

Aus dem Institut für Agrarökonomie  
der Christian-Albrechts-Universität zu Kiel

# Soziale Netze als Determinanten von Transaktionskosten landwirtschaftlicher Betriebe: ein ökonometrischer Ansatz

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*„Your corn is ripe today; mine will be tomorrow. 'Tis profitable for us both, that I shou'd labour with you to-day, and that you shou'd aid me to-morrow. I have no kindness for you and know you have as little for me. I will not, therefore, take any pains on your account, in expectation of a return, I know I shou'd be disappointed, and that I shou'd in vain depend on your gratitude. Here then I leave you to labour alone: You treat me in the same manner. The seasons change; and both of us lose our harvests for want of mutual confidence and security.“*

**David Hume**

*„The average man is destitute of independence of opinion. He is not interested in contriving an opinion of his own, by study and reflection, but is only anxious to find out what his neighbor's opinion is and slavishly adopt it.“*

**Marc Twain**

# Kapitel 1

## Einleitung und Zusammenfassung

Die vorliegende Arbeit ist im Rahmen des EU Projekts „Advanced Eval“ (Contract Number 022708, 6th Framework) entstanden. Ziel des Projektes war es, neue und verbesserte Methoden zur ex ante und ex post Evaluation ländlicher Entwicklungspolitiken zu entwickeln.

Alle hier vorgestellten Arbeiten fallen dabei in den Bereich der ex ante Evaluierung ländlicher Entwicklungspolitiken, und untersuchen einen Teilbereichs des Projektes, der sich schwerpunktmäßig mit dem mikroökonomischen Verhalten landwirtschaftlicher Betriebe beschäftigt. Dabei soll vor allem empirisch die Wirkung sozialer Netze auf Transaktionskosten auf landwirtschaftlichen Güter- und Faktormärkten untersucht werden. Die empirischen Ergebnisse sowie die methodischen Beiträge aus der vorliegenden Arbeit gehen zum Teil im weiteren Verlauf des Projekts in die Simulation einer ländlich geprägten regionalen Ökonomie ein, wobei die empirischen Ergebnisse im Sinne des Coleman'schen Mikro-Makro Modells (COLEMAN, 1990) in die Mikro Ebene eingehen und die erarbeiteten Methoden einen Link zwischen mikro- und makroökonomischer Ebene bilden.

Für die empirische Untersuchung des Einflusses sozialer Netze auf Transaktionskosten werden Elemente aus der soziologischen Netzwerktheorie mit mikroökonomischer Produktionstheorie verbunden. Dieser Ansatz stellt — soweit dem Autor bekannt — eine wissenschaftliche Neuheit dar, da bisherige Arbeiten zu diesem Thema sich auf rein theoretische Aspekte konzentrieren (z.B. BATENBURG et al., 2003) und die wenigen empirischen Studien (z.B. BUSKENS, 1999; ROOKS et al., 2000) zu diesem Thema eher der soziologischen bzw. betriebswirtschaftlichen Forschung zuzuordnen sind und keinen Bezug zur ökonomischen Theorie herstellen.

Die vorliegenden Beiträge untersuchen den Einfluss sozialer Netze auf verschiedene Aspekte von Transaktionskosten. Des weiteren werden empirische Methoden untersucht und vorgestellt, die zur Lösung der oben vorgestellten Forschungsfragen beitragen sollen. Der erste Beitrag steht in der Tradition der empirischen Transaktionskostenforschung und untersucht den Einfluss sozialer Netze auf die Wahl der „Governance Structure“ und ist daher in erster Linie eine empirische Arbeit. Der zweite Beitrag erarbeitet die theoretische Zusammenfüh-

rung von soziologischer Netzwerktheorie und mikroökonomischer Produktionstheorie, sowie ein empirisches Schätzmodell, mit dessen Hilfe der Einfluss sozialer Netze auf die Höhe von Transaktionskosten auf landwirtschaftlichen Güter- und Faktormärkten quantifiziert werden soll. Der Schwerpunkt dieses Beitrags liegt jedoch auf der empirischen Anwendung. In Anlehnung an Beitrag zwei untersucht der dritte Beitrag einen methodischen Aspekt im Bezug auf die Schätzung von stochastischen Frontier-Funktionen mit mehr als einem Output. Der vierte und fünfte Beitrag stellen wiederum eine theoretisch fundierte empirische Anwendung dar und untersuchen nun die Wirkung sozialer Netze auf die Produktivität landwirtschaftlicher Betriebe. Der letzte Beitrag hingegen muss als rein methodischer Forschungsbeitrag gewertet werden, indem er verschiedene ökonometrische Methoden zur Schätzung einer CES-Funktion vorstellt und vergleicht, sowie deren praktische Anwendung demonstriert.

Tabelle 1.1: Einordnung der Beiträge

Nr.	Theorie	empirische Anwendung	Methode	Bereich
1		✓		Governance Structure
2	✓	✓		Messung TAC
3			✓	Multiple Output SFA
4	✓	✓		Effizienzschätzung
5		✓	✓	Effizienzschätzung
6			✓	CES Funktion

### **Influence of Social Networks on Governance Structure**

Basierend auf der klassischen empirischen Transaktionskostenforschung (z.B. RINDFLEISCH and HEIDE, 1997; BOERNER and MACHER, 2001) untersuchen wir die Wahl der „Governance Structure“ (im Weiteren „Regime“ genannt) für einen bestimmten Transaktionstyp. Der vorliegende Artikel untersucht den Bezug von Kraftfuttermitteln für Rinder, Schweine und Geflügel durch *a)* eigene Produktion, *b)* kurz- und längerfristige Verträge oder *c)* direkt über den Markt. Im Gegensatz zu bisherigen Untersuchungen bezieht diese Analyse auch soziale Netze als erklärende Variable für die Wahl des Regimes mit ein. Basierend auf Arbeiten von u.a. RAUB and WEESIE (1990), BUSKENS (1998, 1999) und HENNING (2005) wird die These aufgestellt, dass soziale Netze einen Einfluss auf das Vertrauen zwischen Handelspartnern haben. In Folge dessen haben Betriebe unterschiedliche Präferenzen im Bezug auf die Wahl des Regimes. Die daraus abgeleitete Hypothese nimmt an, dass Betriebe mit vorteilhafter Netzwerkstruktur, d.h. einer Netzwerkstruktur, die das Vertrauen zu Handelspartnern erhöht, tendenziell eher dazu neigen, Kraftfuttermittel über den Markt zu beziehen und tendenziell weniger auf die eigene Produktion setzen.



Da die endogene Variable unserer ökonometrischen Analyse — der Anteil an Kraftfuttermitteln, die über eines der drei Regime bezogen wird — in Prozentanteilen gemessen wird, ist eine Schätzung mit einer einfachen linearen Regression (OLS) inadäquat. Eine weitere Schwierigkeit ist der hohe Anteil an Betrieben, die ausschließlich ein Regime nutzen. Dementsprechend häufig treten in der endogenen Variable die Werte 0 und 1 auf. Wir lösen dieses ökonometrische Problem durch die Verwendung eines „Generalized Linear Models“ (GLM) unter Verwendung einer „Logit“ Link-Funktion. Durch die Verwendung der Logit Link-Funktion wird sichergestellt, dass die auf das Intervall  $[0, 1]$  reduzierte endogene Variable auf die Menge  $\mathbb{R}$  abgebildet wird.

In Anlehnung an MCCULLAGH and NELDER (1983) schätzen wir das Modell mit der Quasi-Maximum-Likelihood (QMLE) Methode, wobei wir den Prozentanteil eines Regimes auf Tierbesatzdichte, landwirtschaftliche Nutzfläche, Bodenqualität sowie die Netzwerkparameter „Anzahl der ausgehenden Verbindungen“ („outdegrees“) und „Dichte“ für vier verschiedene Netzwerktypen (Handelsnetz, Informationsnetz, soziales Netz und Kooperationsnetz) regressieren.

Überraschenderweise zeigen unsere Daten keinen signifikanten Einfluss von Tierbesatzdichte, Land und Bodenqualität auf die Wahl des Regimes. Hingegen finden wir messbare Effekte des Handelsnetzes und Informationsnetzes auf die Wahl des Regimes. Die Ergebnisse bestätigen unsere Hypothese. Ein dichtes Handelsnetz erhöht signifikant die Wahrscheinlichkeit, Kraftfuttermittel über den Markt zu beziehen. Ein großes Informationsnetz hingegen erhöht den Anteil von Kraftfuttermitteln aus Eigenproduktion. Für das Regime „Vertrag“ können wir keinen Einfluss der Netze ausmachen, was evtl. durch die geringe Datenmenge in dieser Regimeform zu erklären ist.

### **Multiple Output Stochastic Frontiers: Two Approaches**

Die vorliegende Arbeit ist in Anlehnung an Beitrag zwei entstanden. Im Rahmen der Analyse des Einflusses sozialer Netze auf die Höhe der Transaktionskosten wird eine Frontier-Produktionsfunktion für mehrere Outputgüter geschätzt. Für diese gibt es mittlerweile zwei gängige Verfahren:

1. das Schätzen einer „Distance Function“ (SHEPHARD, 1953, 1970)
2. das Schätzen einer „Stochastic Ray Production Frontier“ (LÖTHGREN, 2000).

Es liegen bereits Arbeiten vor, die diese beiden Verfahren vergleichen (z.B. FUSEKIS, 2002; ZHANG and GARVEY, 2008). Allerdings werden diese Vergleiche ausschließlich an Realdaten durchgeführt. Damit geben diese Beiträge keinen Aufschluss über die Güte der Schätzung,

sondern sind allenfalls in der Lage zu zeigen, wie stark die Ergebnisse der beiden Verfahren übereinstimmen. Wir untersuchen diese Fragestellung daher anhand einer Monte Carlo Simulation. In Anlehnung an COELLI and PERELMAN (1999) fokussieren wir uns dabei hauptsächlich auf die Qualität der geschätzten Ineffizienzmaße, da ein direkter Vergleich der Parameter beider Schätzfunktionen nur in begrenztem Umfang möglich ist. Der Grund liegt darin, dass beide Funktionen unterschiedliche Maße verwenden, um das Outputverhältnis abzubilden, sodass es nicht sinnvoll ist, in beiden Funktionsformen identische Koeffizienten zu verwenden.

Eine weitere Schwierigkeit ergibt sich aus dem unbekanntem Outputverhältnis der Technologie. Wir lösen dieses Problem, indem wir eine weitere Pseudo-Zufallsvariable aus der Dirichlet Verteilung generieren. Mit Hilfe dieser Variable generieren wir einen Verteilungsschlüssel für die Outputs. Obwohl sich unsere Analyse auf den Zwei-Output Fall beschränkt, lässt sich dieses Verfahren auch auf den n-Output Fall erweitern.

Zur Durchführung der Monte Carlo Simulation generieren wir 200 Beobachtungen. Die „wahren“ Parameter liegen in Wertebereichen, die in Anlehnung an die Literatur, beziehungsweise eigene Schätzungen, gesetzt werden. Der Ineffizienzterm wird, dem Ansatz von BATTESE and COELLI (1995) folgend, aus einer Halbnormalverteilung generiert. Dabei wird die Varianz so gesetzt, dass das Modell eine durchschnittliche Effizienz von ca. 0.83 aufweist. Für den allgemeinen Fehlerterm hingegen nehmen wir eine Normalverteilung mit Erwartungswert 0 und  $\sigma^2 = 0.1$  an. Insgesamt werden 1000 Iterationen durchlaufen.

Die Ergebnisse zeigen für das gegebene Szenario — sofern vergleichbar — nur sehr geringe Unterschiede in der Qualität der Schätzer. Allerdings ist die Qualität der einzelnen Schätzer sehr heterogen und bedarf einer weiteren Untersuchung. Bei der Messung der Ineffizienzen hingegen, liefern beide Schätzfunktionen verlässliche Ergebnisse, wobei kein wesentlicher Unterschied in der Qualität der Schätzer beider Funktionen festgestellt werden kann. Es bedarf weiterer Untersuchungen um festzustellen, inwiefern dieses Ergebnis auch unter weniger optimalen Bedingungen replizierbar ist.

## **Networks and Transaction Costs**

Basierend auf den Arbeiten von BUSKENS (1999) und HENNING (2005) wird zunächst theoretisch hergeleitet, wie die Struktur sozialer Netze sich in der Höhe von Transaktionskosten manifestiert. Anhand polnischer Betriebs- sowie egozentrierter Netzwerkdaten soll empirisch der Einfluss sozialer Netze auf betriebsindividuelle Transaktionskosten gemessen werden. Dabei werden die Märkte für pflanzliche und tierische Erzeugnisse sowie drei Inputmärkte betrachtet.

Das theoretische Modell wird aus der klassischen mikroökonomischen Produktionstheorie abgeleitet. Die zugrunde liegende Idee ist, dass nicht beobachtbare Transaktionskosten zu einer Abweichung zwischen der tatsächlich beobachteten Produktionsentscheidung und einer bei beobachtbaren Preisen optimalen Produktionsentscheidung führen. Die dieser Beobachtung zugrunde liegende Annahme ist, dass der Betriebsleiter die wahren — nicht beobachtbaren — Schattenpreise (d.h. Preise inklusive Transaktionskosten) kennt und in seine Produktionsentscheidung einbezieht. Die beobachtbaren Preise sind dementsprechend bei Outputgütern höher und bei Inputgütern niedriger als die Schattenpreise, so dass die auf den beobachteten Preisen basierende scheinbar optimale Produktionsentscheidung den Zustand ohne Transaktionskosten darstellt. Auch hier gehen wir von der Prämisse aus, dass die Höhe betriebsindividueller Transaktionskosten von der Struktur der sozialen Netze beeinflusst wird. In anderen Worten: die Höhe der Schattenpreise hängt von der Struktur des Netzes ab und ist individuell verschieden. Daher gehen wir davon aus, dass die nicht beobachtbaren Transaktionskosten durch soziale Netze approximiert werden können. Aufbauend auf diesen Annahmen versuchen wir die beobachtete Divergenz zwischen der tatsächlichen und der theoretisch optimalen Produktionsentscheidung durch Strukturparameter sozialer Netze zu erklären.

Das Schätzmodell besteht aus einem zweistufigen Schätzverfahren. Im ersten Schritt wird die beobachtbare Produktionsfunktion als stochastische Frontier-Funktion geschätzt. Da wir Transaktionskosten sowohl für tierische als auch pflanzliche Outputgüter berechnen möchten, greifen wir auf ein Verfahren zur Schätzung von Produktionsfunktionen bei mehreren Outputgütern — der „Stochastic Ray Production Frontier“ — zurück (siehe hierzu auch Beitrag 3). Aus der Produktionsfunktion berechnen wir die partiellen Ableitungen der Multi-Output Produktionsfunktion zu den drei variablen Inputs und den beiden Outputs, aus denen wiederum die verschiedenen Grenzprodukte und Grenzzraten der technischen Substitution sowie die Grenzrate der Transformation berechnet werden können. Im zweiten Schritt wird in einem linearen Gleichungssystem der Einfluss der Transaktionskosten auf die Divergenz zwischen beobachteter und „optimaler“ Produktionsentscheidung gemessen, wobei die variablen Transaktionskosten in proportionale und disproportionale Transaktionskosten getrennt werden. Da die wahren Transaktionskosten nicht bekannt sind, werden sie durch Netzwerkstrukturindikatoren approximiert.

Die Ergebnisse weisen auf einen schwachen bis deutlichen Einfluss der Netze auf proportionale und nichtproportionale Transaktionskosten hin. Die Ergebnisse zeigen auch, dass sowohl proportionale als auch variable disproportionale Transaktionskosten sich in realistischen Wertebereichen bewegen.

## **Measuring the Influence of Social Networks on Transaction Costs Using a Non-parametric Regression Technique**

Dieser Beitrag untersucht den Einfluss sozialer Netze auf die Produktivität landwirtschaftlicher Betriebe. Die zugrunde liegende Annahme ist, dass Betriebe, die mit hohen Transaktionskosten beim Erwerb und Absatz landwirtschaftlicher In- und Outputs konfrontiert sind, eine geringere Produktivität aufweisen. Die Argumentationskette lautet dabei wie folgt: Für den Erwerb und Absatz landwirtschaftlicher Güter muss der Betrieb beim Vorliegen von Transaktionskosten Ressourcen aufwenden, die auch in die Produktion landwirtschaftlicher Produkte eingehen können, z.B. Arbeitsstunden oder Treibstoff. Da ein üblicher Produktionsdatensatz i.d.R. nur aggregierte Inputmengen enthält, deren Verwendung nicht genau spezifiziert ist, wird in Folge dessen der Inputeinsatz bei der Schätzung der technischen Übersetzung von Inputs in Outputs zu hoch angesetzt. Mit anderen Worten, es wird nicht die wahre Technologie geschätzt, sondern eine Technologie mit Transaktionskosten. Ein Betrieb, der viele Ressourcen für die Abwicklung von Transaktionen bereitstellen muss, wird in Relation zu Betrieben mit geringerem Aufwand für Transaktionen, einen höheren Inputeinsatz zur Erreichung eines bestimmten Outputniveaus aufweisen und in Konsequenz unproduktiver (ineffizienter) erscheinen. Dies bedeutet, dass beim Vorliegen von betriebsindividuellen Transaktionskosten und eines Standarddatensatzes an Produktionsdaten, Transaktionskosten in eine geringere Produktivität übersetzt werden.

Wie bereits in vorangegangenen Beiträgen erläutert, besteht theoretisch ein Zusammenhang zwischen der Struktur sozialer Netze und der Höhe von Transaktionskosten. Wir folgern daraus, dass ein Wirkungszusammenhang zwischen sozialen Netzen und Transaktionskosten und ein Wirkungszusammenhang zwischen Transaktionskosten und Produktivität zu einem messbaren Einfluss sozialer Netze auf die Produktivität führen muss.

Wir schätzen diese Annahme in „Reduced Form“, indem wir den aggregierten Outputwert auf die Inputs Arbeit, Land, Kapital und variable Inputs sowie auf Betriebscharakteristika und Netzwerkparameter regressieren. Die Netzwerkparameter beziehen sich dabei nicht nur auf den Betrieb, sondern — aufgrund der Interdependenz zwischen Betrieb und Haushalt — auch auf den Haushalt. Da ein funktionaler Zusammenhang zwischen Netzwerkparametern und Produktivität unbekannt ist, vermeiden wir die Annahme einer bestimmten Funktionsform und schätzen unser Modell mit Hilfe einer nicht parametrischen Regressionsanalyse. Wir verwenden dabei ein „local-linear“ Schätzer, wobei wir für stetige Variablen einen Epanechnikov Kernel, für geordnete kategoriale Variablen einen Kernel von WANG and VAN RYZIN (1981) und für ungeordnete kategoriale Variablen einen Kernel von AITCHISON and AITKEN (1976) verwenden. Die Bandbreiten der einzelnen Regressoren werden mit dem

„cross-validation“ Verfahren von HURVICH et al. (1998) bestimmt, das auf einem korrigierten Akaike Informationskriterium beruht. Die nicht parametrische Schätzung führen wir in der Statistikumgebung *R* (R DEVELOPMENT CORE TEAM, 2009) mit Hilfe des Zusatzpakets *np* (HAYFIELD and RACINE, 2008) durch. Wir berechnen die Signifikanzen mit einem Bootstrap Verfahren von RACINE et al. (2006) und RACINE (1997). Da die Outputmenge und alle Inputmengen logarithmiert in die Schätzung eingehen, können die Gradienten als partielle Produktionselastizität interpretiert werden.

Unsere Ergebnisse zeigen, dass die Outputmenge monoton steigend in allen Inputmengen ist. Für die Netzwerke finden wir einen schwach signifikanten Einfluss der Menge an Kontakten des Betriebsnetzes, sowie einen signifikanten Einfluss der Dichte des Haushaltsnetzes. Alle vier Netzwerkparameter (Menge an Kontakten, bzw. Dichte des Betriebs- und Haushaltsnetzes) zeigen im Mittel einen positiven Einfluss auf die Produktivität des Betriebs.

### **Measuring the Influence of Social Networks on Transaction Costs Using a Non-parametric Regression Technique - Seminarbeitrag**

Dieser Beitrag ist ein Auszug aus dem vierten Artikel. Der Schwerpunkt liegt auf einer näheren Beschreibung der Methode der nicht parametrischen Regression am Beispiel des im vierten Beitrag vorgestellten Schätzmodells. Dabei geht dieser Beitrag stark auf die angewandten Methoden der nicht parametrischen Regressionsanalyse aus dem vierten Beitrag ein. Insbesondere wird intensiv auf die Problemstellung von Modellen mit gemischten Variablentypen (im vorliegenden Fall stetige, geordnete und ungeordnete kategoriale Variablen) eingegangen, sowie auf die Möglichkeit, Bandbreiten zu interpretieren, bzw. Signifikanzen für gemischte Datentypen zu ermitteln.

### **Estimating the CES Function in R**

Die CES Funktion wurde von ARROW et al. (1961) entwickelt. Sie hat vor allem im Bereich der Makroökonomie und der Wachstumstheorie als Alternative zur Cobb-Douglas Funktion an Popularität gewonnen. Im Bereich der Mikroökonomie konnte sie sich bis heute weniger durchsetzen. Im Rahmen des Ad-Eval Projektes wird die CES genutzt, um eine Verbindung zwischen der Mikro- und Makroebene eines Mikro-Makro Modells (COLEMAN, 1990) herzustellen. Während es üblich ist, die CES bei Simulationen auf Makroebene (z.B. eines CGE) zu kalibrieren, ist dies auf Mikroebene nicht möglich. Die Schätzung der Parameter einer CES Funktion ist jedoch nicht unproblematisch.

1. Die nichtlineare CES Funktion lässt sich nicht linearisieren. Daher muss die CES Funktion entweder linear approximiert werden oder sie muss als nicht-lineares Modell geschätzt werden.
2. Die Zielfunktion der nicht-linearen Schätzung der CES-Funktion hat häufig die Eigenschaft, in der Nähe des Minimums sehr flach zu verlaufen. Diese Eigenschaft führt bei vielen Optimierungsalgorithmen zu Konvergenzproblemen.

Das Zusatzpaket „micEconCES“ (HENNINGSEN and HENNINGSEN, 2010) für die Statistikumgebung „R“ (R DEVELOPMENT CORE TEAM, 2009) greift diese Problematik auf und stellt mehrere Methoden zur Schätzung der CES Funktion zur Verfügung. Darunter sind

- die Kmenta Approximation (KMENTA, 1967), eine Taylor Approximation ersten Grades der CES Funktion, wobei „micEconCES“ das Verfahren von HOFF (2004) verwendet
- den Levenberg-Marquardt Algorithmus (MARQUARDT, 1963) als klassisches nichtlineares Schätzverfahren der CES Funktion
- verschiedene lokale, globale und evolutionäre Optimierungsalgorithmen zur direkten Schätzung der nichtlinearen CES Funktion, u.a. Nelder-Mead (NELDER and MEAD, 1965), BFGS (BROYDEN, 1970), Simulated Annealing (KIRKPATRICK et al., 1983) oder Differential Evolution (STORN and PRICE, 1997)
- die Funktion `nls` zur direkten Schätzung der nichtlinearen CES Funktion
- sowie ein „Grid Search“ Verfahren über den Substitutionsparameter  $\rho$ , welches besonders robust in Fällen flacher Zielfunktionen und/oder hoher  $\rho$ -Werte (starker Nicht-linearität) ist. In Verbindung mit einer nicht-linearen Schätzung können die Parameter des „Grid Search“ Verfahrens verfeinert werden, wobei die mit dem „Grid Search“ Verfahren geschätzten Parameter als Startwerte in die nicht-lineare Schätzung eingehen.

Darüber hinaus hat der Nutzer zahlreiche Möglichkeiten, die Optimierungsalgorithmen zu beeinflussen, z.B. im Bezug auf die Festlegung von Startwerten, Konvergenzkriterien oder Iterationsdurchläufen.

Wir demonstrieren die einzelnen Methoden anhand eines Beispieldatensatzes. Anschließend vergleichen wir die Verfahren mit Hilfe einer Monte Carlo Simulation. Für den gegebenen datengenerierenden Prozess liefern alle vorgestellten Methoden gute bis befriedigende Ergebnisse, wobei die Kmenta Approximation die stärksten Abweichungen von den Originalparametern aufweist. Unsere Ergebnisse bestätigen jedoch die Ergebnisse früherer Studien (z.B.

THURSBY, 1980) in Hinsicht auf eine unbefriedigende Qualität des Schätzers für den Substitutionsparameter  $\rho$ . Auch unsere Simulation zeigt für alle Verfahren deutliche Abweichungen des Schätzers vom Originalwert des Parameters. Unsere Ergebnisse zeigen jedoch auch, dass die aus den Schätzern berechnete Substitutionselastizität  $\sigma$  gute Ergebnisse liefert, so dass das Problem nicht überbewertet werden sollte.

Auch wenn die Ergebnisse unter kontrollierten Bedingungen einer Simulationen nur bedingt auf reale Daten übertragbar sind, so stellt das Paket „micEconCES“ eine breite Palette an Methoden zu Verfügung, so dass für die meisten Datenkonstellationen auf adäquate Schätzmethoden zurückgegriffen werden kann.

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## **Kapitel 2**

# **The Influence of Social Networks on the Choice of Governance Structure**

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## **Abstract**

Transaction costs can be divided in physical and institutional transaction costs. While physical transaction costs must be seen as given, institutional transaction costs can be minimised by choosing the right governance structure. The choice of governance structure depends on the level of trust the firm is facing on the market. Theory shows that social networks can affect the level of trust on a market. The level of trust the firm can achieve on a specific market is presumed to depend on the firm's network position and the network structure. We use data from Polish farms collected in 2007 to test our hypothesis. Applying a quasi-maximum likelihood model, we regress the percentage of concentrated feeding stuffs acquired by own production, over contracts, or the market, *inter alia* on two network parameters (size and density) derived from four types of egocentered farmers' networks. Results show a significant influence, particularly of the trade network on the choice of regime.

## **2.1 Introduction**

Sociology often states that social components such as social capital (Putnam et al., 1993; Leonardi, 1995; Landry et al., 2001; Sabatini, 2006) or social networks (Larson, 1992; Talmud and Mesch, 1997; Uzzi, 1996) have a significant impact on the economic success of a region, a firm, or an individual. On the other hand in recent years economists and particularly business economists become increasingly aware of the social components in economic performance, e.g. firms that are well-established in networks are more likely to be chosen as trading partner (Blumberg and Pfann, 2001) and may get better trade conditions (McMillan and Woodruff, 1999).

Although a vast amount of literature of theoretical models from both disciplines explains the influence of social networks on determinants such as trust, cooperation, contracting, and economic success, (e.g. Wilson, 1985; Buskens, 1998a,b; Buskens and Raub, 2002; Clay, 1997; Alvarez et al., 2003; Ding et al., 2004; Raub and Buskens, 2006), empirically-founded links between social network theory and economic theory are sparse (Raub and Weesie, 1990; Rooks et al., 2000). Our work tries to close the gap between theory and and real-world data by empirically connecting social network theory with transaction cost theory. By means of a sample collected from Polish farms in 2007 we analyse the effect of social networks on the acquisition of concentrated feeding stuffs for cattle, pigs, and poultry. We observe three different acquisition sources, namely own production, contract, and market. The amount of concentrated feeding stuffs procured by any of the three sources is modelled as a fractional response variable, representing the importance of the specific source for the individual farm. As exogenous variables we use inter alia two parameters from the farmers' egocentered networks, namely network size and network density.

The paper is structured as follows: section two provides a brief theoretical overview of the connection of transaction costs and social networks, section three describes the econometric method and the data, section four presents and discusses the estimation results, and the fifth section concludes.

## **2.2 Theoretical Link Between Transaction Costs and Social Networks**

Transaction cost theory argues that every economic transaction is more or less burdened by additional costs, i.e. transaction costs that are both of a physical (e.g. physical transport costs) and an institutional nature (Green and Sheshinski, 1975). Both physical and

institutional transaction costs refer to the sacrifice of resources. However, only institutional transaction costs can be lessened or avoided by choosing the right organisational form (Klaes, 2000).

In 1937 Coase introduced the idea of transaction costs arguing that market transactions often involve higher costs than just the market price. Other costs, including search and information costs, bargaining costs, and policing and enforcement costs, may all potentially add to the costs of procuring something from a market. Firms arise to internalise transactions and so to avoid or lower transaction costs. However, there is a natural limit to what can be produced internally. Managerial supervising and administrative costs might outreach the benefits of an internal solution. Coase argues that the size of a firm is the result of optimally balancing both types of costs.

Williamson (1971, 1973, 1975) follows Coase in arguing that as market transactions are more or less costly, choosing the right *governance structure* can lead to a better outcome under market imperfections. Firms can choose among spot market transactions, short-term and long-term contracts as well as vertical integration to manage transactions. In other words, firms facing additional costs with each transaction have to choose among three different regimes and have to decide which regime minimises institutional transaction costs and supervising/administrative costs. One may presume that these costs vary from firm to firm, forcing firms — even if they are in the same branch — to choose different regimes. But why is it so?

Williamson enriches the basic framework by two concepts: bounded rationality and opportunism. Bounded rationality implies that individuals do not have the capacity to foresee every relevant detail in contracting. As a consequence contracts must always be incomplete. By vertical integration firms internalise the “contracting” and thus reduce negotiation costs and inefficiencies. Opportunism is defined as *self-interest with guile* and becomes mostly important in small number bargaining situations. In competitive markets opportunism might play an inferior role, but in situations where one contracting partner made a special investment in future trade, she might be locked into that particular relationship. So the former competitive situation shifts to a bilateral monopoly. To summarise, transaction costs can be seen as:

*“(...) the ex ante costs of contracting cumulated with the opportunity costs of forgone transactions, together with the ex post costs of enforcing the agreement added to the opportunity costs of not shifting to more profitable activities in the light of new information.” (Ménard, 2004, p.354)*

Following this argumentation one can assert that differences in institutional transaction costs are induced by the nature of the traded good but also by the individuals trading. The better informed partners are, the lesser are negotiation and contracting costs (Rooks et al., 2000). The better partners are connected either by a long trading history or by common contacts, the lesser are negotiation, contracting, and enforcement costs, and the incentive of opportunism (Raub and Weesie, 1990). Taking all this into account the explanation of differences in the level of transaction costs is reduced to trust between partners.

Game theory provides the microeconomic foundation for understanding how conflicts evolve in exchange situations and the instruments that can be used as a solution in the sense of social theory (Raub and Buskens, 2006; Parisi, 2000). On the one hand, it can model trust as repeated games with infinite horizon in dyadic relationships, also defined as temporal embeddedness (e.g. Telser, 1980; Parkhe, 1993; Ellison, 1994; Gautschi, 2000). But the more interesting extension of the game is to allow for multiple actors, introducing network embeddedness (e.g. Buskens, 1998a, 1999; Schmidt et al., 2001).

*“In this category of games, there is no temptation to defect unilaterally because there are safeguards that eliminate all the payoff advantages of unilateral defection” (Parisi, 2000, p. 100)*

If actors play strategically in a coordination game with a payoff leading to the dominant strategy “defect”, they need to trust their partner to stay cooperative (Granovetter, 1985; Smith Ring, 1997). The concept of trust in turn heavily relies on two terms, namely reputation and punishment. Actors connected by dyadic and/or network relations form expectations about each others’ behavior, i.e. each actor has a reputation. The reputation is based on information about an actor’s former behaviour. In the dyadic case this information results from a common past. In the network case information can be transferred through third parties so the availability of information no longer depends on a common history between trade partners. Anyway, if an actor defects, this will consequently lead to a loss of reputation. But a bad reputation is only costly if some kind of punishment follows. In the dyadic case the punishment depends only on the strategy of the trading partner. In the network case punishment can become somewhat more complex, depending not only on the players’ strategy but also on information diffusion in the network. If information about an actor’s conduct is easily dispersing through the network, the probability of punishing bad a conduct will be higher than in networks with low information flow. But also an individual actor’s network position can have a crucial influence on how much and what kind of information is available to the actor.



Following this line of argument, we hypothesise that the structure of a firm's network has an influence on the availability of information and hence on the level of trust the firm encounters on the market. We argue that this trust level, visualised by measurable network characteristics, influences the choice of regime for the acquisition of input goods.

## 2.3 Model and Data

As our endogenous variable contains fractional data, applying a simple OLS regression would lead to the problem that predicted values could never be guaranteed to lie in the unit interval. In fact, the drawbacks of linear models for fractional data are analogous to the drawbacks of linear probability models for binary data (Papke and Wooldridge, 1996). One possibility to solve this problem is to use the logit transformation

$$E\left(\log\left[\frac{y}{(1-y)}\right] \mid \mathbf{x}\right) = \mathbf{x}'\boldsymbol{\beta} \quad (2.1)$$

to map the original variable, which was bounded by 0 and 1, to the real line, then applying OLS. Unfortunately our data includes a considerable amount of zeros and ones which would—applying the logit transformation—lead to undefined values and the exclusion of the observation from the sample. But even when this problem would not appear in our data, there is still the problem that the logit transformation cannot recover  $E(y \mid \mathbf{x})$  without further assumptions (Papke and Wooldridge, 1996). Under the logit transformation the expected value of  $y$  given  $\mathbf{x}$  is

$$E(y \mid \mathbf{x}) = \int_{-\infty}^{\infty} \left( \frac{\exp(\mathbf{x}'\boldsymbol{\beta} + v)}{1 + \exp(\mathbf{x}'\boldsymbol{\beta} + v)} \right) f(v \mid \mathbf{x}) dv \quad (2.2)$$

where  $f(u \mid \mathbf{x})$  is the conditional density of  $u = \log[y/(1-y)] - \mathbf{x}'\boldsymbol{\beta}$  given  $\mathbf{x}$  and  $v$  is a dummy argument of integration. Even under the assumption of  $\mathbf{x}$  and  $u$  being independent,  $E(y \mid \mathbf{x}) \neq \exp(\mathbf{x}'\boldsymbol{\beta})/[1 + \exp(\mathbf{x}'\boldsymbol{\beta})]$ . If  $u$  and  $\mathbf{x}$  are not independent, equation (2.2) cannot be estimated without knowing  $f(\cdot \mid \mathbf{x})$ , which is either algebraically difficult or non-robust (Papke and Wooldridge, 1996). So the preferable method is to estimate  $E(y \mid \mathbf{x})$  without having to estimate the conditional density of  $u$  given  $\mathbf{x}$ . Taking this into account there are two common approaches in the literature solving the problem of proportional data. The first approach applies a generalised linear model (GLM) with binomial distribution and logit link function (McCullagh and Nelder, 1983; Hardin and Hilbe, 2001). The second approach

prefers a quasi-maximum likelihood estimator (QMLE) instead of a GLM (McCullagh and Nelder, 1983; Papke and Wooldridge, 1996).

The generalised linear model was first introduced by Nelder and Wedderburn (1972) and unifies several econometric methods for different types of data. The classical linear model for a response variable  $Y$  is  $y = \mathbf{X}'\boldsymbol{\beta} + u$ , with  $\mathbf{X}$  a  $n \times p$  matrix of covariates and  $\boldsymbol{\beta}$  a  $p \times 1$  vector of  $p$  unknown parameters. In the case of GLM it is extended in its applicability by releasing the assumption of additive errors  $u$ . For the generalised case the density function of  $Y$  is given by

$$f(y) = f(y; \mathbf{X}'\boldsymbol{\beta}) \quad (2.3)$$

with  $\mathbf{X}$  and  $\boldsymbol{\beta}$  forming the *linear predictor*  $\eta = \mathbf{X}'\boldsymbol{\beta}$ . The  $E(y) = \mu$  is defined by the linear predictor  $\eta$  through the function  $g(\mu) = \eta$ , where  $g(\mu)$  is the link function. The density function in (2.3) can be of any appropriate family but for convenience mostly density functions of the exponential family are chosen. The GLM has the form

$$\mu = g^{-1} \left( \sum_{j=1}^p \mathbf{x}_j \beta_j \right) = g^{-1} (\mathbf{X}'\boldsymbol{\beta}) \quad (2.4)$$

with  $\beta_1, \dots, \beta_p$  as the unknown parameters and  $\mathbf{x}_1, \dots, \mathbf{x}_p$  as covariates, while  $\mathbf{x}_j$  can be a quantitative as well as an indicator variable. The GLM is not linear in itself but the link function is supposed to be a strictly monotone function. It defines the scale of linearity but the choice of  $g(\cdot)$  is limited by  $\mu$ .

Let  $y$  be a scalar observation from a distribution of an exponential family with the canonical parameter  $\theta$  and dispersion parameter  $\phi$ . Then the log-likelihood function is

$$l = \frac{(\theta y - b(\theta))}{a(\phi)} + c(y, \phi) \quad (2.5)$$

with  $a(\cdot)$ ,  $b(\cdot)$ , and  $c(\cdot)$  as known functions. Maximising equation (2.5) leads to

$$\frac{\partial l}{\partial \theta} = \frac{\{y - b'(\theta)\}}{a(\phi)} = \frac{(y - \mu)}{a(\phi)} \quad (2.6)$$

Thus,

$$\frac{\partial l}{\partial \mu} = \frac{\frac{\partial l}{\partial \theta}}{\frac{d\mu}{d\theta}} = \frac{y - \mu}{V} \quad (2.7)$$

since

$$\frac{d\mu}{d\theta} = b''(\theta) = \frac{V}{a(\pi)} \quad (2.8)$$

and

$$\frac{\partial l}{\partial \eta} = \frac{\partial l}{\partial \mu} \frac{d\mu}{d\eta} \quad (2.9)$$

the final result is

$$\frac{\partial l}{\partial \beta_j} = \frac{\partial l}{\partial \eta} \frac{\partial \eta}{\partial \beta_j} = \frac{\partial l}{\partial \mu} \frac{d\mu}{d\eta} \mathbf{x}_j = \left( \frac{y - \mu}{V} \right) \frac{d\mu}{d\eta} \mathbf{x}_j \quad (2.10)$$

The maximum-likelihood equations for  $\beta_j$  are then

$$\sum_j \left( \frac{y - \mu}{V} \right) \frac{d\mu}{d\eta} \mathbf{x}_j = 0. \quad (2.11)$$

Specifying a GLM in (2.5) leads to

$$g(\mu) = g(b'(\theta)) = \sum_{j=1}^p x_j \beta_j \quad (2.12)$$

which can lead to algebraically very complicated likelihood functions. Thus, applying the *canonical* link function,  $g(\cdot) = 1/b'(\cdot)$ , allows a simplification to  $g(\mu) = \eta = \theta$ . And the log-likelihood function simplifies to

$$l = \sum_{j=1}^p \left( \beta_j \frac{y x_j}{\phi} \right) - \left\{ \frac{b(\theta)}{\phi} - c(y, \phi) \right\}. \quad (2.13)$$

The canonical link function for the binomial distribution is the logit link function,  $g(\mu) = \log(\mu/(1 - \mu))$  (Hardin and Hilbe, 2001). For fractional response data the logit link maps the interval  $[0, 1]$  of  $y$  on the real line. Fractional response data can be seen as an underlying Bernoulli process. It involves two variables identified by the number of successes,  $y_i$ , of a population of  $k_i$  trials, where it is not necessary that the number of trials is the same for each observation. So—in our case—the farmer has to choose for each entity of concentrated feeding stuff whether he purchases the entity by regime *A* or not. The sum of the positive choices for regime *A* will be represented by the outcome  $y_i$  as fractional data. Therefore,  $\mathbf{Y}$  is assumed to follow a binomial distribution.

Thus, assuming that our random variables are  $Y_i \sim B(k_i, \pi_i)$ , with mean  $\mu_i = k_i \pi_i$  and variance  $V(\mu_i) = k_i \pi_i (1 - \pi_i)$  with  $\pi_i$  the probability of success, respectively, then the log-

likelihood function for a sample of  $n$  realisations  $\mathbf{y} = y_1, \dots, y_n$  can be written as,

$$l(\boldsymbol{\pi}; \mathbf{y}) = \sum_{i=1}^n \left( y_i \log \left( \frac{\pi_i}{1 - \pi_i} \right) + k_i \log(1 - \pi_i) \right). \quad (2.14)$$

Applying the link function of the logit form

$$g(\boldsymbol{\pi}) = \log \left\{ \frac{\boldsymbol{\pi}}{(1 - \boldsymbol{\pi})} \right\}, \quad (2.15)$$

equation (2.14) can be expressed as a function of the unknown parameters  $\boldsymbol{\beta}$  and becomes

$$l(\boldsymbol{\beta}; \mathbf{y}) = \sum_i \sum_j y_i x_{ij} \beta_j - \left\{ \sum_i k_i \log \left( 1 + \exp \left( \sum_j x_{ij} \beta_j \right) \right) \right\} \quad (2.16)$$

As regards the logit link function the maximisation of  $l(\boldsymbol{\beta}; \mathbf{y})$  described in general terms in equation (2.11) is

$$\frac{\partial l}{\partial \beta_j} = \sum_i W_i \frac{y_i - k_i \pi_i}{k_i \pi_i (1 - \pi_i)} \mathbf{x}_j \quad (2.17)$$

with  $W_i = k_i \pi_i (1 - \pi_i)$ . The estimation of the  $\beta$ 's is done iteratively by applying the Newton-Raphson algorithm or iterative reweighted least squares (IRLS).

The problem with applying a GLM with binomial distribution on fractional data is that very often the data arises under conditions where the assumption of binomial variation is unrealistic as the variance in the data is greater than that predicted by the binomial model (McCullagh and Nelder, 1983). In other words the assumption for the binomial distribution of the dispersion parameter being  $\phi = 1$  is violated by  $\phi > 1$ . McCullagh and Nelder (1983) call this problem over-dispersion or heterogeneity. They suggest applying QMLE to release the assumption of  $\phi = 1$ . Applying QMLE leads to the same  $\hat{\beta}_1, \dots, \hat{\beta}_p$  as the GLM (with  $\phi = 1$ ) but the  $cov(\hat{\beta})$  is proportional to  $\phi$  and thus all standard errors are multiplied by  $\phi^{1/2}$  (Hatzinger, 1991). The basic assumption of a QMLE is that the score equation

$$\sum_{i=1}^n \frac{y_i - \mu_i}{\phi_i V(\mu)} \cdot \frac{\partial \mu_i}{\partial \beta_j} = 0, \quad \text{with } j = 1, \dots, p \quad (2.18)$$

produces unbiased  $\hat{\beta}_j$ . In the QMLE framework it is no longer necessary that the  $Y_i$  are from an exponential family. Hence, the assumptions of the model focus only on the two first moments,  $E(Y_i) = \mu_i(\boldsymbol{\beta})$  and  $Var(Y_i) = \phi V(\mu_i)$ , respectively, and no longer on the distribution of the  $Y_i$ . The idea of the QMLE is to find out under which conditions a function  $Q(\boldsymbol{\beta}; \mathbf{y})$  with gradient vector  $U(\boldsymbol{\beta}; \mathbf{y})$  exists. If  $Q(\boldsymbol{\beta}; \mathbf{y})$  exists and meets all the

conditions of a log-likelihood function it is called a quasi-likelihood function (McCullagh and Nelder, 1983). Let again  $\mathbf{y}$  be a vector of response variables of the independent random variables  $\mathbf{Y}$  where the  $i$ -th element of  $\mathbf{Y}$  has mean  $\mu_i$  and covariance matrix  $\phi_i V(\mu_i)$ . As the  $Y_i$  are independent,  $V(\mu) = \text{diag}\{V_1(\mu_1), \dots, V_n(\mu_n)\}$  is a positive- semi-definite matrix of known functions of  $\mu$  and the dispersion parameter  $\phi_i$  can be unknown. Again the aim is to find the relationship between  $\mu$  and the explanatory variables  $\mathbf{x}$ , i.e.  $\mu = \mu(\beta)$ . Under these conditions the score equation

$$U(\beta; \mathbf{y}) = \sum_i \frac{y_i - \mu_i}{\phi V_i(\mu_i)} \cdot \frac{\partial \mu_i}{\partial \beta} \quad (2.19)$$

has the same properties as the derivative of a normal log-likelihood, namely,

$$E(U) = 0 \quad (2.20)$$

$$\text{Var}(U) = \frac{1}{\phi V(\mu_i)} \quad (2.21)$$

Hence,

$$Q(\beta; \mathbf{y}) = \sum_i \int_{y_i}^{\mu_i} \frac{y_i - t}{\phi V_i(t)} dt \quad (2.22)$$

can be treated like a usual log-likelihood function with gradient vector  $U(\beta; \mathbf{y})$ . In the case where a distribution of the exponential family for the  $Y_i$  is still presumed, the QMLE focuses on the variance function only. For a binomial distribution with  $V(\mu) = \mu(1 - \mu)$  the quasi-likelihood function is

$$Q(\mu; \mathbf{y}) = y \log \left( \frac{\mu}{1 - \mu} \right) + \log(1 - \mu) \quad (2.23)$$

which equals the normal log-likelihood function of a binomial distribution.

The quasi-likelihood equation to estimate the  $\beta$ 's is called the quasi-score function. Like in the log-likelihood case it is derived by differentiating  $Q(\mu; \mathbf{y})$ ,

$$U(\beta) = D' V^{-1} \frac{Y - \mu}{\phi} \quad (2.24)$$

with  $D$  a  $n \times p$  matrix with elements  $D_{ij} = \partial \mu_i / \partial \beta_j$ . The covariance matrix of  $U(\beta)$  is

$$i_\beta = D' V^{-1} D \phi^{-1} \quad (2.25)$$

and equals the Fisher information in a normal likelihood function. Under the central limit theorem the quasi-score function is approximately normal and the derived  $\hat{\beta}$ 's are efficient and unbiased (Hatzinger, 1991).

