

APPLICATIONS OF AGENT-BASED MODELS AND
NONLINEAR ECONOMETRICS IN FINANCE

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1 Introduction

Several financial crises have shown that our financial system might be fragile and not as efficient as claimed by many financial economist and practitioners and a lot of policy questions have arosen from this crisis, e.g. : Are financial markets inefficient and which regulations increase the efficiency of financial markets?

Academic questions as well emerged from the financial crisis, e.g.: Which economic models are able to explain financial instabilities and stylized facts of financial market data? Which are the building blocks a model of financial fragility should consist of? Which behavioral trading rules destabilize financial markets? How should we build econometric models for modeling and forecasting financial instabilities? Do we need new methodologies, e.g. nonlinear models?

The advocates of agent-based financial market models began in the 1990's with the construction of models for addressing questions about financial instability. This branch of the finance literature is inspired by *(i)* questionnaire surveys among financial market experts about their trading behavior (Taylor and Allen 1992; Menkhoff and Taylor 2007), and *(ii)* by the empirical finance literature (De Vries 1994; Pagan 1996) which has found a catalogue of stylized fact of daily financial market data, which are robust to different asset classes, markets and time periods (Lux 2009a,

2009b). Important contributions to agent-based financial market models are Brock and Hommes (1997, 1998), Kirman (1991, 1993), Lux (1995, 2009a, 2009b), and Lux and Marchesi (1999, 2000). They are surveyed in Westerhoff (2008b), Hommes (2006), LeBaron (2006) and Lux (2009a, 2009b). Figure 1.1 contains a flow chart which represents the integration of this dissertation project into the literature on agent-based models, the literature on regulations of financial markets and the literature on econometric modeling of financial market instabilities. All of these branches of the literature have in common, that multiple equilibria lead to financial market instabilities, which is also claimed by the literature on the second generation of currency crisis models (Jeanne 1997).

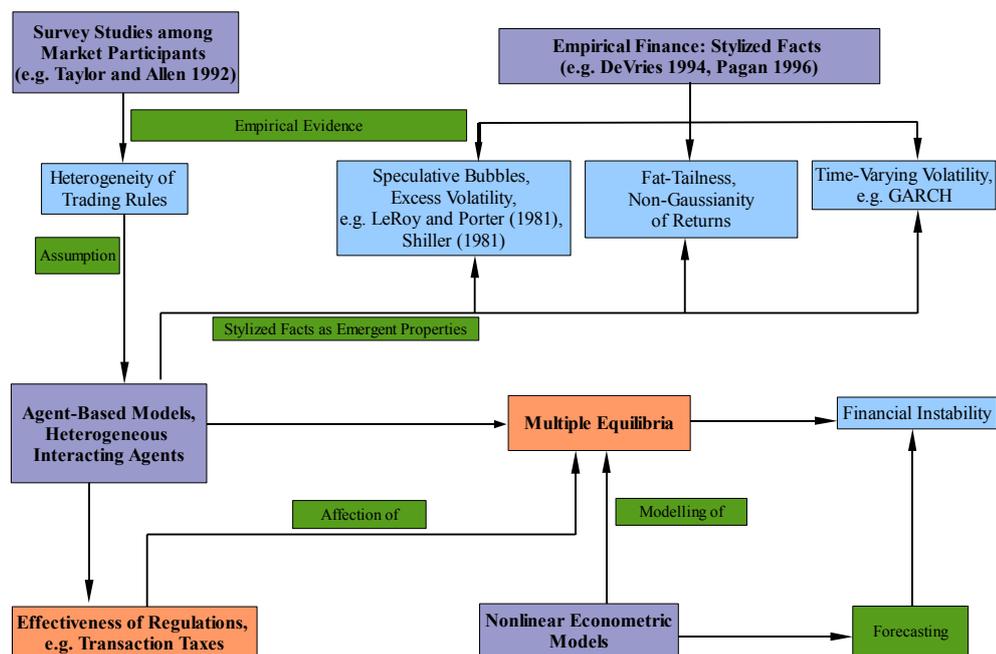


Figure 1.1: Integration of the Dissertation Project into the Literature

Agent-based financial market models base their assumptions on agents' behavior on the empirical evidence of the heterogeneity of trading rules in real financial markets. The first survey among financial market experts is Taylor and Allen (1992) who find that market practitioners use trading rules which are based on economic fundamentals as well as so called technical trading rules or chartist trading rules, which extrapolate past trends and trend reversals into the future, and which ignore economic fundamentals. Taylor and Allen (1992) find that 42 percent of surveyed experts rely on chartist trading rules on short-term investment horizons. This heterogeneity is neglected by classical financial theory.

Agent-based financial market models aim to explain the empirical stylized facts of daily financial market data by the evolution and interaction of competing trading rules. In these models agents switch to the trading strategies, which were most successful in the recent past (Brock and Hommes 1997) or they are influenced by social interactions (Kirman 1991, 1993; Lux 2005; Lux and Marchesi 1999, 2000). The empirical stylized facts emerge from the coordination of the individual trading rules (Lux 2009a). Empirical stylized facts are following Lux (2009a), Westerhoff (2008) and Lux and Marchesi (2000) *(i)* the absence of autocorrelations in daily asset returns, *(ii)* the long-lasting autocorrelations in the magnitude of asset returns which lead to the emergence of volatility clusters, *(iii)* the deviation of the return distribution from a normal distribution with more probability mass in the vicinity of the mean and in the tails compared to the Gaussian distribution, and *(iv)* the frequent emergence and bursts of speculative bubbles, which lead to the observation that asset prices are more volatile compared to their fundamental values (LeRoy and Porter 1981; Shiller 1981). Classical financial theory which concentrates on the optimizing behavior of one representative agent is unable to explain these empirical

stylized facts.

Colander et al. (2009) criticize the neglect of the evolution of heterogeneous trading rules and their contribution to the emergence of stylized facts in classical financial theory. These authors state, that neoclassical financial theory is incapable of addressing questions about the coordination of economic agents and the possibility of coordination failures by neglecting these factors. Hence, Colander et al. (2009) demand the following requirements for theoretical model building and empirical analysis:

- Theoretical and empirical soundness of models applied to economic policy analysis. Models should be based on sound empirical assumptions about traders behavior and they should find support in the data by replicating the empirical stylized facts.
- New time series models should be developed for detecting speculative bubbles and the emergence of non-fundamental equilibria for forecasting future financial instabilities.

Within this dissertation work I try to meet both goals. This cumulative dissertation work consists of four essays, which all possess an abstract, the Journal of Literature Classification and their own bibliography. Two of them are already published, while one is under review and one is still a working paper up to now. The first three papers apply agent-based financial market models to questions about sound regulations of financial markets, here the example of Tobin-type transaction taxes. In the last paper I introduce a simple nonlinear empirical model, which is shown to be a reduced form of the agent-based model of Lux (1995), which I apply for modeling multiple equilibria in real exchange rates and for forecasting financial instabilities.

The first essay (Demary 2008) is based on my earliest research on agent-based financial market models. Its title is *"Who Does a Currency Transaction Tax Harm More: Short-Term Traders or Long-Term Investors?"*, which is published in *Jahrbücher für Nationalökonomie und Statistik*, Vol. 228, pp. 228-250 in the year 2008. Here, I propose a novel model of the foreign exchange market with heterogeneous interacting agents. Traders place their buy or sell orders in response to the interest rate differential between two countries. They are allowed to select endogenously between two different forecasting models and are moreover allowed to be short-term speculators or long-term investors.¹ Within this model framework I study the effectiveness of currency transaction taxes on exchange rate volatility and traders' behavior. The numerical analysis yields the result that these taxes reduce the volatility of exchange rate returns, but increase their kurtosis. Moreover, taxation does not reduce the misalignment. The second result is that the tax harms short-term speculation in favor of longer term investments, while it also harms trading rules based on economic fundamentals in favor of trend-extrapolating trading rules. However, these results are only valid when agents trade aggressively. Otherwise, taxation is not necessary.

The second essay (Demary 2010) is entitled *"Transaction Taxes and Traders with Heterogeneous Investment Horizons in an Agent-Based Financial Market Model"*. It is published in *Economics: The Open-Access, Open-Assessment E-Journal*, Vol. 4, 2010-8 in the year 2010. This agent-based financial market model is a generalization of the model of Westerhoff (2008a) by traders who are allowed to have different in-

¹Arnoufrieu and Botazzi (2004) is an earlier attempt of modeling heterogeneous investment horizons in an agent-based stock market model. However, their model differs from my model in several aspects, e.g. the different formulation of the long-term chartist trading rule. Moreover, it concentrates on the stability properties of the fundamental equilibrium and not onto the working of regulations.

vestment horizons similar to Demary (2008).² My research goals are, first, to study what consequences the introduction of heterogeneous investment horizons has for agent-based financial market models, and second, how effective transaction taxes are in stabilizing financial markets. Numerical simulations reveal that under sufficiently small tax rates traders abstain from short-term trading in favor of longer term investment horizons. This change in behavior leads to less volatility and less mispricings. When the tax rate exceeds a certain threshold, however, mispricings increase as also found in Westerhoff (2003, 2008). This emergent property is due to the fact that taxation reduces short-term fluctuations and causes longer lasting trends in the exchange rate. As a result, the longer term fundamentalist trading rule becomes unpopular in favor of the longer term trend-chasing rule.

The third essay is entitled "*Transaction Taxes, Greed and Risk Aversion in an Agent-Based Financial Market Model*". It is under review in the *Journal of Economic Interaction and Coordination*. Recent agent-based financial market models come to the result that taxing financial transactions does not *per se* increase financial stability (Westerhoff 2003, 2008a; Demary 2008, 2010). However, the response of volatility and misalignments to rising tax rates seem to be *u*-shaped. Moreover, greed and the risk appetite of traders are often blamed for financial instability and there is no evidence how greed and risk aversion affect the effectiveness of regulations in financial markets, e.g. how transaction taxes work in a market with risk averse traders compared to a market with less risk averse traders. I aim to fill this gap in the literature by analyzing how different degrees of risk aversion and aggressiveness measured by behavioral parameters of traders affect the working of transaction taxes

²The difference between the models of Westerhoff (2008a) and Demary (2008) is that traders do not trade on interest differentials in Westerhoff (2008a) but on return expectations.

within an agent-based framework. My simulations indicate that a tax rate of 0.1 percent demarcates the stabilizing tax regime from the destabilizing one. I figure out that the shape of the volatility and misalignment response curves to taxation depend on behavioral parameters of the model. Higher transaction taxes are necessary, either when chartists trade more aggressively, fundamentalists trade less aggressively, agents switch more frequently between trading strategies or only have short memory in their fitness measures. A lower risk aversion of agents, however, makes higher tax rates more effective as indicated by a flatter volatility response curve.

The last essay is entitled "*Speculative Bimodality in Real Exchange Rates: Estimation, Testing and Forecasting*" and deals with the estimation of stochastic differential equations with a cubic drift for modeling and forecasting financial instability. The cubic drift model can be interpreted as a reduced form of the agent-based financial market model of Lux (1995). Bimodal distributions emerge from this type of differential equation for a certain interval of the models' parameters. I estimate these models for 14 monthly real exchange rates taken from the European Central Bank's database covering the time from 1993:01 to 2009:02. Parameter estimates give evidence for bimodal exchange rate distributions for fourteen exchange rates, two cubic-drift models and two estimation periods. Forecasts of the exchange rate density predict the exchange rate to be in the vicinity of one of the two attractors for most of the time. However, there are periods, in which both attractors are covered with probability mass indicating uncertainty about the equilibrium exchange rate. The models identify the beginning of the Exchange Rate Mechanism II and the introduction of the Euro as two major phase transitions. Based on tests on unconditional and conditional predictive accuracy I find that the cubic-drift models outperform the random walk in out-of-sample forecasting for one out of fourteen real

exchange rates. I conclude that bimodality is a good description of real exchange rates.

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2 Who Does a Currency Transaction Tax Harm More: Short-Term Speculators or Long-Term Investors?

Chapter originally published as

Demary, Markus (2008), "Who Does a Currency Transaction Tax Harm More: Short-Term Speculators or Long-Term Investors?", *Jahrbücher für Nationalökonomie und Statistik*, Vol. 228, pp. 228-250.

Abstract. We propose a novel heterogeneous interacting agents model in which traders are allowed to select endogenously between two different forecasting models and are moreover allowed to be short-term speculators or long-term investors. Within this model framework we study the effects of currency transaction taxes on exchange rate volatility and traders' behavior measured by their population fractions. The numerical analysis yields the result that these taxes reduce the variance of exchange rate returns, but increases their kurtosis. Moreover taxation does not lead to a reduction in the misalignment. The second result is, the tax harms short-term speculation in favor of long-term investments, while it also harms trading rules based on economic fundamentals in favor to trend-extrapolating trading rules. But these results are only valid if agents trade very aggressively. Otherwise taxation is not necessary.

Key Words: *Currency Transaction Taxes, Financial Market Volatility, Foreign Exchange Markets, Heterogeneous Interacting Agents*

JEL Classification: *C15, F31, F32, G15, G18*

2.1 Introduction

Foreign exchange markets are excessively volatile and risky due to speculative bubbles and crashes. These transitory bubbles and crashes do not reveal rational arbitrage-free pricing behavior but might be due to irrational and trend-chasing behavior of speculators. Because trend-chasing behavior and short-term speculation leads to excessive risks, policy instruments like currency transaction taxes were proposed for reducing speculative attacks and excessive risks. Tobin (1978) proposes following Keynes (1936) the introduction of an internationally uniform tax on all currency transactions. This tax should reduce the profitability of speculation and should lead to a stabilization of the exchange rate. The proponents of this policy instrument claim that the proportional taxation of foreign currency transactions does not harm international trade but penalizes high frequency speculation. This argument can be summarized in the following proposition:

Currency transaction taxes crowd out short term speculators in favor of long term investors.

In order to analyze this proposition we have to consider the following requirements and assumptions: (i) we need a model framework in which we are able to distinguish between short-term speculators and long-term investors, (ii) the number of short-term speculators and long-term investors should not be fixed, but traders should change groups or leave the market, and (iii) the model should be able to match empirical properties of foreign exchange market data for performing an economic policy analysis which is close to reality.

For fulfilling requirements (i), (ii) and (iii) the most appealing framework is an

agent-based model of the financial market that considers behavioral heterogeneity of agents.

Survey data of foreign exchange markets yields empirical evidence for behavioral heterogeneity and heterogeneous expectations among traders. This empirical finding gives an empirical basis for the chartist-fundamentalist agent-based model framework. Due to survey studies like the one conducted by Taylor and Allen (1992), these short term expectations display trend chasing behavior and are destabilizing for this reason, while long term expectations are regressive and therefore of a stabilizing nature. Based on this empirical fact, several studies like Brock and Hommes (1997, 1998), Chiarella and He (2002), and Lux and Marchesi (2000) start to incorporate heterogeneous expectations into economic models of the financial market, while only a few like Westerhoff (2003), Grimaldi (2004) and DeGrauwe and Grimaldi (2006) concentrate on exchange rates. Econometric tests on rational expectations in the foreign exchange market¹ are usually rejected and cannot explain stylized facts of financial market time series. For this reason researchers switched to the chartist-fundamentalist approach based on the empirical evidence of heterogeneous expectations from survey studies. This model framework uses the alternative expectations hypothesis of behavioral heterogeneity which is an appealing building block for models of the foreign exchange market. It assumes that traders are bounded rational in that they do not use all available information and economic models to forecast the future exchange rate because the true complex nature of the economy is not known to them. Instead agents rely on simple rules of thumb in order to approximate the structure of the model. Most of these interacting agent models assume that the market is populated by two types of traders. The chartist trader

¹See Taylor and Allen (1992) and Menkhoff (1997) among others.

type searches for patterns in past exchange rates like trends and trend reversals for forecasting future rates, while fundamentalist traders search for over- and undervaluations of the exchange rate with respect to its fundamental value and expect the misalignment to be corrected in the distant future. Moreover, this approach allows agents to choose endogenously one of this two views of the world. Moreover, there is also empirical evidence for the chartist-fundamentalist approach². Researchers who use these quantitative models compare model implied statistics and numbers like autocorrelations, standard deviations and distributional properties with those emerging from financial market data. The overall success of heterogeneous agents models to explain empirical facts encourages to elaborate on them.

Studies like Westerhoff (2003) use the chartist-fundamentalist approach for economic policy analysis by analyzing the effects of market regulations in foreign exchange markets. Westerhoff finds that small currency transaction taxes lower exchange rate volatility while a high Tobin tax rate leads to an increase. He explains this finding by a change in the composition of chartists and fundamentalists through the tax levied on financial market transactions. The reason for this finding is that small transaction taxes make destabilizing chartism unprofitable and increase the fraction of fundamentalist traders which stabilizes the exchange rate. If the tax rate exceeds a certain threshold also fundamentalism will be unprofitable and the fraction of chartist traders will rise, such that a destabilization of the exchange rate occurs and volatility will rise again. Mannaro et al. (2006) find in their simulation study within an artificial stock market framework that volatility will fall by 2% for a tax

²Engle and Hamilton (1990) find that there is regime switching in exchange rates in that there are phases of trends and mean-reversion. Vigfusson (1997) finds empirical evidence by estimating parameters of the chartist-fundamentalist model in a Markov-switching framework. Other studies that try to estimate chartist-fundamentalist models are Alfarano et al. (2005), Boswijk et al. (2007), Gilli and Winkler (2003) and Westerhoff and Reitz (2003).

rate of 0.1%, while it will fall by 8% for a tax rate of 0.5% with respect to the reference situation without taxes. Moreover, the percentage fraction of fundamentalists will rise due to the imposition of the transaction tax. In a simulation with only random traders and chartists a small tax can also lead to a small increase in volatility. Bianconi et al. (2006) analyze the effectiveness of Tobin taxes in minority games, while Ehrenstein et al. (2005) analyze the impact of this tax in a model framework that goes back to the percolation model of Bouchaud and Cont (2000). Moreover, Frankel (1996), Palley (1999) and Westerhoff and Dieci (2006) are papers that analyze the effectiveness of currency transaction taxes. The latter one proposes a model with bounded rational agents that apply either technical or fundamental analysis for their trading decisions. In contrast to other models traders are allowed to trade in two different markets. The decision of each agent to enter one of the two markets and to apply either technical or fundamental analysis again depends on profit considerations. This model is able to match empirical findings of financial market data which makes it usable for economic policy analysis. One of the model's implications is that taxation of only one market decreases the volatility of the taxed market while the volatility in the other market increases. The reason for this is that destabilizing technical traders move to the market in which the tax is not levied. The other result of this study is that if the tax is levied in both markets it is able to stabilize both markets.

In this paper we want to introduce an extended version of the chartist-fundamentalist model of the foreign exchange market. Our model is similar to the models of Brock and Hommes (1997, 1998), Chiarella and He (2002) and DeGrauwe and Grimaldi (2006) among others. In contrast to these models we allow agents to choose between different investment horizons, such that there are short-term chartists and funda-

mentalists and long-term chartists and fundamentalists. Moreover, we also deviated from the commonly used discrete choice model for the evolution of trading rules and introduce another evolutionary mechanism that also allows traders to choose between different investment horizons. A typical simulation run of the baseline model without taxes reveals that the model is able to match empirical properties of exchange rate data. This makes the model able for performing an economic policy analysis. This is the second task of our paper. We employ the model for economic policy analysis by introducing taxes on foreign exchange market transactions into the model. Our research goal is to analyze, how these taxes influence traders' behavior and financial market risks. Simulations yield the result that currency transaction taxes reduce the variance of exchange rate returns but also increase their kurtosis. Moreover, the tax harms short-term speculation in favor of long-term investment, while it also harms trading rules based on economic fundamentals in favor to trend-extrapolating trading rules. But these results are only valid under a scenario where the behavioral parameters of fundamentalists and chartists are large. If these parameters are assumed to be small the market will be dominated by fundamentalists and long-term traders so that a transaction tax will not result in more stable exchange rates.

The remainder of this paper is organized as follows: the next section presents the model economy, while section three will present model's fundamental equilibrium. The following section will contain the numerical analysis of the model, while section five concludes.

2.2 The Model Economy

This section presents the model's building blocks. This quantitative model is similar to the one proposed by DeGrauwe and Grimaldi (2006) with the difference that here traders are allowed to choose between being a short-term speculator or a long-term investor. The model's building blocks are (i) the fundamental determinants of the exchange rate, (ii) the agents' portfolio selection problem, (iii) the agents' forecasts of the future exchange rate via different forecasting models, (iv) agents' evaluation of these portfolio rules by a comparison of their past profitability, (v) the exchange rate is set by a market maker in contrast to DeGrauwe and Grimaldi (2006), and (vi) a policy maker sets the currency transaction tax rate. In the following subsections we will discuss these building blocks in more detail.

2.2.1 Fundamental Factors and Arbitrage

We assume that the fundamental factors driving the exchange rate are the gross rates of return on the domestic and foreign bond with one-period maturity. We assume both interest rates $R = (1 + r)$ to follow stochastic mean-reverting processes of the form

$$\ln R_{t+1} = (1 - \alpha) \ln \bar{R} + \alpha \ln R_t + \sigma \varepsilon_{t+1}, \quad \varepsilon_t \sim \mathcal{N}(0, 1), \quad (2.1)$$

where \bar{R} is the long-run average interest rate, $\alpha \in [0, 1]$ is the rate of mean-reversion and ε_t is a random innovation to the interest rate. We choose the logarithmic version in order to guarantee that interest rates remain positive.

Similar, the rate of return on the foreign one-period-bond follows

$$\ln R_{t+1}^* = (1 - \alpha^*) \ln \bar{R}^* + \alpha^* \ln R_t^* + \sigma^* \varepsilon_t^*, \quad \varepsilon_{t+1}^* \sim \mathcal{N}(0, 1), \quad (2.2)$$

where foreign variables are denoted with an asterisk.

Assuming homogeneous interest rate expectations and that all agents know the true data generating processes for the interest rates, long-term bonds can be priced according to the expectations hypothesis of the term structure. The expectations hypothesis of the term structure well known from many macroeconomics textbooks (see Romer 2005 for example) states that no arbitrage should be possible between the rates of return of a long-term bond and the rates of return of a sequence of one-period bonds over the maturity of the long-term bond. This consideration gives us the following valuation formula for long-term bonds

$$\ln R_{t,N} = \frac{1}{N} \sum_{n=0}^{N-1} \mathbb{E}_t \ln R_{t+n}. \quad (2.3)$$

Using the fact that the n -period-ahead forecast of the autoregressive process for the interest rate is given by

$$\mathbb{E}_t \ln R_{t+n} = \alpha^n \ln R_t + (1 - \alpha^n) \ln \bar{R} \quad (2.4)$$

and applying the rule for the finite geometric series yields the following long-term interest rate

$$\ln R_{t,N} = \frac{1}{N} \left\{ \frac{1 - \alpha^N}{1 - \alpha} \cdot \ln R_t + N - \frac{\alpha^N - \alpha}{1 - \alpha} \ln \bar{R} \right\}. \quad (2.5)$$

Figure 2.1 shows the time series of short-term and long-term interest rates of a

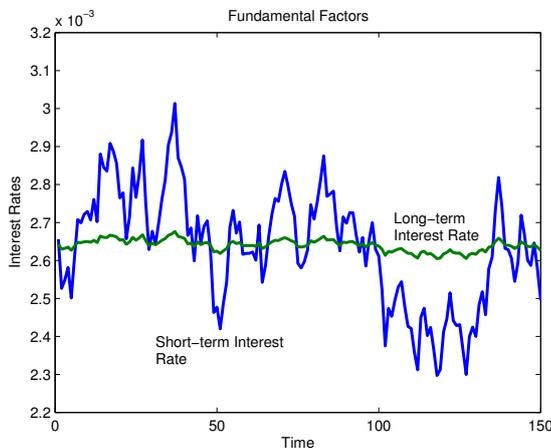


Figure 2.1: Simulated Fundamental Factors

Note: Model generated time series according to the formulas (2.1) and (2.3). The used parameter values are those given in table 2.1.

typical simulation run.

The second building block is the uncovered interest rate parity condition which claims that the interest rate differential between two countries should be offset by the agents' expected exchange rate change. Deviations from the no-arbitrage interest rate parity condition

$$\frac{E_t^i s_{t+n}}{s_t} = \left(\frac{R_{t,n}}{R_{t,n}^*} \right)^n \quad (2.6)$$

arise because interest rates follow stochastic processes. This deviation promises profits for foreign exchange traders and provokes them to demand foreign currency in the financial market. With s_t we denote the bilateral exchange rate. If this equation holds with equality the expected exchange rate change will be offset by the interest rate differential and no trade will occur. We call the foreign exchange rate s_{t+1} under which no arbitrage is possible the fundamental exchange rate. This is due

to the fact that it is completely determined by the model's economic fundamentals which are the domestic and the foreign interest rates.

2.2.2 Traders' Demand for Foreign Currency

The next goal in building a quantitative model of the foreign exchange market is to introduce traders who are characterized by behavioral heterogeneity. Following DeGrauwe and Grimaldi (2006) we assume that each agent is allowed to invest into a domestic asset and a foreign asset. In contrast to DeGrauwe and Grimaldi (2006) both assets are risky due to the randomness of domestic and foreign interest rates and due to exchange rate risks.

We assume overlapping generations of agents, who enter the market for their pertinent investment horizon. After having realized profits traders will leave the market and consume. The timing of each period is defined as follows:

- (i) trader i enters the market and observes interest rates, the exchange rate and the past profits of the other traders. Depending on the past profits of the other traders, he or she decides to be a short-run or long-run fundamentalist or to be a short-run or long-run chartist trader,
- (ii) depending on the interest rate differential and his or her expected depreciation of the exchange rate the trader decides how much to invest in the domestic and the foreign asset,
- (iii) after the trader has realized his profit, he or she leaves the market and consumes.

Following Brock and Hommes (1997) agents are assumed to have preferences towards

risks with constant absolute risk aversion characterized by the following exponential utility function

$$U(W_{t+n}^i, \delta_i) = -\exp\{-\delta_i W_{t+n}^i\}, \quad (2.7)$$

where W_t^i is agent i 's wealth at time t , $n \in \{1, \dots, N\}$ is the set of agents' investment horizons, and δ_i is the agents Arrow-Pratt measure of absolute risk aversion. The agent's wealth is assumed to follow

$$W_{t+n}^i = (R_t^*)^n s_{t+n} d_t^i (1 - \tau)^2 + (R_t)^n (W_t^i - s_t d_t^i), \quad (2.8)$$

where $R = (1+r)$ and $R^* = (1+r^*)$ are the gross returns on the domestic and foreign bond, while s_t is the bilateral exchange rate between both countries. The first part is the return on the foreign asset on which a tax on foreign currency transactions $\tau \in [0, 1]$ is levied, while the second term measures the costs of borrowing in the domestic country. For $n = 1$ and $\tau = 0$ this budget constraint collapses to the one proposed by DeGrauwe and Grimaldi (2006).

If we assume wealth to be normally distributed we can simplify the portfolio selection problem by maximizing the certainty equivalent

$$U(W_t^i, \delta_i) = \mathbf{E}_{t-1}^i[W_t^i] - \frac{\delta_i}{2} \mathbf{Var}_{t-1}^i[W_t^i] \quad (2.9)$$

subject to the same budget constraint. Maximization yields agent i 's demand function for foreign currency for the has investment horizon n

$$d_t^{i,n} = \frac{\mathbf{E}_t^i[W_{t+n}^i]}{\delta_i \mathbf{Var}_t^i[W_{t+n}^i]} = \frac{(R_t^*)^n (1 - \tau)^2 \mathbf{E}_t^i[s_{t+n}] - (R_t)^n s_t}{\delta \sigma_{i,t}^2}. \quad (2.10)$$

Thus, trader i 's demand is decreasing in his degree of risk aversion, in a higher

risk of fluctuations in his wealth $\sigma_{i,t}^2$, decreasing in the transaction tax rate τ , and increasing in the expected profit. For $n = 1$ and $\tau = 0$ the demand function collapses to the one used in DeGrauwe and Grimaldi (2006).

Following Brock and Hommes (1997) we assume that the risk evaluation is the same for all agents and constant over time, the demand function simplifies to

$$d_t^{i,n} = \psi \left((R_t^*)^n (1 - \tau)^2 \mathbf{E}_t^i [s_{t+n}] - (R_t)^n s_t \right), \quad (2.11)$$

with $\psi = 1/\delta\sigma^2$. This simplified demand function only depends on the domestic and foreign interest rates, the currency transaction tax as well as the current and the for the future expected exchange rate.

The trader's demand for foreign currency possesses a term which depends on the agent's individual forecasting model of the future exchange rate. The set of possible forecasting models will be tackled in the next subsection.

2.2.3 Traders' Forecasting Models

We assume that the true data generating process of the exchange rate is unknown to the agents and that they have to approximate it by rules of thumb. Therefore they use ad-hoc rules for forecasting. We assume that two types of forecasting rules are used. A rule which reacts on trends in the exchange rate is commonly called chartist rule or technical trading rule. The other technique called fundamentalist forecasting rule looks for over- and undervaluations of the exchange rate with respect to its arbitrage-free fundamental value s_t^f and expects a reversion back to it.

The fundamentalist forecasting rule for the one-step-ahead prediction of the ex-

change rate can be written as

$$\mathbf{E}_t^f[s_{t+1} - s_t] = \kappa^f \cdot (s_t^f - s_t). \quad (2.12)$$

According to this equation known as error-correction model from the econometrics literature, this rule predicts an exchange rate change such that $\kappa^f \cdot 100\%$ of the disequilibrium $s_t^f - s_t$, which is the deviation of the realized exchange rate s_t from the arbitrage-free exchange rate s_t^f , will be corrected by the subsequent exchange rate change. Note that the two step ahead forecast assumes that $\kappa^f \cdot 100\%$ of the remaining disequilibrium $(1 - \kappa^f) \cdot (s_t^f - s_t)$ will be corrected by the subsequent exchange rate change and so on. From this consideration follows, that the n -step ahead forecast will be

$$\mathbf{E}_t^f[s_{t+n} - s_{t+n-1}] = \kappa^f (1 - \kappa^f)^{n-1} \cdot (s_t^f - s_t). \quad (2.13)$$

For $n = 1$ this forecasting model collapses to the one used in DeGrauwe and Grimaldi (2006), Lux and Marchesi (2000), Chiarella and He (2002) and Brock and Hommes (1997).

The expected exchange rate change $\mathbf{E}_t^f[s_{t+n}] - s_t$ over n periods can be derived from the forecasted one period exchange rate changes as follows

$$\begin{aligned} \mathbf{E}_t^f[s_{t+n}] - s_t &= \mathbf{E}_t^f[s_{t+n} - s_{t+n-1}] + \mathbf{E}_t^f[s_{t+n-1} - s_{t+n-2}] + \dots + \mathbf{E}_t^f[s_{t+1} - s_t] \\ &= [1 - (1 - \kappa^f)^n] \cdot (s_t^f - s_t). \end{aligned} \quad (2.14)$$

Fundamentalists believe that the arbitrage-free exchange rate s_t^f is the exchange rate

under which the uncovered interest rate parity condition holds with equality

$$s_t^f = s_{t-1} \cdot \frac{R_{t-1}}{R_{t-1}^*}. \quad (2.15)$$

Therefore, if this fundamental exchange rate s_t^f realizes, the exchange rate change offsets the possible profits from the interest rate differential and no arbitrage should be possible.

The technical forecasting rule for the one-step-ahead prediction can be specified as follows

$$E_t^c[s_{t+1} - s_t] = (\kappa^c) \cdot (s_t - s_{t-1}). \quad (2.16)$$

This forecasting model known as autoregressive process from the econometrics literature predicts a trend-continuation. If the exchange rate change $s_t - s_{t-1}$ is one, this forecasting model predicts the next exchange rate change to be κ^c . As usual in the theory of autoregressive models we use the last period's forecast to predict the next future exchange rate if we do not have information about realizations. Thus, the two-step-ahead forecast expects an exchange rate change of $(\kappa^c)^2$ and so on. Thus, the n -step-ahead prediction will be

$$E_t^c[s_{t+n} - s_{t+n-1}] = (\kappa^c)^n \cdot (s_t - s_{t-1}). \quad (2.17)$$

Equivalent to the fundamentalists' technique, chartists calculate the expected exchange rate change $E_t^c[s_{t+n} - s_t]$ over n periods as the sum of the predicted one

period exchange rate changes over n periods

$$\mathbf{E}_t^c[s_{t+n} - s_t] = \mathbf{E}_t^c[s_{t+n} - s_{t+n-1}] + \dots + \mathbf{E}_t^c[s_{t+1} - s_t] \quad (2.18)$$

$$= \frac{1 - (\kappa^c)^n}{1 - \kappa^c} \cdot \kappa^c \cdot (s_t - s_{t-1}). \quad (2.19)$$

For $n = 1$ this forecasting model collapses to the one used in DeGrauwe and Grimaldi (2006), Lux and Marchesi (2000), Chiarella and He (2002) and Brock and Hommes (1997).

Now that we modeled demands for foreign currency and the traders forecasting models for predicting the future exchange rate we will have a look at the selection and revision of used trading rules from the set of possibilities. This point will be tackled in the following subsection.

2.2.4 Evolution of Trading Rules

The agents' strategy space consists of five trading rules. The agent can either be a short-run fundamentalist or a short-run chartist, or the trader can be a long-term fundamentalist or a long-term chartist. The fifth possibility for the agents is to stay inactive, that means not to trade. Most studies force agents to trade. Westerhoff (2003a) suggested to allow agents not to trade in his analysis of the effectiveness of currency transaction taxes.

As already said, we assume that agents may have multi-period investment horizons. In the numerical simulation of the model the information concerning the individual agent's investment horizon is saved every period in the matrix Φ_t , which has the

dimension $M \times 5$ and may have for example the following form

$$\Phi_t = \begin{bmatrix} 1 & 1 & 100 & 84 & 1 \\ 1 & 1 & 61 & 31 & 1 \\ 1 & 1 & 87 & 54 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}. \quad (2.20)$$

The first two columns of this investment-horizon matrix identify the short-run fundamentalist trading rule and the short-run chartist trading rule, which have an investment horizon of one by construction. Columns three and four identify the long-term fundamentalist trading rules and the long-term chartist trading rule and the time until their investment matures. Agents are allowed to stay inactive for one period. This information is given in column five. Agents $1, \dots, M$ are given in rows, while strategies $1, \dots, 5$ are given in columns. Thus, this investment horizon matrix reads as follows. If agent 1 is a long-run fundamentalists, then the time to maturity of his investment is 100 periods in this example. If he is a long-term chartist, then the time to maturity is 84 periods. This investment-horizon matrix is updated as follows

$$\Phi_{t+1} = \Phi_t - \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{0} \end{bmatrix}, \quad (2.21)$$

where $\mathbf{0}$ is a $M \times 1$ vector of zeros, $\mathbf{1}$ is a $M \times 1$ vector of ones and M is the number of agents. Thus, the investment horizon of long-term agents decreases by one period until maturity is reached. After that it switches back to the maximum investment horizon of N periods. The starting value for this updating process is generated by a random draw for the columns three and four in order to avoid artificial N -period cycles.

Agents are only allowed to change their trading rules if maturity is reached. Thus short-term traders and inactive traders are allowed to switch every period, long-term traders are only allowed to switch every N th period. The information about which agent is allowed to switch and which agent is not allowed to switch is saved in the matrix \mathbf{S}_t , where we set the element $\mathbf{S}_t(i, j)$ equal to one, when one of agent i 's trading rule j has reached the investment horizon $n_i = 1$ and zero for $n_i > 1$

$$\Phi_t(i, j) = 1 \quad \implies \quad \mathbf{S}_t(i, j) = 1 \quad (2.22)$$

$$\Phi_t(i, j) > 1 \quad \implies \quad \mathbf{S}_t(i, j) = 0 \quad \text{for } i = 1, \dots, M, j = 1, \dots, 5. \quad (2.23)$$

Note, that this matrix only contains information about if an agent is allowed to change a trading rule or not. This matrix contains the information given in this example that agent one is allowed to change his type, if he is type one, two or three, but he is not allowed to switch if he is type four. Thus, this matrix does not tell us, which trading rule the agent is currently using. This information is contained in the population matrix $\mathbf{\Gamma}_t$, where $\mathbf{\Gamma}_t(i, j)$ is one if agent i uses the trading rule j and zero otherwise. Thus, the row sum of this matrix is one, because an agent can only use one trading rule at the same time. We will explain the updating process of this matrix in the subsequent paragraph.

We assume that agents switch to the trading rule, which was the most successful in the past if they are allowed to switch. Therefore agents calculate the profits each trading rule yielded over the last N periods. The 1×5 vector of past profits is given by

$$\mathbf{\Pi}_t = \begin{bmatrix} \pi_t^{f,1} & \pi_t^{c,1} & \pi_t^{f,N} & \pi_t^{c,N} & 0 \end{bmatrix}, \quad (2.24)$$

where the agents realizes a profit of zero if he or she stays inactive.

The profit of agent i is measured by the variable $\pi_t^{i,n}$

$$\pi_t^{i,n} = \{s_t(R_{t-n}^*)^n(1 - \tau)^2 - s_{t-n}(R_{t-n})^n\} \cdot d_t^{i,n}, \quad (2.25)$$

for $i \in \{c, f\}$ and $n = 1, \dots, N$. The first part of this equation is the investment into the foreign bond while the second term measures the loan in the domestic country needed to finance the investment. In this equation we replaced the forecast $\mathbf{E}_{t-n}^i s_t$ with the realized exchange rate s_t . Thus, $\pi_t^{i,n}$ measures the profit per unit currency that results from the exchange rate change and the interest rate differential times the amount of currency demanded. This expression is similar to the one used in Grimaldi (2004) and DeGrauwe and Grimaldi (2006) with the difference to their model that we scale the profit per unit currency by the currency demanded by the agent.

The updating vector $\mathbf{\Pi}_t^*$ has the entry one at the same place, where $\mathbf{\Pi}_t$ has its maximum and zeros at all other entries. Thus, this vector indicates to which trader type the agent has to switch if he is allowed to switch. The switching of agents is conducted, by replacing the pertinent row in the population matrix $\mathbf{\Gamma}_t$ with the vector $\mathbf{\Pi}_t$. This operation is conducted if an agent is allowed to switch. Switching to other trading rules is only possible if agent i uses rule j ($\mathbf{\Gamma}_t(i, j) = 1$) and if agent i using rule j has reached the investment horizon $n_i = 1$ ($\mathbf{S}_t(i, j) = 1$). Hence,

$$\mathbf{S}_t(i, j) = 1 \ \& \ \mathbf{\Gamma}_t(i, j) = 1 \quad \implies \quad \mathbf{\Gamma}_{t+1}(i, j) \leftarrow \mathbf{\Pi}_t(j) \quad \forall \quad j = 1, \dots, 5. \quad (2.26)$$

In this case we replace row i of $\mathbf{\Gamma}_t$ with the vector $\mathbf{\Pi}_t$. If, the condition

$$\mathbf{S}_t(i, j) = 0 \quad \implies \quad \mathbf{\Gamma}_{t+1}(i, j) \leftarrow \mathbf{\Gamma}_t(i, j) \quad (2.27)$$

is met, then agent i using rule j is not allowed to update his or her trading strategy and hence has stick to his or her trading strategy.

All information about the number of agents, who are allowed to trade and the number of agents being using one special trading rule is contained in these matrices.

Now we finished modeling the agents' behavior when trading in the foreign exchange market. What remains is to model institutional properties of the foreign exchange market, for example the price setting mechanism.

2.2.5 Institutional Properties and Price Setting

The market maker collects all individual demands in order to determine the market demand. Individual demands $d_t^{i,n}$ can be aggregated to the market demand D_t by adding them, while weighting them with the population fractions $w_t^{i,n}$ of traders, who are allowed to trade. These population fractions can be calculated as the row sum of the population matrix $\mathbf{\Gamma}_t$ divided by the number of all agents

$$\begin{aligned} D_t &= w_t^{c,1} d_t^{c,1} + w_t^{c,N} d_t^{c,N} + w_t^{f,1} d_t^{f,1} + w_t^{f,N} d_t^{f,N} \\ &- w_{t-1}^{c,1} \pi_{t-1}^{c,1} - w_{t-N}^{c,N} \pi_{t-N}^{c,N} - w_{t-1}^{f,1} \pi_{t-1}^{f,1} - w_{t-N}^{f,N} \pi_{t-N}^{f,N}. \end{aligned} \quad (2.28)$$

Agents are allowed to trade at the beginning of their investment and at the end of their investment. They have to pay back the loan they raised in one country in order to invest in the other country. This effect is captured by the last term of this equation.

If market demand is positive, the market maker will rise the price of the exchange rate, while he will lower it, if market demand is negative. Thus the exchange rate

changes proportional to the sum of all market orders. This behavior of the market maker can be approximated by the following price impact function³

$$s_{t+1} = s_t + \beta s_t D_t, \quad (2.29)$$

while β measures the market response to the order flow D_t . The exchange rate return can be calculated as

$$\rho_{t+1} = \frac{s_{t+1} - s_t}{s_t} = \beta D_t. \quad (2.30)$$

Thus, the model is complete now. Because it cannot be solved analytically, we will rely on quantitative results derived by numerical simulations. This means that we analyze the numbers and statistics implied by the model in comparison with numbers and statistics implied by empirical exchange rate data.

2.3 Fundamental Steady States

In this part we want to describe the non-stochastic steady state. Therefore we restrict all shocks to be zero and concentrate on the deterministic skeleton of the model. A steady state is characterized by the absence of stochastic shocks

$$\varepsilon_t = \varepsilon_t^* = 0 \quad (2.31)$$

³Kyle (1985) derives this price impact function as the solution of his continuous double auction model. Lux and Marchesi (2000) and Westerhoff (2003a) also use this pricing rule within an agent-based-framework.

and by variables equal to their constant long-run values denotes with a bar

$$R_t = R_{t-1} = \bar{R}, R_t^* = R_{t-1}^* = \bar{R}^*, s_t = s_{t-1} = \bar{s}. \quad (2.32)$$

Note, that interest rates are exogenous in this model. The condition of a zero interest rate differential $\bar{R} = \bar{R}^*$ is necessary in order to have a constant steady-state $\bar{s} = \bar{s}^f$. Otherwise, investments in the country with the higher interest rate will be attractive for which reason agents buy or sell foreign currency even in steady-state which leads to a appreciation or depreciation of the currency in steady-state.

From the agents' demand schedule (2.11) and the definition of profits (2.25) follows under the conditions of the steady state

$$d_{i,t} = d_{i,t-1} = 0, \pi_{i,t} = \pi_{i,t-1} = 0, \quad (2.33)$$

while the population fractions are undetermined under the proposed evolutionary mechanism.

Chartists' demand will be zero when $s_{t-1} = s_t = \bar{s}$. However, fundamentalists' demand will only be zero when $\bar{s} = \bar{s}^f$. From the market maker's price impact function (2.29) follows, that \bar{s} is a fundamental steady-state then.

Non-fundamental steady-states $s_{t-1} = s_t = \bar{s}$ can be possible, when no trader uses the fundamentalist trading rules. In this case there is no driving force that brings the exchange rate back to the fundamental equilibrium.

Summing up, the fundamental steady state is characterized by equal rates of return in both countries and no exchange rate change. Therefore we get zero demands and zero profits, because the exchange rate equals its no-arbitrage fundamental value.

2.4 Simulation Results

This section presents the results of the numerical simulation of the model. The model is evaluated by its performance to match numbers and statistics of real world exchange rate data, while the policy analysis is done by inferring how numbers and statistics change due to a change in the policy instrument.

2.4.1 Calibration

As mentioned before, we are neither able to analyze the model's properties nor to conduct the economic policy analysis by means of analytical methods. Therefore we have to rely on quantitative results from numerical simulations of the model. The model is simulated with the parameters given in table 2.1. We assume two scenarios. Scenario 1 is characterized by low behavioral parameters and thus traders will not trade very aggressively. In contrast to this, scenario 2 is characterized by high behavioral parameters implying that traders will trade more aggressively. The reason is that under scenario 1 the market is dominated by stabilizing traders, while there is dominance of destabilizing traders under scenario 2.

In the baseline simulation we set the transaction tax rate to zero in order to have a benchmark for the policy simulations conducted later. This numerical analysis wants to infer if a typical simulation run is able to match empirical properties of exchange rate data. In contrast to this, the policy simulations thereafter are based on on a larger Monte Carlo study with several simulation runs. This baseline case can also be inferred from the larger Monte Carlo simulation by looking at the pertinent column of table 2.4 discussed later on.

Table 2.1: Parameter Calibration for both Scenarios

| (a) Scenario 1 | | | |
|--|-------------------|-------------------------------|-------------------|
| mean reversion parameters | | standard deviations of shocks | |
| $\alpha_1 = 0.95$ | $\alpha_2 = 0.95$ | $\sigma_1 = 0.03$ | $\sigma_2 = 0.03$ |
| risk aversion parameters | | max. horizon | simulation length |
| $\delta^C = 1$ | $\delta^F = 1$ | $N = 30$ | $T = 6\,000$ |
| exchange rate response | | forecasting parameters | |
| $\beta = 0.01$ | | $\kappa^C = 0.3$ | $\kappa^F = 0.2$ |
| transaction tax | | number of agents | |
| $\tau = 0$ | | 300 | |
| (b) Scenario 2 | | | |
| mean reversion parameters | | standard deviations of shocks | |
| $\alpha_1 = 0.95$ | $\alpha_2 = 0.95$ | $\sigma_1 = 0.03$ | $\sigma_2 = 0.03$ |
| risk aversion parameters | | max. horizon | simulation length |
| $\delta^C = 1$ | $\delta^F = 1$ | $N = 30$ | $T = 6\,000$ |
| exchange rate response | | forecasting parameters | |
| $\beta = 0.01$ | | $\kappa^C = 0.9$ | $\kappa^F = 0.9$ |
| transaction tax | | number of agents | |
| $\tau \in \{0.000, 0.005, 0.010, 0.015, 0.020\}$ | | 300 | |

NOTE: These parameters are used for the baseline simulation of the model without transaction taxes. Scenario 1 is characterized by low behavioral parameters $\kappa^C = 0.3$ and $\kappa^F = 0.1$, while scenario 2 is given by high behavioral parameters $\kappa^C = 0.9$ and $\kappa^F = 0.9$ indicating that agents trade more aggressively. The reason is that under scenario 1 the market is dominated by stabilizing traders, while there is dominance of destabilizing traders under scenario 2.

For conducting the numerical simulation we assume the interest rates in both countries to be quite persistent because empirical exchange rate data are also quite near a unit-root process. Thus, we assume the two interest rate processes to follow

$$\ln R_t = 0.05 \cdot 1.005 + 0.95 \cdot \ln R_{t-1} + 0.03 \cdot \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, 1). \quad (2.34)$$

For modeling risk aversion we assume chartist traders and fundamentalist traders to have the same value for the Arrow-Pratt measure of absolute risk aversion. Under

scenario 1, we assume chartists to have an extrapolation parameter less than one

$$\mathbf{E}_t^C[s_{t+n} - s_t] = \frac{1 - (0.3)^n}{1 - 0.3} \cdot 0.3(s_t - s_{t-1}), \quad (2.35)$$

such that their forecasting model predicts a return of 0.3 for the next period, if the current return is one and the two-period return to be 0.09. Under scenario 2 we assume this parameter to be 0.9 so that chartist traders extrapolate more strongly and trade more aggressively.

Under scenario 1 we assume that fundamentalist traders expect exchange rate disequilibria to be corrected with 20% per period. Although many studies assume this parameter to be between zero and 0.1 we made the experience that we get results which match empirical data better if we use a higher value for this parameter. Thus, the fundamentalists' forecasting model becomes

$$\mathbf{E}_t^F[s_{t+n} - s_t] = [1 - (1 - 0.2)^n] \cdot (s_t^f - s_t). \quad (2.36)$$

Under scenario 2 we assume this parameter to be 0.9 such that fundamentalist assume misalignments to be corrected much faster. Therefore, they will trade more aggressively.

Furthermore, we set the exchange rate response to $\beta=0.01$

$$s_{t+1} = s_t + 0.01 \cdot s_t D_t. \quad (2.37)$$

Endowed with this parameter calibration we are able to conduct the numerical simulations. Although some calibrated parameter values differ from the values used in similar models we stick to this parameterization. The reason is on the one hand

that we use another evolutionary mechanism than other studies, on the other hand we want to analyze two scenarios, one in which the financial market is dominated by stabilizing traders and one in which the market is dominated by destabilizing traders.

2.4.2 The Baseline Simulation

This part of our research consists of results of the baseline simulation without taxes. The aim is to provide insights about how good numbers and statistics of a typical simulation run match numbers and statistics implied by empirical exchange rate data.

Statistical Properties

Figures 2.2 and 2.3 show the simulation outcome of the baseline model without taxes under scenario 1, the case where traders are not very aggressive in trading and scenario 2, the case where traders trade aggressively. Figure 2.1 shows the exchange rate under scenario 1. The exchange rate shows a random walk like behavior like empirical financial time series. One can clearly see that the time series displays periods of trends and crashes as we typically find in financial market time series. Although the exchange rate looks more or less the same under both scenarios, the exchange rate returns are characterized by a different behavior as can be seen from figure 2.3. The model is able to reproduce the stylized facts volatility clustering and excess kurtosis which can be seen from this figure. In the model produced time series periods of high volatility and periods of low volatility tend to cluster together like in empirical daily return time series. What can be seen is, that extreme returns

are realized quite frequently and volatility clusters are more realistic under scenario 1.

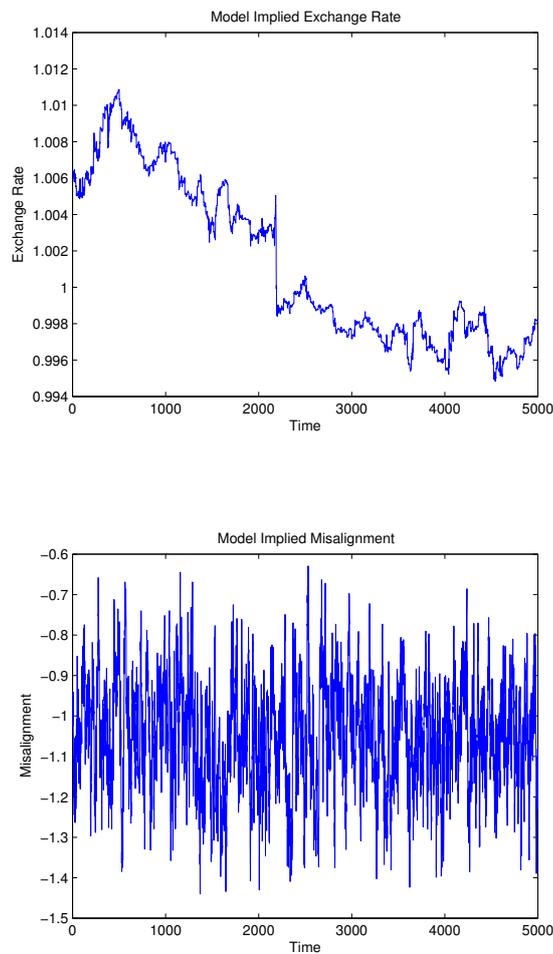


Figure 2.2: Exchange Rate and Misalignment: Scenario 1

Note: Model generated time series from the baseline simulation under scenario 1. The used parameter values are those given in table 2.1. The first 1000 data points were removed.

By looking at figure 2.4 we can analyze this phenomenon in greater detail. The upper subfigures show a quantile-quantile-plot with respect to the normal distribution.

Here quantiles of the standard normal distribution are plotted against the quantiles of the empirical return distribution. If the data is normally distributed all points should lie on the broken line.

From this figures we can infer deviations from the normal distribution in the positive and negative extreme parts. In the lower subfigures the estimated kernel density of the returns is plotted together with the density of a normally distributed random variable with the same mean and variance the input sample has. From this figure can be seen that the density of the model generated data has a higher peak and fatter tails with respect to the normal distribution which means that this distribution is leptokurtic like empirical exchange rate distributions are. These results are valid under scenario 1 and 2.

The phenomenon of volatility clustering can be analyzed in more detail from figure 2.5. This figure plots the autocorrelation function of returns and squared returns for 80 lags. Here, raw returns display only small serial correlations which indicates that exchange rate returns are difficult to predict from their past data. This finding is more or less in line with the efficient market hypothesis. In contrast to this squared returns display strong correlations over 80 lags. This indicates that although returns themselves are uncorrelated they are not independently distributed because squared returns display high serial dependencies. We can interpret squared returns as a noisy measure for volatility because

$$\text{Var}[r_t|\mathcal{I}_{t-1}] = \text{E}[r_t^2|\mathcal{I}_{t-1}] \text{ and } r_t^2 = \text{E}[r_t^2|\mathcal{I}_{t-1}] + v_t \quad (2.38)$$

$$\implies r_t^2 = \text{Var}[r_t^2|\mathcal{I}_{t-1}] + v_t. \quad (2.39)$$

Therefore, high serial correlations of squared returns indicates that volatility is se-

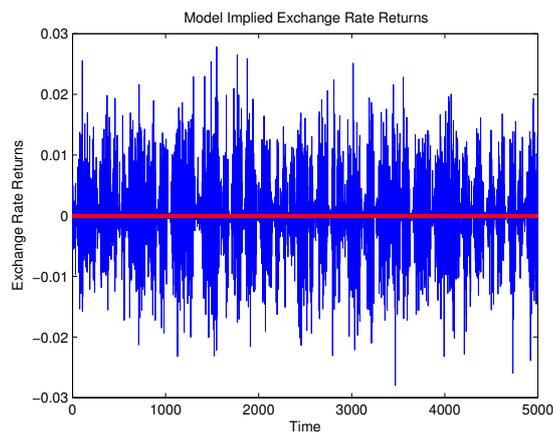
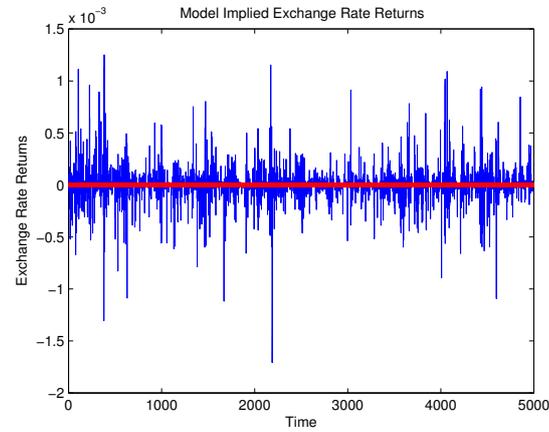


Figure 2.3: Exchange Rate Returns: Scenario 1 and Scenario 2

Note: Model generated time series from the baseline simulation. The used parameter values are those given in table 2.1. The figure above is based on scenario 1, while the figure below is based on scenario 2. The first 1000 data points were removed.

rially correlated and therefore predictable. This strong serial correlation in squared returns leads to the empirical fact of volatility clustering which the model is able to

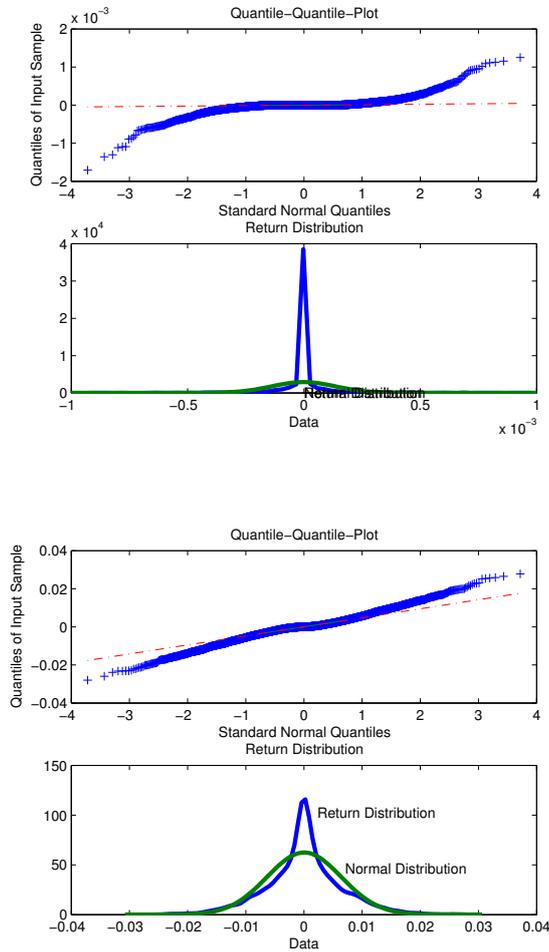


Figure 2.4: Return Distribution: Scenario 1 and Scenario 2

Note: Model generated time series from the baseline simulation under scenario 1. The used parameter values are those given in table 2.1. The figure above is based on scenario 1, while the figure below is based on scenario 2. The dark line represents the kernel density of the model generated exchange rate returns, while the light line is the density of a normally distributed random variable with the same mean and the same variance. The first 1000 data points were removed.

