{11}^{-1}\tilde{w}_0 = Z_{11}^{-1} \begin{pmatrix} w_0 \\ 0_{n_2 \times 1} \end{pmatrix} \quad (\text{B.14})$$

Since $E_0 \nu_{s+1} = 0$ the expected time path of \tilde{z}_t is given by

$$E_0 \tilde{z}_t = (S_{11}^{-1}T_{11})^t Z_{11}^{-1}\tilde{w}_0 \quad (\text{B.15})$$

Premultiplying equation (B.12) with Z_{11} and using $\tilde{w}_t = Z_{11}\tilde{z}_t$ to obtain the VAR(1) process

$$\tilde{w}_{t+1} = \Gamma\tilde{w}_t + \begin{pmatrix} A_{11}^{-1}D_1 \\ 0_{n_2 \times r} \end{pmatrix} \nu_{t+1} \quad (\text{B.16})$$

where

$$\Gamma = Z_{11}(S_{11}^{-1}T_{11})Z_{11}^{-1} \quad (\text{B.17})$$

Then

$$\tilde{w}_t = \Gamma^t \tilde{w}_0 + \sum_{s=0}^{t-1} \Gamma^{t-s-1} \begin{pmatrix} A_{11}^{-1}D_1 \\ 0_{n_2 \times r} \end{pmatrix} \nu_{s+1} \quad (\text{B.18})$$

and the expected future path of \tilde{w}_t is given by

$$E_0 \tilde{w}_t = \Gamma^t \tilde{w}_0 = \Gamma^t \begin{pmatrix} A_{11}^{-1}D_1 \\ 0_{n_2 \times r} \end{pmatrix} \nu_0 \quad (\text{B.19})$$

The solution to the forward-looking vector \tilde{v}_t follows from

$$\tilde{v}_t = Z_{21}\tilde{z}_t = Z_{21}Z_{11}^{-1}\tilde{w}_t = N\tilde{w}_t \quad (N = Z_{21}Z_{11}^{-1}) \quad (\text{B.20})$$

by inserting the solution time path of \tilde{w}_t .

In order to determine the minimum value of the loss function J_0 , set

$$\varepsilon_{t+1} = \begin{pmatrix} A_{11}^{-1} D_1 \\ 0_{n_2 \times r} \end{pmatrix} \nu_{t+1} \quad (\text{B.21})$$

According to (3.48), (3.51), (3.54), and (B.18) we then obtain

$$\begin{aligned} J_0 &= \frac{1}{2} \mathbf{E}_0 \sum_{i=0}^{\infty} \lambda^i \tilde{w}'_i H^* \tilde{w}_i \\ &= \frac{1}{2} \sum_{i=0}^{\infty} \lambda^i \left\{ (\Gamma^i \tilde{w}_0)' H^* (\Gamma^i \tilde{w}_0) + 2 \mathbf{E}_0 (\Gamma^i \tilde{w}_0)' H^* \left(\sum_{s=0}^{i-1} \Gamma^{i-s-1} \varepsilon_{s+1} \right) \right. \\ &\quad \left. + \mathbf{E}_0 \left(\sum_{s=0}^{i-1} \Gamma^{i-s-1} \varepsilon_{s+1} \right)' H^* \left(\sum_{s=0}^{i-1} \Gamma^{i-s-1} \varepsilon_{s+1} \right) \right\} \\ &= \frac{1}{2} \tilde{w}'_0 \left(\sum_{i=0}^{\infty} \lambda^i \Gamma^i H^* \Gamma^i \right) \tilde{w}_0 + \frac{1}{2} \sum_{i=0}^{\infty} \lambda^i \mathbf{E}_0 \left(\sum_{s=0}^{i-1} \Gamma^{i-s-1} \varepsilon_{s+1} \right)' H^* \left(\sum_{s=0}^{i-1} \Gamma^{i-s-1} \varepsilon_{s+1} \right) \quad (\text{B.22}) \end{aligned}$$