Unlike the log-likelihood function the quasi-likelihood function treats the unknown dispersion parameter separately from  $\beta$ . For the estimation of  $\phi$  it is usual to apply a methods of moments estimator, which is based on the residual vector  $y - \mu$ ,

$$\hat{\phi} = \frac{1}{n-p} \sum_i \frac{(Y_i - \mu_i)^2}{V_i(\mu_i)} = \frac{X^2}{n-p} \quad (2.26)$$

with  $X^2$  is the Pearson residual.

Just like in the GLS the  $\hat{\beta}$  can be calculated by the Newton-Raphson algorithm

$$\beta^{(t+1)} = \beta^t + (D'V^{-1}D)^{-1}U(\beta; y) \quad (2.27)$$

or the IRLS method

$$\hat{\beta}^{(t+1)} = \{D'V^{-1}D\}^{-1}D'V^{-1}D\hat{\beta}^{(t)} - (y - \hat{\mu}) \quad (2.28)$$

As our data shows considerable over-dispersion we follow McCullagh and Nelder (1983) and Papke and Wooldridge (1996) and estimate the model by the QMLE with binomial distribution. To estimate the effects of networks on the choice of governance structure we use data from 616 Polish farms collected in 2007 in the regions Chotcza, Siemiątkowo Koziębrodzkie, Wieliszew, and Kamieniec.

To generate the endogenous variable we examined the percentage of concentrated feeding stuffs for cattle, pigs, and poultry obtained by own production, long-term and short-term contracts, and the market. From all 616 observations 223 had to be removed because of missing or wrong answers. To examine each regime in a general picture we calculate a weighted mean over all three concentrated feeding types for the variables percentage procured by own production (**ownProd**), long term and short term contracts (**contract**), and markets (**market**).

The exogenous variables are network parameters calculated from egocentered networks and variables from extended farm accountancy data. Parameters describing the network structure are outdegrees, with  $l$  = the number of all actors in the egocentered network, and density,  $d = t/(m(m-1))/2$ , where  $t$  is the number of ties between the alteri and  $m$  is the number of alteri in the egocentered network. These two parameters are calculated for each of the four network types, i.e. trade network (**outdI0**, **densI0**), information ex-

change network (**outdIE**, **densIE**), social network (**outdS**, **densS**), and cooperation network (**outdC**, **densC**). Other exogenous variables are livestock unit per hectar (**hpHa**), total land cultivated (**land**), and the average quality of the cultivated land (**quality**).

So we estimate the model

$$E(\text{Proportion of } s \mid \mathbf{x}) = g^{-1}(\alpha_1 + \alpha_2 \text{hpHa} + \alpha_3 \text{land} + \alpha_4 \text{quality} + \alpha_5 (\text{land} \cdot \text{quality}) + \gamma \text{outd} + \delta \text{dens}) \quad (2.29)$$

for  $s \in \{\text{ownProd}, \text{contract}, \text{market}\}$ ,

where **outd** = (**outdIO**, **outdIE**, **outdS**, **outdC**), and **dens** = (**densIO**, **densIE**, **densS**, **densC**), and  $g^{-1}$  is the inverse of the link function which is in our case the logistic function.

## 2.4 Estimation Results

Tables 2.A1–2.A3 summarise the estimation results of the model presented in equation (2.27). As can be seen in Tables 2.A1–2.A3 in none of the three cases the variables **hpHa**, **land**, and **quality** are statistically significant. Therefore, we apply a deviance statistics to test for misspecification in equation (2.27)<sup>1</sup>. Table 2.1 shows the results for the case of own production, which is representative for the two other cases. The deviance statistics in Table 2.1 confirms

Table 2.1: Specification test

Model	Deviance	DoF	Diff Deviance	p-value
1	327.5	407	-	-
outd	317.3	403	10.2	0.037
dens	303.0	403	24.5	0.001
hpHa	327.5	406	0	1
land	327.2	406	0.3	0.583
outd + dens	297.2	399	30.3	0.000
hpHa + land	327.1	405	0.4	0.527
outd + dens + land	296.9	398	30.5	0.617
outd + dens + land + hpHa	296.8	397	30.6	0.791
outd + dens + land + hpHa + quality	280.1	377	47.3	0.668

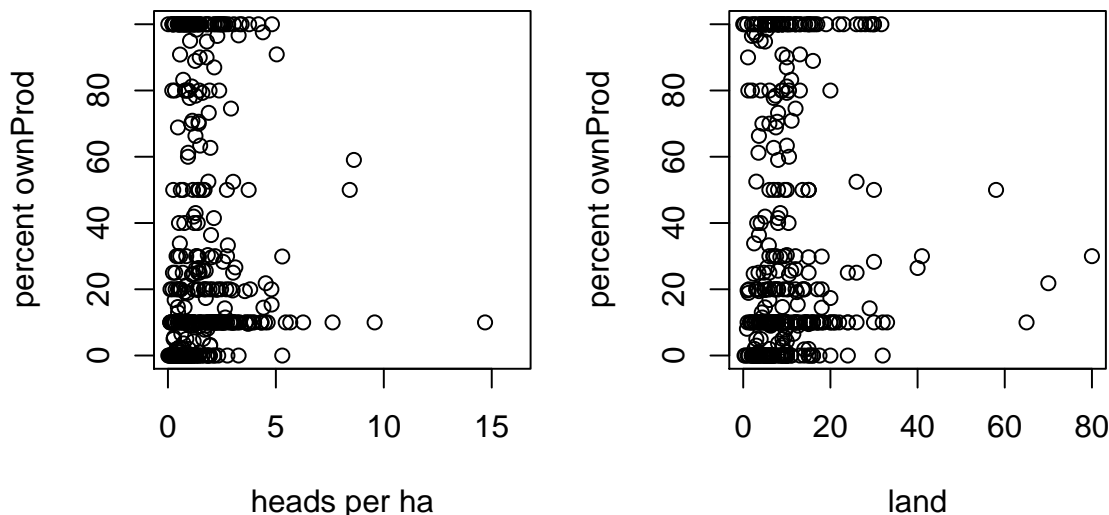
the t-test results. The model does not gain in explanatory quality by adding **hpHa**, **land**, and **quality**, so these variables can be seen as superfluous. Plotting animal heads per hectar and land, respectively on the percentage of own production does not show any significant link, as

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<sup>1</sup>Results can be seen in appendix

figure 2.1 shows. Also the correlation coefficient between percentage of own production and

Figure 2.1: OwnProd plotted against hpHa and land



animal heads per hectare (- .011) and land (.036) shows clearly that there is no connection between the variables. These results are astonishing. As the Polish farm sector is rather small-scaled with an average of 7 hectares per farm (Henningsen, 2009), one possible explanation could be that the data is strongly biased towards small-scale farms in either amount of land and number of animals. This would lead to the phenomenon that we simply don't have enough variance in the data when it comes to bigger farms. Figure 2.2 confirms this assumption. The data is strongly biased towards smaller farms, the distribution has its peak around the Polish average of 7 hectares. Taking this into account we correct our model to the form

$$E(\text{REGIME}_i | \mathbf{x}) = g^{-1}(\beta_1 + \beta_2 \text{outd} + \beta_3 \text{dens}) \quad (2.30)$$

for  $i \in \{\text{ownProd}, \text{contract}, \text{market}\}$ ,

Tables 2.2–2.4 show the final estimation results

Applying a non-linear model we have to calculate the marginal effect for all statistically significant variables on  $E(y_i | \mathbf{x})$ . The marginal effect is calculated by  $G(\mathbf{x}\boldsymbol{\beta})\beta_j$ , with  $G(\cdot)$  as the partial derivative of the link function with respect to  $\beta_j$ . As our link function is of the



Figure 2.2: Histogram of land

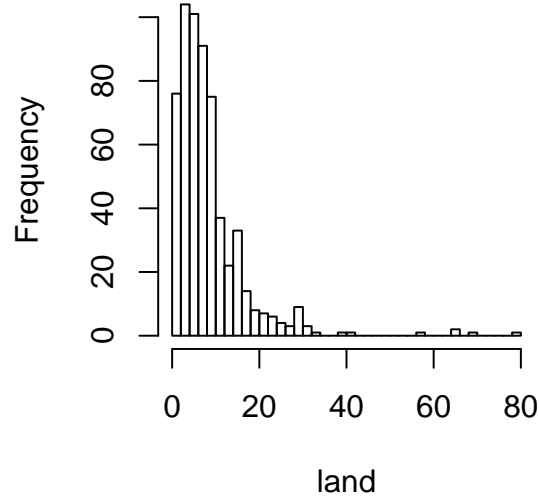


Table 2.2: Corrected results for own production

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.3768	0.2200	-1.71	0.0875
outdIO	-0.0147	0.0579	-0.25	0.8004
outdIE	0.6583	0.2185	3.01	0.0028
outdS	-0.2947	0.2850	-1.03	0.3018
outdC	-0.3046	0.3708	-0.82	0.4119
densIO	-0.9623	0.3186	-3.02	0.0027
densIE	-0.7991	1.0919	-0.73	0.4647
densS	30.6002	1263.7140	0.02	0.9807
densC	0.3781	2.4592	0.15	0.8779

Table 2.3: Corrected results for contract

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-4.2513	0.6985	-6.09	0.0000
outdIO	0.2107	0.1706	1.24	0.2174
outdIE	-1.7834	1.4212	-1.25	0.2103
outdS	0.6616	0.9208	0.72	0.4728
outdC	1.0693	1.0306	1.04	0.3001
densIO	-1.9304	1.1593	-1.67	0.0967
densIE	-54.0851	8693.4040	-0.01	0.9950
densS	-18.5405	3800.6163	-0.00	0.9961
densC	-32.5031	7964.4403	-0.00	0.9967

Table 2.4: Corrected results for market

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	0.3182	0.2178	1.46	0.1449
outdIO	-0.0028	0.0572	-0.05	0.9615
outdIE	-0.5792	0.2169	-2.67	0.0079
outdS	0.2659	0.2808	0.95	0.3442
outdC	0.2237	0.3634	0.62	0.5386
densIO	1.0675	0.3161	3.38	0.0008
densIE	0.6712	1.0893	0.62	0.5381
densS	-30.3722	1277.1179	-0.02	0.9810
densC	-0.1382	2.4237	-0.06	0.9546

logistic form, the marginal effect is calculated by,

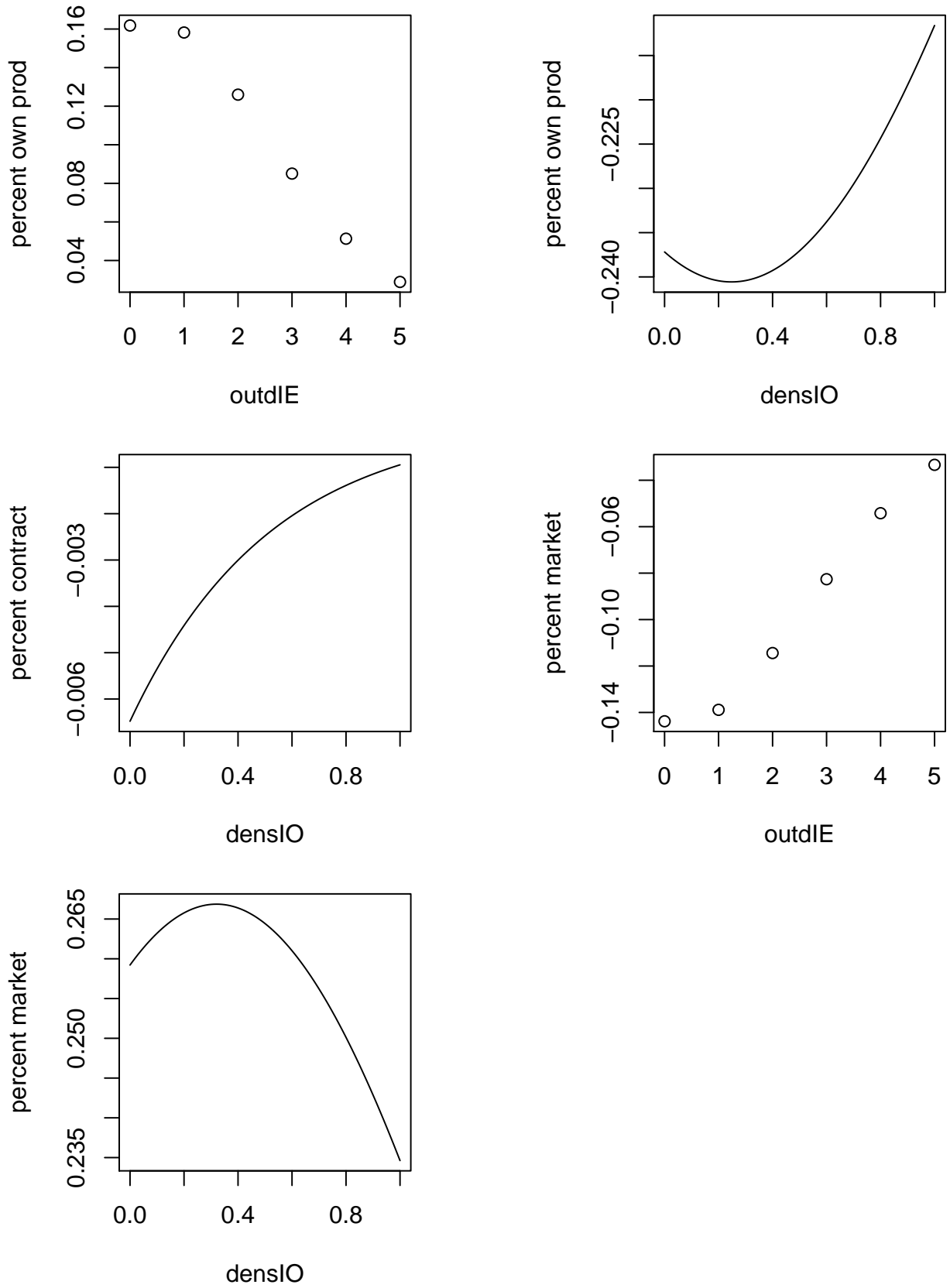
$$ME = \frac{\exp(\mathbf{x}\boldsymbol{\beta})}{(1 + \exp(\mathbf{x}\boldsymbol{\beta}))^2} \beta_j. \quad (2.31)$$

Figure 2.3 shows the plotted results.

In the case of own production we find an economic and highly statistically significant influence of `outdIE` and `densIO`. The outdegree of the information network (`outdIE`) shows a strong but diminishing positive impact on the amount of concentrated feeding stuffs from own production. This can be interpreted in the way that a larger information network provides the farmer with adequate information on production methods and novelties so that a farmer's capacities to produce her own concentrated feeding stuffs increases. These findings confirm Burt (1992) who states that large and loose networks are better able to provide its members with usefull information than small and narrow ones. On the other hand the density of the trade network `densIO` shows a strong negative impact on the amount of concentrated feeding stuffs from own production. This confirms our hypothesis as it shows that farmers with dense trade networks prefer rather to buy their concentrated feeding stuffs. A trade network with high density seems to increase security in trade and significantly augments the incentive to procure the feeding stuffs by other channels than own production.

In the contract regime network parameters show barely any statistically or economic significant influence on the decision of purchase of concentrated feeding stuffs over contracts. A problem with contracts is that only a significantly small amount of farmers chose that kind of regime so the data in this case can be seen as problematic, as there is too little variance in the endogenous variable. Nevertheless, we find a weak statistically significant negative influence of the trade network's density on the decision pro contract. Hence, there is a tendency to denser farmers' trade networks lowering the probability to purchase concentrated

Figure 2.3: Marginal effects



feeding stuffs over contracts. Although the influence is very weak we find an indication for our hypothesis, as it seems that dense trade networks increase the trust level and reduce negotiation and security costs, hence, reduce the incentive to close a contract.

As we found little influence in the contract regime —using fractional data— the market regime is just the mirror image of the regime own production. As expected we find a highly economical and statistically significant influence of `outdIE` and `densIO` just with opposite signs.

To sum up, although not all networks reveal an influence on the choice of governance structure, still, highly economic and statistically significant results especially for the trade network confirm our hypothesis that network structure has an impact on the choice of governance structure. Especially interesting with respect to sociological theory are the strong effects of the trade and information network on the choice of own production.

## **2.5 Conclusion**

In section 2.2 we formulated the hypothesis that a firm’s network structure has an impact on the choice of governance structure in the acquisition of input. To prove our hypothesis we use data from a farmers’ survey collected in 2007. We regress the percentage of concentrated feeding stuffs obtained by one of the three regimes, i.e. own production, contract, and market, on two network parameters, size, and density. The influence of networks show surprisingly strong influence. Hence, we can state that our empirical findings confirm our hypothesis. Particular the trade network shows a significant impact on the choice of regime. Farmers with a dense trade network are to a significant degree less likely to produce their own concentrated feeding stuffs. One could argue that it could be the other way round, i.e. farmers who mainly purchase concentrated feeding stuffs by trade and not by own production consequently have a denser trade network. This, however, is countered by the fact that farmer’s with dense trade networks not only produce a smaller proportion of feeding stuffs themselves, but they also do significantly less contracting. Other interesting but unexpected findings in our data are that farmers with large information networks produce significantly more concentrated feeding stuffs on their own than farmers with smaller information networks. This confirms Burt (1992)’s assumptions about the effect of large and loose networks on information diffusion and indicates that such networks seem to have a positive impact on a farmer’s productivity. For future work these preliminary findings raise further interesting questions which should be addressed more closely, as for example the impact of networks on farm performance or the impact of networks on transaction costs.

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## Appendix

Table 2.A1: Results for own production

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-1.4052	0.7621	-1.84	0.0660
hpHa	-0.0057	0.0601	-0.09	0.9246
land	0.0173	0.0558	0.31	0.7565
quality	0.2258	0.1700	1.33	0.1848
I(land * quality)	-0.0025	0.0134	-0.18	0.8537
outdIO	-0.0190	0.0612	-0.31	0.7559
outdIE	0.6304	0.2211	2.85	0.0046
outdS	-0.3517	0.2950	-1.19	0.2339
outdC	-0.2383	0.3818	-0.62	0.5329
densIO	-0.9580	0.3433	-2.79	0.0055
densIE	-0.6717	1.0987	-0.61	0.5414
densS	30.5543	1264.6457	0.02	0.9807
densC	0.3888	2.4415	0.16	0.8736



Table 2.A2: Results for contract

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-5.5380	2.4348	-2.27	0.0235
hpHa	0.1404	0.1403	1.00	0.3174
land	0.0072	0.1964	0.04	0.9709
quality	0.2977	0.5374	0.55	0.5799
I(land * quality)	-0.0112	0.0482	-0.23	0.8158
outdIO	0.2644	0.1666	1.59	0.1133
outdIE	-1.7289	1.3096	-1.32	0.1876
outdS	0.6082	0.8845	0.69	0.4921
outdC	1.2621	0.9808	1.29	0.1989
densIO	-2.0447	1.1252	-1.82	0.0700
densIE	-51.7944	8315.7907	-0.01	0.9950
densS	-18.4639	3644.6450	-0.01	0.9960
densC	-32.5080	7603.4742	-0.00	0.9966

Table 2.A3: Results for market

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	1.4116	0.7532	1.87	0.0617
hpHa	-0.0063	0.0587	-0.11	0.9144
land	-0.0157	0.0552	-0.28	0.7765
quality	-0.2401	0.1681	-1.43	0.1541
I(land * quality)	0.0026	0.0133	0.20	0.8425
outdIO	-0.0027	0.0603	-0.05	0.9637
outdIE	-0.5490	0.2191	-2.51	0.0126
outdS	0.3191	0.2900	1.10	0.2718
outdC	0.1420	0.3740	0.38	0.7043
densIO	1.0744	0.3401	3.16	0.0017
densIE	0.5474	1.0958	0.50	0.6177
densS	-30.3249	1274.3757	-0.02	0.9810
densC	-0.1150	2.4055	-0.05	0.9619



## **Kapitel 3**

# **Multiple Output Stochastic Frontiers: Two Approaches**

**Geraldine Henningsen**



## 3.1 Introduction

Using multiple output stochastic production frontier applies to situations where:

- Inefficiency in the production is assumed so that the error term is left-skewed.
- Multiple products are produced and it is assumed that the underlying production technologies differ significantly.
- Duality cannot be exploited either because of missing price data or because profit maximisation/cost minimisation cannot be assumed (e.g. in the public sector).

There are two approaches to estimating multiple output production frontiers. The first, the output distance function, is based on Shephard (1953, 1970). A more recent approach is the multiple output stochastic ray production function developed by Löthgren (2000). In the following, we will apply a Monte Carlo simulation to test the performance of both approaches.

The article is organised as follows: section two gives a short introduction to the principles of the multiple output distance and stochastic ray production functions; section three discusses the results of the Monte Carlo simulation; and finally, section four concludes.

## 3.2 Distance vs. stochastic ray

### 3.2.1 Output distance function

The output distance function models the production technology which transforms multiple inputs into multiple outputs.<sup>1</sup> The output distance function is defined as:

$$D_o(\mathbf{x}, \mathbf{y}) = \inf\{\delta : (\mathbf{y}/\delta) \in P(\mathbf{x})\} \quad (3.1)$$

where  $\mathbf{y}$  is the vector of outputs,  $\mathbf{x}$  is the vector of inputs,  $P(\mathbf{x})$  is the output set,  $D_o(\mathbf{x}, \mathbf{y})$  describes the distance from the firm's output vector to the production frontier, and  $\delta \leq 1$  is the corresponding level of efficiency. The output distance function seeks the largest proportional increase in the observed output vector  $\mathbf{y}$ , provided that the expanded vector  $(\mathbf{y}/\delta)$  is still an element of the original output set  $P(\mathbf{x})$  (Kumbhakar and Lovell, 2000). The distance function has the following properties (Coelli et al., 2005):

- i.  $D_o(\mathbf{x}, 0) = 0$  for all non-negative  $\mathbf{x}$

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<sup>1</sup>In the case of single output, the output distance function reduces to the production frontier.

- ii.  $D_o(\mathbf{x}, \mathbf{y})$  is non-decreasing in  $\mathbf{y}$  and non-increasing in  $\mathbf{x}$
- iii.  $D_o(\mathbf{x}, \mathbf{y})$  is linearly homogeneous in  $\mathbf{y}$
- iv.  $D_o(\mathbf{x}, \mathbf{y})$  is quasi-convex in  $\mathbf{x}$  and convex in  $\mathbf{y}$
- v. if  $\mathbf{y} \in P(\mathbf{x})$ , then  $D_o(\mathbf{x}, \mathbf{y}) \leq 1$ , and
- vi.  $D_o(\mathbf{x}, \mathbf{y}) = 1$  if  $\mathbf{y}$  lies on the production frontier.

The econometric specification of the distance function applying the translog form for  $k = 1, 2, \dots, K$  inputs and  $n = 1, 2, \dots, N$  outputs is as follows:

$$\begin{aligned} \ln(D_o) - \ln(|\mathbf{y}|) &= \alpha_0 + \sum_n \alpha_n \ln(y_n/|\mathbf{y}|) + 0.5 \sum_n \sum_m \alpha_{nm} \ln(y_n/|\mathbf{y}|) \ln(y_m/|\mathbf{y}|) \quad (3.2) \\ &+ \sum_k \beta_k \ln(x_k) + 0.5 \sum_k \sum_l \beta_{kl} \ln(x_k) \ln(x_l) \\ &+ \sum_n \sum_k \gamma_{nk} \ln(y_n/|\mathbf{y}|) \ln(x_k), \end{aligned}$$

where the Euclidean distance  $|\mathbf{y}|$  is used for normalising outputs,  $\alpha_0$  is a scalar intercept,  $\alpha = [\alpha_n]$ ,  $n = 1, \dots, N$  and  $\beta = [\beta_k]$ ,  $k = 1, \dots, K$  are vectors of first-order parameters and  $\mathbf{A} = [\alpha_{nm}]$ ,  $n, m = 1, \dots, N$ ,  $\mathbf{B} = [\beta_{kl}]$ ,  $k, l = 1, \dots, K$ , and  $\mathbf{C} = [\gamma_{nk}]$ ,  $n = 1, \dots, N$ ,  $k = 1, \dots, K$  are matrices of second-order parameters, where  $\mathbf{A}$  and  $\mathbf{B}$  are symmetric, i.e.  $\alpha_{nm} = \alpha_{mn} \forall n, m = 1, \dots, N$  and  $\beta_{kl} = \beta_{lk} \forall k, l = 1, \dots, K$ . The restrictions required for homogeneity of degree 1 in outputs are:

$$\sum_n \alpha_n = 1 \quad (3.3)$$

$$\sum_n \alpha_{nm} = 0 \forall m = 1, \dots, N \quad (3.4)$$

$$\sum_n \gamma_{nk} = 0 \forall k = 1, \dots, K \quad (3.5)$$

We use the analogy between the distance measure  $\ln(D_o)$  of output distance function (3.2) and the efficiency term  $u$  of the usual single-input stochastic production frontier  $y = f(\mathbf{x}) + v - u$  and substitute  $-u$  for  $\ln(D_o)$  in equation (3.2). After adding a noise term  $v$  we get:

$$\begin{aligned} -\ln(|\mathbf{y}|) &= \alpha_0 + \sum_n \alpha_n \ln(y_n/|\mathbf{y}|) + 0.5 \sum_n \sum_m \alpha_{nm} \ln(y_n/|\mathbf{y}|) \ln(y_m/|\mathbf{y}|) \quad (3.6) \\ &+ \sum_k \beta_k \ln(x_k) + 0.5 \sum_k \sum_l \beta_{kl} \ln(x_k) \ln(x_l) \\ &+ \sum_n \sum_k \gamma_{nk} \ln(y_n/|\mathbf{y}|) \ln(x_k) + v + u \end{aligned}$$

As in the usual single-input stochastic production frontier, we assume that the noise term  $v$  follows a normal distribution with mean 0 and variance  $\sigma$  and the inefficiency term  $u$  follows a half-normal distribution. Imposing the homogeneity restrictions (3.3) to (3.5)), we get

$$\begin{aligned}
 -\ln(|\mathbf{y}|) = & \alpha_0 + \ln(y_N/|\mathbf{y}|) & (3.7) \\
 & + \sum_{n=1}^{N-1} \alpha_n \ln(y_n/y_N) + 0.5 \sum_{n=1}^{N-1} \sum_{m=1}^{N-1} \alpha_{nm} \ln(y_n/y_N) \ln(y_m/y_N) \\
 & + \sum_k \beta_k \ln(x_k) + 0.5 \sum_k \sum_l \beta_{kl} \ln(x_k) \ln(x_l) \\
 & + \sum_{n=1}^{N-1} \sum_k \gamma_{nk} \ln(y_n/y_N) \ln(x_k) + v + u,
 \end{aligned}$$

which can be estimated as

$$\begin{aligned}
 -\ln(y_N) = & \alpha_0 + \sum_{n=1}^{N-1} \alpha_n \ln(y_n/y_N) + 0.5 \sum_{n=1}^{N-1} \sum_{m=1}^{N-1} \alpha_{nm} \ln(y_n/y_N) \ln(y_m/y_N) & (3.8) \\
 & + \sum_k \beta_k \ln(x_k) + 0.5 \sum_k \sum_l \beta_{kl} \ln(x_k) \ln(x_l) \\
 & + \sum_{n=1}^{N-1} \sum_k \gamma_{nk} \ln(y_n/y_N) \ln(x_k) + v + u.
 \end{aligned}$$

### 3.2.2 Stochastic ray production frontier

The multiple output stochastic ray production function (Löthgren, 2000) in turn directly applies the stochastic production frontier. Multiple outputs are modeled by decomposing the vector of  $n$  output quantities  $\mathbf{y}$  into a scalar distance component  $l$  and a vector of direction measures  $\mathbf{m}(\boldsymbol{\theta})$  so that  $\mathbf{y} = l \cdot \mathbf{m}(\boldsymbol{\theta})$ . The distance component is the Euclidean norm  $l = l(\mathbf{y}) = |\mathbf{y}| = \left(\sum_{i=1}^N y_i^2\right)^{1/2}$ . The direction is measured by polar coordinates, i.e. the angles related to the ratios of one output to the others:

$$m_i(\boldsymbol{\theta}) = \frac{y_i}{l(\mathbf{y})} = \cos(\theta_i) \prod_{j=0}^{i-1} \sin(\theta_j) \quad \text{with } \sin(\theta_0) = \cos(\theta_N) = 1 \quad (3.9)$$

A multiple-output production function can be defined that gives the maximum output norm given inputs and the direction of the output vector, given by the polar coordinates, so that the ray production function is defined as:

$$f(\mathbf{x}, \boldsymbol{\theta}(\mathbf{y})) = \max \{ \iota \in \mathbb{R}_+ : \iota \cdot \mathbf{m}(\boldsymbol{\theta}(\mathbf{y})) \in P(\mathbf{x}) \} \quad (3.10)$$

To specify the econometric estimation equation the first choice of a functional form is again the flexible translog function. Therefore, the final estimation equation is denoted by:

$$\begin{aligned} \ln(|\mathbf{y}|) = & \alpha_0 + \sum_{n=1}^{N-1} \alpha_n \theta_n + 0.5 \sum_{n=1}^{N-1} \sum_{m=1}^{N-1} \alpha_{nm} \theta_n \theta_m \\ & + \sum_k \beta_k \ln(x_k) + 0.5 \sum_k \sum_l \beta_{kl} \ln(x_k) \ln(x_l) \\ & + \sum_{n=1}^{N-1} \sum_k \gamma_{nk} \theta_n \ln(x_k) + v - u \end{aligned} \quad (3.11)$$

### 3.3 Monte Carlo simulation

In the following we compare the output distance function (OD) and the stochastic ray production frontier (SR) by a Monte Carlo simulation. As both approaches use different specifications, a direct comparison of parameter estimates is not possible. Therefore, we follow Coelli and Perelman (2000) and compare the quality of the efficiency measure of both approaches.

We implemented this Monte Carlo experiment in the statistical programming language “R” (R Development Core Team, 2009) and used the add-on packages “frontier” (Coelli and Henningsen, 2010) for stochastic frontier estimations, “fdrtool” (Strimmer, 2009) to generate half-normal random numbers, and “MCMCpack” (Martin, Quinn, and Park, 2009) to generate random numbers from a Dirichlet distribution.

This Monte Carlo analysis is based on a production model with two outputs ( $y_1$  and  $y_2$ ) and three inputs ( $x_1$ ,  $x_2$ , and  $x_3$ ). We generate 200 observations of each input by drawing random numbers from  $\chi^2$  distributions and introducing correlation between the input quantities, as this is also the case in many real data sets.<sup>2</sup> The squared output “proportions”  $m_i^2 = y_i^2/|\mathbf{y}|^2$ , which sum up to one, are generated by drawing random numbers from a Dirichlet distribution with shape parameters equal to vector  $(1, 1)'$  so that  $E[m_1^2] = E[m_2^2] = 0.5$  and  $V[m_1^2] = V[m_2^2] = 1/12$ . We generate the inefficiency term  $u$  from a half-normal distribution with  $E[u] = 0.2$  and  $V[u] = 0.02$ , which results in an average (true) efficiency of roughly 83%.<sup>3</sup>

The parameters chosen for the OD function are shown in the left column of table 3.1. We have checked that the monotonicity conditions derived in section 3.2.1 are fulfilled at all data points used in this Monte Carlo study. The elasticities of scale are around 1.15, which is also the case in many empirical applications. Multiplying the specification of the translog OD

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<sup>2</sup>Details can be obtained from the R script shown in appendix 3.E.

<sup>3</sup>A table with all covariates (input quantities, “proportion” of first output, and efficiencies) can be found in appendix 3.A.



function in (3.7) by  $-1$  and comparing it to the translog SR function in (3.11), shows that both functions are equal at the chosen data points  $\mathbf{y} = \mathbf{y}^0$  and  $\boldsymbol{\theta} = \boldsymbol{\theta}(\mathbf{y}^0)$ , when there is the following relationship between the coefficients of the translog SR function ( $\alpha^R$ ,  $\beta^R$ , and  $\gamma^R$ ) and the coefficients of the translog OD function ( $\alpha^D$ ,  $\beta^D$ , and  $\gamma^D$ ):

$$\alpha_0^R = -\alpha_0^D - \ln(y_N^0/|\mathbf{y}^0|) = -\alpha_0^D - \ln(m_N^0) \quad (3.12)$$

$$\alpha_n^R = -\frac{\alpha_n^D \ln(y_n^0/y_N^0)}{\theta_n^0} = -\frac{\alpha_n^D \ln(m_n^0/m_N^0)}{\theta_n^0} \quad \forall n = 1, \dots, N-1 \quad (3.13)$$

$$\alpha_{nm}^R = -\frac{\alpha_{nm}^D \ln(y_n^0/y_N^0) \ln(y_m^0/y_N^0)}{\theta_n^0 \theta_m^0} \quad (3.14)$$

$$= -\frac{\alpha_{nm}^D \ln(m_n^0/m_N^0) \ln(m_m^0/m_N^0)}{\theta_n^0 \theta_m^0} \quad \forall n, m = 1, \dots, N-1 \quad (3.15)$$

$$\beta_k^R = -\beta_k^D \quad \forall k = 1, \dots, K \quad (3.16)$$

$$\beta_{kl}^R = -\beta_{kl}^D \quad \forall k, l = 1, \dots, K \quad (3.17)$$

$$\gamma_{nk}^R = -\frac{\gamma_{nk}^D \ln(y_n^0/y_N^0)}{\theta_n^0} \quad (3.18)$$

$$= -\frac{\gamma_{nk}^D \ln(m_n^0/m_N^0)}{\theta_n^0} \quad \forall n = 1, \dots, N-1, k = 1, \dots, K \quad (3.19)$$

We choose the parameters of the stochastic ray function so that the OD and the SR function are equal at the sample mean of the output ratio of good one ( $m_1$ ), i.e.  $m_1^0 = E[m_1] = \bar{m}_1$ ,  $m_2^0 = \sqrt{1 - (m_1^0)^2}$  and  $\theta_1^0 = \text{acos}(m_1^0)$ . The elasticity of scale is still around 1.15 and the monotonicity conditions are fulfilled at all data points as well. The coefficients of the SR function are shown in the right column of table 3.1.

The deterministic aggregated output quantities  $|\mathbf{y}|$  for the OD and the SR functions are calculated using equations (3.6) and (3.11), respectively, where the stochastic noise term  $v$  is set to zero and the inefficiency term  $u$  is added, in the case of the OD function, and subtracted in the case of the SR function. In each replication of the Monte Carlo experiment, we add a new vector of noise terms  $v$ , which is drawn from a Gaussian distribution with  $E[v] = 0$  and  $V[v] = 0.1$ . Then, we can calculate the quantities of both outputs by

$$y_i = m_i |\mathbf{y}|, \quad i = 1, 2. \quad (3.20)$$

We run the simulation 1000 times. At each replication, we generate a new vector of random noise terms, calculate the stochastic ‘‘observed’’ output quantities for both the OD and the SR function and estimate equations (3.8) and (3.11) as stochastic frontier models. The script

Table 3.1: True coefficients in Monte Carlo study

	translog OD function	translog SR function
$\alpha_0$	2.900	-2.619
$\alpha_1$	0.400	0.066
$\alpha_{11}$	0.050	-0.001
$\beta_1$	-0.200	0.200
$\beta_2$	-0.400	0.400
$\beta_3$	-0.300	0.300
$\beta_{11}$	0.003	-0.003
$\beta_{12}$	-0.040	0.040
$\beta_{13}$	-0.020	0.020
$\beta_{22}$	0.009	-0.009
$\beta_{23}$	-0.008	0.008
$\beta_{33}$	0.040	-0.040
$\gamma_{11}$	-0.040	-0.007
$\gamma_{12}$	-0.007	-0.001
$\gamma_{13}$	0.067	0.011

used for the simulation is shown in appendix 3.E. Table 3.2 presents the results for the parameter estimates of the OD and the SR function.

Table 3.2: Precision of parameter estimates

	OD: bias	OD: RMSE	OD: MSE	SR: bias	SR: RMSE	SR: MSE
$\alpha_0$	-0.012	0.931	0.868	-0.492	1.111	1.235
$\alpha_1$	0.209	0.244	0.060	0.665	0.755	0.570
$\alpha_{11}$	0.016	0.026	0.001	-0.153	0.221	0.049
$\beta_1$	0.048	0.416	0.173	0.127	0.442	0.196
$\beta_2$	0.051	0.504	0.254	0.043	0.522	0.273
$\beta_3$	-0.082	0.532	0.283	-0.017	0.533	0.284
$\beta_{11}$	-0.167	0.239	0.057	0.163	0.237	0.056
$\beta_{12}$	0.074	0.164	0.027	-0.085	0.167	0.028
$\beta_{13}$	0.091	0.170	0.029	-0.086	0.167	0.028
$\beta_{22}$	-0.154	0.234	0.055	0.156	0.234	0.055
$\beta_{23}$	0.051	0.148	0.022	-0.049	0.147	0.022
$\beta_{33}$	-0.118	0.253	0.064	0.119	0.253	0.064
$\gamma_{11}$	-0.074	0.085	0.007	-0.187	0.213	0.046
$\gamma_{12}$	-0.036	0.052	0.003	-0.091	0.131	0.017
$\gamma_{13}$	0.038	0.059	0.003	0.091	0.141	0.020

The results in table 3.2 should be interpreted very carefully, as both equations must in fact be seen as different models. Anyway, for the parameters that have the same absolute size in both models ( $\beta_n, \beta_{nm}$ ), the two models show rather similar (root) mean square errors, which

indicates that both models are equally capable of finding good estimates for the true parameters. An expedient comparison of the results regarding the parameters that are different in both models ( $\alpha_0, \alpha_1, \alpha_{11}, \gamma_{1k}$ ) is not possible. At this point, we cannot tell whether the observed differences originate in the difference of the parameter value, or in different abilities of the functions to find good estimates. In general, for our given setting, both models return parameter estimates of very mixed quality. Future research should investigate whether this characteristic persists under different settings of the Monte Carlo analysis.

Because of the difficulties mentioned above, we will mainly concentrate on the estimation of the inefficiencies. Table 3.3 presents means and standard errors of the biases, RMSEs, and MSEs of the efficiency estimates. The biases, RMSEs, and MSEs of the efficiency estimates for each individual firm are shown in appendices 3.B, 3.C, and 3.D. For our specification of

Table 3.3: Precision of inefficiency estimates

	OD: bias	OD: RMSE	OD: MSE	SR: bias	SR: RMSE	SR: MSE
mean	-0.00314	0.07123	0.00526	0.00154	0.07140	0.00534
std. dev.	0.03959	0.01367	0.00202	0.04089	0.01564	0.00234

the Monte Carlo simulation, the results show that both approaches are nearly equally capable of determining the efficiency levels of firms. As for the parameter estimates, whether this effect continues under different settings, must be examined.

### 3.4 Conclusion

The analysis with respect to the quality of frontier models suffers from the fact that the comparability of parameter estimates is limited. Therefore, our analysis focuses on the quality of the inefficiency measure to compare the output distance function with the stochastic ray production frontier. Our Monte Carlo analysis indicates that neither of the two models is clearly superior. However, these findings are only valid for the settings of our Monte Carlo experiment and should not be unduly generalised. Anyway, future research should include the extension to the n-output case, instead of the limitation to just two outputs. Furthermore, the output ratio should reflect the input relation, instead of being completely random. The most crucial point is the validation of the presented results by changing the setting of the simulation with respect to sample size, input correlation, different parameter values, increased or reduced general inefficiency, and increased or reduced variance in the error term.