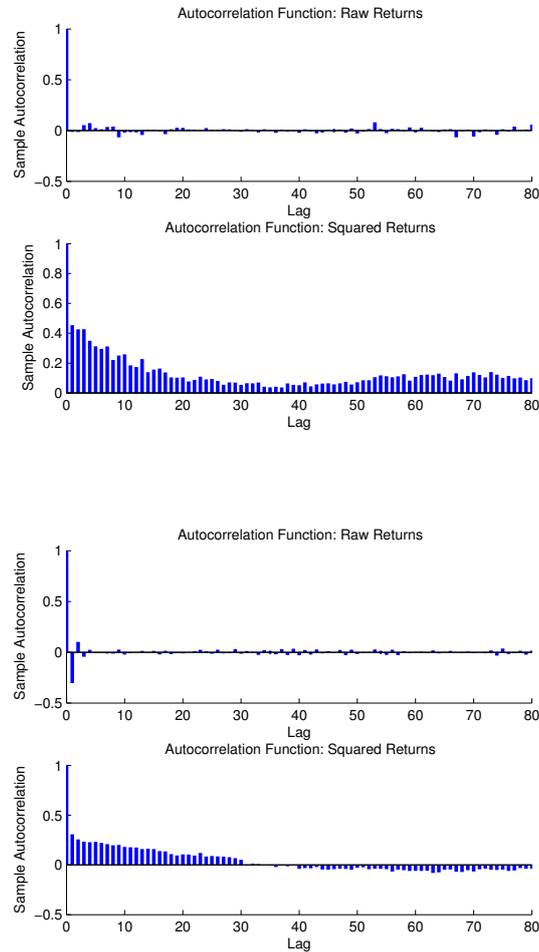


Figure 2.5: Autocorrelation Functions: Scenario 1 and Scenario 2

Note: Model generated time series from the baseline simulation under scenario 1. The used parameter values are those given in table 2.1. The figure above is based on scenario 1, while the figure below is based on scenario 2. The first 1000 data points were removed.

explain. Under scenario 1 autocorrelations look more realistic than under scenario 2. The reason for this is that under scenario 1 returns are uncorrelated while volatility

is highly correlated like in empirical return time series. Under scenario 2 returns are characterized by large correlations while squared returns display negative autocorrelations for lags larger than 40 which is in contrast to empirical autocorrelations. Again, scenario 1 is more realistic.

Table 2.2: Summary Statistics of a Typical Simulation Run

| | Scen. 1 | Scen. 2 | USD-Euro | YEN-USD | GBP-USD |
|---------------|---------|---------|----------|---------|---------|
| mean | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| st. deviation | 0.001 | 0.013 | 0.006 | 0.007 | 0.006 |
| skewness | -0.603 | 0.039 | 0.014 | -0.487 | -0.135 |
| kurtosis | 36.794 | 5.728 | 3.619 | 7.335 | 6.573 |
| ARCH | 0.329 | 0.335 | 0.014 | 0.056 | 0.065 |
| GARCH | 0.670 | 0.666 | 0.977 | 0.942 | 0.922 |

Note: Mean, variance, skewness and kurtosis are calculated from the model generated exchange rate return data by using the parameters of the baseline simulation given in table 2.1. ARCH and GARCH are the coefficients of an GARCH(1,1) model fitted to the model generated return data. The exchange rate data used in columns 3,4 and 5 are taken from the FRED2 database of the Federal Reserve Bank of St. Louis in daily frequency from 04-01-1999 to 05-31-2005. The data is available under the series-ID: DEXUSEU, DEXJPUS, and DEXUSUK.

A quantitative model is a good description of reality if the numbers and statistics produced by a typical numerical simulation run are able to match numbers and statistics from empirical data. Table 2.3 contains summary statistics of the baseline simulation under scenario 1 and scenario 2 in comparison with summary statistics of empirical exchange rate return data. What can be seen is that the mean of simulated returns and empirical returns is always zero, while the model is calibrated in that way that the variance of the model is similar to the variance of empirical returns. The kurtosis of empirical data and of the baseline simulation is always greater than 3, which indicates that this kurtosis is larger than that of a normally distributed random variable. This fact also could be seen from the quantile-quantile-plot and the

kernel density graphs discussed before. Moreover, we fitted a GARCH(1,1) model to the baseline simulation data and the empirical data. The GARCH-model due to Bollerslev (1986) assumes the data to be conditional normally distributed

$$r_t | \mathcal{I}_{t-1} \sim \mathcal{N}(0, \sigma_t^2), \quad \text{with } \mathcal{I}_{t-1} = \{r_{t-1}, r_{t-2}, \dots\}, \quad (2.40)$$

while the variance is assumed to follow an autoregressive process

$$\sigma_t^2 = \omega + ar_{t-1}^2 + b\sigma_{t-1}^2. \quad (2.41)$$

New information about volatility can enter the model through squared returns, while the last term measures the persistence of volatility. Empirical studies usually find a to be less than 0.1 and b approximately 0.9, with $a + b$ close to one. This is an indication of the strong persistence in volatility. From table 2.3 one can infer, that this fact can also be found in estimates for the three exchange rate return time series as well as for the model generated return time series. Thus, our model is also able to display high persistence of volatility. Summing up, summary statistics of both scenarios are in line with empirical findings.

Fundamental Traders and Technical Traders

Figure 2.7 plots the evolution of the population fractions of traders using either the technical trading rule or the fundamental trading rule. This simulation is again based on scenario 1, the case in which traders do not trade very aggressively. What can clearly be seen from this figure is that for this calibration of the model majorities for the fundamentalist trading rule emerge because the members of the pertinent

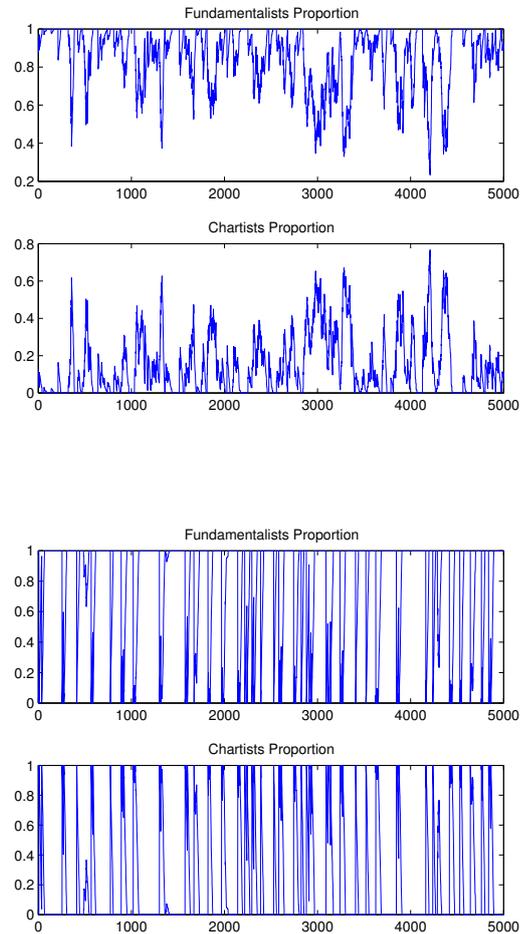


Figure 2.6: Fraction of Fundamentalist and Technical Traders: Scenario 1 and Scenario 2

Note: Model generated time series from the baseline simulation under scenario 1. The used parameter values are those given in table 2.1. The figure above is based on scenario 1, while the figure below is based on scenario 2. The first 1000 data points were removed. Short term and long term traders were aggregated to the two groups of fundamentalist and technical traders.

group approach 100%. What can be learned is that if traders do not trade very aggressively the market is dominated by fundamentalists most of the time. Thus, taxation does not help to reduce volatility because the market is already dominated by stabilizing traders. Therefore, we conduct our economic policy analysis under scenario 2, the case where the fraction of destabilizing trend chasers is larger as can be seen from the figure.

Short-term Speculators and Long-term Investors

Figure 2.8 plots the time variation of population fractions of traders having either short-term investment horizons or long-term investment horizons. This outcome is again based on scenario 1. What can be seen is, that the population fractions are fluctuating but displaying a dominance of long-term traders. Again, under scenario 1 taxation will not yield any reductions in volatility because as can be seen the market is already dominated by long-term traders, while short-term speculators are the minority. Therefore, we use scenario 2 for the policy simulations performed later because we have a larger fraction of destabilizing short-term speculators here.

All in all, this section showed, that our model is able to reproduce deviations of the exchange rate from the fundamental value, random walk or martingale behavior of the exchange rate, volatility clustering and fat tails in the distribution. Moreover, we learn about traders behavior that if they do not trade very aggressively the financial market is characterized by a high dominance of long-term traders and a high dominance of traders using the fundamental-based trading rule. Thus, there is no need for a currency transaction tax under this scenario. For performing the economic policy analysis we use a scenario with a larger fraction of short-term traders and

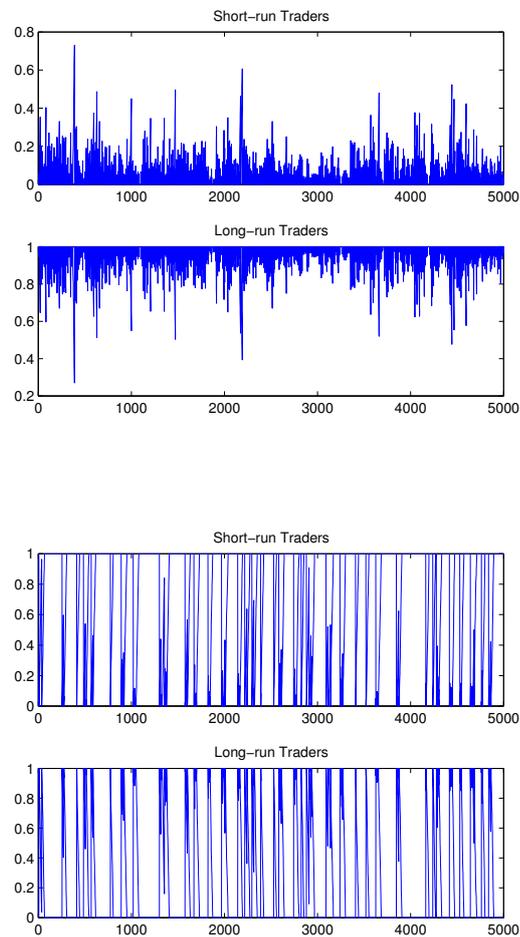


Figure 2.7: Fraction of Short-term and Long-term Traders: Scenario 1 and Scenario 2

Note: Model generated time series from the baseline simulation under scenario 1. The used parameter values are those given in table 2.1. The figure above is based on scenario 1, while the figure below is based on scenario 2. The first 1000 data points were removed. Fundamentalist and chartist traders were aggregated to the two groups short-term traders and long-term traders.

chartist traders. This is the case if we allow agents to trade more aggressively. By varying the transaction tax rate we are able to analyze the effects of this economic policy instrument on financial risks and the composition of trading rules. This analysis will be performed in the next section.

2.4.3 Sensitivity to Transaction Tax Rate Changes

This section presents the economic policy analysis of the model by means of numerical methods. The analysis of properties of a typical simulation run as conducted in the former section will not give us enough information about the overall effects of transaction tax changes. Therefore we use a richer Monte Carlo method for the analysis of the effects of currency transaction taxes. In detail we perform several simulation runs and take averages and standard deviations over these simulation runs for the statistics and numbers of interest. This method will provide more information about how tax rate changes affect properties of the exchange rates in a stochastic environment. For performing the policy simulation we choose scenario 2, the case with higher values for the chartists' extrapolation parameter and the fundamentalists' error-correction parameter. The reason is that this calibration provides us with higher values for the population fractions of destabilizing traders.

Statistical Properties

Table 2.4 shows summary statistics of the model generated exchange rate returns for different values of the transaction tax rate ranging from 0% which represents the baseline case to 2% which is the highest taxation of our study. These values are averages over 100 simulation runs of sample size 1000 while the standard deviation

of these numbers are given in parenthesis. The calibration is the one of scenario 2, the case where agents trade more aggressively and thus, the fraction of destabilizing traders is larger.

Table 2.3: Variation of the Transaction Tax: Scenario 2

| τ | 0.0% | 0.5% | 1.0% | 1.5% | 2.0% |
|--------------|---------|---------|---------|---------|---------|
| mean | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| SE | (0.000) | (0.000) | (0.000) | (0.000) | (0.000) |
| variance | 0.007 | 0.006 | 0.006 | 0.006 | 0.005 |
| SE | (0.002) | (0.003) | (0.003) | (0.003) | (0.003) |
| skewness | 0.064 | 0.079 | 0.070 | 0.061 | 0.069 |
| SE | (0.113) | (0.148) | (0.253) | (0.419) | (0.299) |
| kurtosis | 4.291 | 5.216 | 6.751 | 8.619 | 10.211 |
| SE | (0.744) | (1.219) | (1.591) | (2.319) | (2.660) |
| minimum | -0.027 | -0.026 | -0.028 | -0.028 | -0.026 |
| SE | (0.014) | (0.017) | (0.017) | (0.015) | (0.016) |
| maximum | 0.028 | 0.027 | 0.028 | 0.028 | 0.026 |
| SE | (0.012) | (0.015) | (0.015) | (0.014) | (0.015) |
| misalignment | 0.411 | 0.349 | 0.431 | 0.452 | 0.442 |
| SE | (0.137) | (0.163) | (0.168) | (0.168) | (0.184) |

Note: The statistics and numbers reported are averages of 100 simulation runs of size 1000. Standard errors of the Monte Carlo simulation are reported in parenthesis. All columns are based on the same seed of random variables. A currency transaction tax rate $\tau = 0\%$ defines the baseline case. The simulation outcome is based on the parameterization of scenario 2 given in table 2.1.

From this table one can infer that the mean exchange rate return does not change due to changes in the transaction tax rate. This result implies that taxation of currency transactions does not change the long run exchange rate returns of holding foreign currency. Moreover one can see that positive transaction tax rates are able to reduce the variance of exchange rate returns from 0.7% for the case of the baseline simulation to 0.5% for a tax rate of 2% levied on international currency transactions. Although

decreasing volatility the tax rises the kurtosis of the distribution from 4.3 for the baseline case to 10.2 for the case of a tax rate of 2%. Thus, positive transaction tax rates increase the probability of extreme positive and negative returns which leads to a higher risk of extreme events. This finding limits the success of taxes to reduce risks in foreign exchange markets. Moreover, one can see that the tax does not change the skewness of the distribution significantly as well as the minimum and the maximum return. The misalignment measured as the deviation from the uncovered interest rate parity condition is decreasing from 0.411 to 0.349 and then monotonically increasing. This u-shaped response of the misalignment to increasing tax rates can also be found in Westerhoff (2003). In his study, distortions are decreasing since the number of fundamentalist traders is higher compared to the number of chartist traders under small tax rates, while the number of chartist traders is higher under higher tax rates. Misalignments decrease when fundamentalism is more often used, while misalignments decrease, when chartism gains attractiveness. We will provide a similar behavioral interpretation for this finding in the following lines. What is surprising in our analysis, is that the misalignment decreases for a tax increase from 1.5 percent to 2 percent. Summing up, although the currency transaction tax reduces exchange rate volatility its success in reducing risks is limited because it rises the probability for extreme positive and negative returns.

Fundamental Traders and Technical Traders

From table 2.5 one can infer how positive transaction tax rates influence traders behavior under scenario 2. The numbers belonging to this table are also averages over 100 simulation runs of size 1000 and are based on the same seed of random

numbers like the statistics in table 2.3. From this table one can infer that the number of traders using the fundamental trading rules is increasing from 78.3% to 78.6% for a tax increase from 0.0% to 0.5%. For tax rates above 0.5% the number of fundamentalists decreases. The chartist decrease from 21.7% to 21.2% and then increase for higher tax rates. Hence, we get a similar picture as can be found in Westerhoff (2003). Small taxes favor fundamentalist trading rules, while higher tax rates favor chartist trading rules. From these changes in trading behavior we can derive a similar explanation for the u-shaped response of misalignments to taxation. Misalignments decrease for small tax rates, since the tax harms destabilizing chartist rules more compared to stabilizing fundamentalist rules. However, misalignments increase for higher tax rates, since higher tax rates favor destabilizing chartist trading rules and harm stabilizing fundamentalist rules.

Table 2.4: Average Percentages of Used Trading Rules: Scenario 2

| τ | 0.0% | 0.5% | 1.0% | 1.5% | 2.0% |
|-------------------|------------------|------------------|------------------|------------------|------------------|
| fundamental SE | 0.783 (0.087) | 0.786 (0.093) | 0.731 (0.070) | 0.689 (0.045) | 0.675 (0.046) |
| technical SE | 0.217 (0.087) | 0.212 (0.093) | 0.266 (0.071) | 0.306 (0.046) | 0.320 (0.047) |
| inactive SE | 0.000 (0.000) | 0.002 (0.001) | 0.003 (0.002) | 0.005 (0.002) | 0.005 (0.001) |
| short-term SE | 0.756 (0.090) | 0.633 (0.116) | 0.495 (0.069) | 0.399 (0.087) | 0.331 (0.069) |
| long-term SE | 0.244 (0.090) | 0.356 (0.116) | 0.502 (0.090) | 0.597 (0.086) | 0.664 (0.069) |

NOTE: Average percentage fractions of used trading rules for different transaction tax rates τ . The statistics reported are averages of 100 simulation runs of size 1000. The results of each column are based on the same seed of random variables. Standard errors of the Monte Carlo simulations are reported in parenthesis. The used parameter values are those given in table 2.1 under scenario 2. $\tau=0\%$ represents the baseline case.

Short-term Speculators and Long-term Investors

From table 2.5 one can also infer that the number of short term traders is decreasing from 75.6% in the baseline simulation to 33.1% under a tax rate of 2% while the number of long term traders is increasing from 24.4% to 66.4%. This is in line with the conventional view that a transaction tax makes short-term trading more costly and therefore prevents speculative attacks in favor of long-term investments.

Summing up, we come to the conclusion that levying a tax on foreign exchange market transactions within this model framework will result in a decline of short-term speculators in favor of long-term investors while it will also lead to an increase in chartist trading rules and to a decrease in fundamental based trading. Moreover we come to the conclusion that a currency transaction tax is able to reduce the variance of exchange rate returns while it will increase their kurtosis. Therefore the overall success of this policy instrument in reducing foreign exchange market risks is not for sure. Therefore policy makers should wait for new research results about the effectiveness of this tax before introducing it.

2.5 Conclusion

In paper we want to analyze the effectiveness of a currency transaction tax, especially the following proposition is often heard from proponents of the Tobin tax:

Currency transaction taxes crowd out short term speculators in favor of long term investors.

In order to analyze this proposition we have to consider the following requirements

and assumptions:

- (i) we need a model framework in which we are able to distinguish between short term speculators and long term investors,*
- (ii) the number of short term traders and long term traders should not be fixed but traders should change groups or leave the market*
- (iii) the model should be able to match stylized facts of empirical financial market data for performing a policy analysis which is close to reality.*

For fulfilling requirements (i), (ii) and (iii) the most appealing framework is an agent-based model of the financial market. This model class allows us to distinguish between different types of agents. Moreover this framework allows agents to change their type. Furthermore the overall success of this model class to match stylized facts of financial market data encourages to elaborate on them. Usually agent-based models of the financial market assume two types of agents: chartist traders and fundamentalist traders. In our model we allowed these two groups of traders to be either a short-term speculator or a long term investor. Moreover, agents are allowed not to enter the market as proposed by Westerhoff (2003a).

In contrast to other agent-based financial market models we do not use any abstract fundamental value but use the interest rate differential between the home and the domestic country to be our fundamental value. Stochastic interest rates in both countries lead to temporal arbitrage opportunities and therefore to demand for foreign currency. A market maker aggregates the agents' individual market orders and rises the exchange rate due to positive excess demand and lowers it due to negative excess demand. Moreover, a policy maker levies a tax on international currency

transactions.

Simulations of the baseline model without transaction taxes produce time series which match time series properties of empirical exchange rate data. This makes the model usable for an economic policy analysis which is closer to reality. Under this scenario agents are assumed not to trade too aggressively. Therefore, we chose small numbers for their behavioral parameters. Simulations show that taxation will not reduce volatility under this scenario because the market is already populated by stabilizing fundamentalist traders and long-term investors. Therefore, we chose a scenario with higher values for the behavioral parameters for the economic policy analysis. The reason is that under this scenario the fractions of chartists and short term traders are higher.

This economic policy analysis of our model shows that positive transaction taxes are capable of reducing volatility. The disadvantage of this policy instrument is, that the probability of extreme positive or negative exchange rate returns is increased. That means higher transaction tax rate increases the kurtosis of the return distribution which increases the frequency of sharp trends and crashes in the exchange rate and thusly leads to additional risks. The tax alters traders behavior by reducing short-term speculation in favor of long-term investments, which is in line with the arguments of the proponents of the Tobin tax and our proposition. In contrast to their view, in our model the tax favors trend extrapolating trading rules and punished trading rules based on economic fundamentals. Because trend extrapolating trading rules are a source of destabilization of the exchange rate, this can be the reason why the transaction tax increases the kurtosis of the return distribution. All in all, this model showed that currency transaction taxes have advantages and disadvantages.

A policy maker should consider this when taking into account to introduce a tax of this type.

Summing up, further research should look for analytical solutions to a simplified version of this model and for extensions by the incorporation of other long-term investment strategies into the model in order to get more information about the effectiveness of transaction taxes on traders' behavior and the reduction of risks in financial markets. As a policy advice can be given that policy makers should wait for additional research results before introducing a currency transaction tax because its overall success is not for sure.

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3 Transaction Taxes and Traders with Heterogeneous Investment Horizons in an Agent-Based Financial Market Model

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Abstract. This agent-based financial market model is a generalization of the model of Westerhoff (2008a) by traders who are allowed to have different investment horizons as introduced by Demary (2008). Our research goals are, first, to study what consequences the introduction of heterogeneous investment horizons has for agent-based financial market models, and second, how effective transaction taxes are in stabilizing financial markets. Numerical simulations reveal that under sufficiently small tax rates traders abstain from short-term trading in favour of longer investment horizons. This change in behavior leads to less volatility and less mispricings. When the tax rate exceeds a certain threshold, however, mispricings increase as also found in Westerhoff (2003a, 2008a). This emergent property is due to the fact that taxation reduces short-term fluctuations and causes longer lasting trends in the exchange rate. As a result, the longer term fundamentalist trading rules becomes unpopular in favor of the longer term trend-chasing rule.

Key Words: *Agent-Based Models, Financial Market Stability, Regulation of Financial Markets, Technical and Fundamental Analysis, Transaction Taxes*

JEL Classification: *C15, D84, G01, G15, G18*

3.1 Introduction

Asset prices are excessively volatile and risky (LeRoy and Porter 1981, Shiller 1981) due to speculative bubbles and crashes. Because large asset price crashes lead to severe recessions, research on the occurrence of asset price bubbles and their avoidance through financial market regulations is a highly relevant topic for economic scientists, economists in firms and institutions as well as for economic policy makers. Because empirical case studies of financial market regulations lack of sufficiently rich datasets, agent-based financial market simulations are promising tools for analysing the effectiveness of policy measures.¹

Famous proposals of financial market regulations go back to Keynes (1936) and Tobin (1978) who proposed to introduce taxes on financial markets in order to reduce speculative trading. Both assume that short term traders have a destabilizing impact on prices, while long term traders' trading behavior is stabilizing. Both suggest that the introduction of a transaction tax will harm short-term speculators more compared to longer term investors. The rationale behind this proposal is that a round trip of borrowing money in one country, investing it in another country and consuming the profit in the home country will lead to higher transaction costs the more frequent transactions takes place. Some empirical papers, however, find that the introduction of transaction taxes may be destabilizing.² Westerhoff (2008a) notes that these empirical studies are not without problems. Umlauf (1993), for example, analyzes Sweden's introduction of a 2 percent securities transaction tax,

¹See Westerhoff (2008a) for an survey of the advantages of agent-based financial market models and applications of financial market regulations like transaction taxes, trading halts and central bank interventions.

²See for example Umlauf (1993), Jones and Seguin (1997), Aliber (2003), Hau (2006).

which is following Westerhoff (2008a) a quite high tax rate. Sweden abolished the financial market tax because it did not lead to the expected success. Insights to the failure of this real world policy experiment can be given by referring to the heterogeneous agents model proposed by Westerhoff (2003a). This model predicts that small transaction taxes are stabilizing, while higher transaction taxes are destabilizing. The reason is that different tax rates have a different impact on the composition of stabilizing and destabilizing trading rules. A small tax rate makes destabilizing trading rules unprofitable, while a higher tax rate also makes stabilizing trading rules unprofitable.

Inspired by this result, Westerhoff (2008a) suggests to analyze the effectiveness of small transaction taxes on financial markets by means of agent-based financial market models. Lux (2009b) highlights that agent-based models are preferable tools for doing policy experiments which are close to reality. Westerhoff (2008a) surveys the following advantages of agent-based policy analysis: (i) The researcher is able to generate a huge amount of data, (ii) is able to measure all variables precisely, (iii) is able to control for exogenous shocks and special events and simulate them by varying a policy parameter under otherwise same conditions. In this way the researcher is able to analyze how a certain policy performs under these special events. (iv) Finally, the researcher is able to measure the behavior of artificial agents during the simulations.

Up to now, there is a growing body of articles which employ agent-based models for the analysis of the effectiveness of currency transaction taxes.³ We already

³Other policy applications of agent-based models are Westerhoff (2001), Wieland and Westerhoff (2005), Westerhoff and Wieland (2004), Westerhoff (2008a) who analyze the effectiveness of central bank interventions. Westerhoff (2003b), Westerhoff (2006) and Westerhoff (2008a) analyze the effectiveness of trading halts for stabilizing financial markets. Weidlich and Veit

referred to Westerhoff (2003a) who finds that small currency transaction taxes lower exchange rate volatility, while higher tax rates lead to an increase in volatility. Westerhoff and Dieci (2006) propose an agent-based model in which traders are allowed to trade in two different financial markets. The model predicts that when a policy maker levies a transaction tax only onto one market, the volatility in this market will decline, while the volatility in the second market will increase. The reason is that trend-chasing trading rules are more profitable in the untaxed market. The second result of their article is, that levying the tax on both markets, will lead to a decline in the volatility in both markets. Demary (2008) introduces an artificial foreign exchange market with chartists and fundamentalists who are allowed to choose between being a day trader and being a longer term trader. Levying a transaction tax onto this artificial foreign exchange market leads to an increase in the kurtosis of the return distribution, which means a higher probability of large positive and negative returns. This increased probability of extreme returns emerges from the changed composition of short-term and longer term traders. Under taxation short-term trading becomes unprofitable relative to longer term trading. Short-term traders who normally trade small orders every day now decide to trade larger orders every 30 days. This increase in larger orders leads to an increase in the kurtosis of the return distribution. Pelizzari and Westerhoff (2007) show that transaction taxes are only effective under certain market structures, while they will not work under some other market structures. Summing up, all these studies reveal important insights for economic policy makers into the effects of currency transaction taxes on financial markets.

(2008) introduce an agent-based model for analyzing electricity market regulation, while Haber (2008) uses an agent-based model for monetary and fiscal policy analysis.

In this paper we expand the artificial financial market of Westerhoff (2008a) by the trader types with different investment horizons of Demary (2008).⁴ Our first objective is to study the implications of longer term investment horizons for exchange rate dynamics in agent-based models, the second one is to use this artificial laboratory for analyzing the effectiveness of currency transaction taxes. The analysis of these two objectives can be combined to analyzing the joint hypothesis that *transaction taxes stabilize financial markets by crowding out short-term speculators in favor of longer term investors*. In line with the literature we are interested in how this regulatory policy changes emergent properties that arise from the changed interaction of traders, like bubbles and crashes, excess volatility, volatility clustering and the fat-tailness of the return distribution.

Within our model the following results emerge. Numerical simulations of our artificial financial market reveal that emergent properties and stylized facts still remain when longer term traders are introduced. The economic policy analysis reveals that small transaction taxes make short-term trading unprofitable. Therefore, the number of short-term fundamentalists and short-term chartists decreases to zero. Moreover, volatility and distortions decrease under small transaction taxes. The reason for this result lies in the fact that under small transaction taxes the market is populated by a larger fraction of longer term fundamentalist traders in relation to longer term chartist traders. However, when tax rates are too high, misalignments increase as also found in Westerhoff (2003a, 2008a). The reason for this *u*-shaped response of misalignments to increasing tax rates is caused by the changed composition of trading rules. When tax rates are too high the longer term fundamentalist trading

⁴Note, that under the restriction that all traders have a short-term investment horizon, our model collapses to Westerhoff's model.

rule becomes unpopular, while the number of traders, who favor the longer term chartist trading rule increases. The reason lies in the fact that short-term traders abstain from trading under transaction taxes. The diminishing short-term fluctuations lead to longer swings in the exchange rate, which makes longer term chartist trading rules more profitable. In contrast to Keynes (1936) and Tobin (1978) taxing financial markets is not *per se* stabilizing by making short-term trading unprofitable in favor of longer term trading. Our model shows that the result depends of the composition of the trading rules used in the financial market and depends on tax rate.

The remainder of this paper is organized as follows. The next section introduces the artificial foreign exchange market, while section three will present an analysis of the model's steady state. Section four tackles the validation of this model, while section five discusses the simulation results. Section six ends this paper with conclusions and outlook.

3.2 Transaction Taxes in an Agent-Based Financial Market Model

In this section we introduce the agent-based financial market which is a generalization of the model proposed by Westerhoff (2008a), which can represent either a foreign exchange market, a stock market or a commodity market (Westerhoff 2008a). We interpret it as a foreign exchange market and calibrate it to exchange rate data, here. Following Demary (2008) we introduce longer term traders into this model. If all traders have a daily investment horizon, then our model collapses to

Westerhoff's. Influential contributions to agent-based financial market models⁵ are surveyed in Westerhoff (2008b), Hommes (2006), LeBaron (2006) and Lux (2009a, 2009b). All models have in common that agents choose from a finite set of behavioral rules. Commonly, these are a fundamental trading rule, which reacts to deviations of the asset price from its fundamental value, and a chartist trading rule, which reacts on trends in the asset price. The former one has a centripetal effect on the asset price dynamics, while the later one has a centrifugal effect (Lux 2009a). These heterogenous agents are either assumed to consist of a finite population (Kirman 1991) or of a continuum of agents (Brock and Hommes 1998). Moreover, models may differ in the assumed process for the evolution of heterogeneous trading rules. While in Brock and Hommes (1998) and DeGrauwe and Grimaldi (2006) the popularity of trading rules is governed by the past success of these rules, in Kirman (1991) and Lux and Marchesi (1999, 2000) the evolution of trading rules is governed by social interactions. LeBaron (1999) uses genetic algorithms as evolutionary processes. Macroscopic properties of asset prices like bubbles and crashes, excess volatility, excess kurtosis of the return distribution and volatility clustering emerge from the interaction of agents. Note that these properties cannot simply be deduced by aggregating agents (Westerhoff 2008a) but emerge independently of the microscopic properties (Lux 2009a). An example for an emergent property is the occurrence and burst of a speculative bubble. When the majority of agents relies on chartist rules a speculative bubble can emerge, when this bubble makes fundamental rules more popular and agents switch to this trading strategy, this change in behavior results in a crash back to the fundamental value. Summing up, these models are quite

⁵Important and influential contributions are Day and Huang (1990), Kirman (1991), Chiarella (1992), Chiarella and He (2002), DeGrauwe and Grimaldi (2006), Lux (1995), Lux and Marchesi (1999, 2000), Brock and Hommes (1998), LeBaron (1999) and Farmer and Joshi (2002).

successful in replicating stylized facts of daily financial market data (Lux 2009a).

Similar to Demary (2008) we want to analyze in detail the following proposition - which is often heard from the proponents of transaction taxes and the public media especially in times of financial instability - within our artificial financial market: *transaction taxes stabilize asset prices by crowding out short-term speculators in favor of longer term investors.* In order to analyze this proposition we have to consider the following requirements and assumptions:

- (i) *we need a model in which we are able to distinguish between short-term traders and longer term investors,*
- (ii) *the number of short-term traders and longer term traders should not be fixed but traders should be allowed to change groups or leave the market, and*
- (iii) *the model should be able to match empirical properties of financial market data in order to perform policy experiments which are close to reality.*

For fulfilling requirements (i), (ii) and (iii) the most appealing framework is an agent-based model of a financial market. This artificial foreign exchange market should consist of the following building blocks

- (i) *a fundamental exchange rate s_t^f , which is purely determined by exogenous factors (e.g. monetary aggregates, current accounts, business cycle conditions, ...)*
- (ii) *traders who choose from a finite set of possible trading rules: a short-term fundamentalist rule, a short-term chartist rule, a longer term fundamentalist rule, a longer term chartist rule, or being inactive,*

- (iii) *an evolutionary mechanism for determining the popularity of a certain trading rule according its past performance,*
- (iv) *a market maker who adjusts the exchange rate in response to excess demand,*
- (v) *a policy maker who determines the value of the currency transaction tax rate.*

We will elaborate this building blocks in more detail.

3.2.1 Traders' Demand for Foreign Currency

Westerhoff (2008a) models the agents' demand in line with the literature on heterogeneous agents models of financial markets (Brock and Hommes 1998, Day and Huang 1990, Lux 1995, Lux and Marchesi 2000, DeGrauwe and Grimaldi 2006), but adds random disturbances to the agents' demands in order to account for the empirical variety of trading rules. Orders are assumed to be proportional to the expected exchange rate change

$$d_{i,t} = \beta E_{i,t}[s_{t+1} - s_t], \quad (3.1)$$

with $i \in \{SC, SF, LC, LF\}$ and β as the price impact factor. Short-term chartists' (SC) expectations are given by

$$E_{t,SC}[s_{t+1} - s_t] = \kappa_C(s_t - s_{t-1}) + \varepsilon_t^C, \text{ where } \varepsilon_t^C \sim \mathcal{N}(0, \sigma_C^2), \quad (3.2)$$

while short-term fundamentalists' (SF) expectations are given by

$$E_{t,SF}[s_{t+1} - s_t] = \kappa_F(s_t^f - s_t) + \varepsilon_t^F, \text{ where } \varepsilon_t^F \sim \mathcal{N}(0, \sigma_F^2). \quad (3.3)$$

Chartists trade foreign currency because they expect the recent trend ($s_t - s_{t-1}$) to sustain in the next period. The parameter κ_C governs the strength of the trend extrapolation. Note, that s_t is the exchange rate in logarithmic notation, thus, $s_t - s_{t-1}$ is the percentage change in the exchange rate. Chartists expect to make profits by buying (selling) the exchange rate at s_t and selling (buying) it at the expected higher (lower) value s_{t+1} . Following Westerhoff (2008a), the random disturbance ε_t^C accounts for the variety of possible chartist trading rules. Fundamentalists buy (sell) foreign currency when the current exchange rate s_t is below (above) the fundamental one s_t^f . The reason is that this group expects the exchange rate to return to its fundamental value in the future, where κ_F is the assumed rate of misalignment-correction. Thus, fundamentalist traders expect profits by buying (selling) foreign currency at the exchange rate s_t and selling (buying) it at the higher (lower) one s_{t+1} , which they assume to be close to the fundamental value s_t^f . The fundamental value is assumed to be purely exogenous. Westerhoff (2008a) adds the random disturbance ε_t^F to this equation, which should represent a perception error or a deviation from the strict deterministic trading rule.

In addition to these two trading rules we assume, following Demary (2008), two longer-term trading rules for chartists and fundamentalists. The rationale behind this assumption is that traders assume a longer lasting trend in the exchange rate⁶ or they expect a longer convergence period to the fundamental value. Longer-term chartists (LC) expectations are given by

$$E_{t,LC}[s_{t+N} - s_t] = \left[\frac{1 - (\kappa_C)^N}{1 - \kappa_C} \kappa_C \right] (s_t - s_{t-1}). \quad (3.4)$$

⁶See Engel and Hamilton (2000) for an empirical analysis of long swings in exchange rates.

More precise, it is the chartists' forecast corresponding to an investment horizon of N days. For $N = 1$ this forecasting rule collapses to the conventional one period chartist rule. This trading rule can be derived as follows. When a longer term chartist trader observes the current trend segment $s_t - s_{t-1}$ he or she will expect a trend of $\kappa_C(s_t - s_{t-1})$ for the next period. For calculating the following exchange rate change, the trader uses this forecast and calculates $(\kappa_C)^2(s_t - s_{t-1})$ for the following period. Thus, the expected exchange rate change $s_{t+N} - s_t$ conditional on the chartists' rule the sum over all one period forecasts. By applying the rule for the finite geometric series equation (3.4) can be derived. Following Demary (2008) the longer-term fundamentalists' (LF) expectations are given by

$$E_{t,LF}[s_{t+N} - s_t] = \left[1 - (1 - \kappa_F)^N\right](s_t^f - s_t). \quad (3.5)$$

This trading rule can be derived by the following consideration. When fundamentalist traders observes the misalignment $s_t^f - s_t$ he or she expects $\kappa_F \cdot 100\%$ of this misalignment to be corrected by the next exchange rate change. Thus, he or she expects $(1 - \kappa_F) \cdot 100\%$ of the misalignment to prevail, of which $\kappa_F(1 - \kappa_F) \cdot 100\%$ will be corrected by the subsequent exchange rate change and so on. Thus, the expected exchange rate change $s_{t+N} - s_t$ conditional on the fundamentalist forecasting rule is the sum over all one period forecasts. Again, by applying the formula for the finite geometric series equation (3.5) can be derived. Note, that for $N = 1$ this rule collapses to the conventional one period fundamentalist rule. Furthermore, note that we do not add random disturbances to the longer term trading rules. The reason is that we assume longer term trading rules to be more robust compared to one period rules.

3.2.2 Price Adjustment

Westerhoff (2008a) assumes following Farmer and Joshi (2002) a price impact function which can be interpreted as a stylized description of a risk-neutral market maker. Following Westerhoff (2008a), this market maker mediates transactions out of equilibrium and adjusts prices in response to excess demand. More precisely, the market maker will rise the exchange rate when excess demand for foreign currency is positive, while he will lower the exchange rate in response to negative market demand D_t

$$s_{t+1} = s_t + D_t + \varepsilon_t, \text{ where } \varepsilon_t \sim \mathcal{N}(0, \sigma_s^2). \quad (3.6)$$

Market demand D_t is defined as the sum of orders of fundamentalist traders and chartist traders weighted by their pertinent population weights w_t^{SC} , w_t^{SF} , w_t^{LC} and w_t^{LF}

$$D_t = w_t^{SC} d_t^{SC} + w_t^{SF} d_t^{SF} + w_t^{LC} d_t^{LC} + w_t^{LF} d_t^{LF}. \quad (3.7)$$

Westerhoff (2008a) adds the random disturbance ε_t to the market maker's price adjustment rule, because it only represents a simple representation of real markets. It can be interpreted as a non-fundamental innovation. From equations (3.6) and (3.7) can be inferred that the orders of the four trader groups as well as their population fractions determine exchange rate dynamics in a nonlinear way.

3.2.3 Evolution of Trading Rules

In the model of Westerhoff (2008a) traders have three alternatives. They can either be a fundamentalist trader or a chartist trader. The third possibility for traders is to stay inactive. In our version of this model traders have two additional alternatives.

They can either be a longer term chartist or a longer term fundamentalist. Following Brock and Hommes (1998), DeGrauwe and Grimaldi (2006) and Westerhoff (2008a) the selection of one of these five alternatives depends on the strategies' past performances. The rationale behind this is an evolutionary mechanism in which more agents prefer to follow the trading rule which was most profitable in the past. Note, that this mechanism does not model herding behavior explicitly because there is no social interaction.⁷ Inspired by Westerhoff (2008a) we assume the following fitness functions for short-term fundamentalist and short-term chartists

$$A_t^{SC} = (\exp\{s_t\} - \exp\{s_{t-1}\})d_{t-2}^{SC} - \tau(\exp\{s_t\} + \exp\{s_{t-1}\})|d_{t-2}^{SC}| + \theta A_{t-1}^{SC} \quad (3.8)$$

$$A_t^{SF} = (\exp\{s_t\} - \exp\{s_{t-1}\})d_{t-2}^{SF} - \tau(\exp\{s_t\} + \exp\{s_{t-1}\})|d_{t-2}^{SF}| + \theta A_{t-1}^{SF}. \quad (3.9)$$

The first term of the performance measures A_t^{SC} and A_t^{SF} is the return the agent got by investing in foreign currency by placing his or her order d_{t-2}^{SC} or d_{t-2}^{SF} to the market maker. Here, Westerhoff (2008a) assumes that traders submit orders in period $t-2$, which are executed at period $t-1$. Whether a pertinent trading rule is profitable or not thus depends on the realized price in period t . The second term is the transaction cost the trader has to pay when executing orders. If the trader buys (sells) foreign currency at the price s_{t-1} he or she has to pay a tax amount of $\tau \exp\{s_{t-1}\}|d_{t-2}|$ on this transaction, where τ is the transaction tax rate. The trader only realizes a profit if he or she sells (buys) the currency back at the price s_t . Again, a tax will be levied on this transaction with the tax amount $\tau \exp\{s_t\}|d_{t-2}|$. Thus, Westerhoff

⁷For herding models see Lux and Marchesi (1999, 2000) and Lux (2009a).

(2008a) assumes a round trip where the investors have to pay the transaction tax twice. The parameter θ is a memory parameter. Thus, the last term measures how quickly profits are discounted for strategy selection. If θ is high, then past profits generated by this trading rule will be considered in today's strategy selection, while for $\theta = 0$ past profits do not play any role for today's strategy selection. For $d = 1$ all past profits will play a role for today's selection. The fitness measures for the longer term trading strategies A_t^{LC} and A_t^{LF} are

$$\begin{aligned} A_t^{LC} &= (\exp\{s_t\} - \exp\{s_{t-N}\})d_{t-N-1}^{LC}/N \\ &\quad - \tau(\exp\{s_t\} + \exp\{s_{t-N}\})|d_{t-N-1}^{LC}|/N + \theta A_{t-1}^{LC} \end{aligned} \quad (3.10)$$

$$\begin{aligned} A_t^{LF} &= (\exp\{s_t\} - \exp\{s_{t-N}\})d_{t-N-1}^{LF}/N \\ &\quad - \tau(\exp\{s_t\} + \exp\{s_{t-N}\})|d_{t-N-1}^{LF}|/N + \theta A_{t-1}^{LF}. \end{aligned} \quad (3.11)$$

Following Demary (2008), we divide the profit generated by the multi-period investment through the investment horizon N . Thus, we measure the profit per day. This assumption is necessary in order to have a fair comparison between short-term trading strategies and longer term strategies. Following Westerhoff (2008a) the profit of being inactive for one period is zero. Following Brock and Hommes (1998) and Westerhoff (2008a) the population fractions of agents are given by the discrete choice model proposed by Manski and McFadden (1981)

$$w_t^{SC} = \frac{\exp\{\gamma A_t^{SC}\}}{1 + \exp\{\gamma A_t^{SC}\} + \exp\{\gamma A_t^{SF}\} + \exp\{\gamma A_t^{LC}\} + \exp\{\gamma A_t^{LF}\}} \quad (3.12)$$

$$w_t^{SF} = \frac{\exp\{\gamma A_t^{SF}\}}{1 + \exp\{\gamma A_t^{SC}\} + \exp\{\gamma A_t^{SF}\} + \exp\{\gamma A_t^{LC}\} + \exp\{\gamma A_t^{LF}\}} \quad (3.13)$$

for short term traders,

$$w_t^{LC} = \frac{\exp\{\gamma A_t^{LC}\}}{1 + \exp\{\gamma A_t^{SC}\} + \exp\{\gamma A_t^{SF}\} + \exp\{\gamma A_t^{LC}\} + \exp\{\gamma A_t^{LF}\}} \quad (3.14)$$

$$w_t^{LF} = \frac{\exp\{\gamma A_t^{LF}\}}{1 + \exp\{\gamma A_t^{SC}\} + \exp\{\gamma A_t^{SF}\} + \exp\{\gamma A_t^{LC}\} + \exp\{\gamma A_t^{LF}\}} \quad (3.15)$$

for longer term traders, and finally

$$w_t^I = \frac{1}{1 + \exp\{\gamma A_t^{SC}\} + \exp\{\gamma A_t^{SF}\} + \exp\{\gamma A_t^{LC}\} + \exp\{\gamma A_t^{LF}\}} \quad (3.16)$$

for inactive traders, whose profits are zero by construction. Note that the higher the fitness of one particular strategy, the higher will be the percentage fraction of agents, who use it. The parameter $\gamma \geq 0$ controls how sensitive traders react to a change in the fitness measure of their trading rule. The higher γ the more agents switch to the strategy with the highest fitness. For $\gamma = 0$ all trading strategies will be selected with equal probability, while for $\gamma = \infty$ all agents select the strategy with the highest performance.

Note, that for $N = 1$ (all traders have the same investment horizon) our model collapses to the model of Westerhoff (2008a).

3.3 (Non-)Fundamental Steady-States

In order to analyze the steady-states of the model we have to set all shocks to zero and concentrate on the deterministic skeleton of the model. In order to be in steady

state the restriction

$$(s_t, s_t^f, d_t^i, w_t^i, A_t^i) = (s_{t-1}, s_{t-1}^f, d_{t-1}^i, w_{t-1}^i, A_{t-1}^i) = (\bar{s}, \bar{s}_t, \bar{d}^i, \bar{w}^i, \bar{A}^i) \quad (3.17)$$

should hold ($i \in \{SF, LF, SC, LC, I\}$). Thus, all variables should be equal to their (fundamental) long-run values and all dynamics should rest there. Under the restriction $s_t = s_{t-1} = \bar{s} = \bar{s}^f$ all fitness measures collapse to zero

$$\overline{A^{SC}} = \overline{A^{SF}} = \overline{A^{LC}} = \overline{A^{LF}} = 0 \quad (3.18)$$

leading to a uniform distribution of the popularity of all five trading rules

$$\overline{w^{SC}} = \overline{w^{SF}} = \overline{w^{LC}} = \overline{w^{LF}} = \overline{w^I} = 0.2. \quad (3.19)$$

Chartists' demand will be zero when the exchange rate remains constant

$$\overline{d^{SC}} = \overline{d^{LC}} = 0, \quad (3.20)$$

while the fundamentalists' demand will only be zero when the steady state exchange rate \bar{s} equals the fundamental exchange rate \bar{s}^f

$$\overline{d^{SF}} = \overline{d^{LF}} = 0. \quad (3.21)$$

Thus, the fundamental steady state is characterized by zero demands for foreign currency, zero profits and equal selection of possible trading rules. Note, that this result can often be found in heterogeneous traders models of this type (see DeGrauwe and Grimaldi 2006).

When the exchange rate is in a state \bar{s} , where it is different from the fundamental exchange rate $\bar{s} \neq \bar{s}^f$ fundamentalist traders' demand is always positive in absolute value. These orders will push the exchange rate back to the fundamental value.

3.4 Calibration and Model Validation

In order to use this artificial foreign exchange market as a computer laboratory for the analysis of regulatory policies we have to assume numerical values for the model's parameters first.⁸ This set of parameter values can be found in Table 3.1. Most of the parameter values are taken from Westerhoff (2008a). According to him, parameters are chosen such that the model is able to match numbers and statistics of real world financial market data.⁹ Westerhoff (2008a) assumes both extrapolation parameters to have the value 0.04. By following his suggestions short-term chartists expects a return of 0.04 percent for the next day in response to a return of 1 percent today and a cumulative return of 0.046 percent over the next 30 days. Fundamentalists, however, assume a return of 0.04 in response to a misalignment of 1 percent and a cumulative return of 0.71 over the next 30 days.

Westerhoff (2008a), Lux and Marchesi (2000), Lux (2009a) and Franke and Westerhoff (2009) validate agent-based models by analyzing how good the model is able to reproduce stylized facts of empirical daily financial market data like uncorrelated raw returns, volatility clustering, long memory and fat tails of the return distri-

⁸All programming and computations were done using the free open source software R (R Development Core Team 2009).

⁹Studies that estimate rather than calibrate these models are Gilli and Winker (2003), Westerhoff and Reitz (2003), Lux (2009c), Alfarano, Lux and Wagner (2005), Boswijk, Hommes and Manzan (2007), Winker, Gilli and Jeleskovic (2007), Manzan and Westerhoff (2007) and Ghongadze and Lux (2009). These studies suggest to that the chartist and fundamentalist reaction parameters κ^C and κ^F lie between 0 and 0.1 for daily data (see Westerhoff 2008a).

Table 3.1: Parameter Calibration

| Parameter | Value | Interpretation |
|---------------|-------|---|
| β | 1.00 | price adjustment |
| σ_s | 0.01 | volatility of non-fundamental innovations |
| σ_{sf} | 0.01 | volatility of fundamental innovations |
| κ_C | 0.04 | chartists' reaction |
| σ_C | 0.03 | variety of chartist rules |
| κ_F | 0.04 | fundamentalists' reaction |
| σ_F | 0.005 | variety of fundamentalist rules |
| γ | 800 | intensity of choice |
| θ | 0.985 | memory parameter |
| N | 30 | longer term investment horizon |

Note: Most parameter values are based on Westerhoff (2008a). We set a higher value for the intensity of choice parameter γ . The longer term investment horizon is assumed to be 30 days.

bution. Thus, we analyze how numbers and statistics like distributional moments, autocorrelations and distributional shape parameters of our generated computer laboratory data match numbers and statistics of data generated in the real world. If our models produces data whose properties are close to those of real world data, than we will - as proposed by Lux (2009a) - be able to perform an economic policy analysis which should be close to reality. The economic policy analysis can be done by running the simulations for a given seed of random variables but for different values of the policy parameter. More general results can be achieved by calculating average statistics over several simulation runs. These results can be interpreted as cross-section averages over several artificial financial markets.

Following Westerhoff (2008a) and Lux (2009a) we use the following validation criteria (Lux-Westerhoff criteria hereafter):

- (i) *the model should generate bubbles and crashes (deviations from fundamental value),*

- (ii) *asset prices should be more volatile than their fundamental values (excess volatility),*
- (iii) *the return distribution should deviate from the normal distribution (excess kurtosis),*
- (iv) *absence of autocorrelations in raw returns (non-predictability of daily returns),*
- (v) *hyperbolically decaying autocorrelations of absolute returns (volatility clustering).*

Table 3.2: Summary Statistics: Baseline Simulation versus Empirical Data

| | Model | USD-Euro | YEN-USD | GBP-USD | USD-AusD |
|----------|-------|----------|---------|---------|----------|
| mean | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| st. dev. | 0.015 | 0.007 | 0.007 | 0.006 | 0.009 |
| skewness | 0.000 | 0.179 | -0.509 | -0.330 | -0.757 |
| kurtosis | 6.963 | 5.560 | 6.885 | 9.315 | 17.009 |

NOTE: Mean, variance, skewness and kurtosis are calculated from the model generated exchange rate return data by using the parameters given in Table 3.1. The exchange rate data for the US-Dollar to Euro, Yen to US-Dollar, Great Britain Pound to US-Dollar and US-Dollar to Australian Dollar are taken from the FRED2 database of the Federal Reserve Bank of St. Louis in daily frequency. The data series range from 1999-01-04 to 2009-10-09 and are available under the series-ID: DEXUSEU, DEXJPUS, DEXUSUK and DEXAUUS.

Figure 3.1 contains results of the baseline simulation of our artificial foreign exchange market. The baseline simulation is characterized by the absence of transaction taxes. Moreover, the fundamental exchange rate is assumed to stay constant. Thus, fundamental based trading rules are not affected by the risk that the fundamental rate will change in the future. Fundamental risks make arbitrage more risky (Brunnermeier 2001) and thereby limit arbitrage. As a result, the fundamental trading rules may become less profitable, because fundamental forecasting rules generate larger prediction errors. We will tackle the problem of fundamental risks in section 3.5.2

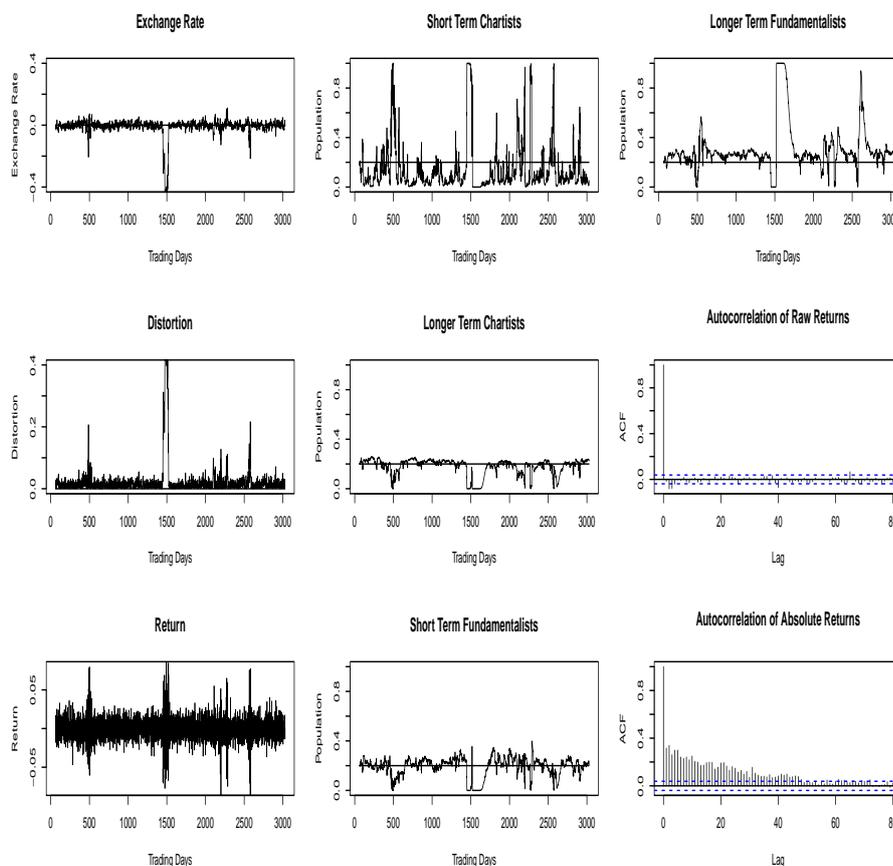


Figure 3.1: Simulation without Transaction Taxes

NOTE: Simulation of 3000 artificial trading days. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.005$, $\sigma_c = 0.03$ and $\sigma_s = 0.01$. The fundamental value is assumed to be constant and normalized to zero. Distortion is measured as the absolute value of the deviation of the exchange rate from its fundamental value.

and abstain from these kinds of risk during the baseline simulation.