where we have used $\mathbf{E}_0 \varepsilon_{s+1} = 0$. $V = \sum_{i=0}^{\infty} \lambda^i \Gamma^i H^* \Gamma^i$ satisfies the matrix equation (cf. (3.74))

$$V = H^* + \lambda \Gamma' V \Gamma \quad (\text{B.23})$$

and

$$\frac{1}{2} \tilde{w}'_0 \left(\sum_{i=0}^{\infty} \lambda^i \Gamma^i H^* \Gamma^i \right) \tilde{w}_0 = \frac{1}{2} \tilde{w}'_0 V \tilde{w}_0 = \frac{1}{2} \text{trace}(V \tilde{w}_0 \tilde{w}'_0) \quad (\text{B.24})$$

To calculate the infinite sum in (B.22) note that

$$\begin{aligned} &\mathbf{E}_0 \left(\sum_{s=0}^{i-1} \Gamma^{i-s-1} \varepsilon_{s+1} \right)' H^* \left(\sum_{s=0}^{i-1} \Gamma^{i-s-1} \varepsilon_{s+1} \right) \\ &= \mathbf{E}_0 (\Gamma^{i-1} \varepsilon_1 + \Gamma^{i-2} \varepsilon_2 + \dots + \Gamma^0 \varepsilon_i)' H^* (\Gamma^{i-1} \varepsilon_1 + \Gamma^{i-2} \varepsilon_2 + \dots + \Gamma^0 \varepsilon_i) \\ &= \mathbf{E}_0 (\Gamma^{i-1} \varepsilon_1)' H^* (\Gamma^{i-1} \varepsilon_1) + \mathbf{E}_0 (\Gamma^{i-2} \varepsilon_2)' H^* (\Gamma^{i-2} \varepsilon_2) + \dots + \mathbf{E}_0 (\Gamma^0 \varepsilon_i)' H^* (\Gamma^0 \varepsilon_i) \\ &= \mathbf{E}_0 \varepsilon'_i (\Gamma^0 H^* \Gamma^0 + \Gamma' H^* \Gamma + \dots + \Gamma^{i-2'} H^* \Gamma^{i-2} + \Gamma^{i-1'} H^* \Gamma^{i-1}) \varepsilon_i \\ &= \mathbf{E}_0 \varepsilon'_i \left(\sum_{s=0}^{i-1} \Gamma^{i-s-1'} H^* \Gamma^{i-s-1} \right) \varepsilon_i \quad (\text{B.25}) \end{aligned}$$

since $\mathbf{E}_0(\varepsilon'_i \varepsilon_j) = 0$ for $i \neq j$. The variance-covariance matrix

$$\mathbf{E}_0(\varepsilon_i \varepsilon'_i) = \mathbf{E}_0(\varepsilon_j \varepsilon'_j) = \Sigma_{\varepsilon\varepsilon} \quad (\text{B.26})$$

is independent of i and j . We then obtain

$$\frac{1}{2} \sum_{i=0}^{\infty} \lambda^i \mathbf{E}_0 \left(\sum_{s=0}^{i-1} \Gamma^{i-s-1} \varepsilon_{s+1} \right)' H^* \left(\sum_{s=0}^{i-1} \Gamma^{i-s-1} \varepsilon_{s+1} \right) = \frac{1}{2} \frac{\lambda}{1-\lambda} \text{trace}(V \Sigma_{\varepsilon\varepsilon}) \quad (\text{B.27})$$

with V defined as in equation (B.23). The optimal value of the loss function J_0 in the stochastic case (with $T = 0$) is then given by

$$J_0 = \frac{1}{2} \text{trace}(V \tilde{w}_0 \tilde{w}'_0) + \frac{1}{2} \frac{\lambda}{1-\lambda} \text{trace}(V \Sigma_{\varepsilon\varepsilon}) \quad (\text{B.28})$$