## Appendix

### 3.A Covariates

Table 3.A1: Covariates

Obs. No.	$x_1$	$x_2$	$x_3$	$y_1/ y $	$\exp(-u)$	Obs. No.	$x_1$	$x_2$	$x_3$	$y_1/ y $	$\exp(-u)$
1	9.363	11.134	10.153	0.367	0.936	2	9.986	13.543	10.682	0.678	0.848
3	10.929	9.615	9.967	0.849	0.903	4	11.523	17.563	13.281	0.361	0.988
5	9.255	15.407	4.648	0.457	0.856	6	8.520	5.429	8.286	0.822	0.826
7	10.700	13.348	27.410	0.674	0.899	8	6.990	19.181	5.782	0.438	0.983
9	16.032	7.054	3.823	0.775	0.813	10	8.583	12.112	10.515	0.842	0.980
11	16.038	10.048	14.082	0.944	0.812	12	6.536	5.943	17.515	0.936	0.909
13	8.410	11.095	9.094	0.873	0.762	14	7.185	9.720	6.221	0.818	0.965
15	5.517	10.363	10.536	0.601	0.655	16	8.851	6.367	14.060	0.662	0.756
17	11.249	7.479	15.560	0.941	0.989	18	17.476	12.908	11.559	0.963	0.831
19	5.246	18.392	12.063	0.817	0.728	20	13.561	9.254	13.672	0.594	0.822
21	4.684	13.131	6.099	0.722	0.802	22	9.160	15.313	15.917	0.827	0.996
23	5.130	9.028	10.139	0.771	0.826	24	10.986	15.742	5.401	0.077	0.874
25	9.771	16.665	9.685	0.698	0.915	26	9.717	8.211	14.951	0.853	0.984
27	11.059	15.866	6.839	0.633	0.835	28	6.713	9.585	11.982	0.956	0.778
29	7.494	3.420	11.055	0.659	0.890	30	6.644	15.775	15.338	0.756	0.866
31	9.948	5.479	6.010	0.725	0.920	32	10.638	10.910	14.887	0.623	0.855
33	13.241	14.669	18.649	0.715	0.910	34	10.762	21.955	6.329	0.332	0.653
35	8.395	10.560	11.892	0.805	0.837	36	20.441	4.179	10.745	0.924	0.912
37	25.131	9.220	10.769	0.216	0.996	38	6.024	6.797	10.170	0.809	0.994
39	10.216	9.895	3.947	0.965	0.924	40	6.989	9.425	14.274	0.451	0.861

### 3.B Biases of efficiency estimates

Table 3.A2: Biases of the efficiency term - output distance function

Obs. No.	bias	Obs. No.	bias	Obs. No.	bias	Obs. No.	bias	Obs. No.	bias
1	0.0503	2	0.1320	3	0.0778	4	0.0034	5	0.1149
6	0.1572	7	0.0915	8	0.0117	9	0.1807	10	0.0166
11	0.1607	12	0.0863	13	0.2117	14	0.0160	15	0.3382
16	0.2267	17	-0.0035	18	0.1587	19	0.2555	20	0.1508
21	0.1818	22	-0.0418	23	0.1643	24	0.1114	25	0.0708
26	-0.0068	27	0.1207	28	0.1932	29	0.0888	30	0.1048
31	0.0681	32	0.1240	33	0.0750	34	0.3397	35	0.1481
36	0.0802	37	-0.0124	38	-0.0049	39	0.0658	40	0.1190

Table 3.A3: Biases of the efficiency term - stochastic ray production frontier

Obs. No.	MSE	Obs. No.	MSE	Obs. No.	MSE	Obs. No.	MSE	Obs. No.	MSE
1	0.0056	2	0.0167	3	0.0078	4	0.0009	5	0.0173
6	0.0227	7	0.0080	8	0.0005	9	0.0311	10	0.0005
11	0.0300	12	0.0067	13	0.0427	14	0.0017	15	0.0992
16	0.0472	17	0.0007	18	0.0250	19	0.0623	20	0.0258
21	0.0364	22	0.0075	23	0.0258	24	0.0135	25	0.0054
26	0.0007	27	0.0193	28	0.0459	29	0.0099	30	0.0146
31	0.0048	32	0.0166	33	0.0065	34	0.1077	35	0.0250
36	0.0062	37	0.0018	38	0.0033	39	0.0046	40	0.0169

### 3.C Root mean square errors of efficiency estimates

Table 3.A4: Root mean square error of the efficiency term - output distance function

Obs. No.	RMSE	Obs. No.	RMSE	Obs. No.	RMSE	Obs. No.	RMSE	Obs. No.	RMSE
1	0.0575	2	0.1373	3	0.0888	4	0.0131	5	0.1284
6	0.1597	7	0.0929	8	0.0144	9	0.1809	10	0.0182
11	0.1718	12	0.0865	13	0.2170	14	0.0385	15	0.3385
16	0.2287	17	0.0253	18	0.1598	19	0.2578	20	0.1611
21	0.1834	22	0.0867	23	0.1650	24	0.1159	25	0.0746
26	0.0471	27	0.1399	28	0.2004	29	0.0951	30	0.1150
31	0.0708	32	0.1304	33	0.0847	34	0.3399	35	0.1505
36	0.0815	37	0.0301	38	0.0187	39	0.0684	40	0.1277

Table 3.A5: Root mean square error of the efficiency term - stochastic ray production frontier

Obs. No.	RMSE	Obs. No.	RMSE	Obs. No.	RMSE	Obs. No.	RMSE	Obs. No.	RMSE
1	0.0749	2	0.1293	3	0.0885	4	0.0306	5	0.1316
6	0.1506	7	0.0894	8	0.0230	9	0.1764	10	0.0214
11	0.1733	12	0.0819	13	0.2066	14	0.0409	15	0.3150
16	0.2172	17	0.0272	18	0.1581	19	0.2496	20	0.1607
21	0.1908	22	0.0864	23	0.1607	24	0.1161	25	0.0732
26	0.0259	27	0.1390	28	0.2141	29	0.0993	30	0.1210
31	0.0691	32	0.1289	33	0.0809	34	0.3282	35	0.1580
36	0.0790	37	0.0422	38	0.0579	39	0.0681	40	0.1300

### 3.D Mean square errors of efficiency estimates

Table 3.A6: Mean square error of the efficiency term - output distance function

Obs. No.	MSE	Obs. No.	MSE	Obs. No.	MSE	Obs. No.	MSE	Obs. No.	MSE
1	0.0033	2	0.0189	3	0.0079	4	0.0002	5	0.0165
6	0.0255	7	0.0086	8	0.0002	9	0.0327	10	0.0003
11	0.0295	12	0.0075	13	0.0471	14	0.0015	15	0.1146
16	0.0523	17	0.0006	18	0.0255	19	0.0664	20	0.0259
21	0.0336	22	0.0075	23	0.0272	24	0.0134	25	0.0056
26	0.0022	27	0.0196	28	0.0402	29	0.0091	30	0.0132
31	0.0050	32	0.0170	33	0.0072	34	0.1156	35	0.0227
36	0.0066	37	0.0009	38	0.0003	39	0.0047	40	0.0163

Table 3.A7: Mean square error of the efficiency term - stochastic ray production frontier

Obs. No.	MSE	Obs. No.	MSE	Obs. No.	MSE	Obs. No.	MSE	Obs. No.	MSE
1	0.0056	2	0.0167	3	0.0078	4	0.0009	5	0.0173
6	0.0227	7	0.0080	8	0.0005	9	0.0311	10	0.0005
11	0.0300	12	0.0067	13	0.0427	14	0.0017	15	0.0992
16	0.0472	17	0.0007	18	0.0250	19	0.0623	20	0.0258
21	0.0364	22	0.0075	23	0.0258	24	0.0135	25	0.0054
26	0.0007	27	0.0193	28	0.0459	29	0.0099	30	0.0146
31	0.0048	32	0.0166	33	0.0065	34	0.1077	35	0.0250
36	0.0062	37	0.0018	38	0.0033	39	0.0046	40	0.0169

### 3.E Script used for the Monte Carlo simulation

```

# load micEcon package
library( "frontier" )
library( "fdrtool" ) # for function rhalfnorm
library( "MCMCpack" ) # for function rdirichlet

set.seed( 200 )

##### Generating Covariates #####

# preliminary variables for input quantities
data <- data.frame(
  XP1 = rchisq( 200, 10 ),
  XP2 = rchisq( 200, 10 ),
  XP3 = rchisq( 200, 10 ) )

# input quantities
data$X1 <- data$XP1 + data$XP2
data$X2 <- data$XP2 + data$XP3
data$X3 <- data$XP3 + data$XP1

# logarithms of input quantities
data$lX1 <- log( data$X1 )
data$lX2 <- log( data$X2 )
data$lX3 <- log( data$X3 )

# inefficiency term u ~ halfnormal
data$u <- rhalfnorm( 200, theta= 5 )

# ratio of output quantities to Euclidean distance of outputs
mSquared <- rdirichlet( 200, c( 1, 1 ) )
data$m1 <- sqrt( mSquared[ , 1 ] )
data$m2 <- sqrt( mSquared[ , 2 ] )

# logarithms of output ratios
data$lm1 <- log( data$m1 )
data$lm2 <- log( data$m2 )

##### Defining Parameters #####

# Output Distance

```

```
a0 <- 2.9
b1 <- -0.2
b2 <- -0.4
b3 <- -0.3
a1 <- 0.4

b11 <- 0.003
b12 <- -0.04
b13 <- -0.02
g11 <- -0.04
b22 <- 0.009
b23 <- -0.008
g12 <- -0.007
b33 <- 0.04
g13 <- 0.067
a11 <- 0.05

a2 <- 1 - a1
g21 <- -g11
g22 <- -g12
g23 <- -g13
a12 <- -a11
a22 <- a11

allCoef_D <- c( a0, b1, b2, b3, a1, b11, b12, b13, g11, b22, b23, g12, b33, g13, a11 )

# Stochastic Ray
m1z <- mean( data$m1 )
m2z <- sqrt( 1 - m1z^2 )

A0 <- -a0 - log( m2z )
B1 <- -b1
B2 <- -b2
B3 <- -b3
A1 <- -a1 * log( m1z / m2z ) / acos( m1z )

B11 <- -b11
B12 <- -b12
B13 <- -b13
G11 <- -g11 * log( m1z / m2z ) / acos( m1z )
B22 <- -b22
B23 <- -b23
G12 <- -g12 * log( m1z / m2z ) / acos( m1z )
```



```

B33 <- -b33
G13 <- -g13 * log( m1z / m2z ) / acos( m1z )
A11 <- -a11 * log( m1z / m2z )^2 / acos( m1z )^2

allCoef_R <- c( A0, B1, B2, B3, A1, B11, B12, B13, G11, B22, B23, G12, B33, G13, A11 )

# for checking if the functions are really equal at m1z, m2z:
# data$m1 <- m1z; data$m2 <- m2z; data$lm1 <- log( data$m1 ); data$lm2 <- log( data$m2 )

##### checking monotonicity & elast. of scale #####
# distance function: d|y| / dy_n >= 0 & d|y| / dx_k <= 0
ela_D <- cbind(
  b1 + b11 * data$1X1 + b12 * data$1X2 + b13 * data$1X3 +
  g11 * data$lm1 + g21 * data$lm2,
  b2 + b12 * data$1X1 + b22 * data$1X2 + b23 * data$1X3 +
  g12 * data$lm1 + g22 * data$lm2,
  b3 + b13 * data$1X1 + b23 * data$1X2 + b33 * data$1X3 +
  g13 * data$lm1 + g23 * data$lm2,
  data$m2^2 * ( a1 + a11 * data$lm1 + a12 * data$lm2 +
  g11 * data$1X1 + g12 * data$1X2 + g13 * data$1X3 ),
  data$m1^2 * ( a2 + a12 * data$lm1 + a22 * data$lm2 +
  g21 * data$1X1 + g22 * data$1X2 + g23 * data$1X3 ) )
# colSums( ela_D > 0 ) # should be 0 0 0 200 200
# hist( -rowSums( ela_D[ , 1:3 ] ) ) # elasticities of scale

# stochastic ray function: d|y| / dx >= 0
ela_R <- cbind(
  B1 + B11 * data$1X1 + B12 * data$1X2 + B13 * data$1X3 +
  G11 * acos( data$m1 ),
  B2 + B12 * data$1X1 + B22 * data$1X2 + B23 * data$1X3 +
  G12 * acos( data$m1 ),
  B3 + B13 * data$1X1 + B23 * data$1X2 + B33 * data$1X3 +
  G13 * acos( data$m1 ) )
# colSums( ela_R > 0 ) # should be 200 200 200
# hist( rowSums( ela_R ) ) # elasticities of scale
# compPlot( -rowSums( ela_D[,1:3] ), rowSums( ela_R ) )

##### Calculating deterministic actual aggregate output quantities #####

data$LYDet_D <- -( a0 + a1 * data$lm1 + a2 * data$lm2 +
  0.5 * a11 * data$lm1^2 + a12 * data$lm1 * data$lm2 +
  0.5 * a22 * data$lm2^2 +

```

```

        b1 * data$1X1 + b2 * data$1X2 + b3 * data$1X3 +
        0.5 * b11 * data$1X1^2 + b12 * data$1X1 * data$1X2 +
        b13 * data$1X1 * data$1X3 + 0.5 * b22 * data$1X2^2 +
        b23 * data$1X2 * data$1X3 + 0.5 * b33 * data$1X3^2 +
        g11 * data$1m1 * data$1X1 + g12 * data$1m1 * data$1X2 +
        g13 * data$1m1 * data$1X3 + g21 * data$1m2 * data$1X1 +
        g22 * data$1m2 * data$1X2 + g23 * data$1m2 * data$1X3 +
        data$u )

data$1YDet_D2 <- -( a0 + data$1m2 + a1 * log( data$1m1 / data$1m2 ) +
        0.5 * a11 * log( data$1m1 / data$1m2 )^2 +
        b1 * data$1X1 + b2 * data$1X2 + b3 * data$1X3 +
        0.5 * b11 * data$1X1^2 + b12 * data$1X1 * data$1X2 +
        b13 * data$1X1 * data$1X3 + 0.5 * b22 * data$1X2^2 +
        b23 * data$1X2 * data$1X3 + 0.5 * b33 * data$1X3^2 +
        g11 * log( data$1m1 / data$1m2 ) * data$1X1 +
        g12 * log( data$1m1 / data$1m2 ) * data$1X2 +
        g13 * log( data$1m1 / data$1m2 ) * data$1X3 +
        data$u )

# all.equal( data$1YDet_D, data$1YDet_D2 )

data$1YDet_R <- A0 + A1 * acos( data$1m1 ) + 0.5 * A11 * acos( data$1m1 )^2 +
        B1 * data$1X1 + B2 * data$1X2 + B3 * data$1X3 +
        0.5 * B11 * data$1X1^2 + B12 * data$1X1 * data$1X2 +
        B13 * data$1X1 * data$1X3 + 0.5 * B22 * data$1X2^2 +
        B23 * data$1X2 * data$1X3 + 0.5 * B33 * data$1X3^2 +
        G11 * acos( data$1m1 ) * data$1X1 + G12 * acos( data$1m1 ) * data$1X2 +
        G13 * acos( data$1m1 ) * data$1X3 - data$u

##### MC Simulation #####

nRep <- 1000

D_results <- matrix( NA, nrow=nRep, ncol=15 )
R_results <- matrix( NA, nrow=nRep, ncol=15 )
D_eff <- matrix( NA, nrow=nRep, ncol=200 )
R_eff <- matrix( NA, nrow=nRep, ncol=200 )

for( i in 1:nRep ) {

##### Generating Error Terms #####

```

---

```

# General Error term  $v \sim N(0, \sigma)$ 
v <- rnorm( 200, 0, 0.1 )

##### Generating Endogenous Variable #####

# Endogenous - Distance function

data$ly_D <- data$lyDet_D + v
data$Y_D <- exp( data$ly_D )

data$Y1_D <- data$m1 * data$Y_D
data$Y2_D <- data$m2 * data$Y_D

data$ly1n_D <- log( data$Y1_D / data$Y2_D )
data$ly2m_D <- - log( data$Y2_D )

# Endogenous - Stochastic Ray
data$ly_R <- data$lyDet_R + v
data$Y_R <- exp( data$ly_R )

data$Y1_R <- data$m1 * data$Y_R
data$Y2_R <- data$m2 * data$Y_R

##### Estimating Model #####

# Distance Function

D_result <- frontierQuad( yName = "ly2m_D", xNames = c( "lx1", "lx2", "lx3", "ly1n_D" ),
                        data = data, ineffDecrease = FALSE )

D_results[ i, ] <- coef( D_result )[1:15]
D_eff[ i, ] <- 1/efficiencies( D_result )

# Stochastic Ray Function

R_result <- frontierTranslogRay( yNames= c( "Y1_R", "Y2_R" ), xNames= c( "X1", "X2", "X3" ),
                               data = data )

R_results[ i, ] <- coef( R_result )[ 1:15 ]
R_eff[ i, ] <- efficiencies( R_result )

```

}

#####

# Bias and MSE for distance function

```
bias_D <- colMeans( D_results ) - allCoef_D
bias_eff_D <- colMeans( D_eff ) - exp( -data$u )
MSE_D <- colMeans( ( D_results - matrix( allCoef_D, nrow = nRep, ncol = 15, byrow = TRUE ) )^2 )
RMSE_D <- sqrt( MSE_D )
MSE_eff_D <- colMeans( ( D_eff -
  matrix( exp( -data$u ), nrow = nRep, ncol = 200, byrow = TRUE ) )^2 )
RMSE_eff_D <- sqrt( MSE_eff_D )
```

# Bias and MSE for Stochastic Ray function

```
bias_R <- colMeans( R_results ) - allCoef_R
bias_eff_R <- colMeans( R_eff ) - exp( -data$u )
MSE_R <- colMeans( ( R_results - matrix( allCoef_R, nrow = nRep, ncol = 15, byrow = TRUE ) )^2 )
RMSE_R <- sqrt( MSE_R )
MSE_eff_R <- colMeans( ( R_eff -
  matrix( exp( -data$u ), nrow = nRep, ncol = 200, byrow = TRUE ) )^2 )
RMSE_eff_R <- sqrt( MSE_eff_R )
```

##### LaTeX tables #####

```
library( "xtable" )
# covariates
numOdd <- seq( 1, 199, 2 )
numEven <- seq( 2, 200, 2 )
isOdd <- (1:200) %in% numOdd
isEven <- !isOdd
tabCov <- cbind(
  numOdd, subset( data, isOdd, c( "X1", "X2", "X3", "m1" ) ),
  exp( -data$u ) [ isOdd ],
  numEven, subset( data, isEven, c( "X1", "X2", "X3", "m1" ) ),
  exp( -data$u ) [ isEven ] )
colnames( tabCov ) <-
  rep( c( "Obs. No.", "$x_1$", "$x_2$", "$x_3$", "$y_1/\\mathbf{y}|$", "$\\exp(-u)$" ), 2 )
xTabCov <- xtable( tabCov, digits = c( 0, 0, rep( 3, 5 ), 0, rep( 3, 5 ) ),
  align = rep( "r", 13 ), caption = "Covariates", label = "tab:cov" )
print( xTabCov, file = "../tables/covariates.tex",
  tabular.environment = "longtable", floating = FALSE,
  caption.placement = "top", include.rownames = FALSE,
  sanitize.colnames.function = function( x ){ x } )
```

```

# true coefficients
tabCoef <- cbind( allCoef_D, allCoef_R )
colnames( tabCoef ) <-
  c( "translog_OD_function", "translog_SR_function" )
rownames( tabCoef ) <- c( "$\alpha_0$",
  paste( "$\beta_", 1:3, "$", sep = " " ), "$\alpha_1$",
  paste( "$\beta_{1}", 1:3, "$", sep = " " ), "$\gamma_{11}$",
  paste( "$\beta_{2}", 2:3, "$", sep = " " ), "$\gamma_{12}$",
  "$\beta_{33}$", "$\gamma_{13}$", "$\alpha_{11}$" )
coefOrder <- c( 1, 5, 15, 2:4, 6:8, 10:11, 13, 9, 12, 14 )
xTabCoef <- xtable( tabCoef[ coefOrder, ], digits = rep( 3, 3 ),
  align = c( "c", rep( "r", 2 ) ) )
print( xTabCoef, file = "../tables/coef.tex", floating = FALSE,
  sanitize.rownames.function = function( x ){ x } )

# biases, RMSEs, and MSEs of coefficients
tabBiasMse <- cbind( bias_D, RMSE_D, MSE_D, bias_R, RMSE_R, MSE_R )
colnames( tabBiasMse ) <-
  c( "OD:bias", "OD:RMSE", "OD:MSE", "SR:bias", "SR:RMSE", "SR:MSE" )
rownames( tabBiasMse ) <- c( "$\alpha_0$",
  paste( "$\beta_", 1:3, "$", sep = " " ), "$\alpha_1$",
  paste( "$\beta_{1}", 1:3, "$", sep = " " ), "$\gamma_{11}$",
  paste( "$\beta_{2}", 2:3, "$", sep = " " ), "$\gamma_{12}$",
  "$\beta_{33}$", "$\gamma_{13}$", "$\alpha_{11}$" )
xTabBiasMse <- xtable( tabBiasMse[ coefOrder, ], digits = rep( 3, 7 ),
  align = c( "c", rep( "r", 6 ) ) )
print( xTabBiasMse, file = "../tables/coefBiasMse.tex", floating = FALSE,
  sanitize.rownames.function = function( x ){ x } )

# mean biases, RMSEs, and MSEs of efficiency estimates
tabEffBiasMse <- rbind(
  c( mean( bias_eff_D ), mean( RMSE_eff_D ), mean( MSE_eff_D ),
    mean( bias_eff_R ), mean( RMSE_eff_R ), mean( MSE_eff_R ) ),
  c( sd( bias_eff_D ), sd( RMSE_eff_D ), sd( MSE_eff_D ),
    sd( bias_eff_R ), sd( RMSE_eff_R ), sd( MSE_eff_R ) ) )
colnames( tabEffBiasMse ) <-
  c( "OD:bias", "OD:RMSE", "OD:MSE", "SR:bias", "SR:RMSE", "SR:MSE" )
rownames( tabEffBiasMse ) <- c( "mean", "std.dev." )
xTabEffBiasMse <- xtable( tabEffBiasMse, digits = rep( 5, 7 ),
  align = c( "c", rep( "r", 6 ) ) )
print( xTabEffBiasMse, file = "../tables/effBiasMse.tex", floating = FALSE,
  sanitize.rownames.function = function( x ){ x } )

```

```
# function to create LaTeX tables that show individual data
xTableInd <- function( results, label, digits, fileName ) {
  tab <- NULL
  for( i in 1:5 ) {
    numSel <- seq( i, length( results ), 5 )
    tab <- cbind( tab, numSel, results[ numSel ] )
  }
  colnames( tab ) <- rep( c( "Obs. No.", label ), 5 )
  xTab <- xtable( tab, digits = c( 0, rep( c( 0, digits ), 5 ) ),
    align = rep( "r", 11 ) )
  print( xTab, file = paste( "../tables/", fileName, sep = "" ),
    floating = FALSE, include.rownames = FALSE )
  invisible( xTab )
}

# biases of the efficiencies of the distance function
xTableInd( bias_eff_D, label = "bias", digits = 4, fileName = "biasEffD.tex" )

# RMSEs of the efficiencies of the distance function
xTableInd( RMSE_eff_D, label = "RMSE", digits = 4, fileName = "rmseEffD.tex" )

# MSEs of the efficiencies of the distance function
xTableInd( MSE_eff_D, label = "MSE", digits = 4, fileName = "mseEffD.tex" )

# biases of the efficiencies of the stochastic ray function
xTableInd( bias_eff_R, label = "bias", digits = 4, fileName = "biasEffR.tex" )

# RMSEs of the efficiencies of the stochastic ray function
xTableInd( RMSE_eff_R, label = "RMSE", digits = 4, fileName = "rmseEffR.tex" )

# MSEs of the efficiencies of the stochastic ray function
xTableInd( MSE_eff_R, label = "MSE", digits = 4, fileName = "mseEffR.tex" )
```

---

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## **Kapitel 4**

### **Networks and Transaction Costs**

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## 4.1 Introduction

Development theory is based on Adam Smith's notion that economic development and wealth depend on specialisation. Specialisation necessarily goes hand in hand with an increase in trade. One of the main assumptions in classical economics is that the cost of exchange is negligible. In fact, we can observe incomplete specialisation in the production of many goods, indicating that the neoclassical assumptions do not apply to every market. Transaction cost theory has proven that many market transactions do not fulfil the neoclassical requirements of costless transfers (Coase, 1937; Williamson, 1971, 1973, 1981). Thus, we consider that transaction costs trigger reduced trade volume and specialisation (Akerlof, 1970; den Butter and Mosch, 2003) and, hence, inhibit development. Transaction costs in trade do not only comprise technical transaction costs, e.g. transportation, but also institutional transaction costs such as search costs, costs of gathering information on non-observable quality characteristics, negotiation costs, and control costs. Studies that try to measure transaction costs indicate that their size is far from immaterial. Wallis and North (1986) state that about 45 per cent of US GNP is devoted to the trading sector. McCloskey and Klammer (1995) find comparable results noting that one quarter of US GDP is related to persuasion in trade. Furthermore, Dalen and van Vuuren (2003) show that approximately 29 per cent of the total workforce in Holland is employed in the trading sector.

A closer look at institutional transaction costs reveals that the main determinant affecting the level of transaction costs in a market is insecurity. The more *ex ante* insecurity in the transaction, the more resources will be devoted to measures to reduce the insecurity. Finally, if costs exceed rents from a transaction, the transaction will not take place, inducing welfare losses on both the seller's and buyer's side. A mechanism that reduces *ex ante* insecurity in a transaction is trust (Greif, 1994; Calvert, 1995; den Butter and Mosch, 2003). If there is a higher *ex ante* trust level in the transaction, fewer resources are needed to reduce insecurity; hence, transaction costs decrease.

Social networks can provide effective mechanisms that help to increase the trust level between two trade partners (Granovetter, 1983; Buskens, 1998, 1999; Buskens and Raub, 2002; Wiebusch et al., 2004). These social networks can be based on formal institutions such as business associations, chambers, professional associations, etc., but can also be locally bounded, e.g. local, regional or national networks in trade, advice, friendship, etc. Most actors are simultaneously part of several types of networks, being more or less socially embedded (Granovetter, 1985; Uzzi, 1996, 1997; Rooks et al., 2000). If two well embedded actors meet for trade, they are contingent on two control mechanisms resulting from their embeddedness: reputation effects and norms. Reputation is the effect of information about

the actor's past behaviour being transferred between actors connected to the network. On the other hand, norms are beliefs that every actor in the network agrees upon and expects his opponent to adhere to (Kandori, 1992). Any conduct diverging from the norm will be punished by the other members of the network. The level of trust that follows from the embeddedness can be seen as an actor's social capital (Coleman, 1988; Granovetter, 1983). The higher the level of trust induced by the embeddedness, the lower the transaction costs that the actor faces.

The main hypothesis made here is that networks are not equally well qualified to provide effective reputational effects and norms (Buskens, 1999; Buskens and Snijders, 2005; Henning, 2002). In fact, different network structures result in different outcomes of reputational effects and norms, inducing varying trust levels. The second assumption is that actors do not know exactly how network embeddedness transfers into trust (Henning, 2002). This leaves the actor insecure about his trading partner's future behaviour, which finally results in higher transaction costs.

The present article describes the empirical framework utilised to measure the effects of different network structures on the level of transaction costs that traders face in an economy. This framework combines the idea of transaction costs from New Institutional Economics (Williamson, 1973, 1981) with ideas from sociology about the influence of trust and social capital on economic markets (Coleman, 1990; Helliwell and Putnam, 1999; Putnam et al., 1993; Knack and Keefer, 1997; Dasgupta, 1999).

The structure of the paper is as follows. Section two analyses the connection between social networks, trust and transaction costs and introduces the formal theoretical background. Section three describes the empirical framework used to estimate the influence of network structure on the level of transaction costs. Section four presents and discusses the empirical results and section five concludes.

## **4.2 Theoretical Framework**

### **4.2.1 Definition of transaction costs within the theoretical framework**

Transaction costs can be divided into two categories, technological transaction costs and institutional transaction costs (Green and Sheshinski, 1975). Both technological and institutional transaction costs refer to the sacrifice of resources. However, only institutional transaction costs can be reduced or avoided by choosing the correct organisational form. Although technological transaction costs are present under all institutional regimes, they are generally unavoidable (Klaes, 2000). Therefore, they are of little interest here.

In 1937, Coase wrote his essay 'The Nature of the Firm' introducing the concept of transaction costs for the first time <sup>1</sup>. Until then, the common economic view was that markets are efficient and always provide goods at the lowest possible price. Coase in turn argued that market transactions often involve higher costs than just the market price. Other costs (e.g. search and information costs, bargaining costs, and policing and enforcement costs) can increase the costs of procuring something from a market. Firms try to internalise transactions to avoid or lower transaction costs. However, there is a natural limit to what can be produced internally. Managerial supervision costs and administrative costs might outweigh the benefits of an internal solution. Coase argues that the size of a firm is the result of optimally balancing both types of costs.

In the early 1970s, Williamson (1971, 1973, 1975) introduced his transaction cost theory, making way to a vast domain in economic literature (e.g. Joskow, 1991; Crocker and Masten, 1996; Boerner and Macher, 2001). Williamson (1971, 1973, 1975) follows Coase (1937) in arguing that — as market transactions are more or less costly — choosing the right *governance structure* can lead to a better outcome under market imperfections. Firms can choose between spot market transaction, short- and long-term contracts, as well as vertical integration to manage transactions.

Williamson (1971, 1973, 1975) enriches the basic framework using two concepts: bounded rationality and opportunism. Bounded rationality implies that individuals do not have the capacity to foresee every relevant detail in contracting. As a consequence contracts must always be incomplete. By vertical integration, firms internalise contracting and thus reduce negotiation costs and inefficiencies. Opportunism is defined as *self-interest with guile* and becomes important in bargaining situations where the number of market participants is small. In competitive markets, opportunism might play a lesser role, but in situations where one contracting partner has made a special investment in future trade, she might become locked into that particular relationship. So the former competitive situation shifts to a bilateral monopoly. The actor not owning the specific asset might extract the *quasi-rents*.

Williamson (1979) states that transaction costs matter if relationships are *i)* frequent, *ii)* uncertain, and *iii)* if specific assets are involved.

- i. Frequent transactions, especially if specific assets are involved, are likely to be integrated, reducing the risk of opportunism;
- ii. Williamson (1979) differentiates between unpredictable environmental uncertainty and behavioural uncertainty which can be reduced by monitoring. However, due to these

---

<sup>1</sup>For a brief introduction into the development of transaction cost theory, see Klaes (2000)

uncertainties, perfect contracts are not achievable and those firms which invest in specific assets are especially at risk of high renegotiation costs.

- iii. Specific assets can comprise every special investment in a transaction, e.g. machinery, human capital, dedicated assets, etc.

If these conditions occur, transaction costs take place in three different stages of the transaction: *i*) contact phase, *ii*) contracting phase, and *iii*) control phase (den Butter and Mosch, 2003).

In the *contact* phase of a potential transaction, the actor is looking for information on potential trade partners (buyers or sellers), information on non-observable quality characteristics of his preferred product, and prices of the product (either seller or buyer prices). These searching costs occur because the search for information is not free, nor is information complete or easily accessible. Information costs are reduced if information is more easily accessible. Well functioning information networks can provide their members with information on business opportunities. As a further benefit, they increase the reliability of the information. In fact, with more information available in the network, and with easier transfers to all interested members, the probability that the information is of high quality increases, i.e. the information can be trusted to be relevant and true (Casson, 1997; Fafchamps, 2001).

The *contract* phase starts when two trade partners agree to make a deal. Transaction costs in this phase are mainly negotiation costs. Both partners have to agree on how to divide potential rents from trade, i.e. negotiation of trading conditions. Because of bounded rationality, a perfect contract that accounts for all eventualities is unachievable. First, not all arrangements are verifiable by third parties (verification problem). Second, many eventualities cannot be foreseen (environmental and behavioural uncertainty). Again, the more the trading partners trust each other — either by being well embedded in the same network or by agreeing on the same norms — the lower the necessity to negotiate every detail of the transaction. Hence, the costs of negotiation are reduced (Fafchamps, 2001).

Finally, if contracts are signed, the *control* phase starts. This consists of monitoring and enforcing the contract. Both actions involve many resources and, thus, induce high transaction costs. The first step is the monitoring of the partner to ensure that she meets the arrangement manifested in the contract. If it is seen that one of the partners behaves opportunistically by not keeping to the agreements, the next step is the enforcement of the contract. In most cases, legal procedures are troublesome, expensive and of long duration. Informal punishment systems, such as the loss of a good reputation or exclusion from future trade possibilities (Greif, 1994), can reduce the costs of contract enforcement. The better

these informal mechanisms work, the lower the incentive to defect in a transaction and, hence, the lower the monitoring and enforcement costs.

### 4.2.2 Modelling trust in games

It has become obvious that transaction costs can be reduced by a higher *ex ante* trust level in all three phases of the transaction. The question is now, how can trust be generated and modelled? In fact, every transaction can be modelled as a one-sided prisoner’s dilemma (PD). In a classical Seller-Buyer relationship, one can assume asymmetric strategy sets. From the buyer’s perspective, it is important that the seller keeps to the agreement, e.g. concerning characteristics such as quality and quantity of a product or punctuality of delivery. In this case, she has a clear preference for  $\langle cooperate, cooperate \rangle$  which does not necessarily apply to the seller. However, from the seller’s perspective, the emphasis is on other aspects of the transaction. For the seller, it is most important that the buyer pays the agreed price on time. So in this respect, it is the seller who has a clear preference for  $\langle cooperate, cooperate \rangle$ , while the buyer refrains. Figure 4.1 shows an example of the game from the Buyer’s perspective.

		<b>Buyer</b>	
		<i>Buy</i> <i>(Cooperate)</i>	<i>Boycott</i> <i>(Defect)</i>
<b>Seller</b>	<i>Honest (Cooperate)</i>	5, 5	0, 0
	<i>Cheat (Defect)</i>	10, -5	<b>0, 0</b>

Figure 4.1: Payoffs to Seller and Buyer from the Buyer’s Perspective

In a one-shot game, the outcome will be  $\langle defect, defect \rangle$ , as in this example, defect is still weakly dominant for the seller (see figure 4.1). To introduce trust into the one-sided PD, there are three mechanisms in game theory. The first is the assumption of an infinite horizon. The resulting shadow of the future increases the price of defection, so that a one-time gain from defection will never cover the loss of future trade opportunities. Hence, the dominant strategies of the seller or buyer change to  $\langle cooperate, cooperate \rangle$ .

Enhancing the dyadic framework by multiple actors leads to the introduction of social networks. As in the infinite horizon game, repeated interactions with an actor form expectations about the actor’s future behaviour, i.e. every actor in a network achieves a reputation based on her former conduct. On the other hand, the reputation effect can only work if loss of a good reputation —due to bad conduct— has negative consequences for the actor. The aggravating effect of the punishment in a network compared to the dyadic case is that a bad

reputation not only leads to the exclusion from future trade, but also to the exclusion from trade options with other community members. For this effect to work, network members must be able and willing to transfer information about an actor's reputation from one to another. The ability of a social network to provide this mechanism can differ substantially with the consequence that the outcome of the level of trust in a social network varies significantly.

### **4.2.3 Formally introducing networks in trust games with reputation**

Focusing on the reputation mechanism, in a simple trust game involving the reputation transfer of information is the one crucial factor in making reputation effects work. If information about an actor's conduct does not pass from actor to actor in an appropriate period of time, neither the punishment of bad conduct nor the reputation effect will work properly. Consequently, the quality of the reputation mechanism depends on the structure of the information network or—more precisely—on the global and local <sup>2</sup> network structure.

Buskens (1999) shows the impact of an information network on the equilibrium of trade in a simple trust game with reputation. He defines global network parameters as:

1. The density of a network.
2. The extent to which a network is centralised around one or several actors.
3. The transitivity of a network, which measures the existence of dense subgroups.
4. Network size.

and local network parameters as:

1. An actor's outdegree and indegree.
2. Degree quality, i.e. the extent to which an actor's neighbours are connected to others.
3. Local density, i.e. the extent to which an actor's neighbours are connected to each other.

On the basis of these network parameters and further properties of information exchange between actors—which will not be explained in detail here— Buskens (1999) derives a Pareto-optimal sub-game perfect equilibrium. This equilibrium defines an exact threshold up to which two actors can exchange goods without the interference of opportunism. The threshold

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<sup>2</sup>Global network structure describes the overall structure of a network, e.g. average degree, density, centrality. Local network structure describes an actor's position in the network, i.e. his connection to other actors in the network



level, i.e. the amount of goods two actors can safely exchange, depends on both the actors' local position in the network and the global network structure.

Results show, among others, that the threshold increases with sanction costs, which confirms earlier assumptions. So the better the information flow, and consequently the efficiency of the reputation effect, the higher the amount of goods that can be traded.

#### 4.2.4 Extending the trust game with reputation by transaction costs

In its dependence on classical game theory, Buskens (1999) model assumes that if an actor has a fortunate position in the information network and receives all relevant information about his trading partner, she is fully informed about her opponent's future conduct. This implies that the actor can ascertain exactly how her opponent's future behaviour will be affected by the network structure, and knows the exact amount of goods that can safely be exchanged with any other actor.

Starting from a one-sided prisoner's dilemma game based on a model from Greif (1994), Henning (2002) extends the trust game with reputation making two assumptions:

1. Network embeddedness is extended by introducing more than one exchange network and one information network. Actors are allowed to have several kinds of relationship including social relationships such as friendship, religious communities, etc. (Portes and Sensenbrenner, 1993; Rooks et al., 2000; Richman, 2006). The more networks an actor is embedded in, the more the shadow of the future increases, because misbehaviour in one relationship, for example trade, will have negative consequences not only among members of the trade network, but also among members of all other networks. So the costs of a one time gain from cheating increase by the potential loss of for example friendships, exclusion from religious communities, etc. (Richman, 2006). Consequently, if one allows for network embeddedness over several networks, one extends the chance of increasing trade volume more than by just focusing on the information network.
2. The actor no longer knows the exact threshold of the quantity of a commodity that is exchangeable without defection. He only knows a probability distribution to cheat at a given transfer quantity for each actor.

Formally this probability can be written in form of a cumulated distribution (see figure 4.3) as follows:

$$F(t_{ij}) = \begin{cases} 1 - \left[ \frac{z_{ij} - t_{ij}}{z_{ij}} \right]^{X_{ij}} & \text{for } t_{ij} < z_{ij}, 0 < X_{ij} < 1 \\ 1 & \text{otherwise} \end{cases} \quad (4.1)$$

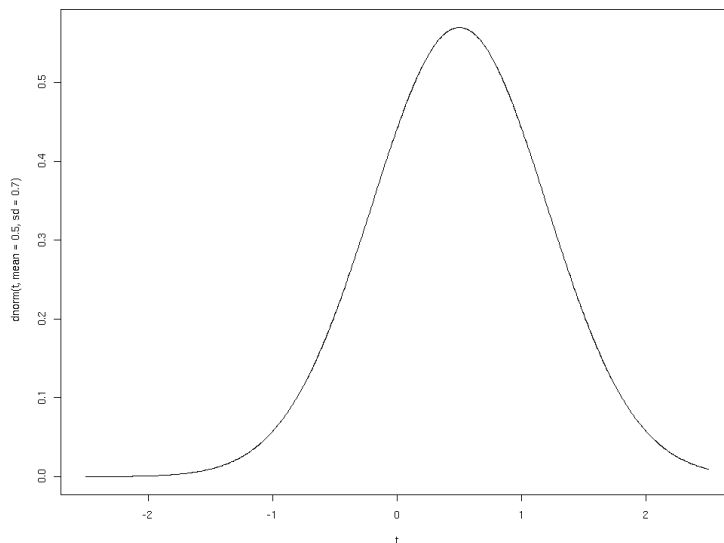


Figure 4.2: Normal distribution over expectancy value of defection

with  $F(t_{ij})$  = cumulative distribution function,  $t_{ij}$  = transfer from  $i$  to  $j$ ,  $z_{ij}$  = threshold of actors pair  $i$  and  $j$ .

The level of transaction costs depends on two variables: the threshold  $z_{ij}$  and the variance of the distribution  $X_{ij}$ . Both variables in turn depend on the network structure.  $z_{ij}$  depends on the embeddedness of actors  $i$  and  $j$  in different networks and on the efficiency of the reputation mechanism. The variance parameter  $X_{ij}$  mostly depends on the structure of the information network. The better the actor is embedded into the information network, the more information she receives about the actor's type of her opponent and hence, variation decreases (see figure 4.3).

The right part of figure 4.3 shows that a low variation (0.01) leads to a situation comparable to the model in Buskens (1999), where information about the opponent's type, and thus the threshold, is nearly complete. So the better the information available about an actor's conduct, the more the distribution about the threshold approaches a single peak distribution.

Henning (2002) shows that transaction costs depend on two variables. First, transaction costs increase with the variance of the distribution, as shown in figure 4.3. Second, transaction costs increase with decreasing level  $z_{ij}$ . The lower  $z_{ij}$  at given variance, the more the distribution is jolted. Hence, insecurity increases at any level  $t_{ij}$  which leads to higher transaction costs.

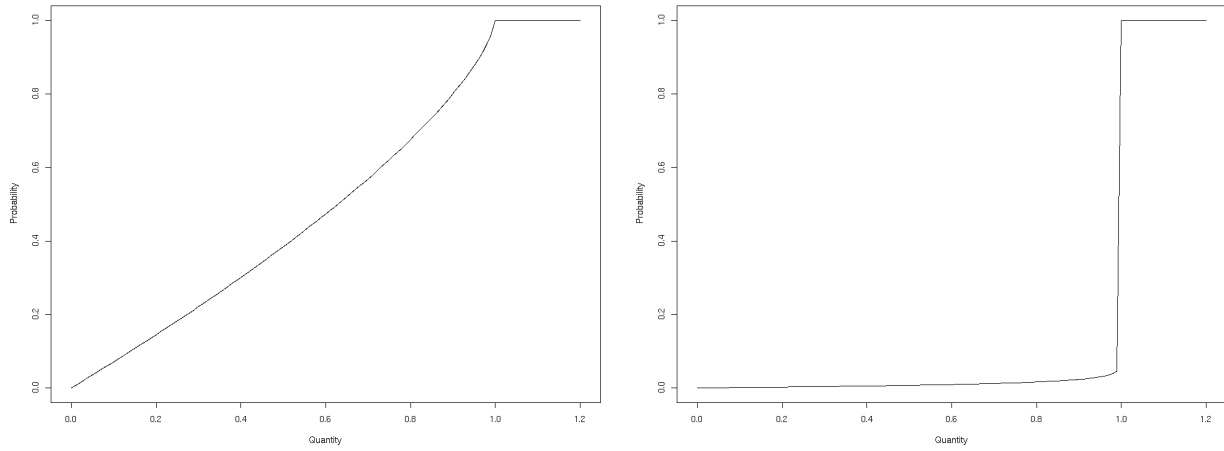


Figure 4.3: Cumulated distribution with variance  $X_{ij} = 0.7$  and  $X_{ij} = 0.01$

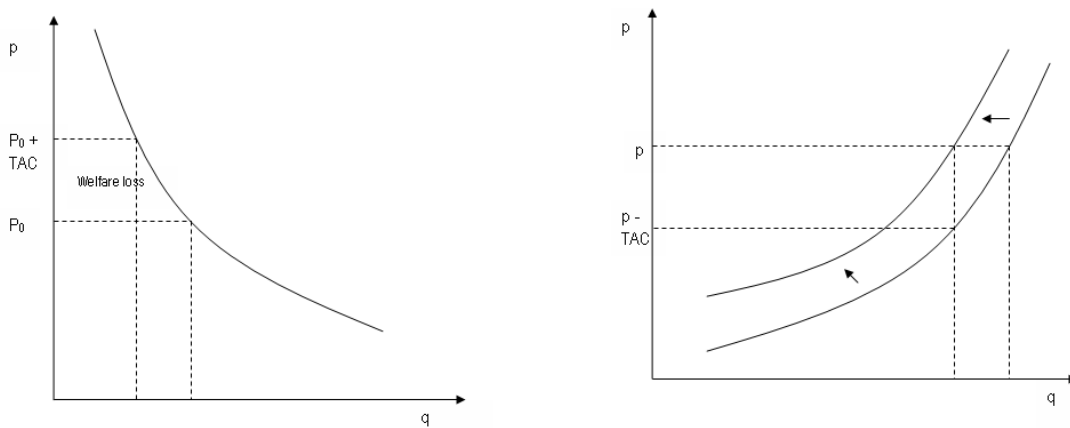


Figure 4.4: Welfare loss on consumers' and suppliers' side

Increasing insecurity about an opponent's conduct in a transaction, induced by incomplete information about the threshold, forces an actor to dedicate resources to measures reducing the insecurity. These measures are costly and increase the price of a commodity for the buyer and decrease the price for the seller. Hence, the amount of trade, as well as the actor's welfare, decreases (see figure 4.4).

## **4.3 Empirical Framework**

### **4.3.1 Specification of transaction costs in the empirical framework**

Transaction cost theory as formulated by Williamson (1971, 1975) is well designed and has a reasonable fit, but it leaves the question about how asset specificity and dependence hang together unclear. Dependence, and thus insecurity, in a transaction must not necessarily be symmetric among trading partners. On the contrary, Noteboom (1993) shows that dependence in a transaction depends strongly on which side of the transaction depends on the transaction specific asset. Assuming that the dependent side faces increased insecurity and thus higher transaction costs, it is an essential point in empirical studies to specify dependence in transactions for the concrete case.

Noteboom (1993) provides a helpful classification of different forms of asymmetric and symmetric dependencies in transactions, which will be applied to the case of Polish farms:

- A. Transaction specific investments, not specific to the good exchanged. These investments may be symmetric or unilateral.
- B. Transaction specific investments to the product and the buyer causing high dependence on supplier's side and lesser dependence on the buyer's side.
- C. Dependence is given by the buyer demanding a specific technology which does not allow for the production of another alternative product.
- D. Dependence is given as in C above, and by a very narrow market (monopsony).
- E. Strong dependence on the buyer's side on a product with no substitute which is, in the most extreme case, supplied by a monopolist.
- F. Strong dependence on the buyer's side, as in E above, but with no dependence on the supplier's side. This situation can occur if the product is specific to the buyer, but production technology is not specific to the product which leaves the supplier independent from a concrete transaction.

The list shows that dependence in a transaction is by no means always symmetric and can achieve increasing levels of dependence both on the supplier's side (B, C, D), as well as on the buyer's side (E, F). As seen in section 4.2.1, transaction costs appear in different forms at different stages of the transaction. By matching both approaches, we are now able to concretise situations that might be affected by transaction costs.

Generally, one can assume that transactions on agricultural markets are not affected by high asset specificity, and cause little concern about opportunistic behaviour. As most goods such as fertiliser, herbicides and seeds are more or less homogeneous mass products, there is little concern about quality aspects either. Still, information about prices and other factors, such as time and credibility of delivery, are aspects that can cause insecurity. Therefore, farmers mostly face transaction costs on all levels which apply to situation A. These are mainly search costs, negotiation costs, and control/enforcement costs which depend purely on the transaction and are unrelated to the traded product.

### 4.3.2 Calculation of transaction costs on output and input markets

The central idea to measure transaction costs as a gap between MC and output prices and VMP and input prices, respectively, is quite straightforward. Assuming a simple linear cost definition  $c(y) = v(y) + F$  where  $v(y)$  denotes the variable and  $F$  denotes fixed costs, one derives MC as  $\frac{\partial c(y)}{\partial y} = \frac{\partial v(y)}{\partial y}$ . Assuming an output market with transaction costs, the cost definition extends by  $c(y) = v(y) + F + tac^y(y)$  and so do the MC  $\frac{\partial c(y)}{\partial y} = \frac{\partial v(y)}{\partial y} + tac_y$ .

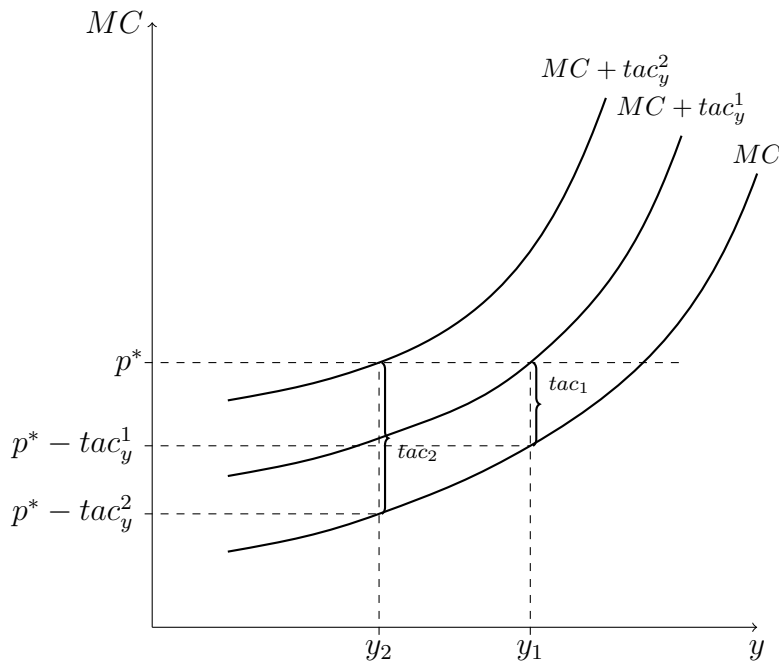


Figure 4.5: Transaction costs on output market

Figure 4.5 shows an output market with transaction costs and two suppliers. As transaction costs are a function of the traded good, location, time and the individuals trading, it is obvious that in this case the transaction costs must also differ c.p. between actors one and two. As

actor two faces higher transaction costs than actor one, she consequently produces only the quantity  $y_2 < y_1$ . On the input market an enterprise faces similar problems. Assuming again a simple profit definition  $\Pi = pf(x) - wx$  the factor demand without transaction costs is defined as  $\frac{\partial \Pi}{\partial x_i} = p * \frac{\partial f(x)}{\partial x_i} - w_i = 0$ . Allowing for transaction costs on the input market leads to an input demand function  $p * \frac{\partial f(x)}{\partial x_i} = w_i + tac_x$  and thus, reduced input demand as can be seen in figure 4.6 for the case of two actors.