From Figure 3.1 we can infer that most of the chartist traders prefer to be short-term traders, while fundamental traders prefer the longer term investment horizon. This results is in line with the argument of Keynes (1936) and Tobin (1978) that short-term traders are destabilizing, while longer term traders are stabilizing. Moreover, this result is also in line with the empirical evidence from questionnaire studies like

Taylor and Allen (1992)¹⁰. There are periods with sharp increases in the number of short-term chartist traders. These periods correspond to periods with high volatility and large misalignments in the exchange rate. Thus, short-term chartists lead to additional risks. These periods of high volatility are followed by periods with a low volatility and a high popularity of the longer term fundamentalists trading strategy. Raw returns display two small negative autocorrelations. These mean reverting dynamics may result from the dominance of longer term fundamental traders who trade large orders against the mispricing. The autocorrelation function of absolute returns shows slowly decaying serial correlations in the magnitude of returns which correspond to the persistent phases of high and low volatility in the artificial return data. Summing up, the model is able to generate bubbles and crashes which can be inferred from the time series plot of misalignments (Figure 3.1). Moreover, it is able to generate non-predictable returns, which can be inferred from the small serial correlations in the artificial exchange rate returns. Furthermore, the model is able to generate volatility clustering. This, can be inferred by just eyeballing the time series of returns or more elaborate by the slowly decaying serial correlations in absolute returns. Moreover, within the model a return distribution emerges which is characterized by excess kurtosis. As can be inferred from Table 3.2, the model is able to generate statistics, which are in line with the statistical properties of the Yen, Euro, the Great Britain Pound and the Australian Dollar versus US-Dollar daily exchange rate data. Note, that empirical data as well as data generated by our artificial foreign exchange market are characterized by zero means, standard deviations in the range of 0.006 to 0.015 and a kurtosis measures that ranges from 5.6 to 17.0. Thus, our model for daily exchange rate fluctuations can be regarded as

¹⁰See Menkhoff and Taylor (2007) for an survey article on questionnaire studies in financial markets.

validated by the Lux-Westerhoff criteria (Westerhoff 2008a, Lux 2009a). Moreover, we can conclude that all stylized facts remain unaffected by the introduction of longer term traders.

3.5 The Effectiveness of Transaction Taxes

Lux (2009a) and Westerhoff (2008a) suggest to use agent-based models as computer laboratories for performing economic policy experiments which are prohibitively costly to perform in real world markets. The advantages of agent-based modeling referred in Westerhoff (2008a) apply to the agent-based experiments presented in this paper in the following way:

- (i) We are able to generate as much data points as needed for our policy analysis. In our agent-based policy analysis we choose following Westerhoff (2008a) a simulation horizon of 5000 data points, which corresponds to a time horizon of 20 years since the model is calibrated to daily data. In more detail, we simulate 100 simulation runs of 5000 artificial trading days and take averages over all numbers and statistics for each value of the currency transaction tax. The advantage of this Monte-Carlo procedure is that our results do not only depend on one certain seed of random variables. One can interpret the generated sample as a panel of 100 foreign exchange markets over 5000 time steps.
- (ii) Westerhoff (2008a) notes that the second advantage of agent-based modeling is that we are able to measure all variables precisely during our policy analysis. Within the agent-based experiments presented here we are able to measure the fundamental value as well as the decisions of our artificial traders precisely.

(iii) We are able to control for exogenous shocks. Within our simulation we introduce three types of exogenous events. These are random deviations from the market maker's price adjustment rule, fundamental news, and random deviations from the chartists' and fundamentalists' trading rules. Other exogenous events like a large drop in the fundamental value (e.g. a big recession) are not introduced. Thus, we analyze the effectiveness of the currency transaction tax during "normal" trading days.

(iv) We perform the simulations under the same conditions (the same seed of random variables), but with different values of the policy instrument. In this way we are able to get an inference on how the policy instrument changes macroscopic properties - measured by numbers and statistics - of our artificial market. Westerhoff (2003a, 2008a) suggests the following evaluation criteria

(a) volatility, defined as the average absolute change in the exchange rate

$$\text{Vol} = \frac{1}{T} \sum_{t=1}^T |s_t - s_{t-1}|, \quad (3.22)$$

and

(b) distortion

$$\text{Dis} = \frac{1}{T} \sum_{t=1}^T |s_t^f - s_t|, \quad (3.23)$$

which is defined as the average absolute deviation of the exchange rate s_t from its fundamental value s_t^f .

Moreover, Demary (2008) and Westerhoff (2008a) suggest to analyze the change the average percentage fraction of used trading rules as a measure of traders' behavior, which will change in response to policy changes, while

Demary (2008) suggests also to analyze the change in the kurtosis of the exchange rate returns

$$\text{Kurt} = \frac{\frac{1}{T} \sum_{t=1}^T (s_t - s_{t-1})^4}{\left(\frac{1}{T} \sum_{t=1}^T (s_t - s_{t-1})^2\right)^2} \quad (3.24)$$

as a measure of catastrophic risks.

3.5.1 Simulations without Fundamental Risk

Figure 3.2 to 3.4 contain snapshots of 3000 trading days within our artificial foreign exchange market. Figure 3.2 is based on a tax rate of 0.1 percent on each currency transaction, while Figure 3.3 and Figure 3.4 are based on the tax rates of 0.3 and 0.5 percent. From Figure 3.2 one can infer by just visual inspection that bubbles and crashes, volatility clusters and distortions still prevail under a small transaction tax. Absolute returns are still characterized by persistent serial correlations. Similar to Figure 3.1 most of the traders prefer to be short-term chartist or longer term fundamentalist. Figure 3.3 shows a simulation run of our artificial foreign exchange market under a transaction tax rate of 0.3 percent. What can be seen is that short-term chartism now has the lowest popularity. The reason is that taxation makes this trading rule too expensive. The large fluctuations and amplitudes in short-term chartist and longer term fundamentalists population fractions are now absent. As a result, the occurrence of spectacular bubbles and crashes also diminishes, as can be inferred from the time series plot of distortions. Moreover, taxation reduces the autocorrelation of absolute returns. Thus, volatility clusters are absent under this tax rate. From Figure 3.4 can be inferred that taxation of financial transactions smoothes the fluctuations in the popularity of trading rules. However, it seems that

the highest stabilizing impact is achieved by increasing the tax rate from 0 to 0.1 percent.

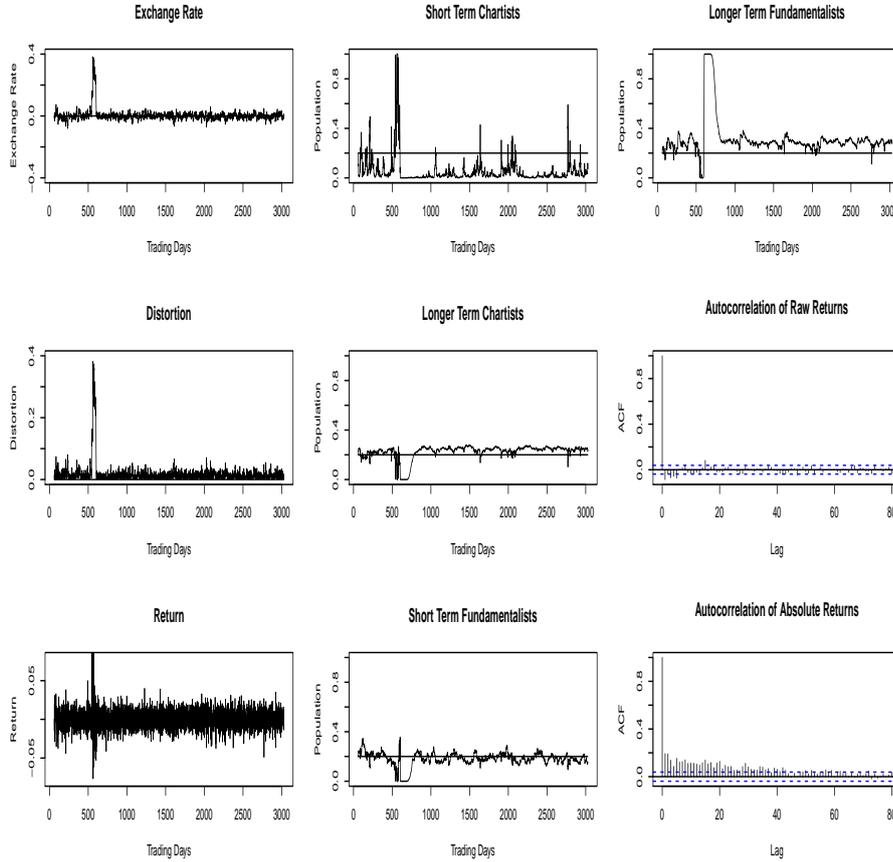


Figure 3.2: Simulation with a 0.1 Percent Transaction Tax Rate

NOTE: Simulation of 3000 artificial trading days. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.01$, $\sigma_c = 0.03$ and $\sigma_s = 0.01$. The fundamental value is normalized to zero. The transaction tax rate is 0.1 percent. Distortion is measured as the absolute value of the deviation of the exchange rate from its fundamental value.

Figure 3.5 contains summarized results of our policy experiments for small transaction taxes between 0 and 1 percent. Each reported number is the average over 5000 artificial trading days and 100 simulation runs. The simulations for different tax rates are based on the same seed of random variables. The tax rate is varied

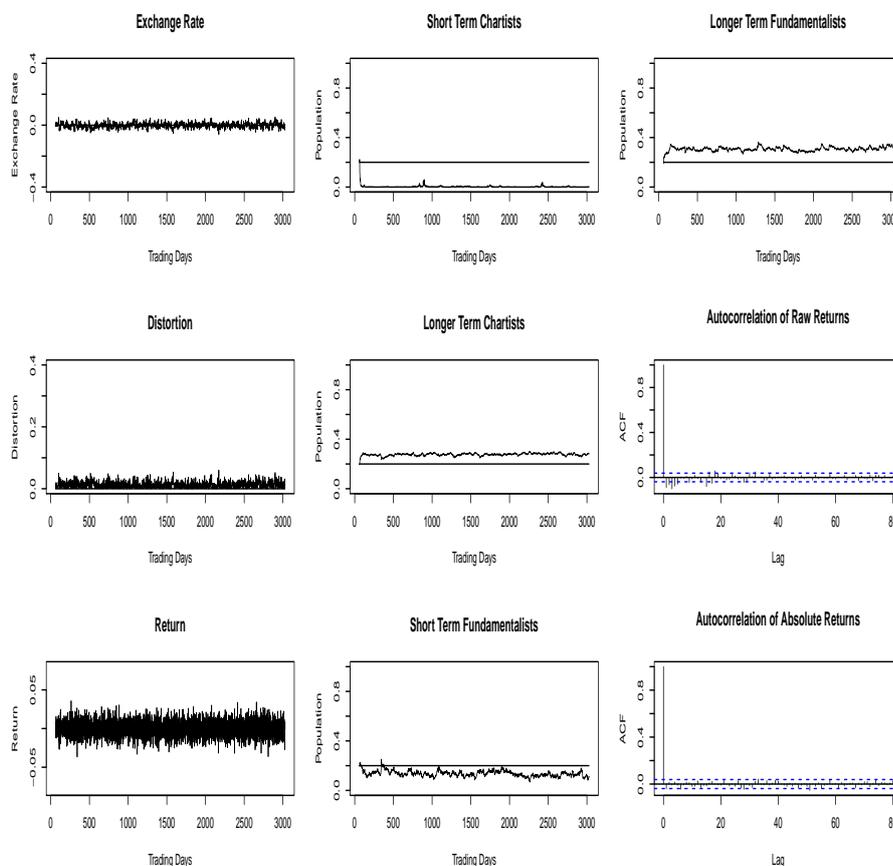


Figure 3.3: Simulation with a 0.3 Percent Transaction Tax Rate

NOTE: Simulation of 3000 artificial trading days. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.01$, $\sigma_c = 0.03$ and $\sigma_s = 0.01$. The fundamental value is normalized to zero. The transaction tax rate is 0.3 percent. Distortion is measured as the absolute value of the deviation of the exchange rate from its fundamental value.

from 0 (the baseline case) in 0.1 percentage point steps and measured by the x-axis of the plot. As one can see, taxation of foreign currency transactions with small tax rates does not change the average returns on holding foreign currency. However, it decreases the volatility of exchange rate returns and the distortion of the market (the average misalignment). Thus, the transaction tax helps to decrease price volatility and brings the exchange rate on average closer to its fundamental value.

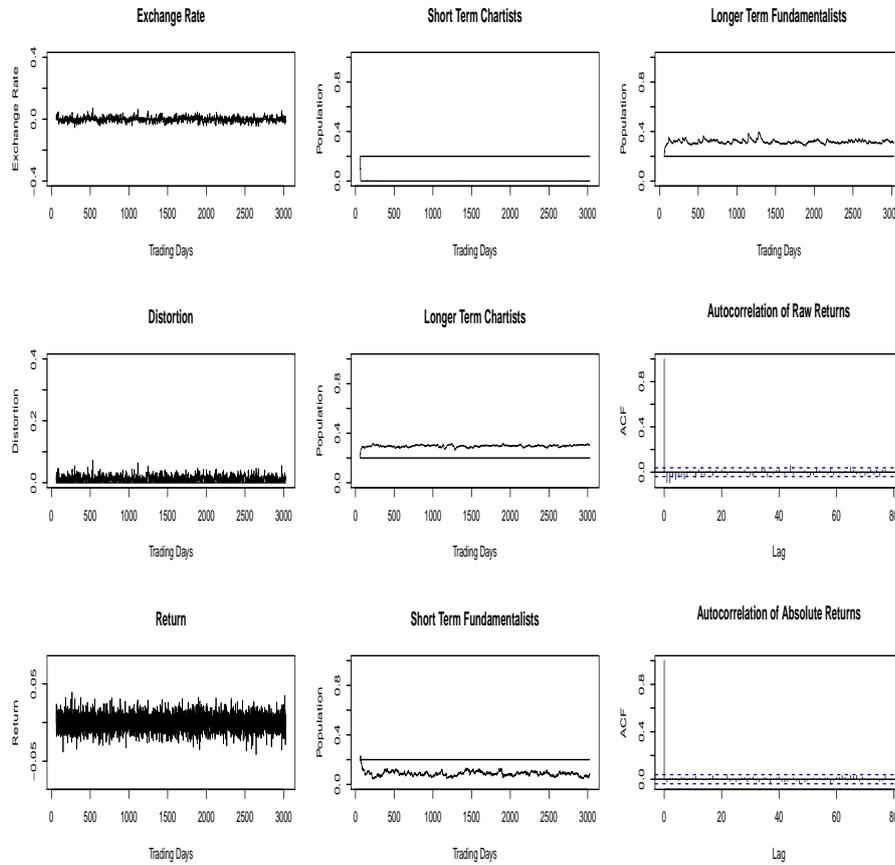


Figure 3.4: Simulation with a 0.5 Percent Transaction Tax Rate

NOTE: The numbers and statistics of this figure are based on 3000 artificial trading days. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.01$, $\sigma_c = 0.03$ and $\sigma_s = 0.01$. The fundamental value is normalized to zero. The transaction tax rate is 0.5 percent. Distortion is measured as the absolute value of the deviation of the exchange rate from its fundamental value.

However, the tax has no effect on the kurtosis of the exchange rate return distribution. The reason for this can be inferred from the change in the composition of trading rules. The number of short-term chartists and short-term fundamentalists decreases to zero on average, while the number of longer-term chartists and longer-term fundamentalists is increasing. Moreover, the number of inactive traders is

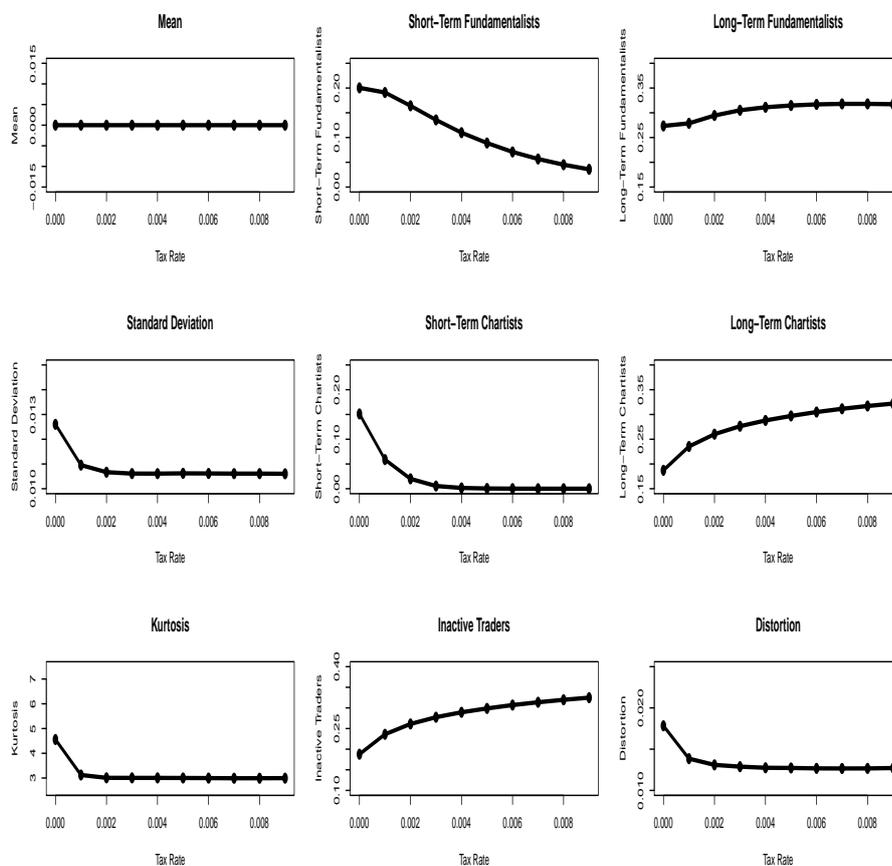


Figure 3.5: The Effectiveness of Small Currency Transaction Taxes

NOTE: The numbers and statistics of this figure are averages over 5000 artificial trading days over 100 simulation runs. They can be interpreted as averages over time and (artificial) markets. Note that all simulations for different values of the transaction tax rate are based on the same seed of random variables. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.005$, $\sigma_c = 0.03$ and $\sigma_s = 0.01$. The fundamental value is normalized to zero. Distortion is defined as the average absolute deviation of the exchange rate from its fundamental value.

increasing because some short-term traders decide not to trade because transaction costs are higher than the returns of trading in the foreign exchange market. This is also the reason why the kurtosis of the exchange rate return distribution is not increasing. Suppose that all short-term traders switch to the longer-term trading

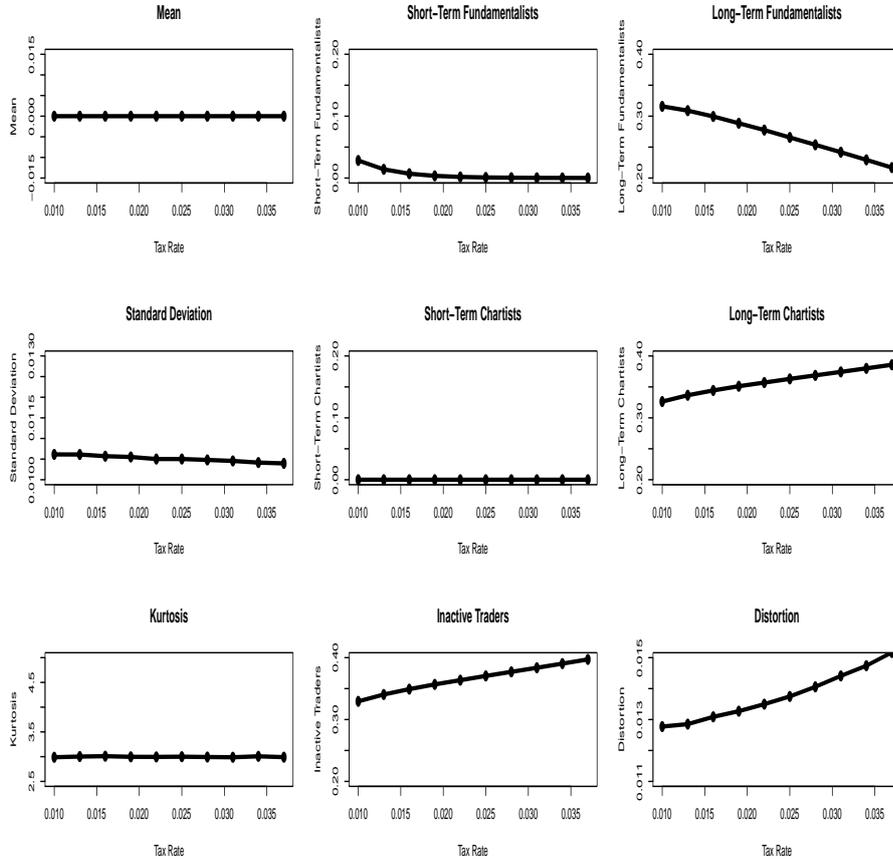


Figure 3.6: The Effectiveness of Higher Currency Transaction Taxes

NOTE: The numbers and statistics of this figure are averages 5000 artificial trading days over 100 simulation runs. They can be interpreted as averages over time and (artificial) markets. Note that all simulations for different values of the transaction tax rate are based on the same seed of random variables. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.005$, $\sigma_c = 0.03$ and $\sigma_s = 0.02$. The fundamental value is normalized to zero. Distortion is defined as the average absolute deviation of the exchange rate from its fundamental value.

strategies. This change in behavior will result in the fact that some traders place larger orders every 30 days instead of placing small orders every day. Because returns are proportional to market demand in our model, larger orders in a 30 days cycle lead to more frequent large returns and thus to a larger kurtosis of the return distribution. In the artificial foreign exchange market proposed by Demary (2008) a

lot of traders change from short-term trading to longer term trading instead of becoming inactive. Therefore, the kurtosis of the exchange rate distribution increases in his simulations, which means a higher probability of large positive and negative returns. In the model presented here, however, enough short-term traders decide not to trade instead of becoming longer-term traders, which leads to the observed negligible effect of transaction taxes on the extreme parts of the return distribution measured by the kurtosis.

Figure 3.6 contains summary results for tax rates between 1 and 4 percent. As one can see, higher tax rates have no significant impact onto the standard deviation and the kurtosis of the exchange rate returns. Within this interval for the tax rate the number of short-term chartists is zero, while the number of short-term fundamentalists declines to zero. For tax rates above 2.5 percent there are no short-term fundamentalists in the market. The number of inactive traders is monotonically increasing under these tax scenarios and lies between 30 and 40 percent. When the tax rate takes the value 4 percent, then 40 percent of the artificial traders decide not to trade. Moreover, it can be inferred that the number of longer term fundamentalists is decreasing under rising tax rates. While more than 30 percent of all traders use the longer term fundamentalist strategy for a tax rate of 1 percent, only 20 percent use this strategy for a tax rate of 4 percent. However, higher tax rates lead to a higher popularity of the longer term chartist trading rule. Under a 1 percent transaction tax approximately 30 percent of all traders use the longer term chartist rule, while approximately 40 percent use the chartist rule under a tax rate of 4 percent. The reason for the increasing popularity of the chartist rule is the following. Short-term trading is prohibitively costly under these high tax rates. When there is no short-term trading, then high frequency fluctuations in the

exchange rate are absent and longer lasting trends emerge. When there are longer lasting trends with less noise, then the longer term chartist forecasting technique becomes more precise and therefore more agents prefer to choose the chartist trading rule. Similar, longer term fundamentalist rules become more unprecise when the exchange rate exhibits longer lasting trends. As a result, which emerges from the changed composition of trading rules, distortion is rising under higher tax rates. The reason is, that under higher tax rates chartist rules are more frequently used compared to fundamentalist trading rules. A similar result can also be found in Westerhoff (2003a). Furthermore, it can be inferred from Figure 3.6, that higher tax rates do not have any significant impact on volatility because the number of short-term traders is zero. In contrast to Demary (2008) higher tax rates do not have any impact on the kurtosis of the return distribution. As explained above, the kurtosis is increasing in his model because a lot of short-term traders decide to become longer term traders. This results in a higher frequency of larger orders which leads to a higher kurtosis of the return distribution. In this model, however, 40 percent of all traders decide not to trade instead of becoming longer term traders. Thus, higher tax rates have no significant impact on the kurtosis of the return distribution here.

3.5.2 Simulations under Fundamental Risk

In Figure 3.7 the fundamental exchange rate is assumed to follow a random walk $s_{t+1}^f = s_t^f + 0.01\varepsilon_{t+1}$, where ε_t is standard normally distributed fundamental news. When the fundamental rate follows a random walk, then the fundamental-based trading strategies becomes more risky. The reason is that fundamentalists can only assume that the exchange rate changes in order to correct to the fundamental rate

s_t^f , because it is the best forecast of the uncertain future fundamental rate s_{t+N}^f . When there is a large fundamental innovation, then fundamentalists make a large prediction error. As a consequence, fundamentalists lose money which may result in a higher popularity of chartist rules for the next period. Brunnermeier (2001) notes that this fundamental risk limits arbitrage and in this way leads to persistent speculative bubbles. From Figure 3.7 can be inferred, that similar to the simulations without fundamental risk most of the chartist traders are short term traders, while most of the fundamentalists are long- term traders. Again, periods with a clear dominance of chartist traders correspond to periods with large fluctuations in the exchange rate and large misalignments. These periods are followed by longer correction periods with a clear dominance of longer term fundamentalist traders. The emergent properties are again excess kurtosis of the return distribution and volatility clustering, which can be inferred from the slowly decaying autocorrelations of absolute returns. In contrast to the simulations without fundamental risk, raw returns seem to exhibit more persistent autocorrelations.

Figure 3.8 contains simulation results of our artificial foreign exchange market with fundamental risk in which a policy maker introduces a currency transaction tax rate of 0.5 percent. Under taxation short term chartism becomes unprofitable, thus, the number of traders who use this trading rule becomes zero over the whole simulation horizon. In reaction to missing chartists misalignments diminish, which is the reason for the diminishing periods with a clear dominance of longer term fundamental traders. Short-term fundamentalism also becomes unprofitable, however, longer term chartism rises in popularity compared to the case without taxation. Emergent properties from this changed interaction are diminishing autocorrelations of raw returns and diminishing autocorrelations of absolute returns. Thus, under taxation

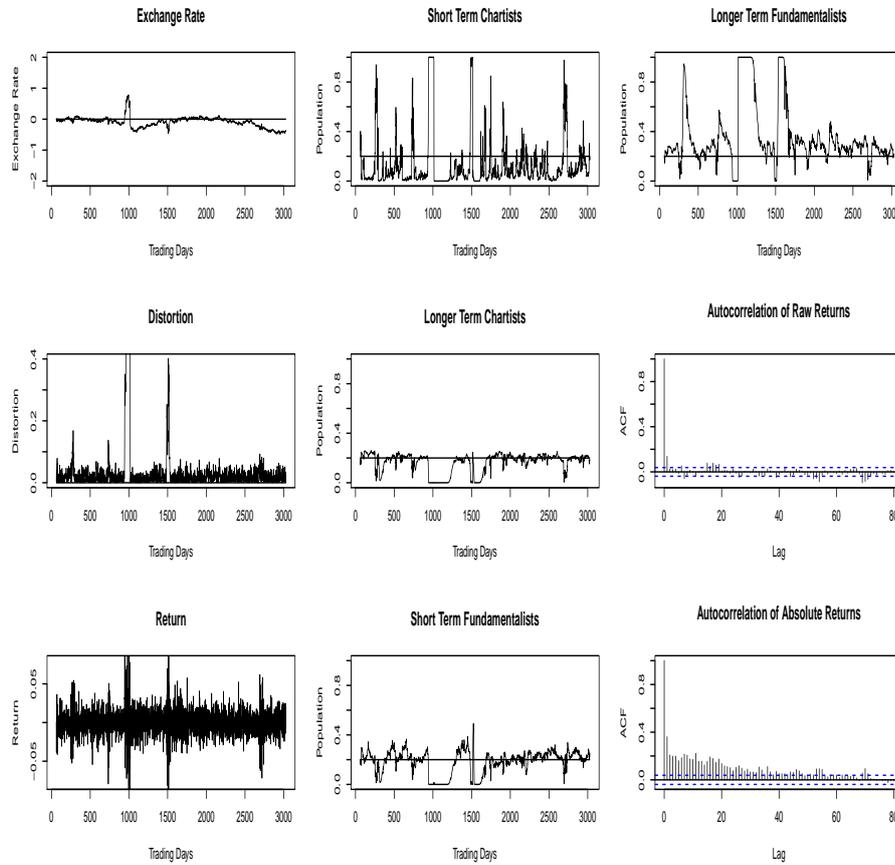


Figure 3.7: Simulation with Fundamental Risk and no Transaction Taxes

NOTE: Simulation of 3000 artificial trading days. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.005$, $\sigma_c = 0.03$ and $\sigma_s = 0.02$. The fundamental value is normalized to zero. The transaction tax rate is 0 percent.

volatility clusters diminish. Moreover, the misalignments decrease in amplitude.

Figure 3.9 contains figures with summary statistics over 100 simulation runs of size 5000. These can be interpreted as statistics over a panel of 100 artificial markets and 5000 trading days. These summary statistics are plotted for different values of the currency transaction tax rate. Similar to other configurations taxation does not change average daily returns. The standard deviation of returns and the kurtosis of

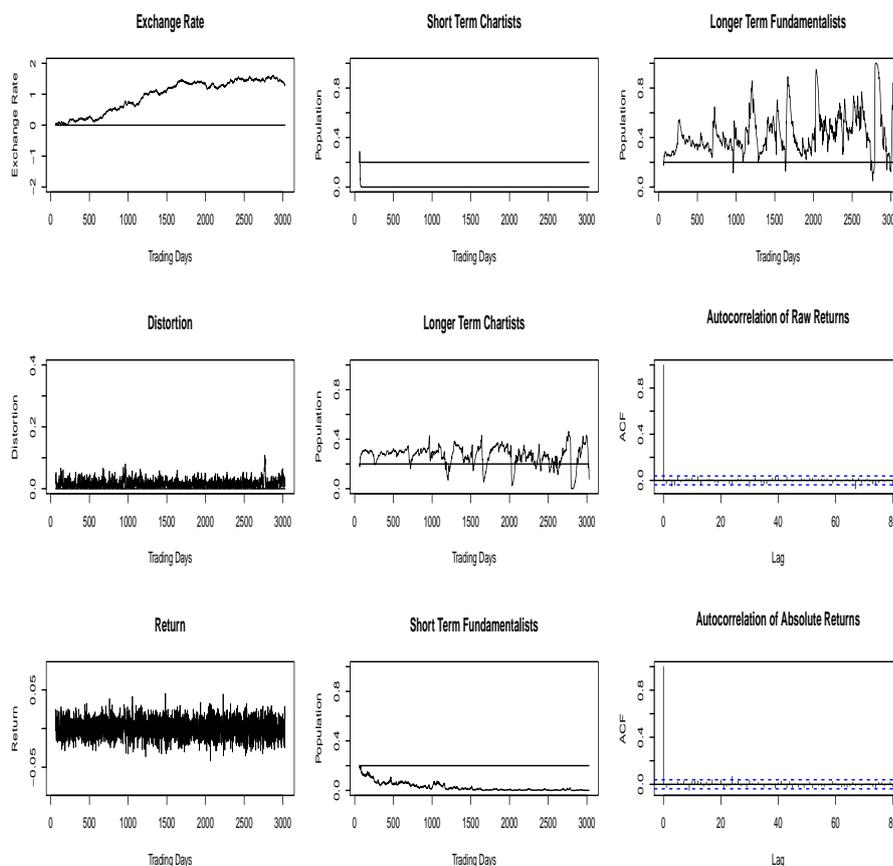


Figure 3.8: Simulation with Fundamental Risk and a 0.5 percent Transaction Tax
 NOTE: Simulation of 3000 artificial trading days. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.005$, $\sigma_c = 0.03$ and $\sigma_s = 0.02$. The fundamental value is normalized to zero. The transaction tax rate is 0.5 percent. Distortion is defined as the absolute deviation of the exchange rate from its fundamental value.

the return distribution are decreasing under taxation. The lower kurtosis measure is due to the diminishing volatility clusters under taxation. Taxation of round trips increases the costs of speculation. In response the number of inactive traders is increasing in the transaction tax rate. As already indicated Figure 3.8, the number of short-term fundamentalists and short-term chartists are decreasing in the tax rate, while the number of longer term chartists and longer term fundamentalists is at least

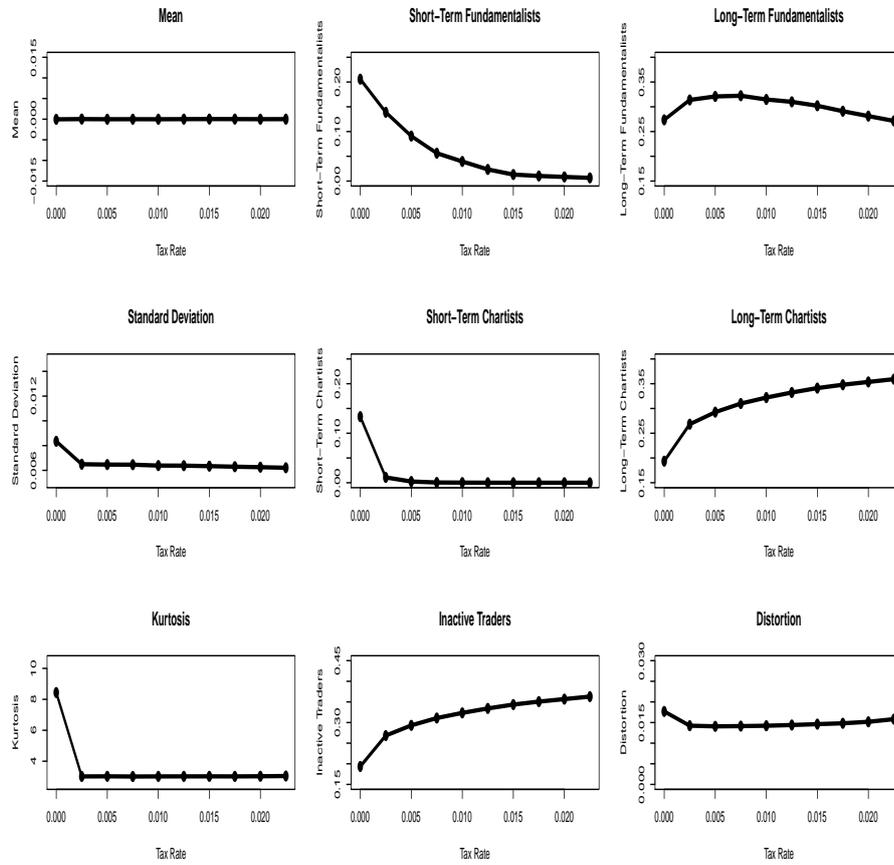


Figure 3.9: The Effectiveness of Small Transaction Taxes under Fundamental Risk

NOTE: The numbers and statistics of this figure are averages over 5000 artificial trading days over 100 simulation runs. They can be interpreted as averages over time and (artificial) markets. Note that all simulations for different values of the transaction tax rate are based on the same seed of random variables. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.005$, $\sigma_c = 0.03$ and $\sigma_s = 0.02$. Distortion is defined as the average absolute deviation of the exchange rate from its fundamental value.

for small tax rates increasing. Note, that the response of longer term fundamentalist traders is hump-shaped again. For tax rates below 0.5 percent the number of longer term term fundamentalists is increasing, while fundamentalists decrease in number when transaction taxes are higher than 0.5 percent. However, the number of longer term chartist traders is monotonically increasing in the currency transaction tax

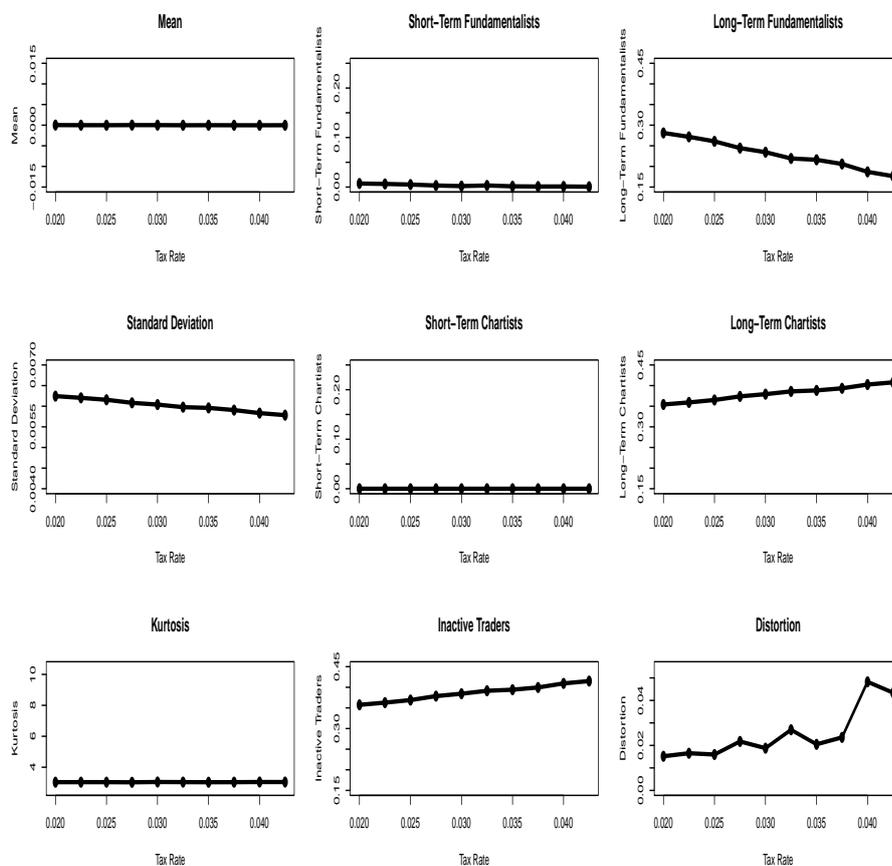


Figure 3.10: The Effectiveness of Higher Transaction Taxes under Fundamental Risk

NOTE: The numbers and statistics of this figure are averages over 5000 artificial trading days over 100 simulation runs. They can be interpreted as averages over time and (artificial) markets. Note that all simulations for different values of the transaction tax rate are based on the same seed of random variables. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.005$, $\sigma_c = 0.03$ and $\sigma_s = 0.01$. Distortion is defined as the average absolute deviation of the exchange rate from its fundamental value.

rate. The rationale is the following. Under taxation the number of short-term traders decrease in magnitude, which leads to less short-term fluctuations in the exchange rate. Thus, longer lasting trends in the exchange rate emerge, which make the longer term chartist trading rule more profitable compared to the longer term fundamentalist rule. As can be inferred from Figure 3.10 misalignments increase for

tax rates above 2 percent. This *u*-shaped response of misalignment are similar to Westerhoff (2003a, 2008a) due to the fact that the number of longer term chartists is increasing in the transaction tax rate, while the number of fundamentalists rises for small tax rates, while the popularity of this trading strategy decreases in number for higher tax rates.

3.5.3 Sensitivity Analysis

Table 3.3 and Table 3.4 contain results of a sensitivity analysis. The reported numbers and statistics are averages over 5000 artificial trading days and 50 simulation runs. We report the volatility and the distortion for the tax rates $\tau \in \{0.0\%, 1.0\%, 2.0\%, 3.0\%, 4.0\%\}$ and for different calibrations of the model's parameters like the fundamentalists' misalignment-correction parameter $\kappa_F \in \{0.02, 0.04, 0.06, 0.08\}$, the chartists' trend-extrapolation parameter $\kappa_C \in \{0.02, 0.04, 0.06, 0.08\}$, the intensity of choice parameter $\gamma \in \{400, 600, 800, 1000\}$, the memory parameter $\theta \in \{0.75, 0.80, 0.975, 0.985\}$ and the longer term traders investment horizon $N \in \{1, 10, 20, 30\}$.

Table 3.3 contains the sensitivity analysis under a constant fundamental value, while the results in Table 3.4 are based on the assumption that the fundamental value follows a random walk. From Table 3.3 we can infer that the volatility seems to be robust for different calibrations of the model. It is slightly declining in the tax rate for all 20 calibrations. Results for the distortion are robust for tax rates below 3.0% at least for variations in the fundamentalist and chartist behavioral parameters as well as for the memory parameter and the intensity of choice parameter. For tax rate above 3.0% we find that values of the distortion measure vary with changes in the

Table 3.3: Sensitivity Analysis: Constant Fundamental Value

| Tax Rate | Constant Fundamental Value | | | | | | | | | |
|-------------------|----------------------------|-------|-------|-------|-------|------------|-------|-------|-------|-------|
| | Volatility | | | | | Distortion | | | | |
| | 0.0% | 1.0% | 2.0% | 3.0% | 4.0% | 0.0% | 1.0% | 2.0% | 3.0% | 4.0% |
| $\kappa^F = 0.02$ | 0.013 | 0.011 | 0.010 | 0.010 | 0.010 | 0.018 | 0.013 | 0.014 | 0.016 | 0.083 |
| $\kappa^F = 0.04$ | 0.012 | 0.011 | 0.010 | 0.010 | 0.010 | 0.019 | 0.013 | 0.014 | 0.016 | 0.068 |
| $\kappa^F = 0.06$ | 0.012 | 0.011 | 0.010 | 0.010 | 0.010 | 0.018 | 0.013 | 0.014 | 0.016 | 0.041 |
| $\kappa^F = 0.08$ | 0.013 | 0.011 | 0.010 | 0.010 | 0.010 | 0.018 | 0.013 | 0.014 | 0.016 | 0.061 |
| $\kappa^C = 0.02$ | 0.012 | 0.011 | 0.010 | 0.010 | 0.010 | 0.016 | 0.013 | 0.014 | 0.020 | 0.075 |
| $\kappa^C = 0.04$ | 0.012 | 0.011 | 0.010 | 0.010 | 0.010 | 0.017 | 0.013 | 0.014 | 0.016 | 0.092 |
| $\kappa^C = 0.06$ | 0.013 | 0.011 | 0.010 | 0.010 | 0.010 | 0.018 | 0.013 | 0.014 | 0.016 | 0.053 |
| $\kappa^C = 0.08$ | — | 0.011 | 0.010 | 0.010 | 0.010 | — | 0.013 | 0.014 | 0.016 | 0.055 |
| $\gamma = 400$ | 0.012 | 0.011 | 0.011 | 0.011 | 0.011 | 0.016 | 0.013 | 0.013 | 0.013 | 0.014 |
| $\gamma = 600$ | 0.012 | 0.011 | 0.011 | 0.011 | 0.010 | 0.017 | 0.013 | 0.013 | 0.014 | 0.015 |
| $\gamma = 800$ | 0.013 | 0.011 | 0.011 | 0.010 | 0.010 | 0.017 | 0.013 | 0.013 | 0.014 | 0.016 |
| $\gamma = 1000$ | 0.013 | 0.011 | 0.011 | 0.010 | 0.010 | 0.020 | 0.013 | 0.014 | 0.015 | 0.017 |
| $\theta = 0.750$ | 0.012 | 0.011 | 0.011 | 0.011 | 0.011 | 0.018 | 0.014 | 0.014 | 0.013 | 0.013 |
| $\theta = 0.800$ | 0.012 | 0.011 | 0.011 | 0.011 | 0.011 | 0.018 | 0.014 | 0.013 | 0.013 | 0.013 |
| $\theta = 0.975$ | 0.013 | 0.011 | 0.011 | 0.010 | 0.010 | 0.018 | 0.013 | 0.013 | 0.014 | 0.016 |
| $\theta = 0.985$ | 0.013 | 0.011 | 0.010 | 0.010 | 0.010 | 0.018 | 0.013 | 0.014 | 0.016 | 0.052 |
| $N = 1$ | 0.014 | 0.010 | 0.010 | 0.010 | 0.010 | 0.069 | 0.323 | 0.364 | 0.426 | 0.360 |
| $N = 10$ | 0.013 | 0.010 | 0.010 | 0.010 | 0.010 | 0.025 | 0.018 | 0.053 | 0.224 | 0.336 |
| $N = 20$ | 0.013 | 0.010 | 0.010 | 0.010 | 0.010 | 0.020 | 0.014 | 0.015 | 0.017 | 0.046 |
| $N = 30$ | 0.013 | 0.011 | 0.011 | 0.010 | 0.010 | 0.018 | 0.013 | 0.013 | 0.014 | 0.016 |

NOTE: The numbers and statistics are averages over 5000 artificial trading days over 50 simulations runs. Note that all simulations for different values of the transaction tax rate are based on the same seed of random variables. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.005$, $\sigma_c = 0.03$ and $\sigma_s = 0.005$ except otherwise stated.

calibration. Interestingly, we find that the u -shaped reaction of the distortion measure to changes in the tax rate is robust to all 20 parameterizations. However, the convexity of this curve is smaller for higher values of the chartists' and fundamentalists' behavioral parameters as well as for smaller intensity of choice parameters and smaller memory parameter values. Thus, mispricings are reduced when fundamentalist traders as well as chartist traders act more aggressively. Moreover, we find that the convexity of the distortion-tax-curve is smaller, the higher the longer

Table 3.4: Sensitivity Analysis: Fundamental Value Follows Random Walk

| Tax Rate | Fundamental Value Follows Random Walk | | | | | | | | | |
|-------------------|---------------------------------------|-------|-------|-------|-------|------------|-------|-------|-------|-------|
| | Volatility | | | | | Distortion | | | | |
| | 0.0% | 1.0% | 2.0% | 3.0% | 4.0% | 0.0% | 1.0% | 2.0% | 3.0% | 4.0% |
| $\kappa^F = 0.02$ | 0.013 | 0.011 | 0.011 | 0.011 | 0.010 | 0.019 | 0.014 | 0.016 | 0.023 | 0.081 |
| $\kappa^F = 0.04$ | 0.012 | 0.011 | 0.011 | 0.011 | 0.010 | 0.018 | 0.014 | 0.017 | 0.019 | 0.101 |
| $\kappa^F = 0.06$ | 0.013 | 0.011 | 0.011 | 0.011 | 0.010 | 0.018 | 0.014 | 0.016 | 0.021 | 0.109 |
| $\kappa^F = 0.08$ | 0.013 | 0.011 | 0.011 | 0.011 | 0.010 | 0.019 | 0.014 | 0.016 | 0.019 | 0.095 |
| $\kappa^C = 0.02$ | 0.013 | 0.011 | 0.011 | 0.010 | 0.010 | 0.018 | 0.014 | 0.016 | 0.023 | 0.068 |
| $\kappa^C = 0.04$ | 0.013 | 0.011 | 0.011 | 0.010 | 0.010 | 0.019 | 0.014 | 0.016 | 0.024 | 0.111 |
| $\kappa^C = 0.06$ | 0.013 | 0.011 | 0.011 | 0.010 | 0.010 | 0.019 | 0.014 | 0.016 | 0.030 | 0.139 |
| $\kappa^C = 0.08$ | — | 0.011 | 0.011 | 0.010 | 0.010 | — | 0.014 | 0.016 | 0.026 | 0.091 |
| $\gamma = 400$ | 0.012 | 0.011 | 0.011 | 0.011 | 0.010 | 0.017 | 0.014 | 0.015 | 0.016 | 0.017 |
| $\gamma = 600$ | 0.012 | 0.011 | 0.011 | 0.011 | 0.010 | 0.018 | 0.014 | 0.015 | 0.017 | 0.023 |
| $\gamma = 800$ | 0.013 | 0.011 | 0.011 | 0.010 | 0.010 | 0.018 | 0.014 | 0.016 | 0.021 | 0.101 |
| $\gamma = 1000$ | — | 0.011 | 0.011 | 0.010 | 0.010 | — | 0.014 | 0.017 | 0.040 | 0.203 |
| $\theta = 0.750$ | 0.012 | 0.011 | 0.011 | 0.011 | 0.011 | 0.019 | 0.016 | 0.015 | 0.015 | 0.015 |
| $\theta = 0.800$ | 0.012 | 0.011 | 0.011 | 0.011 | 0.011 | 0.020 | 0.016 | 0.015 | 0.015 | 0.015 |
| $\theta = 0.975$ | 0.013 | 0.011 | 0.011 | 0.011 | 0.010 | 0.019 | 0.014 | 0.015 | 0.016 | 0.021 |
| $\theta = 0.985$ | — | 0.011 | 0.011 | 0.010 | 0.010 | — | 0.014 | 0.016 | 0.031 | 0.115 |
| $N = 1$ | 0.015 | 0.010 | 0.010 | 0.010 | 0.010 | 0.084 | 0.417 | 0.439 | 0.431 | 0.452 |
| $N = 10$ | 0.013 | 0.010 | 0.010 | 0.010 | 0.010 | 0.024 | 0.021 | 0.236 | 0.375 | 0.410 |
| $N = 20$ | 0.013 | 0.011 | 0.010 | 0.010 | 0.010 | 0.020 | 0.015 | 0.018 | 0.119 | 0.346 |
| $N = 30$ | 0.013 | 0.011 | 0.011 | 0.010 | 0.010 | 0.019 | 0.014 | 0.016 | 0.020 | 0.115 |

NOTE: The numbers and statistics are averages over 5000 artificial trading days over 50 simulations runs. Note that all simulations for different values of the transaction tax rate are based on the same seed of random variables. The underlying parameter values are $\kappa_f = \kappa_c = 0.04$, $N = 30$, $\gamma = 800$, $\theta = 0.975$, $\sigma_f = 0.005$, $\sigma_c = 0.03$ and $\sigma_s = 0.005$ except otherwise stated.

term traders' investment horizon. Furthermore, the distortion is smaller in value for longer investment horizons. This finding is robust for all tax rates considered here. This findings also hold, when we assume the fundamental value to follow a random walk (Table 3.4). Thus, traders with longer investment horizons act as a stabilizing force as proposed by Keynes (1936) and Tobin (1978). We conclude, that the finding that small tax rates decrease distortion, while higher tax rates increase it is a robust finding for 20 calibrations of this agent-based financial market model.

3.6 Conclusion

Agent-based models with heterogeneous interacting agents are powerful tools for economic policy analysis. Their success in replicating stylized facts of financial markets data like bubbling and crashing asset prices, non-predictability of returns, excessively high volatilities of returns, excessively high probabilities of extreme large absolute returns and volatility clustering makes them preferable tools for analyzing regulations in financial markets. The empirical stylized facts emerge within agent-based models from the interaction of heterogeneous traders. By affecting the individual agents' decisions, these regulations have effects on emergent properties like the ones cited before which cannot simply be deduced by aggregating over individual agents.

Within this paper we introduced the longer term traders of Demary (2008) into the foreign exchange market model proposed by Westerhoff (2008a). Our first result is that the stylized facts of financial market data also emerge when longer term traders are introduced into these models. Because our model is able to reproduce stylized facts of financial market data, we regard it as validated by the Lux-Westerhoff criteria (Lux 2009a and Westerhoff 2008a). The success of this artificial foreign exchange market in replicating stylized facts of foreign exchange market data makes it a powerful tool for analyzing the following proposition which is often heard from the proponents of financial market taxes and the public media in times of financial instability: *transaction taxes stabilize financial markets by crowding out short-term traders in favor of longer term investors.*

The economic policy analysis leads to the following results.

- (i) Small transaction taxes make short-term trading unprofitable. Therefore, the

number of short-term fundamentalists and short-term chartists decreases to zero. One emergent property of this change in behavior are the diminishing volatility clusters. Moreover, volatility and distortions decrease under small transaction taxes. The reason for this result lies in the fact that under small transaction taxes the market is populated by a higher number of longer term fundamentalist traders compared to longer term chartist traders.

(ii) However, when tax rates are too high, misalignments increase as also found in Westerhoff (2003a, 2008a). The reason for this *u*-shaped response of misalignments to increasing tax rates is caused by the changed composition of trading rules. In his model tax rates above a certain threshold make fundamental trading unprofitable relative to trend-chasing trading. Within our artificial foreign exchange market a similar result emerges. Here, the longer term fundamentalist trading rule becomes unpopular under tax rates above a certain threshold, while the number of traders, who favor the longer term chartist trading rule increases. In contrast to Keynes (1936) and Tobin (1978) taxing financial markets is not *per se* stabilizing by making short-term trading unprofitable in favor of longer term trading. Our model shows that this result depends on the composition of the trading rules used in the financial market and on the tax rate.

(iii) A sensitivity analysis indicates that these findings are robust for 20 different calibrations of the model. Furthermore, it indicates that higher investment horizons of longer term traders lead to less misalignments. We conclude that longer term traders act as a stabilizing force within this model.

This model shows that taxing financial markets has complex effects caused by be-

havioral heterogeneity and the interaction of agents. Therefore policy makers should pay attention to recent research in the area of agent-based financial market models. Within a world in which heterogeneous agents interact the effects of currency transaction taxes are complex and their effects on markets emerge from the change in the composition of popular trading rules in the market. The stabilizing or destabilizing effects of regulatory policies thus emerge by changing the composition of trading rules. This study and Westerhoff (2003a, 2008a) come to the result that there is a threshold tax rate where transaction taxes becomes destabilizing. Because this threshold is not known in reality introducing a tax rate on financial transactions which is above this threshold may have destabilizing effects. Moreover, when a policy maker wants to set a tax rate below this threshold, he or she has to consider other transaction costs in the financial market. Therefore, the tax threshold might be lower in reality due to additional transaction costs. Furthermore, Demary (2008) finds that the kurtosis of the return distribution may increase due to taxation because agents trade larger orders instead of a sequence of smaller orders. This results may also emerge in similar agent-based financial market models, when it is more profitable for traders to change to longer term trading rules instead of deciding not to trade. Furthermore Pelizzari and Westerhoff (2007) find that the effectiveness of transaction taxes depends on the underlying market microstructure. During phases of financial instability policy makers often refer to the introduction of financial market taxes. Recent research, like this study and the papers cited therein show, that changes in the composition in trading rules has important consequences for the effectiveness of financial market taxes. Therefore policy makers should be aware of the danger of setting the wrong tax rate. The Swedish experience with transaction taxes was perhaps caused by tax rates, which were simply set too high. Agent-based

models may therefore give important insights into the working of financial market regulations.

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4 Transaction Taxes, Greed and Risk Aversion in an Agent-Based Financial Market Model

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Abstract. Recent agent-based financial market models came to the result that taxing financial transactions does not *per se* increase financial stability and that the response of volatility and misalignments to rising tax rates seem to be *u*-shaped. Moreover, greed and the risk appetite of traders are often blamed for financial instability and there is no evidence how greed and risk aversion affect the effectiveness of regulations in financial markets. We aim to add to this gap in the literature by analyzing how the effectiveness of transaction taxes depend on different behavioral patterns within an agent-based framework. Our simulations indicate that a tax rate of 0.1 percent demarcates the stabilizing tax regime from the destabilizing one. We figure out that transaction taxes are less effective, either when chartists trade more aggressively, fundamentalists trade less aggressively, agents switch more frequently between trading strategies or only have short memory in their fitness measures. Lower risk aversion of agents, however, makes higher tax rates more effective as indicated by a flatter volatility response curve. We conclude that additional regulations should concentrate on the traders' responsibilities for their risk-exposure.