Note that (B.28) is a generalization of equation (3.72) where we have assumed a deterministic shock in $t = 0$ ($\Sigma_{\varepsilon\varepsilon} = 0$). The formula (B.28) holds for a discount factor λ with $0 < \lambda < 1$.¹ The right-hand side of (B.28) is not defined in the special case $\lambda = 1$. If the discount factor λ approaches unity we must scale the intertemporal loss function J_0 by the factor $(1 - \lambda)$ (Rudebusch and Svensson, 1999). Equation (B.28) then implies

$$(1 - \lambda)J_0 = \frac{1}{2}(1 - \lambda) \text{trace}(V\tilde{w}_0\tilde{w}'_0) + \frac{1}{2}\lambda \text{trace}(V\Sigma_{\varepsilon\varepsilon}) \quad (\text{B.29})$$

The scaled intertemporal loss function $(1 - \lambda)J_0$ converges as λ approaches unity. (B.29) implies

$$\lim_{\lambda \rightarrow 1} (1 - \lambda)J_0 = \frac{1}{2} \text{trace}(V\Sigma_{\varepsilon\varepsilon}) \quad (\text{B.30})$$

Note that in the case $T = 0$ and $\lambda = 1$ the RHS of (B.30) equals the RHS of (3.72) provided $w_0w'_0 = \Sigma_{\varepsilon\varepsilon}$. In this special case the stochastic and deterministic case are equivalent. If the off-diagonal elements of W_1 and W_2 in the loss function (3.3) are equal to zero, then the limit value of $(1 - \lambda)J_0$ can be expressed as

$$\lim_{\lambda \rightarrow 1} (1 - \lambda)J_0 = \frac{1}{2} E(L_t) \quad (\text{B.31})$$

where $E(L_t)$ is the unconditional mean of the period-loss-function

$$L_t = (s'_t, u'_t) \begin{pmatrix} W_1 & 0 \\ 0 & W_2 \end{pmatrix} \begin{pmatrix} s_t \\ u_t \end{pmatrix} = \sum_{i=1}^{n_3} w_{ii,1} s_{i,t}^2 + \sum_{i=1}^m w_{ii,2} u_{i,t}^2 \quad (\text{B.32})$$

Then

$$E(L_t) = \sum_{i=1}^{n_3} w_{ii,1} \text{VAR } s_{i,t} + \sum_{i=1}^m w_{ii,2} \text{VAR } u_{i,t} \quad (\text{B.33})$$

The period-loss-function can also be written as

$$L_t = Y'_t H Y_t \quad (\text{B.34})$$

where $Y'_t = (k'_t, u'_t)$ and H as defined in (3.49). Then the unconditional period loss also fulfills

$$E(L_t) = E(Y'_t H Y_t) = \text{trace}(H\Sigma_{YY}) \quad (\text{B.35})$$

where Σ_{YY} is the unconditional variance-covariance matrix of the vector Y .

¹In the deterministic case, where $\Sigma_{\varepsilon\varepsilon} = 0$, (B.28) also holds for $\lambda = 1$.

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Personal

Born on January 26, 1977 in Bremen.

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Education

Doctoral Programme “Quantitative Economics”, Christian-Albrechts-Universität zu Kiel, since October 2002.

Diplom-Volkswirt (Diploma in Economics), Christian-Albrechts-Universität zu Kiel, 2002.

Abitur (General Higher Education Entrance Qualification), Hermann-Böse-Gymnasium Bremen, 1996.

Research Experience

Research and Teaching Assistant, Christian-Albrechts-Universität zu Kiel, Department of Economics, Professorship of Macroeconomics (H.-W. Wohltmann), since April 2003.

Student Assistant, Christian-Albrechts-Universität zu Kiel, Department of Economics, Professorship of Macroeconomics (H.-W. Wohltmann), April 2001 – March 2003.