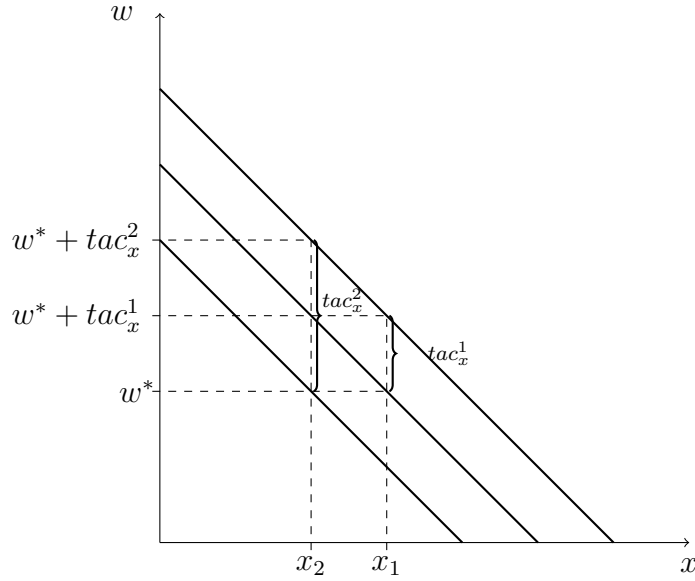


Figure 4.6: Transaction costs on input market

As we are only able to measure the MC and input demand curve, the quantities  $y_1$ ,  $y_2$ ,  $x_1$ , and  $x_2$ , and the prices  $p^*$  and  $w^*$ , respectively, we find a gap of  $p^* > MC$  on the output and  $w^* < VMP$  on the input market, which we theoretically interpret as transaction costs. Problems related to this assumption will be discussed in the next chapter.

Unfortunately, by assuming transaction costs on both output and input markets one can no longer distinguish between market specific transaction costs.

$$\Pi = pf(x) - tac^y(f(x)) - wx - tac^x(x) \tag{4.2}$$

$$\begin{aligned} \frac{\partial \Pi}{\partial x} &= (p - tac_y) \cdot \frac{\partial f(x)}{\partial x} - (x + tac_x) = 0 \\ \Leftrightarrow (p - tac_y) * \frac{\partial f(x)}{\partial x} &= w + tac_x \end{aligned} \tag{4.3}$$

As figure 4.7 demonstrates, is it not possible to denote market specific transaction costs simply by applying  $P > MC$  or  $w < VMP$ . In not considering transaction costs at all, we measure only function  $a$  in figure 4.7, although the supplier taking transaction costs on the

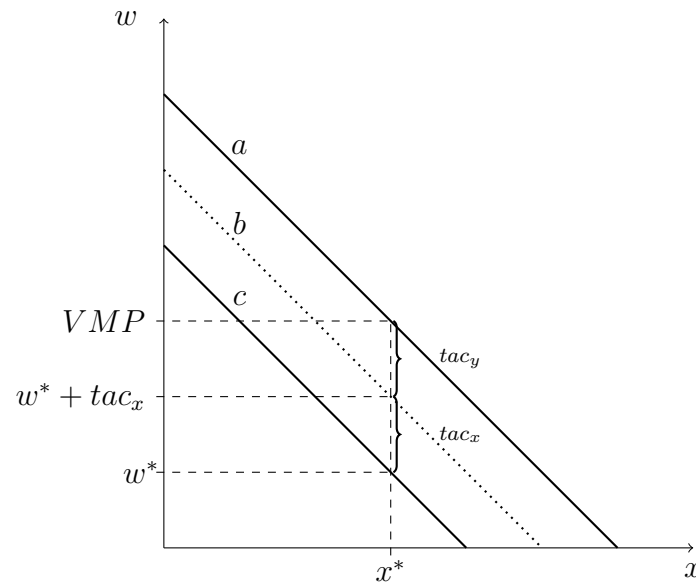


Figure 4.7: Transaction costs on input market with transaction costs on output market

output side into account bases her input demand decision on  $(p - tac_y) * \frac{\partial y}{\partial x}$  pictured by the dotted line *b*. As transaction costs on the output side are unknown, we do not know this function, hence we cannot denote whether, or to what extent, the gap we measure between the VMP and the measured price *w* is due to transaction costs on the output or input side. In the following chapter, we will demonstrate how we solve this problem.

### 4.3.3 Estimation procedure

In the following, we will derive the empirical model to estimate the influence of social networks on the level of transaction costs on two output and three input markets (outputs crops, outputs animal, variable inputs crop, variable inputs animal, and general variable inputs). To estimate our model, we apply a two step procedure.

#### 4.3.3.1 First step: estimation of a multi-output stochastic frontier production function

To be able to estimate farm specific transaction costs for single products or product groups, we apply the stochastic frontier approach (SFA) introduced by Aigner et al. (1977), Meeusen and van den Broeck (1977) and Battese and Coelli (1988). As we are interested in estimating the transaction costs for two output commodities, we use a multiple output frontier approach.

We follow the approach introduced by Löthgren (1997, 2000) and apply a multiple output stochastic ray production frontier.

The key idea of the multiple output generalisation of the stochastic production function is to represent the output as an output vector  $y = l * m(\theta)$  of  $n$  outputs represented by the Euclidean norm (i.e. distance component)  $l = l(y) = \|y\| = (\sum_{i=1}^n y_i^2)^{1/2}$  and a direction measure (i.e. the polar coordinates, or angle related to ratio of one output to the others)  $m(\theta)$  with:

$$m_i(\theta) = \frac{y_i}{l(y)} = \cos(\theta_i) \prod_{j=0}^{i-1} \sin(\theta_j) \quad \text{textwith } \sin(\theta_0) = \cos(\theta_p) = 1 \quad (4.4)$$

Analogously to the single output production function that gives the maximum output given inputs, a multiple output production function can be defined that gives the maximum output norm given inputs and the direction of the output vector, given by the polar coordinates, so that the ray production function is defined as:

$$f(x, \theta(y), \omega) = \max \{ \iota \in R_+ : \iota * m(\theta(y)) \in P(x, \omega), \omega \in \Omega \} \quad (4.5)$$

where  $x$  denotes input factors and  $\omega \in \Omega$  the actual state of the world derived from the set of all possible states of the world.

As well as in the single output case (see Shepard, 1970), the output distance function in the multiple case is given by the ratio of the output norm to the frontier output norm. The distance function can be expressed in terms of the ray production function as follows:

$$D_0(x, y, \omega) = \frac{\|l(y) * m(\theta(y))\|}{\|f(x, \theta(y), \omega) * m(\theta(y))\|} \quad (4.6)$$

As well as in the single case the error term is divided into two components, the first  $v$  accounting for state-of-the-world effects, the second  $u$  for technical efficiency, distributed half-normal or exponentially. The general stochastic ray frontier production model is given by

$$\ln l(y) = \ln f(x, \theta(y)) + v - u \quad (4.7)$$

As in most single output models, the best applicable functional form is the flexible translog function which in this particular case is defined as:

$$\ln l = \beta_0 + z' \beta + \frac{1}{2} z' \Gamma z + v - u \quad (4.8)$$



where  $\beta_0$  is a scalar intercept,  $z = (\ln x', \ln \theta)'$  is the matrix of covariates,  $\beta$  is a vector of first-order parameters and  $\Gamma$  a symmetric matrix of second-order parameters.

To estimate the parametric ray frontier model least squares or maximum likelihood estimators can be used. We estimate the Stochastic Ray Production frontier with the help of the "R" package "frontier" (Coelli and Henningsen, 2010).

#### 4.3.3.2 Second step: modelling the influence of network structural parameters on transaction costs

The last step in our empirical framework is to estimate the influence of network structural parameters on the level of transaction costs. Following the theoretical considerations we expect that an actor's network embeddedness facilitates trade and lowers transaction costs.

$$tac_{ijk_r} = f(\text{network structural parameters, others}) \quad (4.9)$$

where  $tac_{ijk_r}$  denotes specific transaction costs between actors  $i$  and  $j$ , trading commodity  $k$  in region  $r$ .

A problem related to the measurement of transaction costs using the method mentioned in section 4.3.2, is the theoretical assumption that the gap between price and MC or VMP, respectively, is only related to transaction costs. In fact, there are other disturbances in markets that might affect the assumption  $p = MC$  or  $w = VMP$ . In our concrete case we can single out two main factors that might have a significant influence on the gap. First, markets can suffer from strong environmental insecurity, namely price fluctuations or—which applies especially to farmers—extreme and unforeseen weather conditions or other external shocks. Both effects can lead to difficulties in the production decision, especially in cases where the production decision and price formation on output markets happen at different times. This environmental insecurity in markets might lead to a discrepancy between MC (production decision) and realised price on the market. How strong the influence of these effects is on the gap between prices and MC or VMP, respectively, depends on two factors. First, the qualification of the management, i.e. the ability to foresee market developments and to react appropriately. This effect is captured by the efficiency term in the stochastic ray production frontier. Second, unforeseeable external shocks, i.e. events that affect the market and are unpredictable. As our data covers only a limited local area these effects might affect all enterprises, thus, must lead to relatively similar impact.

The second problem relates to market power, which, consequently, in the case of monopolistic situations annuls the assumption  $p = MC$ , or in the case of monopsony, the assumption

$w = VMP$ . Market power — as affecting our model assumption — can be separated in three different forms:

- Monopolistic concurrence: as asset specificity increases, the level of market power for a specific good also increases. Investing in transaction specific assets implies that the producer extracts a rent from the transaction that exceeds the costs. Therefore in monopolistic concurrence situations the assumption  $p = MC$  is no longer valid for a given product. Hence, measuring a gap can imply either transaction costs and/or a rent. To which level the transaction is affected by transaction costs is then no longer perceivable.
- Market power due to increasing firm size: firms with a very large market share might be able to exert pressure on market partners, both on the supplier's and buyer's side. The bigger the market share on the given firm level compared to that on upstream and downstream market levels, the more power the firm can wield, dictating prices and delivery conditions.
- Monopoly and monopsony: in cases of classical monopoly or monopsony the assumption of  $p = MC$  or  $w = VMP$  is *ex ante* foiled. The consequences are the same as in the first and second case.

To include these aspects in our analysis we have to consider that the measured gap is not only affected by network structural parameters, but also by insecurity in prices and market power. Therefore, we include proxy variables into our analysis. By asking for output and input related data, each farmer had to rate on a scale from 1 to 10 the level of price volatility and concentration on the market, respectively. Later we accumulate the product specific information to product group specific indices.

The estimation model is derived as follows:

$$\Pi = \sum_i p_i \cdot y_i - \sum_k w_k \cdot x_k - T(\mathbf{y}) - T(\mathbf{x}) - R(v_i)y_i - R(v_k)x_k - C(c_k)x_k - \lambda(F(\mathbf{y}, \mathbf{x})) \quad (4.10)$$

with  $y_i$  the  $i = 1, \dots, n$  output commodities;  $x_k$  the  $k = 1, \dots, l$  input commodities;  $p_i$  the  $i$ th output price;  $w_k$  the  $k$ th input price;  $T(\mathbf{y})$  transaction costs on the output side;  $T(\mathbf{x})$  transaction costs on the input side;  $R(v_{i,k})$  are functions of deduction for risk due to price volatility on input and output markets, respectively;  $C(c_k)$  is a function reflecting the surcharge on input markets with market concentration on suppliers side; and,  $F(\cdot)$  is the technology. Deriving first order conditions:

$$\frac{\partial \Pi}{\partial y_1} = p_1 - T_{y_1} - R(v_1) - \lambda F_{y_1} \equiv 0 \quad (4.11)$$

$$\frac{\partial \Pi}{\partial y_2} = p_2 - T_{y_2} - R(v_2) - \lambda F_{y_2} \equiv 0 \quad (4.12)$$

$$\frac{\partial \Pi}{\partial x_k} = -w_k - T_{x_k} - R(v_k) - C(c_k) - \lambda F_{x_k} \equiv 0 \quad (4.13)$$

some calculus leads to:

$$\text{i. } p_1 \frac{F_{y_2}}{F_{y_1}} - T_{y_1} \frac{F_{y_2}}{F_{y_1}} - R(v_1) \frac{F_{y_2}}{F_{y_1}} = p_2 - T_{y_2} - R(v_2) \quad (4.14)$$

$$\text{ii. } -p_1 \frac{F_{x_k}}{F_{y_1}} + T_{y_1} \frac{F_{x_k}}{F_{y_1}} + R(v_2) \frac{F_{x_k}}{F_{y_1}} = w_k + T_{y_2} + R(v_k) + C(c_k) \quad (4.15)$$

and:

$$\text{i. } \frac{F_{y_2}}{F_{y_1}} = \frac{p_2}{p_1} \frac{\left(1 - \frac{T_{y_2}}{p_2} - \frac{R(v_2)}{p_2}\right)}{\left(1 - \frac{T_{y_1}}{p_1} - \frac{R(v_1)}{p_1}\right)} \quad (4.16)$$

$$\text{ii. } -\frac{F_{x_k}}{F_{y_1}} = \frac{w_k}{p_1} \frac{\left(1 + \frac{T_{x_k}}{w_k} + \frac{R(v_k)}{w_k} + \frac{C(c_k)}{w_k}\right)}{\left(1 - \frac{T_{y_1}}{p_1} - \frac{R(v_1)}{p_1}\right)} \quad (4.17)$$

taking the logarithm leads to:

$$\text{i. } \ln\left(\frac{F_{y_2}}{F_{y_1}}\right) - \ln\left(\frac{p_2}{p_1}\right) = \ln\left(1 - \frac{T_{y_2}}{p_2} - \frac{R(v_2)}{p_2}\right) - \ln\left(1 - \frac{T_{y_1}}{p_1} - \frac{R(v_1)}{p_1}\right) \quad (4.18)$$

$$\text{ii. } \ln\left(-\frac{F_{x_k}}{F_{y_1}}\right) - \ln\left(\frac{w_k}{p_1}\right) = \ln\left(1 + \frac{T_{x_k}}{w_k} + \frac{R(v_k)}{w_k} + \frac{C(c_k)}{w_k}\right) - \ln\left(1 - \frac{T_{y_1}}{p_1} - \frac{R(v_1)}{p_1}\right). \quad (4.19)$$

Approximating the logarithmic function  $g(x, y, z) = \ln(1 + x + y + z)$  for small values  $x, y, z$  at  $x = y = z = 0$  leads to:

$$\begin{aligned} g(x, y, z) &\approx g(0, 0, 0) + x \cdot \left. \frac{\partial g(x, y, z)}{\partial x} \right|_{x=y=z=0} + y \cdot \left. \frac{\partial g(x, y, z)}{\partial y} \right|_{x=y=z=0} + z \cdot \left. \frac{\partial g(x, y, z)}{\partial z} \right|_{x=y=z=0} \\ &= 0 + x + y + z = x + y + z \end{aligned} \quad (4.20)$$

So we replace the logarithmic terms on the right side by their linear approximation:

$$\text{i. } \ln\left(\frac{F_{y_2}}{F_{y_1}}\right) - \ln\left(\frac{p_2}{p_1}\right) = -\frac{T_{y_2}}{p_2} - \frac{R(v_2)}{p_2} + \frac{T_{y_1}}{p_1} + \frac{R(v_1)}{p_1} \quad (4.21)$$

$$\text{ii. } \ln\left(-\frac{F_{x_k}}{F_{y_1}}\right) - \ln\left(\frac{w_k}{p_1}\right) = \frac{T_{x_k}}{w_k} + \frac{R(v_k)}{w_k} + \frac{C(c_k)}{w_k} + \frac{T_{y_1}}{p_1} + \frac{R(v_1)}{p_1} \quad (4.22)$$

As we do not know the transaction costs  $T_{y_i}$  and  $T_{x_k}$  we replace them by the following function:

$$T(y) = \alpha_0 + \alpha_1 y_i + \alpha_2 y_i^2 \quad (4.23)$$

$$T(x) = \beta_0 + \beta_1 x_k + \beta_2 x_k^2 \quad (4.24)$$

taking derivatives leads us to:

$$T_{y_i} = \alpha_1 + \alpha_2 2 \cdot y_i \quad (4.25)$$

$$T_{x_k} = \beta_1 + \beta_2 2 \cdot x_k \quad (4.26)$$

So we get:

$$\text{i. } \ln\left(\frac{F_{y_2}}{F_{y_1}}\right) - \ln\left(\frac{p_2}{p_1}\right) = -\frac{(\alpha_1 + \alpha_2 2 \cdot y_2)}{p_2} - \frac{R(v_2)}{p_2} + \frac{(\alpha_1 + \alpha_2 2 \cdot y_1)}{p_1} + \frac{R(v_1)}{p_1} \quad (4.27)$$

$$\begin{aligned} \text{ii. } \ln\left(-\frac{F_{x_k}}{F_{y_1}}\right) - \ln\left(\frac{w_k}{p_1}\right) &= \frac{(\beta_1 + \beta_2 2 \cdot x_k)}{w_k} + \frac{R(v_k)}{w_k} + \frac{C(c_k)}{w_k} \\ &+ \frac{(\alpha_1 + \alpha_2 2 \cdot y_1)}{p_1} + \frac{R(v_1)}{p_1}. \end{aligned} \quad (4.28)$$

To estimate the influence of network structural parameters on the endogenous variable we further parametrise the parameters of the transaction cost function by:

$$\alpha^{y_1} = \delta_0 + \delta_1 z \quad (4.29)$$

$$\alpha^{y_2} = \zeta_0 + \zeta_1 z \quad (4.30)$$

$$\beta = \gamma_0 + \gamma_1 z \quad (4.31)$$

where  $z$  denotes the network structural parameters *outdegree*, referring to the number  $m$  of contacts an ego has, and *density* referring to the number of contacts between the alteri  $t$  in relation to the possible number of contacts  $d = t/(m(m-1))/2$ ,  $\alpha^{y_1}$  denotes the parameters of the function of  $T_{y_1}$ , and  $\alpha^{y_2}$  denotes the parameters of the function of  $T_{y_2}$ . Therefore, the final estimation equation is defined as:

$$\begin{aligned} \text{i. } Y_y &= \zeta_0 \left( -\frac{1}{p_2} \right) + \zeta_1 \left( -\frac{z}{p_2} \right) + \zeta_2 \left( -2 \cdot \frac{y_2}{p_2} \right) + \zeta_3 \left( -2 \cdot z \frac{y_2}{p_2} \right) + \mu_1 \left( -\frac{R(v_2)}{p_2} \right) \\ &\quad \delta_0 \frac{1}{p_1} + \delta_1 \frac{z}{p_1} + \delta_2 2 \cdot \frac{y_1}{p_1} + \delta_3 2 \cdot z \frac{y_1}{p_1} + \mu_2 \frac{R(v_1)}{p_1} + \epsilon \end{aligned} \quad (4.32)$$

$$\begin{aligned} \text{ii. } Y_x &= \gamma_0 \frac{1}{w_k} + \gamma_1 \frac{z}{w_k} + \gamma_2 2 \cdot \frac{x_k}{w_k} + \gamma_3 2 \cdot z \frac{x_k}{w_k} + \mu_3 \frac{R(v_k)}{w_k} + \mu_4 \frac{C(c_k)}{w_k} \\ &\quad \delta_0 \frac{1}{p_1} + \delta_1 \frac{z}{p_1} + \delta_2 2 \cdot \frac{y_1}{p_1} + \delta_3 2 \cdot z \frac{y_1}{p_1} + \mu_2 \frac{R(v_1)}{p_1} + \epsilon \end{aligned} \quad (4.33)$$

with  $Y_y = \ln \left( \frac{F_{y_2}}{F_{y_1}} \right) - \ln \left( \frac{p_2}{p_1} \right)$  and  $Y_x = \ln \left( -\frac{F_{x_k}}{F_{y_1}} \right) - \ln \left( \frac{w_k}{p_1} \right)$ . The endogenous variables  $Y_y$  and  $Y_x$  can be interpreted as follows:

$$\text{i. } \ln \left( \frac{F_{y_2}}{F_{y_1}} \right) - \ln \left( \frac{p_2}{p_1} \right) = \ln \left( \frac{F_{y_2}}{F_{y_1}} \cdot \frac{p_1}{p_2} \right) \quad (4.34)$$

$$\text{ii. } \ln \left( -\frac{F_{x_k}}{F_{y_1}} \right) - \ln \left( \frac{w_k}{p_1} \right) = \ln \left( \frac{F_{x_k}}{F_{y_1}} \cdot \frac{w_k}{p_2} \right) \quad (4.35)$$

applying the implicit function theorem leads to:

$$\text{i. } \frac{F_{y_2}}{F_{y_1}} = -\frac{\partial y_1}{\partial y_2} \quad (4.36)$$

$$\text{i. } -\frac{F_{x_k}}{F_{y_1}} = \frac{\partial y_1}{\partial x_k} \quad (4.37)$$

so that we find:

$$\ln \left( -\frac{\partial y_1}{\partial y_2} \cdot \frac{p_2}{p_1} \right) = 0 \quad (4.38)$$

the logarithm of the rate of transformation multiplied with the reciprocal ratio of output prices for the endogenous variable  $Y_y$  and:

$$\ln \left( \frac{\partial y_1}{\partial x_k} \cdot \frac{p_2}{w_k} \right) = 0 \quad (4.39)$$

the optimality condition for the input demand for the endogenous variable  $Y_x$ . Hence, if we assume that divergence from zero is caused by transaction costs we can derive the following conditions for  $Y_y$ :

$$Y_y = \begin{cases} 0 & \text{for } \frac{T_{y_1}}{F_{y_1}} = \frac{T_{y_2}}{F_{y_2}} \\ > 0 & \text{for } \frac{T_{y_2}}{F_{y_2}} > \frac{T_{y_1}}{F_{y_1}} \\ < 0 & \text{for } \frac{T_{y_2}}{F_{y_2}} < \frac{T_{y_1}}{F_{y_1}} \end{cases} \quad (4.40)$$

It is not possible to derive the same information out of  $Y_x$ , as  $Y_x$  is always equal to or greater than zero.

## 4.4 Results

We conduct the estimation by applying accountancy data and data of egocentered networks of Polish farms collected in 2007. We have a sample of 322 observations entering the first step of the estimation.

As described in section 4.3.3.1 on the first step we estimate a stochastic ray production frontier, where  $\|y\|$  is the euclidean distance of the two outputs *crops* and *animal products*, that enters the estimation as dependent variable, and *labour*, *land*, *capital*, and the variable inputs  $x_{crop}$ ,  $x_{animal}$ ,  $x_{general}$ , as well as  $\theta$  as exogenous variables. Furthermore, we include an ordered categorical variable *education*<sup>3</sup>, *experience* measured in years worked, a dummy variable for mixed mode farms, as well as a second ordered categorical variable *risk behaviour*<sup>4</sup>, as explanatory variables of the inefficiency term. Table 4.1 shows the parameters and standard errors for the stochastic ray production function.

To enforce monotonicity in inputs we apply a three step estimation procedure proposed by Henningsen and Henning (2009). Table 4.2 compares the estimates of the unrestricted model

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<sup>3</sup>where 1 = none, 2 = apprenticeship, 3 = vocational school, 4 = university diploma

<sup>4</sup>four categories, where 1 equals risk neutral behaviour and 4 equals strong risk aversion

Table 4.1: Summary of the unrestricted Stochastic Ray Frontier

	Estimate	Std. Error	z value	Pr(> z )
$\alpha_0$	13.481897	7.774840	1.734042	0.082911
$\alpha_1$	0.176360	0.214775	0.821140	0.411566
$\alpha_2$	0.929493	1.402296	0.662836	0.507435
$\alpha_3$	-0.684397	1.016755	-0.673118	0.500872
$\alpha_4$	0.049177	0.607509	0.080948	0.935483
$\alpha_5$	-0.447344	0.366349	-1.221085	0.222054
$\alpha_6$	0.861075	1.100633	0.782345	0.434012
$\alpha_t$	-10.954101	1.232760	-8.885832	0.000000
$\beta_{11}$	0.002589	0.020540	0.126028	0.899710
$\beta_{12}$	-0.021356	0.025794	-0.827949	0.407699
$\beta_{13}$	0.007706	0.017883	0.430886	0.666552
$\beta_{14}$	-0.010174	0.011409	-0.891762	0.372520
$\beta_{15}$	0.005167	0.005905	0.875055	0.381544
$\beta_{16}$	-0.011004	0.023197	-0.474396	0.635218
$\beta_{1t}$	-0.046283	0.024723	-1.872044	0.061201
$\beta_{22}$	0.514259	0.245099	2.098167	0.035890
$\beta_{23}$	0.107363	0.129769	0.827333	0.408048
$\beta_{24}$	-0.220001	0.111270	-1.977184	0.048021
$\beta_{25}$	-0.151216	0.049351	-3.064106	0.002183
$\beta_{26}$	0.041178	0.150120	0.274301	0.783854
$\beta_{2t}$	-0.044447	0.165407	-0.268711	0.788152
$\beta_{33}$	0.002245	0.087857	0.025556	0.979612
$\beta_{34}$	0.034939	0.054740	0.638270	0.523298
$\beta_{35}$	-0.033709	0.033249	-1.013820	0.310669
$\beta_{36}$	0.011078	0.106147	0.104363	0.916881
$\beta_{3t}$	0.369111	0.110796	3.331453	0.000864
$\beta_{44}$	0.006347	0.027508	0.230723	0.817530
$\beta_{45}$	0.002171	0.022900	0.094823	0.924456
$\beta_{46}$	0.018307	0.061065	0.299797	0.764332
$\beta_{4t}$	-0.005803	0.078283	-0.074131	0.940906
$\beta_{55}$	0.027168	0.013258	2.049130	0.040449
$\beta_{56}$	0.113577	0.039863	2.849204	0.004383
$\beta_{5t}$	0.034587	0.038239	0.904496	0.365733
$\beta_{66}$	-0.254676	0.114312	-2.227893	0.025888
$\beta_{6t}$	0.289399	0.127877	2.263116	0.023629
$\beta_{tt}$	3.067377	0.551665	5.560220	0.000000
$Z_{education}$	224.959871	448.437985	0.501652	0.615912
$Z_{exper}$	44.078577	87.866180	0.501656	0.615910
$Z_{delta_3FALSE}$	-2224.633373	4445.031138	-0.500476	0.616740
$Z_{delta_3TRUE}$	-2191.346657	4375.178579	-0.500859	0.616471
$Z_{risk2}$	-649.154135	1298.590187	-0.499891	0.617152
$\sigma^2$	1111.405179	2229.860324	0.498419	0.618189
$\gamma$	0.999706	0.000625	1600.799491	0.000000

( see table 4.1 ) with the estimates of the restricted model and the estimates of an adjusted restricted model.

Using the parameters of the adjusted restricted model, we estimate the endogenous variable from equations (27) and (28). The calculated endogenous variables enter the second step of the estimation procedure (see section 4.3.3.2). We estimate equations (32) and (33) for crop related inputs, animal related inputs, and general inputs, as well as for animal products, in a linear system of equations applying seemingly unrelated regressors (SUR). Tables 4.3 to 4.6 present the results for each estimation equation.

Because of missing or unreliable price data, we are forced to reduce our dataset considerably<sup>5</sup>. The small data volume and the noticeable correlation between the regressors contribute to the low significances we find for  $T_{x_c}$ ,  $T_{x_a}$ ,  $T_{x_g}$ , and  $T_{y_a}$ . This becomes especially apparent in comparison with  $T_{y_c}$ , which enter all four equations and additionally are restricted on parameters in the way that all parameters entering  $T_{y_c}$  have to be the same over all four estimation equations. Therefore,  $T_{y_c}$  is estimated with far more observations than the other transaction cost equations, which as a consequence leads to considerably higher significance levels.

- In table 4.3, we find little influence of the network parameters on  $T_{x_c}$ . There is a small, but significant influence of *density* on proportional transaction costs. Otherwise, we find no effects of network structural parameters on  $T_{x_c}$ , neither do we find a significant influence of price volatility  $R(v_c)$  on the dependent variable.
- Table 4.4 shows, inter alia, the results for  $T_{x_a}$  the transaction costs on animal related inputs. Also, in this case, we find little effects of the network structural parameters on  $T_{x_a}$ . There is a significantly positive effect of *outdegree* on the proportional part of  $T_{x_a}$ , as well as a significantly negative effect of *density* on non proportional transaction costs. As in the case of  $T_{x_c}$ , we find no significant effect of price volatility  $R(v_a)$  on the depended variable.
- The estimation equation of  $T_{x_g}$  (table 4.5) resembles the two former cases in the fact that there is little influence of the network structural parameters, except for a weakly significant influence of *density* on the non proportional part of  $T_{x_g}$ .
- We find no significant parameters for  $T_{y_a}$  (table 4.6). Anyway, our data displays a considerably negative impact of *density* on the proportional transaction costs, as well as a strong significant impact of both *density* and *outdegree* on the non proportional part of  $T_{y_a}$ .

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<sup>5</sup>Missing data is also the reason for the exclusion of the variable for market concentration



Table 4.2: Coefficients of the unrestricted, restricted and adj. restricted models

	unrestricted coefficients	restricted coefficients	adj. restricted coefficients
$\alpha_0$	13.4819	8.7127	8.7333
$\alpha_1$	0.1764	0.1451	0.1453
$\alpha_2$	0.9295	0.6429	0.6439
$\alpha_3$	-0.6844	-0.0231	-0.0231
$\alpha_4$	0.0492	0.1603	0.1606
$\alpha_5$	-0.4473	-0.0776	-0.0778
$\alpha_6$	0.8611	0.3503	0.3508
$\alpha_t$	-10.9541	-9.4343	-9.4491
$\beta_{11}$	0.0026	0.0005	0.0005
$\beta_{12}$	-0.0214	0.0006	0.0006
$\beta_{13}$	0.0077	0.0019	0.0019
$\beta_{14}$	-0.0102	-0.0031	-0.0031
$\beta_{15}$	0.0052	0.0008	0.0008
$\beta_{16}$	-0.0110	-0.0084	-0.0084
$\beta_{1t}$	-0.0463	-0.0398	-0.0399
$\beta_{22}$	0.5143	0.1913	0.1916
$\beta_{23}$	0.1074	-0.0007	-0.0007
$\beta_{24}$	-0.2200	-0.0530	-0.0531
$\beta_{25}$	-0.1512	-0.0368	-0.0368
$\beta_{26}$	0.0412	-0.0067	-0.0067
$\beta_{2t}$	-0.0444	0.1185	0.1187
$\beta_{33}$	0.0022	-0.0003	-0.0003
$\beta_{34}$	0.0349	0.0023	0.0023
$\beta_{35}$	-0.0337	-0.0004	-0.0004
$\beta_{36}$	0.0111	-0.0005	-0.0005
$\beta_{3t}$	0.3691	0.2357	0.2361
$\beta_{44}$	0.0063	-0.0145	-0.0145
$\beta_{45}$	0.0022	0.0030	0.0030
$\beta_{46}$	0.0183	0.0149	0.0150
$\beta_{4t}$	-0.0058	0.0488	0.0489
$\beta_{55}$	0.0272	0.0055	0.0055
$\beta_{56}$	0.1136	0.0147	0.0147
$\beta_{5t}$	0.0346	0.0266	0.0267
$\beta_{66}$	-0.2547	-0.0519	-0.0519
$\beta_{6t}$	0.2894	0.1616	0.1619
$\beta_{tt}$	3.0674	3.4907	3.4962
$Z_{education}$	224.9599		220.7778
$Z_{exper}$	44.0786		27.9750
$Z_{\delta_3 FALSE}$	-2224.6334		-1799.4540
$Z_{\delta_3 TRUE}$	-2191.3467		-1843.5906
$Z_{risk2}$	-649.1541		-388.9322
$\sigma^2$	1111.4052		832.8322
$\gamma$	0.9997		0.9996

Table 4.3: Equation Input Crops

	Estimate	Std. Error	t value	Pr(> t )
$\gamma_0$	-0.018972	0.127076	-0.149298	0.881410
$\gamma_{11}$	0.676902	0.408737	1.656081	0.098662
$\gamma_{12}$	0.030193	0.031699	0.952474	0.341558
$\gamma_2$	-0.000001	0.000003	-0.468878	0.639468
$\gamma_{31}$	-0.000004	0.000005	-0.819819	0.412914
$\gamma_{32}$	-0.000000	0.000001	-0.643170	0.520563
$\mu_3$	-0.009144	0.031565	-0.289696	0.772232
$\delta_0$	0.948279	0.335250	2.828573	0.004963
$\delta_{11}$	-1.732925	0.798947	-2.169010	0.030800
$\delta_{12}$	-0.105918	0.080941	-1.308588	0.191590
$\delta_2$	0.000003	0.000001	4.825695	0.000002
$\delta_{31}$	0.000004	0.000002	1.747129	0.081551
$\delta_{32}$	-0.000000	0.000000	-2.341483	0.019806
$\mu_2$	-0.123414	0.078182	-1.578553	0.115402
	N:	99	DF:	85
	$R^2$ :	0.57414	Adj. $R^2$ :	0.509008

Table 4.4: Equation Input Animal

	Estimate	Std. Error	t value	Pr(> t )
$\gamma_0$	-0.169611	0.098451	-1.722809	0.085838
$\gamma_{11}$	0.373214	0.310661	1.201354	0.230455
$\gamma_{12}$	0.057648	0.024139	2.388224	0.017478
$\gamma_2$	0.000001	0.000000	1.309595	0.191222
$\gamma_{31}$	-0.000002	0.000001	-1.736610	0.083367
$\gamma_{32}$	-0.000000	0.000000	-1.534554	0.125829
$\mu_3$	-0.072521	0.100681	-0.720307	0.471834
$\delta_0$	1.196577	0.331491	3.609687	0.000353
$\delta_{11}$	-1.732925	0.798947	-2.169010	0.030800
$\delta_{12}$	-0.105918	0.080941	-1.308588	0.191590
$\delta_2$	0.000003	0.000001	4.825695	0.000002
$\delta_{31}$	0.000004	0.000002	1.747129	0.081551
$\delta_{32}$	-0.000000	0.000000	-2.341483	0.019806
$\mu_2$	-0.123414	0.078182	-1.578553	0.115402
	N:	83	DF:	69
	$R^2$ :	0.461281	Adj. $R^2$ :	0.359784

Table 4.5: Equation Input General

	Estimate	Std. Error	t value	Pr(> t )
$\gamma_0$	0.836533	0.564013	1.483181	0.138987
$\gamma_{11}$	-0.210445	1.230014	-0.171092	0.864257
$\gamma_{12}$	0.196645	0.138629	1.418495	0.156996
$\gamma_2$	-0.000039	0.000022	-1.795980	0.073418
$\gamma_{31}$	0.000058	0.000027	2.141497	0.032970
$\gamma_{32}$	-0.000004	0.000005	-0.773158	0.439986
$\delta_0$	0.948279	0.335250	2.828573	0.004963
$\delta_{11}$	-1.732925	0.798947	-2.169010	0.030800
$\delta_{12}$	-0.105918	0.080941	-1.308588	0.191590
$\delta_2$	0.000003	0.000001	4.825695	0.000002
$\delta_{31}$	0.000004	0.000002	1.747129	0.081551
$\delta_{32}$	-0.000000	0.000000	-2.341483	0.019806
$\mu_2$	-0.123414	0.078182	-1.578553	0.115402
	N:	97	DF:	84
	$R^2$ :	0.372618	Adj. $R^2$ :	0.282992

Table 4.6: Equation Output Animal

	Estimate	Std. Error	t value	Pr(> t )
$\zeta_0$	-0.184430	0.524839	-0.351402	0.725512
$\zeta_{11}$	-0.534653	1.321052	-0.404717	0.685949
$\zeta_{12}$	-0.188434	0.127484	-1.478092	0.140343
$\zeta_2$	0.000002	0.000005	0.481992	0.630133
$\zeta_{31}$	-0.000003	0.000007	-0.398473	0.690541
$\zeta_{32}$	0.000001	0.000001	0.678144	0.498159
$\mu_1$	0.001924	0.006961	0.276411	0.782407
$\delta_0$	0.948279	0.335250	2.828573	0.004963
$\delta_{11}$	-1.732925	0.798947	-2.169010	0.030800
$\delta_{12}$	-0.105918	0.080941	-1.308588	0.191590
$\delta_2$	0.000003	0.000001	4.825695	0.000002
$\delta_{31}$	0.000004	0.000002	1.747129	0.081551
$\delta_{32}$	-0.000000	0.000000	-2.341483	0.019806
$\mu_2$	-0.123414	0.078182	-1.578553	0.115402
	N:	83	DF:	69
	$R^2$ :	0.511398	Adj. $R^2$ :	0.419343

To figure out the influence of the two network parameters *density* and *outdegree* on the different forms of transaction costs we calculate

$$T(s) = \phi_0 + \phi_1 z + \phi_2 2S + \phi_3 z 2S \quad (4.41)$$

$$\frac{\partial T(s)}{\partial z} = \phi_1 + \phi_3 2S \quad (4.42)$$

where  $S = \{y_i, x_k\}$  <sup>6</sup>. Table 4.7 presents the results.

Table 4.7: Effects of network structural parameters on transaction costs

	Mean Density	Mean Outdegree	Median Density	Median Outdegree	Min Density	Min Outdegree	Max Density	Max Outdegree
Input Crops	0.5420	0.0164	0.5935	0.0217	-0.5983	-0.0998	0.6768	0.0302
Input Animal	-0.0922	0.0416	-0.0912	0.0444	-0.1084	-0.0036	-0.0911	0.0448
Input General	0.9263	0.1174	0.5309	0.1449	-0.1699	-0.6354	11.7209	0.1938
Output Cops	-1.0522	-0.1624	-1.7278	-0.1063	-1.7329	-2.8601	31.4869	-0.1059
Output Animal	-0.7496	-0.1267	-0.6546	-0.1540	-2.4740	-0.1884	-0.5347	0.3682

- Input crops: Both the mean and the median of the parameters show a positive impact of networks on  $T_{x_c}$ . If *density* increases from 0 to 1, the relative  $T_{x_c}$  increases by 54 % of the price, i.e. the shadow price raises by 150 %. If *outdegree* increases by one contact, the relative  $T_{x_c}$ , in turn increase by 1.6 % of the price. Hence, in the case of  $T_{x_c}$  we cannot state a positive impact of networks on the reduction of transaction costs. Quite contrary, our results seem to indicate that it is preferable to reduce to dyadic trade relationships in the purchase of crop related variable inputs.
- Input animal: Both mean and median for the parameters of *density* display a negative impact on  $T_{x_a}$ . If *density* increases from 0 to 1 the relative  $T_{x_a}$  reduce by 9 % of the price. On the other hand, *outdegree* still increases relative  $T_{x_a}$ . An additional contact augments the relative  $T_{x_a}$  by approximately 4 % of the price value. Our results indicate that small and dense networks seem to be able to reduce the relative  $T_{x_a}$  on markets for animal related variable inputs.

<sup>6</sup>As we have normalised the prices to unity we can interpret  $T(s)$  as relative transaction costs

- Input general: As well as in the case of  $T_{x_c}$ , we find no positive impact of social networks on the reduction of transaction costs.
- Output crop and output animal: On the other hand we find our theoretical assumptions confirmed by the parameters measuring the impact of social networks on  $T_{y_c}$  and  $T_{y_c}$ . Networks show a negative impact on both  $T_{y_c}$  and  $T_{y_c}$ . Increasing *density* from 0 to 1 leads to a reduction of the relative  $T_{y_c}$  and  $T_{y_c}$  by more than 100 % and 70 % of the price value, respectively. An additional contact decreases  $T_{y_c}$  by 16 % and  $T_{y_c}$  by approximately 12 % of the price value.

Hence, we can conclude that given our results we can confirm the theoretical assumptions derived in earlier chapters with respect to output markets, while we find opposite effects on input markets. In the latter case, our data seems to indicate that it is beneficial to reduce the number of the suppliers to the smallest amount possible. A possible explanation could be that small and tight village networks have a tendency to bind their members tightly. A break out, for example by searching for additional suppliers outside the local area, could lead to social consequences, being so severe that the net-benefit from social networks is reversed. On the other hand, our results indicate that output markets seem to benefit from tight and huge networks. Obviously they are better able to provide social control mechanisms than other network structures so that in consequence transaction costs reduce.

Table 4.9 and table 4.8 show the calculated proportional, disproportional and total transaction costs for the different commodity markets, respectively.

Table 4.8: Calculated transaction costs

	Mean	Median	StDev
Input Crops	0.3422	0.3009	1.3827
Input Animal	-1.9448	0.1203	21.3190
Input General	0.8591	0.9531	0.8536
Output Cops	0.6019	0.4505	0.8414
Output Animal	0.6713	0.6230	0.5580

As can be seen from the standard errors the data is considerably afflicted by outliers. Therefore, we will concentrate our discussion on the median instead of the mean. We stated in equation (40) that for the case of  $Y_y$  the dependent variable displays the ratio of the  $T_{y_1}$  and  $T_{y_2}$ . Because of opposite effects of the transaction costs on  $p_i$  and  $w_k$  this does not apply to  $y_x$ . Comparing the conditions denounced in equation (40) with the estimated transaction cost values of  $T_{y_1}$  ( $-52.3225$ ) and  $T_{y_2}$  ( $-20.356$ ) we find that the estimated transaction costs confirm our assumptions derived under equation (40).

Table 4.9: Proportional and non proportional transaction costs

	Proportional TAC			Non proportional TAC		
	Mean	Median	StDev	Mean	Median	StDev
Input Crops	3.7530	0.5459	26.2501	-0.5969	-0.1996	1.7935
Input Animal	-0.1770	0.2160	16.8709	-1.4803	-0.0201	11.0797
Input General	1.5777	1.5352	0.4145	-0.7186	-0.4710	0.8991
Output Cops	0.0127	0.1716	0.4649	0.5893	0.1726	0.7189
Output Animal	1.0481	0.9574	0.4734	-0.3768	-0.2000	0.4475

Our findings show that relative transaction costs for markets for crop and animal related inputs, as well as the market for crop products range in realistic domains, while transaction costs for general input markets and animal products are out of scale, so that the linear approximation assumed in equation (20) does not in fact apply in these two cases.

One reason for high transaction costs for the  $T_{x_g}$  could be that the input quantity and price are a conglomerate of heterogeneous and unrelated inputs. Under these conditions, the question remains whether it is correct to model these inputs in an aggregate form.

In the case of markets for animal products, we have a bit of the same problem, as our sample of animal products is much more diverse than it is in the case of crops (which mainly comprises of grains and oilseeds). Hence, both production technology and price index in the case of animal products will be afflicted by a higher variance than there is in the case of crop products. This effect, can result in an inferior approximation of the given technology, larger effects of price volatility, and more heterogeneity in the characteristics of the products.

## 4.5 Conclusions

We have derived a theoretically founded structural equation model to measure the effect of the structure of social networks on transaction costs. To the authors' knowledge, this or a similar approach cannot be found in the literature. Given the novelty of the approach, it is unsurprising that not all problems are solved to an optimal extent. The measurement of other effects than transaction costs on the divergence between measured technology and observed price data is very ad hoc and should be further investigated in future research.

Anyway, we have been able to measure some influence of social networks on transaction costs and have achieved quantity values for transaction costs that, given noisy real world data, in most cases lie in reasonable ranges.

We consider this method to be an innovative tool to approach the problem of markets with transaction costs. We hope to give some inspiration for future research in this interesting research field.

## Appendix

### 4.A Calculation of output specific marginal costs

#### 4.A.1 General form

Production function

$$F(y, x) = 0 \quad (4.43)$$

where  $y = (y_1, \dots, y_I)'$  is a vector of  $I$  outputs and  $x = (x_1, \dots, x_J)'$  is a vector of  $J$  inputs.