Key Words: *Agent-Based Financial Market Models, Regulations of Financial Markets, Financial Stability, Monte Carlo Analysis, Technical and Fundamental Analysis*

JEL Classification: *C15, D84, G01, G15, G18*

4.1 Introduction

Financial instability is often said to be caused by greed and the risk appetite of market participants. Newspaper articles which comment financial market outcomes are full of phrases like "bull markets" or "hot spots" during financial bubble periods. Westerhoff (2004a) suggest that agents' greed and fear has important implications for asset price dynamics. He notes that agents optimistically believe in booming markets, while panic dominates when prices change abruptly. Menkhoff and Schmidt (2005) find in a questionnaire survey among fund managers that buy-&-hold traders behave fundamentally oriented and risk averse, while momentum traders are less risk-averse and follow aggressively the trend. We should expect that the higher risk-aversion limits the stabilizing impact of fundamentalists, while less risk-aversion enhances the destabilizing impact of chartists. Thereby greed and risk-aversion might affect the effectiveness of regulations through the stabilizing behavior of fundamentalists. Up to now, there is less evidence how financial regulations work under different behavioral patterns. The scope of this paper is to fill this gap in the literature by analyzing how traders' risk aversion and aggressiveness affects the ability of transaction taxes to stabilize financial markets within an agent-based model.¹

Up to now, there is a growing body of literature which analyzes the effectiveness of transaction taxes in agent-based financial market models (Westerhoff 2003a, 2008b; Westerhoff and Dieci 2006; Pelizzari and Westerhoff 2007; Demary 2008, 2010). Lux (2009a) highlights that the success of agent-based models in replicating empirical stylized facts makes them preferable tools for economic policy analysis.² Wester-

¹Keynes (1936) blamed traders' "*animal spirits*" for the occurrence of bubbles and crashes in asset prices. He proposed to tax financial markets in order to reduce speculative trading. His argument was concretized by Tobin (1978), who proposed to tax international capital flows.

²The effectiveness of central bank interventions were analyzed with agent-based models in West-

hoff (2008a) highlights the following advantages of agent-based policy analysis: (i) the researcher is able to generate as much data as needed, (ii) is able to measure all variables precisely, (iii) is able to control any exogenous events, (iv) is able to replicate the policy experiment under the same conditions, and (v) the researcher is able to measure the behavior of the artificial agents. The last point is crucial for understanding the coordination of individual decisions which determine the aggregate behavior of the system. Demary (2008, 2010) highlights that agent-based models are preferable tools for analyzing financial regulations, since they meet the necessary requirements and assumptions for performing these policy experiments: (i) these models distinguish between different trading strategies, (ii) traders are allowed to change their trading strategy or leave the market in response to regulations, (iii) the models are based on realistic behavioral assumptions, (iv) are able to generate realistic asset price dynamics, and (v) might therefore be able to generate realistic policy implications as also highlighted in Lux (2009a).

Most studies that analyze the effectiveness of financial market taxes within agent-based models figure out that taxing financial transactions is not stabilizing *per se*. Westerhoff (2003a, 2008a) finds that small transaction taxes have the potential to stabilize financial markets, while higher tax rates will destabilize the market. Demary (2010) introduces additional longer term traders into the model of Westerhoff (2008a) and replicates this *u*-shaped response of mispricings to taxation. His simulations reveal that under sufficiently small tax rates traders abstain from short-term trading in favor of longer term investment horizons. Similar to Westerhoff (2003a,

erhoff (2001), Wieland and Westerhoff (2005), Westerhoff (2008a). Weidlich and Veit (2008) use agent-based models for analyzing regulations in the electricity market, while Haber (2008) applies an agent-based model for monetary and fiscal policy analysis. Westerhoff (2003b, 2006, 2008a) analyze the effectiveness of trading halts in financial markets.

2008a), mispricings increase when the tax rate exceeds a certain threshold. This outcome emerges, since taxation reduces short-term fluctuations and longer lasting trends emerge. As a result, the longer term fundamentalist rule becomes unpopular and the longer term chartist rule rises in popularity, which leads to a destabilization of the artificial financial market. Westerhoff and Dieci (2006) analyze financial market taxes when traders are allowed to trade in two different markets. Their simulations reveal that the volatility in the taxed market will decline, while the volatility in the untaxed market will increase, since the trend-chasing trading rules are more profitable in untaxed markets. However, the volatility in both markets will decline when the tax is levied onto both markets. Pelizzari and Westerhoff (2007) show that transaction taxes work in a dealership market since liquidity is provided by specialists, while they are not effective in continuous double auction markets since the reduction in market liquidity increases the market impact of orders.

Most studies do not analyze how the effectiveness of policy measures depend on behavioral patterns of traders. Therefore, we introduce financial market taxes into the model proposed by DeGrauwe and Grimaldi (2006), which explicitly assumes risk averse traders. We expand the original model by structural stochastic volatility as proposed by Franke and Westerhoff (2009), Westerhoff (2008a) and Westerhoff and Dieci (2006). As Franke and Westerhoff (2009) note, structural stochastic volatility is an important generator of volatility clusters. This expansion leads to more realistic daily fluctuations of returns compared to the original model formulation. In addition to that, we are able to analyze the impact of taxation onto the empirically observed volatility clusters as done in Demary (2010).

Within our analysis we figure out which behavioral patterns make taxation more ef-

fective as indicated by a change in the shape of the u -shaped volatility and misalignment response curves to taxation. Within the model the following results emerge: (i) The tax rate 0.1 percent demarcates the stabilizing tax regime from the destabilizing tax regime. (ii) The volatility and misalignment response curves become flatter the less risk averse agents are. (iii) More aggressive fundamental trading makes taxes more effective, while (iv) more aggressive chartists makes them less effective. (v) The more agents switch to the most profitable trading rule in response to changes in the performance measures, the less effective is taxation. (vi) Longer memory in the performance measures makes low transaction taxes more effective as indicated by a flatter volatility response curve. We conclude that additional regulations should concentrate on the traders' responsibilities for their risk-exposure.

The remainder of this essay is structured as follows. Section two presents the model, where section three analyses some properties of the model. Section four deals with calibration, validation and the simulation design. Section five presents the results of the policy analysis, while section six ends this paper with conclusions and outlook.

4.2 The Artificial Financial Market

The artificial financial market is modeled in spirit of DeGrauwe and Grimaldi (2006) who assume risk averse traders.³ In contrast to the original formulation, we abstract from the time varying risk evaluation of traders and expand the model to a structural stochastic volatility model (SSV) as proposed by Westerhoff (2008a) and Franke and Westerhoff (2009). As Franke and Westerhoff (2009) note, SSV might be a relevant

³Influential contributions to agent-based financial market models are surveyed in Westerhoff (2008b), Hommes (2006), LeBaron (2006) and Lux (2009a).

generator of volatility clusters. Under the introduction of SSV the DeGrauwe and Grimaldi (2006) model is able to generate more realistic daily fluctuations of asset prices compared to the original formulation even when we abstract from the time-varying risk-evaluation of traders. The rationale for this restriction is that although we want the model to be a good description of observed data, we wish to keep it sophisticated simple as well in order to understand easier how financial regulations work within the model.

The model presented in this paper consists of the following building blocks

- (i) an exogenous fundamental exchange rate,
- (ii) traders who choose between a fundamental and a trend-chasing trading rule, who are allowed not to trade, when the transaction costs associated with trading in the foreign exchange market are higher compared to the returns of trading,
- (iii) an evolutionary process that determines the popularity of the trading rules according to their past performance,
- (iv) market demand that meets market supply for determining the value of the exchange rate, and
- (v) a policy maker who sets the currency transaction tax rate.

4.2.1 Portfolio Allocation

Following DeGrauwe and Grimaldi (2006) we assume that traders are myopic one-period investors who only care about the return and the risk of their investment.

Traders allocate their investment positions by investing in either a riskless domestic asset with return r or into a risky foreign asset with return r^* . The foreign asset is risky due to fluctuations in the exchange rate s_t . Agents have to buy foreign currency, when they decide to invest into the foreign asset. They determine their demand for the foreign currency d_t^i by maximizing their mean-variance-utility

$$\mathcal{U}(W_{t+1}^i) = \mathcal{E}_t^i(W_{t+1}^i) - \frac{\mu}{2} \mathcal{V}_t^i(W_{t+1}^i) \quad (4.1)$$

subject to their wealth constraint

$$\mathcal{E}_t^i W_{t+1} = (1+r)W_t + (1+r^*)\mathcal{E}_t^i s_{t+1} d_t^i - (1+r)s_t d_t^i, \quad (4.2)$$

and the fluctuations in wealth

$$\mathcal{V}_t^i(W_{t+1}) = (1+r^*)^2 d_{i,t}^2 \mathcal{V}_t^i(s_{t+1}).^4 \quad (4.3)$$

The agent's wealth at time $t+1$ is denoted by W_{t+1}^i and consists of the compounded wealth endowment from the previous period $(1+r)W_t$, the investment in the foreign asset $(1+r^*)\mathcal{E}_t^i s_{t+1} d_t^i$ and the domestic component $-(1+r)s_t d_t^i$. When the agent borrows money in his or her home country for an investment into the foreign asset, then $d_t^i > 0$, since he or she needs to convert the domestic money into foreign currency, while $d_t^i < 0$ indicates that the agent borrows money abroad, which he or she has to convert to domestic currency for investing it into the home country. Since d_t^i is the demand for foreign currency, the exchange rate s_t is given as the price of

⁴At this stage of the maximization problem agents treat $\mathcal{E}_t^i(\cdot)$ and $\mathcal{V}_t^i(\cdot)$ as expectation operator and variance operator. Hence, we can pull $(1+r^*)^2$ and $d_{i,t}^2$ off the variance operator, since they are realized at time t . When forming their demands $d_{i,t}$, agents change the expected exchange rate $\mathcal{E}_t^i(s_{t+1})$ and the exchange rate risk $\mathcal{V}_t^i(s_{t+1})$ by bounded-rational rules-of-thumb.

the foreign currency in domestic currency units. Note, that DeGrauwe and Grimaldi (2006) assume agents to be bounded rational. Hence, $\mathcal{E}(\cdot)$ and $\mathcal{V}(\cdot)$ are not the objective expectation and variance operators based on objective probabilities, but they are the individual traders subjective expectations and the subjective variance assessments based on bounded-rational rules of thumb. By solving this portfolio allocation problem we arrive at the following demand for foreign currency

$$d_t^i = \frac{(1 + r^*)\mathcal{E}_t^i(s_{t+1}) - (1 + r)s_t}{\mu(1 + r^*)^2\mathcal{V}_t^i(s_{t+1})}. \quad (4.4)$$

Typically for utility functions with constant absolute risk aversion the demand for the risky asset does not depend on initial wealth W_t , is rising in the return of the risky asset and declining in the risk of the asset. Note, that μ is the degree of risk aversion, a parameter crucial for our analysis. Since we abstract from interest rate risk, we set following DeGrauwe and Grimaldi (2006) the interest rates to zero ($r = r^* = 0$). Under this restriction we arrive at a demand function, which is in line with the ones used in Westerhoff (2003a, 2003b, 2003c, 2004b) and Manzan and Westerhoff (2003)

$$d_t^i = \frac{\mathcal{E}_t^i(s_{t+1}) - s_t}{\mu\mathcal{V}_t^i(s_{t+1})}. \quad (4.5)$$

Traders buy foreign currency when they expect the exchange rate to rise ($\mathcal{E}_t^i(s_{t+1}) > s_t$), while they sell foreign currency when they expect the exchange rate to decline in the future ($\mathcal{E}_t^i(s_{t+1}) < s_t$). However, these demands are augmented by risk aversion in the DeGrauwe and Grimaldi (2006) model. This means, that the demands will be smaller in absolute value when there is more risk associated with trading in the foreign exchange market.

Buy or sell orders generate transaction costs $\tau s_t d_t^i$, since the policy maker levies a

proportional transaction tax τ in spirit of the one proposed by Tobin (1978). A round trip of investing in the foreign country and consuming the profit in the home country leads to the following profit after taxes (see Westerhoff 2008a and Demary 2010)

$$\tilde{\pi}_t^i = (s_t - s_{t-1})d_{t-1}^i - \tau(s_t + s_{t-1})|d_{t-1}^i|. \quad (4.6)$$

The first part of this formulation is the return of buying (selling) foreign currency at s_{t-1} and selling (buying) it at s_t , while the last part is the transaction costs associated with these two transactions. When agents face transaction costs they will only adjust their portfolios, when the profit from adjusting the portfolio is higher compared to the transaction costs

$$x_t^i = \begin{cases} d_t^i, & \text{if } (s_t - s_{t-1})d_{t-1}^i > \tau(s_t + s_{t-1})|d_{t-1}^i|; \\ 0, & \text{otherwise.} \end{cases} \quad (4.7)$$

Hence, the transaction tax creates a neutral band around the no-arbitrage condition $(s_t - s_{t-1})d_{t-1}^i = 0$, which is in line with the literature on financial market taxes.

4.2.2 Agents' Forecasting Rules

The traders' forecasting rules are modeled following the literature on agent-based financial market models. These trading rules are based on empirical evidence from survey studies among market participants which were originally performed by Taylor and Allen (1992).⁵ One result is that professional traders either rely on trading rules that take explicitly economic fundamentals into account, while others use chartist techniques that rely on trends or other patterns in past asset prices and ignore

⁵Menkhoff and Taylor (2007) give an overview over these studies and their key results.

economic fundamentals. As already noted, Menkhoff and Schmidt (2005) find that buy-&-hold traders behave fundamentally oriented and risk averse, while momentum traders are less risk averse and follow aggressively the trend.

The fundamentalist forecasting rule expects that the exchange rate s_t will revert to the fundamental exchange rate s_t^f in the distant future, with a rate of error-correction of $\kappa_F \cdot 100$ percent.

$$\mathcal{E}_t^F(\Delta s_{t+1}) = \kappa_F(s_{t-1}^f - s_{t-1}) + \sigma_F \varepsilon_t^F, \quad \text{with } \varepsilon_t^F \sim \mathcal{N}(0, 1). \quad (4.8)$$

The random disturbance ε_t^F can be interpreted as the fundamentalists' misperception of the fundamental value (Westerhoff 2008a). Its volatility σ_F accounts for the uncertainty associated with using this expectational rule. This assumption is based on the fact that neither the fundamental value nor the reversion speed of the exchange rate toward this fundamental can be inferred without observational error.

Chartists believe that the past trend segment $s_{t-1} - s_{t-2}$ can be extrapolated into the future

$$\mathcal{E}_t^C(\Delta s_{t+1}) = \kappa_C(s_{t-1} - s_{t-2}) + \sigma_C \varepsilon_t^C, \quad \text{with } \varepsilon_t^C \sim \mathcal{N}(0, 1). \quad (4.9)$$

The parameter κ_C is the strength of the trend-extrapolation. For $\kappa_C > 0$ this rule is trend-chasing, while chartists act as contrarians for $\kappa_C < 0$. It can also be interpreted as the chartists' aggressiveness. The random disturbance ε_t^C can be interpreted as a deviation from this strict deterministic rule. The volatility parameter σ_C accounts for the variety of chartist rules (Westerhoff 2008a).

The volatilities σ_C and σ_F are crucial for the SSV mechanism.

4.2.3 Risk Evaluation

DeGrauwe and Grimaldi (2006) assume agents to calculate a moving average of past forecast errors in order to account for the uncertainty associated with their trading rule

$$\mathcal{V}_t^i(s_{t+1}) = \omega + \alpha(s_t - \mathcal{E}_{t-1}^i(s_t))^2 + \beta\mathcal{V}_{t-1}^i(s_t). \quad (4.10)$$

When agents assume a constant risk, as in Brock and Hommes (1998), then $\alpha = \beta = 0$. We assume this restriction, since we concentrate on SSV as a generator for volatility clusters.⁶

4.2.4 Evaluation of Trading Rules

Traders are allowed to change their trading rules when other rules are more successful compared to theirs. This mechanism was originally proposed by Brock and Hommes (1998). Following Westerhoff (2008a) and Demary (2010) we assume the following fitness measures

$$\pi_t^i = (s_t - s_{t-1})x_{t-1}^i - \tau(s_t + s_{t-1})|x_{t-1}^i| + \theta\pi_{t-1}^i. \quad (4.11)$$

The memory parameter θ can often be found in agent-based models (see Westerhoff 2008a and Westerhoff and Dieci 2006). It measures how strong agents discount past

⁶The time-varying risk-evaluation (TVRE) of traders as well as the heterogeneity in the variation of the trading rules of chartists and fundamentalists (which leads to SSV) are both generators of volatility clustering. When I abstract from the TVRE and only use the SSV, volatility clusters and slowly declining autocorrelations of absolute returns remain. However, when I abstract from SSV and only use the TVRE, the model produces less persistent autocorrelations compared to empirical data. I conclude that SSV is the more important generator of volatility clustering. Moreover, this simplification is not crucial for the models ability to explain stylized facts nor for the implications of the policy analysis.

profits for strategy selection. When $\theta = 0$, then only today's profits determine the popularity of the trading strategies, while for $\theta = 1$ all past profits will be considered.

Following Westerhoff (2003a, 2008a) and Demary (2010) we assume that traders can choose between using the chartist trading rule, the fundamentalist trading rule or they abstain from trading. The last possibility is crucial when traders act under transaction costs. When the transaction costs of trading in the foreign exchange market are higher compared to the return of trading then traders decide to stay inactive as already highlighted in the subsection before. Brock and Hommes (1998) suggest the percentage fractions of traders to be determined by the discrete choice model proposed by Manski and McFadden (1981). Hence, the percentage fraction of traders using the fundamentalist rule is determined as

$$w_t^F = \frac{\exp\{\gamma\pi_{t-1}^F\}}{1 + \exp\{\gamma\pi_{t-1}^F\} + \exp\{\gamma\pi_{t-1}^C\}}, \quad (4.12)$$

the percentage fraction of traders using the chartist rule is

$$w_t^C = \frac{\exp\{\gamma\pi_{t-1}^C\}}{1 + \exp\{\gamma\pi_{t-1}^F\} + \exp\{\gamma\pi_{t-1}^C\}}, \quad (4.13)$$

while the percentage fraction of traders, who abstain from trading is given by

$$w_t^I = \frac{1}{1 + \exp\{\gamma\pi_{t-1}^F\} + \exp\{\gamma\pi_{t-1}^C\}}, \quad (4.14)$$

since the profit generated by abstaining from trade is zero by construction. The parameter γ can be interpreted as the intensity-of-choice parameter. The higher γ the more agents switch to the trading rule which was the most profitable one in the past. For $\gamma = 0$ we have the special case of constant population fractions, while for

$\gamma = \infty$ all agents switch to the most profitable trading strategy.

4.2.5 Market Equilibrium

Market demand for foreign currency X_t is given by the sum over all individual demands x_t^i multiplied by the number of agents n_t^i , who use one of the possible trading strategies

$$X_t \equiv n_t^C x_t^C + n_t^F x_t^F + n_t^I x_t^I. \quad (4.15)$$

Here n_t^C is the number of agents, who use the chartist trading rule to determine their demand, n_t^F is the number of agents who use the fundamentalist rule to determine their demand, while n_t^I is the number of inactive agents. Since these agents abstain from trading, their demand is zero. Since DeGrauwe and Grimaldi (2006) normalize the supply of foreign assets to zero, the market clearing condition is given by

$$n_t^C x_t^C + n_t^F x_t^F = 0. \quad (4.16)$$

The rationale for setting the supply for foreign currence to zero is that we abstract from central bank interventions which are beyond the scope of this paper.

We follow the derivation of the model explained in DeGrauwe and Grimaldi (2006). By dividing through the total number of agents $N \equiv n_t^C + n_t^F + n_t^I$ and by defining the percentage fractions as $w_t^i \equiv n_t^i/N$, we arrive at

$$w_t^C x_t^C + w_t^F x_t^F = 0. \quad (4.17)$$

Inserting the traders' demand functions (4.7) results in

$$w_t^C \cdot \frac{s_{t-1} + \mathcal{E}_t^C(\Delta s_{t+1}) - s_t}{\mu \mathcal{V}_t^C(s_{t+1})} + w_t^F \cdot \frac{s_{t-1} + \mathcal{E}_t^F(\Delta s_{t+1}) - s_t}{\mu \mathcal{V}_t^F(s_{t+1})} = 0, \quad (4.18)$$

where we assume following DeGrauwe and Grimaldi (2006) that $\mathcal{E}_t^i(s_{t+1}) = s_{t-1} + \mathcal{E}_t^i(\Delta s_{t+1})$ ⁷

Dividing by $\{w_t^C/\mathcal{V}_t^C(s_{t+1}) + w_t^F/\mathcal{V}_t^F(s_{t+1})\}$ yields

$$s_t = s_{t-1} + \Theta_t^C \mathcal{E}_t^C(\Delta s_{t+1}) + \Theta_t^F(\mathcal{E}_t^F(\Delta s_{t+1})), \quad (4.19)$$

where we define the risk-adjusted population fractions in line with DeGrauwe and Grimaldi (2006) as

$$\Theta_t^C \equiv \frac{w_t^C/\mathcal{V}_t^C(s_{t+1})}{w_t^C/\mathcal{V}_t^C(s_{t+1}) + w_t^F/\mathcal{V}_t^F(s_{t+1})} \quad (4.20)$$

$$\Theta_t^F \equiv \frac{w_t^F/\mathcal{V}_t^F(s_{t+1})}{w_t^C/\mathcal{V}_t^C(s_{t+1}) + w_t^F/\mathcal{V}_t^F(s_{t+1})}. \quad (4.21)$$

Inserting the fundamentalist and chartist forecasting models yields

$$s_t = s_{t-1} + \Theta_t^C(\kappa_C(s_{t-1} - s_{t-2}) + \sigma_C \varepsilon_t^C) + \Theta_t^F(\kappa_F(s_{t-1}^F - s_{t-1}) + \sigma_F \varepsilon_t^F) + \sigma \varepsilon_t^S, \quad (4.22)$$

where $\varepsilon_t^S \sim \mathcal{N}(0, 1)$ is a constant fraction of noise traders in the market, which ensure a minimum fraction of liquidity in the market for the case that a large fraction of chartists and fundamentalists abstain from trading under transaction taxes.

⁷DeGrauwe and Grimaldi (2006) assume that s_t is not known to the agents, when they form their expectations.

Rearranging this equation results in

$$s_t = s_{t-1} + \Theta_t^F \kappa_F (s_{t-1}^F - s_{t-1}) + \Theta_t^C \kappa_C (s_{t-1} - s_{t-2}) + \sigma_t \varepsilon_t^S \quad (4.23)$$

$$\sigma_t \equiv \sigma + \Theta_t^F \sigma_F + \Theta_t^C \sigma_C. \quad (4.24)$$

This model is characterized by SSV, which is according to Franke and Westerhoff (2009) an important generator of volatility clustering. The mean value dynamics (4.22) are characterized by a centrifugal component $\kappa_C (s_{t-1} - s_{t-2})$ and a centripetal component $\kappa_F (s_{t-1}^F - s_{t-1})$ (see Lux 2009). The centrifugal component is caused by the trend-chasing behavior of chartist traders, while the centripetal force is caused by the stabilizing behavior of fundamentalists. The exchange rate dynamics is characterized by an on-off-intermittency due to fluctuations in the population weights Θ_t^F and $\Theta_t^C = 1 - \Theta_t^F$ (see Lux and Marchesi 2000). Thus, the exchange rate s_t deviates from its fundamental value s_t^F and a speculative bubble emerges when Θ_t^C is large, while the exchange rate will be attracted by its fundamental value, when Θ_t^F is large. These fluctuations in the population weights also lead to volatility clusters due to the SSV formulation of the volatility dynamics (4.23), when the variability in chartist rules is larger compared to the variability in fundamentalist rules $\sigma_C > \sigma_F$. When Θ_t^C is large, the market is in a high volatility state, while the market will be in a low volatility state, when Θ_t^F is large. By affecting the population weights, the transaction tax rates has effects onto the mean value dynamics as well as onto the volatility dynamics.

4.3 The Fundamental Equilibrium

For analyzing the long-term properties of the model we follow the derivations in Westerhoff and Dieci (2006) and concentrate on the deterministic skeleton of the model. Hence, we set all shocks to zero. The long-term solution is given when

$$(s_{t-1}, s^F, x_{t-1}^i, w_{t-1}^i, \pi_{t-1}^i) = (s_t, s^F, x_t^i, w_t^i, \pi_t^i) = (\bar{s}, s^F, \bar{x}^i, \bar{w}^i, \bar{\pi}^i) \quad (4.25)$$

for $i \in \{C, F, I\}$.

Within the long-term fundamental steady-state $s_{t-1} = s_t = \bar{s} = s^F$ traders demand for foreign currency will be zero

$$\bar{x}^C = \bar{x}^F = \bar{x}^I = 0, \quad (4.26)$$

and all fitness measures will be zero as well

$$\bar{\pi}^C = \bar{\pi}^F = \bar{\pi}^I = 0. \quad (4.27)$$

Zero fitness measures for all agents leads to a uniform distribution in the popularity of all three trading rules

$$\bar{w}^C = \bar{w}^F = \bar{w}^I = \frac{1}{3}, \quad (4.28)$$

which leads to

$$\bar{\Theta}^C = \bar{\Theta}^F = \frac{1}{2}. \quad (4.29)$$

Comparable results can often be found in chartist-fundamentalist models (DeGrauwe and Grimaldi 2006, Westerhoff 2008a, Westerhoff and Dieci 2006).

Stability properties for the fundamental steady-state can be found in the appendix.

4.4 Calibration, Validation and Simulation Design

4.4.1 Calibration

Before we start the simulation of the model and the policy analysis, we have to find reasonable values for the model's parameters. Most parameter values are in the range as the ones used in Westerhoff (2008a) and Demary (2010), which are chosen such that the model is able to match numbers and statistics of real world financial market data.⁸ Westerhoff (2008a) and Demary (2010) assume chartists' and fundamentalists' aggressiveness as $\kappa^C = \kappa^F = 0.04$. Hence, chartists expect an exchange rate change of 0.04 for the next trading day in response to a past exchange rate change of 1, while fundamentalists expect an exchange rate change of 0.04 in response to a misalignment of 1. The variation in the chartist trading rules $\sigma^C = 0.055$ is higher compared to the fundamentalists misperception $\sigma^F = 0.005$, in order to achieve volatility clusters through the SSV mechanism. The memory parameter $\theta = 0.98$ indicates that agents orientate on recent as well as on past performances of all trading rules. The intensity of choice parameter $\gamma = 600$ ensures that enough traders change their views of the world in response to changes in the performance measures. The agents' subjective volatility estimate $\omega = 0.03$ seems to be high, since its value is larger compared to the volatility of the exchange rate. However, this high value is necessary for the simulations to be computationally

⁸Studies in which agent-based financial market models were estimated are Gilli and Winker (2003), Westerhoff and Reitz (2003), Lux (2009b), Alfarano, Lux and Wagner (2005), Boswijk, Hommes and Manzan (2007), Winker, Gilli and Jeleskovic (2007), Manzan and Westerhoff (2007) and Ghonghadze and Lux (2009).

stable and to avoid unrealistic large crashes in the exchange rate due to unrealistic high demands of chartists. Another is, that it is the compound parameter $\mu \times \omega$ that determines risk aversion and not the value of ω alone. We calibrate the risk aversion parameter $\mu = 50$ such that the simulations yield realistic return properties. The reason for this assumption is, that there is no empirical evidence on reasonable values for this parameter. The fundamental value of the exchange rate is assumed to be constant and normalized to $s^F = 10$. Parameters are summarized in table 4.1.

Table 4.1: Calibration of the Baseline Simulation

| Parameter | Value | Interpretation |
|------------|-------|---|
| κ_C | 0.04 | chartists' aggressiveness |
| κ_F | 0.04 | fundamentalists' aggressiveness |
| σ_C | 0.055 | variety of chartist rules |
| σ_F | 0.005 | variety of fundamentalist rules |
| γ | 600 | intensity of choice |
| θ | 0.98 | memory parameter |
| σ_S | 0.005 | volatility of non-fundamental innovations |
| μ | 50 | degree of risk aversion |
| ω | 0.03 | subjective volatility estimate |
| s^F | 10 | fundamental exchange rate |

NOTES: Most parameter values are chosen following the parameterization of Westerhoff (2008a) and Demary (2010). All remaining parameters are given such that the model reproduces properties of real world exchange rate data quite well.

4.4.2 Validation

Following Westerhoff (2008a), Lux (2009a), Westerhoff and Franke (2009) and Demary (2010) we use the following validation criteria, which are based on the stylized facts of financial market data: (i) asset prices are characterized by bubbles and crashes (deviations from fundamental value), (ii) asset prices should be more

volatile compared to their fundamental values (excess volatility or distortions), *(iii)* the return distribution should deviate from the normal distribution (excess kurtosis), *(iv)* absence of autocorrelation in raw returns (non-predictability of daily returns), *(v)* persistent and slowly decaying autocorrelations in absolute returns (volatility clustering).

There are additional stylized facts from which we abstract. We concentrate on the minimum required stylized facts, that a model should be able to match instead. Unfortunately, models with less economic structure are better in replicating stylized facts. However, we need a model with an economic foundation to do an economic policy analysis. Since the aim of this paper is not to reproduce stylized facts as precise as possible, but to do an economic policy analysis within a model that is able to reproduce realistic exchange rate fluctuations, we are satisfied when our model is able to match the most relevant (or minimum required) stylized facts of financial market data. When the model is able to meet the Franke-Lux-Westerhoff validation criteria, then our model is a sufficient representation of daily foreign exchange market dynamics and we should expect to derive realistic policy implications from it.

Figure 4.1 shows a snapshot of a typical simulation run.⁹ The first 1000 data points were removed in order to avoid phenomena that depend on the initial conditions. The exchange rate is characterized by persistent bubbles and crashes. As can be seen in the figure, large distortions (large deviations from the fundamental value) correspond to periods with a large number of chartist traders, while periods with the exchange rate close to the fundamental value correspond to periods dominated by fundamentalist traders. The observed distortions indicates that asset prices are

⁹All programming and computations were done using the free open source software R (R Development Core Team 2009).

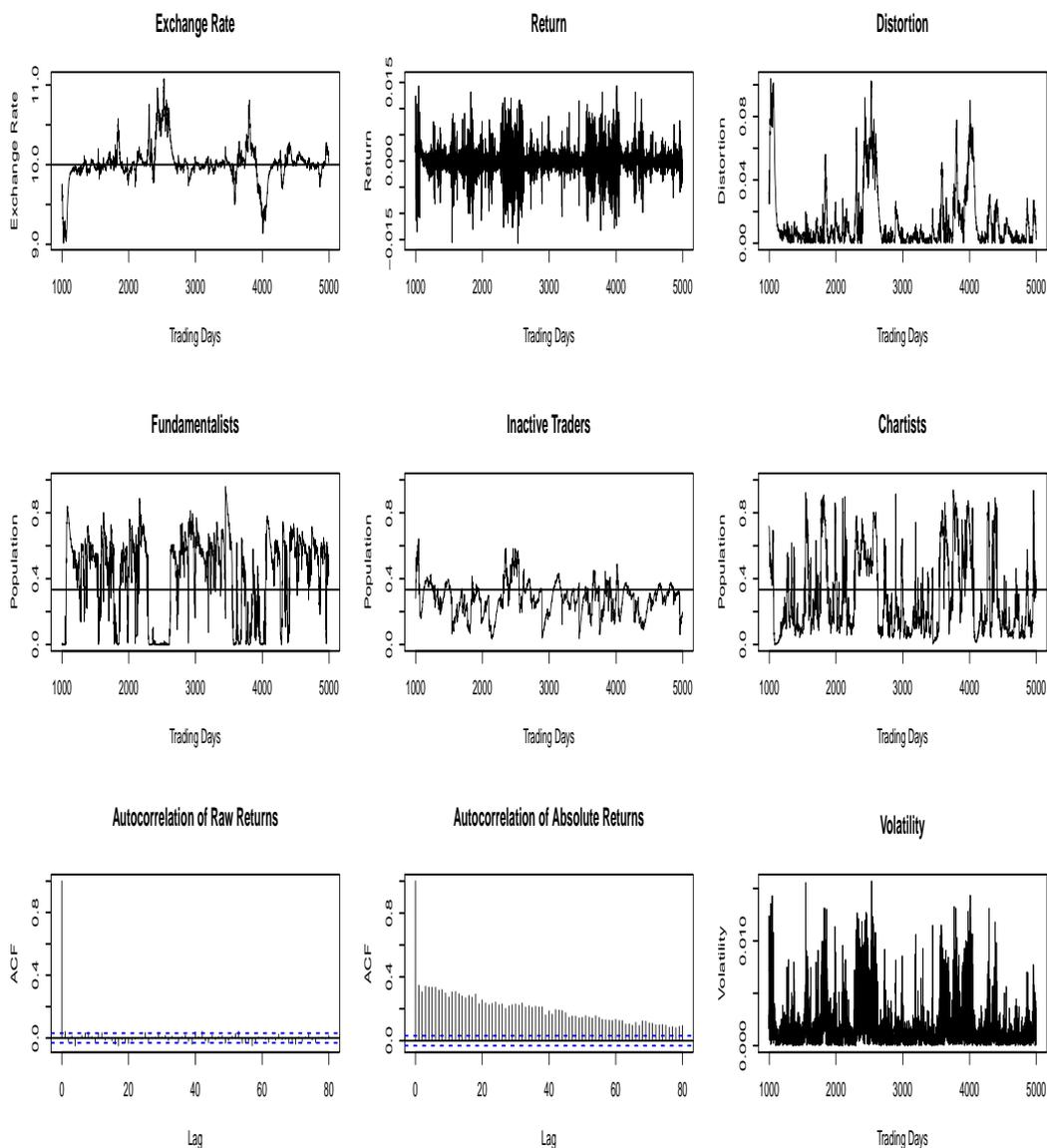


Figure 4.1: Baseline Simulation: No Taxes Levied

NOTES: The output of a typical simulation run. The underlying parameter values are those given in Table 4.1. Fundamental values are represented by horizontal lines at $\bar{s}^F = 10$ and $\bar{w}^F = \bar{w}^C = \bar{w}^I = 1/3$. Distortion is the average absolute percentage deviation of the exchange rate from its fundamental value, while volatility is the average absolute percentage time change in the exchange rate.

more volatile than their fundamental values. Another empirical fact the model is able to replicate can be inferred from the autocorrelations of raw returns and absolute returns. Raw returns are characterized by negligible autocorrelations which indicate an absence of predictability on a daily basis. The autocorrelations of absolute returns, however, are persistent and slowly decaying. This high autocorrelations in absolute returns indicate the presence of volatility clusters. These changing periods of high and low volatility can be inferred as well by just eyeballing the time series of returns. In line with our intuition, periods of high volatility tend to correspond to periods with a large number of chartists, while periods with low volatility correspond to periods with a high dominance of fundamentalist traders.

Table 4.2: Summary Statistics: Model versus Data

| | USD-Euro | Yen-USD | GBP-USD | USD-AusD |
|------------|----------|---------|---------|----------|
| mean | 0.000 | 0.000 | 0.000 | 0.000 |
| volatility | 0.007 | 0.007 | 0.006 | 0.009 |
| skewness | 0.179 | -0.509 | -0.330 | -0.757 |
| kurtosis | 5.560 | 6.885 | 9.315 | 17.009 |
| | Model 1 | Model 2 | Model 3 | Model 4 |
| mean | 0.000 | 0.000 | 0.000 | 0.000 |
| volatility | 0.002 | 0.003 | 0.002 | 0.007 |
| skewness | 0.000 | 0.000 | 0.000 | 0.000 |
| kurtosis | 7.283 | 7.545 | 12.005 | 8.057 |

NOTES: The exchange rate data are taken from the FRED2 database of the Federal Reserve Bank of St. Louis in daily frequency. The data series range from 1999-01-04 to 2009-10-09 and are available under the series-ID: DEXUSEU, DEXJPUS, DEXUSUK, DEXCAUS and DEXAUUS. Model 1 to Model 4 are characterized by different parameter sets. Model 1 represents the baseline case given in table 1, while model 2 assumes more aggressive chartists $\kappa^C = 0.09$, model 3 assumes more aggressive fundamentalists $\kappa^F = 0.09$, while model 4 assumes variabilities in forecasting rules of $\sigma_C = 0.12$ and $\sigma_F = 0.03$. All remaining parameters are assigned to the values given in table 1.

Table 4.2 compares summary statistics of the model to those of US-Dollar to Euro exchange rate returns and those of the Yen - US-Dollar exchange rate, the Great Britain Pound - US-Dollar and the US-Dollar - Australian Dollar exchange rates. All four exchange rate return time series are characterized by an average daily return near zero, a standard deviation between 0.7 percent and 0.9 percent and a kurtosis between 5.6 and 17.0 which indicate that the return distribution deviates from a normal distribution. Table 4.2 contains summary statistics of model generated exchange rate returns of a typical simulation run for a snapshot of 6,000 artificial trading days.¹⁰ We report summary statistics for four calibrations of the model. The model generated daily returns are as well characterized by a mean return near zero, a standard deviation of 0.2 to 0.3 percent and a kurtosis in the range of 7.3 to 13.1 indicating deviations from the normal distribution. In contrast to the empirical returns, we find a skewness of zero in the artificial return distributions. The reason lies in the fact that we abstract from persistent interest rate differentials and central bank interventions, which can cause the skewness in the empirical return distributions. The volatility of models 1 to 3 are smaller compared to the empirical volatility, while model 4 is able to mimick the empirical volatility. However, model 4 assumes a higher variation of the chartists' and fundamentalists' trading rules ($\sigma_C = 0.12$ and $\sigma_F = 0.03$) as applied in the literature (Westerhoff 2008a, Demary 2010). However, we perform the policy analysis under the parameterization of model 1, since this parameterization is more common in the literature. However, one should keep in mind that results only apply to the 1/3 of actual volatility the model mimicks.

¹⁰ Actually, we simulated 7,000 trading days from which we skipped the first 1,000 in order to avoid artefacts that depend on initial conditions.

4.4.3 Simulation Design

Westerhoff (2008a) lists advantages of agent-based policy analysis which apply to our experiments in the following way:

- (i) We choose a simulation horizon of 7,000 artificial trading days, where we skip the first 1,000 data points since they are biased by fluctuations that depend on the chosen initial conditions. We calculate numbers and statistics over 6,000 trading days and present the variation in these numbers and statistics over the 50 independent simulation runs in box plots.¹¹ Although these box plots are more difficult to interpret compared to just average properties over 50 simulation runs, they deliver useful information about the variation of the model's properties for different simulation runs.
- (ii) Westerhoff (2008a) also highlights that we are able to measure all variables precisely during our simulations like the fundamental value and the decisions of agents.
- (iii) Westerhoff (2008a) notes that the researcher is able to account for all exogenous shocks. Within our simulations we introduce the shocks we already highlighted in the description of the model. We do not introduce other exogenous events like a large drop in the fundamental value (e.g. a big recession) or large non-fundamental innovations (e.g. panic). The reason is that we want to analyze the effectiveness of financial market taxes during "normal" trading days.
- (iv) Following Demary (2010) we perform the simulations under the same con-

¹¹The box plots indicate the median over 50 simulation runs, the first and the third quartile. The whiskers indicate the most extreme points which are in a range of 1.5 from the box, while outliers are highlighted as dots.

ditions (the same seed of random variables), but under different tax rates. Simulations for each value of the currency transaction tax are based on the same seed of $50 \times 6,000 = 300,000$ random numbers.

The policy analysis should give us more insights in how greed and risk aversion affect the effectiveness of financial market taxes. Therefore, we have to vary the following model parameters:

- (a) Transaction tax rate.

- (b) Risk aversion: A large value of μ indicates that the agent is more risk averse. However, we are only able to measure constant absolute risk aversion. This means that the degree of risk aversion does not depend on the agent's wealth. However, more complicated utility functions are necessary for analyzing decreasing or increasing absolute risk aversion.

- (c) Greed and aggressiveness: We define a greedy or aggressive agent as one, who has a higher aggressiveness parameter either as a chartist (κ_C) or as a fundamentalist (κ_F) or the agent's response to changes in profits (γ) is high. In more detail, when a chartist is greedy, he or she expects a stronger trend compared to a less greedy chartist and therefore places larger orders. A greedy fundamentalist expects the exchange rate to revert faster to its fundamental value and therefore places larger orders compared to a less greedy fundamentalist. Moreover, a greedy agent switches more frequently between different trading rules in order to increase his or her profits.

4.4.4 Policy Evaluation

Westerhoff (2003a, 2008a) suggests the following market (in-)efficiency measures for evaluating the effectiveness of policy measures:

- (i) volatility, defined as the average absolute percentage change in the asset price.
- (ii) distortion, defined as the average absolute deviation of the asset price from its fundamental value, which measures the degree of excess volatility.
- (iii) Westerhoff (2008a) and Demary (2008, 2010) suggest to analyze the change in the average percentage fraction of trading rules used by agents as a measure of the change in traders' behavior in response to policy measures, while
- (iv) Westerhoff (2008a) and Demary (2008, 2010) suggests to use the change in the kurtosis of the return distribution as a measure of catastrophic risks caused by large outliers. There are better measures for the tail behavior of the distribution of returns like the Hill estimator for the tail index (Lux and Ausloos 2002). However, the kurtosis is a statistic which is very sensitive to outliers. Hence, although not measuring tail behavior it might be a good statistic to measure the occurrence of extreme events directly, like a crash for example.

4.5 The Effectiveness of Transaction Taxes

4.5.1 Simulation Runs under Taxation

Stabilizing Tax Regime: Figure 4.2 shows the model outcome of a typical simulation run under a tax rate of 0.1 percent. Under this tax rate all chartists abstain

from trading, while the number of fundamentalists fluctuates around 20 percent. Thus, the number of inactive traders fluctuates around 80 percent. Hence, there are more fundamentalist traders left compared to trend-chasing traders indicating that this low tax rate might have the potential to stabilize financial markets. However, 80 percent of traders are not trading which indicates a loss of liquidity. An absence of autocorrelation in raw as well as in absolute returns can be inferred for the autocorrelation diagrams. Thus, volatility clusters are absent under taxation, a result which can also be found in Demary (2010). This result is caused by the decline in the number of chartist traders, who account for a large fraction of the variation of returns through the SSV mechanism. Moreover, bubbles are less persistent compared to the baseline simulation. This outcome is due to the fact that there are no chartist traders left in the market. Furthermore, returns look like white noise, while distortions seem to be unsystematic. The model outcome indicates that the tax rate 0.1 percent lies within a stabilizing tax regime.

Destabilizing Tax Regime: Figure 4.3 shows a simulation run under a tax rate of 1 percent. Under this tax rate chartists as well as fundamentalists abstain from trading. Since fundamentalists are absent, there is no driving force that connects the exchange rate to its fundamental value. As a result, the time series of distortions is highly persistent and seems to follow a random walk. Thus, the exchange rate seems to return to its fundamental value only with a small probability. Moreover, volatility is rising in time, a phenomenon which is a characteristic of a random walk. The results indicate that the tax rate of 1 percent lies within a destabilizing tax regime. The results from figures 4.2 and 4.3 are in line with the *u*-shaped volatility and misalignment response curves found in the models of Westerhoff (2003a, 2008a) and Demary (2010). In the following lines we will get more insights into this point.

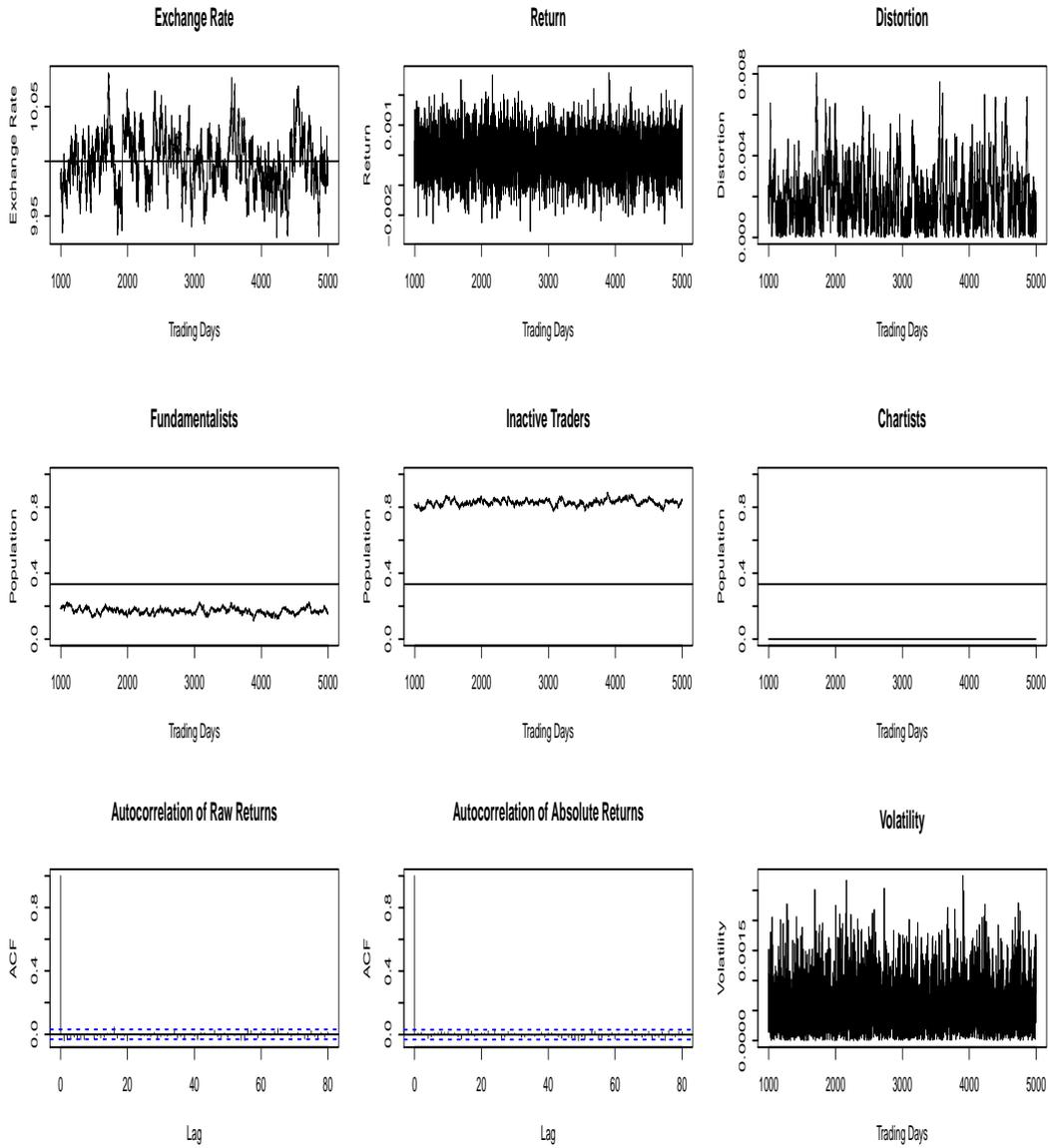


Figure 4.2: Simulation of a Stabilizing Tax Regime: 0.1 Percent Tax

NOTES: The output of a typical simulation run of a stabilizing tax regime. The underlying parameter values are those given in table 1. Fundamental equilibrium values from the baseline case are given by the horizontal lines located at $\bar{s}^F = 10$ and $\bar{w}^F = \bar{w}^C = \bar{w}^I = 1/3$.

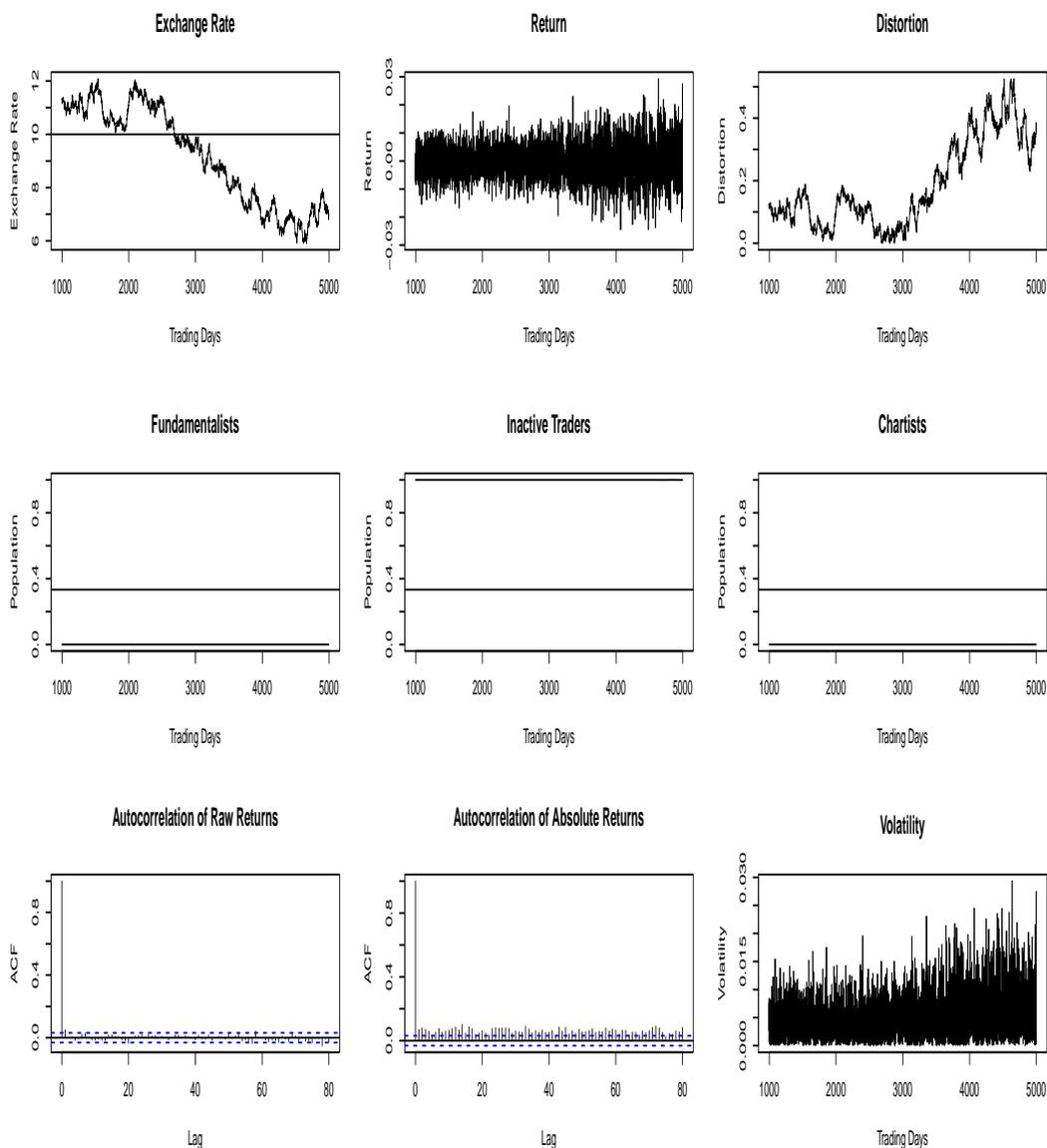


Figure 4.3: Simulation of a Destabilizing Tax Regime: 1 Percent Tax

NOTES: The output of a typical simulation run of a destabilizing tax regime. The underlying parameter values are those given in table 1. Fundamental equilibrium values from the baseline case are given by the horizontal lines located at $\bar{s}^F = 10$ and $\bar{w}^F = \bar{w}^C = \bar{w}^I = 1/3$.

4.5.2 Comparison of Different Tax Regimes

Figure 4.4 contains a comparison of tax regimes. The tax rate ranges from 0 (baseline case) to 1 percent (destabilizing tax regime) and is varied in 0.05 percentage point

steps. All simulations for different tax rates are based on the same seed of random variables. The figure shows volatility, distortion and the kurtosis, as well as the population fractions of chartists, fundamentalists and inactive traders as a function of the transaction tax rate. Remind, that 100 percent inactive traders indicate that there are no chartists and fundamentalists left in the market, while a constant fraction of noise traders is still trading.

From these figures can be inferred, that the number of chartists and fundamentalists are declining on average in the tax rate. Moreover, all chartists and fundamentalists are inactive on average for tax rates above 0.5 percent. Furthermore, the variation in these population fractions is increasing in the tax rate. This can be inferred from the larger boxes for higher values of the transaction tax rates. Moreover, there is a larger number of outliers for higher tax rates as well. However, the number of outliers for the number of chartists is larger compared to the outliers in the number of fundamentalists. These results indicate several speculative attacks of chartists.

The volatility of the exchange rate returns is decreasing on average in the tax rate. However, the variation in this statistic is increasing in the tax rate as well. This can be inferred from the larger boxes. Note that the whisker of the baseline case lies within the boxes for higher tax rates. Thus, there is a certain probability that taxation will increase the volatility of exchange rate returns as well. This is especially the case for the simulations in which the number of fundamentalists is near zero, which limits the reversion of the exchange rate towards its fundamental values. From the variation in the results of these policy experiments we can conclude that taxation might also lead to higher volatility with a certain probability.

The kurtosis statistic is decreasing in the tax rate on average. There are small

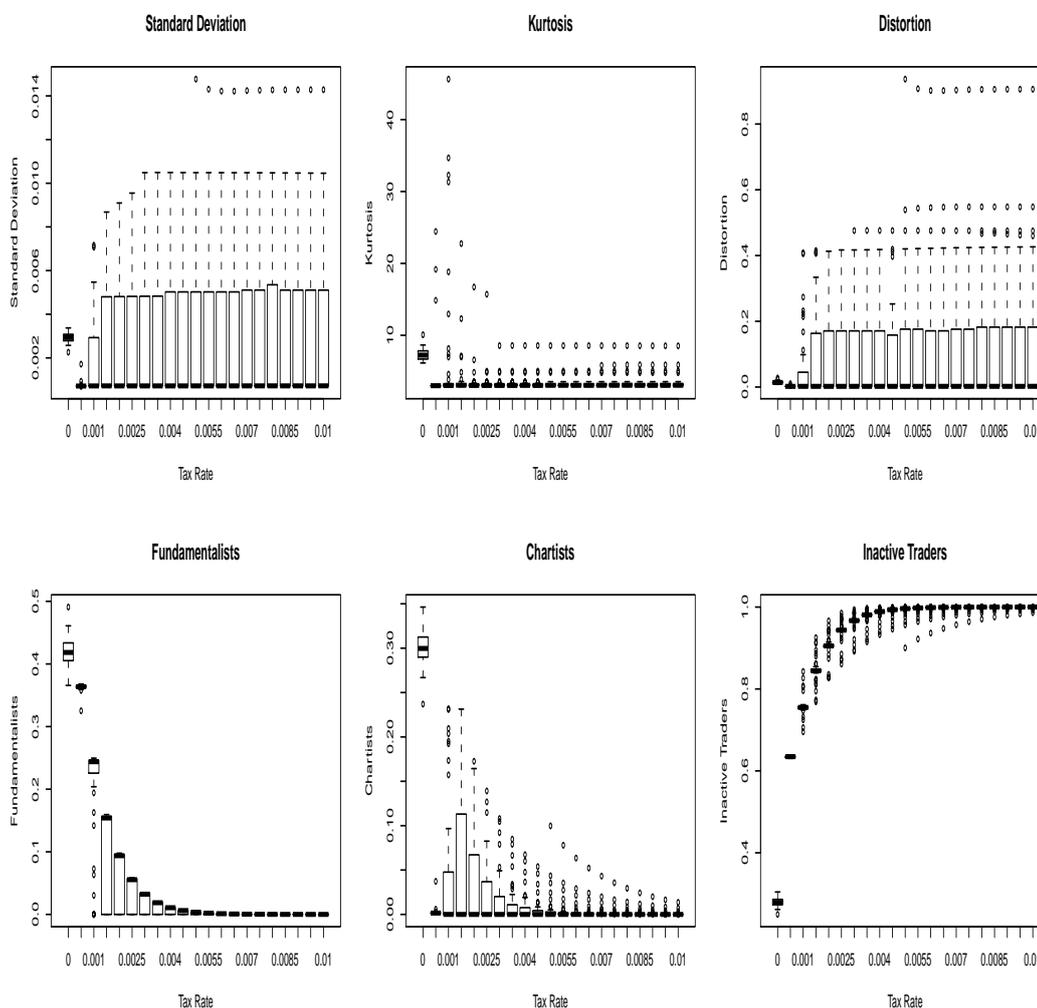


Figure 4.4: Comparison of Tax Regimes

NOTES: All boxplots are based on 6,000 artificial trading days and 50 simulation runs. The middle line of the box plots represent the median over 50 simulation runs, while the ends of the boxes represent the first and third quartiles. The whiskers indicate the most extreme point that is with a factor of 1.5 of the hinge. Any points beyond the whiskers are outliers. Note, that simulations for different tax rates are based on the same seed of random variables. The underlying parameter values are those given in table 4.1.

boxes indicating that there is a small process error due to different seeds of random variables. Hence, taxing financial markets lead to a smaller number of large returns,

that means to less crashes. However, there is as well a significant number of outliers. Thus, there are also simulation runs in which taxing financial markets lead to a higher frequency of crashes.