Teaching Experience

Introduction to Macroeconomic Theory (basic study period), Winter 02/03, Winter 03/04, Winter 04/05, Winter 05/06, Winter 06/07, Winter 07/08, Winter 08/09.

Macroeconomics I (advanced study period), Summer 04, Summer 05, Summer 06, Summer 07, Summer 08.

Dynamic Macroeconomics with Mathematica (advanced study period), Summer 06.

Dynamic Macroeconomics with Matlab/Dynare (advanced study period), Winter 06/07, Summer 07, Winter 07/08, Winter 08/09.

Seminar on Macroeconomic Dynamics, Summer 05.

Seminar on Macroeconomics, Summer 06, Winter 06/07, Summer 07, Winter 07/08, Winter 08/09.

Seminar on Stabilization Policy in Open Economies, Winter 05/06, Summer 08.

Seminar on Transmission Mechanisms, Summer 06.

Working Papers

“Rational Expectations Models with Anticipated Shocks and Optimal Policy: A General Solution Method and a New Keynesian Example,” with Hans-Werner Wohltmann (2009).

“On the Non-Optimality of Information: An Analysis of the Welfare Effects of Anticipated Shocks in the New Keynesian Model,” with Hans-Werner Wohltmann (2008).

“Anticipated and Unanticipated Oil Price Shocks and Optimal Monetary Policy,” with Hans-Werner Wohltmann (2008).

“Solution of RE Models with Anticipated Shocks and Optimal Policy,” with Hans-Werner Wohltmann (2007).

“Anticipated Raw Materials Price Shocks and Monetary Policy Response - A New Keynesian Approach,” with Hans-Werner Wohltmann (2006).

“Monetary Policy Dynamics in Large Oil-Dependent Economies,” with Hans-Werner Wohltmann (2005).

“Oil Price Shocks and Currency Denomination,” with Hans-Werner Wohltmann (2005).

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Publications

“Das Grundmodell der Neukeynesianischen Makroökonomik” (The Baseline New Keynesian Model), *Das Wirtschaftsstudium* 37, 1210 - 1220, with Hans-Werner Wohltmann (2008).

“Optimale Zinspolitik im Grundmodell der Neuen Keynesianischen Makroökonomik” (Optimal Monetary Policy in the Baseline New Keynesian Model), *Das Wirtschaftsstudium*, with Hans-Werner Wohltmann (forthcoming).

Conferences and Workshops

Presentation at the Annual Meeting of the Verein für Socialpolitik (German Economic Association) 2007, Munich, October 9 - 12, “Anticipated Raw Materials Price Shocks and Simple Monetary Policy Rules”, October 11, 2007.

Dynare Workshop, Deutsche Bundesbank, Frankfurt (Main), April 2007.

Memberships

Econometric Society, European Economic Association, Verein für Socialpolitik (German Economic Association)

Honors & Awards

Erich-Schneider-Award (awarded by the directors of the Department of Economics, CAU Kiel) for the Diploma thesis “Schocks und Schockverarbeitung in einer asymmetrischen Währungsunion - Dynamische Ansätze und empirische Überprüfung” (Shocks and Stabilization Policy in an Asymmetric Monetary Union), 2003.

Erich-Schneider-Award (awarded by the directors of the Department of Economics, CAU Kiel) for the best Diploma in the summer term 2002.

Further Qualifications

Computer Skills: MATLAB, DYNARE, *Mathematica*, T_EX, L^AT_EX.

Foreign Language: English

Kiel, February 26, 2009

Eidesstattliche Erklärung

Ich erkläre hiermit an Eides Statt, dass ich meine Doktorarbeit “Anticipated Shocks, Optimal Monetary Policy, and Welfare” selbständig und ohne fremde Hilfe angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen nach den mir angegebenen Richtlinien zitiert habe.

Kiel, 27. Februar 2009

(Roland Winkler)