Cost function

$$C(y, w) = \min_x w'x \text{ s.t. } F(y, x) = 0 \quad (4.44)$$

Lagrangian

$$L = w'x + \lambda F(y, x) \quad (4.45)$$

We get  $J + 1$  FOC:

$$\frac{\partial L}{\partial x_i} = w_i + \lambda \frac{\partial F(y, x)}{\partial x_i} = 0 \quad (4.46)$$

$$\frac{\partial L}{\partial \lambda} = F(y, x) = 0 \quad (4.47)$$

The  $J$ th FOC is solved for  $\lambda$

$$\lambda = - \frac{w_J}{\frac{\partial F(y, x)}{\partial x_J}} \quad (4.48)$$

Replacing  $\lambda$  in the first  $J - 1$  FOC with equation (4.48) we derive the following system of  $J$  equations

$$g_1(y, x) = w_1 - w_J \frac{\frac{\partial F(y, x)}{\partial x_1}}{\frac{\partial F(y, x)}{\partial x_J}} = 0 \quad (4.49)$$

$$\vdots \quad (4.50)$$

$$g_{J-1}(y, x) = w_{J-1} - w_J \frac{\frac{\partial F(y, x)}{\partial x_{J-1}}}{\frac{\partial F(y, x)}{\partial x_J}} = 0 \quad (4.51)$$

$$g_J(y, x) = F(y, x) = 0 \quad (4.52)$$

Applying the implicit function theorem on the system  $J$  functions  $g_1(y, x), \dots, g_J(y, x)$ , we derive the Jacobian matrix consisting of the derivatives of the output quantities ( $y$ ) with



respect to the input quantities ( $x$ )

$$\frac{\partial x}{\partial y} = - \begin{pmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_J} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_J}{\partial x_1} & \cdots & \frac{\partial g_J}{\partial x_J} \end{pmatrix}^{-1} \begin{pmatrix} \frac{\partial g_1}{\partial y_1} & \cdots & \frac{\partial g_1}{\partial y_I} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_J}{\partial y_1} & \cdots & \frac{\partial g_J}{\partial y_I} \end{pmatrix} \quad (4.53)$$

Finally the output-specific marginal costs can be derived by

$$\frac{\partial C(y, w)}{\partial y} = w' \frac{\partial x}{\partial y} \quad (4.54)$$

#### 4.A.2 Translog form for the multiple stochastic ray production frontier

Lagrange function

$$\begin{aligned} L = & \sum_i w_i x_i + \lambda \left( \alpha_0 + \sum_i \alpha_i \log(x_i) \right. \\ & + \frac{1}{2} \sum_i \sum_m \beta_{im} \log(x_i) \log(x_m) + \alpha_\theta \theta \\ & \left. + \frac{1}{2} \beta_{\theta\theta} \theta^2 + \sum_i \beta_{i\theta} \theta \log(x_i) - \log(\|y\|) \right) \end{aligned} \quad (4.55)$$

The resulting  $J$  functions  $g_1(y, x), \dots, g_J(y, x)$  are

$$g_i(y, x) = w_i - w_J \cdot \frac{x_J}{x_i} \cdot \frac{\alpha_i + \sum_m \beta_{im} \log(x_m) + \beta_{i\theta} \theta}{\alpha_J + \sum_m \beta_{Jm} \log(x_m) + \beta_{J\theta} \theta} = 0 \quad \forall i = 1, \dots, J-1 \quad (4.56)$$

$$\begin{aligned} g_J(y, x) = & \alpha_0 + \sum_i \alpha_i \log(x_i) \\ & + \frac{1}{2} \sum_i \sum_m \beta_{im} \log(x_i) \log(x_m) + \alpha_\theta \theta \\ & + \frac{1}{2} \beta_{\theta\theta} \theta^2 + \sum_i \beta_{i\theta} \theta \log(x_i) - \log(\|y\|) = 0 \end{aligned} \quad (4.57)$$

The elements of the Jacobian matrices  $\partial g(y, x)/\partial x$  and  $\partial g(y, x)/\partial x$  are defined by

$$\frac{\partial g_i}{\partial x_j} = \begin{cases} \Delta_{ij} w_J \cdot \frac{x_J}{x_j^2} \cdot \frac{\alpha_i + \sum_m \beta_{im} \log(x_m) + \beta_{i\theta} \theta}{\alpha_J + \sum_m \beta_{Jm} \log(x_m) + \beta_{J\theta} \theta} \\ -\Delta_{jJ} w_J \cdot \frac{1}{x_i} \cdot \frac{\alpha_i + \sum_m \beta_{im} \log(x_m) + \beta_{i\theta} \theta}{\alpha_J + \sum_m \beta_{Jm} \log(x_m) + \beta_{J\theta} \theta} \\ -w_J \cdot \frac{x_J}{x_i x_j} \cdot \frac{\beta_{im}}{\alpha_J + \sum_m \beta_{Jm} \log(x_m) + \beta_{J\theta} \theta} \\ +w_J \cdot \frac{x_J \beta_{Jj}}{x_i x_j} \cdot \frac{\alpha_i + \sum_m \beta_{im} \log(x_m) + \beta_{i\theta} \theta}{(\alpha_J + \sum_m \beta_{Jm} \log(x_m) + \beta_{J\theta} \theta)^2} & \text{for } i < J \\ \frac{\alpha_j + \sum_m \beta_{jm} \log(x_m) + \beta_{j\theta} \theta}{x_j} & \text{for } i = J \end{cases} \quad (4.58)$$

and

$$\frac{\partial g_i}{\partial y_z} = \begin{cases} -w_J \cdot \frac{x_J \beta_{i\theta}}{x_i} \cdot \frac{1}{\alpha_J + \sum_m \beta_{Jm} \log(x_m) + \beta_{J\theta} \theta} \cdot \frac{\partial \theta}{\partial y_z} \\ +w_J \cdot \frac{x_J \beta_{J\theta}}{x_i} \cdot \frac{\alpha_i + \sum_m \beta_{im} \log(x_m) + \beta_{i\theta} \theta}{(\alpha_J + \sum_m \beta_{Jm} \log(x_m) + \beta_{J\theta} \theta)^2} \cdot \frac{\partial \theta}{\partial y_z} & \text{for } i < J \\ \left( \alpha_\theta + \beta_{\theta\theta} \theta + \sum_m \beta_{m\theta} \log(x_m) \right) \cdot \frac{\partial \theta}{\partial y_z} - \frac{\partial \log(\|y\|)}{\partial y_z} & \text{for } i = J \end{cases} \quad (4.59)$$

with

$$\frac{\partial \log(\|y\|)}{\partial y_z} = \frac{y_z}{y_1^2 + y_2^2} \quad \text{with } z = 1, 2 \quad (4.60)$$

$$\frac{\partial \theta}{\partial y_1} = \frac{1}{\sqrt{1 - \left( \frac{y_1}{\|y\|} \right)^2}} \cdot \left( \frac{y_1^2}{(y_1^2 + y_2^2)^{3/2}} - \frac{1}{\|y\|} \right) \quad (4.61)$$

$$\frac{\partial \theta}{\partial y_2} = \frac{y_1 \cdot y_2 \cdot (y_1^2 + y_2^2)^{-3/2}}{\sqrt{1 - \left( \frac{y_1}{\|y\|} \right)^2}} \quad (4.62)$$

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## Kapitel 5

# Measuring the Influence of Information Networks on Transaction Costs Using a Non-parametric Regression Technique

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## Abstract

All business transactions as well as achieving innovations take up resources, subsumed under the concept of transaction costs (TAC). One of the major factors in TAC theory is information. Information networks can catalyse the interpersonal information exchange and hence, increase the access to non-public information. Our analysis shows that information networks have an impact on the level of TAC. Many resources that are sacrificed for TAC are inputs that also enter the technical production process. As most production data do not distinguish between these two usages of inputs, high transaction costs occur due to reduced productivity. A cross-validated local linear non-parametric regression shows that good information networks increase the productivity of farms. A bootstrapping procedure confirms that this result is statistically significant.

## 5.1 Transaction Costs and Social Networks

Traditional neoclassical economics assumes that the exchange of goods is costless so that—in this respect—markets are efficient and always provide goods at the lowest possible price. However, more than 70 years ago, Coase (1937) argued in his essay “The Nature of the Firm” that market transactions often involve higher costs than just the market price. Other costs (e.g. search and information costs, bargaining costs, and policing and enforcement costs) can increase the costs of procuring something from a market. His theory became manifest in the concept of transaction costs, which has become a major field in institutional economics, especially during the past 30 years.<sup>1</sup>

Transaction costs can be divided into two main categories: technological transaction costs and institutional transaction costs (Green and Sheshinski, 1975). Both technological and institutional transaction costs refer to the sacrifice of resources. Furthermore, institutional transaction costs include search, negotiation, and control costs, while technological transaction costs can be divided into innovation transaction costs and physical transportation costs.

Institutional transaction costs can occur at three different stages of the transaction: *i*) contact phase, *ii*) contracting phase, and *iii*) control phase (den Butter and Mosch, 2003).

In the *contact* phase of a potential transaction, the actor is looking for information on potential trade partners (buyers or sellers), information on non-observable quality characteristics of his preferred product, and prices of the product (either seller or buyer prices). These

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<sup>1</sup>For a brief introduction into the development and concepts of transaction cost theory, see Rindfleisch and Heide (1997), Klaes (2000) and Macher and Richman (2008).

searching costs occur because the search for information is not free, nor is information always complete, reliable, or easily accessible. Akerlof (1970) shows in his classical lemons problem that information asymmetry can be so severe and access to reliable information so costly and difficult that a market collapses. Searching costs are reduced if information is more easily accessible. Well functioning information networks can provide their members with information on business opportunities by giving cheap access to the above mentioned information (Granovetter, 1983; Dekker, 2001; Henning and Zuckerman, 2006), whilst as a further benefit, they increase the reliability of the information. In fact, with more information available in the network, and with easier transfers to all interested members, the probability that the information is of high quality increases, i.e. the information can be trusted to be relevant and true (Casson, 1997; Fafchamps, 2001). Based on the theory of weak ties (Granovetter, 1973), Montgomery (1992) demonstrates that weak ties are positively related to higher wages and higher aggregate employment rates. Actors with many loose ties (gatekeeper) are superior regarding access to reliable information on market opportunities and perform better on the market.

The *contract* phase starts when two trade partners agree to make a deal, in this phase transaction costs are mainly negotiation costs. Both partners have to agree on how to divide potential rents from trade, i.e. negotiation of trading conditions (Braun and Gautschi, 2000). Because of bounded rationality, a perfect contract that accounts for all eventualities is unachievable. First, not all arrangements are verifiable by third parties (verification problem). Second, many eventualities cannot be foreseen (environmental and behavioural uncertainty). The higher the ex ante trust level between trading partners the lower is the necessity to negotiate every detail of the transaction (Portes and Sensenbrenner, 1993; Uzzi, 1996). Hence, negotiation costs can be reduced considerably (Nee, 1998; Fafchamps, 2001). In fact, Rooks, Raub, and Tazelaar (2000) showed, in a vignette study, that socially well embedded transactions led a purchase manager to put less effort into the management of a transaction. In total, less time was invested in the transaction and fewer departments were involved when the ex ante trust level was high because of social embeddedness, which included temporal, network, and institutional embeddedness. The study shows that network embeddedness has a significant influence and reduces the amount of resources invested in a transaction.

Finally, if contracts are signed, the *control* phase starts. This consists of monitoring and enforcing the contract. Both actions involve many resources and, thus, induce high transaction costs. The first step is the monitoring of the partner to ensure that she meets the arrangement manifested in the contract. If one of the partners behaves opportunistically by not keeping to the agreements, the next step is the enforcement of the contract. In most

cases, legal procedures are troublesome, expensive and of long duration. Informal punishment systems, such as the loss of a good reputation or exclusion from future trade possibilities (Kandori, 1992; Greif, 1994; Buskens, 1998), can reduce the costs of contract monitoring and enforcement (Ménard, 2000; Rooks, Raub, and Tazelaar, 2000). The better these informal mechanisms work, the lower the incentive to defect in transaction and, hence, the lower the monitoring and enforcement costs (Buskens, 1999; Richman, 2006).

*Innovation* transaction costs in turn refer to resources which are sacrificed to gather reliable information on novelties and innovative production methods and processes. Although information on innovations is accessible via public resources such as consulting or professional journals, a considerable amount of information is private (e.g. a competitor's experience with a new production method).

However, this does not mean that private information is completely unavailable. Managers might have close business and social contacts who possess this information and are willing to share it. Hence, the quality and quantity of relationships with other professionals and the relevance of these partners may have an important impact on a firm's innovation transaction costs (Castilla et al., 2000). It is quite straightforward that information networks which allow this information to spread among entrepreneurs can have a significant impact on the productivity of the entities that have better access to the network, i.e. have better access to reliable non-public information about innovative production methods and processes (Jenssen and Koenig, 2002).

It has become apparent that a driving force which explains the level of transaction costs an entity faces is information (Greif, 1994; Noorderhaven, 1996; Calvert, 1995; Levi, 2000). Both institutional as well as innovation transaction costs are therefore materially dependent on access to information. As shown above, information networks can provide an efficient and opportune way of gathering especially non-publicly available information (Granovetter, 1983; Raub and Weesie, 1990; Moschandreas, 1997; Buskens, 1999; Burt, 2001; den Butter and Mosch, 2003; Wiebusch, Henning, and Henningsen, 2004).

As the structure of personal and interpersonal networks differs, the ability to gather information via information networks may be limited for some entities and may be amplified for others depending on their individual situation in a network (Buskens, 1999). Hence, we conclude that the individual network position should have an effect on an entity's transaction costs.

The question remains which network structures are beneficial regarding the reduction of transaction costs. Apart from physical transportation that is only determined by local distance and infrastructure, all sources of transaction costs —searching, negotiation, control,

and innovation transaction costs— are related to information networks. The difficulty that arises is that something that might be beneficial for one kind of transaction cost might be disadvantageous to others. The literature is not clear-cut on the effects of network structures on certain problems.

Beginning with searching costs, the literature suggests that beneficiary network structures are characterised by weak ties, i.e. from the ego centered point of view, a high number of outdegrees with lower density. In the case of negotiation and enforcement costs on the other hand, the literature states that they profit from strong social control as a consequence of tight information networks, i.e. networks with high density. The *closure argument* (Granovetter, 1985; Coleman, 1990) states that dense networks increase social control, develop common norms, and provide the possibility of punishment in the case of misbehaviour. Thus, dense networks supply their members with high levels of trust and reliable information. This hypothesis is supported by a simulation study by Buskens (1998) which shows that the level of trust increases with both outdegree and density. But in cases where trust is low on the dyadic level, the importance of density exceeds the influence of outdegrees.

Contrary to the closure argument, the *gossip argument* (Coleman, 1990; Burt, 2001) states that high density is not necessarily beneficial to increase the level of trust and can in fact reduce the reliability of the available information. The argument is that networks affected by the gossip effect show a tendency to self-enforcing exaggeration, which leads to very extreme positions about other actors, i.e. actors are characterised being either extremely good or extremely bad. Hence, the information becomes unreliable and the trust level decreases. Both Burt (2001) and Dekker (2001) provide empirical evidence that very dense networks are dominated by the gossip effect and can show lower trust levels than less dense networks.

Finally, in the case of innovation costs —as in the case of searching costs— Granovetter (1983) states that weak ties are the key to innovation diffusion. Loose ties between the egos' clustered core networks increase the diffusion of relevant information on innovation. According to Granovetter (1983), actors that are enclosed in a tight and locked-in network have no or minimised access to “new” information and lag behind when it comes to innovative production technology, while actors with many and loose contacts profit from an increased access to “new” information. Although Granovetter's theory is straightforward, empirical evidence is not definite. In a study among Norwegian entrepreneurs, Jenssen and Koenig (2002) find no empirical evidence to support the theory of weak ties. Contrary to what might be expected, Jenssen and Koenig (2002) show that strong ties are important channels for information and can influence entrepreneurial success.

As our data leaves no possibility to separate between the different kinds of transaction costs named above, and additionally we find no definite evidence from the literature on the effects of information networks on the different sources of transaction costs, we have to limit our empirical analysis to check whether information networks have a noticeable impact on a firm's productivity, and which network structures are most beneficial for obtaining relevant information that transfers into high productivity.

The article is organised as follows. In the next section, we will give a brief overview of the microeconomic foundation of our model. Section three explains the data and the empirical model, and section four explains the results of the empirical analysis. Finally, section five concludes.

## 5.2 Microeconomic Foundation

We assume that a firm uses a vector of  $n$  input quantities  $x = (x_1, \dots, x_n)'$  to produce the output quantity  $y$ , where the transformation of the inputs into the output can be described by the production function

$$y = f(x, T) \tag{5.1}$$

and depends on the state of the technology  $T$ .<sup>2</sup>

### 5.2.1 Production technology and innovation

We assume that the firm can use resources to improve its production technology  $T$ , where these resources can be of the same type as the inputs used for the production (e.g. labour, office supplies, IT technology, fuel). We denote these resources by  $\bar{x} = (\bar{x}_1, \dots, \bar{x}_n)'$ , where the elements of  $\bar{x}$  correspond to the elements of  $x$ <sup>3</sup> so that we can calculate the total input quantities that the firm uses for production and for improving the production technology by  $x^* = x + \bar{x}$ . Furthermore, the firm can utilise its information networks to improve its production technology by gathering information from peers, which is otherwise difficult or costly to obtain or even unavailable. We assume that these relationships can be described by the function

$$T = k(\bar{x}, z, u), \tag{5.2}$$

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<sup>2</sup>The following derivations can also be calculated for multiple outputs, but for simplicity we only use a single output here.

<sup>3</sup>Of course, some elements of  $\bar{x}$  might be zero (e.g. raw materials). If some inputs are only used for improving the technology, but not in the actual production (e.g. advisory services or consulting), we can add further elements to the vector  $x$  and set these elements to zero.

where  $z$  is a vector of network parameters characterising the firm's information networks and  $u$  is a vector of other factors that might affect the firm's state of technology (e.g. the education of the management). Substituting the function in (5.2) for  $T$  in equation (5.1), we get

$$y = f(x, k(\bar{x}, z, u)) \equiv f^*(x, \bar{x}, z, u). \quad (5.3)$$

With respect to resources used for both production and the improvement of the production technology, data sets that are used for estimating production functions generally do not separate between input quantities used for the actual production and input quantities used to improve the production technology. Therefore, the following approximation is necessary for empirical applications:

$$y = f^*(x, \bar{x}, z, u) \approx \hat{f}^*(x + \bar{x}, z, u) = \hat{f}^*(x^*, z, u). \quad (5.4)$$

### 5.2.2 Transaction costs in trade

In addition to the resources required for the production  $x$  and for improving the production technology  $\bar{x}$ , the firm needs further resources for trading goods, i.e. purchasing the inputs and selling the output. These resources can be of the same type as the inputs used for production (e.g. labour, capital, office supplies, IT technology, fuel). We denote the vector of resources used for trading goods by  $\tilde{x} = (\tilde{x}_1, \dots, \tilde{x}_n)'$ , where the elements of  $\tilde{x}$  correspond to the elements of  $x$ ,  $\bar{x}$ , and  $x^*$ .<sup>4</sup> Hence, we can calculate the total input quantities that the firm acquires to produce the output, improve the production technology, and to trade the goods by  $x^{**} = x^* + \tilde{x} = x + \bar{x} + \tilde{x}$ . We expect that the quantities of the resources required for trading goods depend on the quantities of the traded goods. Furthermore, our considerations in the previous section suggest that good information networks can reduce the input quantities that are sacrificed for trading goods ( $\tilde{x}$ ). We assume that the above mentioned relationships can be described by the (implicit) functions

$$\tilde{x}_i = g_i(x^{**}, y, z, v) \quad \forall i, \quad (5.5)$$

where  $z$  is—again—the vector of network parameters and  $v$  is a vector of other factors that might influence the resources required to trade the goods (e.g. heterogeneity of goods, distance to potential sellers and buyers). Now, we rearrange the above system to get a system

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<sup>4</sup>Of course, some elements of  $\tilde{x}$  might be zero (e.g. raw materials).

of implicit functions for  $x^*$

$$x_i^{**} - x_i^* = g_i(x^{**}, y, z, v) \quad \forall i \quad (5.6)$$

$$x_i^* = x_i^{**} - g_i(x^{**}, \hat{f}^*(x^*, z, u), z, v) \quad \forall i \quad (5.7)$$

$$x_i^* \equiv g_i^*(x^{**}, x^*, z, u, v) \quad \forall i, \quad (5.8)$$

which we can solve to get a system of explicit functions for  $x^*$

$$x_i^* \equiv h_i(x^{**}, z, u, v) \quad \forall i. \quad (5.9)$$

Substituting these functions for  $x^*$  in the production function that accounts for activities to improve the production technology (5.4), we get

$$y = \hat{f}^*(h(x^{**}, z, u, v), z, u) \equiv \hat{f}^{**}(x^{**}, z, u, v). \quad (5.10)$$

As data sets generally do not separate input quantities that are used for the actual production, for improving the production technology, and for trading goods into these three parts, production economists usually do not estimate the *real* production function  $f(x)$ , but an *augmented* production function  $\hat{f}^{**}(x^{**}, z, u, v)$  that not only includes the production process, but also the trading of goods and activities to improve the production technology. Hence, transaction costs and innovation costs are usually included in the estimated production technology. According to our assumptions, firms with better information networks need less resources for trading goods and can improve their production technology more easily and at less cost (see discussion in the previous section). Hence, these firms should be able to produce the same amount of output ( $y$ ) with smaller (total) input quantities ( $x^{**}$ ), i.e. they should appear to be more productive.

### 5.3 Model and Data

If our considerations about transaction costs and information networks are correct and we use a typical data set, where the input quantities include resources used for the production  $x$ , resources used to improve the production technology  $\bar{x}$ , and resources used for trading goods  $\tilde{x}$ , the production function should not only depend on the input quantities, but also on the entity's network position. Hence, we can test the hypothesis that information networks influence transaction costs by estimating the *augmented* production function  $\hat{f}^{**}(x^{**}, z, u, v)$  defined in (5.10) and testing if the network parameters  $z$  have a significant influence.

Given our microeconomic model derived above, the relationship between the total input quantities  $x^{**}$ , the network parameters  $z$ , the other factors  $u$  and  $v$ , and the output quantity  $y$  is unknown and could be rather complex. To avoid specifying a parametric functional form, we estimate this *augmented* production function by a non-parametric regression technique.<sup>5</sup> We apply the non-parametric local-linear estimation method for both continuous and categorical explanatory variables described in Li and Racine (2004); Racine and Li (2004), where the second-order Epanechnikov kernel is used for continuous regressors, the kernel proposed by Aitchison and Aitken (1976, p. 29) is used for unordered categorical explanatory variables, and the kernel proposed by Wang and van Ryzin (1981) is used for ordered categorical explanatory variables. The bandwidths of the regressors are selected according to the expected Kullback-Leibler cross-validation criterion (Hurvich, Simonoff, and Tsai, 1998). The estimation was performed within the statistical software environment “R” (R Development Core Team, 2009) using the add-on package “np” (Hayfield and Racine, 2008).

In our empirical analysis, we use a data set of Polish farms. The data were collected within the framework of the “Advanced-Eval” project financed by the European Union within the Sixth Framework Programme (contract number 022708). The data set includes detailed farm accountancy data and information on the farms’ ego centered networks. We take the total value of all produced goods as output (in Zloty) and we distinguish between four inputs: labour (in working hours), land (in ha), capital (in Zloty), and intermediate inputs (in Zloty), where the last category mainly consists of seeds, fertilisers, pesticides, purchased feed, fuel, and electricity. We take the logarithm of the output and all the input quantities so that the individual values of these variables are more equally distributed within the range of observed values. If we did not do this, there would be many observations within the bandwidths for small values (farms), but only very few observations within the bandwidth for large values (farms), which usually causes problems in non-parametric regression.

Since Polish farms usually have a single farm manager, we do not have to model intra-firm networks, which can play an important role in information diffusion. Hence, our data set has the advantage that we can neglect intra-firm networks when modeling information networks. We apply two common information network parameters for ego-centered networks to model

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<sup>5</sup>We estimated the *augmented* production function also using stochastic frontier analysis (SFA) with the model specification of Battese and Coelli (1995), where we assumed that the network parameters influence the firm’s (in)efficiency. The results were rather similar, but the SFA approach generally returned higher marginal significance levels (smaller  $P$ -values) of the regressors than the non-parametric approach. Since the marginal significance levels of the SFA are probably incorrect due to erroneous assumptions about the parametric specification (translog function for the total input quantities  $x^{**}$ ; random errors follow a normal distribution; inefficiencies follow a truncated normal distribution, where the effect of the network parameters on the expected inefficiency is linear), we decided to only present the results of the non-parametric analysis.



the structure of the farms' information networks, namely the number of outdegrees and the density of the network. The first network parameter refers to the total number of contacts  $n$  (alteri) an ego—in our case the farm—has. The second network parameter, density, describes the degree of interconnectedness between ego's alteri,  $h/[m(m-1)/2]$ , where  $h$  is the actual number of ties between the alteri and  $m(m-1)/2$  is the number of possible ties.

The variables that might affect the firm's state of technology ( $u$ ) include management characteristics, namely level of education (ordered categorical variable), work experience (in years) and attitude to risk. The latter is the average response to several questions about risk attitude, where larger positive values indicate higher risk aversion.

We have only one variable in our data set that might influence the resources required to trade the goods ( $v$ ), namely the region where the farm is located (unordered categorical variable). Our data include farms from four different municipalities (Gminas). The municipalities Chotcza and Wieliszew are close to urban areas, while Siemiatkowo and Kamieniec are rather remote. While Wieliszew and Kamieniec perform well economically, Chotcza and Siemiatkowo's economic performance is weak. Hence, the four municipalities cover all possible combinations of location and economic performance. Of course, this regional variable also accounts for differences in climate and soil, but we cannot differentiate between these effects. Whilst a separation of these effects would be interesting, it is not essential for our study.

## 5.4 Results

Table 5.1: Bandwidths

Variable	Bandwidth	Scale Factor
lLabor	64083	185067
lLand	103683	184643
lCapital	177112	253857
lIntermed	439205	628217
education	1.000	2.202
exper	15471269	2294050
risk	1421294	2882712
municip	0.591	1.300
outdFarm	4870844	4875115
outdHH	2731927	2921081
densFarm	503279	6998705
densHH	4870860	16505559

The cross-validated bandwidths obtained by the method of Hurvich, Simonoff, and Tsai (1998) are presented in table 5.1. The bandwidths of the continuous explanatory variables are very large, which indicates that the relationship between these independent variables and the dependent variable is approximately linear. However, in contrast to a parametric linear regression (e.g. OLS), our non-parametric regression with large bandwidths still allows for interaction effects between the regressors, i.e. the effect of one regressor on the dependent variable may depend on the values of all other regressors.

Table 5.2: Gradients: minimum, mean, median, and maximum

Variable	Min	Mean	Median	Max	Sig
lLabor	0.11	0.15	0.15	0.18	*
lLand	0.26	0.37	0.37	0.45	***
lCapital	0.13	0.20	0.22	0.25	**
lIntermed	0.31	0.41	0.40	0.50	***
sum: all inputs	1.09	1.14	1.15	1.17	
education: 1 → 2	-0.27	-0.00	-0.01	0.31	
education: 2 → 3	-0.14	0.02	0.01	0.23	
education: 3 → 4	-0.09	-0.01	-0.01	0.06	
exper	-0.01	-0.01	-0.01	0.00	
risk	-0.08	0.02	0.01	0.10	
municip: chot → kami	-0.06	0.09	0.08	0.28	*
municip: chot → siem	-0.19	-0.01	-0.01	0.14	*
municip: chot → wiel	-0.18	0.14	0.15	0.41	*
outdFarm	0.05	0.08	0.08	0.11	.
outdHH	0.02	0.06	0.05	0.09	
densFarm	-0.16	0.41	0.45	0.88	
densHH	0.24	0.37	0.31	0.54	**

The gradients of the independent variable with respect to the explanatory variables are summarised in table 5.2. All input quantities (`lLabor`, `lLand`, `lCapital`, `lIntermed`) have a positive effect on the output quantity at all observations. Hence, the monotonicity condition derived from microeconomic production theory is fulfilled in our analysis even though the input quantities include transaction costs. As all input and output quantities are logarithmised, the gradients can be interpreted as partial production elasticities of the inputs. However, in contrast to their usual definition, they not only account for the actual production process but also for activities for improving technology and trading goods. While intermediate inputs and land have a relatively large marginal effect on output, the use of labour has only a small marginal effect—probably owing to the abundant use of labour on most Polish farms (Henningson, 2009). The elasticities of scale, which are equal to the sums over the four partial

production elasticities, range from 1.09 to 1.17, which indicates that all farms operate under increasing returns to scale.

The gradients with respect to the farm manager's education (**education**) describe the effect of increasing education by one level, i.e. from level 1 to 2, from 2 to 3, and from 3 to 4. The estimated gradients in table 5.2 show that the effect of the farm manager's education on the output is ambiguous and on average higher education neither increases nor decreases the output. The effect of the farm manager's experience (**exper**) on the output is negative for most farms, where each year of experience can reduce the output by a maximum of 1%. The farm manager's risk aversion (**risk**) has an ambiguous effect, which is positive for some farms and negative for others. The gradients with respect to the municipality where the farm is located (**municip**) describe the expected differences in output that are due to farms lying in different municipalities. We take the municipality Chotcza (**chot**) as the base for our comparison. Farms that are located in the municipality Siemiatkowo (**siem**) need on average roughly as many resources for improving technology, trading goods, and producing the same output as farms in the municipality Chotcza. In contrast, farms that are located in the municipalities Kamieniec (**kami**) and Wieliszew (**wiel**) can produce on average 9% and 14% more output, respectively, with the same amount of input. Given our model and data, we cannot distinguish if the above-mentioned effects of the management characteristics and the farms' location are due to differences in the production process, differences in the resources used to improve the production technology, or differences in the resources required for trade but we only analyse the combined effect.

The number of outdegrees of the farm network (**outdFarm**) has a positive and rather large effect for all farms; an additional contact increases the farm output by on average 8%. The number of outdegrees of the household network (**outdHH**) also has a positive effect for all farms, but this effect is generally smaller than the effect of the contacts in the farm network. In this respect, our results support theoretical and empirical conclusions derived from the literature.

The density of the farm network (**densFarm**) increases the output for most farms, but decreases the output for some farms. Increasing the density of the farm network from zero (a totally loose network without any connection between the alteri) to one (a totally dense network with all alteri connected) would increase the output by approximately 41%. The density of the household network (**densHH**) clearly increases the output of all farms. Increasing this density from zero to one would increase the output by approximately 37%.

As a complement to the gradients shown in table 5.2, we present the estimation results in figure 5.1 graphically. While the gradients shown in table 5.2 are calculated at all data

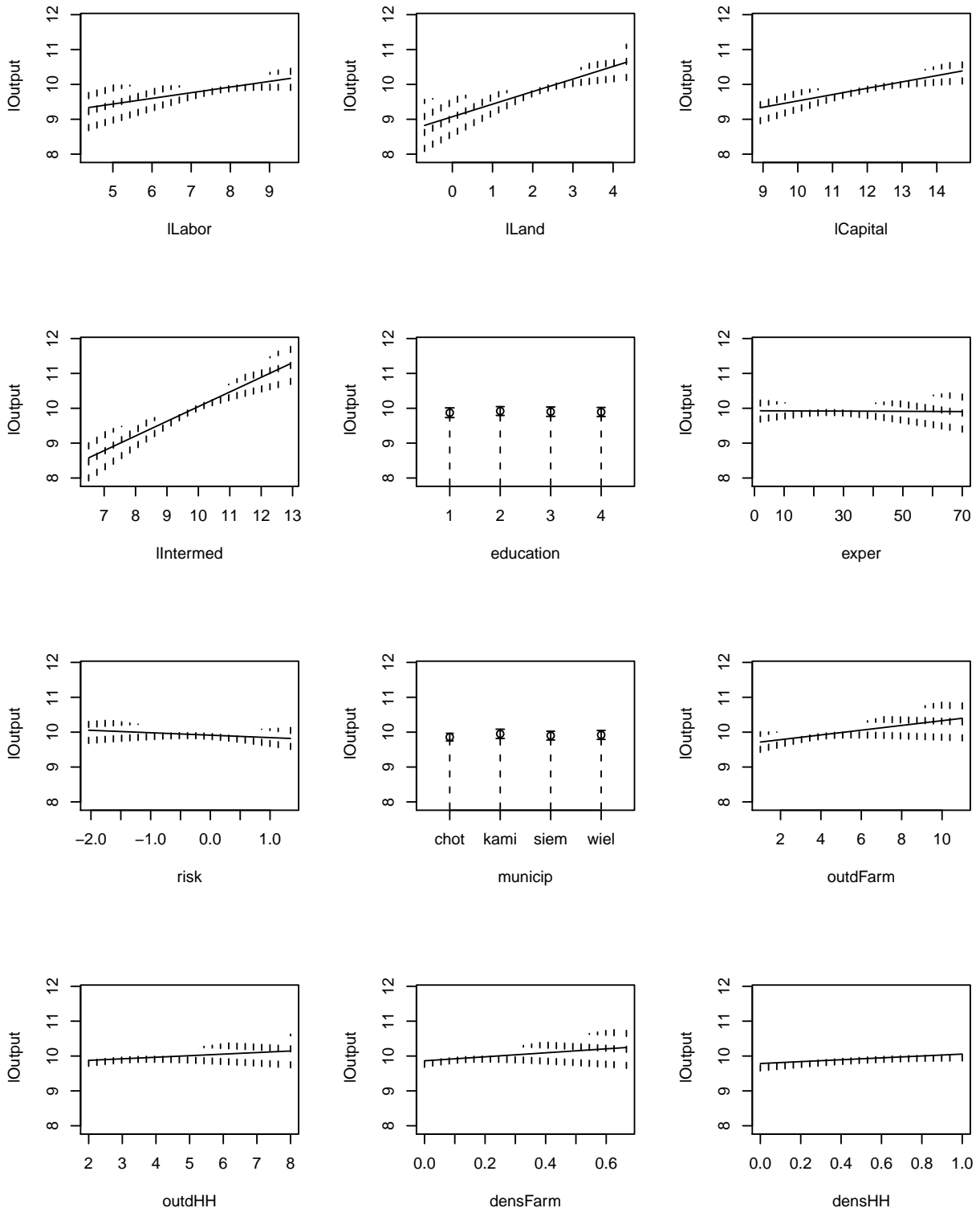


Figure 5.1: Estimation results

points that are in the sample, the estimated relationships displayed in figure 5.1 are calculated by holding the other explanatory variables constant at their medians (numeric variables) or their modal values (categorical variables). Furthermore, the figure shows the 95% variability bounds obtained by bootstrapping (see Hayfield and Racine, 2008, p. 17). Most findings derived from the gradients shown in table 5.2 are confirmed in figure 5.1, e.g. the output monotonically increases in all inputs and all four network parameters have a positive effect on the output. However, in contrast to the gradients in table 5.2, figure 5.1 suggests that the farm manager’s experience (**exper**) has virtually no effect and the farm manager’s risk aversion (**risk**) actually decreases the output. These contradictory results and the variation bounds, which are relatively large compared to the small effects of the two variables, indicate that the variables do not have a clear and significant effect.

Table 5.3: Statistical significance of regressors

Variable	P value	
lLabor	0.02757	*
lLand	0.00000	***
lCapital	0.00752	**
lIntermed	0.00000	***
education	0.22306	
exper	0.14787	
risk	0.69173	
municip	0.04261	*
outdFarm	0.07268	.
outdHH	0.19048	
densFarm	0.48120	
densHH	0.00251	**

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

We use the bootstrapping method suggested by Racine (1997) and Racine, Hart, and Li (2006) to test the statistical significance of all explanatory variables (see Hayfield and Racine, 2008, p. 9). The results are presented in table 5.3. All four inputs (labour, land, capital, intermediate inputs) as well as the location (municipality) of the farm, but none of the three management variables (education, experience, risk attitudes), have a statistically significant effect on the output. While the (positive) effects of the outdegrees of the farm network (**outdFarm**) and the density of the household network (**densHH**) are statistically significant at the 10% and 1% level, respectively, the (positive) effects of the outdegrees of the household network (**outdHH**) and the density of the farm network (**densFarm**) are not statistically significant.

Our results confirm the findings in the literature that the number of outdegrees should decrease transaction costs and increase productivity. As the literature provides partly contradictory results regarding the effect of the density of the firm's network, it is difficult to hypothesise the effect of density on total transaction costs. Since our results indicate a positive influence of dense networks on productivity, our empirical study supports the *closure argument* (Granovetter, 1985; Coleman, 1990). The linear relationship between output and density indicates that our data do not support the *gossip argument* (Coleman, 1990; Burt, 2001) to the extent that a very high density has no negative effect on productivity. We cannot make any valid statement about the *weak ties* hypothesis (Granovetter, 1973; Montgomery, 1992) as we cannot separate the effect of density on technical and institutional transaction costs and we cannot exclude the possibility that the effect of weak ties is overlaid by the closure effect.

## 5.5 Conclusion

As most data sets do not allow a distinction to be made between inputs used for production and resources dedicated to gather information and to trade goods, the variables that are typically used for estimating production functions generally include technical and institutional transaction costs. We showed that this results in estimating an "augmented" production function that also includes the trading of goods and activities to improve the production technology. A vast literature shows that information networks can promote the gathering of reliable information in an economical way. Our empirical study generally supports these results. Dense farm household networks and large farm business networks have a positive impact on a farm's productivity. Our results regarding the size of the network (outdegrees) support the conclusions derived from the literature. On the other hand, the literature shows a very diffuse picture regarding the density of a network. Our results show that increasing density is beneficial to the firm's productivity, whilst we find no evidence for any negative effects of the gossip effect in our data.

Still, further research should be conducted in this field, especially further empirical studies are needed to obtain more reliable information about the coherency between information networks and the specific types of transaction costs. Since the farming sector has some very special characteristics (e.g. close connection between household and farm, mainly located in rural areas which includes special norms and a special culture due to small and closed communities), the representativeness of our results is generally limited. In this context, it would be interesting to also study other sectors to see whether the effects of information

networks differ between sectors. Furthermore, future research should include more advanced network parameters and additional types of networks, not just ego centered. Finally, future work should focus on the separation of technical and the different forms of institutional transaction costs. However, the last two suggestions require data that are difficult and costly to collect.

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## Kapitel 6

# Measuring the Influence of Social Networks on Transaction Costs Using a Non-parametric Regression Technique

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## **Abstract**

All business transactions as well as achieving innovations take up resources, subsumed under the concept of transaction costs (TAC). One of the major factors in TAC theory is information. Information networks can catalyse the interpersonal information exchange and hence, increase the access to non-public information. Our analysis shows that information networks have an impact on the level of TAC. Many resources that are sacrificed for TAC are inputs that also enter the technical production process. As most production data do not separate between these two usages of inputs, high transaction costs are unveiled by reduced productivity. A cross-validated local linear non-parametric regression shows that good information networks increase the productivity of farms. A bootstrapping procedure confirms that this result is statistically significant.

## **6.1 Introduction**

In 1937 Coase argued in his essay “The Nature of the Firm” that market transactions often involve higher costs than just the market price. Other costs (e.g. search and information costs, bargaining costs, and policing and enforcement costs) can increase the costs of procuring something from a market. His theory became manifest in the concept of transaction costs, which has become a major field in institutional economics especially during the past 30 years.

His scholar Williamson (1971) carried on with this idea. He argued that firms can choose between two regimes of governance structure for transacting goods and services, namely markets and internal solutions. Later Williamson and other scholars (e.g. Grandori and Soda, 1995) extended the approach by integrating social networks into the theory. For instance, Williamson (1991) included inter-firm networks as “hybrid form” while other scholars (e.g. Powell, 1990; Johanisson, 1987) stated that networks are a third type organisational arrangement.

A vast literature shows that social networks provide well functioning mechanisms to reduce the resources that a firm needs for transacting goods and services and for accessing innovation. Hence, social networks reduce transaction costs (e.g. Henning, 2002).

Most of these resources used for transactions and accessing innovation are inputs also used in production. As most data sets do not separate between inputs used for technical production and resources dedicated to trade and innovativeness, increasing unobservable transaction costs are translated into lower productivity. Hence, we conclude that firms with “good” social networks show higher productivity and vice versa. Our analysis tries to capture the effect of social networks on productivity.

In studies like this, the normal procedure to measure the influence of social networks on the efficiency of production would be to apply a stochastic frontier analysis (SFA) using the specification of Battese and Coelli (1995). The drawback of this approach is its rigid assumptions about the distributions of the error terms<sup>1</sup> and about linearity between the effects of social networks and the inefficiency. If these assumptions are false—which can easily be the case in such complex relationships—the estimated parameters and the statistical tests will be misleading. Therefore, to avoid specifying a parametric functional form, we estimate the influence of social networks on productivity by a non-parametric regression technique. In the following section we describe our methodological approach. The results are presented in the third section and the last section concludes.

## **6.2 Data and Methodology**

In our empirical analysis we use a data set on Polish farms. The data were collected within the framework of the “Advanced-Eval” project financed by the European Union within the Sixth Framework Programme (contract number 022708). The data set includes detailed farm accountancy data and information on the farms’ ego centred networks. We take the total value of all produced goods as output (in Zloty) and we distinguish between four inputs: labour (in working hours), land (in ha), capital (in Zloty), and intermediate inputs (in Zloty). Furthermore, we include management characteristics: education (as an ordered categorical variable), experience (in years), and risk attitudes (as a continuous variable, where increasing values indicate increasing risk aversion). In addition to the above-mentioned variables, we include four network parameters as well as an unordered categorical variable for the four regions, where the farms in our data set are located.

We take the logarithm of the output and all the input quantities so that the individual values of these variables are more equally distributed within the range of observed values. Otherwise, there were many observations within the bandwidths for small values (farms) but only very few observations within the bandwidth for large values (farms), which usually causes problems in non-parametric regression when fixed bandwidths are used.<sup>2</sup>

We apply a non-parametric local-linear estimation method, which was initially suggested by Stone (1977) and Cleveland (1979). Since we have both continuous and categorical explanatory variables, we use the extension of this estimator for mixed data types proposed by Li and Racine (2004) and Racine and Li (2004). In the presence of both continuous and

---

<sup>1</sup>A normal distribution for the general error term and a truncated normal distribution for the efficiency term is assumed under most stochastic frontier analyses (Battese and Coelli, 1995).

<sup>2</sup>An alternative would be to use a “nearest neighbor”-method.



categorical explanatory variables, this estimator outperforms the local-constant estimator (Li and Racine, 2004).

We use the second-order Epanechnikov kernel for continuous regressors, the kernel proposed by Aitchison and Aitken (1976, p. 29) for unordered categorical explanatory variables, and the kernel proposed by Wang and van Ryzin (1981) for ordered categorical explanatory variables. Since the bandwidths of the regressors are pivotal for the estimation, we use a data-driven bandwidth selection method, which has been proposed by (Hurvich, Simonoff, and Tsai, 1998) and is based on a corrected Akaike information. It is an expected Kullback-Leibler cross-validation method and has very good finite sample properties (Li and Racine, 2004, p. 501).

We also used the second-order Gaussian kernel for the continuous regressors, but the bandwidths suggested by the cross-validation strongly depended on the starting values, where the bandwidths of some (varying) regressors were very small, which resulted in extreme under-smoothing.

The estimation was done within the statistical software environment “R” (R Development Core Team, 2009) using the add-on package “np” (Hayfield and Racine, 2008).

### 6.3 Results

Table 6.1: Bandwidths

Variable	Bandwidth	Scale Factor
lLabor	64083	185067
lLand	103683	184643
lCapital	177112	253857
lIntermed	439205	628217
education	1.000	2.202
exper	15471269	2294050
risk	1421294	2882712
municip	0.591	1.300
outdFarm	4870844	4875115
outdHH	2731927	2921081
densFarm	503279	6998705
densHH	4870860	16505559

The cross-validated bandwidths obtained by the method of Hurvich, Simonoff, and Tsai (1998) are presented in table 6.1. The bandwidths of the continuous explanatory variables are very large, indicating that the relationship between these independent variables and the

dependent variable is approximately linear. However, in contrast to a parametric linear regression (e.g. OLS), our non-parametric regression with large bandwidths still allows for interaction effects between the regressors, i.e. the effect of one regressor on the dependent variable may depend on the values of all other regressors.

Table 6.2: Gradients: minimum, mean, median, and maximum

Variable	Min	Mean	Median	Max
lLabor	0.11	0.15	0.15	0.18
lLand	0.26	0.37	0.37	0.45
lCapital	0.13	0.20	0.22	0.25
lIntermed	0.31	0.41	0.40	0.50
sum: all inputs	1.09	1.14	1.15	1.17
education: 1 → 2	-0.27	-0.00	-0.01	0.31
education: 2 → 3	-0.14	0.02	0.01	0.23
education: 3 → 4	-0.09	-0.01	-0.01	0.06
exper	-0.01	-0.01	-0.01	0.00
risk	-0.08	0.02	0.01	0.10
municip: chot → kami	-0.06	0.09	0.08	0.28
municip: chot → siem	-0.19	-0.01	-0.01	0.14
municip: chot → wiel	-0.18	0.14	0.15	0.41

The gradients of the independent variable with respect to the explanatory variables are summarized in table 6.2. All input quantities (`lLabor`, `lLand`, `lCapital`, `lIntermed`) have a positive effect on the output quantity at all observations. Hence, the monotonicity condition derived from microeconomic production theory is fulfilled in our analysis even though the input quantities include transaction costs. As all input and output quantities are logarithmised, the gradients can be interpreted as partial production elasticities of the inputs. The elasticities of scale, which are equal to the sums over the four partial production elasticities, range from 1.09 to 1.17, indicating that all farms operate under increasing returns to scale.

The gradients with respect to the farm manager’s education (`education`) describe the effect of increasing education by one level, i.e. from level 1 to 2, from 2 to 3, and from 3 to 4. The estimated gradients in table 6.2 show that the effect of the farm manager’s education on the output is ambiguous and on average higher education neither increases nor decreases the output. The effect of the farm manager’s experience (`exper`) on the output is negative for most farms, where each year of experience can reduce the output by a maximum of 1%. The farm manager’s risk aversion (`risk`) has an ambiguous effect, which is positive for some farms and negative for others. The gradients with respect to the municipality where the farm is located (`municip`) describe the expected differences in output that are due to farms lying in different municipalities. We take the municipality Chotcza (`chot`) as the base for

our comparison. Farms that are located in the municipality Siemiatkowo (*siem*) need on average roughly as many resources for improving technology, trading goods, and producing the same output as farms in the municipality Chotcza. In contrast, farms that are located in the municipalities Kamieniec (*kami*) and Wieliszew (*wiel*) can produce on average 9% and 14% more outputs, respectively, with the same amount of inputs.

As a complement to the gradients shown in table 6.2, we present the estimation results in figure 6.1 graphically. While the gradients shown in table 6.2 are calculated at all data points that are in the sample, the estimated relationships displayed in figure 6.1 are calculated by holding the other explanatory variables constant at their medians (continuous variables) or their modal values (categorical variables). Furthermore, the figure shows the 95% variability bounds obtained by bootstrapping (see Hayfield and Racine, 2008, p. 17). Most findings derived from the gradients shown in table 6.2 are confirmed in figure 6.1. However, in contrast to the gradients in table 6.2, figure 6.1 suggests that the farm manager’s experience (*exper*) has virtually no effect and the farm manager’s risk aversion (*risk*) even decreases the output. These contradicting results and the variation bounds, which are relatively large compared to the small effects of these two variables, indicate that these variables do not have a clear and significant effect.