A similar picture can be found for the average distortion or excess volatility. Distortions are decreasing in the tax rate on average but the variation in this number is as well increasing in the tax rate. This can be inferred from the larger boxes for higher tax rates. Moreover, there is a significant number of large outliers. These results indicate that distortion might be declining on average. However, the large number of simulation runs in which it is increasing indicates that the stabilizing effect is not for sure and that there are significant cases where taxation has destabilizing effects. This is especially the case for the simulation runs, where the number of fundamentalists is near zero, while there is a larger number of chartist traders left. In these cases the reversion of the exchange rate towards its fundamental value is limited.

From these results we conclude that small taxes have the ability to stabilize the financial market, while higher tax rates have not. The stabilizing tax regime is departed from the destabilizing one by a tax rate of 0.1 percent. For tax rates below this threshold value the number of fundamentalists is higher compared to the number of chartists, while volatility and distortions are declining in the tax rate. The reason for the destabilizing nature of higher tax rates is due to the fact, that these tax regimes are characterized by a higher uncertainty around the average behavior of volatility and distortions over several simulation runs as indicated by the larger boxes. The uncertainty around these numbers is due to the larger outliers in the number of chartists for higher tax rates. In these cases, their number is higher compared to the number of fundamentalists, what explains the destabilization of the

market. It seems that chartism might be more profitable compared to fundamentalism under higher transaction cost at least for some cases. Comparable results can also be found in Westerhoff (2003a, 2008a) and Demary (2010). The variation in our result might be due to the nonlinearity of the model and its sensitivity to different seeds of random variables.

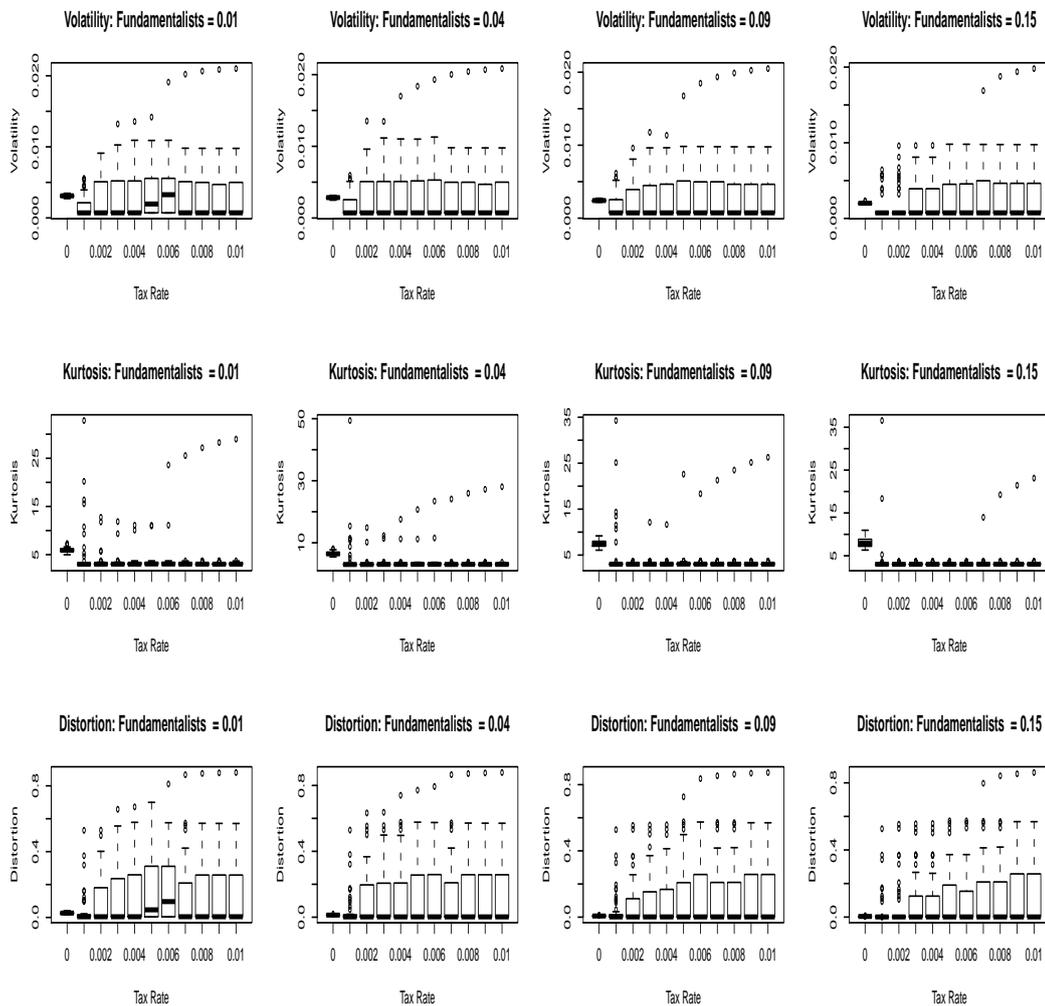


Figure 4.5: Comparisons of Tax Regimes for Different Degrees of Fundamentalists' Aggressiveness κ_F

NOTES: See Figure 4.4.

4.5.3 Aggressiveness of Fundamentalists

Figure 4.5 contains a comparison of different tax regimes for different degrees of the aggressiveness of fundamentalists. Results for volatility, kurtosis and distortions are given in boxplots. These boxplots contain averages over 6,000 artificial trading day plotted for 50 independent simulation runs. In contrast to Figure 4.4 we provide plots for different degrees of aggressiveness with which fundamentalists trade. More precisely, we differentiate between the cases $\kappa_F \in \{0.01, 0.04, 0.09, 0.15\}$, while the tax rate is varied from 0 percent to 1 percent in 0.1 percentage point steps. Note, that $\tau = 0.01$ means that the tax rate is 1 percent.

We figure out the u -shaped volatility response on average for $\kappa^F = 0.01$, the case in which fundamentalists act less aggressive compared to the baseline case. Moreover, there is large uncertainty around this median for larger tax rates, which is indicated by the wider boxes and whiskers. The kurtosis seems to decline in the tax rate on average, however, there are large outliers especially for larger tax rates. A u -shaped response of distortions to higher tax rates can be found as well. However, there are larger boxes and whiskers for higher tax rates indicating more variation in this measure over the 50 simulation runs.

For larger values of κ^F we find that volatility is declining especially for higher tax rates. However, fluctuations in this statistics are not decreasing in the fundamentalists aggressiveness. For $\kappa^F = 0.15$ we find smaller boxes for lower tax rates, but a larger number of outliers. Distortions are declining on average in the tax rate when fundamentalists act more aggressively. Moreover, we find smaller boxes and whiskers at least for small tax rates. Furthermore, we find a reduction in the number of outliers for this measure, when fundamentalists act more aggressively.

Summing up, these results are in line with the conventional view, that more aggressive fundamentalists act as a stabilizing force. This can be inferred from the volatility and misalignment response curves which become flatter the more aggressively fundamentalists trade. A comparable result can be found in the model used in Demary (2010). We conclude, that taxation is more effective, when fundamentalists trade more aggressively.

4.5.4 Aggressiveness of Chartists

Figure 4.6 contains a comparison of different tax regimes for different degrees of chartists' aggressiveness. A higher value of this parameter indicates that chartists place larger orders for a given trend segment and, hence, trade more aggressively. We consider the cases $\kappa^C \in \{0.01, 0.04, 0.06, 0.09\}$.

The u -shaped volatility response to taxation becomes flatter for small tax rates the less aggressively chartists trade. The smallest box can be found for a tax rate of 0.1 percent, while boxes increase in size for tax rates above this value. Larger boxes indicate a larger variation through different simulation runs. For increasing tax rates we figure out that whiskers become more narrow, while outliers of the volatility increase in magnitude, when chartists become more aggressively. The higher frequency of outliers might be caused by more frequent speculative attacks when chartist act more aggressively.

For $\kappa^C = 0.09$ we find that chartists destabilize since we find the highest number of outliers of volatility for this value of the trend-extrapolation parameter. Large outliers of the kurtosis measure increase especially for higher transaction tax rates, when trend-chasing agents act more aggressive. The large outliers of a statistic

which indicates large average returns in a sample indicates that there are infrequent speculative attacks caused by chartists' aggressiveness.

We figure out a u -shaped distortion response curve for rising tax rates as well. The picture which emerges from the box plot is more or less robust to different values of the chartists' aggressiveness. For the case $\kappa^C = 0.09$ we find more outliers and more variation as indicated by a larger box for the case of a 0.1 percent transaction tax.

Summing up, the results indicate that a higher aggressiveness of chartists leads to a destabilization and a smaller effectiveness of transaction taxes in financial markets. The smaller effectiveness can be inferred from the volatility response curve which becomes flatter on average the less aggressively chartists act. Similar findings can be found in the sensitivity analysis in Demary (2010).

4.5.5 Intensity of Choice

Figure 4.7 contains a comparison of different tax regimes for different values of the agent's intensity-of-choice parameter. A higher value of this parameter indicates that agents switch more often between trading strategies for maximizing profits.

The case $\gamma = 1$ is the case where agents react more sluggish to changes in the performance measures. As a result, flat volatility, kurtosis, and distortion response curves to taxation emerge. Moreover, the kurtosis measure corresponds to the one of the normal distribution.

For the case $\gamma = 50$ and for higher values of this parameter a u -shaped volatility response curve emerges. Volatility reductions can be found for tax rates between 0.3 and 0.8 but not for tax rates below 0.3 percent. For rising tax rates boxes

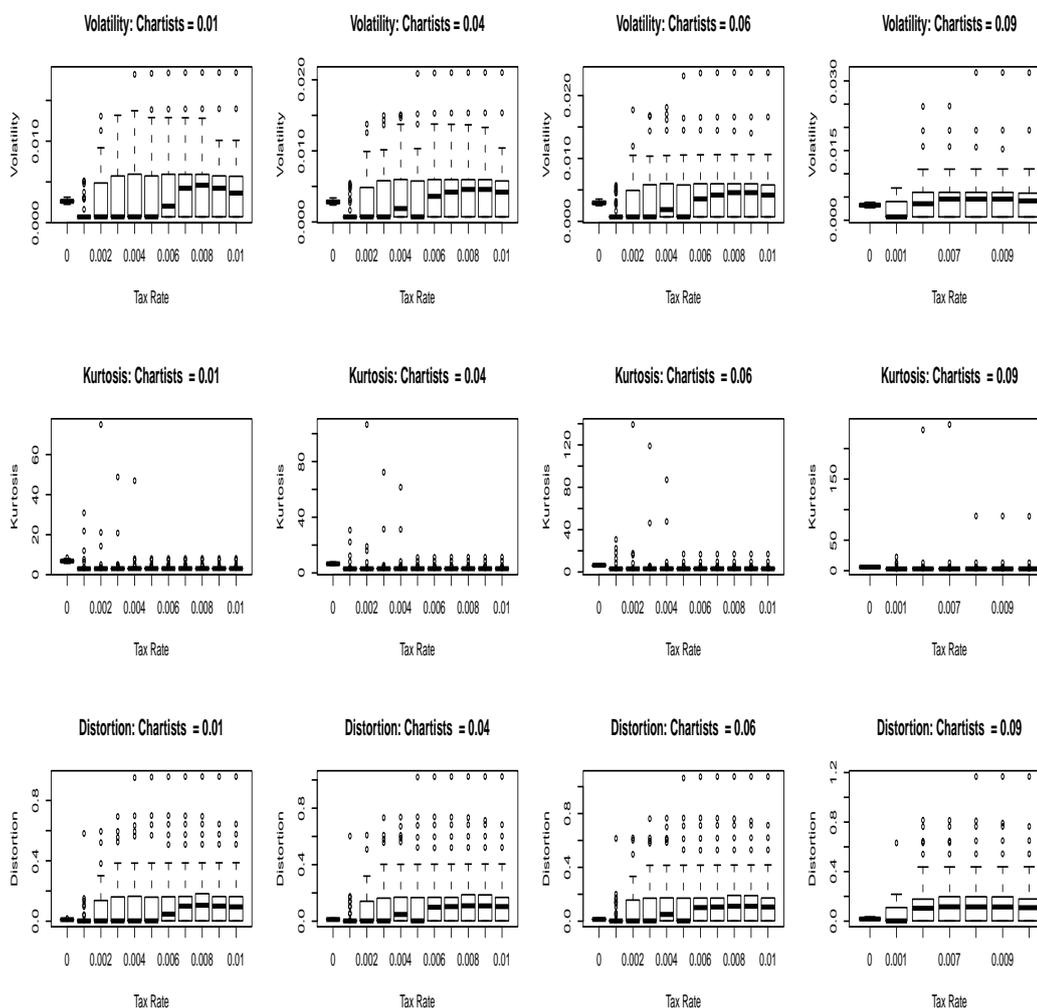


Figure 4.6: Comparison of Tax Regimes for Different Degrees of Chartists' Aggressiveness K_C

NOTES: See Figure 4.4.

and whiskers increase indicating more variation for different simulation runs. For $\gamma = 100$ we find that the tax rates 0.1 percent and 0.2 percent become stabilizing, since they decrease volatility now. However, we figure out that the tax rate 0.3 percent increases volatility when we enlarge γ to the value 600.

As already mentioned, we find that the kurtosis of the exchange rate return distri-

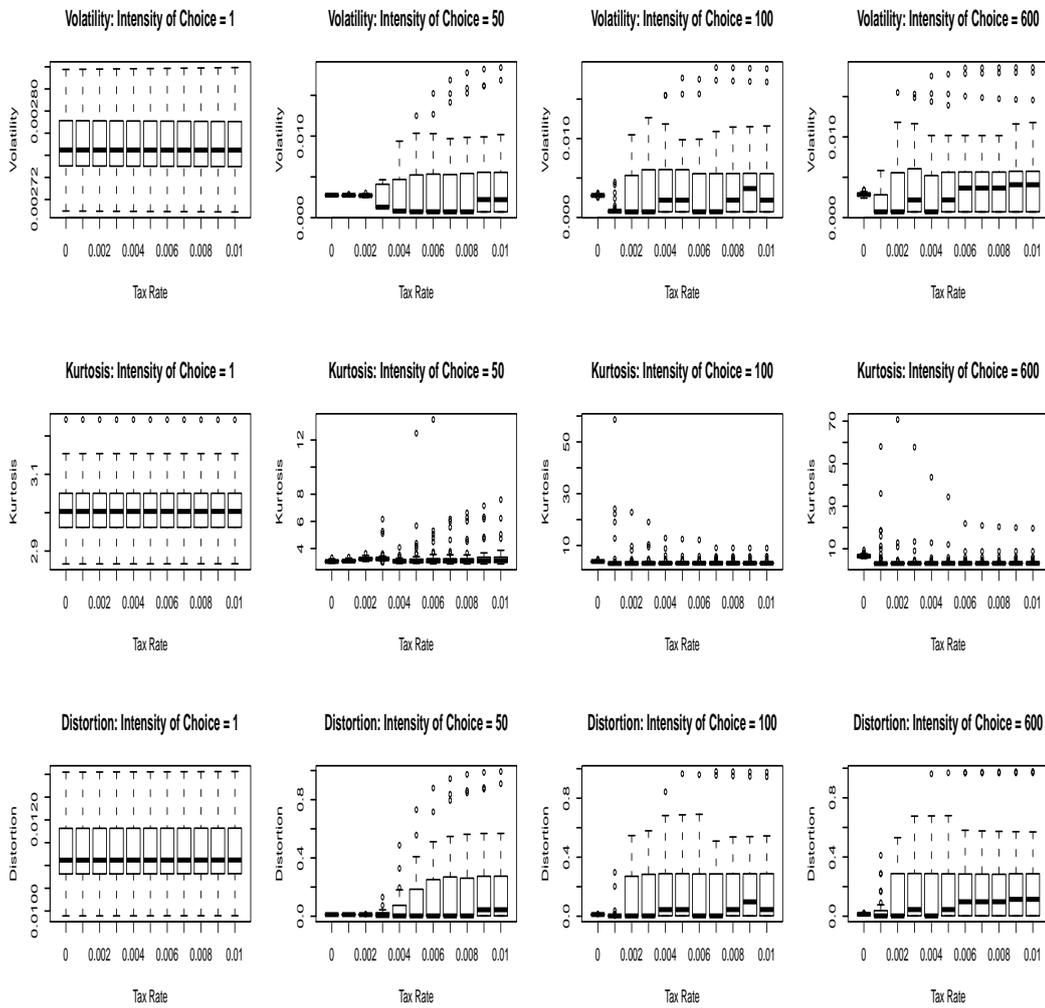


Figure 4.7: Comparison of Tax Regimes and Different Intensity-of-Choice Parameters γ

NOTES: See Figure 4.4.

bution equals the one of the normal distribution on average for $\gamma = 1$, while the kurtosis increases when the parameter γ takes higher values. Moreover, we figure out more outliers for this statistic. Thus, higher values of the intensity-of-choice measure might cause large trends and crashes in returns. This finding makes sense since more agents switch to the most profitable trading rule under a higher value

of γ . From this more aggressive trading behavior more turbulent and more quite phases emerge, which summarizes in a more leptokurtic exchange rate return distribution. Moreover, we find larger variations in the distortion measure and more outliers, as indicated by the larger boxes and whiskers.

We conclude, that a more aggressive switching of agents between different trading strategies destabilize the financial market and lowers the effectiveness of higher transaction taxes, which is indicated by the shapes of the volatility response curve.

4.5.6 Profit Discounting

Figure 4.8 contains a comparison of different tax regimes for different values of the agents' memory parameter $\theta \in \{0, 0.5, 0.9, 0.98\}$. A higher value of this parameter θ indicates that past profits are taken into account for the current strategy selection, while a lower value of θ indicates that agents orientate only on recent profits for determining their investment strategy. When $\theta = 0$, then agents consider the last periods profit only, hence, the agent will react abruptly to changes in the performance measures. From figure 4.8 can be inferred, that volatility, kurtosis as well as distortions are rising in the transaction tax rate for the case $\theta = 0$. Thus, taxation leads to a destabilization of the market, when agents are very aggressive and only consider their short-term profits for strategy selection.

When the memory parameter takes the value $\theta = 0.5$ the profits of the last days are as well relevant for strategy selection. In this case, volatility, kurtosis and distortions are as well increasing in the tax rate, while also the dispersion of these statistics is increasing in the tax rate as indicated by the larger boxes and whiskers. Hence, there is increasing variability over several simulation runs. For tax rates near 1 percent

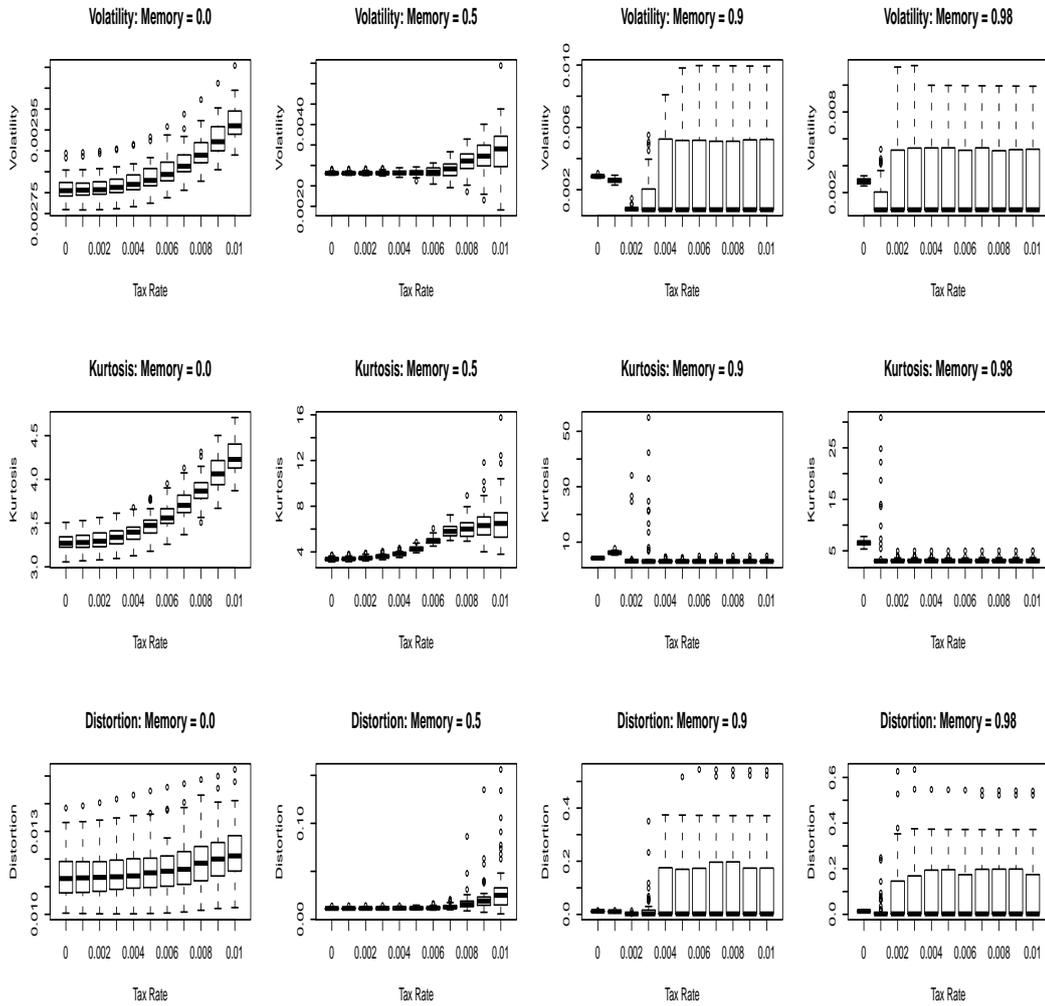


Figure 4.8: Comparison of Tax Regimes for Different Memory Parameters θ

NOTES: See Figure 4.4.

there are large outliers of distortions and kurtosis indicating erratic fluctuations for some simulation runs.

For the case of a memory parameter $\theta = 0.9$ traders' performance measure takes even the profits some weeks ago into account. In this case volatility is declining for tax rates below 0.3 percent, while this decline is not significant for higher tax rates

which can be inferred from the larger boxes which indicate a larger variability of this statistic. From the larger whiskers can be inferred, that there are a lot of simulation runs in which volatility is increasing for higher transaction tax rates. A similar result can be found for the kurtosis measure. Here, the number of outliers is increasing for tax rates around 0.3 percent indicating that this regime can be destabilizing. The variation in the average distortions are increasing in the tax rate as well as indicated by the larger boxes and whiskers. As a result, higher tax rates lead to larger deviations from the fundamental values for a large part of our simulation runs.

For a memory parameter $\theta = 0.98$ only the tax rate 0.1 percent seems to have stabilizing power, while higher tax rates lead to large variations of this statistic.

We conclude, that for a large part of our simulations volatility seems to increase in the tax rate. A similar result can be found for distortions, although a large number of outliers can even be found for a 0.1 percent tax. The results indicate that agents' discounting of past profits plays a role for the stabilizing impact of transaction taxes. Although taxation works on average, when agents take a large part of past profits into account, we find in a significant part of our simulations that taxation can also have destabilizing effects. We conclude from the shape of the volatility response curve to taxation that taxation becomes more effective the longer the memory of the traders is.

4.5.7 Risk Aversion

Figure 4.9 contains a comparison of different tax regimes for different degrees of risk aversion. A larger value of this parameter indicates that agents are more

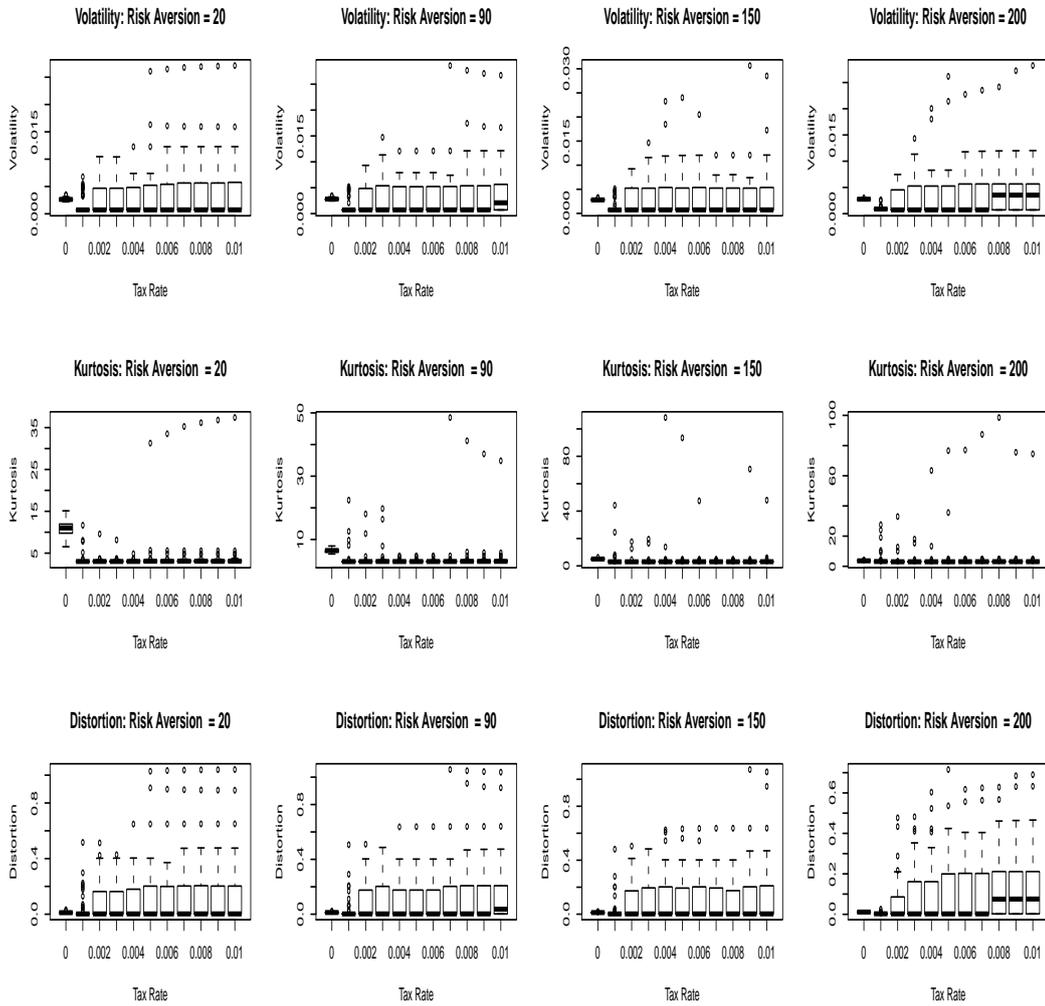


Figure 4.9: Comparison of Tax Regimes under Different Degrees of Risk Aversion μ

NOTES: See Figure 4.4.

careful and place smaller orders. Hence, agents that are less risk averse tend to be more aggressive, since they tend to trade larger orders. We analyze the cases $\mu \in \{20, 90, 150, 200\}$. For $\mu = 200$ we find the *u*-shaped volatility response curve, which diminishes for lower values of this parameter. In more detail, we find that volatility is smaller on average for tax rates above 0.7 percent, when agents are less

risk averse. Hence, higher transaction taxes seems to be more effective, when agents are less risk averse. When traders have a higher degree of risk aversion, we find larger outliers in the kurtosis measure. Moreover, we find less outliers for smaller taxes, when traders are less risk averse. We find a *u*-shaped distortions response curve on average, when traders are risk averse ($\mu = 200$), while distortions decrease in magnitude for higher tax rates, when traders are less risk averse.

Thus, higher transaction taxes are more effective as indicated by the flatter volatility response curve, when traders are less risk averse. This result adds important insights into the working of financial market taxes, since the curvature of the volatility as well as the distortions response curve might depend on the agents degree of risk aversion. We conclude that taxation becomes more effective the less risk averse traders are.

4.6 Conclusion and Outlook

Recent agent-based financial market models came to the result that taxing financial transactions does not *per se* increase financial stability and that the response of volatility and distortions to rising tax rates seem to be *u*-shaped. Moreover, greed and the risk appetite of traders are often blamed for financial instability and there is no evidence how behavioral patterns affect the effectiveness of regulations in financial markets. We aim to fill this gap in the literature by analyzing how different degrees of risk aversion and aggressiveness of traders affect the working of transaction taxes within an agent-based framework.

We introduce financial market taxes into the model proposed by DeGrauwe and Grimaldi (2006), which explicitly assumes risk averse traders. Moreover, we expand

the original model by structural stochastic volatility as proposed by Franke and Westerhoff (2009), Westerhoff (2008a) and Westerhoff and Dieci (2006), since it is an important generator of volatility clusters (Franke and Westerhoff 2009).

Within the model the following results emerged: (i) The tax rate 0.1 percent seems to demarcate the stabilizing tax regime from the destabilizing tax regime. (ii) The shapes of the volatility-response-curve and the distortion-response-curve depend on the agents risk aversion. Both are *u*-shaped when agents are highly risk averse, while the curves become flatter the less risk averse agents are. From these results we can conclude that transaction taxes are more effective the less risk averse traders are. (iii) More aggressive fundamental trading makes financial market taxes more effective, which is indicated by a flatter volatility response curve. (iv) More aggressive chartists makes small taxes less effective. This results can be inferred from the flatter volatility response curve for lower degrees of chartists' aggressiveness. (v) The more agents switch to the most profitable trading rule, the steeper the response curves are for higher tax rates. We conclude that transaction taxes are less effective the higher this parameter is. (vi) Longer memory in the performance measures makes transaction taxes more effective as indicated by a flatter response curves for lower values of this parameter.

Further research should concentrate on behavioral heterogeneity in risk aversion. We find that transaction taxes are more effective the less risk-averse traders are. This finding might depend on a configuration with more fundamentalists compared to chartists and might not be valid under a configuration with less fundamentalists than chartists. We should expect that markets with less risk-averse fundamentalists and more risk-averse chartists are more stable compared to markets with more risk-

averse fundamentalists and less risk-averse chartists. Moreover, transaction taxes should be more effective in markets with more aggressive fundamentalists. The reason lies in the fact that less risk-averse fundamentalists place larger orders against a mispricing compared to a less risk-averse ones, while more risk-averse chartists places smaller orders to a given trend segment compared to a less risk-averse ones. Hence, introducing heterogeneity in risk aversion might give additional insights to the working of financial markets and the effectiveness of regulations.

This heterogeneity in risk aversion of chartists and fundamentalists might have important implications for the implementation of regulations. As highlighted in the introduction, Menkhoff and Schmidt (2005) finds that buy-&-hold traders behave fundamentally oriented and risk averse, while momentum traders are less risk averse and follow aggressively the trend. Hence, policy measures for ensuring financial stability should delimit the risk-exposure of chartists, while they should allow fundamentalists to aggressively arbitrage away mispricings. Therefore, additional regulations should concentrate on the traders' responsibilities for their risk-exposure.

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4.A Appendix: Stability Properties

For analyzing the stability properties of the steady-state we set all shocks to zero and concentrate on the deterministic skeleton of the model. The system is described by the following state equations

$$s_t = s_{t-1} + \Theta_t^F \kappa_F (s_{t-1}^F - s_{t-1}) + \Theta_t^C \kappa_C (s_{t-1} - s_{t-2}) \quad (4.30)$$

$$\Theta_t^C \equiv \frac{w_t^C / \mathcal{V}_t^C (s_{t+1})}{w_t^C / \mathcal{V}_t^C (s_{t+1}) + w_t^F / \mathcal{V}_t^F (s_{t+1})} \quad (4.31)$$

$$\Theta_t^F \equiv \frac{w_t^F / \mathcal{V}_t^F (s_{t+1})}{w_t^C / \mathcal{V}_t^C (s_{t+1}) + w_t^F / \mathcal{V}_t^F (s_{t+1})} \quad (4.32)$$

$$w_t^F = \frac{\exp\{\gamma \pi_{t-1}^F\}}{1 + \exp\{\gamma \pi_{t-1}^F\} + \exp\{\gamma \pi_{t-1}^C\}} \quad (4.33)$$

$$w_t^C = \frac{\exp\{\gamma \pi_{t-1}^F\}}{1 + \exp\{\gamma \pi_{t-1}^F\} + \exp\{\gamma \pi_{t-1}^C\}} \quad (4.34)$$

$$w_t^I = \frac{1}{1 + \exp\{\gamma \pi_{t-1}^F\} + \exp\{\gamma \pi_{t-1}^C\}} \quad (4.35)$$

$$\pi_t^i = (s_t - s_{t-1}) x_{t-1}^i - \tau (s_t + s_{t-1}) |x_{t-1}^i| + \theta \pi_{t-1}^i. \quad (4.36)$$

Case 1: No Taxes Levied. Since we assume $\mathcal{V}_t^F (s_{t+1}) = \mathcal{V}_t^C (s_{t+1}) = \omega$ and zero transaction taxes ($\tau = 0$), the system reduces to

$$s_t = s_{t-1} + w_t^F \kappa_F (s_{t-1}^F - s_{t-1}) + w_t^C \kappa_C (s_{t-1} - s_{t-2}) \quad (4.37)$$

$$w_t^F = \frac{\exp\{\gamma \pi_{t-1}^F\}}{1 + \exp\{\gamma \pi_{t-1}^F\} + \exp\{\gamma \pi_{t-1}^C\}} \quad (4.38)$$

$$w_t^C = \frac{\exp\{\gamma \pi_{t-1}^F\}}{1 + \exp\{\gamma \pi_{t-1}^F\} + \exp\{\gamma \pi_{t-1}^C\}} \quad (4.39)$$

$$w_t^I = \frac{1}{1 + \exp\{\gamma \pi_{t-1}^F\} + \exp\{\gamma \pi_{t-1}^C\}} \quad (4.40)$$

$$\pi_t^i = (s_t - s_{t-1}) x_{t-1}^i + \theta \pi_{t-1}^i. \quad (4.41)$$

The stability of the fundamental steady-state can be inferred from the eigenvalues of the Jacobian matrix evaluated at the steady-state. By defining $y_{t-1} \equiv s_{t-2}$, we arrive at the following partial derivatives evaluated at the long-term solution (LTS)

$$\left. \frac{\partial s_t}{\partial s_{t-1}} \right|_{LTS} = 1 + \frac{1}{2}(\kappa_C - \kappa_F) \quad (4.42)$$

$$\left. \frac{\partial s_t}{\partial y_{t-1}} \right|_{LTS} = -\frac{1}{2}\kappa_C \quad (4.43)$$

$$\left. \frac{\partial s_t}{\partial \pi_{t-1}} \right|_{LTS} = \kappa_C(s_{t-1} - y_{t-1}) \left. \frac{\partial \Theta_t^C}{\partial \pi_t^C} \right|_{LTS} + \kappa_F(s_{t-1}^F - s_{t-1}) \left. \frac{\partial \Theta_t^F}{\partial \pi_t^F} \right|_{LTS} = 0 \quad (4.44)$$

since $\bar{s}^F = \bar{s} = \bar{y}$ at the long-term steady-state.

Moreover, we get the following derivatives which are comparable to those calculated in the appendix of Westerhoff and Dieci (2006)

$$\left. \frac{\partial \pi_t^F}{\partial s_{t-1}} \right|_{LTS} = 0 \quad (4.45)$$

$$\left. \frac{\partial \pi_t^C}{\partial s_{t-1}} \right|_{LTS} = 0 \quad (4.46)$$

$$\left. \frac{\partial \pi_t^C}{\partial y_{t-1}} \right|_{LTS} = 0 \quad (4.47)$$

since $\bar{x}^i = 0$ at the long-term steady-state.

Furthermore, we have

$$\left. \frac{\partial \pi_t^F}{\partial \pi_{t-1}^F} \right|_{LTS} = \left. \frac{\partial \pi_t^C}{\partial \pi_{t-1}^C} \right|_{LTS} = \theta, \quad (4.48)$$

while

$$\left. \frac{\partial \pi_t^I}{\partial \pi_{t-1}^I} \right|_{LTS} = \left. \frac{\partial \pi_t^I}{\partial s_{t-1}} \right|_{LTS} = 0. \quad (4.49)$$

All remaining derivatives with respect to y_{t-1} are zero.

Under the restriction $\alpha = \beta = 0$ the model has a similar, but has an even simpler structure compared to the model of Westerhoff and Dieci (2006), since Westerhoff and Dieci (2006) analyze a model with two markets and we have only one market here. Note that the analysis of the stability properties in the original DeGrauwe and Grimaldi (2006) model are much more complicated due to the the risk-assessment of agents $\mathcal{V}_t^i(s_{t+1})$, which we restrict to a constant. Following the derivations in the appendix of Westerhoff and Dieci (2006) we can exploit the block structure of the Jacobian matrix

$$J = \begin{bmatrix} \frac{\partial s_t}{\partial s_{t-1}} & \frac{\partial s_t}{\partial y_{t-1}} & \frac{\partial s_t}{\partial \pi_{t-1}^C} & \frac{\partial s_t}{\partial \pi_{t-1}^F} \\ \frac{\partial y_t}{\partial s_{t-1}} & \frac{\partial y_t}{\partial y_{t-1}} & \frac{\partial y_t}{\partial \pi_{t-1}^C} & \frac{\partial y_t}{\partial \pi_{t-1}^F} \\ \frac{\partial \pi_t^C}{\partial s_{t-1}} & \frac{\partial \pi_t^C}{\partial y_{t-1}} & \frac{\partial \pi_t^C}{\partial \pi_{t-1}^C} & \frac{\partial \pi_t^C}{\partial \pi_{t-1}^F} \\ \frac{\partial \pi_t^F}{\partial s_{t-1}} & \frac{\partial \pi_t^F}{\partial y_{t-1}} & \frac{\partial \pi_t^F}{\partial \pi_{t-1}^C} & \frac{\partial \pi_t^F}{\partial \pi_{t-1}^F} \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}(\kappa_C - \kappa_F) & -\frac{1}{2}\kappa_C & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & \theta \end{bmatrix} \quad (4.50)$$

and compute the eigenvalues of the blocks separately in our model as well. Two eigenvalues are equal to θ and therefore less than one by construction. The computation of the remaining eigenvalues can be calculated from the submatrix Q of the Jacobian matrix as shown in Westerhoff and Dieci (2006)

$$Q = \begin{bmatrix} 1 + \frac{1}{2}(\kappa^C + \kappa^F) & -\frac{1}{2}\kappa^C \\ 1 & 0 \end{bmatrix}. \quad (4.51)$$

Hence, the stability conditions in the absence of transaction costs reduce to

$$1 - \text{Tr}(Q) + \text{Det}(Q) = \frac{1}{2}\kappa_F > 0 \quad (4.52)$$

$$1 + \text{Tr}(Q) + \text{Det}(Q) = 2 + \frac{1}{2}\kappa_F > 0 \quad (4.53)$$

$$\text{Det}(Q) = \frac{1}{2}\kappa_F < 1 \quad (4.54)$$

Thus, when these conditions are met, the fundamental steady-state will stable in the absence of transaction taxes.

Case 2: Transaction Taxes Levied.

In case of transaction costs the performance measures change to

$$\pi_t^i = (s_t - s_{t-1})x_{t-1}^i - \tau(s_t + s_{t-1})|x_{t-1}^i| + \theta\pi_{t-1}^i. \quad (4.55)$$

In this case we cannot analyze the stability properties of the model by analyzing the eigenvalues of the Jacobian matrix at the steady state, since this equation is not differentiable due to the non-differentiability of the absolute value function. We can only draw intuitive conclusions by comparing the profits of chartists, fundamentalists and inactive traders. Note, that the demand of fundamentalist, chartists and inactive traders are zero within the steady-state, while it is different from zero for chartists and fundamentalists in the vicinity of the long-term solution. In the case, when the transaction cost of trading on an expected mean reversion are higher compared to the transaction costs of using this trading rule, while the chartist rule and being inactive is less profitable, the market will be dominated by fundamentalist traders. Hence, the fundamental steady-state will be attracting. In case where the transaction cost of using the fundamentalist trading rule is higher compared to the

returns of using this rule, while the after tax returns of using the trend-chasing rule will be more profitable, the market might be dominated by chartist traders. This will lead to a destabilization of the financial market by making the fundamental equilibrium repelling. We will get a clearer picture of the stabilizing or destabilizing nature of financial market taxes by calculating volatility response curves and misalignment response curves with respect to transaction taxes by Monte-Carlo simulation methods. Our simulations show, that the exchange rate can be decoupled from its fundamental value under sufficiently high tax rates.

5 Speculative Bimodality in Real Exchange Rates: Estimation, Testing and Forecasting

Abstract. We estimate stochastic differential equations with with a cubic-drift for modeling and forecasting financial instability. Bimodal distributions emerge from this type of differential equation for a certain interval of the models' parameters. We estimate these models for 14 monthly real exchange rates taken from the ECB's database covering the time from 1993:01 to 2009:02. Parameter estimates give evidence for bimodal exchange rate distributions for fourteen exchange rates, two cubic-drift models and two estimation periods. Forecasts of the exchange rate density predict the exchange rate to be in the vicinity of one of the two attractors for most of the time. However, there are periods, in which both attractors are covered with probability mass indicating uncertainty about the equilibrium exchange rate. The models identify the beginning of the Exchange Rate Mechanism II and the introduction of the Euro as two major phase transitions. Based on tests on unconditional and conditional predictive accuracy we find that the cubic-drift models outperform the random walk in out-of-sample forecasting for one out of fourteen real exchange rates. We conclude that bimodality is a good description of real exchange rates.

Key Words: *Exchange Rate Distributions, Fokker-Planck Equation, Forecasting, Real Exchange Rates, Speculative Dynamics*

JEL Classification: *C51, C53, F31, F37*

5.1 Introduction

Asset prices are excessively volatile (Shiller 1981, LeRoy and Porter 1981) due to speculative bubbles and crashes. Several financial crises in the recent past indicate historical evidence for financial instability, which could not be predicted by linear econometric models. Hence, econometric modeling and forecasting of financial instabilities is demanding.

Going back to the famous critique of Meese and Rogoff (1983) there is no evidence that linear structural exchange rate models outperform a random walk in forecasting exchange rates. The assumption that the exchange rate is exclusively driven by one single equilibrium, which is determined by economic fundamentals, cannot not explain large movements in asset prices or even financial instabilities. However, the recent literature, especially the second generation of currency crises models (Jeanne 1997) and the stochastic behavioral asset pricing models (Lux 2005, 2009a; Lux and Marchesi 1999, 2000), model financial instability by multiple equilibria.

Jeanne (1997) suggests a model of a currency crash. He assumes a zone of multiple equilibria, in which market participants do not share a consensus about the true equilibrium exchange rate, whereas agents know the fundamental equilibrium outside this interval. He concludes that exchange rates are most vulnerable to expectations within this zone of multiple equilibria. A similar rationale arises from Gennotte and Leland's (1990) model of the October 1978 stock market crash. Cremer and Salehi-Isfahani (1989) develop a multiple equilibrium model for the oil market with a backward-bending supply curve. A crash occurs as a jump between equilibria. The stochastic behavioral asset pricing models of Lux (1995) and Lux and Marchesi

(1999, 2000) are characterized by one stable fundamental equilibrium for certain parameter values, while two additional stable speculative equilibria emerge around the now unstable fundamental one for a different parameter set. Hence, a zone of instability emerges.

Multiple equilibria and discrete jumps in asset prices are nonlinear phenomena. Creedy et al. (1996) suggested to model a financial crash as a jump between two modes of the asset price distribution. They suggest the generalized normal distribution (see Cobb et al. 1983, Lye and Martin 1999), which they derive by solving a cubic continuous time diffusion process under the assumption that the parameters are time-varying functions of the economic fundamentals. Hence, fundamental variables might cause a crash by changing the distributional shape from unimodal (unique equilibrium) to bimodal (three equilibria).¹

Nonlinear stochastic differential equations which lead to bimodal distributions have a wide array of applications in financial economics. Examples are Aït-Sahalia (1996, 1999), Conley et al. (1997), Gallant and Tauchen (1998) and Chan et al. (1992), who assume interest rate processes to have a cubic drift which leads to a bimodal interest rate distribution. Inspired by the oil market model of Cremer and Salehi-Isfahani (1989), Jäger (2008) estimates a stochastic differential equation with a cubic drift for modeling jumps and crashes in oil prices. Lux (2006, 2009b) and Lux and Ghonghaze (2008) estimate nonlinear stochastic differential equations for the dynamics of market sentiment indices, which lead to bimodal distributions. Bimodal sentiment distributions imply no consensus among agents and herding behavior as

¹A related literature models jumps between different regimes by a latent Markov-process (Engle 1994 and Engle and Hamilton 2000). However, these models do not significantly outperform a random walk in out-of-sample forecasting (Engel 1994, Martin et al. 1997 and Koh et al. 2007).

implied by the models of Kirman (1993), Lux (1995, 2009a) and Lux and Marchesi (1999, 2000).

The aim of this paper is to model zones of instability in exchange rates by the cubic-drift model, which can be interpreted as a reduced form of the behavioral asset pricing models of Lux (2005) and Lux and Marchesi (1999, 2000). We use the numerical solution to the Kolmogorov forward equation for estimation and forecasting as suggested in Lux (2006, 2009b) and applied in Ghonghaze and Lux (2009), since the transition densities from this approach contain more information compared to the stationary approach of Creedy et al. (1996). We estimate these models for 14 real exchange rates taken from the ECB's database covering the time from 1993:01 to 2009:02.

Within this paper the following results emerge: *(i)* The parameter estimates give evidence for bimodality in exchange rates for two cubic-drift models. Bimodality cannot be rejected for fourteen real exchange rates for the period 1993:01 to 2009:02 as well as for the period 1993:01 to 2003:12 for both models. *(ii)* Our density forecasts identify phase transitions in exchange rates. We identify two major phase transitions as the beginning of ERM II and the introduction of the Euro. *(iii)* We find most exchange rates closer to their attractors and confidence regions becoming smaller after the beginning of ERM II. Moreover, bimodal distributions fit exchange rates better in the period before the introduction of the Euro, while unimodality is more evident afterwards. *(iv)* Statistical tests on equal unconditional predictive accuracy as well as on equal conditional predictive accuracy lead to the result that the cubic-drift models outperform the random walk prediction for seven of fourteen real exchange rates.

The remainder of this essay is structured as follows. The next section introduces the nonlinear model and analyzes its dynamics and distributional properties. The following parts contain a description of the numerical solution to the Fokker-Planck equation as well as a description of the approximate maximum likelihood estimation. After that, we present estimation results and the results of our forecasting exercise. The last section concludes.

5.2 The Nonlinear Diffusion Process for Speculative Dynamics

5.2.1 The Structural Model

Lux (1995) suggests a stochastic behavioral model of herd behavior and contagion in financial markets which serves as an economic foundation to bimodality in exchange rates. This model explains the emergence of speculative bubbles as a self-organizing process of contagion among heterogeneous speculators.

Lux (1995) assumes $2N$ speculative traders, who can be subdivided into n_+ optimistic traders and n_- pessimistic traders. By defining $n \equiv (n_+ - n_-)/2$ and $x \equiv n/N \in [-1, 1]$, we arrive at an index for the market opinion. For $x > 0$ the market is dominated by optimistic agents, while the market is dominated by pessimistic agents for $x < 0$. For $x = 0$ there are equal numbers of optimists and pessimists, while the market is completely dominated by optimists for $x = 1$, while it is completely dominated by chartists for $x = -1$.

Lux (1995) defines the transition probabilities from optimism to pessimism p_{+-} and

from pessimism to optimism p_{-+} as a function of the change in the asset price $\dot{s} = ds/dt$ and the opinion-index x

$$p_{+-} = v \exp\{a_1 \dot{s}/v + a_2 x\}, \quad \text{and} \quad p_{-+} = v \exp\{-a_1 \dot{s}/v - a_2 x\}, \quad (5.1)$$

where v is a variable for the speed of change, a_1 measures how much information agents draw from price changes, while a_2 measures how strong agents respond to the behavior of others as measured by the opinion index x .

Under this assumption for the transition rates, the dynamics of the mean value of the opinion-index $\dot{x} = dx/dt$ become

$$\dot{x} = \frac{1}{N} \left\{ (N - n)p_{+-} - (N + n)p_{-+} \right\} \quad (5.2)$$

$$= (1 - x)v \exp\{a_1 \dot{s}/v + a_2 x\} - (1 + x)v \exp\{-a_1 \dot{s}/v - a_2 x\} \quad (5.3)$$

$$= 2v(\text{Tanh}(a_1 \dot{s}/v + a_2 x) - x) \text{Cosh}(a_1 \dot{s}/v + a_2 x). \quad (5.4)$$

Lux (1995) divides the excess demand of speculators into the excess demand of optimists $n_+ t_N$ and the excess demand of pessimists $n_- t_N$, where t_N is a fixed amount of the asset which the traders may either buy or sell. Hence, net excess demand D_N is given by

$$D_N = n_+ t_N - n_- t_N = 2n t_N. \quad (5.5)$$

From the definition of the opinion index follows $n = Nx$ which can be used to express net excess demand as

$$D_N = 2Nxt_N = xT_N, \quad \text{with} \quad T_N \equiv 2Nt_N. \quad (5.6)$$

The group of fundamental traders determine their excess demand proportional to the difference between the fundamental price s_f and the actual price

$$D_F = T_F(s_f - s), \quad (5.7)$$

where T_F is a measure for the trading volume of fundamentalists.

Lux (1995) assumes that a market maker adjusts prices in response to the sum of fundamentalists' and speculators' excess demand

$$\frac{ds}{dt} = \beta(D_N + D_F) = \beta(xT_N + T_F(s_f - s)), \quad (5.8)$$

where β is the market maker's price-adjustment.

Hence, we arrive at the following dynamic system

$$\dot{x} = 2v(\text{Tanh}(a_1\dot{s}/v + a_2x) - x)\text{Cosh}(a_1\dot{s}/v + a_2x) \quad (5.9)$$

$$\dot{s} = \beta(xT_N + T_F(s_f - s)). \quad (5.10)$$

Proposition 2 of Lux (1995) describes the dynamic behavior of the system:

- (i) For $a_2 \leq 1$ there exists a unique equilibrium $E_0(0, s_f)$, which is attracting if $2(a_1\beta T_N + v(a_2 - 1)) - \beta T_F < 0$. Otherwise all trajectories converge to a stable limit cycle.
- (ii) For $a_2 > 0$ three equilibria $E_+ = (x_+, s_+)$, $E_0 = (0, s_f)$ and $E_- = (x_-, s_-)$ exists with $x_- = -x_+$ and $s_f - s_- = s_+ - s_f$. The two speculative equilibria E_+ and E_- are attracting, while the fundamental equilibrium E_0 is repelling.

In the following line we present a simple cubic-drift model, which is characterized

by a similar dynamics and which can be interpreted as a reduced form to the model of Lux (1995). The cubic-drift model is a reduced form, since we cannot identify the parameters of the model of Lux (1995) from the cubic drift model. However, we are able to estimate and to test the existence of multiple speculative equilibria against a single fundamental equilibrium by a simple statistical test, which will provide empirical evidence for the speculative dynamics described in Lux (1995).

5.2.2 The Cubic-Drift Model as a Reduced Form

The double-well potential model of Ait-Sahalia (1999) is a simple cubic model for the drift term and is given by the following nonlinear differential equation

$$ds_t = (\alpha s_t - s_t^3)dt, \quad (5.11)$$

which summarizes the dynamics of the model of Lux (1995) in a simple reduced form. However, it cannot describe the stable limit cycle. The equilibria \bar{s}_j of the process can be derived by setting the mean value dynamics to zero

$$\bar{s}(\alpha - \bar{s}^2) = 0, \quad (5.12)$$

which yields

$$s_- = -\sqrt{\alpha}, s_0 = 0, s_+ = +\sqrt{\alpha}. \quad (5.13)$$

For parameter values $\alpha \leq 0$ there is one unique equilibrium $s_0 = 0$, while it exhibits three equilibria $s_+ = +\sqrt{\alpha}$, $s_0 = 0$ and $s_- = -\sqrt{\alpha}$ for parameter values $\alpha > 0$. We allow only real-valued solutions. Otherwise, multiple equilibria with complex numbers are possible under $\alpha \leq 0$. The critical value $\alpha_c = 0$ is called the bifurcation

threshold. In order to compare these equilibria to the ones of the model of Lux (1995) we have to determine their stability properties first.

5.2.3 Stationary Points and Distributional Features

The potential function $V(s_t; \theta)$ is defined as the primitive of a vector field. A function $V(s_t; \theta)$ that satisfies $\partial V(s_t; \theta)/\partial s_t = -\mu(s_t; \theta)$ is called a potential of $\mu(s_t; \theta)$. The potential contains all information about steady-states, the stability of steady-states and the dynamics of the state variable s_t . All trajectories of s_t are graphs on $V(s_t; \theta)$. Minima of the potential correspond to stable states, while maxima correspond to unstable states. In order to get information about the stability of the stationary points of the process we have to calculate the derivatives of the drift function with respect to s_t evaluated at each of the stationary points \bar{s}_j . Note, that $\theta = \alpha$ here. For s_0 we get

$$\left. \frac{\partial \mu(s_t, \alpha)}{\partial s_t} \right|_{\bar{s}=0} = \alpha, \quad (5.14)$$

which is a minimum for $\alpha < 0$ (hence a stable state) and a maximum for $\alpha > 0$ (hence an unstable state). The remaining equilibria s_+ and s_- are only existent when $\alpha \geq 0$. They are stable, since

$$\left. \frac{\partial \mu(s_t; \alpha)}{\partial s_t} \right|_{\bar{s}=-\sqrt{\alpha}} = -2\alpha. \quad (5.15)$$

and

$$\left. \frac{\partial \mu(s_t, \alpha)}{\partial s_t} \right|_{\bar{s}=\sqrt{\alpha}} = -2\alpha. \quad (5.16)$$

As a result, the lower and the higher equilibria are stable states, while the equilibrium between them is a repelling equilibrium. Thus, the interval $[-\sqrt{\alpha}, \sqrt{\alpha}]$ defines the

zone of instability for the state space of the exchange rate s_t . Multiple equilibria and the zone of instability only emerge, when the parameter α exceeds the threshold value $\alpha_c = 0$.

Figure 5.1 shows the features of our model as a function of the parameter α . The upper figure contains the potential as a function of α in a 3D-plot, while the lower subfigure shows the stationary points as a function of α in a so-called bifurcation plot. The bifurcation plot is just a projection of the stationary states of the potential into (\bar{s}, α) -space.

Figure 5.2 shows characteristics of the cubic-drift model for the case $\alpha \leq 0$ in a stochastic environment. The upper subfigure contains the drift $\mu(s_t; \theta) = (\alpha s_t - s_t^3)dt$, while the lower subfigure contains a histogram of realizations of this process $ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t$ together with the kernel density estimate. From the phase plot can be inferred that the process is characterized by one single attractor located at $\bar{s} = 0$ for our example with $\alpha = 0$. The drift is positive for values of the state variable which are smaller compared to the attractor, while it is negative for values greater than the attractor. Hence, the dynamics revert to the attractor after the system is shocked. In a stochastic environment, where the state variable is continuously hit by shocks, the process should be located in the vicinity of this attractor for most of the time. Hence, a unimodal distribution emerges as can be seen in the histogram in the lower subfigure.