Table 6.3: Statistical significance of regressors

Variable	P value	
lLabor	0.02757	*
lLand	0.00000	***
lCapital	0.00752	**
lIntermed	0.00000	***
education	0.22306	
exper	0.14787	
risk	0.69173	
municip	0.04261	*

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

We use the bootstrapping method suggested by Racine (1997) and Racine, Hart, and Li (2006) to test the statistical significance of all explanatory variables (see Hayfield and Racine, 2008, p. 9). The results are presented in table 6.3. All four inputs (labour, land, capital, intermediate inputs) as well as the location (municipality) of the farm but none of the three management variables (education, experience, risk attitudes) have a statistically significant effect on the output.

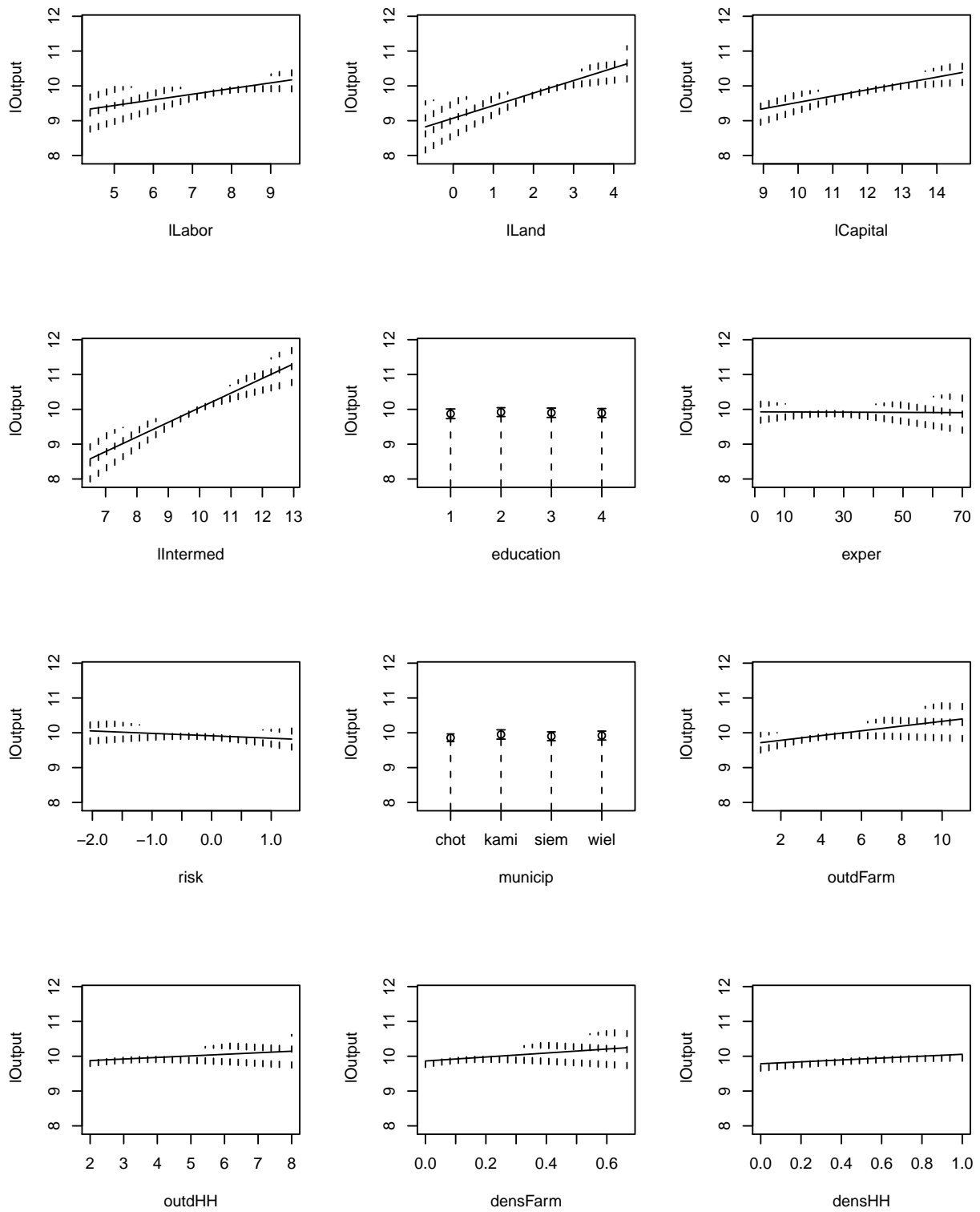


Figure 6.1: Estimation results

## 6.4 Conclusions

Our study has shown that a non-parametric regression is proper approach to model unknown complex relationships. The estimation of individual gradients allows for further analyses of the results, which would not be possible in classical parametric regression analysis.

## Acknowledgements

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## **Kapitel 7**

### **Estimating the CES Function in R: Package micEconCES**

**Géraldine Henningsen, Arne Henningsen**





## 7.1 Introduction

The Constant-Elasticity-of-Substitution (CES) function was developed as a generalisation of the Cobb-Douglas function by the Stanford group around Arrow *et al.* (1961). In recent years, the CES has gained in importance in macroeconomics (e.g. Amras, 2004; Bentolila and Gilles, 2006) and growth theory (e.g. Caselli, 2005; Caselli and Coleman, 2006) as an alternative to the Cobb-Douglas function and it can be applied in many other fields. In microeconomics, the CES function gained less popularity most likely because of its restrictive assumptions, especially in the case of more than two explanatory variables.

The formal specification of a CES production function<sup>1</sup> with two inputs is

$$y = \gamma \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}}, \quad (7.1)$$

where  $y$  is the output quantity,  $x_1$  and  $x_2$  are the input quantities, and  $\gamma$ ,  $\delta$ ,  $\rho$ , and  $\nu$  are parameters. Parameter  $\gamma \in (0, \infty)$  determines the productivity,  $\delta \in (0, 1)$  determines the optimal distribution of the inputs,  $\rho \in (-1, 0) \cup (0, \infty)$  determines the (constant) elasticity of substitution, which is  $\sigma = 1 / (1 + \rho)$ , and  $\nu \in (0, \infty)$  is equal to the elasticity of scale.<sup>2</sup>

The CES function includes three special cases: for  $\rho \rightarrow 0$ ,  $\sigma$  approaches 1 and the CES turns into the Cobb-Douglas form; for very large  $\rho$ ,  $\sigma$  approaches 0 and the CES turns into the Leontief production function; and for  $\rho \rightarrow -1$ ,  $\sigma$  approaches infinity and the CES turns into a linear function if  $\nu$  is equal to 1.

As the CES function is non-linear in parameters and cannot be linearised analytically, it is not possible to estimate it with the usual linear estimation techniques. Therefore, the CES is usually approximated by the so-called “Kmenta approximation” (Kmenta, 1967) or estimated by non-linear least-squares using different optimisation algorithms. In this paper, we describe and compare these estimation approaches, explain how we implemented them in the R package **micEconCES** (Henningsen and Henningsen, 2010), and show how they can be used for economic analysis and modelling. The **micEconCES** package is developed as part of the “micEcon” project on R-Forge (<http://r-forge.r-project.org/projects/micecon/>). Stable versions of this package are available for download from the Comprehensive R Archive Network (CRAN, <http://CRAN.R-Project.org/package=micEconCES>).

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<sup>1</sup>The CES functional form can be used to model different economic relationships (e.g. as production function or utility function). However, as the CES functional form is mostly used to model production technology, we name the independent (right-hand side) variables “inputs” and the dependent (left-hand side) variable “output” to keep the notation simple.

<sup>2</sup>Originally, the CES function of Arrow *et al.* (1961) could only model constant returns to scale, but later Kmenta (1967) added the parameter  $\nu$ , which allows for variable returns to scale if  $\nu \neq 1$ .

The paper is structured as follows. In the next section, we discuss several approaches to estimating the CES production function and show how they can be applied in R. The third section describes the implementation of these methods in the R package **micEconCES**. Section four presents the results of a Monte Carlo study to compare the various estimation approaches, and the last section concludes.

## 7.2 Estimation of the CES production function

Tools for economic analysis with CES function are available in the R package **micEconCES** (Henningsen and Henningsen, 2010). If this package is installed, it can be loaded with the command

```
> library("micEconCES")
```

We demonstrate the usage of this package by estimating a CES model with an artificial data set, because this avoids several problems that usually occur with real-world data.

```
> set.seed(123)
> cesData <- data.frame(x1 = rchisq(200, 10), x2 = rchisq(200,
+   10))
> cesData$y <- cesCalc(xNames = c("x1", "x2"), data = cesData,
+   coef = c(gamma = 1, delta = 0.6, rho = 0.5, nu = 1.1))
> cesData$y <- cesData$y + 2.5 * rnorm(200)
```

The first line sets the “seed” for the random number generator so that these examples can be replicated with exactly the same data set. The second line creates a data set with two input variables (called `x1` and `x2`) that have 200 observations each and are generated from a random  $\chi^2$  distribution with 10 degrees of freedom. The third line uses the command `cesCalc` that is included in the **micEconCES** package and calculates the deterministic output variable (called `y`) given the CES production function with the two input variables `x1` and `x2` and the coefficients  $\gamma = 1$ ,  $\delta = 0.6$ ,  $\rho = 0.5$ , and  $\nu = 1.1$ . The last line generates the stochastic output variable by adding normally distributed random errors to the deterministic output variable.

As the CES function is non-linear in its parameters, the most straightforward way to estimate the CES function in R would be to use `nls`, which performs non-linear least-squares estimations.

```
> cesNls <- nls(y ~ gamma * (delta * x1^(-rho) + (1 - delta) *
+   x2^(-rho))^(phi/rho), data = cesData, start = c(gamma = 0.5,
```

```
+      delta = 0.5, rho = 0.25, phi = 1))
> print(cesNls)
```

Nonlinear regression model

```
model: y ~ gamma * (delta * x1^(-rho) + (1 - delta) * x2^(-rho))^(phi/rho)
data: cesData
gamma delta rho phi
1.0102 0.6271 0.6398 1.0955
residual sum-of-squares: 1175
```

Number of iterations to convergence: 6

Achieved convergence tolerance: 4.385e-07

While the `nls` routine works well in this ideal artificial example, it does not perform well in many applications with real data, either because of non-convergence, convergence to a local minimum, or theoretically unreasonable parameter estimates. Therefore, we show alternative ways of estimating the CES function in the following subsections.

## 7.2.1 Kmenta approximation

Given that non-linear estimation methods are often troublesome—particularly during the 1960s and 1970s when computing power was very limited—Kmenta (1967) derived an approximation of the classical two-input CES production function that can be estimated by ordinary least-squares techniques.

$$\begin{aligned} \log y = & \log \gamma + \nu \delta \log x_1 + \nu (1 - \delta) \log x_2 \\ & - \frac{\rho \nu}{2} \delta (1 - \delta) (\log x_1 - \log x_2)^2 \end{aligned} \quad (7.2)$$

While Kmenta (1967) obtained this formula by logarithmising the CES function and applying a second-order Taylor series expansion to  $\log(\delta x_1^{-\rho} + (1 - \delta)x_2^{-\rho})$  at the point  $\rho = 0$ , the same formula can be obtained by applying a first-order Taylor series expansion to the entire logarithmised CES function at the point  $\rho = 0$  (Uebe, 2000). As the authors consider the latter approach to be more straight-forward, the Kmenta approximation is called—in contrast to Kmenta (1967, p. 180)—first-order Taylor series expansion in the remainder of this paper.

The Kmenta approximation can also be written as a restricted translog function (Hoff, 2004):

$$\log y = \alpha_0 + \alpha_1 \log x_1 + \alpha_2 \log x_2 \quad (7.3)$$

$$+ \frac{1}{2} \beta_{11} (\log x_1)^2 + \frac{1}{2} \beta_{22} (\log x_2)^2 + \beta_{12} \log x_1 \log x_2,$$

where the two restrictions are

$$\beta_{12} = -\beta_{11} = -\beta_{22}. \quad (7.4)$$

If constant returns to scale should be imposed, a third restriction

$$\alpha_1 + \alpha_2 = 1 \quad (7.5)$$

must be enforced. These restrictions can be utilised to test whether the linear Kmenta approximation of the CES (7.2) is an acceptable simplification of the translog functional form.<sup>3</sup> If this is the case, a simple *t*-test for the coefficient  $\beta_{12} = -\beta_{11} = -\beta_{22}$  can be used to check if the Cobb-Douglas functional form is an acceptable simplification of the Kmenta approximation of the CES.<sup>4</sup>

The parameters of the CES function can be calculated from the parameters of the restricted translog function by

$$\gamma = \exp(\alpha_0) \quad (7.6)$$

$$\nu = \alpha_1 + \alpha_2 \quad (7.7)$$

$$\delta = \frac{\alpha_1}{\alpha_1 + \alpha_2} \quad (7.8)$$

$$\rho = \frac{\beta_{12} (\alpha_1 + \alpha_2)}{\alpha_1 * \alpha_2} \quad (7.9)$$

The Kmenta approximation of the CES function can be estimated by the function `cesEst`, which is included in the **micEconCES** package. If argument `method` of this function is set to "Kmenta", it (a) estimates an unrestricted translog function (7.3), (b) carries out a Wald test of the parameter restrictions defined in equation (7.4) and eventually also in equation (7.5) using the (finite sample) *F* statistic, (c) estimates the restricted translog function (7.3, 7.4), and finally, (d) calculates the parameters of the CES using equations (7.6–7.9) as well as their covariance matrix using the delta method.

The following code estimates a CES function with the endogenous variable `y` (specified in argument `yName`), the two explanatory variables `x1` and `x2` (argument `xNames`), the artificial

---

<sup>3</sup>Note that this test does *not* check whether the *non-linear* CES function (7.1) is an acceptable simplification of the translog functional form or whether the *non-linear* CES function can be approximated by the Kmenta approximation.

<sup>4</sup>Note that this test does *not* compare the Cobb-Douglas function with the (non-linear) CES function but only with its linear approximation.

data set `cesData` that we generated above (argument `data`) using the `Kmenta` approximation (argument `method`) and allowing for variable returns to scale (argument `vrs`).

```
> cesKmenta <- cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData,
+   method = "Kmenta", vrs = TRUE)
```

Summary results can be obtained applying the `summary` method to the returned object.

```
> summary(cesKmenta)
```

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,
      method = "Kmenta")
```

Estimation by the linear `Kmenta` approximation

Test of the null hypothesis that the restrictions of the Translog function required by the `Kmenta` approximation are true:

P-value = 0.6135929

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
gamma	0.74252	0.11009	6.745	1.53e-11	***
delta	0.60864	0.03373	18.043	< 2e-16	***
rho	0.71527	0.31722	2.255	0.0241	*
nu	1.21865	0.06617	18.416	< 2e-16	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.481868

Multiple R-squared: 0.7643218

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )	
E_1_2 (all)	0.5830	0.1078	5.407	6.4e-08	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

The Wald test indicates that the restrictions on the Translog function implied by the Kmenta approximation cannot be rejected at any reasonable significance level.

To see whether the underlying technology is of the Cobb-Douglas form, we can check if the coefficient  $\beta_{12} = -\beta_{11} = -\beta_{22}$  significantly differs from zero. As the estimation of the Kmenta approximation is stored in component `kmenta` of the object returned by `cesEst`, we can obtain summary information on the estimated coefficients of the Kmenta approximation by

```
> coef(summary(cesKmenta$kmenta))
```

	Estimate	Std. Error	t value	Pr(> t )
eq1_(Intercept)	-0.2977003	0.14826207	-2.007933	0.04602347
eq1_a_1	0.7417197	0.05337124	13.897367	0.00000000
eq1_a_2	0.4769324	0.05156227	9.249638	0.00000000
eq1_b_1_1	-0.2076294	0.08907193	-2.331030	0.02076840
eq1_b_1_2	0.2076294	0.08907193	2.331030	0.02076840
eq1_b_2_2	-0.2076294	0.08907193	-2.331030	0.02076840

Given that  $\beta_{12} = -\beta_{11} = -\beta_{22}$  significantly differs from zero at the 5% level, we can conclude that the underlying technology is not of the Cobb-Douglas form. Alternatively, we can check if the parameter  $\rho$  of the CES, which is calculated from the coefficients of the Kmenta approximation, significantly differs from zero. This should—as in our case—deliver similar results (see above).

Finally, we plot the fitted values against the actual endogenous variable ( $y$ ) to check whether the parameter estimates are reasonable.

```
> compPlot(cesData$y, fitted(cesKmenta), xlab = "actual values",
+         ylab = "fitted values")
```

Figure 7.1 shows that the parameters produce reasonable fitted values.

However, the Kmenta approximation encounters several problems. First, it is a truncated Taylor series, whose remainder term must be seen as an omitted variable. Second, the Kmenta approximation converges to the underlying CES function only in a region of convergence, that is dependent on the true parameters of the CES function (Thursby and Lovell, 1978).

Although, Maddala and Kadane (1967) and Thursby and Lovell (1978) find estimates for  $\nu$  and  $\delta$  with small bias and MSE, results for  $\gamma$  and  $\rho$  are estimated with generally large bias and MSE (Thursby and Lovell, 1978; Thursby, 1980). More reliable results can only be obtained if  $\rho \rightarrow 0$ , and thus,  $\sigma \rightarrow 1$  which increases the convergence region, i.e. if the underlying CES

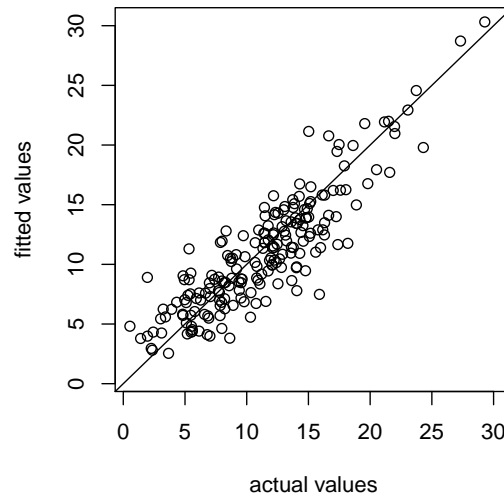


Figure 7.1: Fitted values from the Kmenta approximation against  $y$

is of the Cobb-Douglas form. This is a major drawback of the Kmenta approximation as its purpose is to facilitate the estimation of functions with non-unitary  $\sigma$ .

### 7.2.2 Levenberg-Marquardt algorithm

Initially, the Levenberg-Marquardt algorithm (Marquardt, 1963) was most commonly used for estimating the parameters of the CES function by non-linear least-squares. This iterative algorithm is performed by using an optimum interpolation between the Gauss-Newton method that involves a linearisation by a first-order Taylor series approximation and the gradient method (steepest-descent method).

In a Monte Carlo study by Thursby (1980) the Levenberg-Marquardt algorithm outperforms the other methods and gives the best estimates of the CES parameters. However, the Levenberg-Marquardt algorithm performs as poorly as the other methods in estimating the elasticity of substitution ( $\sigma$ ), meaning that the estimated  $\sigma$  tends to be biased towards infinity, unity, or zero.

Although the Levenberg-Marquardt algorithm does not live up to modern standards, we include it for reasons of completeness, as it has proven to be a standard method to estimate the CES technology. The Levenberg-Marquardt algorithm can be seen as a maximum neighbourhood method which performs an optimum interpolation between a first-order Taylor approximation (Gauss-Newton) and a steepest descent method (gradient method) (Marquardt, 1963). By combining these two non-linear optimisation algorithms, the developers want to increase conversion probability by reducing the weaknesses of each of the two methods.

The objective function  $\Phi = \|Y - \hat{Y}\|^2$  of a non-linear least-squares estimation does not fulfill the theoretical criteria of a well behaved function, unless the function value is close to its minimum. This feature becomes more severe the more non-linear the function. Therefore, it is crucial to find starting values close to the minimum. However, this is not always possible in practice. Choosing non-optimal starting values, the Gauss-Newton as well as the steepest descent method show a tendency to fail to converge. If the starting values are too far from the minimum, the Gauss-Newton algorithm has difficulties in determining an appropriate step size, which can lead to step sizes which are either too big (cutting across the minimum), or too small (slow rates of convergence). On the other hand, the steepest descent method can handle suboptimal starting values very well, but it shows a failure to convergence mostly due to very slow convergence when it gets close to the minimum (Kelley, 1999).

In contrast to the Gauss-Newton and the steepest descent algorithms, the Levenberg-Marquardt algorithm determines the direction and the step size simultaneously, and thus, the algorithm proves to be more robust with higher rates of convergence, even if starting values are not optimal. If the Levenberg-Marquardt parameter  $\lambda$  is set to zero, the algorithm turns into Gauss-Newton, for  $\lambda \rightarrow \infty$  on the other hand, it turns into the steepest descent. Hence, as  $\lambda$  is defined in every iteration, the Levenberg-Marquardt algorithm uses the good global properties of the steepest descent method and—by approaching the minimum of the objective function—recovers the Gauss-Newton’s fast convergence for small residual problems.

In the following we will give a rough outline of the algorithm.<sup>5</sup> We let

$$\langle Y_i(\mathbf{X}_i, \boldsymbol{\beta} + \boldsymbol{\gamma}) \rangle = f(\mathbf{X}_i, \boldsymbol{\beta}) + \sum_{j=1}^k \left( \frac{\partial f_i}{\partial \beta_j} \right) \gamma_j, \quad (7.10)$$

or shorter

$$\langle \mathbf{Y} \rangle = f_0 + P\boldsymbol{\gamma}, \quad (7.11)$$

be the first-order Taylor series approximation, where  $Y_i$  is the  $i$ th value of the dependent variable, here output,  $\mathbf{X}_i$  is the  $i$ th vector of covariates,  $\boldsymbol{\beta}$  is a vector of parameters to be estimated,  $\boldsymbol{\gamma}$  is a vector of small correction parameters to  $\boldsymbol{\beta}$  calculated from a Taylor series with  $j$ th element  $\gamma_j$ ,  $f$  is a differentiable function,  $k$  is the number of parameters to be estimated,  $f_0$  is a vector of the first terms of the Taylor series, and  $P = \partial f / \partial \boldsymbol{\beta}$  is a Jacobian matrix. Then  $\boldsymbol{\gamma}$  can be found by

$$(A + \lambda I)\boldsymbol{\gamma} = \mathbf{g}, \quad (7.12)$$

---

<sup>5</sup>For a more detailed introduction to the Levenberg-Marquardt algorithm see Marquardt (1963) or Soda and Vichi (1976).



where  $I$  is an identity matrix and

$$A = P^\top P \tag{7.13}$$

and

$$\mathbf{g} = P^\top (\mathbf{Y} - f_0). \tag{7.14}$$

Finally, let

$$\Phi(\gamma) = \|\mathbf{Y} - f_0 - P\gamma\|^2 \tag{7.15}$$

The algorithm is then as follows: `marquardt` ( $\mu, \lambda, \Phi, r$ )

1. Let  $\mu > 1$  be a tolerance parameter
2. Let  $\lambda^{(r-1)}$  be the value from the previous iteration. Initially let  $\lambda^{(0)} = 10^{-2}$ .
3. Compute  $\Phi(\lambda^{(r-1)})$  and  $\Phi(\lambda^{(r-1)}/\mu)$ 
  - a) if  $\Phi(\lambda^{(r-1)}/\mu) \leq \Phi^{(r)}$ , let  $\lambda^{(r)} = \lambda^{(r-1)}/\mu$ .
  - b) if  $\Phi(\lambda^{(r-1)}/\mu) > \Phi^{(r)}$ , and  $\Phi(\lambda^{(r-1)}) \leq \Phi^{(0)}$ , let  $\lambda^{(r)} = \lambda^{(r-1)}$ .
  - c) if  $\Phi(\lambda^{(r-1)}/\mu) > \Phi^{(r)}$ , and  $\Phi(\lambda^{(r-1)}) > \Phi^{(0)}$ , increase  $\lambda$  by successive multiplication by  $\mu$  until for some smallest  $w$   $\Phi(\lambda^{(r-1)}\mu^w) \leq \Phi^{(r)}$ . Let  $\lambda^{(r)} = \lambda^{(r-1)}\mu^w$ .

To estimate a CES function by non-linear least-squares using the Levenberg-Marquardt algorithm, one can call the `cesEst` function with argument `method` set to "LM" or without this argument, as the Levenberg-Marquardt algorithm is the default estimation method used by `cesEst`. The user can modify a few details of the algorithm (e.g. different criteria for convergence) by adding argument `control` as described in the documentation of `nls.lm.control`. Argument `start` can be used to specify a vector of starting values, where the order must be  $\gamma, \delta, \rho$  (only if  $\rho$  is not fixed, e.g. during grid search), and  $\nu$  (only if the model has variable returns to scale). If no starting values are provided, they are determined automatically (see section 7.3.7). We estimate the same example as before but now with the Levenberg-Marquardt algorithm.

```
> cesLm <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE)
> summary(cesLm)
```

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE)
```

Estimation by non-linear least-squares using the 'LM' optimizer  
assuming an additive error term  
Convergence achieved after 4 iterations  
Message: Relative error in the sum of squares is at most `ftol`.

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
gamma	1.01020	0.11244	8.984	<2e-16 ***
delta	0.62711	0.02834	22.126	<2e-16 ***
rho	0.63975	0.29705	2.154	0.0313 *
nu	1.09545	0.04500	24.346	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.424083

Multiple R-squared: 0.7751686

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )
E_1_2 (all)	0.6098	0.1105	5.52	3.39e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Finally we plot the fitted values against the actual values  $y$  to see whether the estimated parameters are reasonable. The result is presented in figure 7.2.

```
> compPlot(cesData$y, fitted(cesLm), xlab = "actual values", ylab = "fitted values")
```

### 7.2.3 Alternative gradient-based optimisation algorithms

Several further gradient-based optimisation algorithms that are suitable for non-linear least-squares estimations are implemented in R. Function `cesEst` can use some of them to estimate a CES function by non-linear least-squares. As a proper application of these estimation methods requires the user to be familiar with the main characteristics of the different algorithms, we will briefly discuss some practical issues of the algorithms that will be used to estimate

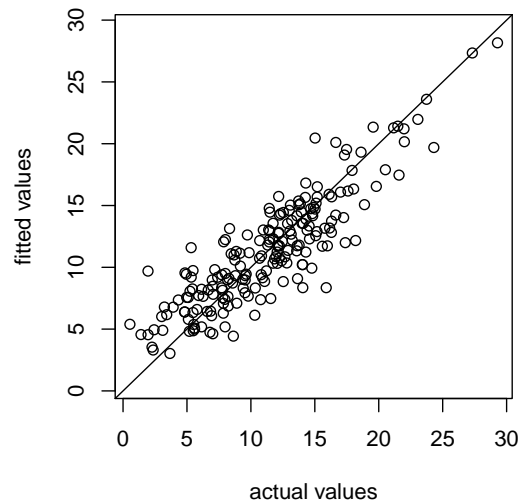


Figure 7.2: Fitted values from the LM algorithm against  $y$

the CES function. However, it is not the aim of this paper to thoroughly discuss these algorithms. A detailed discussion of iterative optimisation algorithms is available, e.g., in Kelley (1999) or Mishra (2007).

One of the gradient-based optimisation algorithms that can be used by `cesEst` is the “Conjugate Gradients” method based on Fletcher and Reeves (1964). This iterative method is mostly applied to optimisation problems with many parameters and a large and possibly sparse Hessian matrix, because this algorithm requires neither storing nor inverting the Hessian matrix. The “Conjugated Gradient” method works best for objective functions that are approximately quadratic and it is sensitive to objective functions that are not well-behaved and have a non-positive semidefinite Hessian, i.e. convergence within the given number of iterations is less likely the more the level surface of the objective function differs from spherical (Kelley, 1999). Given that the CES function has only few parameters and the objective function is not approximately quadratic and shows a tendency to “flat surfaces” around the minimum, the “Conjugated Gradient” method is probably less suitable than other algorithms for estimating a CES function. Setting argument `method` of `cesEst` to “CG” selects the “Conjugate Gradients” method for estimating the CES function by non-linear least-squares. The user can modify this algorithm (e.g. replacing the update formula of Fletcher and Reeves (1964) by the formula of Polak and Ribière (1969) or the one based on Sorenson (1969) and Beale (1972)) or some details (e.g. convergence tolerance level) by adding a further argument `control` as described in the “Details” section of the documentation of `optim`.

```
> cesCg <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "CG")
> summary(cesCg)
```

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,
      method = "CG")
```

Estimation by non-linear least-squares using the 'CG' optimizer  
assuming an additive error term

Convergence NOT achieved after 406 function and 101 gradient calls

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
gamma	0.99847	0.11124	8.976	<2e-16	***
delta	0.62574	0.02807	22.294	<2e-16	***
rho	0.60680	0.29187	2.079	0.0376	*
nu	1.09985	0.04503	24.427	<2e-16	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.424191

Multiple R-squared: 0.7751486

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )	
E_1_2 (all)	0.6224	0.1130	5.505	3.69e-08	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Although the estimated parameters are similar to the estimates from the Levenberg-Marquardt algorithm, the “Conjugated Gradient” algorithm reports that it did not converge. Increasing the maximum number of iterations and the tolerance level leads to convergence. This indicates a slow convergence of the Conjugate Gradients algorithm for estimating the CES function.

```
> cesCg2 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "CG",
+   control = list(maxit = 1000, reltol = 1e-05))
> summary(cesCg2)
```

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,
      method = "CG", control = list(maxit = 1000, reltol = 1e-05))
```

Estimation by non-linear least-squares using the 'CG' optimizer  
assuming an additive error term

Convergence achieved after 1559 function and 387 gradient calls

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
gamma	1.01020	0.11244	8.984	<2e-16 ***
delta	0.62711	0.02834	22.126	<2e-16 ***
rho	0.63975	0.29705	2.154	0.0313 *
nu	1.09545	0.04500	24.346	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.424083

Multiple R-squared: 0.7751686

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )
E_1_2 (all)	0.6098	0.1105	5.52	3.39e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Another algorithm supported by `cesEst` that is probably more suitable for estimating a CES function is an improved Newton-type method. As the original Newton method, this algorithm uses first and second derivatives of the objective function to determine the direction of the shift vector and searches for a stationary point until the gradients are (almost) zero.

However, in contrast to the original Newton method, this algorithm does a line search at each iteration to determine the optimal length of the shift vector (step size) as described in Dennis and Schnabel (1983) and Schnabel *et al.* (1985). Setting argument `method` of `cesEst` to "Newton" selects this improved Newton-type method. The user can modify a few details of the algorithm (e.g. the maximum step length) by adding further arguments that are described in the documentation of `nlm`. The following commands estimate a CES function by non-linear least-squares using the algorithm and print summary results.

```
> cesNewton <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE,  
+   method = "Newton")  
> summary(cesNewton)
```

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,  
      method = "Newton")
```

Estimation by non-linear least-squares using the 'Newton' optimizer  
assuming an additive error term  
Convergence achieved after 27 iterations

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
gamma	1.01020	0.11244	8.984	<2e-16 ***
delta	0.62711	0.02834	22.126	<2e-16 ***
rho	0.63975	0.29705	2.154	0.0313 *
nu	1.09545	0.04500	24.346	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.424083

Multiple R-squared: 0.7751686

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )
E_1_2 (all)	0.6098	0.1105	5.52	3.39e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Furthermore, a quasi-Newton method developed independently by Broyden (1970), Fletcher (1970), Goldfarb (1970), and Shanno (1970) can be used by `cesEst`. This so-called BFGS algorithm also uses first and second derivatives and searches for a stationary point of the objective function where the gradients are (almost) zero. In contrast to the original Newton method, the BFGS method performs a line search for the best step size and uses a special procedure to approximate and update the Hessian matrix in every iteration. The problem with BFGS can be that although the current parameters are close to the minimum, the algorithm does not converge because the Hessian matrix at the current parameters is not close to the Hessian matrix at the minimum. However, in practice BFGS proves robust convergence (often superlinear) (Kelley, 1999). If argument `method` of `cesEst` is "BFGS", the BFGS algorithm is used for the estimation. The user can modify a few details of the BFGS algorithm (e.g. the convergence tolerance level) by adding the further argument `control` as described in the "Details" section of the documentation of `optim`.

```
> cesBfgs <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "BFGS")
> summary(cesBfgs)
```

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,
      method = "BFGS")
```

Estimation by non-linear least-squares using the 'BFGS' optimizer  
assuming an additive error term

Convergence achieved after 71 function and 15 gradient calls

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
gamma	1.01020	0.11244	8.984	<2e-16	***
delta	0.62711	0.02834	22.126	<2e-16	***
rho	0.63975	0.29705	2.154	0.0313	*
nu	1.09545	0.04500	24.346	<2e-16	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.424083

Multiple R-squared: 0.7751686

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )
E_1_2 (all)	0.6098	0.1105	5.52	3.39e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

## 7.2.4 Global optimisation algorithms

While the gradient-based (local) optimisation algorithms described above are designed to find local minima, global optimisation algorithms, which are also known as direct search methods, are designed to find the global minimum. They are more tolerant to not well-behaved objective functions but they usually converge more slowly than the gradient-based methods. However, increasing computing power has made these algorithms suitable for day-to-day use.

One of these global optimisation routines is the so-called Nelder-Mead algorithm (Nelder and Mead, 1965), which is a downhill simplex algorithm. In every iteration  $n + 1$  vertices are defined in the  $n$ -dimensional parameter space. The algorithm converges by successively replacing the “worst” point by a new vertice in the  $n$ -dimensional parameter space. The Nelder-Mead algorithm has the advantage of being a simple and robust algorithm, and is especially suitable for residual problems with non-differentiable objective functions. However, the heuristic nature of the algorithm causes slow convergence, especially close to the minimum, and can lead to convergence to non-stationary points. As the CES function is easily twice differentiable the advantage of the Nelder-Mead algorithm reduces to its robustness. As a consequence of the heuristic optimisation technique, the results should be handled with care. However, the Nelder-Mead algorithm is much faster than the other global optimisation algorithms described below. Function `cesEst` estimates a CES function with the Nelder-Mead algorithm if argument `method` is set to "NM". The user can tweak this algorithm (e.g. the reflection factor, contraction factor, or expansion factor) or change some details (e.g. convergence tolerance level) by adding a further argument `control` as described in the “Details” section of the documentation of `optim`.



```
> cesNm <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "NM")
> summary(cesNm)
```

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,
      method = "NM")
```

Estimation by non-linear least-squares using the 'Nelder-Mead' optimizer  
assuming an additive error term

Convergence achieved after 359 iterations

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
gamma	1.01024	0.11244	8.984	<2e-16 ***
delta	0.62710	0.02834	22.126	<2e-16 ***
rho	0.63961	0.29703	2.153	0.0313 *
nu	1.09544	0.04499	24.346	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.424083

Multiple R-squared: 0.7751686

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )
E_1_2 (all)	0.6099	0.1105	5.52	3.39e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

The Simulated Annealing algorithm was initially proposed by Kirkpatrick *et al.* (1983) and Cerny (1985) and is a modification of the Metropolis-Hastings algorithm. Every iteration chooses a random solution close to the current solution, while the probability of the choice is driven by a global parameter  $T$  which decreases as the algorithm moves on. Unlike other iterative optimisation algorithms, Simulated Annealing also allows  $T$  to increase which

makes it possible to leave local minima. Therefore, Simulated Annealing is a robust global optimiser and can be applied to a large search space, where it provides fast and reliable solutions. Setting argument `method` to "SANN" selects a variant of the "Simulated Annealing" algorithm given in Bélisle (1992). The user can modify some details of the "Simulated Annealing" algorithm (e.g. the starting temperature or the number of function evaluations at each temperature) by adding a further argument `control` as described in the "Details" section of the documentation of `optim`. The only criterion for stopping this iterative process is the number of iterations and it does not indicate whether convergence occurs.

```
> cesSann <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "SANN")
> summary(cesSann)
```

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,
      method = "SANN")
```

Estimation by non-linear least-squares using the 'SANN' optimizer  
assuming an additive error term

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
gamma	0.98474	0.10942	9.000	<2e-16 ***
delta	0.63192	0.02822	22.395	<2e-16 ***
rho	0.67107	0.29777	2.254	0.0242 *
nu	1.10819	0.04488	24.693	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.425541

Multiple R-squared: 0.774898

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )
E_1_2 (all)	0.5984	0.1066	5.612	2e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

As the Simulated Annealing algorithm makes use of random numbers, the solution generally depends on the initial “state” of R’s random number generator. To ensure replicability, `cesEst` “seeds” the random number generator before it starts the “Simulated Annealing” algorithm with the value of argument `random.seed`, which defaults to 123. Hence, the estimation of the same model using this algorithm always returns the same estimates as long as argument `random.seed` is not altered (at least using the same software and hardware components).

```
> cesSann2 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "SANN")
> all.equal(cesSann, cesSann2)
```

```
[1] TRUE
```

It is recommended to start this algorithm with different values of argument `random.seed` and check whether the estimates differ considerably.

```
> cesSann3 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "SANN",
+   random.seed = 1234)
> cesSann4 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "SANN",
+   random.seed = 12345)
> cesSann5 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "SANN",
+   random.seed = 123456)
> m <- rbind(cesSann = coef(cesSann), cesSann3 = coef(cesSann3),
+   cesSann4 = coef(cesSann4), cesSann5 = coef(cesSann5))
> rbind(m, stdDev = sd(m))
```

	gamma	delta	rho	nu
cesSann	0.98473949	0.631924860	0.67107287	1.10818592
cesSann3	1.03244315	0.638666280	0.80004041	1.08781915
cesSann4	1.09853191	0.640687668	0.74022400	1.06285986
cesSann5	1.03797816	0.633285471	0.63724351	1.08296884
stdDev	0.04665818	0.004202345	0.07259749	0.01861008

If the estimates differ considerably, the user can try to increase the number of iterations, which is 10,000 by default. Now we will re-estimate the model a few times with 100,000 iterations each.

```

> cesSannB <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "SANN",
+   control = list(maxit = 1e+05))
> cesSannB3 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE,
+   method = "SANN", random.seed = 1234, control = list(maxit = 1e+05))
> cesSannB4 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE,
+   method = "SANN", random.seed = 12345, control = list(maxit = 1e+05))
> cesSannB5 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE,
+   method = "SANN", random.seed = 123456, control = list(maxit = 1e+05))
> m <- rbind(cesSannB = coef(cesSannB), cesSannB3 = coef(cesSannB3),
+   cesSannB4 = coef(cesSannB4), cesSannB5 = coef(cesSannB5))
> rbind(m, stdDev = sd(m))

```

	gamma	delta	rho	nu
cesSannB	1.019018933	0.626396980	0.62656297	1.091731388
cesSannB3	1.012203547	0.629279618	0.66705294	1.094746865
cesSannB4	1.017438463	0.630137829	0.65935962	1.092449539
cesSannB5	1.000831695	0.634353090	0.64772478	1.099822550
stdDev	0.008227146	0.003289482	0.01763287	0.003656927

The estimates are much more similar now—only the estimates of  $\rho$  still differ somewhat.

In contrary to the other algorithms described in this paper, the Differential Evolution algorithm (Storn and Price, 1997) belongs to the class of evolution strategy optimisers and convergence cannot be proven analytically. However, the algorithm has proven to be effective and accurate on a large range of optimisation problems, inter alia the CES function (Mishra, 2007). For some problems it has proven to be more accurate and more efficient than Simulated Annealing, Quasi-Newton, or other genetic algorithms (Storn and Price, 1997; Ali and Törn, 2004; Mishra, 2007). Function `cesEst` uses a Differential Evolution optimiser for the non-linear least-squares estimation of the CES function, if argument `method` is set to "DE". The user can modify the Differential Evolution algorithm (e.g. the differential evolution strategy or selection method) or change some details (e.g. the number of population members) by adding a further argument `control` as described in the documentation of `DEoptim.control`. In contrary to the other optimisation algorithms, the Differential Evolution method requires finite boundaries for the parameters. By default, the bounds are  $0 \leq \gamma \leq 10^{10}$ ,  $0 \leq \delta \leq 1$ ,  $-1 \leq \rho \leq 10$ , and  $0 \leq \nu \leq 10$ . Of course, the user can specify own lower and upper bounds by setting arguments `lower` and `upper` to numeric vectors.

```
> cesDe <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "DE",
+   control = list(trace = FALSE))
> summary(cesDe)
```

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,
  method = "DE", control = list(trace = FALSE))
```

Estimation by non-linear least-squares using the 'DE' optimizer  
assuming an additive error term

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
gamma	1.01012	0.11256	8.974	<2e-16 ***
delta	0.62777	0.02841	22.094	<2e-16 ***
rho	0.64216	0.29785	2.156	0.0311 *
nu	1.09500	0.04505	24.307	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.424131

Multiple R-squared: 0.7751597

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )
E_1_2 (all)	0.6090	0.1105	5.513	3.52e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Similar to the “Simulated Annealing” algorithm, the Differential Evolution algorithm makes use of random numbers and `cesEst` “seeds” the random number generator with the value of argument `random.seed` before it starts the algorithm to ensure replicability.

```
> cesDe2 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "DE",
+   control = list(trace = FALSE))
> all.equal(cesDe, cesDe2)
```

[1] TRUE

It is also recommended for this algorithm to check if different values of argument `random.seed` result in remarkably different estimates.

```
> cesDe3 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "DE",
+   random.seed = 1234, control = list(trace = FALSE))
> cesDe4 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "DE",
+   random.seed = 12345, control = list(trace = FALSE))
> cesDe5 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "DE",
+   random.seed = 123456, control = list(trace = FALSE))
> m <- rbind(cesDe = coef(cesDe), cesDe3 = coef(cesDe3), cesDe4 = coef(cesDe4),
+   cesDe5 = coef(cesDe5))
> rbind(m, stdDev = sd(m))
```

	gamma	delta	rho	nu
cesDe	1.010119217	0.6277687357	0.642157064	1.095001892
cesDe3	1.008841004	0.6287090120	0.637607137	1.096142323
cesDe4	1.011146256	0.6273823595	0.641914728	1.094396769
cesDe5	1.000593563	0.6268241309	0.640145999	1.098865426
stdDev	0.004814219	0.0007932049	0.002100288	0.001979579

These estimates are rather similar, which generally indicates that all estimates are close to the optimum (minimum of the sum of squared residuals). However, if the user wants to obtain more precise estimates than derived by with the default settings of the algorithm, e.g. if the estimates differ considerably, the user can try increasing the number of iterations, which is 200 by default. Now we will re-estimate the model a few times with 1,000 iterations each.

```
> cesDeB <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "DE",
+   control = list(trace = FALSE, itermax = 1000))
> cesDeB3 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "DE",
+   random.seed = 1234, control = list(trace = FALSE, itermax = 1000))
> cesDeB4 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "DE",
+   random.seed = 12345, control = list(trace = FALSE, itermax = 1000))
> cesDeB5 <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "DE",
+   random.seed = 123456, control = list(trace = FALSE, itermax = 1000))
> rbind(cesDeB = coef(cesDeB), cesDeB3 = coef(cesDeB3), cesDeB4 = coef(cesDeB4),
+   cesDeB5 = coef(cesDeB5))
```

	gamma	delta	rho	nu
cesDeB	1.0102	0.6271122	0.6397527	1.095452
cesDeB3	1.0102	0.6271122	0.6397527	1.095452
cesDeB4	1.0102	0.6271122	0.6397527	1.095452
cesDeB5	1.0102	0.6271122	0.6397527	1.095452

The estimates are now virtually identical.

## 7.2.5 Constraint parameters

As a meaningful analysis based on a CES function requires that the function is consistent with economic theory, it is often desirable to constrain the parameter space to the economically meaningful region.

Function `cesEst` can estimate a CES function under parameter constraints using a modification of the BFGS algorithm suggested by Byrd *et al.* (1995). In contrast to the ordinary BFGS algorithm summarised above, the so-called L-BFGS-B algorithm allows for box-constraints on the parameters and also does not explicitly form or store the Hessian matrix, but instead relies on the past (often less than 10) values of the parameters and the gradient vector. Therefore, the L-BFGS-B algorithm is especially suitable for high dimensional optimisation problems, but—of course—it can also be used for optimisation problems with only a few parameters (such as the CES function). Function `cesEst` estimates a CES function with parameter constraints using the L-BFGS-B algorithm if argument `method` is set to "L-BFGS-B". The user can tweak some details of this algorithm (e.g. the number of BFGS updates) by adding a further argument `control` as described in the “Details” section of the documentation of `optim`. By default, the restrictions on the parameters are  $0 \leq \gamma \leq \infty$ ,  $0 \leq \delta \leq 1$ ,  $-1 \leq \rho \leq \infty$ , and  $0 \leq \nu \leq \infty$ . The user can specify own lower and upper bounds by setting arguments `lower` and `upper` to numeric vectors.