Figure 5.3 contains a phase diagram of the drift for $\alpha = 0.5$ as well as a histogram and the kernel density of a simulation of this process. From the phase diagram can be inferred that the process is characterized by three stationary points located at $s_- = -\sqrt{0.5}$, $s_0 = 0$ and $s_+ = +\sqrt{0.5}$. The process is repelled by $\bar{s}_2 = 0$, and

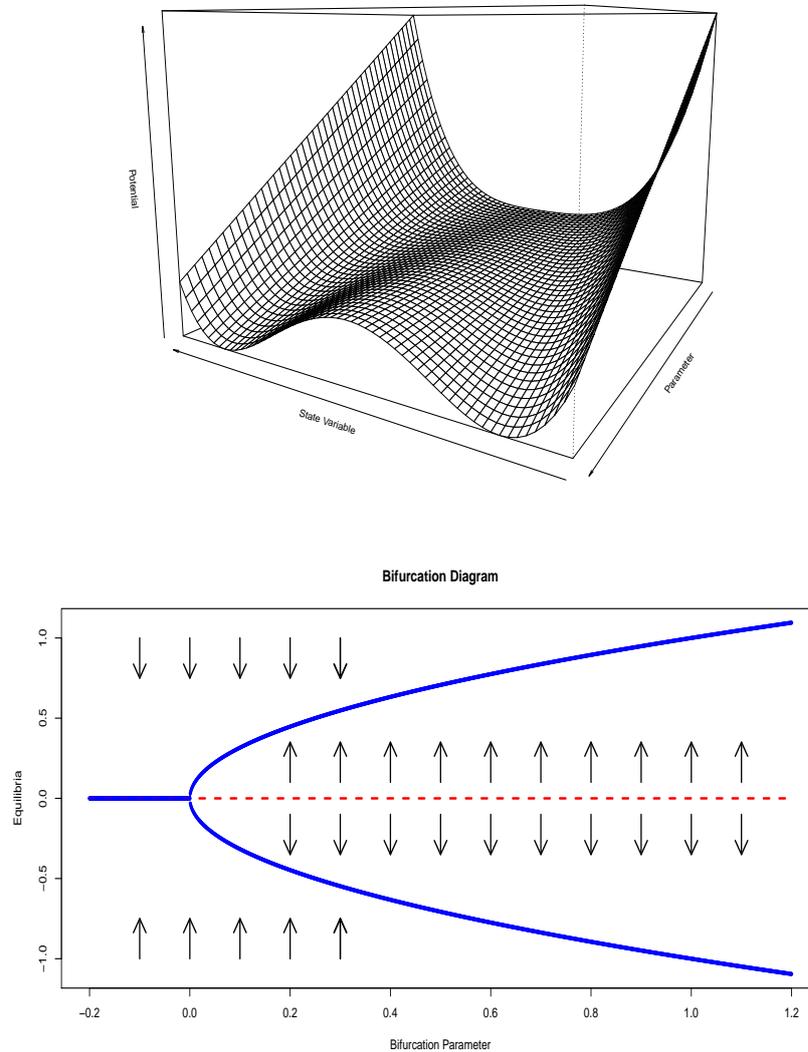


Figure 5.1: Stationary Points of the Cubic-Drift Model

NOTES: The upper figure shows the potential as a function of the parameter α . The lower bifurcation diagram is just a projection of the stationary points of the potential into (\bar{s}, α) -space.

attracted by the stationary points $-\sqrt{0.5}$ and $+\sqrt{0.5}$. As can be seen from the histogram and the kernel density estimate, a bimodal distribution emerges, since the process will be in the vicinity of the two attractors for most of the time, while

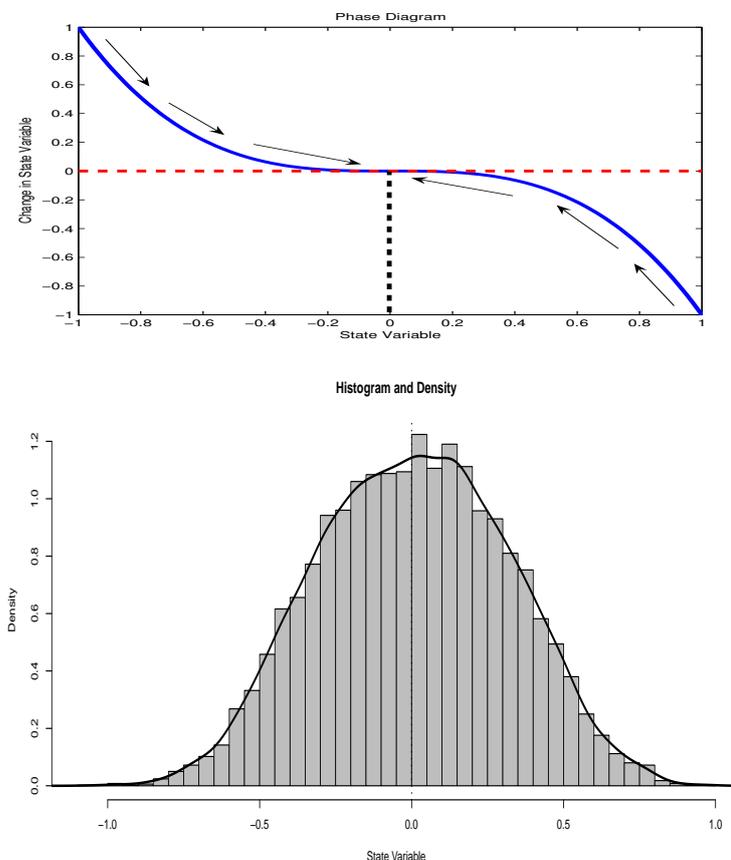


Figure 5.2: Drift and Emergent Distribution for $\alpha = 0$

NOTES: The phase diagram contains the drift of $ds_t = -s_t^3 + 0.3dW_t$, while the lower figure contains a histogram and a kernel density estimate generated by a simulation of size 10000 of the differential equation.

it will visit the repeller less frequently. The modes of the distribution correspond to the attractors for $\sigma = 1$, while they will be located closer to the repeller for $\sigma < 1$ as in this example here.

Distributional Dynamics

The stochastic differential equation $ds_t = \mu(s_t; \theta)dt + \sigma(s_t; \theta)dW_t$ fully describes the evolution of s_t over each infinitesimal time increment dt . However, the derivation

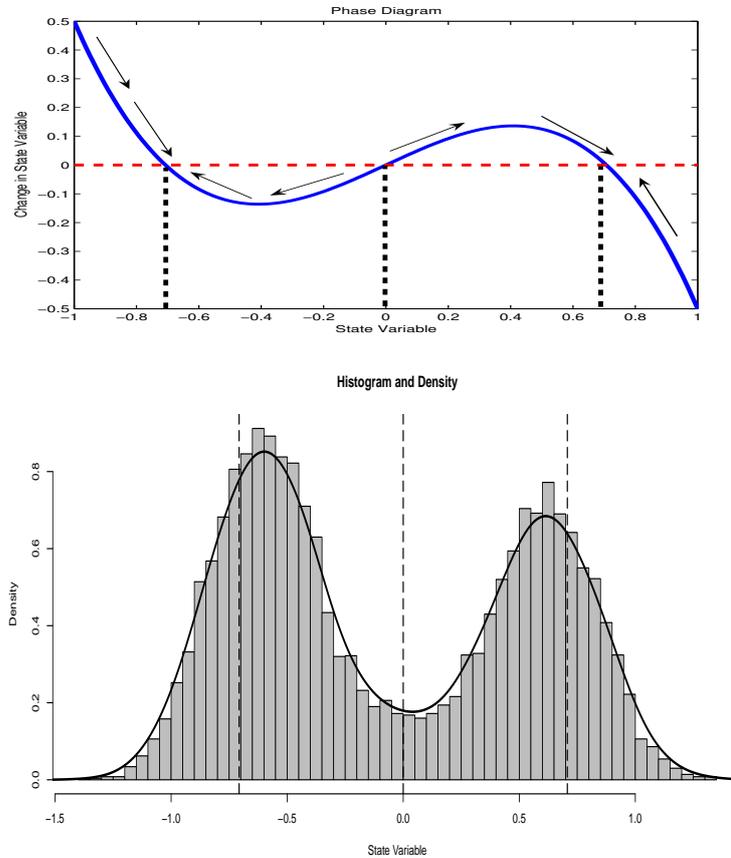


Figure 5.3: Drift and Emergent Distribution for $\alpha = 0.5$

NOTES: The phase diagram contains the drift of $ds_t = 0.5s_t - s_t^3 + 0.3dW_t$, while the lower figure contains a histogram and a kernel density estimate generated by a simulation of size 10000 of the differential equation.

of the conditional density of $s_{t+\Delta t}$ given s_t cannot be given in closed form except for some simple cases (Aït-Sahalia 1999). Closed form solutions for the transient density can only be given in closed form for the Black and Scholes (1973) model, the Vasicek (1977) model and the Cox, Ingersoll and Ross (1985) model. Aït-Sahalia (1998) develops closed form approximations for the unknown transitional density of nonlinear diffusion processes, while Lo (1988), Lux (2006, 2009a) and Poulsen (1999) and Pederson (1995) propose numerical solutions to the Fokker-Planck equation, which

is a parabolic differential equation for the dynamics of the transition density of a differential equation and which we introduce in the following lines. Honore (1997), Santa-Clara (1995) approximate the transition density by simulating numerous future paths of the state variable as also applied in derivative pricing.

The transition density f_t of a stochastic differential equation follows the Kolmogorov forward equation, which is also called Fokker-Planck equation (see Lo 1988, Ait-Sahalia 1999, Poulsen 1999 and Lux 2009b)

$$\frac{\partial f}{\partial t} = -\frac{\partial(\mu_t f_t)}{\partial s_t} + \frac{1}{2} \frac{\partial^2(\sigma_t^2 f_t)}{\partial s_t^2}. \quad (5.17)$$

Solutions to this deterministic differential equation for the time development of the density function cannot be given in closed except for very simple expressions for the drift and the diffusion term. However, the evolution of the transitional density ends in a stationary or equilibrium density $f^*(s)$ when there is no stochasticity. Following Creedy and Martin (1994), the closed-form solution to the stationary density is given by

$$f^*(s) = \exp \left\{ - \int_{-\infty}^s \left[\frac{2\mu(w) + d\sigma^2(w)/dw}{\sigma^2(w)} \right] dw - \eta \right\}, \quad (5.18)$$

where

$$\eta = \log \int_{-\infty}^{\infty} \left[- \frac{2\mu(w) + d\sigma^2(w)/dw}{\sigma^2(w)} dw \right] ds \quad (5.19)$$

is a normalizing constant, which is determined by the boundary conditions of the Fokker-Planck equation. This density has two modes and one antimode due to the cubic form of the drift function.

Testing Bimodality

Tests for bimodality of the asset price distribution against a unimodal distribution can be performed by estimating the model via a maximum likelihood procedure

$$ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t. \quad (5.20)$$

Since the model nests unimodality as a special case, we formulate the null and the alternative hypothesis as

$$\begin{aligned} H_0 : \text{Unimodality: } \alpha \leq 0 & \text{ versus} \\ H_1 : \text{Bimodality: } \alpha > 0. \end{aligned} \quad (5.21)$$

This test can be performed as a simple t-test due to the asymptotic normality of the maximum likelihood estimator, which even holds under multimodality (Poulsen 1999).

5.2.4 The Generalized Cubic-Drift Model

Another way to model exchange rates and multiple equilibria is suggested by Jäger (2008) and Jäger and Kostina (2005, 2006), which Jäger (2008) calls generalized mean-reversion

$$\mu(s_t; \theta) = (x_L - s_t)(x_M - s_t)(x_H - s_t), \quad (5.22)$$

with equilibria $x_L \leq x_M \leq x_H$ and $\theta = (x_L, x_M, x_H)'$. The drift is a known function up to the parameters $\theta = (x_L, x_M, x_H)'$. This process has three stationary points x_L , x_M and x_H to which the exchange rate can be attracted or repelled. This formulation

of the drift term is as well a polynomial of order three and, hence, has two stable states and one unstable state between them

$$\left. \frac{\partial \mu(s_t; \theta)}{\partial s_t} \right|_{s_t=x_L} = -(x_L - x_H)(x_L - x_M) < 0 \quad (5.23)$$

$$\left. \frac{\partial \mu(s_t; \theta)}{\partial s_t} \right|_{s_t=x_M} = -(x_M - x_H)(x_M - x_L) > 0 \quad (5.24)$$

$$\left. \frac{\partial \mu(s_t; \theta)}{\partial s_t} \right|_{s_t=x_H} = -(x_H - x_M)(x_H - x_L) < 0. \quad (5.25)$$

The formal derivation can be found in the appendix. Hence, the zone of instability is defined by the region $[x_L, x_H]$ in the state space of the exchange rate.

This model nests the simple double well potential as a special case. However, it is less restrictive, since the attractors are not restricted to have equal distance to the antimode. Moreover, the antimode is not restricted to be equal to zero. Under the restriction $x_L = -\sqrt{\alpha}$, $x_M = 0$ and $x_H = +\sqrt{\alpha}$ this model collapses to

$$\mu(s_t; \theta) = -s_t(s_t + \sqrt{\alpha})(s_t - \sqrt{\alpha}) = -s_t(s_t^2 - \alpha) = \alpha s_t - s_t^3. \quad (5.26)$$

Hence, a test of the double well potential model against the nonlinear mean-reversion model can be performed by testing the following hypotheses

$$\begin{aligned} H_0 : \text{ Double Well Potential: } & \quad x_L = -\sqrt{\alpha}, x_M = 0, x_H = +\sqrt{\alpha} \text{ versus} \\ H_1 : \text{ Generalized Mean Reversion: } & \quad x_L, x_M, x_H \in \mathcal{R} \end{aligned} \quad (5.27)$$

The resulting likelihood-ratio statistic is

$$\text{LR} = 2 \cdot (\log L(s_t|x_L = -\sqrt{\alpha}, x_M = 0, x_H = +\sqrt{\alpha}) - \log L(s_t|x_L, x_M, x_H)) \sim \chi_{(3)}^2 \quad (5.28)$$

and is χ^2 -distributed with three degrees of freedom, since we have three restrictions under the null hypothesis.

We apply bifurcation theory in order to derive a statistical test on the shape of the exchange rate distribution. Under the restriction $x_L = x_M = x_H = x$ the drift function collapses to

$$\mu(s_t, \theta) = (x - s_t)^3, \quad (5.29)$$

which is the case of one single equilibrium. Note that the generalized mean-reversion model does not nest the linear mean-reversion as a special case, since the drift function is still a polynomial of order three. A test of one single equilibrium against multiple equilibria in exchange rates can be done via a likelihood-ratio statistic of the null hypothesis

$$\begin{aligned} H_0 : \text{ Unimodality: } & \quad x_L = x_M = x_H = x \quad \text{versus} \\ H_1 : \text{ No Unimodality: } & \quad x_L, x_M, x_H \in \mathcal{R} \end{aligned} \quad (5.30)$$

The resulting likelihood-ratio statistic is

$$\text{LR} = 2 \cdot (\log L(s_t | x_L = x_M = x_H = x) - \log L(s_t | x_L, x_M, x_H)) \sim \chi_{(2)}^2 \quad (5.31)$$

and is χ^2 -distributed with two degrees of freedom, since we have two restrictions under the null hypothesis. When the value of the LR-statistic exceeds the critical value 5.99 we are able to reject the null hypothesis of one single equilibrium. However, this model is characterized by a richer dynamics compared to the simple cubic-drift model, since it nests the cases of two as well as of three equilibria. A formal derivation of the existence and the stability properties can be found in the appendix.

5.3 Approximate Maximum Likelihood Estimation

5.3.1 Numerical Solution to the Fokker-Planck Equation

Aït-Sahalia (1996, 1999) and Lux (2009b) suggest to use approximate solutions of the Fokker-Planck equation for maximum likelihood estimation. These authors argue that continuous-time diffusion processes are Markov processes. Since this property applies to any discrete subsample from the continuous-time path, the likelihood function can be derived as the product of the transitional densities calculated from the Fokker-Planck equation (Aït-Sahalia 1999). While Aït-Sahalia (1999) derives approximate closed-form solutions of the Fokker-Planck equation, Lux (2009b) suggests to approximate the differentials by finite-differences and to calculate the approximate solution numerically.

Following Aït-Sahalia (1999), two approaches for calculating approximate solutions to the Fokker-Planck equation exist. *(i)* Numerical solutions of this partial differential equation as done in Lo (1988) and Lux (2006, 2009b), and *(ii)* simulation of a large number of sample paths of the process as done in Honore (1997) and Santa-Clara (1995). As noted by Aït-Sahalia (1999) as well, both approaches do not produce expressions to be maximized over the parameter θ . Hence, all calculations have to be done for every value of θ until convergence of the maximum likelihood estimator is achieved.

The Kolmogorov Forward equation or Fokker-Planck equation is given by

$$\frac{\partial f_t}{\partial t} = -\frac{\partial(\mu_t f_t)}{\partial s_t} + \frac{1}{2} \frac{\partial^2(\sigma_t f_t)}{\partial s_t^2}, \quad (5.32)$$

where f_t is the unknown density, while $\mu_t = \mu(s_t; \theta)$ and $\sigma_t = \sigma(s_t; \theta)$ are the drift and diffusion, which are known up to a parameter vector θ , which we intent to estimate via maximum likelihood.

We construct the space grid on the interval $[s_{min}, s_{max}] \times [t, t + 1]$ on which we evaluate the approximate solution as suggested by Lux (2009b). The space dimension contains $N_x = (s_{max} - s_{min})/h$ equidistant grid points with distance h . These points are calculated as

$$x_j = x_0 + j \cdot h, \quad \text{with } j = 0, 1, \dots, N_x, \quad (5.33)$$

and $x_0 = s_{min}$, $x_{N_x} = s_{max}$. Moreover, we decompose the interval $[t, t + 1]$ of the observed data points into $N_t + 1$ equidistant points with distance $k = 1/N_t$

$$t_i = i \cdot k, \quad \text{with } i = 0, \dots, N_t. \quad (5.34)$$

For calculating the approximate solution to the Fokker-Planck equation by numerical methods, we have to discretize this law of motion by the method proposed in Lux (2009b). Hence, we substitute the time differential by the following finite difference approximation

$$\frac{\partial f_t}{\partial t} \approx \frac{f_j^{i+1} - f_j^i}{k}, \quad (5.35)$$

while we substitute the first and second derivatives by

$$\frac{\partial(\mu_t f_t)}{\partial s_t} \approx \frac{\mu_{j+1} f_{j+1}^i - \mu_j f_j^i}{h}, \quad (5.36)$$

and

$$\frac{\partial^2(\sigma_t f_t)}{\partial s^2} \approx \frac{\sigma_{j+1} f_{j+1}^i - 2\sigma_j f_j^i + \sigma_{j-1} f_{j-1}^i}{h^2}. \quad (5.37)$$

There are three possible discretization schemes which differ in accuracy and stability.

The first possible discretization scheme is the forward discretization or explicit method which approximates the Fokker-Planck equation with the following expression

$$\frac{f_j^{i+1} - f_j^i}{k} = \frac{\mu_{j+1}f_{j+1}^i - \mu_j f_j^i}{h} + \frac{1}{2} \frac{\sigma_{j+1}f_{j+1}^i - 2\sigma_j f_j^i + \sigma_{j-1}f_{j-1}^i}{h^2}, \quad (5.38)$$

where $f_j^i = f(jh, ik)$, $\mu_j = \mu(jh; \theta)$ and $\sigma_j = \sigma(jh; \theta)$ while θ is the parameter vector which we intend to estimate. This method uses the known points f_{j-1}^i , f_j^i and f_{j+1}^i as well as the points μ_{j+1} , μ_j and σ_{j+1} , σ_j , σ_{j-1} to calculate the subsequent density f_j^{i+1} . Hence, this discretization scheme is a closed form solution for the law of motion of the density function from which f_j^{i+1} can be solved explicitly. A shortcoming of the forward discretization is that one has to consider stability properties (Lux 2006).

In contrast to the explicit method the backward discretization scheme is an implicit method and uses the points f_{j-1}^{i+1} , f_j^{i+1} and f_{j+1}^{i+1} as well as the points μ_{j+1} , μ_j and σ_{j+1} , σ_j , σ_{j-1} to calculate f_j^{i+1} . We get the backward discretization or implicit finite difference scheme by replacing $f_j^{i+1} - f_j^i$ with $f_j^i - f_j^{i-1}$ (see Lux 2009b)

$$\frac{f_j^i - f_j^{i-1}}{k} = \frac{\mu_{j+1}f_{j+1}^i - \mu_j f_j^i}{h} + \frac{1}{2} \frac{\sigma_{j+1}f_{j+1}^i - 2\sigma_j f_j^i + \sigma_{j-1}f_{j-1}^i}{h^2}. \quad (5.39)$$

Leading this equation for one time step yields

$$\frac{f_j^{i+1} - f_j^i}{k} = \frac{\mu_{j+1}f_{j+1}^{i+1} - \mu_j f_j^{i+1}}{h} + \frac{1}{2} \frac{\sigma_{j+1}f_{j+1}^{i+1} - 2\sigma_j f_j^{i+1} + \sigma_{j-1}f_{j-1}^{i+1}}{h^2}. \quad (5.40)$$

This scheme is implicit, since the term f_j^{i+1} appears on both sides of the equation. Hence, we do not have a closed form solution, but have to solve a system of equa-

tions for calculating f_j^{i+1} . Although the approximate solution to the Fokker-Planck equation is more complicated to calculate, the implicit method has the advantage of being globally stable (Lux 2006).

Accuracy of the approximation can be achieved without high computational costs by taking the average of the forward and backward approximation which is called the Crank-Nicolson method (Poulsen 1999, Lux 2009b). This method is an implicit method as can be seen from figure 5.4, since it uses the points f_{j-1}^{i+1} , f_j^{i+1} , f_{j+1}^{i+1} , f_{j-1}^i , f_j^i and f_{j+1}^i as well as the points μ_{j+1} , μ_j and σ_{j+1} , σ_j , σ_{j-1} to calculate f_j^{i+1} in order to approximate the density on artificial intermediate points $(i + 1/2)k$ instead of using the grid itself (Lux 2006).

Finally the fully-implicit finite-difference scheme with Chang-Cooper-correction (Chang and Cooper 1970) guarantees a solution which is non-negative, which is particle conserving in the absence of sources and sinks, and which gives an exact representation of the analytic solution upon equilibration (Banks et al. 2009). This scheme uses the monotonically decreasing correction factor

$$f(\tau) = \frac{1}{\tau} - \frac{1}{\exp\{\tau\} - 1}, \quad (5.41)$$

which has the properties

$$\lim_{\tau \rightarrow -\infty} f(\tau) = 1 \quad \text{and} \quad \lim_{\tau \rightarrow \infty} f(\tau) = 0. \quad (5.42)$$

Following Banks et al. (2009) this scheme continuously shifts from a backward difference to a central difference and to a forward difference

$$f_{j+0.5}^{i+1} = \delta_j^{i+1} f_{j+1}^{i+1} + (1 - \delta_j^{i+1}) f_j^{i+1}. \quad (5.43)$$

with

$$\delta_j^{i+1} = \frac{1}{w_j^{i+1}} - \frac{1}{\exp\{w_j^{i+1}\} - 1} \quad \text{and} \quad w_j^{i+1} = h \frac{\mu_{j+0.5}^{i+1} + \sigma_{j+0.5}^{\prime i+1}}{\sigma_{j+0.5}^{i+1}}. \quad (5.44)$$

A formal derivation of the Crank-Nicolson finite-difference scheme and the Chang-Cooper finite-difference scheme as well as the iteration rule for the approximate Fokker-Plank equation can be found in the appendix.

Because we restrict our analysis to the finite interval $[s_{min}, s_{max}]$ we need boundary conditions for preventing transitions to inaccessible states (see Ait-Salahia 1999 and Lux 2006, 2009b). More precisely, we have to prevent a leakage of probability mass to the points outside the support of the transitional density. Following Lux (2006) these conditions are

$$f_{-\frac{1}{2}}^i = f(-\frac{1}{2}h, jk) = 0 \quad (5.45)$$

and

$$f_{N_x + \frac{1}{2}}^i = f(N_x + \frac{1}{2}h, jk) = 0. \quad (5.46)$$

For the derivation of the density $f(s_{t+1}|s_t; \theta)$ we need s_t as a starting value. Lux (2006) and Poulsen (1999) suggest to approximate the initial condition $(f_0^0, \dots, f_{N_x}^0)'$ by the Euler approximation (see Ait-Salahia 1999), which is a normal distribution with density $\Phi(s_t - \mu(s_t; \theta)k, \sigma(s_t; \theta)k)$ evaluated at the grid points $0, \dots, N_x$. The values for the approximate density $f(s_{t+1}|s_t; \theta)$ can now be calculated by iterating the Fokker-Planck equation up to N_t and are given by the values $f_0^{N_t}, \dots, f_{N_x}^{N_t}$.

Figure 5.4 shows the time evolution of the density calculated by iterating the Fokker-Planck equation. The x -axis of this 3D-plot represents the state space of the exchange rate, while the y -axis represents time. The density values can be inferred

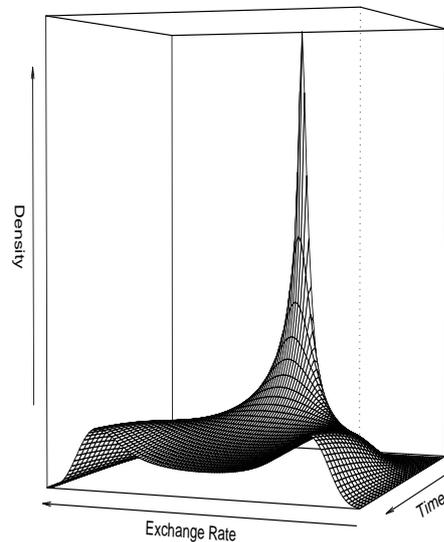


Figure 5.4: Evolution of the Transitional Density Function (a)

NOTES: This figure shows the evolution of the density of the cubic drift model in the time interval $[t, t + 1]$. When we start from a normal distribution, the density evolves to a bimodal distribution.

from the z -axis. As explained in the paragraph before, we start with a very steep normal distribution around our starting value. After some iterations the single mode of our initial condition diminishes and two modes arise. Figure 5.5 shows starting values at the beginning of the time interval $[t, t + 1]$ and the transition density at the end of the time interval. From these figures can be inferred that the density at $t + 1$ is bimodal for starting values around zero at time t , while we get a skewed distribution at time $t + 1$ for starting values away from zero at time t . Hence, when the process is near the repeller at time t , a transition to the attractor at $-\sqrt{\alpha}$ and $+\sqrt{\alpha}$ is equal likely. When the process is in the vicinity of one attractor at time t ,

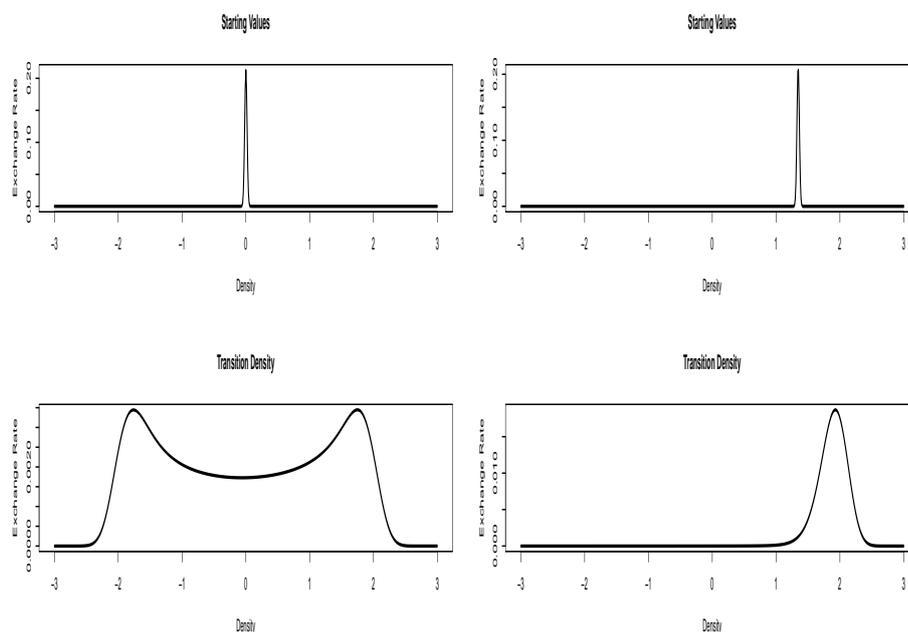


Figure 5.5: Evolution of the Transitional Density Function (b)

NOTES: This figure shows the evolution of the density for the cubic drift model in the time interval $[t, t + 1]$. When we start from a normal distribution with a mean equal to the repeller, the density evolves to a bimodal distribution, making both attractors equal likely. When we start in the vicinity of an attractor, the density becomes skewed, making this attractor more likely compared to the other attractor.

the process will be most likely be in the vicinity of this attractor at time $t + 1$, while the other attractor is less likely.

5.3.2 The Estimation Algorithm

The estimation algorithm suggested by Lux (2006) consists of the following steps

- (i) Given starting values for the parameter vector θ^0 and the discrete observation s_t , evaluate starting values for the density at the grid points for the support of the density $f_0^0, \dots, f_{N_x}^0$ by the Euler approximation $\Phi(s_t - \mu(s_t; \theta^0)k, \sigma(s_t; \theta^0)k)$.

(ii) Calculate the transition density $f(s_{t+1}|s_t; \theta^0)$ by iterating the Fokker-Planck equation until the end of $[t, t + 1]$. Save the values $f_0^{N_t}, \dots, f_{N_x}^{N_t}$.

(iii) Search the corresponding value for the observation s_{t+1} in the grid.

(iv) Extract the approximate value for $f(s_{t+1}|s_t; \theta^0)$ from the values $f_0^{N_t}, \dots, f_{N_x}^{N_t}$.

(v) Calculate the log-likelihood function by applying steps (i) to (vi) to all observations $\{s_t\}_{t=0}^T$.

$$\log \mathcal{L} = \sum_{i=1}^T \log f(s_{t+1}|s_t; \theta^0). \quad (5.47)$$

(vi) Change values of θ and repeat steps (i) to (v) until the maximum of the likelihood is reached.

We call $\theta_{AML} = \operatorname{argmin} \log \mathcal{L}$ the approximate maximum likelihood estimator. As suggested in Bolker (2007, chapter 7), we use the Broyden-Fletcher-Goldfarb-Schanno-algorithm for maximization, which is built in R's `optim` library². This quasi-Newton method for optimization can be use in our case, since the approximate likelihood is smooth and well behaved, which can be inferred by calculating the approximate likelihood over a reasonable parameter space. Monte-Carlo simulations showed that the method yields consistent and unbiased estimation results as also proved in Poulsen (1999).

Poulsen (1999) and Pederson (1999) note, the approximate maximum likelihood estimator follows a normal distribution even in the presence of nonlinear drifts. Hence, asymptotic standard errors of the maximum likelihood estimator are given by the square root of the diagonal elements of the inverse of the Hessian Matrix evaluated at the approximate maximum likelihood estimates.

²See R Development Core Team (2009).

5.4 Data and Estimation Results

The empirical research strategy consists of the following steps

- (i) *Model Identification:* By looking at the histogram and the kernel density estimate of the exchange rate data we determine if a unimodal or a bimodal distribution might fit the data better.
- (ii) *Estimation:* Estimation of the model's parameters via the methodology described before.
- (iii) *Model Evaluation:* We check significance of the model's parameters which determine the shape of the distribution. We calculate the model's stationary points from the estimates and compare them to the modes and the antimodes of the kernel density estimates. The parametric density might be misspecified, while the non-parametric density estimate is robust against misspecification. The model might be misspecified, when modes and antimodes of the kernel density estimate deviate significantly from the estimated attractors and repellers.
- (iv) *Forecasting and Forecast Evaluation:* We compare the predictive accuracy of our models in relation to a naive random walk prediction. We calculate two predictors: the predicted mean of the distribution and the predicted mode. The mean might evolve more sluggish, while the mode might switch between attractors. Moreover, we calculate the 95% confidence interval from the numerically derived densities, by extracting the 2.5%- and 97.5%-quantiles. We test on equal predictive accuracy by means of the tests proposed by Diebold and Mariano (1995), Giacomini and White (2006) and Mitchell and Hall (2005),

which we explain later on.

5.4.1 Exchange Rate Data

Time series of real exchange rates versus the European Area are taken from the European Central Bank's database. The dataset comprises 14 monthly real exchange rates (real harmonized competitive indicators, CPI deflated) covering the Austrian Shilling, the Belgian Franc, the Cyprus Pound, the German Mark, the Spanish Peseta, the Finnish Markka, the French Franc, the Greek Drachma, the Irish Pound, the Luxembourg Franc, the Netherland Guilder, the Portuguese Escudo, the Slovene Tolar and the Slovak Koruna. All real exchange rate data series are given at a monthly frequency and span the period 1993:01 to 2009:02. We use the subsample 1993:01 to 2003:12 for estimation, while we use the remaining observations from 2004:01 to 2009:02 for the evaluation of the out-of-sample forecasting performance of our model. Hence, our out-of-sample period comprises 5 years, since we lose one observation when calculating $f(s_{t+1}|s_t)$. Following Creedy et al. (1996) and Aït-Sahalia (1999), we standardize the data series by subtracting the unconditional mean and dividing by the unconditional standard deviation. We can compare the estimates of the attractors and repellers across different real exchange rates, since all of them are measured in standard units now. Time series plots and histograms of the data series are given in figures 5.6 to 5.16 and will be discussed in section 4.3.

We experimented with different grid sizes. We came to the result that the state space of the grid has to be much larger compared to the minimum and the maximum value of the data. Otherwise we ran into the trap of truncating the density, when a data point lies near to a boundary of our grid, which leads to upward biases of

the transition densities at the lower boundary and downward biases at the upper boundary. A grid spanning $[-6,+6]$ leaves reasonable space for avoiding these problems at the boundaries. We also experimented with different grid sizes like $h = 0.05$, $h = 0.02$ and $h = 0.01$. Note, that we apply the grid to the standardized exchange rates. In case of the Austrian Shilling which has a standard deviation of 0.035 a grid size $h = 0.05$ applied to the standardized series means actually a grid of 0.0018 applied to the original data series, while a grid size $h = 0.01$ means actually a grid of 0.0004 applied to the original data. A grid size of $h = 0.05$ implies 241 grid points on our grid spanning $[-6,+6]$, while a grid size $h = 0.01$ implies 1201 grid points on that grid. We found comparable coefficient estimates for $h = 0.05$ and $h = 0.01$. However, reducing the grid to the interval $[-4,+4]$ leads to truncated transition densities and thereby to biased estimates.

5.4.2 Parameter Estimates

Table 5.1 contains parameter estimates of the model $ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t$. The estimates of the bifurcation parameter α are positive and significantly different from zero for all 14 real exchange rates indicating a bimodal long-run exchange rate distribution over the estimation period 1993:01 to 2009:02 (panel a) as well as for the estimation period 1993:01 to 2003:12 (panel b). The estimates of the diffusion parameter are positive and significantly different from zero as they should.

Table 5.2 and 5.3 contain estimation results for the less restrictive cubic model $ds_t = (x_L - s_t)(x_M - s_t)(x_H - s_t)dt + \sigma dW_t$ for the period 1993:01 to 2009:02 (table 5.2) and for 1993:01 to 2003:12 (table 5.3). The estimates of the model indicate that one stationary point is negative and significantly different from zero, while one stationary

point is positive and significantly different from zero for all 14 real exchange rates and for both estimation periods. Hence, there is evidence from bimodality in real exchange rates. We find that the repellor located between the two stationary points is not significantly different from zero for some exchange rates. However, this result does not indicate that we have to reject bimodality, since it only indicates that the repellor is located at zero or more precisely, we cannot reject the repellor to lie at zero for some real exchange rates. Estimates of the diffusion parameter are positive and significantly different from zero for all 14 time series. Moreover, all estimates lie in the range of estimates of the more restrictive double well potential model. However, the algorithm did not converge for the Greek Drachma, the Slovene Tolar and the Slovak Koruna, but remained in a local maximum of the likelihood, which we inferred from the lower likelihood values compared to the less restrictive model. The non-convergence might be caused by the persistent trends in these real exchange rates.

5.5 Phase Transitions and Uncertainty about the Equilibrium

Figure 5.6 to 5.16 contain the results of our forecasting exercise for 12 example exchange rates. The figure in the upper left shows the exchange rate time series together with the three estimated equilibria highlighted as broken horizontal lines.

The upper right contains a histogram of the time series together with a kernel density estimate of the unconditional exchange rate distribution. The kernel density estimate is a non-parametric estimate, which can be interpreted as a smoothed his-

Table 5.1: Parameter Estimates: Model 1

Model 1: $ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t$

| Real Exchange Rate | (a) 1993:01 - 2009:02 | | | (b) 1993:01 - 2003:12 | | |
|-------------------------------------|-----------------------|--------------------|---------|-----------------------|-------------------|---------|
| | $\hat{\alpha}$ | $\hat{\sigma}$ | Log-L | $\hat{\alpha}$ | $\hat{\sigma}$ | Log-L |
| Austrian Shilling (t-statistic) | 2.187** (20.86) | 0.271** (2.58) | -751.79 | 2.525** (21.42) | 0.330** (8.24) | -494.87 |
| Belgian Franc (t-statistic) | 2.062** (22.49) | 0.232** (9.60) | -720.37 | 2.288** (20.56) | 0.273** (8.55) | -487.27 |
| Cyprus Pound (t-statistic) | 1.535** (18.83) | 0.148** (8.13) | -657.14 | 0.940** (48.89) | 0.063** (6.53) | -408.01 |
| German Mark (t-statistic) | 2.213** (22.43) | 0.232** (8.47) | -733.20 | 2.472** (24.19) | 0.269** (8.52) | -477.57 |
| Spanish Peseta (t-statistic) | 1.688** (19.63) | 0.189** (9.47) | -684.75 | 1.064** (33.54) | 0.061** (7.30) | -400.90 |
| Finnish Markka (t-statistic) | 2.178** (9.26) | 0.999** (6.18) | -837.38 | 3.052** (13.15) | 0.951** (6.72) | -572.50 |
| French Franc (t-statistic) | 2.241** (16.70) | 0.408** (8.12) | -780.56 | 2.725** (17.83) | 0.425** (8.21) | -528.83 |
| Greek Drachma (t-statistic) | 2.004** (20.96) | 0.248** (9.81) | -722.82 | 1.620** (14.32) | 0.182** (6.16) | -485.78 |
| Irish Pound (t-statistic) | 1.728** (19.10) | 0.203** (8.61) | -696.51 | 1.046** (44.28) | 0.064** (6.37) | -414.60 |
| Luxembourg Franc (t-statistic) | 1.798** (18.85) | 0.190** (6.16) | -703.34 | 1.343** (24.02) | 0.093** (7.69) | -431.23 |
| Netherland Guilder (t-statistic) | 1.354** (48.32) | 0.090** (7.50) | -712.01 | 2.038** (12.41) | 0.385** (5.33) | -525.70 |
| Portogese Escudo (t-statistic) | 1.597** (26.43) | 0.112** (10.22) | -618.76 | 1.225** (27.81) | 0.086** (8.00) | -420.22 |
| Slovene Tolar (t-statistic) | 1.497** (12.50) | 0.202** (5.56) | -819.27 | 1.432** (2.29) | 1.913** (3.04) | -576.95 |
| Slovak Koruna (t-statistic) | 1.750** (18.79) | 0.225** (8.60) | -708.25 | 0.900** (102.71) | 0.022** (6.34) | -342.64 |

Notes: */** denotes significance of the coefficient estimate at the 5%/1% level. Standard errors are calculated from the inverse of the Hessian matrix evaluated at the point estimates. Real exchange rate data is taken from the ECB's database (www.ecb.int).

Table 5.2: Parameter Estimates: Model 2 (a)

$$\text{Model 2: } ds_t = (x_L - s_t)(x_M - s_t)(x_L - s_t)dt + \sigma dW_t$$

| Country | 1993:01 - 2009:02 | | | | |
|--------------------------------------|----------------------|---------------------|--------------------|--------------------|---------|
| | x_L | x_M | x_H | σ | Log-L |
| Austrian Shilling (t-statistic) | -1.306** (-25.54) | 0.115 (1.62) | 1.643** (30.89) | 0.281** (8.88) | -741.97 |
| Belgian Franc (t-statistic) | -1.593** (-35.38) | -0.122 (-1.94) | 1.263** (27.85) | 0.211** (9.48) | -707.48 |
| Cyprus Pound (t-statistic) | -1.013** (-35.43) | 0.244** (3.26) | 1.448** (36.98) | 0.131** (9.88) | -628.52 |
| German Mark (t-statistic) | -1.440** (-29.86) | 0.025 (0.40) | 1.562** (34.13) | 0.235** (8.80) | -730.37 |
| Spanish Peseta (t-statistic) | -1.027** (-35.37) | 0.286** (5.33) | 1.589** (42.69) | 0.134** (10.17) | -635.88 |
| Finnish Markka (t-statistic) | -1.147** (19.81) | 0.551** (4.42) | 2.074** (21.10) | 0.612** (7.25) | -803.00 |
| French Franc (t-statistic) | -1.941** (-41.67) | -0.365** (-5.30) | 0.973** (28.69) | 0.200** (8.91) | -681.72 |
| Greek Drachma (t-statistic) | -1.348** (-29.34) | 0.051** (0.79) | 1.488** (29.54) | 0.242** (9.64) | -720.92 |
| Irish Pound (t-statistic) | -1.036** (-30.25) | 0.257** (4.30) | 1.587** (31.08) | 0.169** (10.13) | -661.30 |
| Luxembourg Franc (t-statistic) | -1.179** (-34.75) | 0.206** (3.46) | 1.571** (31.84) | 0.183** (9.71) | -683.79 |
| Netherlands Guilder (t-statistic) | -1.742** (-28.64) | -0.457** (-6.41) | 1.028** (33.45) | 0.191** (9.24) | -672.88 |
| Portuguese Escudo (t-statistic) | — — | — — | — — | — — | — |
| Slovene Tolar (t-statistic) | -2.160** (-23.81) | -0.674** (-6.37) | 1.063** (25.40) | 0.444** (8.34) | -763.75 |
| Slovak Koruna (t-statistic) | -1.165** (-36.92) | -0.074** (-1.49) | 1.316** (61.24) | 0.103** (9.87) | -611.63 |

Notes: */** denotes significance of the coefficient estimate at the 5%/1% level. Standard errors are calculated from the inverse of the Hessian matrix evaluated at the point estimates. Real exchange rate data is taken from the ECB's database (www.ecb.int).

Table 5.3: Parameter Estimates: Model 2 (b)

$$ds_t = (x_L - s_t)(x_M - s_t)(x_L - s_t)dt + \sigma dW_t$$

| Country | 1993:01 - 2003:12 | | | | Log-L |
|--------------------------------------|----------------------|----------------------|--------------------|-------------------|---------|
| | x_L | x_M | x_H | σ | |
| Austrian Shilling (t-statistic) | -1.441** (-28.70) | 0.159 (1.89) | 1.717** (31.65) | 0.311** (8.80) | -488.29 |
| Belgian Franc (t-statistic) | -1.628** (-33.38) | -0.145 (-1.90) | 1.319** (24.37) | 0.238** (8.42) | -479.39 |
| Cyprus Pound (t-statistic) | -1.056** (-55.45) | -0.310** (-7.66) | 0.616** (31.90) | 0.048** (7.23) | -393.52 |
| German Mark (t-statistic) | -1.506** (-32.05) | 0.075 (0.95) | 1.631** (34.35) | 0.264** (8.63) | -475.83 |
| Spanish Peseta (t-statistic) | -1.111** (-31.12) | -0.381** (-5.71) | 0.364** (5.91) | 0.025** (7.01) | -381.50 |
| Finnish Markka (t-statistic) | -1.232** (-22.63) | 0.449** (3.78) | 2.265** (26.37) | 0.595** (7.53) | -538.37 |
| French Franc (t-statistic) | -1.965** (-41.22) | -0.393** (-5.30) | 1.072** (24.82) | 0.219** (8.39) | -466.13 |
| Greek Drachma (t-statistic) | — — | — — | — — | — — | — |
| Irish Pound (t-statistic) | -1.177** (-48.06) | -0.414** (-11.24) | 0.675** (14.32) | 0.032** (6.97) | -384.19 |
| Luxembourg Franc (t-statistic) | -1.321** (-34.75) | 0.341** (3.46) | 0.433** (31.84) | 0.034** (9.71) | -370.23 |
| Netherlands Guilder (t-statistic) | -1.821** (-24.01) | -0.639** (-7.28) | 0.683** (13.00) | 0.167** (7.78) | -465.71 |
| Portuguese Escudo (t-statistic) | -1.253** (-53.38) | -0.340** (-13.00) | 0.748** (78.28) | 0.043** (7.65) | -392.72 |
| Slovene Tolar (t-statistic) | — — | — — | — — | — — | — |
| Slovak Koruna (t-statistic) | — — | — — | — — | — — | — |

Notes: */** denotes significance of the coefficient estimate at the 5%/1% level. Standard errors are calculated from the inverse of the Hessian matrix evaluated at the point estimates. Real exchange rate data is taken from the ECB's database (www.ecb.int).

togram. We plot the estimated equilibria into the histogram in order to compare them to the non-parametric estimate of the modes of the exchange rate distribution. This plot might indicate possible misspecifications of our model, since the non-parametric density estimate is robust against misspecification. We will present a test on misspecification in section 6.

The figure in the lower left shows the one-month-ahead forecast of the exchange rate distribution calculated by iterating the Fokker-Planck equation of the estimated differential equation. Probabilities are given in a temperature plot. Light colors represent regions with high probability, while dark colors represent regions with low probability. Moreover, light colors indicate that the distribution is very steep indicating that the exchange rate will only fluctuate in a narrow region around its actual value, while light blue indicates that the distribution is narrow indicating that a large region of the state space is covered by probability mass. Hence, larger fluctuations of the exchange rate are predicted by light blue regions. The estimated equilibria are given as broken horizontal lines.

The figure in the lower right contains the actual time series highlighted through circles, together with the predicted average exchange rate highlighted as a solid line as well as a forecast of the mode of the exchange rate distribution highlighted as dots. The average exchange rate and the mode need not necessarily be equal in nonlinear models. When the mode lies above the expected value, then values above the average are more likely, which indicates an upward trend. Conversely, a downward trend is more likely, when the mode of the distribution lies below the expected value. However, when the exchange rate distribution becomes bimodal, the expected value has less probability compared to the two modes indicating that there

is uncertainty about the equilibrium exchange rate. Confidence bands are calculated by extracting the 2.5- and the 97.5-percentiles of the density. The upper and the lower confidence bounds are given as broken lines.

Austrian Shilling:

Figure 5.6 contains our results for the Austrian Shilling. We identify three phases in the exchange rate by just eyeballing the time series (upper left subfigure). In the first phase around 1995 we find the exchange rate to fluctuate in the vicinity of the attractor x_H . Approximately when Sweden has entered EMU in 1995 a second period starts which is characterized by an downward trend and a phase transition to the lower attractor. The exchange rate reaches the attractor at x_L in 1999 when the Exchange Rate Mechanism II applies. After the introduction of the Euro in 2002 the exchange rate fluctuates in the vicinity of x_M .

The histogram as well indicates the existence of three modes, while two of them are located in the vicinity of the estimates of the attractors x_H and x_L . The first mode is due to the fluctuations of the Shilling around the upper attractor, while the lower mode is caused by the fluctuations around the lower one. Moreover, there seems to be a mode below zero. This one is due to exchange rate fluctuations around zero after the introduction of the Euro. When we split the sample into the period before the introduction of the Euro and a period after the introduction of the Euro, we find evidence for bimodality before 2002 and evidence for unimodality after 2002.

The temperature plot of the predicted density values indicates a steep exchange rate distribution around x_H before Sweden has entered the EMU in 1995. Hence, the model does not predict large exchange rate movements during this time period. Around 1995, the distribution seems to cover the whole interval $[x_L, x_H]$ between the

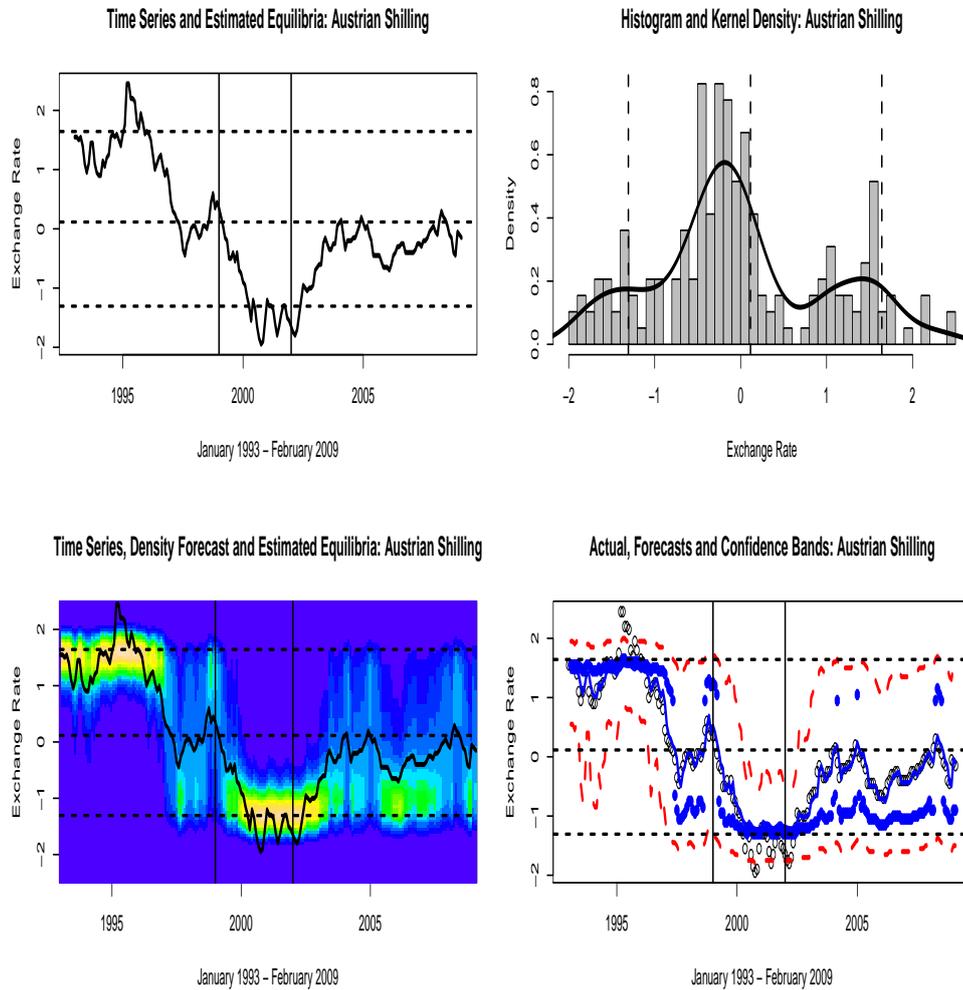


Figure 5.6: One-Step-Ahead Forecast: Austrian Shilling

NOTES: Upper left: The exchange rate (solid line) and its three equilibria (broken lines). Solid horizontal lines indicate the beginning of the Exchange Rate Mechanism II in 1999 and the introduction of the Euro in 2002. Upper right: Histogram, kernel density estimate and estimated equilibria of the exchange rate (broken lines). Lower left: Density forecast of the exchange rate. Light colors indicate high probability values, while dark colors indicate low probability values. Lower right: Actual exchange rate (circle), mean forecast (solid line) and mode forecast (dots). The 95% confidence interval is given by the broken lines. Equilibria are given by the horizontal broken lines.

attractors as indicated by the light gray region. Hence, the model predicts uncertainty about the equilibrium exchange rate here. The model predicts the exchange

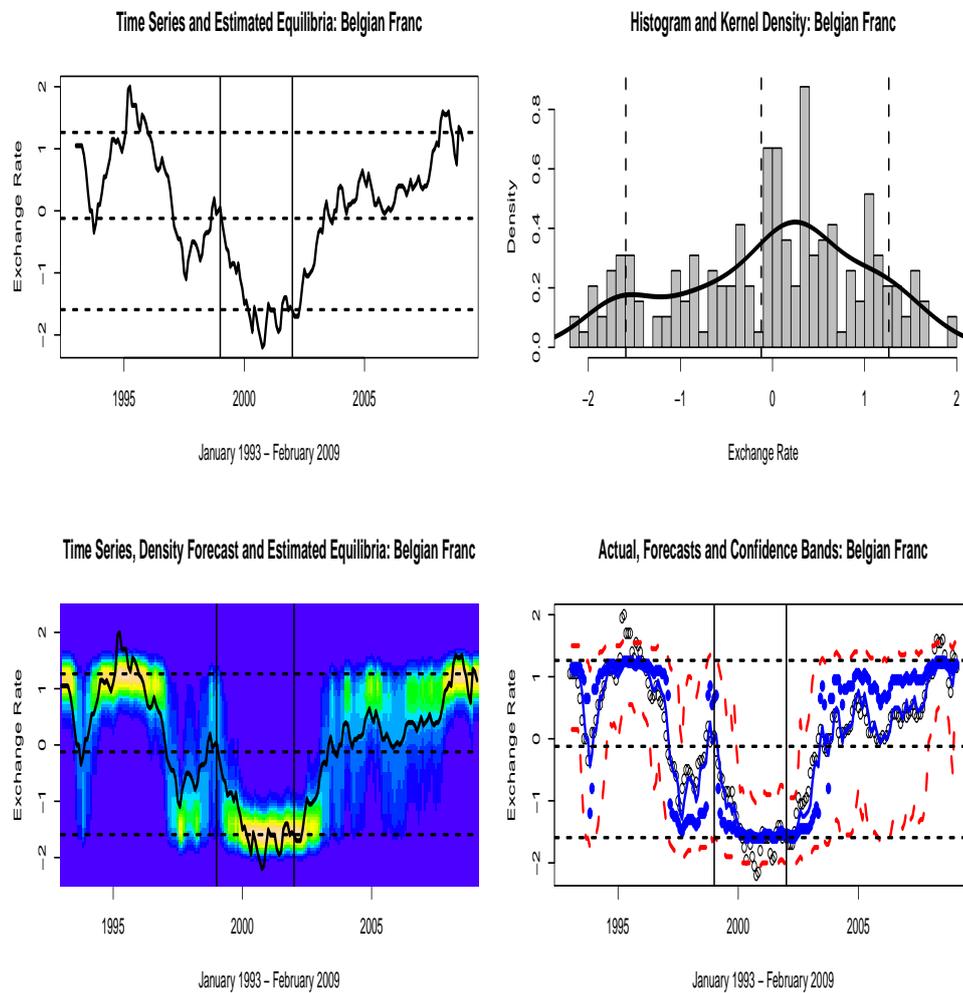


Figure 5.7: One-Step-Ahead Forecast: Belgian Franc

NOTES: See notes of figure 5.6.

rate to be in the vicinity of this attractor until the introduction of the Euro as indicated by the steep distribution indicated by yellow colors. The exchange rate distribution becomes narrow between the introduction of the Euro and today as indicated by the light blue color of the temperature plot. Again, the probability mass covers the whole area $[x_L, x_H]$.