```
> cesLbfgsb <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE,
+   method = "L-BFGS-B")
> summary(cesLbfgsb)
```

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,
      method = "L-BFGS-B")
```

Estimation by non-linear least-squares using the 'L-BFGS-B' optimizer  
assuming an additive error term

Convergence achieved after 36 function and 36 gradient calls

Message: CONVERGENCE: REL\_REDUCTION\_OF\_F <= FACTR\*EPSMCH

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )	
gamma	1.01020	0.11244	8.984	<2e-16	***
delta	0.62711	0.02834	22.126	<2e-16	***
rho	0.63975	0.29705	2.154	0.0313	*
nu	1.09545	0.04500	24.346	<2e-16	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.424083

Multiple R-squared: 0.7751686

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )	
E_1_2 (all)	0.6098	0.1105	5.52	3.39e-08	***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

The so-called PORT routines (Gay, 1990) include a quasi-Newton optimisation algorithm that allows for box constraints on the parameters and has several advantages over traditional Newton routines, e.g. trust regions and reverse communication. Setting argument `method` to "PORT" selects the optimisation algorithm of the PORT routines. The user can modify a few details of the Newton algorithm (e.g. the minimum step size) by adding a further argument `control` as described in section "Control parameters" of the documentation of `nlminb`. The lower and upper bounds of the parameters have the same default values as for the L-BFGS-B method.

```
> cesPort <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, method = "PORT")  
> summary(cesPort)
```

Estimated CES function with variable returns to scale



Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,
      method = "PORT")
```

Estimation by non-linear least-squares using the 'PORT' optimizer  
 assuming an additive error term  
 Convergence achieved after 27 iterations  
 Message: relative convergence (4)

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
gamma	1.01020	0.11244	8.984	<2e-16 ***
delta	0.62711	0.02834	22.126	<2e-16 ***
rho	0.63975	0.29705	2.154	0.0313 *
nu	1.09545	0.04500	24.346	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.424083  
 Multiple R-squared: 0.7751686

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )
E_1_2 (all)	0.6098	0.1105	5.52	3.39e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

### 7.2.6 Grid search for $\rho$

As the objective function for estimating the CES by non-linear least-squares shows a tendency to “flat surfaces” around the minimum—in particular for a wide range of values for  $\rho$ —many optimisation algorithms have problems in finding the minimum of the objective function. This problem can be alleviated by performing a one-dimensional grid search, where a sequence of values for  $\rho$  is pre-selected and the remaining parameters are estimated by non-linear least-squares holding  $\rho$  fixed at each of the pre-defined values. Later, the estimation with the value of  $\rho$  that results in the smallest sum of squared residuals is chosen.

The function `cesEst` carries out this grid search procedure, if the user sets its argument `rho` to a numeric vector containing the values of  $\rho$  that should be used in the grid search. The estimation of the other parameters during the grid search can use all non-linear optimisation algorithms described above. Since the “best” value of  $\rho$  that was found in the grid search is not known but estimated (as the other parameters, but with a different method), the covariance matrix of the estimated parameters includes  $\rho$  and is calculated as if  $\rho$  was estimated as usual. The following command estimates the CES function by a one-dimensional grid search for  $\rho$ , where the pre-selected values for  $\rho$  are the values from  $-0.3$  to  $1.5$  with an increment of  $0.1$  and the default optimisation method, the Levenberg-Marquardt algorithm, is used to estimate the remaining parameters.

```
> cesGrid <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE, rho = seq(from = -0.3,
+   to = 1.5, by = 0.1))
> summary(cesGrid)
```

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,
      rho = seq(from = -0.3, to = 1.5, by = 0.1))
```

Estimation by non-linear least-squares using the 'LM' optimizer  
and a one-dimensional grid search for coefficient 'rho'  
assuming an additive error term

Convergence achieved after 4 iterations

Message: Relative error in the sum of squares is at most `ftol`.

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
gamma	1.00527	0.11192	8.982	<2e-16 ***
delta	0.62568	0.02809	22.272	<2e-16 ***
rho	0.60000	0.29151	2.058	0.0396 *
nu	1.09699	0.04501	24.374	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.424194

Multiple R-squared: 0.775148

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )
E_1_2 (all)	0.6250	0.1139	5.489	4.05e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

An overview of the relationship between the pre-selected values of  $\rho$  and the corresponding sums of the squared residuals can be obtained by applying the `plot` method.<sup>6</sup>

```
> plot(cesGrid)
```

This overview is shown in figure 7.3.

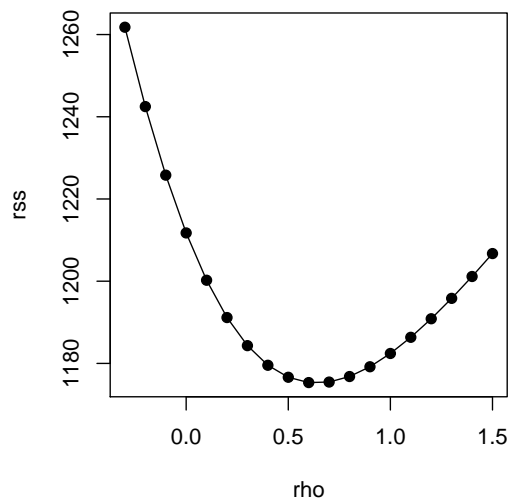


Figure 7.3: Values of  $\rho$  and corresponding sums of squared residuals

The results of this grid search algorithm can be either used directly or used as starting values for a non-linear least-squares estimation so that  $\rho$  values between the grid points can also be estimated. Starting values can be set by argument `startVal`.

```
> cesStartGrid <- cesEst("y", c("x1", "x2"), cesData, vrs = TRUE,
+   start = coef(cesGrid))
> summary(cesStartGrid)
```

<sup>6</sup>This plot method can only be applied if the model is estimated by grid search.

Estimated CES function with variable returns to scale

Call:

```
cesEst(yName = "y", xNames = c("x1", "x2"), data = cesData, vrs = TRUE,  
      start = coef(cesGrid))
```

Estimation by non-linear least-squares using the 'LM' optimizer  
assuming an additive error term

Convergence achieved after 3 iterations

Message: Relative error in the sum of squares is at most `ftol`.

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
gamma	1.01020	0.11244	8.984	<2e-16 ***
delta	0.62711	0.02834	22.126	<2e-16 ***
rho	0.63975	0.29705	2.154	0.0313 *
nu	1.09545	0.04500	24.346	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.424083

Multiple R-squared: 0.7751686

Elasticity of Substitution:

	Estimate	Std. Error	t value	Pr(> t )
E_1_2 (all)	0.6098	0.1105	5.52	3.39e-08 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

## 7.3 Implementation

The function `cesEst` is the primary user interface of the **micEconCES** package (Henningsen and Henningsen, 2010). However, the actual estimations are carried out by internal helper functions or functions from other packages.

### 7.3.1 Kmenta approximation

The estimation of the Kmenta approximation (7.2) is implemented in the internal function `cesEstKmenta`. This function uses `translogEst` from the **micEcon** package (Henningsen, 2010) for estimating the unrestricted translog function (7.3). The test of the parameter restrictions defined in equation (7.4) is performed by the function `linear.hypothesis` of the **car** package (Fox, 2009). The restricted translog model (7.3, 7.4) is estimated with function `systemfit` from the **systemfit** package (Henningsen and Hamann, 2007).

### 7.3.2 Non-linear least-squares estimation

The non-linear least-squares estimations are carried out by various optimisers from other packages. Estimations with the Levenberg-Marquardt algorithm are performed by function `nls.lm` of the **minpack.lm** package (Elzhov and Mullen, 2009), which is an R interface to the FORTRAN package **MINPACK** (Moré *et al.*, 1980). Estimations with the Conjugate Gradients (CG), BFGS, Nelder-Mead (NM), Simulated Annealing (SANN), and L-BFGS-B algorithms use the function `optim` from the **stats** package (R Development Core Team, 2009). Estimations with the Newton-type algorithm are performed by function `nlm` from the **stats** package (R Development Core Team, 2009), which uses the FORTRAN library **UNCMIN** (Schnabel *et al.*, 1985) with line search as step selection strategy. Estimations with the Differential Evolution (DE) algorithm are performed by function `DEoptim` from the **DEoptim** package (Ardia and Mullen, 2009). Estimations with the PORT routines use function `nlminb` from the **stats** package (R Development Core Team, 2009), which uses the FORTRAN library **PORT** (Gay, 1990).

### 7.3.3 Grid search

The grid search procedure is implemented in the internal function `cesEstGridRho`. This function consecutively calls `cesEst` for each of the pre-selected values of  $\rho$ , where argument `rho` of `cesEst` is set to one of the pre-selected values at each call. If argument `rho` of `cesEst` is a single scalar value, `cesEst` does not perform a grid search, but estimates the CES function by non-linear least-squares with parameter  $\rho$  fixed at the value of argument `rho`.

### 7.3.4 Calculating output

Function `cesCalc` can be used to calculate the output quantity of the CES function given input quantities and parameters. An example of using `cesCalc` is shown in the beginning of

section 7.2, where the output variable of an artificial data set that is used to demonstrate the usage of `cesEst` is generated with this function. Furthermore, the `cesCalc` function is used by the internal function `cesRss`, that calculates and returns the sum of squared residuals, which is the objective function in the non-linear least-squares estimations. As the CES function is not defined for  $\rho = 0$ , `cesCalc` calculates in this case the output quantity with the limit of the CES function for  $\rho \rightarrow 0$ , which is the Cobb-Douglas function.

We noticed that the calculations with `cesCalc` using equation (7.1) are imprecise when  $\rho$  is close to 0. This is caused by rounding errors that are unavoidable on digital computers, but are usually negligible. However, rounding errors can become large in specific circumstances, e.g. in the CES function with very small  $\rho$ , when very small (in absolute terms) exponents ( $-\rho$ ) are applied first and then a very large (in absolute terms) exponent ( $-\nu/\rho$ ) is applied. Therefore, `cesCalc` uses a first-order Taylor series approximation at the point  $\rho = 0$  for calculating the output of the CES function, if the absolute value of  $\rho$  is smaller than or equal to argument `rhoApprox`, which is  $5 \cdot 10^{-6}$  by default. This first-order Taylor series approximation is the Kmenta approximation defined in (7.2). We illustrate this in the left panel of figure 7.4, which was created by the following commands.

```
> rhoData <- data.frame(rho = seq(-2e-06, 2e-06, 5e-09), yCES = NA,
+   yLin = NA)
> for (i in 1:nrow(rhoData)) {
+   cesCoef <- c(gamma = 1, delta = 0.6, rho = rhoData$rho[i],
+     nu = 1.1)
+   rhoData$yLin[i] <- cesCalc(xNames = c("x1", "x2"), data = cesData[1,
+     ], coef = cesCoef, rhoApprox = Inf)
+   rhoData$yCES[i] <- cesCalc(xNames = c("x1", "x2"), data = cesData[1,
+     ], coef = cesCoef, rhoApprox = 0)
+ }
> rhoData$yCES <- rhoData$yCES - rhoData$yLin[rhoData$rho == 0]
> rhoData$yLin <- rhoData$yLin - rhoData$yLin[rhoData$rho == 0]
> plot(rhoData$rho, rhoData$yCES, type = "l", col = "red", xlab = "rho",
+   ylab = "y (normalised, red = CES, black = linearised)")
> lines(rhoData$rho, rhoData$yLin)
```

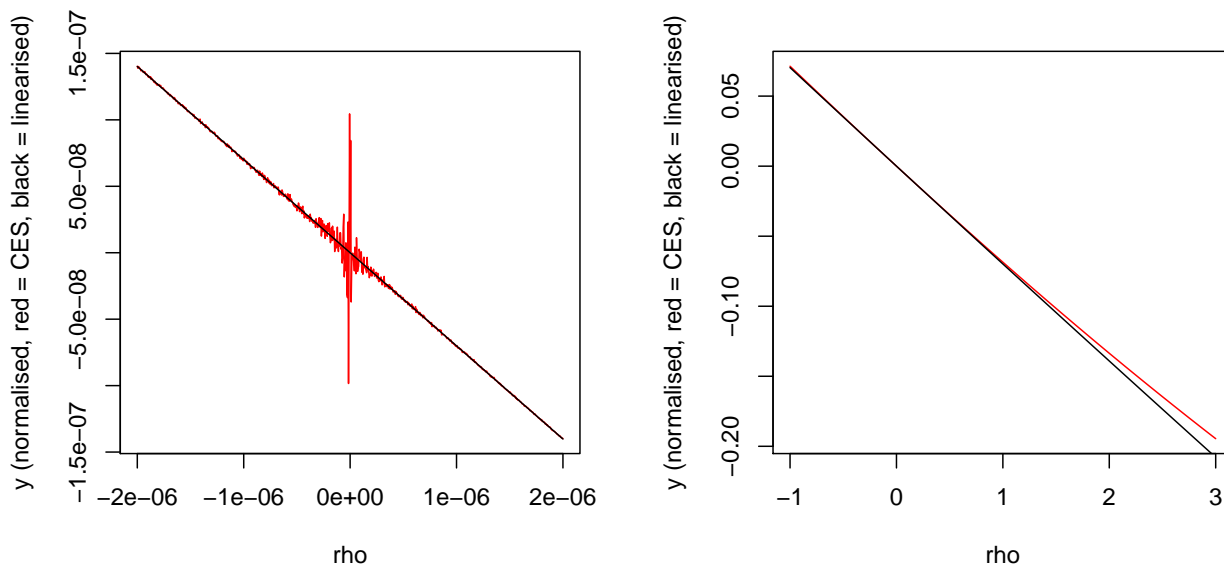


Figure 7.4: Calculated output for different values of  $\rho$

The right panel of figure 7.4 shows that the relationship between  $\rho$  and the output  $y$  can be rather precisely approximated by a linear function, because it is nearly linear for a wide range of  $\rho$  values.<sup>7</sup>

When estimating a CES function with function `cesEst`, the user can use argument `rhoApprox` to modify the threshold for calculating the endogenous variable by the Kmenta approximation (7.2), as the first element of the vector `rhoApprox` is passed to `cesCalc`, partly through `cesRss`. This might not only affect the fitted values and residuals returned by `cesEst`, but also the estimation results, because the endogenous variable is used to calculate the sum of squared residuals, which is the objective function of the non-linear least-squares estimations.

### 7.3.5 Partial derivatives with respect to coefficients

The internal function `cesDerivCoef` returns the partial derivatives of the CES function with respect to all coefficients at all provided data points. These partial derivatives are:

$$\frac{\partial y}{\partial \gamma} = \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}} \quad (7.16)$$

$$\frac{\partial y}{\partial \delta} = -\frac{\gamma \nu}{\rho} \left( x_1^{-\rho} - x_2^{-\rho} \right) \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho} - 1} \quad (7.17)$$

<sup>7</sup>The commands for creating the right panel of figure 7.4 are not shown here, because they are the same as the commands for the left panel of this figure except for the command for creating the vector of  $\rho$  values.

$$\begin{aligned} \frac{\partial y}{\partial \rho} &= \frac{\gamma \nu}{\rho^2} \log(\delta x_1^{-\rho} + (1-\delta)x_2^{-\rho}) (\delta x_1^{-\rho} + (1-\delta)x_2^{-\rho})^{-\frac{\nu}{\rho}} \\ &\quad + \frac{\gamma \nu}{\rho} (\delta \log(x_1)x_1^{-\rho} + (1-\delta) \log(x_2)x_2^{-\rho}) (\delta x_1^{-\rho} + (1-\delta)x_2^{-\rho})^{-\frac{\nu}{\rho}-1} \end{aligned} \quad (7.18)$$

$$\frac{\partial y}{\partial \nu} = -\frac{\gamma}{\rho} \log(\delta x_1^{-\rho} + (1-\delta)x_2^{-\rho}) (\delta x_1^{-\rho} + (1-\delta)x_2^{-\rho})^{-\frac{\nu}{\rho}} \quad (7.19)$$

These derivatives are not defined for  $\rho = 0$  and are imprecise if  $\rho$  is close to zero (similar to the output variable of the CES function, see section 7.3.4). Therefore, we calculate these derivatives by first-order Taylor series approximations at the point  $\rho = 0$  if  $\rho$  is zero or close to zero:

$$\frac{\partial y}{\partial \gamma} = x_1^{\nu\delta} x_2^{\nu(1-\delta)} \exp\left(-\frac{\rho}{2} \nu \delta (1-\delta) (\log x_1 - \log x_2)^2\right) \quad (7.20)$$

$$\begin{aligned} \frac{\partial y}{\partial \delta} &= (\log x_1 - \log x_2) x_1^{\nu\delta} x_2^{\nu(1-\delta)} \\ &\quad \left(1 - \frac{\rho}{2} [1 - 2\delta + \nu\delta(1-\delta) (\log x_1 - \log x_2)] (\log x_1 - \log x_2)\right) \end{aligned} \quad (7.21)$$

$$\begin{aligned} \frac{\partial y}{\partial \rho} &= \gamma \nu \delta (1-\delta) x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left(-\frac{1}{2} (\log x_1 - \log x_2)^2\right. \\ &\quad \left.+ \frac{\rho}{3} (1-2\delta) (\log x_1 - \log x_2)^3 + \frac{\rho}{4} \nu \delta (1-\delta) (\log x_1 - \log x_2)^4\right) \end{aligned} \quad (7.22)$$

$$\begin{aligned} \frac{\partial y}{\partial \nu} &= \gamma x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left(\delta \log x_1 + (1-\delta) \log x_2\right. \\ &\quad \left.- \frac{\rho}{2} \delta (1-\delta) (\log x_1 - \log x_2)^2 [1 + \nu (\delta \log x_1 + (1-\delta) \log x_2)]\right) \end{aligned} \quad (7.23)$$

Function `cesDerivCoef` has an argument `rhoApprox` that can be used to set the threshold levels for defining when  $\rho$  is “close” to zero. This argument must be a numeric vector with exactly four elements that define the thresholds for  $\partial y/\partial \gamma$ ,  $\partial y/\partial \delta$ ,  $\partial y/\partial \rho$ , and  $\partial y/\partial \nu$ , respectively. By default, these thresholds are  $5 \cdot 10^{-6}$  for  $\partial y/\partial \gamma$ ,  $5 \cdot 10^{-6}$  for  $\partial y/\partial \delta$ ,  $10^{-3}$  for  $\partial y/\partial \rho$ , and  $5 \cdot 10^{-6}$  for  $\partial y/\partial \nu$ .

Function `cesDerivCoef` is used to provide argument `jac` to function `nls.lm` so that the Levenberg-Marquardt algorithm can use analytical derivatives of each residual with respect to the coefficients. Furthermore, this function is used by the internal function `cesRssDeriv`, which calculates the partial derivatives of the sum of squared residuals (RSS) with respect to the coefficients by

$$\frac{\partial \text{RSS}}{\partial \theta} = -2 \sum_{i=1}^N \left(u_i \frac{\partial y_i}{\partial \theta}\right), \quad (7.24)$$



where  $N$  is the number of observations,  $u_i$  is the residual of the  $i$ th observation,  $\theta \in \{\gamma, \delta, \rho, \nu\}$  is a coefficient of the CES function, and  $\partial y_i / \partial \theta$  is the partial derivative of the CES function with respect to coefficient  $\theta$  evaluated at the  $i$ th observation as defined in equations (7.16) to (7.19) or—depending on the value of  $\rho$  and argument `rhoApprox`—equations (7.20) to (7.23). Function `cesRssDeriv` is used to provide analytical gradients for the gradient-based optimisation algorithms, i.e. Conjugate Gradients, Newton-type, BFGS, L-BFGS-B, and PORT. Finally, function `cesDerivCoef` is used to obtain the gradient matrix for calculating the asymptotic covariance matrix of the non-linear least-squares estimator (see section 7.3.6).

When estimating a CES function with function `cesEst`, the user can use argument `rhoApprox` to modify the thresholds for calculating the derivatives with respect to the coefficients by the linear approximations (7.20) to (7.23), as a vector containing the second to the fifth element of argument `rhoApprox` is passed to `cesDerivCoef`, partly through `cesRssDeriv`. This might not only affect the covariance matrix of the estimates, but also the estimation results obtained by a gradient-based optimisation algorithm.

### 7.3.6 Covariance matrix

The asymptotic covariance matrix of the non-linear least-squares estimator obtained by the various iterative optimisation methods is calculated by (Greene, 2008, p. 292):

$$\hat{\sigma}^2 \left( \begin{pmatrix} \frac{\partial y}{\partial \theta} \end{pmatrix}^\top \frac{\partial y}{\partial \theta} \right)^{-1}, \quad (7.25)$$

where  $\partial y / \partial \theta$  denotes the  $N \times k$  gradient matrix defined in equations (7.16) to (7.19),  $N$  is the number of observations,  $k$  is 3 for CES functions with constant returns to scale ( $\nu$  not estimated but fixed at 1) and 4 for CES functions with variable returns to scale ( $\nu$  estimated), and  $\hat{\sigma}^2$  denotes the estimated variance of the residuals. As equation (7.25) is only valid asymptotically, we calculate the estimated variance of the residuals by:

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^N u_i^2, \quad (7.26)$$

i.e. without correcting for degrees of freedom.

### 7.3.7 Starting values

If the user calls `cesEst` with argument `start` set to a vector of starting values, the internal function `cesEstStart` checks if the number of starting values is correct and if the individual starting values are in the appropriate range of the corresponding parameter. If no starting values are provided by the user, function `cesEstStart` determines the starting values automatically. The starting value of  $\delta$  is always set to 0.5. If the coefficient  $\rho$  is estimated (not fixed as, e.g., during grid search), the starting value of  $\rho$  is set to 0.25, which corresponds to an elasticity of substitution of 0.8. If the estimation allows for a model with variable returns to scale, the starting value of  $\nu$  is set to 1, which corresponds to constant returns to scale. Finally, the starting value of  $\gamma$  is set to a value so that the mean of the endogenous variable is equal to the mean of its fitted values, i.e.

$$\gamma = \frac{\frac{1}{N} \sum_{i=1}^N y_i}{\frac{1}{N} \sum_{i=1}^N \left(0.5 x_{1i}^{-\rho_0} + 0.5 x_{2i}^{-\rho_0}\right)^{-\frac{1}{\rho_0}}}, \quad (7.27)$$

where  $\rho_0$  is either the pre-selected value of  $\rho$  (if  $\rho$  is fixed) or the starting value of  $\rho$ , i.e. 0.25 (if  $\rho$  is estimated).

### 7.3.8 Other internal functions

The internal function `cesCoefAddRho` is used to add the value of  $\rho$  to the vector of coefficients, when  $\rho$  is fixed (e.g. during grid search for  $\rho$ ) and hence, not included in the vector of estimated coefficients.

If the user selects the optimisation algorithm Differential Evolution, L-BFGS-B, or PORT but does not specify lower or upper bounds of the coefficients, the internal function `cesCoefBounds` creates and returns the default bounds depending on the optimisation algorithm as described in sections 7.2.4 and 7.2.5.

The internal function `cesCoefNames` returns a vector of character strings, which are the names of the coefficients of the CES function.

### 7.3.9 Methods

The `micEconCES` package makes use of the “S3” class system of the R language introduced in Chambers and Hastie (1992). Objects returned by function `cesEst` are of class “`cesEst`” and the `micEconCES` package includes several methods for objects of this class. The `print` method prints the call and the estimated coefficients. The `coef`, `vcov`, `fitted`,

and `residuals` methods extract and return the estimated coefficients, their covariance matrix, the fitted values, and the residuals, respectively. The `plot` method can be applied only if the model was estimated by grid search; it plots a scatter plot of the pre-selected values of  $\rho$  against the corresponding sums of the squared residuals (see section 7.2.6) by using the `plot.default` and `points` commands of the **graphics** package (R Development Core Team, 2009).

The `summary` method calculates the estimated standard error of the residuals ( $\hat{\sigma}$ ), the covariance matrix of the coefficients estimated by non-linear least-squares, the  $R^2$  value as well as the standard errors,  $t$ -values, and marginal significance levels (P values) of the estimated parameters. The object returned by the `summary` method is of class `"summary.cesEst"`. The `print` method for objects of class `"summary.cesEst"` prints the call, the estimated coefficients, their standard errors,  $t$ -values, and marginal significance levels as well as some information on the estimation procedure (e.g. algorithm, convergence). The `coef` method for objects of class `"summary.cesEst"` returns a matrix with four columns containing the estimated coefficients, their standard errors,  $t$ -values, and marginal significance levels, respectively.

## 7.4 Monte Carlo study

In this section we perform a Monte Carlo study to compare the different estimation methods described above. These are the estimation by R's standard tool for non-linear least-squares estimations, `nls`, as well as the linear estimation of the Kmenta approximation and the non-linear least-squares estimations using the various optimisation algorithms described in sections 7.2.2 to 7.2.5 using function `cesEst`.<sup>8</sup> The data set used in this Monte Carlo study has 100 observations, where the input quantities are drawn from a  $\chi^2$  distribution with 10 degrees of freedom. We generate the "deterministic" output quantity by a CES function with variable returns to scale, where the parameters are  $\gamma = 1$ ,  $\delta = 0.6$ ,  $\rho = 1/3$ , and  $\nu = 1.1$ . This function has an elasticity of substitution of  $\sigma = 0.75$ . In each of the 1000 replications, a new set of disturbance terms is drawn from a normal distribution with a mean of zero and standard deviation of 1.5. This results in  $R^2$  values of the estimated models of around 0.915.

Function `cesEst` is generally called with the default values of all arguments (except for argument `method`, of course). However, we override the following default settings:

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<sup>8</sup>The estimation by `nls` in this Monte Carlo study is performed through function `cesEst`, which uses `nls` for the estimation if argument `method` is set to `"nls"`. This feature is not mentioned in the documentation of `cesEst`, because it is not completely implemented yet.

- Function `nls`:  
we set the control parameter `warnOnly` to `TRUE` so that this function returns coefficients (rather than just an error message) if the optimisation does not converge.
- Levenberg-Marquardt, Newton, BFGS, L-BFGS-B:  
we increased the maximum number of iterations to 250 to increase the chance that these algorithms reach convergence.
- Conjugate Gradients:  
we changed control parameter `type` to 2 so that the update formula of Polak and Ribière (1969) is used, increased the maximum number of iterations to 1000 and increased the tolerance level (argument `reltol`) to  $10^{-4}$  so that the algorithm reaches convergence in most replications (see example in section 7.2.3)
- Simulated Annealing:  
we increased the number of iterations to 50,000 so that the estimate is closer to the global minimum of the objective function (see section 7.2.4)
- Differential Evolution:  
we increased the number of iterations to 1,000 so that the estimate is closer to the global minimum of the objective function (see section 7.2.4)

The script used for the Monte Carlo simulation is shown in appendix 7.B. The general results of the Monte Carlo study are shown in table 7.1. Function `nls` reported 29 times that the non-linear minimisation of the squared residuals did not converge. The Newton and the Nelder-Mead algorithms reported this 5 and 3 times, respectively. All other algorithms always reported convergence. Even if `nls` or the Newton or Nelder-Mead algorithm reported non-convergence, the coefficients estimated by these methods were very close to the coefficients estimated by most other methods. Moreover, the sum of squared residuals of the “non-converged” estimations was virtually the same as the sum of squared residuals of most other algorithms in the same replication. Hence, it seems that only the default values of the convergence tolerance of `nls` and the Newton and Nelder-Mead algorithms, which are used in this Monte Carlo study, are a little too low for this optimisation problem. The average sums of the squared residuals were virtually identical for most estimation methods; only the Simulated Annealing method had on average slightly larger sums of the squared residuals and the Kmenta approximation, which does *not* aim at minimising the sum of squared residuals of the (non-linear) CES function, had a somewhat larger average sum of squared residuals.

Table 7.1: General results of the Monte Carlo simulation

	nNoConv	nConv	rssAll	rssConv
Kmenta	0	1000	228.2826052	228.6947658
nls	29	971	216.7175798	217.0249514
LM	0	1000	216.7175777	217.0249514
CG	0	1000	216.7175778	217.0249514
Newton	5	995	216.7175779	217.0249515
BFGS	0	1000	216.7175823	217.0249561
Nelder-Mead	3	997	216.7176217	217.0249958
SANN	0	1000	216.9664636	217.2736183
DE	0	1000	216.7175777	217.0249514
L-BFGS-B	0	1000	216.7175782	217.0249518
PORT	0	1000	216.7175777	217.0249514

Description of columns:

nNoConv: number of replications, where the estimation procedure with the corresponding method warned about non-convergence

nConv: number of replications, where the estimation with the corresponding method converged

rssAll: mean sum of squared residuals of all replications

rssConv: mean sum of squared residuals of the replications, where all methods converged

We summarise the results of the Monte Carlo study by presenting the biases and root mean square errors (RMSE) of the coefficients and the elasticity of substitution. The bias of a parameter  $\theta$  estimated by method  $m$  is

$$\frac{1}{K} \sum_{i=1}^K \hat{\theta}_{i,m} - \theta, \quad (7.28)$$

where  $K$  is the number of replications in the Monte Carlo study,  $\hat{\theta}_{i,m}$  is the estimate of parameter  $\theta$  estimated by method  $m$  in the  $i$ th replication, and  $\theta$  is the true value of this parameter. The root mean square error of this parameter  $\theta$  estimated by method  $m$  is

$$\sqrt{\frac{1}{K} \sum_{i=1}^K (\hat{\theta}_{i,m} - \theta)^2}, \quad (7.29)$$

where all variables are as defined above.

The biases of the estimated coefficients of the CES function and of the elasticity of substitution determined in our Monte Carlo study are shown in table 7.2. These biases are generally very small, which means that the *means* of the estimated parameters are very close to their

true values, no matter which estimation method is used. Only the Kmenta approximation returns on average a  $\gamma$  that is somewhat too small and a  $\nu$  that is a little too large but the bias of  $\delta$  and  $\rho$  is even smaller than the corresponding biases from the non-linear least-squares estimations. The estimated elasticities of substitution are on average a little larger than the true value—particularly for the Kmenta approximation.<sup>9</sup>

Table 7.2: Bias of the estimates

	gamma	delta	rho	nu	sigma
Kmenta	-0.06507	-0.00042	0.00312	0.02987	0.04740
nls	0.00140	0.00113	0.00569	0.00093	0.02469
LM	0.00140	0.00113	0.00569	0.00093	0.02468
CG	0.00140	0.00113	0.00569	0.00093	0.02468
Newton	0.00140	0.00113	0.00569	0.00093	0.02468
BFGS	0.00139	0.00113	0.00568	0.00094	0.02469
Nelder-Mead	0.00139	0.00113	0.00565	0.00094	0.02470
SANN	0.00246	0.00117	0.00703	0.00063	0.02462
DE	0.00140	0.00113	0.00569	0.00093	0.02468
L-BFGS-B	0.00140	0.00113	0.00569	0.00093	0.02468
PORT	0.00140	0.00113	0.00569	0.00093	0.02468

Note: the biases are calculated based on all replications, i.e. including replications, where the algorithm warned about non-convergence; the biases calculated only with the replications, where all estimation methods converged, are mostly rather similar to the reported biases but the biases of  $\rho$  are about 3 times larger than the reported biases. The column “sigma” represents the biases of the elasticity of substitution.

The root mean square errors (RMSE) of the estimated coefficients of the CES function and of the elasticity of substitution obtained by our Monte Carlo study are shown in table 7.3. The RMSEs of  $\gamma$ ,  $\delta$ , and  $\nu$  are mostly rather small, which means that these coefficients are estimated rather precisely, i.e. the estimated coefficients are mostly very close to their true values. In contrast, the RMSEs of  $\rho$  are rather large, which means that the estimation of this coefficient is rather imprecise. However, the elasticities of substitution calculated from the estimated  $\rho$ s have rather small RMSEs, i.e. are mostly rather close to their true values. As the elasticities of substitution—and not the  $\rho$ s—are usually used for interpreting the substitutability of inputs, the imprecise estimation of  $\rho$  is not a major problem. The RMSEs of most algorithms for non-linear least-squares estimations are virtually identical. The RMSEs of the Simulated Annealing algorithm are slightly larger than the RMSEs of the other algorithms for non-linear least-squares but these differences are so small that they

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<sup>9</sup>This is a little surprising as the Kmenta approximation has the smallest bias of the corresponding parameter  $\rho$  but this can be explained by the non-linear relationship between  $\rho$  and  $\sigma$  (see section 7.1).

are negligible in practical work. The estimates of the Kmenta approximation are less precise than the estimates from the non-linear least-squares estimations.

Table 7.3: Root mean square error of the estimates

	gamma	delta	rho	nu	sigma
Kmenta	0.15584	0.02825	0.33360	0.07037	0.04774
nls	0.09131	0.02228	0.25347	0.03647	0.02393
LM	0.09131	0.02228	0.25346	0.03647	0.02393
CG	0.09131	0.02228	0.25346	0.03647	0.02393
Newton	0.09131	0.02228	0.25347	0.03647	0.02393
BFGS	0.09131	0.02228	0.25347	0.03647	0.02393
Nelder-Mead	0.09130	0.02228	0.25344	0.03647	0.02393
SANN	0.09324	0.02242	0.25645	0.03724	0.02463
DE	0.09131	0.02228	0.25347	0.03647	0.02393
L-BFGS-B	0.09131	0.02228	0.25347	0.03647	0.02393
PORT	0.09131	0.02228	0.25347	0.03647	0.02393

Note: the root mean square errors are calculated based on all replications, i.e. including replications, where the algorithm warned about non-convergence; the root mean square errors calculated only with the replications, where all estimation methods converged, are very close to the reported root mean square errors.

## 7.5 Conclusion

We have demonstrated several approaches to estimating the CES function, e.g. the Kmenta approximation, the Levenberg-Marquardt algorithm, several other gradient-based and global optimisation algorithms, a grid search, and the standard tool for non-linear least-squares estimations in R, `nls`. We compared the performance of these methods in a Monte Carlo simulation. For the given data generating process, all methods returned satisfying results. Anyway, our simulation confirms other simulation studies (e.g. Thursby, 1980) in respect to the unsatisfying result for the estimate of  $\rho$ . However, our results show that the elasticity of substitution  $\sigma$ —which is generally of interest—is close to the “true” value. Hence, one should not rank this problem as too severe.

The results were derived under the ideal lab-conditions of a simulation. Clearly, not all methods will return such satisfying results if they face real-world data. Given the econometric problems that are often caused by real-world data, the presented methods will more clearly display their strength and weaknesses in empirical applications.

However, the **micEconCES** package provides the user with a multitude of instruments to address common econometric problems in estimating the CES function with real-world

data. Therefore, the user should be able to find a satisfying solution for estimating the CES function in most cases.



## Appendix

### 7.A Derivations of Taylor series approximations

The derivation of the Taylor series (Kmenta) approximation of the CES function in section 7.A.1 is based on Uebe (2000). The derivation of the Talor series approximation of the partial derivatives of the CES function with respect to the coefficients in section 7.A.2 is novel but inspired by Uebe (2000).

#### 7.A.1 CES function (Kmenta approximation)

$$y = \gamma \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}} \quad (7.30)$$

Logarithmised CES function:

$$\ln y = \ln \gamma - \frac{\nu}{\rho} \ln \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right) \quad (7.31)$$

Define function

$$f(\rho) \equiv -\frac{\nu}{\rho} \ln \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right) \quad (7.32)$$

so that

$$\ln y = \ln \gamma + f(\rho). \quad (7.33)$$

Now we can approximate the logarithmised CES by a first-order Taylor series approximation around  $\rho = 0$  :

$$\ln y \approx \ln \gamma + f(0) + \rho f'(0) \quad (7.34)$$

We define function

$$g(\rho) \equiv \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \quad (7.35)$$

so that

$$f(\rho) = -\frac{\nu}{\rho} \ln(g(\rho)). \quad (7.36)$$

Now we can calculate the first partial derivative of  $f(\rho)$ :

$$f'(\rho) = \frac{\nu}{\rho^2} \ln(g(\rho)) - \frac{\nu g'(\rho)}{\rho g(\rho)} \quad (7.37)$$

and the first three derivatives of  $g(\rho)$

$$g'(\rho) = -\delta x_1^{-\rho} \ln x_1 - (1 - \delta) x_2^{-\rho} \ln x_2 \quad (7.38)$$

$$g''(\rho) = \delta x_1^{-\rho} (\ln x_1)^2 + (1 - \delta) x_2^{-\rho} (\ln x_2)^2 \quad (7.39)$$

$$g'''(\rho) = -\delta x_1^{-\rho} (\ln x_1)^3 - (1 - \delta) x_2^{-\rho} (\ln x_2)^3. \quad (7.40)$$

At the point of approximation  $\rho = 0$  we have

$$g(0) = 1 \quad (7.41)$$

$$g'(0) = -\delta \ln x_1 - (1 - \delta) \ln x_2 \quad (7.42)$$

$$g''(0) = \delta (\ln x_1)^2 + (1 - \delta) (\ln x_2)^2 \quad (7.43)$$

$$g'''(0) = -\delta (\ln x_1)^3 - (1 - \delta) (\ln x_2)^3 \quad (7.44)$$

Now we calculate the limit of  $f(\rho)$  for  $\rho \rightarrow 0$ :

$$f(0) = \lim_{\rho \rightarrow 0} f(\rho) \quad (7.45)$$

$$= \lim_{\rho \rightarrow 0} \frac{-\nu \ln(g(\rho))}{\rho} \quad (7.46)$$

$$= \lim_{\rho \rightarrow 0} \frac{-\nu \frac{g'(\rho)}{g(\rho)}}{1} \quad (7.47)$$

$$= \nu (\delta \ln x_1 + (1 - \delta) \ln x_2) \quad (7.48)$$

and the limit of  $f'(\rho)$  for  $\rho \rightarrow 0$ :

$$f'(0) = \lim_{\rho \rightarrow 0} f'(\rho) \quad (7.49)$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{\nu}{\rho^2} \ln(g(\rho)) - \frac{\nu g'(\rho)}{\rho g(\rho)} \right) \quad (7.50)$$

$$= \lim_{\rho \rightarrow 0} \frac{\nu \ln(g(\rho)) - \nu \rho \frac{g'(\rho)}{g(\rho)}}{\rho^2} \quad (7.51)$$

$$= \lim_{\rho \rightarrow 0} \frac{\nu \frac{g'(\rho)}{g(\rho)} - \nu \frac{g'(\rho)}{g(\rho)} - \nu \rho \frac{g''(\rho)g(\rho) - (g'(\rho))^2}{(g(\rho))^2}}{2\rho} \quad (7.52)$$

$$= \lim_{\rho \rightarrow 0} -\frac{\nu}{2} \frac{g''(\rho)g(\rho) - (g'(\rho))^2}{(g(\rho))^2} \quad (7.53)$$

$$= -\frac{\nu}{2} \frac{g''(0)g(0) - (g'(0))^2}{(g(0))^2} \quad (7.54)$$

$$= -\frac{\nu}{2} \left( \delta (\ln x_1)^2 + (1 - \delta) (\ln x_2)^2 - (-\delta \ln x_1 - (1 - \delta) \ln x_2)^2 \right) \quad (7.55)$$

$$= -\frac{\nu}{2} \left( \delta (\ln x_1)^2 + (1 - \delta) (\ln x_2)^2 - \delta^2 (\ln x_1)^2 - 2\delta(1 - \delta) \ln x_1 \ln x_2 - (1 - \delta)^2 (\ln x_2)^2 \right) \quad (7.56)$$

$$= -\frac{\nu}{2} \left( (\delta - \delta^2) (\ln x_1)^2 + ((1 - \delta) - (1 - \delta)^2) (\ln x_2)^2 - 2\delta(1 - \delta) \ln x_1 \ln x_2 \right) \quad (7.57)$$

$$= -\frac{\nu}{2} \left( \delta(1 - \delta) (\ln x_1)^2 + (1 - \delta)(1 - (1 - \delta)) (\ln x_2)^2 - 2\delta(1 - \delta) \ln x_1 \ln x_2 \right) \quad (7.58)$$

$$= -\frac{\nu\delta(1 - \delta)}{2} \left( (\ln x_1)^2 - 2 \ln x_1 \ln x_2 + (\ln x_2)^2 \right) \quad (7.59)$$

$$= -\frac{\nu\delta(1 - \delta)}{2} (\ln x_1 - \ln x_2)^2 \quad (7.60)$$

so that we get following first-order Taylor series approximation around  $\rho = 0$  :

$$\ln y \approx \ln \gamma + \nu\delta \ln x_1 + \nu(1 - \delta) \ln x_2 - \nu\rho\delta(1 - \delta) (\ln x_1 - \ln x_2)^2 \quad (7.61)$$

## 7.A.2 Derivatives with respect to coefficients

$$\frac{\partial y}{\partial \gamma} = \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}} \quad (7.62)$$

$$\frac{\partial y}{\partial \delta} = -\gamma \frac{\nu}{\rho} \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho} - 1} (x_1^{-\rho} - x_2^{-\rho}) \quad (7.63)$$

$$\frac{\partial y}{\partial \nu} = -\frac{\gamma}{\rho} \ln \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right) \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}} \quad (7.64)$$

$$\begin{aligned} \frac{\partial y}{\partial \rho} &= \frac{\gamma\nu}{\rho^2} \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}} \ln \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right) \\ &\quad + \frac{\gamma\nu}{\rho} \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\left(\frac{\nu}{\rho} + 1\right)} \left( \delta x_1^{-\rho} \ln x_1 + (1 - \delta) x_2^{-\rho} \ln x_2 \right) \end{aligned} \quad (7.65)$$

### 7.A.2.1 Derivatives with respect to Gamma

$$\frac{\partial y}{\partial \gamma} = \exp\left(-\frac{\nu}{\rho} \ln(\delta x_1^{-\rho} + (1-\delta) x_2^{-\rho})\right) \quad (7.66)$$

$$= \exp(f(\rho)) \quad (7.67)$$

$$\approx \exp(f(0) + \rho f'(0)) \quad (7.68)$$

$$= \exp\left(\nu \delta \ln x_1 + \nu(1-\delta) \ln x_2 - \frac{1}{2} \nu \rho \delta(1-\delta) (\ln x_1 - \ln x_2)^2\right) \quad (7.69)$$

$$= x_1^{\nu \delta} x_2^{\nu(1-\delta)} \exp\left(-\frac{1}{2} \nu \rho \delta(1-\delta) (\ln x_1 - \ln x_2)^2\right) \quad (7.70)$$

### 7.A.2.2 Derivatives with respect to Delta

$$\frac{\partial y}{\partial \delta} = -\gamma \frac{\nu}{\rho} (\delta x_1^{-\rho} + (1-\delta) x_2^{-\rho})^{-\frac{\nu}{\rho}-1} (x_1^{-\rho} - x_2^{-\rho}) \quad (7.71)$$

$$= -\gamma \nu \frac{x_1^{-\rho} - x_2^{-\rho}}{\rho} (\delta x_1^{-\rho} + (1-\delta) x_2^{-\rho})^{-\frac{\nu}{\rho}-1} \quad (7.72)$$

Now we define the function  $f_\delta(\rho)$

$$f_\delta(\rho) = \frac{x_1^{-\rho} - x_2^{-\rho}}{\rho} (\delta x_1^{-\rho} + (1-\delta) x_2^{-\rho})^{-\frac{\nu}{\rho}-1} \quad (7.73)$$

$$= \frac{x_1^{-\rho} - x_2^{-\rho}}{\rho} \exp\left(-\left(\frac{\nu}{\rho} + 1\right) \ln(\delta x_1^{-\rho} + (1-\delta) x_2^{-\rho})\right) \quad (7.74)$$

so that we can approximate  $\partial y / \partial \delta$  by using the first-order Taylor series approximation of  $f_\delta(\rho)$ :

$$\frac{\partial y}{\partial \delta} = -\gamma \nu f_\delta(\rho) \quad (7.75)$$

$$\approx -\gamma \nu (f_\delta(0) + \rho f'_\delta(0)) \quad (7.76)$$

Now we define the helper functions  $g_\delta(\rho)$  and  $h_\delta(\rho)$

$$g_\delta(\rho) = \left(\frac{\nu}{\rho} + 1\right) \ln(\delta x_1^{-\rho} + (1-\delta) x_2^{-\rho}) \quad (7.77)$$

$$= \left(\frac{\nu}{\rho} + 1\right) \ln(g(\rho)) \quad (7.78)$$

$$h_\delta(\rho) = \frac{x_1^{-\rho} - x_2^{-\rho}}{\rho} \quad (7.79)$$

with first derivatives

$$g'_\delta(\rho) = -\frac{\nu}{\rho^2} \ln(g(\rho)) + \left(\frac{\nu}{\rho} + 1\right) \frac{g'(\rho)}{g(\rho)} \quad (7.80)$$

$$h'_\delta(\rho) = \frac{-\rho(\ln x_1 x_1^{-\rho} - \ln x_2 x_2^{-\rho}) - x_1^{-\rho} + x_2^{-\rho}}{\rho^2} \quad (7.81)$$

so that

$$f_\delta(\rho) = h_\delta(\rho) \exp(-g_\delta(\rho)) \quad (7.82)$$

and

$$f'_\delta(\rho) = h'_\delta(\rho) \exp(-g_\delta(\rho)) - h_\delta(\rho) \exp(-g_\delta(\rho)) g'_\delta(\rho) \quad (7.83)$$