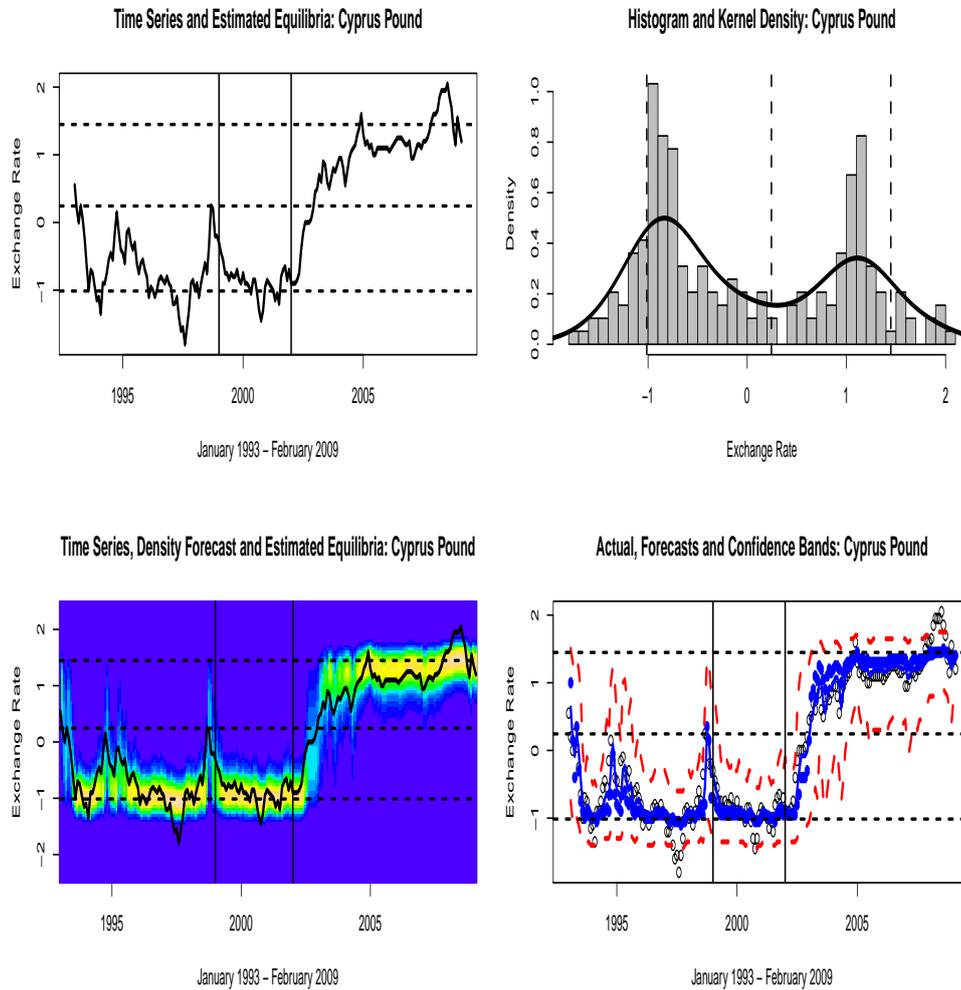


Figure 5.8: One-Step-Ahead Forecast: Cyprus Pound

NOTES: See notes of figure 5.6.

As indicated by the lower right figure, the model predicts switches between both attractors after the Euro as indicated by the modes, while the predicted average tracks the data quite well. This result might indicate that the exchange rate distribution has become unimodal after the introduction of the Euro, which indicates some kind of stabilization. Although we find that the predicted average exchange rate seems to track the data quite well, we find that in periods when the exchange

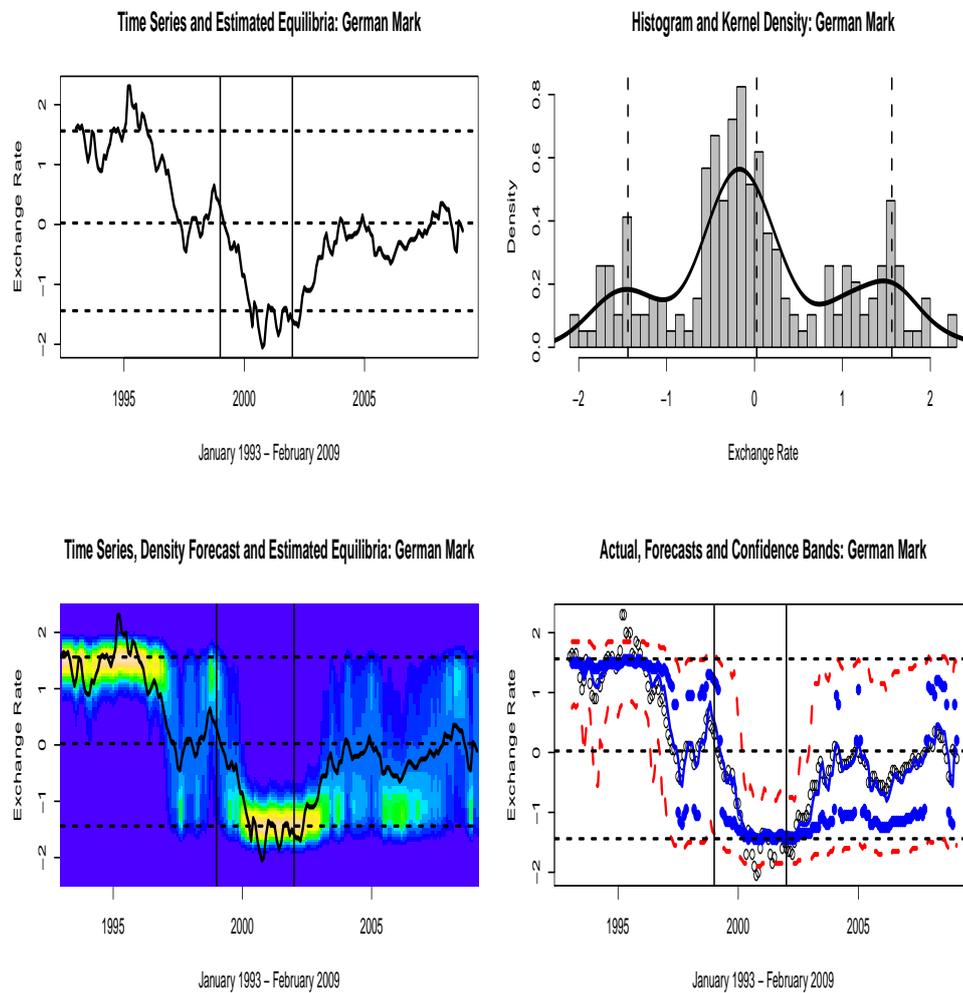


Figure 5.9: One-Step-Ahead Forecast: German Mark

NOTES: See notes of figure 5.6.

rate rises above the upper attractor or falls below the lower one, the model loses its forecasting power, since it predicts the exchange rate to be near the attractors. All in all, the cubic drift model seems to be a good representation of the data.

Belgian Franc:

Figure 5.7 contains our results for the Belgian Franc. The Franc is characterized

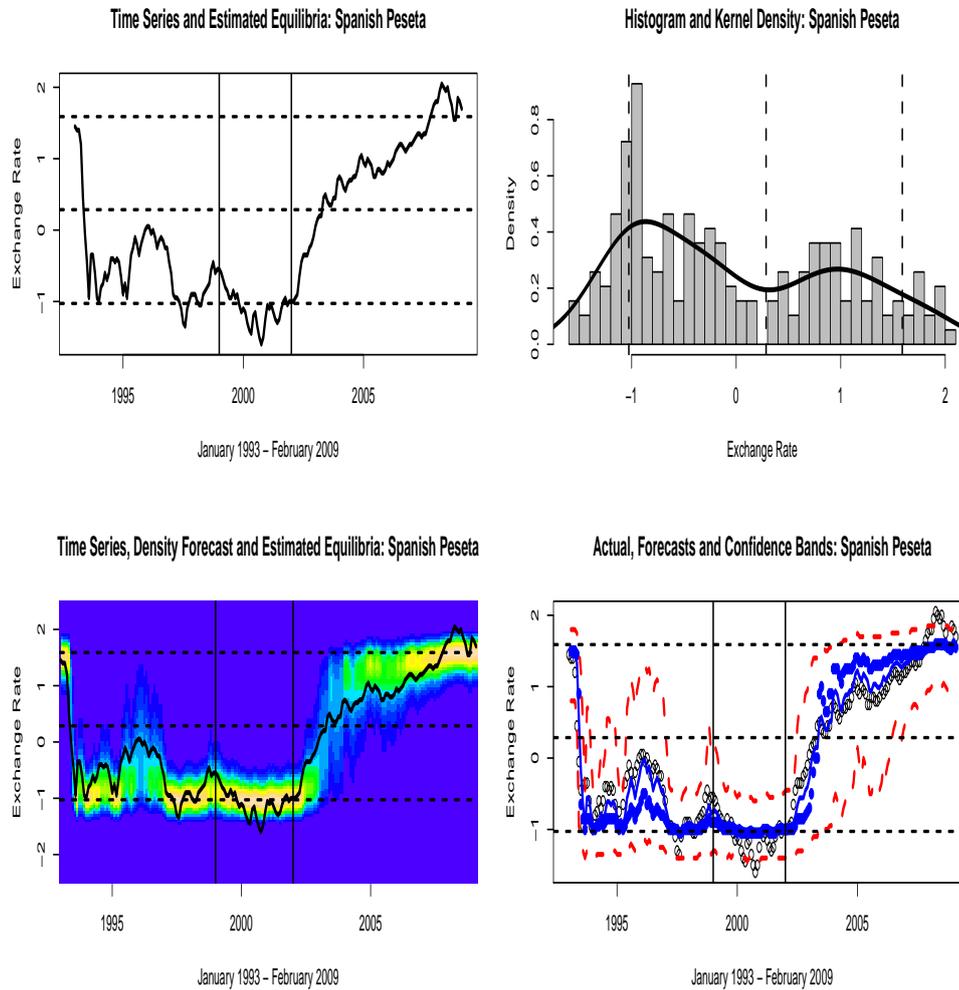


Figure 5.10: One-Step-Ahead Forecast: Spanish Peseta

NOTES: See notes of figure 5.6.

by three large swings. The first one is an upward trend to the attractor x_H which endures until 1995. There is a phase transition from the higher attractor to the lower attractor x_L which spans the period from 1995 to 2002. After 2002 we infer a phase transition back to the attractor x_H .

The kernel density estimate provides evidence for bimodality. The estimate of the

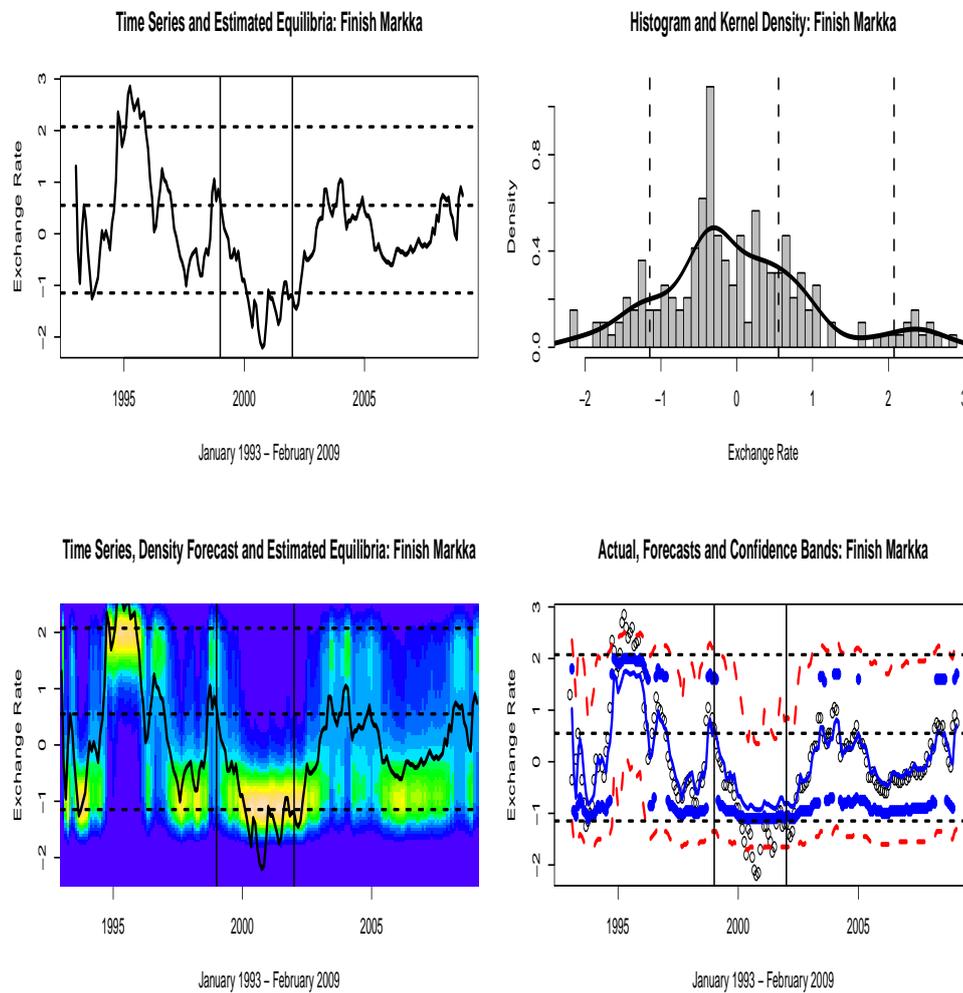


Figure 5.11: One-Step-Ahead Forecast: Finnish Markka

NOTES: See notes of figure 5.6.

lower attractor corresponds to the lower mode of the density, while the second mode does not correspond to the higher attractor. It lies nearer to the repellor located at -0.122 . However, the estimate of the mode located at 1.263 corresponds to a spike of the histogram. Nevertheless, we conclude that the cubic drift model is a good representation of the data.

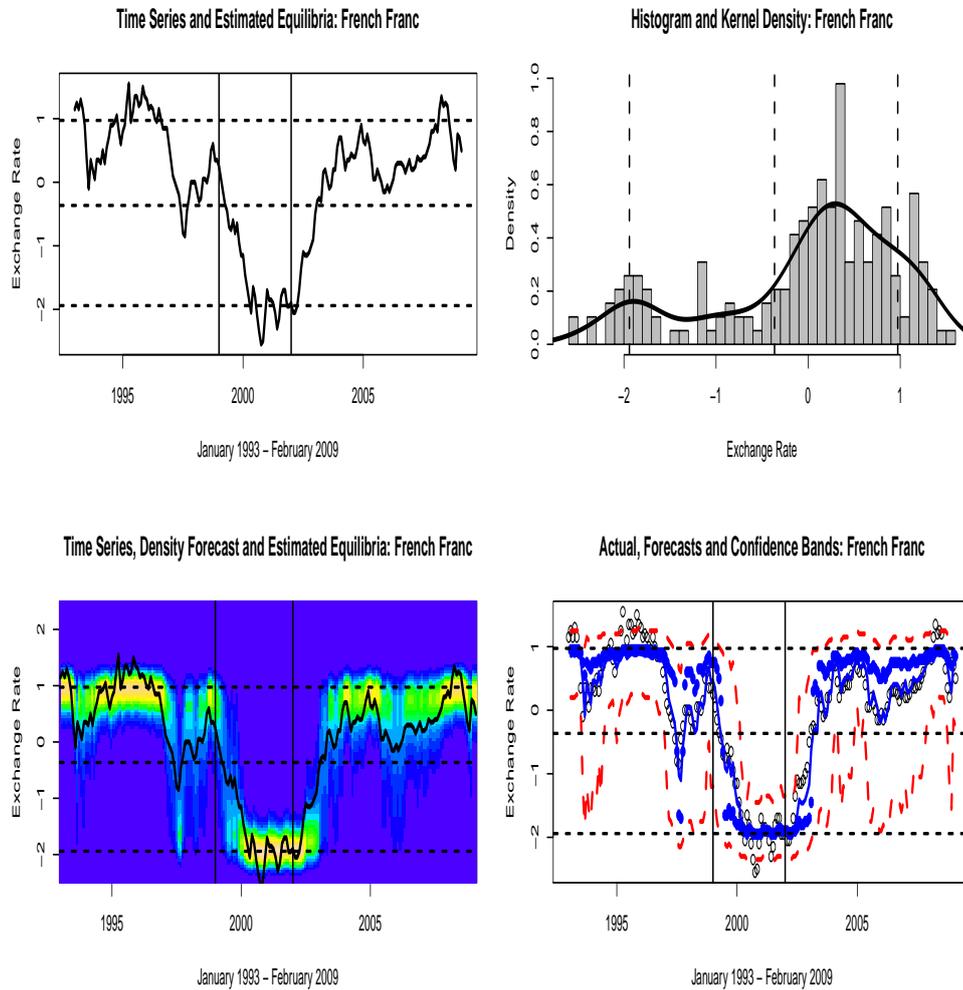


Figure 5.12: One-Step-Ahead Forecast: French Franc

NOTES: See notes of figure 5.6.

The temperature plot of the density predicts periods of uncertainty around 1993, 1997, 1999 and after 2002. However, density values are high, when the exchange rate is near the attractors. The large uncertainty around the exchange rate might be due to large fluctuations and short-run trends in the Belgian Franc.

The mean predictor tracks the data quite well, while the model predictors switch

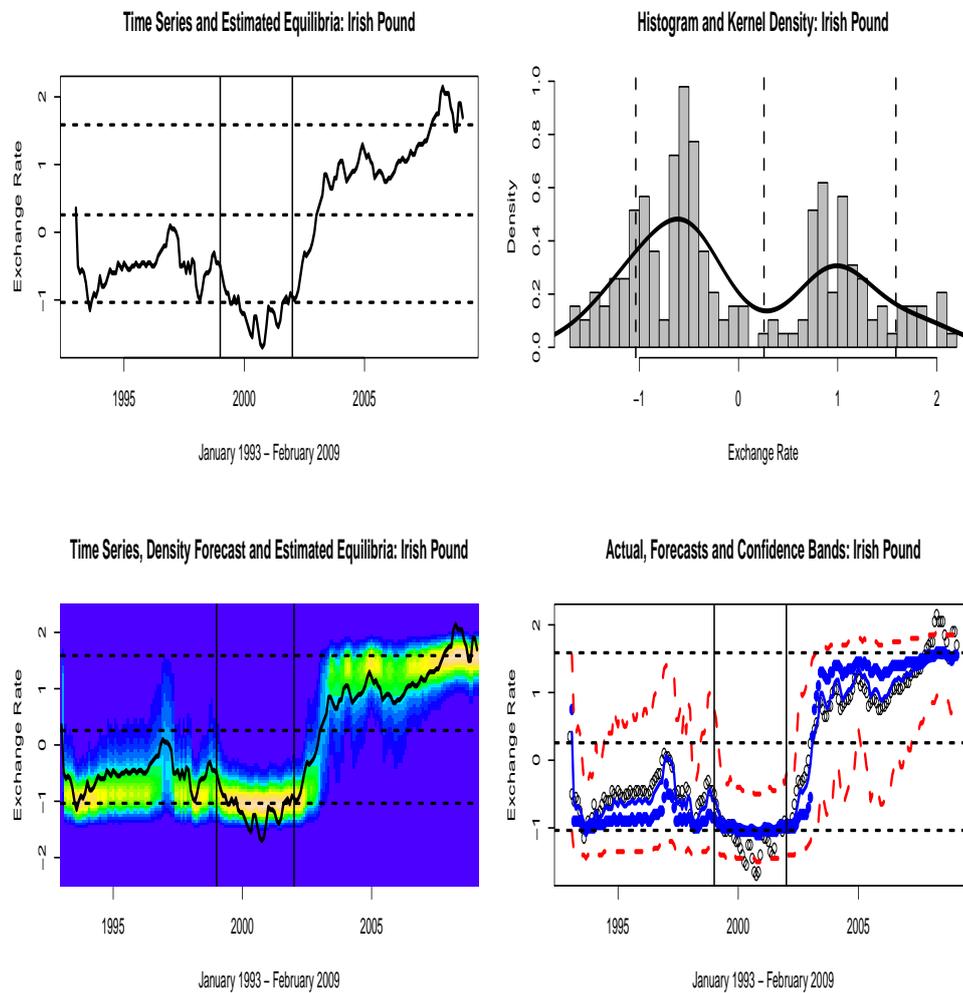


Figure 5.13: One-Step-Ahead Forecast: Irish Pound

NOTES: See notes of figure 5.6.

often between attractors but predict trends and phase transitions quite well. There are some data points which lie outside the confidence bounds when the exchange rate crosses the attractors.

Cyprus Pound:

Figure 5.8 contains our results for the Cyprus Pound. The kernel density plot

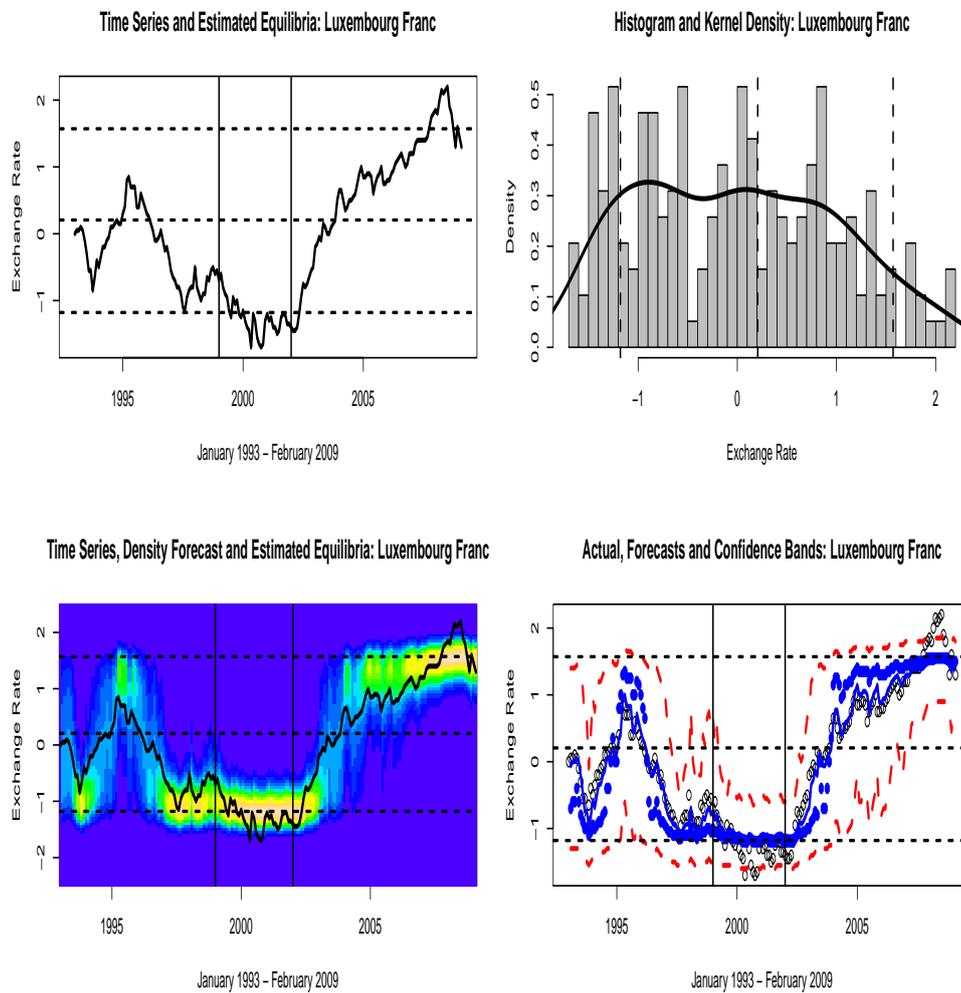


Figure 5.14: One-Step-Ahead Forecast: Luxembourg Franc

NOTES: See notes of figure 5.6.

provides evidence for a bimodal exchange rate distribution here. The estimates of the attractors lie in the vicinity of the estimated modes, while the estimate of the repeller corresponds to the estimate of the antimode. We conclude that the cubic drift model is a good representation of the data.

The Cyprus Pound is characterized by two phases. The first one reaches until the

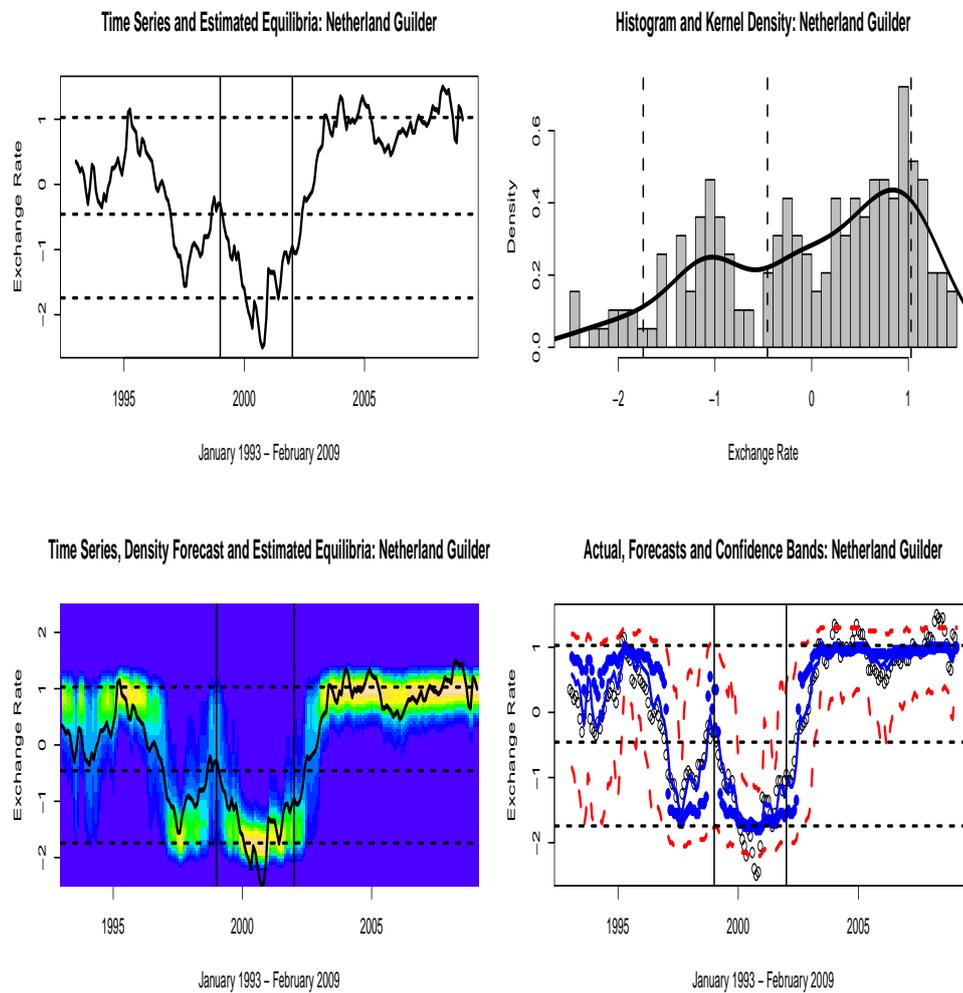


Figure 5.15: One-Step-Ahead Forecast: Netherland Guilder

NOTES: See notes of figure 5.6.

introduction of the Euro, where the Cyprus Pound fluctuates erratically around the lower attractor. After the introduction of the Euro a phase transition occurs in which the real exchange rate is attracted by the attractor located at 1.288. The model predicts this phase transition quite well as can be inferred from the small confidence intervals, while the predicted modes indicate a regime shift.

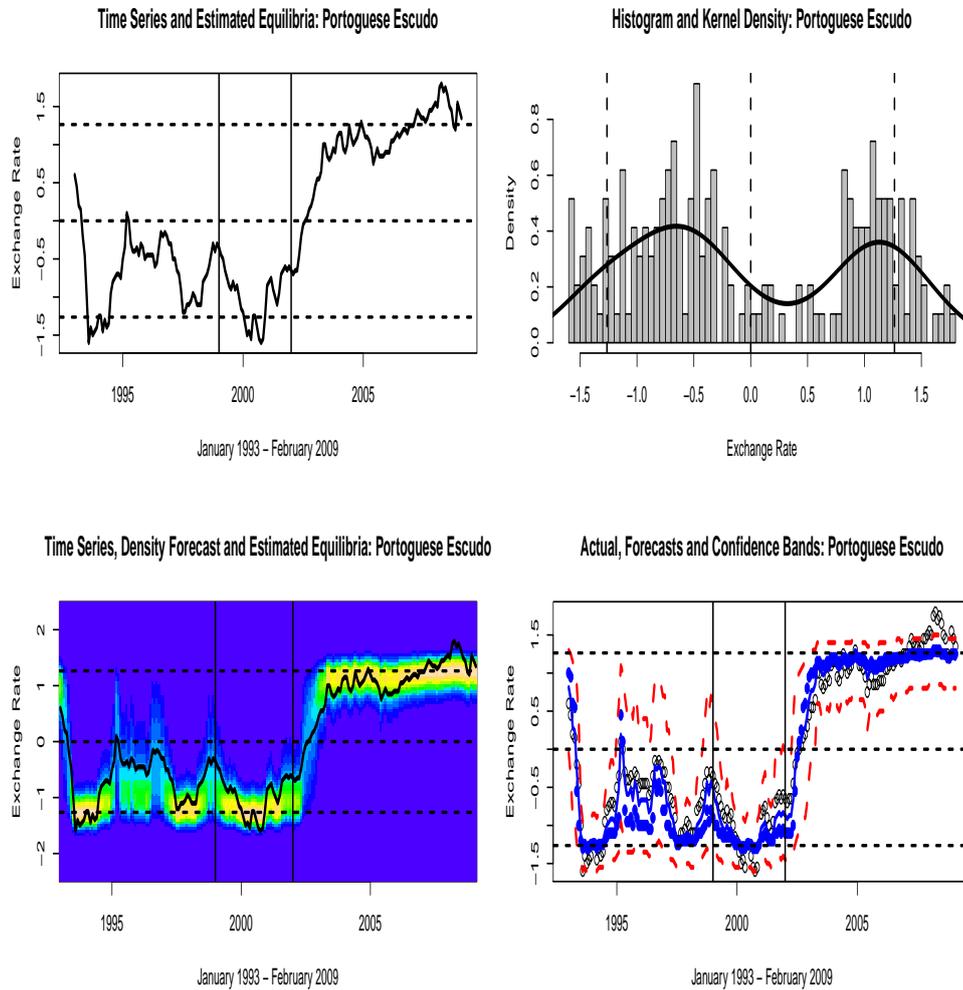


Figure 5.16: One-Step-Ahead Forecast: Portuguese Escudo

NOTES: See notes of figure 5.6.

German Mark:

The kernel density estimate of the German Mark real exchange rate indicates three modes (Figure 5.9). When we split the sample into a pre-Euro period and a post-Euro period we get the result that the histogram is bimodal for the period before the introduction of the Euro, while it is unimodal after the introduction of the

Euro. However, the density plot indicates that our estimates of the two attractors correspond to the modes of the exchange rate distribution.

The time series plot indicates that the exchange rate fluctuates in the vicinity of the attractor x_H in the period before 1995, while a phase transition to the lower attractor located at x_L occurs thereafter. This downward trend endures until the introduction of the Euro in 2002. After that the German Mark seems to be attracted by a state near zero, which can also be seen from the histogram. The results indicate that bimodality might be evident in the period before the introduction of the Euro, while unimodality might be a better representation of the German Mark after the introduction of the Euro.

The temperature plot of the density predicts the exchange rate to be in the vicinity of x_H in the period around 1995. After that a period of uncertainty occurs during which both attractors are covered with probability mass. During this period of uncertainty a phase transition from the attractor x_H to the attractor x_L occurs.

The lower right figure indicates this uncertainty as well. Although the predicted average exchange rate tracks the data quite well, the mode of the exchange rate distribution switches between both attractors indicating uncertainty about the equilibrium exchange rate. After the introduction of the Euro we find the same phenomenon that the modes switch between both attractors. These phase transitions might be due to a misspecification of the model due to a structural break in 2002, since the histogram is unimodal after the introduction of the Euro.

Spanish Peseta:

The histogram of the Spanish Peseta looks bimodal (Figure 5.10), where the lower

mode in the vicinity of -1 is visited more frequently compared to the mode located at around +1. The estimates of the model indicate attractors located at -1.027 and +1.589. The cubic drift model seems to be a good representation of the data.

When we look at the time series and the estimated attractors we find a sharp phase transition from x_H to x_L after 1993 and a phase transition from x_L to x_H after the introduction of the Euro. The Spanish Peseta fluctuates in the vicinity of x_L in the period between 1993 and 2002. There is less uncertainty concerned with this exchange rate, since the 95% confidence interval contains only one attractor at the same time. Hence, phase transitions are unlikely. However, there is a time period of uncertainty in 1995 as found in other exchange rates as well, where the confidence interval become more narrow. However, the mode prediction does not indicate a phase transition.

The figure in the lower right indicates that the predicted average exchange rate tracks the data quite well inside the zone of instability, while the predicted modes indicate phase transitions correctly and do not indicate false alarms. However, the model loses its forecasting power in 2007, when the exchange rate crosses the attractor x_H . The diminishing forecasting power might be caused by events of the current financial crisis.

Finnish Markka:

By looking at the time series of the Finish Markka and the histogram it is not easy to judge if a unimodal distribution or a bimodal distribution fits the data better (figure 5.11). However, the kernel density estimate indicates two small modes in the vicinity of the estimates of the attractors. This uncertainty is due to the fact that the Finnish Markka is characterized by erratic fluctuations and several short-term

trends. The uncertainty about possible attractors are as well manifested in the two lower plots which indicate large uncertainty concerned with this exchange rate. Here, both attractors are covered with probability mass for several periods. Moreover, the mode of the exchange rate distribution switches between the two attractors.

When we split the sample into the time before 2002 and the time after the introduction of the Euro, we find evidence for bimodality in the first time period, while a unimodal distribution seems to fit better in the second one as indicated by the kernel density estimate.

French Franc:

A bimodal distribution seems to characterized the behavior of the French Franc quite well as indicated by the kernel density estimate (figure 5.12). The estimate of the lower attractor fits the lower mode, while the estimate of the second attractor is higher compared to the second mode. Moreover, the estimate of the repeller is larger compared to the antimode.

The behavior of the Franc is characterized by a long trend from the upper attractor located at x_H to the lower attractor located at x_L . There is a period of uncertainty around 1998, when both attractors are covered with probability mass. However, a stabilization occurs in the period between ERM II and the introduction of the Euro, during which period only one attractor is covered with probability mass. However, there is a period of uncertainty in 2006, while the modes predict the exchange rate to be near x_H .

The predicted modes indicate phase transitions quite well. Modes are near x_H in the period before ERM II, while they lie near x_L in the period between 1999 and 2002.

After 2002 the modes predict a phase transition to x_H , in whose vicinity they lie up to now. Although the predicted modes track the data not so well compared to the predicted mean, they contain important information about regime shifts. Therefore, we regard the bimodal model as validated for the French Franc.

Irish Pound:

The kernel density estimate and the histogram indicate that the Irish Pound is characterized by a bimodal distribution (figure 5.13). The estimate of the repeller fits the antimode quite well, while the estimates of the attractors are larger in absolute value compared to the modes of the distribution.

The Pound fluctuates quite erratically in the period before ERM II, is attracted by x_L in the period between ERM II and 2002, while a phase transition to x_H occurs after the introduction of the Euro. The density forecasts indicate certainty about the equilibria, since only one attractor is covered with probability mass for most of the time. There is just one period with a larger confidence region in 1997.

The predicted means track the data quite well, while the predicted modes indicate the phase transition after the introduction of the Euro quite well. Moreover, the predicted modes do not give false alarms.

Luxembourg Franc:

The Luxembourg Franc is characterized by short-term trends in the period before ERM II, while it is attracted by the stable state x_L after ERM II and attracted by the stable state x_H after the introduction of the Euro (figure 5.14). This phase transition has more the character of a longer term trend than that of a phase transition. There is no clear-cut picture of bimodality in the plot of the kernel density estimate due

to the longer term trend and the short-term fluctuations. While there is one mode in the vicinity of the estimate of the lower attractor, there is no equivalent to the repeller and the second attractor in the estimate of the exchange rate distribution.

Although the model seems to be not well specified, the predicted means track the data quite well, while the predicted modes indicate medium-run trends. However, there is larger uncertainty as indicated by the larger confidence region.

Netherland Guilder:

The Netherland Guilder seems to have a bimodal distribution as indicated by the kernel density estimate and the histogram (figure 5.15). The positive attractor lies in the range of the positive mode, while the estimate of the repeller lies in the range of the antimode. However, the estimate of the negative attractor under predicts the negative mode.

Similar to other exchange rates there is a period of erratic fluctuations before ERM II, while the guilder is attracted by the stable state x_L during ERM II and is attracted by x_H after the introduction of the Euro. The density forecast indicates uncertainty in 1994 as well as 1997 and around 1999. However, after ERM II only one attractor is covered by probability mass at the same time. The mean predictor seems to track the data quite well, while the mode predictor indicates trends timely.

Portoguese Escudo:

The estimate of the kernel density indicates bimodality of the distribution of the Portoguese Escudo (figure 5.16). The estimate of the positive attractor lies in the vicinity of the positive mode, while the estimate of the repeller lies in the vicinity of the antimode. However, the estimate of the negative attractor under predicts the

negative mode of the real exchange rate distribution.

The Escudo has a similar time development compared to other real exchange rates. There is a period of erratic fluctuations before ERM II, an attraction to x_L after ERM II as well as an attraction to x_H after the introduction of the Euro. However, the model predicts less uncertainty concerned with this exchange rate, since only one attractor is covered with probability mass at the same time. The mean predictor and the mode predictor under predict the exchange rate before ERM II, while both seem to track the data quite well after ERM II. Even the phase transition after the introduction of the Euro is tracked by both predictors with more or less equal accuracy.

5.6 Forecast Evaluation

The forecast evaluation consists of *(i)* tests for biased forecasts, and *(ii)* the comparison of the losses through prediction errors between different models (Elliot and Timmerman 2008).

5.6.1 Forecast Efficiency Regressions

Predictions of uncertain future realizations of the real exchange rates should be unbiased, which means that the predictions should be right on average. Moreover, the forecast errors should be uncorrelated with any information available up to the time the forecast is made (see Elliot and Timmerman 2008, p. 34). Unbiasedness is

usually tested by applying the regression proposed by Mincer and Zarnowitz (1969)

$$s_t = \beta_0 + \beta_1 \hat{s}_{t|t-1} + v_t, \quad (5.48)$$

where s_t is the realization of the real exchange rate, while $\hat{s}_{t|t-1}$ is the forecast of the exchange rate using information available at time $t - 1$. The error of this regression is denoted by v_t . Under the null hypothesis of unbiasedness the restriction $\beta_0 = 0$ and $\beta_1 = 1$ should hold (Elliot and Timmerman 2008, p. 34). The null hypothesis can simply be tested by applying an F-test.

Table 5.4: Forecast Efficiency Regression: Model 1 (a)

| Test Equation: $s_t = \beta_0 + \beta_1 \hat{s}_{t t-1} + v_t$ Null hypothesis: $H_0 : \beta_0 = 0$ and $\beta_1 = 1$ Model 1: $ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t$ | | | | | | | | |
|--|---------------------|--------------------|-------|--------------------|---------------------|--------------------|-------|-------------------|
| | Mean Predictor | | | | Mode Predictor | | | |
| | β_0 ♣ | β_1 ♣ | R^2 | H_0 ♠ | β_0 ♣ | β_1 ♣ | R^2 | H_0 ♠ |
| Austrian Shilling | 0.055** (2.97) | 1.095** (51.49) | 0.933 | 13.50** (0.00) | 0.122** (3.76) | 0.822** (27.63) | 0.800 | 28.84** (0.00) |
| Belgian Franc | -0.036* (-2.05) | 1.065** (55.80) | 0.942 | 7.62** (0.00) | -0.114** (-3.52) | 0.758** (27.78) | 0.802 | 49.93** (0.00) |
| Cyprus Pound | 0.079** (4.72) | 1.041** (58.10) | 0.946 | 12.93** (0.00) | 0.110** (5.11) | 0.890** (44.65) | 0.913 | 32.09** (0.00) |
| German Mark | 0.029 (1.77) | 1.052** (58.38) | 0.947 | 5.58** (0.00) | 0.108** (2.88) | 0.690** (22.91) | 0.733 | 62.27** (0.00) |
| Spanish Peseta | 0.076** (4.77) | 1.070** (61.37) | 0.952 | 17.98** (0.00) | 0.119** (5.09) | 0.857** (40.56) | 0.896 | 41.10** (0.00) |
| Finnish Markka | 0.091** (3.87) | 2.165** (40.36) | 0.895 | 237.11** (0.00) | 0.100* (2.32) | 0.735** (18.83) | 0.650 | 28.43** (0.00) |
| French Franc | -0.061** (-3.52) | 1.260** (55.55) | 0.942 | 69.70** (0.00) | -0.218** (-5.90) | 0.724** (24.09) | 0.752 | 76.57** (0.00) |

Notes: The real exchange rate data were taken from the ECB's statistical database (www.ecb.int). The dataset covers the period 1993:01 to 2009:02. ♣: Values of the t-statistic given in parenthesis. */** indicate significance at the 5%/1% level. ♠: Values of the F-statistic. Marginal significance levels given in parenthesis. */** indicate significance at the 5%/1% level.

Table 5.5: Forecast Efficiency Regression: Model 1 (b)

| Test Equation: $s_t = \beta_0 + \beta_1 \hat{s}_{t t-1} + v_t$ Null hypothesis: $H_0 : \beta_0 = 0$ and $\beta_1 = 1$ Model 1: $ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t$ | | | | | | | | |
|--|---------------------|--------------------|-------|-------------------|---------------------|--------------------|-------|-------------------|
| | Mean Predictor | | | | Mode Predictor | | | |
| | β_0 ♣ | β_1 ♣ | R^2 | H_0 ♠ | β_0 ♣ | β_1 ♣ | R^2 | H_0 ♠ |
| Greek Drachma | 0.026 (1.49) | 1.087** (54.63) | 0.940 | 10.54** (0.00) | 0.035 (1.07) | 0.748** (27.57) | 0.799 | 44.19** (0.00) |
| Irish Pound | 0.080** (5.18) | 1.085** (63.04) | 0.954 | 23.69** (0.00) | 0.156** (6.50) | 0.846** (39.99) | 0.893 | 57.04** (0.00) |
| Luxembourg Franc | 0.045** (2.78) | 1.066** (59.83) | 0.949 | 10.28** (0.00) | 0.029 (1.08) | 0.832** (35.19) | 0.866 | 25.92** (0.00) |
| Netherland Guilder | -0.078** (-3.87) | 1.074** (47.74) | 0.923 | 11.45** (0.00) | -0.086** (-3.61) | 0.967** (39.67) | 0.892 | 7.92** (0.00) |
| Portoguese Escudo | 0.064** (4.21) | 0.989** (64.26) | 0.956 | 9.42** (0.00) | 0.106** (5.10) | 0.870** (46.49) | 0.919 | 41.80** (0.00) |
| Slovene Tolar | -0.043 (-1.78) | 1.240** (39.24) | 0.890 | 29.84** (0.00) | -0.054 (-1.64) | 0.899** (27.02) | 0.793 | 6.32** (0.00) |
| Slovak Koruna | 0.112** (8.83) | 1.121** (78.07) | 0.970 | 67.13** (0.00) | 0.188** (8.26) | 0.852** (42.44) | 0.852 | 75.32** (0.00) |

Notes: The real exchange rate data were taken from the ECB's statistical database (www.ecb.int). The dataset covers the period 1993:01 to 2009:02. ♣: Values of the t-statistic given in parenthesis. */** indicate significance at the 5%/1% level. ♠: Values of the F-statistic. Marginal significance levels given in parenthesis. */** indicate significance at the 5%/1% level.

Tables 5.4 and 5.5 contains the Mincer-Zarnowitz regression for the predictions produced by model 1 $ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t$. We differentiate between the mean predictor and the mode predictor. When the forecast is unbiased the constant term β_0 of the forecast accuracy regression should not be significantly different from zero. For the predicted mean values we find unbiasedness only in the case of the German Mark, the Greek Drachma and the Slovene Tolar, while the coefficients are significantly different from zero for the remaining exchange rates. We find the coefficient β_0 of the test equation to be significantly different from zero for the mode predictor

in all cases but the Greek Drachma, the Luxembourg Franc and the Slovene Tolar. This bias results from the fact that the model has problems in forecasting values outside the zone of instability, since most of the probability mass is concentrated in the vicinity of the attractors. Hence, the distribution is heavily skewed there, which causes systematic over- and under predictions of these extreme values. For the German Mark, the Greek Drachma, the Luxembourg Franc and the Slovene Tolar we find less values lying outside the zone of instability compared to the other real exchange rates, which reduces the bias in forecasting these exchange rates.

The estimates of β_1 are significantly different from zero and in the range of one indicating that the forecast is able to explain the actual data. The only exception is the Finnish Markka, whose time evolution is characterized by more volatile fluctuations compared to the other exchange rates. Hence, we find an estimate of β_1 in the range of two here, indicating that the forecasts are less volatile compared to the actual data. The values of β_1 are smaller for the mode predictor compared to the values of β_1 for the mean predictor. These results are in line with our visual inspections of figures 5.6 to 5.16, where we can see that the mode predictor is more volatile compared to the mean predictor, since it frequently jumps between the two attractors.

The goodness-of-fit measures are in the range of 0.9 in all cases. Although the explanatory power seems to be very high, we find smaller values for the mode predictor which is due to the fact that the mode predictor frequently jumps between both attractors, while the mean predictor is closer to the actual data. However, we reject the joint hypothesis of $H_0 : \beta_0 = 0$ and $\beta_1 = 1$ in all cases.

Table 5.6 and 5.7 contains the results of the Mincer-Zarnowitz regression for model 2

$ds_t = (x_L - s_t)(x_M - s_t)(x_L - s_t)dt + \sigma dW_t$. The results are more or less comparable of those we got for model 1 in table 5.4 and 5.5. We find the coefficient β_0 to be significantly different from zero except as for the Cyprus Pound, the German Mark, the Spanish Peseta, the Irish Pound and the Luxembourg Franc. Hence, model 2 leads to unbiased forecasts of the real exchange rate for more cases compared to model 1 which can be due to the fact that model 2 is less restrictive compared to model 1. Moreover, we cannot reject the null hypothesis of unbiasedness for the Belgian Franc, the Cyprus Pound, the Greek Drachma, the Irish Pound and the Luxembourg Franc in case of the mode predictor. The goodness-of-fit measures are again in the range of 0.9 for the mean predictor and lower for the mode predictor, since it is more volatile compared to the mean predictor. This higher volatility is again reflected in the lower values of β_1 in the test regression. However, we reject the joint null hypothesis $H_0 : \beta_0 = 0$ and $\beta_1 = 1$ in all cases.

5.6.2 Forecasting Performance

As Elliot and Timmerman (2008) note, forecast do not have any intrinsic value. They are only helpful in improving economic and financial decisions. Hence, the value of a forecast depends on how costly prediction errors are. Let $u_{t+h} \equiv s_{t+h} - s_{t+h|t}$ be the forecast error associated by the h -step ahead point forecast $s_{t+h|t}$. The loss generated by this sequence of forecast errors is defined as $L(u_{t+h})$. Typical loss functions are the squared loss $L(u_{t+h}) = (u_{t+h})^2$ and the absolute loss $L(u_{t+h}) = |u_{t+h}|$. Forecasts are often evaluated over certain time periods. Prominent loss measures over a certain

Table 5.6: Forecast Efficiency Regression: Model 2 (a)

| Test Equation: $s_t = \beta_0 + \beta_1 \hat{s}_{t t-1} + v_t$ Null hypothesis: $H_0 : \beta_0 = 0$ and $\beta_1 = 1$ Model 2: $ds_t = (x_L - s_t)(x_M - s_t)(x_H - s_t) + \sigma dW_t$ | | | | | | | | |
|---|----------------------|--------------------|-------|--------------------|---------------------|--------------------|-------|-------------------|
| | Mean Predictor | | | | Mode Predictor | | | |
| | β_0 ♣ | β_1 ♣ | R^2 | H_0 ♠ | β_0 ♣ | β_1 ♣ | R^2 | H_0 ♠ |
| Austrian Shilling | -0.045** (-2.89) | 1.163** (63.03) | 0.954 | 42.23** (0.00) | 0.155** (4.30) | 0.736** (24.50) | 0.757 | 56.86** (0.00) |
| Belgian Franc | 0.042** (2.63) | 1.116** (60.46) | 0.950 | 22.536** (0.00) | -0.065 (-1.95) | 0.754** (26.81) | 0.790 | 41.68** (0.00) |
| Cyprus Pound | -0.002 (-0.08) | 0.923** (52.57) | 0.935 | 9.55** (0.00) | 0.007 (0.36) | 0.870** (46.59) | 0.919 | 24.31** (0.00) |
| German Mark | -0.010 (-0.66) | 1.077** (61.87) | 0.953 | 9.80** (0.00) | 0.096* (2.53) | 0.679** (22.38) | 0.724 | 63.62** (0.00) |
| Spanish Peseta | 0.014 (0.74) | 0.904** (49.26) | 0.927 | 14.15** (0.00) | 0.026** (-10.86) | 0.836** (41.55) | 0.897 | 33.07** (0.00) |
| Finnish Markka | -0.256** (-10.86) | 1.805** (41.55) | 0.900 | 191.96** (0.00) | 0.184** (3.84) | 0.695** (16.44) | 0.586 | 42.53** (0.00) |
| French Franc | 0.221** (14.40) | 1.278** (65.37) | 0.957 | 166.77** (0.00) | -0.084** (-2.78) | 0.828** (30.23) | 0.827 | 25.12** (0.00) |

Notes: The real exchange rate data were taken from the ECB's statistical database (www.ecb.int). The dataset covers the period 1993:01 to 2009:02. ♣: Values of the t-statistic given in parenthesis. */** indicate significance at the 5%/1% level. ♠: Values of the F-statistic. Marginal significance levels given in parenthesis. */** indicate significance at the 5%/1% level.

out-of-sample time period $[T_0, T_1]$ are the root mean squared error (RMSE)

$$\text{RMSE} = \frac{1}{T_1 - T_0} \sum_{t=T_0-h}^{T_1-h} (s_{t+h} - s_{t+h|t})^2 \quad (5.49)$$

and the mean absolute error (MAE)

$$\text{MAE} = \frac{1}{T_1 - T_0} \sum_{t=T_0-h}^{T_1-h} |s_{t+h} - s_{t+h|t}|. \quad (5.50)$$

Table 5.7: Forecast Efficiency Regression: Model 2 (b)

| Test Equation: $s_t = \beta_0 + \beta_1 \hat{s}_{t t-1} + v_t$ | | | | | | | | |
|---|--------------------|--------------------|-------|--------------------|---------------------|--------------------|-------|-------------------|
| Null hypothesis: $H_0 : \beta_0 = 0$ and $\beta_1 = 1$ | | | | | | | | |
| Model 2: $ds_t = (x_L - s_t)(x_M - s_t)(x_H - s_t) + \sigma dW_t$ | | | | | | | | |
| | Mean Predictor | | | | Mode Predictor | | | |
| | β_0 ♣ | β_1 ♣ | R^2 | H_0 ♠ | β_0 ♣ | β_1 ♣ | R^2 | H_0 ♠ |
| Irish Pound | 0.022 (1.06) | 0.891** (46.70) | 0.920 | 17.18** (0.00) | 0.033 (1.40) | 0.825** (40.22) | 0.894 | 37.55** (0.00) |
| Luxembourg Franc | 0.013 (0.66) | 0.926** (49.01) | 0.926 | 7.83** (0.00) | 0.021 (0.79) | 0.811** (35.53) | 0.869 | 34.97** (0.00) |
| Netherlands Guilder | 0.056** (4.02) | 1.296** (70.90) | 0.963 | 135.61** (0.00) | -0.062* (-2.13) | 0.841** (31.70) | 0.840 | 21.07** (0.00) |
| Portuguese Escudo | — — | — — | — | — — | — — | — — | — | — — |
| Slovene Tolar | 0.370** (24.83) | 1.786** (70.17) | 0.963 | 588.09** (0.00) | -0.098** (-2.29) | 0.720** (18.97) | 0.653 | 32.48** (0.00) |
| Slovak Koruna | 0.084** (4.38) | 0.923** (50.47) | 0.930 | 20.00** (0.00) | 0.089** (4.37) | 0.898** (47.21) | 0.921 | 26.15** (0.00) |

Notes: The real exchange rate data were taken from the ECB's statistical database (www.ecb.int). The dataset covers the period 1993:01 to 2009:02. ♣: Values of the t-statistic given in parenthesis. */** indicate significance at the 5%/1% level. ♠: Values of the F-statistic. Marginal significance levels given in parenthesis. */** indicate significance at the 5%/1% level.

In order to determine if model 1 predicts future exchange rates more accurate compared to model 2, the RMSE and the MAE can be used. Since a comparison of RMSE and MAE depends on a certain sample only, Diebold and Mariano (1995) propose a statistical test on equal predictive accuracy. The authors suggest to test the null of equal predictive accuracy against the alternative hypothesis of unequal predictive accuracy

$$\begin{aligned}
 H_0 : & \text{ equal predictive accuracy} & E[L(u_{t+h}^1)] &= E[L(u_{t+h}^2)] \\
 H_1 : & \text{ unequal predictive accuracy} & E[L(u_{t+h}^1)] &\neq E[L(u_{t+h}^2)]
 \end{aligned}
 \tag{5.51}$$

based on the difference between the quadratic loss (QL) of model 1 and the loss of model 2

$$d_t^{QL} = L(u_{t+h}^1) - L(u_{t+h}^2). \quad (5.52)$$

Hence, null and alternative hypotheses can be transformed to

$$\begin{aligned} H_0 : & \text{ equal predictive accuracy} & E[d_t^{QL}] &= 0 \\ H_1 : & \text{ unequal predictive accuracy} & E[d_t^{QL}] &\neq 0 \end{aligned} \quad (5.53)$$

The Diebold-Mariano test-statistic for the quadratic-loss function (QL-DM) is computed as

$$\text{QL-DM} = \frac{\overline{d^{QL}}}{\sqrt{\widehat{avar}(\overline{d^{QL}})}} = \frac{\overline{d^{QL}}}{\sqrt{LRV_d/T}} \stackrel{H_0}{\approx} \mathcal{N}(0, 1), \quad (5.54)$$

with

$$\overline{d^{QL}} = \frac{1}{T_1 - T_0} \sum_{t=T_0}^{T_1} d_t^{QL} \quad (5.55)$$

and

$$LRV_d = Var(d_t^{QL}) + 2 \sum_{j=1}^{\infty} Cov(d_t^{QL}, d_{t-j}^{QL}) \quad (5.56)$$

as a consistent estimate of the long-run variance of $\overline{d^{QL}}$. For our application we chose a truncation based on Andrew's (1991) heteroskedasticity and autocorrelation consistent (HAC) variance-covariance estimator. This variance estimator is used, since the loss differentials are serially correlated due to overlapping time windows. We calculate the QL-DM-test-statistic as the t -statistic of the estimate of a regression of the loss differential on a constant with Andrew's (1991) HAC-estimator. Hence, the null of equal predictive accuracy can be rejected on a 5%-level if $|\text{QL-DM}| > 1.96$.

The evaluation of point predictions using the Diebold-Mariano-framework implies

the following decision rule for discriminating between competing forecasting models

$$\begin{aligned} d_t^{QL} < 0 &: \text{ use model 1 for forecasting } s_{t+1} \\ d_t^{QL} = 0 &: \text{ equal predictive accuracy} \\ d_t^{QL} > 0 &: \text{ use model 2 for forecasting } s_{t+1}, \end{aligned} \quad (5.57)$$

since $d_t^{QL} < 0$, when model 2 leads to larger losses compared to model 1 and vice versa.

A different concept for forecast comparison is the Kullback-Leibler information criterion (KLIC) which is more appropriate in case of density forecasts. Mitchell and Hall (2005) suggest a test on equal predictive accuracy of two competing density forecasts, which is related to the Berkowitz (2001) likelihood-ratio test for the statistical adequacy of an individual density forecast. Following the suggestions of Bao et al. (2004), Mitchell and Hall (2005) re-interpret the Berkowitz LR-test as a test on a zero KLIC distance between the true and unknown density and the forecast density. The KLIC distance between the true density $g_t(s_t)$ and the density forecast $f_t(s_t)$ is defined as

$$\text{KLIC}_t = \int g_t(s_t) \ln \left\{ \frac{g_t(s_t)}{f_t(s_t)} \right\} ds_t = E[\ln g_t(s_t) - \ln f_t(s_t)], \quad (5.58)$$

while the expected value is evaluated using the probabilities from the true density $g_t(s_t)$. The sample equivalent is given by

$$\widehat{\text{KLIC}} = \frac{1}{T_1 - T_0} \sum_{t=T_0}^{T_1} [\ln g_t(s_t) - \ln f_t(s_t)]. \quad (5.59)$$

Mitchell and Hall (2005) propose the following test on equal predictive accuracy of

two competing density forecast $f_t^1(s_t)$ and $f_t^2(s_t)$

$$d_t^{KLIC} = [\ln g_t(s_t) - \ln f_t^1(s_t)] - [\ln g_t(s_t) - \ln f_t^2(s_t)] \quad (5.60)$$

$$= \ln f_t^2(s_t) - \ln f_t^1(s_t). \quad (5.61)$$

The second equation might be more convenient, since it avoids assumptions of a true density $g_t(s_t)$ (Mitchell and Hall 2005). The null hypothesis is following Mitchell and Hall (2005)

$$\begin{aligned} H_0 : & \text{ equal predictive accuracy} & E[d_t^{KLIC}] = 0 & \rightarrow \text{KLIC} = 0 \\ H_1 : & \text{ unequal predictive accuracy} & E[d_t^{KLIC}] \neq 0 & \rightarrow \text{KLIC} > 0. \end{aligned} \quad (5.62)$$

Following the framework of Diebold and Mariano (1995), this test is equivalent to regressing d_t^{KLIC} on a constant and calculating the t-statistic by using Andrews (1991) HAC variance covariance estimator.

From the KLIC-DM-test on equal predictive accuracy follows the following predictor decision rule

$$\begin{aligned} d_t^{KLIC} < 0 : & \text{ use model 1 for forecasting } s_{t+1} \\ d_t^{KLIC} = 0 : & \text{ equal unconditional predictive accuracy} \\ d_t^{KLIC} > 0 : & \text{ use model 2 for forecasting } s_{t+1}, \end{aligned} \quad (5.63)$$

since $d_t^{KLIC} < 0$, when model 2 produces larger losses compared to model 1.

The tests we proposed here are unconditional tests on equal predictive accuracy, since they provide answers to the question which forecast was more accurate on average. Giacomini and White (2006) propose the concept of conditional predictive accuracy. This framework provides answers to the question if we can predict which forecast

will be more accurate on a future date. While the unconditional method makes a statement on forecasting models, the second one makes a statement about forecasting methods (Giacomini and White 2006). Hence, we change the null hypothesis to

$$\begin{aligned} H_0 : & \text{ equal predictive accuracy} & E_t[L(u_{t+h}^1)] &= E_t[L(u_{t+h}^2)] \\ H_1 : & \text{ unequal predictive accuracy} & E_t[L(u_{t+h}^1)] &\neq E_t[L(u_{t+h}^2)], \end{aligned} \quad (5.64)$$

where E_t is the conditional expectation operator based on the information available up to time t . In terms of the loss differential d_t , which can either be a loss differential based on a MSE or MAE as in Diebold and Mariano (1995) or it can be a loss differential based on the KLIC as in Mitchell and Hall (2005)

$$\begin{aligned} H_0 : & \text{ equal predictive accuracy} & E[d_t^{KLIC} | h_{t-1}] &= E[h_{t-1} d_t^{KLIC}] = 0 \\ H_1 : & \text{ unequal predictive accuracy} & E[d_t^{KLIC} | h_{t-1}] &= E[h_{t-1} d_t^{KLIC}] \neq 0 \end{aligned}, \quad (5.65)$$

where $h_{t-1} = (1, d_{t-1})'$ is the test function applied in Giacomini and White (2006). Hence, we can simply estimate an AR(1)-model for the loss differential

$$d_t^{KLIC} = \alpha_0 + \alpha_1 d_{t-1}^{KLIC} + \varepsilon_t, \quad (5.66)$$

and test the null hypothesis $\alpha_0 = \alpha_1 = 0$. Following Giacomini and White (2006), the resulting test statistic is a simple Wald statistic

$$GW = T \left(\frac{1}{T} \sum_{t=2}^T h_{t-1} d_t^{KLIC} \right) \hat{\Sigma}^{-1} \left(\frac{1}{T} \sum_{t=2}^T h_{t-1} d_t^{KLIC} \right) \sim \chi_{(2)}^2 \quad (5.67)$$

and

$$\hat{\Sigma} = \frac{1}{T_1 - T_0} \sum_{t=T_0}^{T_1} (h_{t-1} d_t^{KLIC}) \times (h_{t-1} d_t^{KLIC})' \quad (5.68)$$

is a robust HAC estimator under the null-hypothesis of equal unconditional predictive accuracy (see Giacomini and White 2006, p. 1553).

Note, that the GW-statistic is a squared test-statistic. Hence, a value of $GM > 5.99$ just implies that we reject the null of equal conditional predictive accuracy. It does not imply a decision rule. The decision rule can be derived via our AR(1)-model which predicts which of the two forecasting models will be more precise in the next month

$$\hat{d}_{t+1}^{KLIC} = \hat{\alpha}_0 + \hat{\alpha}_1 d_t^{KLIC}. \quad (5.69)$$

When $\hat{d}_{t+1}^{KLIC} > 0$, then we predict forecasting model 1 to produce larger losses in the future, while when $\hat{d}_{t+1}^{KLIC} < 0$, we predict forecasting model 2 to produce larger losses in the future. Following Giacomini and White (2006) we obtain the following predictor decision rule

$$\begin{aligned} \hat{d}_{t+1}^{KLIC} < 0 &: \text{ use model 1 for forecasting } s_{t+1} \\ \hat{d}_{t+1}^{KLIC} = 0 &: \text{ equal conditional predictive accuracy} \\ \hat{d}_{t+1}^{KLIC} > 0 &: \text{ use model 2 for forecasting } s_{t+1}, \end{aligned} \quad (5.70)$$

since $\hat{d}_{t+1}^{KLIC} < 0$ predicts model 2 to produce larger losses in $t + 1$ compared to model 1.

Table 5.8 and 5.9 contains the out-of-sample point prediction performance of the mean and the mode predictor of model 1 $ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t$ (panel (a)) and model 2 $ds_t = (x_L - s_t)(x_M - s_t)(x_H - s_t)dt + \sigma dW_t$ (panel (b)) as well as for the random walk prediction for the quadratic loss function. Moreover, table 5.8 and 5.9 contains the Diebold-Mariano test statistic on the null hypothesis of equal predictive accuracy between the models and the random walk prediction. For the

Table 5.8: Out-of-Sample Forecasting: Quadratic Loss (a)

| Estimation Period: 1993:01 - 2003:12 Out-of-Sample Period: 2004:01 - 2009:02 | | | | | | | |
|---|-------|-------|-------|-------------------------------|--------|----------|--------|
| Decision Rule: $QL-DM(M_i, M_j) < 0 (> 0)$, then $M_i \succ (<) M_j$ ♣ | | | | | | | |
| (a) Model 1: $ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t$ | | | | | | | |
| | RMSE | | | QL-DM(M_i, M_{RW})-Test ♣ | | | |
| | Mean | Mode | RW | Mean | | Mode | |
| Austrian Shilling | 0.130 | 1.013 | 0.117 | 1.049 | (0.29) | 17.904** | (0.00) |
| Belgian Franc | 0.161 | 0.710 | 0.156 | 0.387 | (0.70) | 3.582** | (0.00) |
| Cyprus Pound | 0.995 | 0.365 | 0.134 | 1.739 | (0.08) | 1.578 | (0.11) |
| German Mark | 0.154 | 0.993 | 0.121 | 1.894 | (0.06) | 22.227** | (0.00) |
| Spanish Peseta | 0.409 | 0.395 | 0.086 | 1.489 | (0.14) | 1.563 | (0.12) |
| Finnish Markka | 0.238 | 1.065 | 0.192 | 1.303 | (0.19) | 13.713** | (0.00) |
| French Franc | 0.165 | 0.906 | 0.165 | -0.010 | (0.99) | 4.990** | (0.00) |
| Greek Drachma | 0.337 | 0.367 | 0.107 | 1.286 | (0.20) | 2.630* | (0.01) |
| Irish Pound | 0.426 | 0.411 | 0.108 | 1.359 | (0.17) | 1.343 | (0.18) |
| Luxembourg Franc | 0.383 | 0.395 | 0.123 | 1.379 | (0.17) | 1.730 | (0.08) |
| Netherlands Guilder | 0.197 | 0.336 | 0.138 | 2.311* | (0.02) | 2.613* | (0.01) |
| Portuguese Escudo | 0.275 | 0.249 | 0.106 | 1.746 | (0.08) | 1.649 | (0.10) |
| Slovene Tolar | 0.656 | 0.309 | 0.097 | 3.120** | (0.00) | 1.610 | (0.11) |
| Slovak Koruna | 0.486 | 0.490 | 0.056 | 1.503 | (0.13) | 1.465 | (0.14) |

Note: ♣: QL-DM is the value of the Diebold-Mariano-statistic based on the quadratic-loss differential d_t^{QL} . Marginal significance levels in parenthesis. */** denotes rejection of the null hypothesis of equal predictive accuracy at the 5%/1% level. The QL-DM-statistic is calculated by the Andrews HAC-variance estimator.

mean predictor of model 1 we can reject the null of equal predictive accuracy only in cases of the Netherlands Guilder and the Slovene Tolar, for which the random walk prediction produces smaller quadratic losses on average. In all other cases we cannot reject the null-hypothesis of equal predictive accuracy for the point prediction based on the quadratic loss function. However, we reject the null-hypothesis in favor of the random walk for the mode prediction of the Austrian Shilling, the Belgian Franc, the German Mark, the Finnish Markka, the French Franc and the Netherlands Guilder. The random walk outperforms model 2 significantly in eleven of fourteen cases as indicated by the Diebold-Mariano statistic (panel (b)). A reason might be that

Table 5.9: Out-of-Sample Forecasting: Quadratic Loss (b)

| Estimation Period: 1993:01 - 2003:12 | | | | | | | |
|---|-------|-------|-------|-------------------------------|--------|----------|--------|
| Out-of-Sample Period: 2004:01 - 2009:02 | | | | | | | |
| Decision Rule: $QL-DM(M_i, M_j) < 0 (>0)$, then $M_i \succ (<) M_j$ ♣ | | | | | | | |
| (b) Model 2: $ds_t = (x_L - s_t)(x_M - s_t)(x_H - s_t)dt + \sigma dW_t$ | | | | | | | |
| | RMSE | | | QL-DM(M_i, M_{RW})-Test ♣ | | | |
| | Mean | Mode | RW | Mean | | Mode | |
| Austrian Shilling | 0.142 | 0.938 | 0.117 | 2.681* | (0.01) | 9.974** | (0.00) |
| Belgian Franc | 0.182 | 0.606 | 0.156 | 1.021 | (0.31) | 3.521** | (0.00) |
| Cyprus Pound | 0.635 | 0.609 | 0.134 | 2.804* | (0.01) | 2.582* | (0.01) |
| German Mark | 0.164 | 0.972 | 0.121 | 3.272** | (0.00) | 13.589** | (0.00) |
| Spanish Peseta | 0.290 | 0.376 | 0.086 | 5.067** | (0.00) | 3.164** | (0.00) |
| Finnish Markka | 0.298 | 0.950 | 0.192 | 2.470** | (0.00) | 4.655** | (0.00) |
| French Franc | 0.235 | 0.471 | 0.165 | 2.126* | (0.03) | 3.045* | (0.01) |
| Greek Drachma | — | — | — | — | — | — | — |
| Irish Pound | 0.655 | 0.630 | 0.108 | 1.903 | (0.06) | 1.809 | (0.07) |
| Luxembourg Franc | 0.694 | 0.625 | 0.123 | 1.959 | (0.05) | 1.724 | (0.08) |
| Netherlands Guilder | 0.595 | 0.417 | 0.138 | 4.440** | (0.00) | 2.770** | (0.00) |
| Portuguese Escudo | 0.522 | 0.482 | 0.106 | 2.396* | (0.02) | 2.165* | (0.03) |
| Slovene Tolar | — | — | — | — | — | — | — |
| Slovak Koruna | — | — | — | — | — | — | — |

Note: ♣: QL-DM is the value of the Diebold-Mariano-statistic based on the quadratic-loss differential d_t^{QL} . Marginal significance levels in parenthesis. */** denotes rejection of the null hypothesis of equal predictive accuracy at the 5%/1% level. The QL-DM-statistic is calculated by the Andrews HAC-variance estimator.

although model 2 is less restrictive compared to model 1, there is more parameter estimation uncertainty concerned with model 2. In light of the results of Meese and Rogoff (1983), who found that the random walk outperforms linear exchange rate models, we conclude from our results that the cubic drift model seems to represent the data quite well in numerous cases.

Table 5.10 and 5.11 contains out-of-sample density forecasting performance comparisons between competing models based on the KLIC-loss. We calculate the Diebold-Mariano statistic based on the KLIC (KLIC-DM) by regressing d_t^{KLIC} on a constant

Table 5.10: Out-of-Sample Forecasting: KLIC-Loss (a)

| Estimation Period: 1993:01 - 2003:12 Out-of-Sample Period: 2004:01 - 2009:02 H_0 : Equal Unconditional Forecast Accuracy | | | | | | |
|---|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| M1: $ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t$ M2: $ds_t = (x_L - s_t)(x_M - s_t)(x_H - s_t)dt + \sigma dW_t$ RW: $ds_t = cdt + \sigma dW_t$ | | | | | | |
| Decision Rule: KLIC-DM(M_i, M_j) < 0 (> 0), then $M_i \succ (<) M_j$ ♣ | | | | | | |
| | (a) M1 vs. RW | | (b) M2 vs. RW | | (c) M1 vs. M2 | |
| | Mean | Mode | Mean | Mode | Mean | Mode |
| Austrian Shilling (p-value) | 34.182** (0.00) | 4.822** (0.00) | 22.625** (0.00) | 3.297** (0.00) | 0.101 (0.92) | 4.246** (0.00) |
| Belgian Franc (p-value) | 1.898 (0.06) | -1.288 (0.20) | 1.238 (0.22) | -1.445 (0.15) | 7.203** (0.00) | 1.393 (0.16) |
| Cyprus Pound (p-value) | 2.719* (0.01) | 2.665* (0.01) | 3.941** (0.00) | 3..919** (0.00) | -1.954 (0.05) | -1.960 (0.05) |
| German Mark (p-value) | 30.023** (0.00) | 1.393 (0.16) | 24.973** (0.00) | 0.581 (0.56) | 0.295 (0.77) | 2.479* (0.01) |
| Spanish Peseta (p-value) | 2.190* (0.03) | 2.014* (0.04) | 0.926** (0.00) | -0.850 (0.40) | 2.056* (0.04) | 2.323* (0.02) |
| Finnish Markka (p-value) | 29.045** (0.00) | 19.005** (0.00) | 14.789** (0.00) | 6.705** (0.00) | 1.861 (0.06) | 11.794** (0.00) |
| French Franc (p-value) | 10.138** (0.00) | 3.439** (0.00) | 5.728** (0.00) | -0.054 (0.96) | 15.340** (0.00) | 1.942 (0.05) |

Note: ♣: KLIC-DM is the Diebold-Mariano statistic based on the KLIC-loss differential d_t^{KLIC} . The statistic is calculated using Andrew's HAC variance-covariance-matrix estimator.

and by calculating the t-statistic using Andrew's HAC variance-covariance-matrix estimator. We calculate this statistic for model 1: $ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t$, model 2: $ds_t = (x_L - s_t)(x_M - s_t)(x_H - s_t)dt + \sigma dW_t$ and the random walk prediction: $ds_t = cdt + \sigma dW_t$. Although the random walk is a very simple model we calculate its density predictions using the same finite-difference scheme as we employ for models 1 and 2 in order to guarantee a comparison of the three models under otherwise same conditions. From the KLIC-DM statistic in panel (a) can be inferred that

Table 5.11: Out-of-Sample Forecasting: KLIC-Loss (b)

| Estimation Period: 1993:01 - 2003:12 Out-of-Sample Period: 2004:01 - 2009:02 H_0 : Equal Unconditional Forecast Accuracy | | | | | | |
|---|--------------------|--------------------|--------------------|-------------------|--------------------|--------------------|
| M1: $ds_t = (\alpha s_t - s_t^3)dt + \sigma dW_t$ M2: $ds_t = (x_L - s_t)(x_M - s_t)(x_H - s_t)dt + \sigma dW_t$ RW: $ds_t = cdt + \sigma dW_t$ | | | | | | |
| Decision Rule: KLIC-DM(M_i, M_j) < 0 (> 0), then M_i > (<) M_j ♣ | | | | | | |
| | (a) M1 vs. RW | | (b) M2 vs. RW | | (c) M1 vs. M2 | |
| | Mean | Mode | Mean | Mode | Mean | Mode |
| Greek Drachma (p-value) | 1.144 (0.25) | 0.024 (0.98) | — — | — — | — — | — — |
| Irish Pound (p-value) | 2.636* (0.01) | 2.488* (0.01) | 6.747** (0.00) | 6.744** (0.00) | -5.961** (0.00) | -6.013** (0.00) |
| Luxembourg Franc (p-value) | -3.337** (0.00) | -3.727** (0.00) | 5.941** (0.00) | 5.854** (0.00) | -7.011** (0.00) | -7.002** (0.00) |
| Netherland Guilder (p-value) | 13.851 (0.39) | 6.040** (0.00) | 10.191** (0.00) | 8.552** (0.00) | 7.028** (0.00) | -4.214** (0.00) |
| Portoguese Escudo (p-value) | 3.485** (0.00) | 3.107** (0.00) | 5.654** (0.00) | 5.644** (0.00) | -4.952** (0.00) | -5.013** (0.00) |
| Slovene Tolar (p-value) | 7.682** (0.00) | 4.041** (0.00) | — — | — — | — — | — — |
| Slovak Koruna (p-value) | 5.818** (0.00) | 5.828** (0.00) | — — | — — | — — | — — |

Note: ♣: KLIC-DM is the Diebold-Mariano statistic based on the KLIC-loss differential d_t^{KLIC} . The statistic is calculated using Andrew's HAC variance-covariance-matrix estimator.

model 1 outperforms the random walk significantly only for the Luxembourg Franc, since the KLIC-DM is negative and significantly different from zero in this case. For the Belgian Franc, the Greek Drachma and the Netherland Guilder we cannot reject the null of equal predictive accuracy. From panel (b) we can infer that that the random walk outperforms model 2 in all cases, except for the Belgian Franc for which we cannot reject the null of equal predictive accuracy. Moreover, we find that model 1 outperforms model 2 in cases of the Irish Pound, the Luxembourg Franc,

the Netherland Guilder and the Portuguese Escudo, while model 2 is superior for the Belgian Franc, the German Mark, the Spanish Peseta as well as the Finnish Markka. These results are in line with the findings of Meese and Rogoff (1983) that it is difficult to beat a random walk in out-of-sample forecasting of exchange rates.

Table 5.12: Giacomini-White-Test: Equal Conditional Predictive Accuracy (a)

| Estimation Period: 1993:01 - 2003:12 Out-of-Sample Period: 2004:01 - 2009:02 H_0 : Equal Conditional Forecast Accuracy | | | | | | |
|--|-------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| M1: $ds_t = (\alpha s_t - s_t^3) + \sigma dW_t$ | | | | | | |
| M2: $ds_t = (x_L - s_t)(x_M - s_t)(x_H - s_t)dt + \sigma dW_t$ | | | | | | |
| RW: $ds_t = cdt + \sigma dW_t$ | | | | | | |
| Decision Rule: $E_t[\text{KLIC}_{t+1}(M_i, M_j)] < 0 (> 0)$, then $M_i \succ (<) M_j$ | | | | | | |
| | (a) M1 vs. RW | | (b) M2 vs. RW | | (c) M1 vs. M2 | |
| | Mean | Mode | Mean | Mode | Mean | Mode |
| Austrian Shilling ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 59.792** 0.00% | 45.524** 0.00% | 59.662** 0.00% | 28.220** 0.00% | 17.428** 42.61% | 27.691** 0.00% |
| Belgian Franc ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 29.566** 0.00% | 38.903** 83.61% | 12.847** 0.00% | 45.745** 81.97% | 30.970** 0.00% | 29.968** 22.95% |
| Cyprus Pound ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 10.804** 0.00% | 10.505** 0.00% | 10.929** 0.00% | 10.828** 0.00% | 3.156 100.00% | 3.177 100.00% |
| German Mark ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 59.583** 0.00% | 23.218** 26.23% | 59.623** 0.00% | 21.590** 0.00% | 11.460** 36.07% | 21.718** 14.75% |
| Spanish Peseta ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 4.802* 0.00% | 4.140* 0.00% | 14.904** 24.59% | 12.148** 77.05% | 4.275 0.00% | 5.499 0.00% |
| Finnish Markka ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 59.490** 0.00% | 59.414** 0.00% | 59.555** 0.00% | 45.531** 0.00% | 22.251** 0.00% | 54.584** 0.00% |
| French Franc ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 58.900** 0.00% | 39.267** 0.00% | 54.231** 0.00% | 24.369** 60.66% | 55.428** 0.00% | 23.534** 0.00% |

Note: ♣: Value of the Giacomini-White statistic $\text{GW} \sim \chi_{(2)}^2$ as defined in equation (56). */** denote rejection of the null hypothesis of equal conditional predictive accuracy at the 5%/1% level. Critical values for GW are 5.99/9.21. ♠: Percentage number of $E_t[\text{KLIC}_{t+1}] > 0$ within the period 2004:02 - 2009:02.

Table 5.12 and 5.13 contains results of the Giacomini-White tests on equal conditional predictive accuracy. Its null hypothesis states that we should not be able to predict which model will perform better in the next month. We calculated this

Table 5.13: Giacomini-White-Test: Equal Conditional Predictive Accuracy (b)

| Estimation Period: 1993:01 - 2003:12 Out-of-Sample Period: 2004:01 - 2009:02 H_0 : Equal Conditional Forecast Accuracy | | | | | | |
|--|--------------------|--------------------|-------------------|-------------------|---------------------|---------------------|
| M1: $ds_t = (\alpha s_t - s_t^3) + \sigma dW_t$ | | | | | | |
| M2: $ds_t = (x_L - s_t)(x_M - s_t)(x_H - s_t)dt + \sigma dW_t$ | | | | | | |
| RW: $ds_t = cdt + \sigma dW_t$ | | | | | | |
| Decision Rule: $E_t[\text{KLIC}_{t+1}(M_i, M_j)] < 0$ (> 0), then $M_i \succ$ (\prec) M_j | | | | | | |
| | (a) M1 vs. RW | | (b) M2 vs. RW | | (c) M1 vs. M2 | |
| | Mean | Mode | Mean | Mode | Mean | Mode |
| Greek Drachma ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 15.896** 0.00% | 36.567** 59.02% | — — | — — | — — | — — |
| Irish Pound ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 11.128** 0.00% | 10.441* 0.00% | 29.406** 0.00% | 29.406** 0.00% | 25.133** 100.00% | 25.377** 100.00% |
| Luxembourg Franc ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 19.783** 96.72% | 22.028** 96.72% | 25.411** 0.00% | 25.131** 0.00% | 27.557** 100.00% | 27.593** 100.00% |
| Netherland Guilder ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 59.015** 0.00% | 54.831** 0.00% | 51.965** 0.00% | 49.882** 0.00% | 27.038** 0.00% | 13.495** 100.00% |
| Portoguese Escudo ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 10.225* 0.00% | 8.405* 0.00% | 59.634** 0.00% | 57.933** 0.00% | 20.945** 100.00% | 21.267** 100.00% |
| Slovene Tolar ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 58.207** 0.00% | 40.671** 29.51% | — — | — — | — — | — — |
| Slovak Koruna ♣ % ($E_t[\text{KLIC}_{t+1} < 0]$) ♠ | 39.632** 0.00% | 39.629** 0.00% | — — | — — | — — | — — |

Note: ♣: Value of the Giacomini-White statistic $GW \sim \chi_{(2)}^2$ as defined in equation (56). */** denote rejection of the null hypothesis of equal conditional predictive accuracy at the 5%/1% level. Critical values for GW are 5.99/9.21. ♠: Percentage number of $E_t[\text{KLIC}_{t+1}] > 0$ within the period 2004:02 - 2009:02.

statistic again based on the KLIC-loss together with the percentage number of out performances of model i versus model j over the out-of-sample horizon 2004:01 to 2009:02. Note, that $GW > 5.99$ just implies that we reject the null of equal conditional predictive accuracy. However, e.g. $\%E_t[\text{KLIC}_{t+1} < 0] = 30\%$ implies, that model 1 outperformed model 2 in 30% of the months in the out-of-sample period. Our results indicate that we can reject the null hypothesis of equal conditional predictive accuracy in all cases. The reason lies in the fact that the null of equal conditional

predictive accuracy places one additional restriction compared to the null of equal unconditional predictive accuracy. Panel (a) of table 5.10 contains a comparison of model 1 against the random walk. The decision rule suggest to prefer model one over the random walk for the Belgian Franc in 83.61 percent of all months of the out-of-sample period for the mode predictor of model 1, while it prefers model 1 in 26.23 percent of all months for the German Mark. Moreover, model 1 outperforms the random walk in 96.72% of all cases under this criterion for the Luxembourg Franc and in 29.51 percent of all months for the Slovene Tolar. The mode predictor of model 2 outperforms the random walk in 77.05 percent of all months for the Spanish Peseta and in 60.66 percent of all months for the French Franc.

Summing up, our forecasting exercise gives evidence that models from which bimodal distributions emerge have at least for some time periods forecasting power for real exchange rates, while various test only gave evidence that these models significantly outperform the random walk in out-of-sample predictions for one out of fourteen real exchange rates.

5.7 Conclusion and Outlook

In this paper we estimate nonlinear diffusion processes for modeling exchange rate dynamics. Following the empirical finance literature we assume a cubic drift-term from which a bimodal exchange rate distribution emerges (Creedy et al. 1993, Aït-Sahalia 1999). Hence, large swings and crashes in exchange rates are modeled as jumps between attractors (Creedy et al. 1996). The economic interpretation of bimodality is that there is no consensus in the market about the true equilibrium

exchange rate as proposed in the model of Jeanne (1997). When exchange rates lie within a zone of multiple equilibria, they are vulnerable to self-fulfilling expectations (Koh et al. 2007). This approach wants to overcome the difficulties arising in linear exchange rate models in explaining and forecasting exchange rates.

Within this paper the following results emerged:

- (i) The parameter estimates give evidence for bimodality in exchange rates for both cubic-drift models. Bimodality could not be rejected for fourteen real exchange rates over the period 1993:01 to 2009:02 as well as for the period 1993:01 to 2003:12 for both models.
- (ii) Our density forecasts identify phase transitions in exchange rates during which the distribution changes its shape from unimodal to bimodal. We identify two major phase transitions as the beginning of ERM II and the introduction of the Euro.
- (iii) We find exchange rates closer to their attractors and confidence regions becoming smaller after the beginning of ERM II. Moreover, bimodal distributions fit exchange rates better in the period before the introduction of the Euro, while unimodality is more evident afterwards.
- (iv) Statistical tests on equal unconditional predictive accuracy as well as on equal conditional predictive accuracy lead to the result that the cubic-drift models outperform the random walk prediction for one out of fourteen real exchange rates.

Estimation of and forecasting with nonlinear stochastic differential equations might as well have applications in other areas of financial economics. Since linear models

cannot predict periods of financial instability, nonlinear econometrics seems to be a promising field in empirical finance. However, a test of the performance of Value-at-Risk under these more flexible distributions might be of importance for addressing financial risk.

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5.A Appendix

5.A.1 Properties of the Generalized Cubic Drift Model

(a) Three Stationary Points:

The generalized cubic-drift model is given by the following equation

$$\mu(s; \theta) = (x_L - s)(x_M - s)(x_H - s) \quad (5.71)$$

$$= (x_L - s)(x_M x_H - s x_H - s x_M + s^2) \quad (5.72)$$

$$= -(s^3 - s^2(x_H + x_M + x_L) + s(x_H x_L + x_M x_L + x_M x_H) - x_L x_M x_H),$$

with equilibrium points $x^L \leq x^M \leq x^H$ and $\theta = (x^L, x^M, x^H)'$. Since attractors and repellers are minima and maxima of the potential V , which is defined as

$$\frac{\partial V}{\partial s} = -\mu(s; \theta), \quad (5.73)$$

we derive the stability properties by calculating the first derivative of the negative drift and evaluate it at the stationary points x^L , x^M and x^H . The first derivative of the negative drift with respect to the state variable s is given by

$$-\frac{\partial \mu(s; \theta)}{\partial s} = 3s^2 - 2(x_L + x_M + x_H)s + (x_M x_H + x_M x_L + x_L x_H). \quad (5.74)$$

For the special case $x_L = -\sqrt{\alpha}$, $x_M = 0$ and $x_H = +\sqrt{\alpha}$ this derivative collapses to

$$-\frac{\partial \mu(s; \theta)}{\partial s} = 3s^2 - 2(-\sqrt{\alpha} + \sqrt{\alpha})s + \alpha = 3s^2 + \alpha, \quad (5.75)$$

which is the first derivative of the double-well potential.

The curvature of the potential at the lower stationary point is

$$-\frac{\partial\mu(s;\theta)}{\partial s}\Big|_{s=x_L} = 3x_L^2 - 2(x_L + x_M + x_H)x_L + (x_Mx_H + x_Mx_L + x_Lx_H) \quad (5.76)$$

$$= x_L^2 - x_Mx_L - x_Lx_H + x_Mx_H \quad (5.77)$$

$$= (x_L - x_H)(x_L - x_M) > 0, \quad (5.78)$$

which indicates a minimum of the potential function and, hence, a stable state.

The curvature of the potential at the higher stationary point is

$$-\frac{\partial\mu(s;\theta)}{\partial s}\Big|_{s=x_H} = 3x_H^2 - 2(x_L + x_M + x_H)x_H + (x_Mx_H + x_Mx_L + x_Lx_H) \quad (5.79)$$

$$= x_H^2 - x_Lx_H - x_Mx_H + x_Mx_L \quad (5.80)$$

$$= (x_H - x_M)(x_H - x_L) > 0, \quad (5.81)$$

which indicates a minimum of the potential function and, hence, a stable state.

The curvature of the potential at the stationary point between both attractors is

$$-\frac{\partial\mu(s;\theta)}{\partial s}\Big|_{s=x_M} = 3x_M^2 - 2(x_L + x_M + x_H)x_M + (x_Mx_H + x_Mx_L + x_Lx_H) \quad (5.82)$$

$$= x_M^2 - x_Mx_L - x_Mx_H + x_Lx_H \quad (5.83)$$

$$= (x_M - x_H)(x_M - x_L) < 0, \quad (5.84)$$

which indicates a maximum of the potential function and, hence, an unstable state.

(b) Two Stationary Points:

In case of two equilibria $x_L < x_H$ we get the following drift

$$\mu(s; \theta) = -(s - x_L)^2(s - x_H) \quad (5.85)$$

$$= -(s^3 - (2x_L + x_H)s^2 + (x_L^2 + 2x_Lx_H)s + x_L^2x_H). \quad (5.86)$$

The derivative of the negative drift with respect to the state variable is given by

$$-\frac{\partial \mu(s; \theta)}{\partial s} = 3s^2 - (4x_L + 2x_H)s + (x_L^2 + 2x_Lx_H). \quad (5.87)$$

The curvature of the potential at the stationary point x_L is given by

$$-\frac{\partial \mu(s; \theta)}{\partial s} \Big|_{s=x_L} = 3x_L^2 - 4x_L^2 + 2x_Lx_H + x_L^2 + 2x_Lx_H \quad (5.88)$$

$$= 0. \quad (5.89)$$

while the curvature of the potential at the stationary point between both attractors is

$$-\frac{\partial \mu(s; \theta)}{\partial s} \Big|_{s=x_H} = 3x_H^2 - 4x_Lx_H - 2x_H^2 + x_L^2 + 2x_Lx_H \quad (5.90)$$

$$= x_H^2 - 2x_Lx_H + x_L^2 \quad (5.91)$$

$$= (x_H - x_L)^2 > 0 \quad (5.92)$$

which indicates a minimum of the potential function and, hence, a stable state.

Alternatively, we might have

$$\mu(s; \theta) = -(s - x_H)^2(s - x_L) \quad (5.93)$$

$$= -(s^2 - 2x_H s + x_H^2)(s - x_L) \quad (5.94)$$

$$= -(s^3 - (2x_H + x_L)s^2 + (x_H^2 + 2x_L x_H)s + x_H^2 x_L). \quad (5.95)$$

The derivative of the negative drift with respect to the state variable is given by

$$-\frac{\partial \mu(s; \theta)}{\partial s} = 3s^2 - (4x_L + 2x_H)s + (x_L^2 + 2x_L x_H). \quad (5.96)$$

The curvature of the potential at the stationary point x_L is given by

$$-\frac{\partial \mu(s; \theta)}{\partial s} \Big|_{s=x_L} = 3x_L^2 - 4x_L^2 + 2x_L x_H + x_L^2 + 2x_L x_H \quad (5.97)$$

$$= 0. \quad (5.98)$$

while the curvature of the potential at the stationary point between both attractors is

$$\frac{\partial \mu(s; \theta)}{\partial s} \Big|_{s=x_H} = 3x_H^2 - 4x_L x_H - 2x_H^2 + x_L^2 + 2x_L x_H \quad (5.99)$$

$$= x_H^2 - 2x_L x_H + x_L^2 \quad (5.100)$$

$$= (x_H - x_L)^2 > 0 \quad (5.101)$$

which indicates a minimum of the potential function and, hence, a stable state.

(c) One Stationary Point:

In case of one equilibrium the drift collapses to

$$\mu(s; \theta) = -(s - x)^3 \quad (5.102)$$

$$= -(s^3 - 3xs^2 + 3sx^2 - x^3) \quad (5.103)$$

The curvature of the potential at the unique stationary point

$$-\frac{\partial \mu(s; \theta)}{\partial s} \Big|_{s=x} = 3x^2 - 6x^2 + 3x^2 = 0 \quad (5.104)$$

which indicates a flat potential function at $s = x$. However, we have $\mu(s; \theta) > 0$ for $s < x$ and $\mu(s; \theta) < 0$ for $s > 0$. Hence, the unique equilibrium is attracting.

5.A.2 Crank-Nicolson Discretization Scheme

The similar derivation of the Crank-Nicolson Discretization of the Fokker-Planck equation can also be found in the appendix of Lux (2009c).

The Fokker-Planck equation for the density $f(s_t; \theta)$ of the state variable s_t is given by

$$\frac{\partial f(s_t; \theta)}{\partial t} = \frac{\partial (\mu(s_t; \theta) f(s_t; \theta))}{\partial s} + \frac{\partial^2 \sigma(s_t; \theta) f(s_t; \theta)}{\partial s^2}, \quad (5.105)$$

which is defined up to a parameter vector $\theta \in R^n$, which we suppress in order to ease notation.

Lux (2009c) defines the probability flux $F(s_t)$ and writes the Fokker-Planck equation as

$$\frac{\partial f(s_t)}{\partial t} = \frac{\partial F(s_t)}{\partial s}, \quad (5.106)$$

with

$$F(s_t) = \sigma(s_t) \frac{\partial f(s_t)}{\partial s} + (\mu(s_t) + \frac{\partial \sigma(s_t)}{\partial s}) f(s_t). \quad (5.107)$$

The Crank-Nicolson approximation substitutes the differential by the following central differences

$$\frac{\partial f(s_t)}{\partial t} \approx \frac{f_j^{i+1} - f_j^i}{k} \quad (5.108)$$

$$\frac{\partial F(s_t)}{\partial s} \approx \frac{1}{h} \left(\frac{F_{j+0.5}^{i+1} + F_{j+0.5}^i}{2} - \frac{F_{j-0.5}^{i+1} + F_{j-0.5}^i}{2} \right). \quad (5.109)$$

Hence, we get the following finite difference approximation to the Fokker-Planck equation

$$\frac{f_j^{i+1} - f_j^i}{k} = \frac{1}{h} \left(\frac{F_{j+0.5}^{i+1} + F_{j+0.5}^i}{2} - \frac{F_{j-0.5}^{i+1} + F_{j-0.5}^i}{2} \right) \quad (5.110)$$

and

$$F_j^i = \sigma(s_j) \frac{f_{j+0.5}^i - f_{j-0.5}^i}{h} + (\mu(s_j) + \sigma'(s_j)) f_j^i. \quad (5.111)$$

By plugging the second equation into the first we arrive at

$$\frac{f_j^{i+1} - f_j^i}{k} = \frac{1}{2h^2} \sigma_{j+0.5} \left(f_{j+1}^{i+1} - f_j^{i+1} + f_{j+1}^i - f_j^i \right) \quad (5.112)$$

$$- \frac{1}{2h^2} \sigma_{j-0.5} \left(f_j^{i+1} - f_{j-1}^{i+1} + f_j^i - f_{j-1}^i \right) \quad (5.113)$$

$$+ \frac{1}{4h} (\mu_{j+0.5} + \sigma'_{j+0.5}) \left(f_{j+1}^{i+1} + f_j^{i+1} + f_{j+1}^i + f_j^i \right) \quad (5.114)$$

$$- \frac{1}{4h} (\mu_{j-0.5} + \sigma'_{j-0.5}) \left(f_j^{i+1} + f_{j-1}^{i+1} + f_j^i + f_{j-1}^i \right) \quad (5.115)$$

By defining $v = k/h^2$ we obtain

$$f_j^{i+1} - f_j^i = 0.5v \left[\sigma_{j+0.5} \left(f_{j+1}^{i+1} - f_j^{i+1} + f_{j+1}^i - f_j^i \right) \right] \quad (5.116)$$

$$- \sigma_{j-0.5} \left(f_j^{i+1} - f_{j-1}^{i+1} + f_j^i - f_{j-1}^i \right) \quad (5.117)$$

$$+ 0.5h(\mu_{j+0.5} + \sigma'_{j+0.5}) \left(f_{j+1}^{i+1} + f_j^{i+1} + f_{j+1}^i + f_j^i \right) \quad (5.118)$$

$$- 0.5h(\mu_{j-0.5} + \sigma'_{j-0.5}) \left(f_j^{i+1} + f_{j-1}^{i+1} + f_j^i + f_{j-1}^i \right) \quad (5.119)$$

Putting all variables indexed by $i + 1$ to the left-hand-side (LHS)

$$\begin{aligned} \text{LHS} = & f_j^{i+1} - 0.5v \left[\sigma_{j+0.5} \left(f_{j+1}^{i+1} - f_j^{i+1} \right) - \sigma_{j-0.5} \left(f_j^{i+1} - f_{j-1}^{i+1} \right) \right] \quad (5.120) \\ & + 0.5h(\mu_{j+0.5} + \sigma'_{j+0.5}) \left(f_{j+1}^{i+1} + f_j^{i+1} \right) - 0.5h(\mu_{j-0.5} + \sigma'_{j-0.5}) \left(f_j^{i+1} + f_{j-1}^{i+1} \right) \end{aligned}$$

and all variables indexed by i to the right-hand-side (RHS) we arrive at

$$\begin{aligned} \text{RHS} = & f_j^i + 0.5v \left[\sigma_{j+0.5} \left(f_{j+1}^i - f_j^i \right) - \sigma_{j-0.5} \left(f_j^i - f_{j-1}^i \right) \right] \quad (5.121) \\ & + 0.5h(\mu_{j+0.5} + \sigma'_{j+0.5}) \left(f_{j+1}^i + f_j^i \right) - 0.5h(\mu_{j-0.5} + \sigma'_{j-0.5}) \left(f_j^i + f_{j-1}^i \right) \end{aligned}$$

We define the left-hand side as

$$\text{LHS} = a_j f_{j-1}^{i+1} + b_j f_j^{i+1} + c_j f_{j+1}^{i+1}, \quad (5.122)$$

with

$$\begin{aligned} a_j &= 0.5v(\sigma_{j-0.5} + 0.5h(\mu_{j-0.5} + \sigma'_{j-0.5})) \\ b_j &= 1 + 0.5v((\sigma_{j+0.5} + \sigma_{j-0.5}) - 0.5h(\mu_{j+0.5} - \mu_{j-0.5} + \sigma'_{j+0.5} - \sigma'_{j-0.5})) \\ c_j &= -0.5v(\sigma_{j+0.5} - 0.5h(\mu_{j+0.5} + \sigma'_{j+0.5})). \end{aligned} \quad (5.123)$$

The right hand side can be written as

$$\text{RHS} = k_j f_{j-1}^i + l_j f_j^i + m_j f_{j+1}^i, \quad (5.124)$$

with

$$\begin{aligned} k_j &= 0.5v(\sigma_{j-0.5} - 0.5h(\mu_{j-0.5} + \sigma'_{j-0.5})) \\ l_j &= 1 - 0.5v((\sigma_{j+0.5} + \sigma_{j-0.5}) + 0.5h(\mu_{j+0.5} - \mu_{j-0.5} + \sigma'_{j+0.5} - \sigma'_{j-0.5})) \\ m_j &= 0.5v(\sigma_{j+0.5} - 0.5h(\mu_{j+0.5} + \sigma'_{j+0.5})). \end{aligned} \quad (5.125)$$

Boundary conditions have to be imposed in order to guaranty that all probability mass is conserved within the state space (see Lux 2009). These can be imposed as

$$F_{-0.5}^i = F_{N_x+0.5}^i = 0, \quad \forall i \quad (5.126)$$

This changes the values of the coefficients at $j = 0$ to

$$a_j = 0 \quad (5.127)$$

$$b_j = 1 + 0.5v((\sigma_{j+0.5} + \sigma_{j-0.5}) - 0.5h(\mu_{j+0.5} + \sigma'_{j+0.5})) \quad (5.128)$$

$$c_j = -0.5v(\sigma_{j+0.5} - 0.5h(\mu_{j+0.5} + \sigma'_{j+0.5})) \quad (5.129)$$

$$k_j = 0 \quad (5.130)$$

$$l_j = 1 - 0.5v((\sigma_{j+0.5} + 0.5h(\mu_{j+0.5} + \sigma'_{j+0.5})) \quad (5.131)$$

$$m_j = 0.5v(\sigma_{j+0.5} - 0.5h(\mu_{j+0.5} + \sigma'_{j+0.5})), \quad (5.132)$$

and at $j = N_x$ to

$$a_j = 0.5v(\sigma_{j-0.5} + 0.5h(\mu_{j-0.5} + \sigma'_{j-0.5})) \quad (5.133)$$

$$b_j = 1 + 0.5v((\sigma_{j-0.5}) - \mu_{j-0.5} - \sigma'_{j-0.5})) \quad (5.134)$$

$$c_j = 0 \quad (5.135)$$

$$k_j = 0.5v(\sigma_{j-0.5} - 0.5h(\mu_{j-0.5} + \sigma'_{j-0.5})) \quad (5.136)$$

$$l_j = 1 - 0.5v((\sigma_{j-0.5}) + 0.5h(-\mu_{j-0.5} - \sigma'_{j-0.5})) \quad (5.137)$$

$$m_j = 0. \quad (5.138)$$

All equations can be summarized in a system of equations of the form

$$\mathbf{V}\mathbf{f}^{i+1} = \mathbf{W}\mathbf{f}^i, \quad (5.139)$$

with

$$\mathbf{f}^i = (f_0^i, \dots, f_{N_x}^i)', \quad (5.140)$$

and coefficient matrices

$$\mathbf{V} = \begin{bmatrix} b_0 & c_0 & 0 & \dots & 0 & 0 & 0 \\ a_1 & b_1 & c_1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & a_{N_x-1} & b_{N_x-1} & c_{N_x-1} \\ 0 & 0 & 0 & \dots & 0 & a_{N_x} & b_{N_x} \end{bmatrix} \quad (5.141)$$

and

$$\mathbf{W} = \begin{bmatrix} j_0 & m_0 & 0 & \dots & 0 & 0 & 0 \\ k_1 & l_1 & m_1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & k_{N_x-1} & l_{N_x-1} & m_{N_x-1} \\ 0 & 0 & 0 & \dots & 0 & k_{N_x} & l_{N_x} \end{bmatrix}. \quad (5.142)$$

Hence, we can calculate the transition densities \mathbf{f}^{i+1} by means of the following iteration rule

$$\mathbf{f}^{i+1} \leftarrow \mathbf{V}^{-1} \mathbf{W} \mathbf{f}^i. \quad (5.143)$$

5.A.3 Implicit Discretization Scheme with Chang-Cooper Correction

For a similar derivation see also Banks et al. (2009). The Chang-Cooper scheme replaces

$$F_j^i = \sigma(s_j) \frac{f_{j+1}^i - f_j^i}{h} + \left(\mu(s_j) + \sigma'(s_j) \right) f_j^i. \quad (5.144)$$

by

$$F_j^i = \sigma(s_j) \frac{f_{j+1}^i - f_j^i}{h} + \left(\mu(s_j) + \sigma'(s_j) \right) \left(\tau_j f_{j+1}^i + (1 - \tau_j) f_j^i \right), \quad (5.145)$$

with

$$\tau_j = \frac{1}{w_j} - \frac{1}{\exp\{w_j\} - 1}, \quad (5.146)$$

and w_j is defined as

$$w_j = h \frac{\mu_j + \sigma'_j}{\sigma_j}, \quad (5.147)$$

By using the following definition of the probability flux

$$\frac{f_j^{i+1} - f_j^i}{k} = \frac{F_{j+1}^{i+1} - F_j^{i+1}}{h}, \quad (5.148)$$

and rearranging it to

$$f_j^{i+1} - \frac{k}{h}(F_{j+1}^{i+1} - F_j^{i+1}) = f_j^i \quad (5.149)$$

and plugging in the definition of the definition of F_j^i under the Chang-Cooper correction we arrive at

$$\begin{aligned} f_j^i &= f_j^{i+1} - \frac{k}{h} \left(\sigma_{j+1} \frac{f_{j+1}^{i+1} - f_j^{i+1}}{h} - (\mu_{j+1} + \sigma'_{j+1})(\tau_{j+1} f_{j+1}^{i+1} + (1 - \tau_{j+1}) f_j^{i+1}) \right) \\ &+ \frac{k}{h} \left(\sigma_j \frac{f_j^{i+1} - f_{j-1}^{i+1}}{h} + (\mu_j + \sigma'_j)(\tau_j f_j^{i+1} + (1 - \tau_j) f_{j-1}^{i+1}) \right). \end{aligned} \quad (5.151)$$

Sorting terms yields

$$f_j^i = -\frac{k}{h} \left(\frac{1}{h} \sigma_{j+1} + (\mu_{j+1} + \sigma'_{j+1})(\tau_{j+1}) \right) f_{j+1}^{i+1} \quad (5.152)$$

$$+ f_j^{i+1} + \frac{k}{h} \left(\frac{1}{h} \sigma_{j+1} + (\mu_{j+1} + \sigma'_{j+1})(1 - \tau_{j+1}) \right) f_j^{i+1} \quad (5.153)$$

$$+ \frac{k}{h} \left(\frac{1}{h} \sigma_j + (\mu_j + \sigma'_j) \tau_j \right) f_j^{i+1} \quad (5.154)$$

$$- \frac{k}{h} \left(\frac{1}{h} \sigma_j f_{j-1}^{i+1} + (\mu_j + \sigma'_j)((1 - \tau_j)) f_{j-1}^{i+1} \right). \quad (5.155)$$

Using the definitions of w_j and τ_j yields

$$f_j^i = -\frac{k}{h} \left(\frac{\mu_{j+1} + \sigma'_{j+1}}{\exp\{w_{j+1}\} - 1} \right) f_{j+1}^{i+1} \quad (5.156)$$

$$+ f_j^{i+1} + \frac{k}{h} \left(\frac{(\mu_{j+1} + \sigma'_{j+1}) \exp\{w_{j+1}\}}{\exp\{w_{j+1}\}} \right) f_j^{i+1} + \frac{k}{h} \left(\frac{(\mu_j + \sigma'_j)}{\exp\{w_j\}} \right) f_j^{i+1} \quad (5.157)$$

$$- \frac{k}{h} \left(\frac{(\mu_{j-1} + \sigma'_{j-1}) \exp\{w_{j-1}\}}{\exp\{w_{j-1} - 1\}} \right) f_{j-1}^{i+1}. \quad (5.158)$$

Under the Chang-Cooper discretization scheme of the Fokker-Planck equation the system of equations changes following e.g. Banks et al. (2009) to the following form

$$a_j f_{j-1}^{i+1} + b_j f_j^{i+1} + c_j f_{j+1}^{i+1} = f_j^i, \quad (5.159)$$

with

$$a_j = -v\sigma_j w_j \cdot \frac{1}{\exp\{w_j\} - 1} \quad (5.160)$$

$$b_j = 1 + v\sigma_j w_j \cdot \frac{\exp\{w_j\}}{\exp\{w_j\} - 1} + v\sigma_{j+1} w_{j+1} \cdot \frac{1}{\exp\{w_{j+1}\} - 1} \quad (5.161)$$

$$c_j = -v\sigma_j w_j \cdot \frac{\exp\{w_j\}}{\exp\{w_j\} - 1}. \quad (5.162)$$

with boundary conditions at $j = 0$

$$a_j = 0 \quad (5.163)$$

$$b_j = 1 + v\sigma_{j+1} w_{j+1} \cdot \frac{1}{\exp\{w_{j+1}\} - 1} \quad (5.164)$$

$$c_j = -v\sigma_j w_j \cdot \frac{\exp\{w_j\}}{\exp\{w_j\} - 1}. \quad (5.165)$$

and $j = Nx + 1$

$$a_j = -v\sigma_j w_j \cdot \frac{1}{\exp\{w_j\} - 1} \quad (5.166)$$

$$b_j = 1 + v\sigma_j w_j \cdot \frac{\exp\{w_j\}}{\exp\{w_j\} - 1} \quad (5.167)$$

$$c_j = 0. \quad (5.168)$$

All equations can be summarized in a system of equations of the form

$$\mathbf{V}\mathbf{f}^{i+1} = \mathbf{f}^i, \quad (5.169)$$

with

$$\mathbf{f}^i = (f_0^i, \dots, f_{N_x}^i)', \quad (5.170)$$

and the coefficient matrix

$$\mathbf{V} = \begin{bmatrix} b_0 & c_0 & 0 & \dots & 0 & 0 & 0 \\ a_1 & b_1 & c_1 & \dots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & a_{N_x-1} & b_{N_x-1} & c_{N_x-1} \\ 0 & 0 & 0 & \dots & 0 & a_{N_x} & b_{N_x}. \end{bmatrix} \quad (5.171)$$

Hence, we can calculate the transition densities \mathbf{f}^{i+1} by means of the following iteration rule

$$\mathbf{f}^{i+1} \leftarrow \mathbf{V}^{-1}\mathbf{W}\mathbf{f}^i. \quad (5.172)$$

6 Conclusion and Outlook

Within this dissertation I aimed to meet the two goals of Colander et al.' (2009) proposal on economic modeling:

- Theoretical and empirical soundness of models applied to economic policy analysis. Models should be based on sound empirical assumptions about traders behavior and they should find support in the data by replicating the empirical stylized facts.
- New time series models should be developed for detecting speculative bubbles and the emergence of non-fundamental equilibria for forecasting future financial instabilities.

I presented four papers, from which three papers deal with the analysis of transaction taxes in agent-based financial market models, while the last paper deals with the econometric modeling and forecasting of financial instabilities.

Within this dissertation work the following results emerged:

- (i) The first paper showed that the working of transaction taxes has complex effects within an artificial market which is populated by traders with different trading rules and different investment horizons. I find that although transac-

tion taxes have the ability to reduce parts of the exchange rate volatility, they increase the kurtosis of exchange rate returns and thereby lead to a different kind of risk. The second result is that the tax harms short-term speculation in favor of longer term investments, while it also harms trading rules based on economic fundamentals in favor of trend-extrapolating trading rules. The increase in the kurtosis of the return distribution emerges from the fact that short-term traders become long-term traders, who trade large orders less frequently instead of trading small orders more frequently.

- (ii) Within the second paper I find that under sufficiently small tax rates traders abstain from short-term trading in favor of longer investment horizons. This change in behavior leads to less volatility and less mispricings. When the tax rate exceeds a certain threshold, however, mispricings increase as also found in Westerhoff (2003, 2008). This emergent property is due to the fact that taxation reduces short-term fluctuations and causes longer lasting trends in the exchange rate. As a result, the longer term fundamentalist trading rule becomes unpopular in favor of the longer term trend-chasing rule.
- (iii) In the third paper on economic policy analysis with agent-based financial market models I addressed the fact that recent agent-based financial market models came to the result that taxing financial transactions does not *per se* increase financial stability (Westerhoff 2003a, 2008a; Demary 2008, 2010). I add to the literature by analyzing how different degrees of risk aversion and aggressiveness of traders affect the shape of the volatility response curve to taxation. My simulations indicate that a tax rate of 0.1 percent demarcates the stabilizing tax regime from the destabilizing one. I figure out that the shape of the volatility

and misalignment response curves to taxation depend on behavioral parameters of the model. Transaction taxes are less effective, either when chartists trade more aggressively, fundamentalists trade less aggressively, agents switch more frequently between trading strategies or only have short memory in their fitness measures. A lower risk aversion of agents, however, makes higher tax rates more effective as indicated by a flatter volatility response curve.

- (iv) The last paper deals with the econometric modeling and forecasting of financial instabilities. I estimate stochastic differential equations with a cubic drift from which bimodal distributions emerge for a certain interval of the models' parameters. Parameter estimates give evidence for bimodal exchange rate distributions for fourteen exchange rates, two cubic-drift models and two estimation periods. Forecasts of the exchange rate density predict the exchange rate to be in the vicinity of one of the two attractors for most of the time. However, there are periods, in which both attractors are covered with probability mass indicating uncertainty about the equilibrium exchange rate. The models identify the beginning of the Exchange Rate Mechanism II and the introduction of the Euro as two major phase transitions. Based on tests on unconditional and conditional predictive accuracy I find that the cubic-drift models outperform the random walk in out-of-sample forecasting for one out of fourteen real exchange rates. I conclude that bimodality is a good description of real exchange rates.

There are additional policy questions which can be dealt with by agent-based modeling. Agent-based models can also be applied for addressing questions about the consequences of asymmetric information in financial markets. There is a current dis-

cussion in the public media that hedge funds should provide information about their trading strategies to the public. Agent-based models might handle the question if this regulation will increase the efficiency of financial markets. Another interesting question might be what consequences the introduction of traders with market power might have for financial stability.

Within empirical economics there might be several applications of the cubic-drift model. Inflation rates might have a bimodal distribution which results from two different long-term inflation targets. Periods in which both attractors are covered with probability mass might indicate uncertainty about the inflation target of the central bank. The bimodal model might also be evident for interest rates, again, reflecting two different monetary regimes or a structural break in the risk-premia process.

Summing up, I presented applications of a growing fields of research that is contributing to the understanding of the working of financial markets and the potential implications of different regulatory measures.

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Eidesstattliche Erklärung

Ich erkläre hiermit an Eides Statt, dass ich meine Doktorarbeit "Applications of Agent-Based Models and Nonlinear Econometrics in Finance" selbstständig angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen zitiert habe.

Köln, 28.06.2010

Markus Demary