Now we can calculate the limits of  $g_\delta(\rho)$ ,  $g'_\delta(\rho)$ ,  $h_\delta(\rho)$  and  $h'_\delta(\rho)$  for  $\rho \rightarrow 0$  by

$$g_\delta(0) = \lim_{\rho \rightarrow 0} g_\delta(\rho) \quad (7.84)$$

$$= \lim_{\rho \rightarrow 0} \left( \left( \frac{\nu}{\rho} + 1 \right) \ln(g(\rho)) \right) \quad (7.85)$$

$$= \lim_{\rho \rightarrow 0} \frac{(\nu + \rho) \ln(g(\rho))}{\rho} \quad (7.86)$$

$$= \lim_{\rho \rightarrow 0} \frac{\ln(g(\rho)) + (\nu + \rho) \frac{g'(\rho)}{g(\rho)}}{1} \quad (7.87)$$

$$= \ln(g(0)) + \nu \frac{g'(0)}{g(0)} \quad (7.88)$$

$$= -\nu\delta \ln x_1 - \nu(1 - \delta) \ln x_2 \quad (7.89)$$

$$g'_\delta(0) = \lim_{\rho \rightarrow 0} \frac{-\nu \ln(g(\rho)) + \rho(\nu + \rho) \frac{g'(\rho)}{g(\rho)}}{\rho^2} \quad (7.90)$$

$$= \lim_{\rho \rightarrow 0} \frac{-\nu \frac{g'(\rho)}{g(\rho)} + (\nu + \rho) \frac{g'(\rho)}{g(\rho)} + \rho \frac{g'(\rho)}{g(\rho)} + \rho(\nu + \rho) \frac{g''(\rho)g(\rho) - (g'(\rho))^2}{(g(\rho))^2}}{2\rho} \quad (7.91)$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{-\nu \frac{g'(\rho)}{g(\rho)} + \nu \frac{g'(\rho)}{g(\rho)} + \rho \frac{g'(\rho)}{g(\rho)} + \rho \frac{g'(\rho)}{g(\rho)} + \rho(\nu + \rho) \frac{g''(\rho)g(\rho) - (g'(\rho))^2}{(g(\rho))^2}}{2\rho} \right) \quad (7.92)$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{g'(\rho)}{g(\rho)} + \frac{1}{2}(\nu + \rho) \frac{g''(\rho)g(\rho) - (g'(\rho))^2}{(g(\rho))^2} \right) \quad (7.93)$$

$$= \frac{g'(0)}{g(0)} + \frac{1}{2}\nu \frac{g''(0)g(0) - (g'(0))^2}{(g(0))^2} \quad (7.94)$$

$$= -\delta \ln x_1 - (1 - \delta) \ln x_2 + \frac{\nu\delta(1 - \delta)}{2} (\ln x_1 - \ln x_2)^2 \quad (7.95)$$

$$h_\delta(0) = \lim_{\rho \rightarrow 0} \frac{x_1^{-\rho} - x_2^{-\rho}}{\rho} \quad (7.96)$$

$$= \lim_{\rho \rightarrow 0} \frac{-\ln x_1 x_1^{-\rho} + \ln x_2 x_2^{-\rho}}{1} \quad (7.97)$$

$$= -\ln x_1 + \ln x_2 \quad (7.98)$$

$$h'_\delta(0) = \lim_{\rho \rightarrow 0} \frac{-\rho (\ln x_1 x_1^{-\rho} - \ln x_2 x_2^{-\rho}) - x_1^{-\rho} + x_2^{-\rho}}{\rho^2} \quad (7.99)$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{- (\ln x_1 x_1^{-\rho} - \ln x_2 x_2^{-\rho}) + \rho ((\ln x_1)^2 x_1^{-\rho} - (\ln x_2)^2 x_2^{-\rho})}{2\rho} + \frac{\ln x_1 x_1^{-\rho} - \ln x_2 x_2^{-\rho}}{2\rho} \right) \quad (7.100)$$

$$= \lim_{\rho \rightarrow 0} \frac{1}{2} ((\ln x_1)^2 x_1^{-\rho} - (\ln x_2)^2 x_2^{-\rho}) \quad (7.101)$$

$$= \frac{1}{2} ((\ln x_1)^2 - (\ln x_2)^2) \quad (7.102)$$

so that we can calculate the limit of  $f_\delta(\rho)$  and  $f'_\delta(\rho)$  for  $\rho \rightarrow 0$  by

$$f_\delta(0) = \lim_{\rho \rightarrow 0} f_\delta(\rho) \quad (7.103)$$

$$= \lim_{\rho \rightarrow 0} (h_\delta(\rho) \exp(-g_\delta(\rho))) \quad (7.104)$$

$$= \lim_{\rho \rightarrow 0} h_\delta(\rho) \lim_{\rho \rightarrow 0} \exp(-g_\delta(\rho)) \quad (7.105)$$

$$= \lim_{\rho \rightarrow 0} h_\delta(\rho) \exp\left(-\lim_{\rho \rightarrow 0} g_\delta(\rho)\right) \quad (7.106)$$

$$= h_\delta(0) \exp(-g_\delta(0)) \quad (7.107)$$

$$= (-\ln x_1 + \ln x_2) \exp(\nu\delta \ln x_1 + \nu(1 - \delta) \ln x_2) \quad (7.108)$$

$$= (-\ln x_1 + \ln x_2) x_1^{\nu\delta} x_2^{\nu(1-\delta)} \quad (7.109)$$

$$f'_\delta(0) = \lim_{\rho \rightarrow 0} f'_\delta(\rho) \quad (7.110)$$

$$= \lim_{\rho \rightarrow 0} (h'_\delta(\rho) \exp(-g_\delta(\rho)) - h_\delta(\rho) \exp(-g_\delta(\rho)) g'_\delta(\rho)) \quad (7.111)$$

$$= \lim_{\rho \rightarrow 0} h'_\delta(\rho) \lim_{\rho \rightarrow 0} \exp(-g_\delta(\rho)) \quad (7.112)$$

$$\begin{aligned}
 & - \lim_{\rho \rightarrow 0} h_\delta(\rho) \lim_{\rho \rightarrow 0} \exp(-g_\delta(\rho)) \lim_{\rho \rightarrow 0} g'_\delta(\rho) \\
 = & \lim_{\rho \rightarrow 0} h'_\delta(\rho) \exp\left(-\lim_{\rho \rightarrow 0} g_\delta(\rho)\right) \tag{7.113}
 \end{aligned}$$

$$\begin{aligned}
 & - \lim_{\rho \rightarrow 0} h_\delta(\rho) \exp\left(-\lim_{\rho \rightarrow 0} g_\delta(\rho)\right) \lim_{\rho \rightarrow 0} g'_\delta(\rho) \\
 = & h'_\delta(0) \exp(-g_\delta(0)) - h_\delta(0) \exp(-g_\delta(0)) g'_\delta(0) \tag{7.114}
 \end{aligned}$$

$$= \exp(-g_\delta(0)) (h'_\delta(0) - h_\delta(0) g'_\delta(0)) \tag{7.115}$$

$$\begin{aligned}
 = & \exp(\nu\delta \ln x_1 + \nu(1-\delta) \ln x_2) \left(\frac{1}{2} \left((\ln x_1)^2 - (\ln x_2)^2\right) \right. \\
 & \left. - (-\ln x_1 + \ln x_2) \right) \tag{7.116}
 \end{aligned}$$

$$\begin{aligned}
 & \left(-\delta \ln x_1 - (1-\delta) \ln x_2 + \frac{\nu\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2\right) \\
 = & x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left(\frac{1}{2} (\ln x_1)^2 - \frac{1}{2} (\ln x_2)^2 \right. \\
 & \left. + (\ln x_1 - \ln x_2) \right) \tag{7.117}
 \end{aligned}$$

$$\begin{aligned}
 & \left(-\delta \ln x_1 - (1-\delta) \ln x_2 + \frac{\nu\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2\right) \\
 = & x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left(\frac{1}{2} (\ln x_1)^2 - \frac{1}{2} (\ln x_2)^2 - \delta (\ln x_1)^2 \right. \\
 & \left. - (1-\delta) \ln x_1 \ln x_2 + \frac{\nu\delta(1-\delta)}{2} \ln x_1 (\ln x_1 - \ln x_2)^2 \right) \tag{7.118}
 \end{aligned}$$

$$\begin{aligned}
 & + \delta \ln x_1 \ln x_2 + (1-\delta) (\ln x_2)^2 - \frac{\nu\delta(1-\delta)}{2} \ln x_2 (\ln x_1 - \ln x_2)^2 \\
 = & x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left(\left(\frac{1}{2} - \delta\right) (\ln x_1)^2 + \left(\frac{1}{2} - \delta\right) (\ln x_2)^2 \right. \\
 & \left. - 2\left(\frac{1}{2} - \delta\right) \ln x_1 \ln x_2 + \frac{\nu\delta(1-\delta)}{2} (\ln x_1 - \ln x_2) (\ln x_1 - \ln x_2)^2\right) \tag{7.119}
 \end{aligned}$$

$$\begin{aligned}
 = & x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left(\left(\frac{1}{2} - \delta\right) (\ln x_1 - \ln x_2)^2 \right. \\
 & \left. + \frac{\nu\delta(1-\delta)}{2} (\ln x_1 - \ln x_2) (\ln x_1 - \ln x_2)^2\right) \tag{7.120}
 \end{aligned}$$

$$= x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left(\frac{1}{2} - \delta + \frac{\nu\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)\right) (\ln x_1 - \ln x_2)^2 \tag{7.121}$$

$$= \frac{1 - 2\delta + \nu\delta(1-\delta)(\ln x_1 - \ln x_2)}{2} x_1^{\nu\delta} x_2^{\nu(1-\delta)} (\ln x_1 - \ln x_2)^2 \tag{7.122}$$

and approximate  $\partial y / \partial \delta$  by

$$\frac{\partial y}{\partial \delta} \approx -\gamma\nu (f_\delta(0) + \rho f'_\delta(0)) \tag{7.123}$$

$$= -\gamma\nu \left(-\ln x_1 + \ln x_2\right) x_1^{\nu\delta} x_2^{\nu(1-\delta)} \tag{7.124}$$

$$\begin{aligned}
 & +\rho \frac{1-2\delta+\nu\delta(1-\delta)(\ln x_1-\ln x_2)}{2} x_1^{\nu\delta} x_2^{\nu(1-\delta)} (\ln x_1-\ln x_2)^2 \Big) \\
 = & \gamma\nu \left( (\ln x_1-\ln x_2) x_1^{\nu\delta} x_2^{\nu(1-\delta)} \right. \\
 & \left. -\rho \frac{1-2\delta+\nu\delta(1-\delta)(\ln x_1-\ln x_2)}{2} x_1^{\nu\delta} x_2^{\nu(1-\delta)} (\ln x_1-\ln x_2)^2 \right) \tag{7.125}
 \end{aligned}$$

$$\begin{aligned}
 = & \gamma\nu (\ln x_1-\ln x_2) x_1^{\nu\delta} x_2^{\nu(1-\delta)} \\
 & \left( 1-\rho \frac{1-2\delta+\nu\delta(1-\delta)(\ln x_1-\ln x_2)}{2} (\ln x_1-\ln x_2) \right) \tag{7.126}
 \end{aligned}$$

### 7.A.2.3 Derivatives with respect to Nu

$$\frac{\partial y}{\partial \nu} = -\frac{\gamma}{\rho} \ln \left( \delta x_1^{-\rho} + (1-\delta) x_2^{-\rho} \right) \left( \delta x_1^{-\rho} + (1-\delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}} \tag{7.127}$$

Now we define the function  $f_\nu(\rho)$

$$f_\nu(\rho) = \frac{1}{\rho} \ln \left( \delta x_1^{-\rho} + (1-\delta) x_2^{-\rho} \right) \left( \delta x_1^{-\rho} + (1-\delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}} \tag{7.128}$$

$$= \frac{1}{\rho} \ln \left( \delta x_1^{-\rho} + (1-\delta) x_2^{-\rho} \right) \exp \left( -\frac{\nu}{\rho} \ln \left( \delta x_1^{-\rho} + (1-\delta) x_2^{-\rho} \right) \right) \tag{7.129}$$

so that we can approximate  $\partial y/\partial \nu$  by using the first-order Taylor series approximation of  $f_\nu(\rho)$ :

$$\frac{\partial y}{\partial \nu} = -\gamma f_\nu(\rho) \tag{7.130}$$

$$\approx -\gamma (f_\nu(0) + \rho f'_\nu(0)) \tag{7.131}$$

Now we define the helper function  $g_\nu(\rho)$

$$g_\nu(\rho) = \frac{1}{\rho} \ln \left( \delta x_1^{-\rho} + (1-\delta) x_2^{-\rho} \right) \tag{7.132}$$

$$= \frac{1}{\rho} \ln (g(\rho)) \tag{7.133}$$

with first and second derivative

$$g'_\nu(\rho) = \frac{\rho \frac{g'(\rho)}{g(\rho)} - \ln(g(\rho))}{\rho^2} \tag{7.134}$$

$$= \frac{1}{\rho} \frac{g'(\rho)}{g(\rho)} - \frac{\ln(g(\rho))}{\rho^2} \tag{7.135}$$



$$g''_{\nu}(\rho) = -\frac{1}{\rho^2} \frac{g'(\rho)}{g(\rho)} + \frac{1}{\rho} \frac{g''(\rho)}{g(\rho)} - \frac{1}{\rho} \frac{(g'(\rho))^2}{(g(\rho))^2} + 2 \frac{\ln(g(\rho))}{\rho^3} - \frac{1}{\rho^2} \frac{g'(\rho)}{g(\rho)} \quad (7.136)$$

$$= \frac{-2\rho \frac{g'(\rho)}{g(\rho)} + \rho^2 \frac{g''(\rho)}{g(\rho)} - \rho^2 \frac{(g'(\rho))^2}{(g(\rho))^2} + 2 \ln(g(\rho))}{\rho^3} \quad (7.137)$$

and use the function  $f(\rho)$  defined above so that

$$f_{\nu}(\rho) = g_{\nu}(\rho) \exp(f(\rho)) \quad (7.138)$$

and

$$f'_{\nu}(\rho) = g'_{\nu}(\rho) \exp(f(\rho)) + g_{\nu}(\rho) \exp(f(\rho)) f'(\rho) \quad (7.139)$$

Now we can calculate the limits of  $g_{\nu}(\rho)$ ,  $g'_{\nu}(\rho)$ , and  $g''_{\nu}(\rho)$  for  $\rho \rightarrow 0$  by

$$g_{\nu}(0) = \lim_{\rho \rightarrow 0} g_{\nu}(\rho) \quad (7.140)$$

$$= \lim_{\rho \rightarrow 0} \frac{\ln(g(\rho))}{\rho} \quad (7.141)$$

$$= \lim_{\rho \rightarrow 0} \frac{\frac{g'(\rho)}{g(\rho)}}{1} \quad (7.142)$$

$$= -\delta \ln x_1 - (1 - \delta) \ln x_2 \quad (7.143)$$

$$g'_{\nu}(0) = \lim_{\rho \rightarrow 0} g'_{\nu}(\rho) \quad (7.144)$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{1}{\rho} \frac{g'(\rho)}{g(\rho)} - \frac{\ln(g(\rho))}{\rho^2} \right) \quad (7.145)$$

$$= \lim_{\rho \rightarrow 0} \frac{\rho \frac{g'(\rho)}{g(\rho)} - \ln(g(\rho))}{\rho^2} \quad (7.146)$$

$$= \lim_{\rho \rightarrow 0} \frac{\frac{g'(\rho)}{g(\rho)} + \rho \frac{g''(\rho)g(\rho) - (g'(\rho))^2}{(g(\rho))^2} - \frac{g'(\rho)}{g(\rho)}}{2\rho} \quad (7.147)$$

$$= \lim_{\rho \rightarrow 0} \frac{g''(\rho)g(\rho) - (g'(\rho))^2}{2(g(\rho))^2} \quad (7.148)$$

$$= \frac{g''(0)g(0) - (g'(0))^2}{2(g(0))^2} \quad (7.149)$$

$$= \frac{1}{2} \left( \delta (\ln x_1)^2 + (1 - \delta) (\ln x_2)^2 - (-\delta \ln x_1 - (1 - \delta) \ln x_2)^2 \right) \quad (7.150)$$

$$= \frac{1}{2} \left( \delta (\ln x_1)^2 + (1 - \delta) (\ln x_2)^2 - \delta^2 (\ln x_1)^2 \right. \quad (7.151)$$

$$\left. - 2\delta(1 - \delta) \ln x_1 \ln x_2 - (1 - \delta)^2 (\ln x_2)^2 \right)$$

$$= \frac{1}{2} \left( (\delta - \delta^2) (\ln x_1)^2 + ((1 - \delta) - (1 - \delta)^2) (\ln x_2)^2 \right. \quad (7.152)$$

$$\left. - 2\delta(1 - \delta) \ln x_1 \ln x_2 \right)$$

$$= \frac{1}{2} \left( \delta(1 - \delta) (\ln x_1)^2 + (1 - \delta)(1 - (1 - \delta)) (\ln x_2)^2 \right. \quad (7.153)$$

$$\left. - 2\delta(1 - \delta) \ln x_1 \ln x_2 \right)$$

$$= \frac{\delta(1 - \delta)}{2} \left( (\ln x_1)^2 - 2 \ln x_1 \ln x_2 + (\ln x_2)^2 \right) \quad (7.154)$$

$$= \frac{\delta(1 - \delta)}{2} (\ln x_1 - \ln x_2)^2 \quad (7.155)$$

$$g''_{\nu}(0) = \lim_{\rho \rightarrow 0} g''_{\nu}(\rho) \quad (7.156)$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{-2\rho \frac{g'(\rho)}{g(\rho)} + \rho^2 \frac{g''(\rho)}{g(\rho)} - \rho^2 \frac{(g'(\rho))^2}{(g(\rho))^2} + 2 \ln(g(\rho))}{\rho^3} \right) \quad (7.157)$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{-2\rho \frac{g'(\rho)}{g(\rho)} - 2\rho \frac{g''(\rho)}{g(\rho)} + 2\rho \frac{(g'(\rho))^2}{(g(\rho))^2} + 2\rho \frac{g''(\rho)}{g(\rho)} + \rho^2 \frac{g'''(\rho)}{g(\rho)}}{3\rho^2} \right. \quad (7.158)$$

$$\left. + \frac{-\rho^2 \frac{g''(\rho)g'(\rho)}{(g(\rho))^2} - 2\rho \frac{(g'(\rho))^2}{(g(\rho))^2} - 2\rho^2 \frac{g'(\rho)g''(\rho)}{(g(\rho))^2} + 2\rho^2 \frac{(g'(\rho))^3}{(g(\rho))^3} + 2\frac{g'(\rho)}{g(\rho)}}{3\rho^2} \right)$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{\rho^2 \frac{g'''(\rho)}{g(\rho)} - 3\rho^2 \frac{g''(\rho)g'(\rho)}{(g(\rho))^2} + 2\rho^2 \frac{(g'(\rho))^3}{(g(\rho))^3}}{3\rho^2} \right) \quad (7.159)$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{1}{3} \frac{g'''(\rho)}{g(\rho)} - \frac{g''(\rho)g'(\rho)}{(g(\rho))^2} + \frac{2}{3} \frac{(g'(\rho))^3}{(g(\rho))^3} \right) \quad (7.160)$$

$$= \frac{1}{3} \frac{g'''(0)}{g(0)} - \frac{g''(0)g'(0)}{(g(0))^2} + \frac{2}{3} \frac{(g'(0))^3}{(g(0))^3} \quad (7.161)$$

$$= \frac{1}{3} \left( -\delta (\ln x_1)^3 - (1 - \delta) (\ln x_2)^3 \right) \quad (7.162)$$

$$- \left( \delta (\ln x_1)^2 + (1 - \delta) (\ln x_2)^2 \right) (-\delta \ln x_1 - (1 - \delta) \ln x_2)$$

$$+ \frac{2}{3} (-\delta \ln x_1 - (1 - \delta) \ln x_2)^3$$

$$= -\frac{1}{3}\delta(\ln x_1)^3 - \frac{1}{3}(1-\delta)(\ln x_2)^3 + \delta^2(\ln x_1)^3 \quad (7.163)$$

$$+ \delta(1-\delta)(\ln x_1)^2 \ln x_2 + \delta(1-\delta) \ln x_1 (\ln x_2)^2 + (1-\delta)^2 (\ln x_2)^3 \\ + \frac{2}{3} \left( \delta^2 (\ln x_1)^2 + 2\delta(1-\delta) \ln x_1 \ln x_2 + (1-\delta)^2 (\ln x_2)^2 \right) \\ (-\delta \ln x_1 - (1-\delta) \ln x_2)$$

$$= \left( \delta^2 - \frac{1}{3}\delta \right) (\ln x_1)^3 + \delta(1-\delta)(\ln x_1)^2 \ln x_2 \quad (7.164)$$

$$+ \delta(1-\delta) \ln x_1 (\ln x_2)^2 + \left( (1-\delta)^2 - \frac{1}{3}(1-\delta) \right) (\ln x_2)^3 \\ + \frac{2}{3} \left( \delta^2 (\ln x_1)^2 + 2\delta(1-\delta) \ln x_1 \ln x_2 + (1-\delta)^2 (\ln x_2)^2 \right) \\ (-\delta \ln x_1 - (1-\delta) \ln x_2)$$

$$= \left( \delta^2 - \frac{1}{3}\delta \right) (\ln x_1)^3 + \delta(1-\delta)(\ln x_1)^2 \ln x_2 \quad (7.165)$$

$$+ \delta(1-\delta) \ln x_1 (\ln x_2)^2 + \left( (1-\delta)^2 - \frac{1}{3}(1-\delta) \right) (\ln x_2)^3 \\ - \frac{2}{3} \left( \delta^3 (\ln x_1)^3 + \delta^2(1-\delta)(\ln x_1)^2 \ln x_2 + 2\delta^2(1-\delta)(\ln x_1)^2 \ln x_2 \right. \\ \left. + 2\delta(1-\delta)^2 \ln x_1 (\ln x_2)^2 + \delta(1-\delta)^2 \ln x_1 (\ln x_2)^2 + (1-\delta)^3 (\ln x_2)^3 \right)$$

$$= \left( \delta^2 - \frac{1}{3}\delta \right) (\ln x_1)^3 + \delta(1-\delta)(\ln x_1)^2 \ln x_2 \quad (7.166)$$

$$+ \delta(1-\delta) \ln x_1 (\ln x_2)^2 + \left( (1-\delta)^2 - \frac{1}{3}(1-\delta) \right) (\ln x_2)^3 \\ - \frac{2}{3} \delta^3 (\ln x_1)^3 - 2\delta^2(1-\delta)(\ln x_1)^2 \ln x_2 - 2\delta(1-\delta)^2 \ln x_1 (\ln x_2)^2 \\ - \frac{2}{3} (1-\delta)^3 (\ln x_2)^3$$

$$= \left( \delta^2 - \frac{1}{3}\delta - \frac{2}{3}\delta^3 \right) (\ln x_1)^3 + \left( \delta(1-\delta) - 2\delta^2(1-\delta) \right) (\ln x_1)^2 \ln x_2 \quad (7.167)$$

$$+ \left( \delta(1-\delta - 2\delta(1-\delta)^2) \right) \ln x_1 (\ln x_2)^2 \\ + \left( (1-\delta)^2 - \frac{1}{3}(1-\delta) - \frac{2}{3}(1-\delta)^3 \right) (\ln x_2)^3$$

$$= \left( \delta - \frac{1}{3} - \frac{2}{3}\delta^2 \right) \delta (\ln x_1)^3 + (1-2\delta)\delta(1-\delta)(\ln x_1)^2 \ln x_2 \quad (7.168)$$

$$+ (1-2(1-\delta))\delta(1-\delta) \ln x_1 (\ln x_2)^2 \\ + \left( (1-\delta) - \frac{1}{3} - \frac{2}{3}(1-\delta)^2 \right) (1-\delta) (\ln x_2)^3$$

$$= \left( -\frac{1}{3} + \frac{2}{3}\delta \right) \delta(1-\delta)(\ln x_1)^3 + (1-2\delta)\delta(1-\delta)(\ln x_1)^2 \ln x_2 \quad (7.169)$$

$$+ (2\delta-1)\delta(1-\delta) \ln x_1 (\ln x_2)^2 \\ + \left( 1-\delta - \frac{1}{3} - \frac{2}{3} + \frac{4}{3}\delta - \frac{2}{3}\delta^2 \right) (1-\delta) (\ln x_2)^3$$

$$= \left(-\frac{1}{3} + \frac{2}{3}\delta\right) \delta(1-\delta)(\ln x_1)^3 + (1-2\delta)\delta(1-\delta)(\ln x_1)^2 \ln x_2 \quad (7.170)$$

$$+ (2\delta-1)\delta(1-\delta)\ln x_1(\ln x_2)^2 + \left(\frac{1}{3} - \frac{2}{3}\delta\right) \delta(1-\delta)(\ln x_2)^3$$

$$= -\frac{1}{3}(1-2\delta)\delta(1-\delta)(\ln x_1)^3 + (1-2\delta)\delta(1-\delta)(\ln x_1)^2 \ln x_2 \quad (7.171)$$

$$- (1-2\delta)\delta(1-\delta)\ln x_1(\ln x_2)^2 + \frac{1}{3}(1-2\delta)\delta(1-\delta)(\ln x_2)^3$$

$$= -\frac{1}{3}(1-2\delta)\delta(1-\delta) \quad (7.172)$$

$$\left((\ln x_1)^3 + 3(\ln x_1)^2 \ln x_2 + 3\ln x_1(\ln x_2)^2 - (\ln x_2)^3\right)$$

$$= -\frac{1}{3}(1-2\delta)\delta(1-\delta)(\ln x_1 - \ln x_2)^3 \quad (7.173)$$

so that we can calculate the limit of  $f_\nu(\rho)$  and  $f'_\nu(\rho)$  for  $\rho \rightarrow 0$  by

$$f_\nu(0) = \lim_{\rho \rightarrow 0} f_\nu(\rho) \quad (7.174)$$

$$= \lim_{\rho \rightarrow 0} (g_\nu(\rho) \exp(f(\rho))) \quad (7.175)$$

$$= \lim_{\rho \rightarrow 0} g_\nu(\rho) \lim_{\rho \rightarrow 0} \exp(f(\rho)) \quad (7.176)$$

$$= \lim_{\rho \rightarrow 0} g_\nu(\rho) \exp\left(\lim_{\rho \rightarrow 0} f(\rho)\right) \quad (7.177)$$

$$= g_\nu(0) \exp(f(0)) \quad (7.178)$$

$$= (-\delta \ln x_1 - (1-\delta) \ln x_2) \exp(\nu(\delta \ln x_1 + (1-\delta) \ln x_2)) \quad (7.179)$$

$$= -(\delta \ln x_1 + (1-\delta) \ln x_2) x_1^{\nu\delta} x_2^{\nu(1-\delta)} \quad (7.180)$$

$$f'_\nu(0) = \lim_{\rho \rightarrow 0} f'_\nu(\rho) \quad (7.181)$$

$$= \lim_{\rho \rightarrow 0} (g'_\nu(\rho) \exp(f(\rho)) + g_\nu(\rho) \exp(f(\rho)) f'(\rho)) \quad (7.182)$$

$$= \lim_{\rho \rightarrow 0} g'_\nu(\rho) \exp\left(\lim_{\rho \rightarrow 0} f(\rho)\right) + \lim_{\rho \rightarrow 0} g_\nu(\rho) \exp\left(\lim_{\rho \rightarrow 0} f(\rho)\right) \lim_{\rho \rightarrow 0} f'(\rho) \quad (7.183)$$

$$= g'_\nu(0) \exp(f(0)) + g_\nu(0) \exp(f(0)) f'(0) \quad (7.184)$$

$$= \exp(f(0)) (g'_\nu(0) + g_\nu(0) f'(0)) \quad (7.185)$$

$$= \exp(\nu(\delta \ln x_1 + (1-\delta) \ln x_2)) \left( \frac{\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2 \right) \quad (7.186)$$

$$+ (-\delta \ln x_1 - (1-\delta) \ln x_2) \left( -\frac{\nu\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2 \right)$$

$$= x_1^{\nu\delta} x_2^{\nu(1-\delta)} \frac{\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2 (1 + \nu(\delta \ln x_1 + (1-\delta) \ln x_2)) \quad (7.187)$$

and approximate  $\partial y/\partial \nu$  by

$$\frac{\partial y}{\partial \nu} \approx -\gamma (f_\nu(0) + \rho f'_\nu(0)) \quad (7.188)$$

$$\begin{aligned} &= \gamma (\delta \ln x_1 + (1 - \delta) \ln x_2) x_1^{\nu\delta} x_2^{\nu(1-\delta)} \\ &\quad - \gamma \rho x_1^{\nu\delta} x_2^{\nu(1-\delta)} \frac{\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2 \\ &\quad (1 + \nu (\delta \ln x_1 + (1 - \delta) \ln x_2)) \end{aligned} \quad (7.189)$$

$$\begin{aligned} &= \gamma x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( \delta \ln x_1 + (1 - \delta) \ln x_2 \right. \\ &\quad \left. - \frac{\rho\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2 (1 + \nu (\delta \ln x_1 + (1 - \delta) \ln x_2)) \right) \end{aligned} \quad (7.190)$$

#### 7.A.2.4 Derivatives with respect to Rho

$$\begin{aligned} \frac{\partial y}{\partial \rho} &= \frac{\gamma\nu}{\rho^2} (\delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho})^{-\frac{\nu}{\rho}} \ln (\delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho}) \\ &\quad + \frac{\gamma\nu}{\rho} (\delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho})^{-\left(\frac{\nu}{\rho}+1\right)} (\delta x_1^{-\rho} \ln x_1 + (1 - \delta) x_2^{-\rho} \ln x_2) \end{aligned} \quad (7.191)$$

$$\begin{aligned} &= \gamma\nu \left( \frac{1}{\rho^2} (\delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho})^{-\frac{\nu}{\rho}} \ln (\delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho}) \right. \\ &\quad \left. + \frac{1}{\rho} (\delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho})^{-\left(\frac{\nu}{\rho}+1\right)} (\delta x_1^{-\rho} \ln x_1 + (1 - \delta) x_2^{-\rho} \ln x_2) \right) \end{aligned} \quad (7.192)$$

Now we define the function  $f_\rho(\rho)$

$$\begin{aligned} f_\rho(\rho) &= \frac{1}{\rho^2} (\delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho})^{-\frac{\nu}{\rho}} \ln (\delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho}) \\ &\quad + \frac{1}{\rho} (\delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho})^{-\left(\frac{\nu}{\rho}+1\right)} (\delta x_1^{-\rho} \ln x_1 + (1 - \delta) x_2^{-\rho} \ln x_2) \end{aligned} \quad (7.193)$$

so that we can approximate  $\partial y/\partial \rho$  by using the first-order Taylor series approximation of  $f_\rho(\rho)$ :

$$\frac{\partial y}{\partial \rho} = \gamma\nu f_\rho(\rho) \quad (7.194)$$

$$\approx \gamma\nu (f_\rho(0) + \rho f'_\rho(0)) \quad (7.195)$$

We define the helper function  $g_\rho(\rho)$

$$g_\rho(\rho) = \delta x_1^{-\rho} \ln x_1 + (1 - \delta) x_2^{-\rho} \ln x_2 \quad (7.196)$$

with first and second derivative

$$g'_\rho(\rho) = -\delta x_1^{-\rho} (\ln x_1)^2 - (1 - \delta) x_2^{-\rho} (\ln x_2)^2 \quad (7.197)$$

$$g''_\rho(\rho) = \delta x_1^{-\rho} (\ln x_1)^3 + (1 - \delta) x_2^{-\rho} (\ln x_2)^3 \quad (7.198)$$

and use the functions  $g(\rho)$  and  $g_\nu(\rho)$  all defined above so that

$$f_\rho(\rho) = \frac{1}{\rho^2} \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}} \ln \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right) \quad (7.199)$$

$$+ \frac{1}{\rho} \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\left(\frac{\nu}{\rho} + 1\right)} \left( \delta x_1^{-\rho} \ln x_1 + (1 - \delta) x_2^{-\rho} \ln x_2 \right)$$

$$= \frac{1}{\rho^2} \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}} \ln \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right) \quad (7.200)$$

$$+ \frac{1}{\rho} \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}} \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-1} \left( \delta x_1^{-\rho} \ln x_1 + (1 - \delta) x_2^{-\rho} \ln x_2 \right)$$

$$= \frac{1}{\rho} \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-\frac{\nu}{\rho}} \left( \frac{1}{\rho} \ln \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right) \right) \quad (7.201)$$

$$+ \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-1} \left( \delta x_1^{-\rho} \ln x_1 + (1 - \delta) x_2^{-\rho} \ln x_2 \right)$$

$$= \frac{1}{\rho} \exp \left( -\frac{\nu}{\rho} \ln \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right) \right) \left( \frac{1}{\rho} \ln \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right) \right) \quad (7.202)$$

$$+ \left( \delta x_1^{-\rho} + (1 - \delta) x_2^{-\rho} \right)^{-1} \left( \delta x_1^{-\rho} \ln x_1 + (1 - \delta) x_2^{-\rho} \ln x_2 \right)$$

$$= \frac{\exp(-\nu g_\nu(\rho)) \left( g_\nu(\rho) + g(\rho)^{-1} g_\rho(\rho) \right)}{\rho} \quad (7.203)$$

and we can calculate its first derivative

$$f'_\rho(\rho) = \frac{-\rho \nu \exp(-\nu g_\nu(\rho)) g'_\nu(\rho) \left( g_\nu(\rho) + g(\rho)^{-1} g_\rho(\rho) \right)}{\rho^2} \quad (7.204)$$

$$+ \frac{\rho \exp(-\nu g_\nu(\rho)) \left( g'_\nu(\rho) - g(\rho)^{-2} g'(\rho) g_\rho(\rho) + g(\rho)^{-1} g'_\rho(\rho) \right)}{\rho^2}$$

$$- \frac{\exp(-\nu g_\nu(\rho)) \left( g_\nu(\rho) + g(\rho)^{-1} g_\rho(\rho) \right)}{\rho^2}$$

$$= \frac{\exp(-\nu g_\nu(\rho)) \rho \left( -\nu g'_\nu(\rho) g_\nu(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \right)}{\rho^2} \quad (7.205)$$

$$+ \frac{\exp(-\nu g_\nu(\rho)) \rho \left( g'_\nu(\rho) - g(\rho)^{-2} g'(\rho) g_\rho(\rho) + g(\rho)^{-1} g'_\rho(\rho) \right)}{\rho^2}$$

$$\begin{aligned} & \frac{\exp(-\nu g_\nu(\rho)) \left( g_\nu(\rho) + g(\rho)^{-1} g_\rho(\rho) \right)}{\rho^2} \\ = & \frac{\exp(-\nu g_\nu(\rho))}{\rho^2} \left( \rho \left( -\nu g'_\nu(\rho) g_\nu(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \right) \right. \end{aligned} \quad (7.206)$$

$$\begin{aligned} & \left. + g'_\nu(\rho) - g(\rho)^{-2} g'(\rho) g_\rho(\rho) + g(\rho)^{-1} g'_\rho(\rho) \right) \\ & - g_\nu(\rho) - g(\rho)^{-1} g_\rho(\rho) \\ = & \frac{\exp(-\nu g_\nu(\rho))}{\rho^2} \left( -\nu \rho g'_\nu(\rho) g_\nu(\rho) - \nu \rho g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \right) \end{aligned} \quad (7.207)$$

$$\begin{aligned} & + \rho g'_\nu(\rho) - \rho g(\rho)^{-2} g'(\rho) g_\rho(\rho) + \rho g(\rho)^{-1} g'_\rho(\rho) \\ & - g_\nu(\rho) - g(\rho)^{-1} g_\rho(\rho) \end{aligned}$$

Now we can calculate the limits of  $g_\rho(\rho)$ ,  $g'_\rho(\rho)$ , and  $g''_\rho(\rho)$  for  $\rho \rightarrow 0$  by

$$g_\rho(0) = \lim_{\rho \rightarrow 0} g_\rho(\rho) \quad (7.208)$$

$$= \lim_{\rho \rightarrow 0} \left( \delta x_1^{-\rho} \ln x_1 + (1 - \delta) x_2^{-\rho} \ln x_2 \right) \quad (7.209)$$

$$= \delta \ln x_1 \lim_{\rho \rightarrow 0} x_1^{-\rho} + (1 - \delta) \ln x_2 \lim_{\rho \rightarrow 0} x_2^{-\rho} \quad (7.210)$$

$$= \delta \ln x_1 + (1 - \delta) \ln x_2 \quad (7.211)$$

$$g'_\rho(0) = \lim_{\rho \rightarrow 0} g'_\rho(\rho) \quad (7.212)$$

$$= \lim_{\rho \rightarrow 0} \left( -\delta x_1^{-\rho} (\ln x_1)^2 - (1 - \delta) x_2^{-\rho} (\ln x_2)^2 \right) \quad (7.213)$$

$$= -\delta (\ln x_1)^2 \lim_{\rho \rightarrow 0} x_1^{-\rho} - (1 - \delta) (\ln x_2)^2 \lim_{\rho \rightarrow 0} x_2^{-\rho} \quad (7.214)$$

$$= -\delta (\ln x_1)^2 - (1 - \delta) (\ln x_2)^2 \quad (7.215)$$

$$g''_\rho(0) = \lim_{\rho \rightarrow 0} g''_\rho(\rho) \quad (7.216)$$

$$= \lim_{\rho \rightarrow 0} \left( \delta x_1^{-\rho} (\ln x_1)^3 + (1 - \delta) x_2^{-\rho} (\ln x_2)^3 \right) \quad (7.217)$$

$$= \delta (\ln x_1)^3 \lim_{\rho \rightarrow 0} x_1^{-\rho} + (1 - \delta) (\ln x_2)^3 \lim_{\rho \rightarrow 0} x_2^{-\rho} \quad (7.218)$$

$$= \delta (\ln x_1)^3 + (1 - \delta) (\ln x_2)^3 \quad (7.219)$$

so that we can calculate the limit of  $f_\rho(\rho)$  for  $\rho \rightarrow 0$  by

$$f_\rho(0) = \lim_{\rho \rightarrow 0} f_\rho(\rho) \quad (7.220)$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{\exp(-\nu g_\nu(\rho)) (g_\nu(\rho) + g(\rho)^{-1} g_\rho(\rho))}{\rho} \right) \quad (7.221)$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{-\nu \exp(-\nu g_\nu(\rho)) g'_\nu(\rho) (g_\nu(\rho) + g(\rho)^{-1} g_\rho(\rho))}{1} \right. \\ \left. + \frac{\exp(-\nu g_\nu(\rho)) (g'_\nu(\rho) - g(\rho)^{-2} g'(\rho) g_\rho(\rho) + g(\rho)^{-1} g'_\rho(\rho))}{1} \right) \quad (7.222)$$

$$= -\nu \exp(-\nu g_\nu(0)) g'_\nu(0) (g_\nu(0) + g(0)^{-1} g_\rho(0)) \\ + \exp(-\nu g_\nu(0)) (g'_\nu(0) - g(0)^{-2} g'(0) g_\rho(0) + g(0)^{-1} g'_\rho(0)) \quad (7.223)$$

$$= \exp(-\nu g_\nu(0)) (-\nu g'_\nu(0)) (g_\nu(0) + g(0)^{-1} g_\rho(0)) \\ + \exp(-\nu g_\nu(0)) (g'_\nu(0) - g(0)^{-2} g'(0) g_\rho(0) + g(0)^{-1} g'_\rho(0)) \quad (7.224)$$

$$= \exp(-\nu g_\nu(0)) ((-\nu g'_\nu(0)) (g_\nu(0) + g(0)^{-1} g_\rho(0)) \\ + g'_\nu(0) - g(0)^{-2} g'(0) g_\rho(0) + g(0)^{-1} g'_\rho(0)) \quad (7.225)$$

$$= \exp(-\nu(-\delta \ln x_1 - (1-\delta) \ln x_2)) \left( -\frac{\nu\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2 \right. \\ \left. (-\delta \ln x_1 - (1-\delta) \ln x_2 + \delta \ln x_1 + (1-\delta) \ln x_2) \right. \\ \left. + \frac{\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2 \right. \\ \left. - (-\delta \ln x_1 - (1-\delta) \ln x_2) (\delta \ln x_1 + (1-\delta) \ln x_2) \right. \\ \left. - \delta (\ln x_1)^2 - (1-\delta) (\ln x_2)^2 \right) \quad (7.226)$$

$$= x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( \frac{1}{2} \delta (1-\delta) (\ln x_1)^2 - \delta (1-\delta) \ln x_1 \ln x_2 \right. \\ \left. + \frac{1}{2} \delta (1-\delta) (\ln x_2)^2 + \delta^2 (\ln x_1)^2 + 2\delta (1-\delta) \ln x_1 \ln x_2 \right. \\ \left. + (1-\delta)^2 (\ln x_2)^2 - \delta (\ln x_1)^2 - (1-\delta) (\ln x_2)^2 \right) \quad (7.227)$$

$$= x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( \left( \frac{1}{2} \delta (1-\delta) + \delta^2 - \delta \right) (\ln x_1)^2 \right. \\ \left. + \left( \frac{1}{2} \delta (1-\delta) + (1-\delta)^2 - (1-\delta) \right) \right. \\ \left. (\ln x_2)^2 + \delta (1-\delta) \ln x_1 \ln x_2 \right) \quad (7.228)$$

$$= x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( \left( \frac{1}{2} \delta - \frac{1}{2} \delta^2 + \delta^2 - \delta \right) (\ln x_1)^2 \right. \\ \left. + \left( \frac{1}{2} \delta - \frac{1}{2} \delta^2 + 1 - 2\delta + \delta^2 - 1 + \delta \right) (\ln x_2)^2 \right. \\ \left. + \delta (1-\delta) \ln x_1 \ln x_2 \right) \quad (7.229)$$

$$= x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( \left( -\frac{1}{2} \delta + \frac{1}{2} \delta^2 \right) (\ln x_1)^2 \right. \\ \left. + \left( \frac{1}{2} \delta - \frac{1}{2} \delta^2 \right) (\ln x_2)^2 + \delta (1-\delta) \ln x_1 \ln x_2 \right) \quad (7.230)$$



$$\begin{aligned}
 & + \left( -\frac{1}{2}\delta + \frac{1}{2}\delta^2 \right) (\ln x_2)^2 + \delta(1-\delta) \ln x_1 \ln x_2 \Big) \\
 = & x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( -\frac{1}{2}\delta(1-\delta) (\ln x_1)^2 \right. \\
 & \left. -\frac{1}{2}\delta(1-\delta) (\ln x_2)^2 + \delta(1-\delta) \ln x_1 \ln x_2 \right) \tag{7.231}
 \end{aligned}$$

$$\begin{aligned}
 = & -\frac{1}{2}\delta(1-\delta) x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( (\ln x_1)^2 - 2 \ln x_1 \ln x_2 + (\ln x_2)^2 \right) \tag{7.232}
 \end{aligned}$$

$$= -\frac{1}{2}\delta(1-\delta) x_1^{\nu\delta} x_2^{\nu(1-\delta)} (\ln x_1 - \ln x_2)^2 \tag{7.233}$$

Before we can apply de l'Hospital's rule to  $\lim_{\rho \rightarrow 0} f'_\rho(\rho)$ , we have to check whether the numerator also converges to zero. We do this by defining a helper function  $h_\rho(\rho)$ , where the numerator converges to zero if  $h_\rho(\rho)$  converges to zero for  $\rho \rightarrow 0$

$$h_\rho(\rho) = -g_\nu(\rho) - g(\rho)^{-1} g_\rho(\rho) \tag{7.234}$$

$$h_\rho(0) = \lim_{\rho \rightarrow 0} h_\rho(\rho) \tag{7.235}$$

$$= \lim_{\rho \rightarrow 0} \left( -g_\nu(\rho) - g(\rho)^{-1} g_\rho(\rho) \right) \tag{7.236}$$

$$= -g_\nu(0) - g(0)^{-1} g_\rho(0) \tag{7.237}$$

$$= -(-\delta \ln x_1 - (1-\delta) \ln x_2) - (\delta \ln x_1 + (1-\delta) \ln x_2) \tag{7.238}$$

$$= 0 \tag{7.239}$$

As both the numerator and the denominator converge to zero, we can calculate  $\lim_{\rho \rightarrow 0} f'_\rho(\rho)$  by using de l'Hospital's rule.

$$f'_\rho(0) = \lim_{\rho \rightarrow 0} f'_\rho(\rho) \tag{7.240}$$

$$= \lim_{\rho \rightarrow 0} \left( \frac{\exp(-\nu g_\nu(\rho))}{\rho^2} (-\nu \rho g'_\nu(\rho) g_\nu(\rho) \right) \tag{7.241}$$

$$\begin{aligned}
 & -\nu \rho g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) + \rho g'_\nu(\rho) - \rho g(\rho)^{-2} g'(\rho) g_\rho(\rho) \\
 & + \rho g(\rho)^{-1} g'_\rho(\rho) - g_\nu(\rho) - g(\rho)^{-1} g_\rho(\rho) \Big)
 \end{aligned}$$

$$= \lim_{\rho \rightarrow 0} (\exp(-\nu g_\nu(\rho))) \lim_{\rho \rightarrow 0} \left( \frac{1}{\rho^2} (-\nu \rho g'_\nu(\rho) g_\nu(\rho) \right) \tag{7.242}$$

$$\begin{aligned}
 & -\nu \rho g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) + \rho g'_\nu(\rho) - \rho g(\rho)^{-2} g'(\rho) g_\rho(\rho) \\
 & + \rho g(\rho)^{-1} g'_\rho(\rho) - g_\nu(\rho) - g(\rho)^{-1} g_\rho(\rho) \Big)
 \end{aligned}$$

$$= \lim_{\rho \rightarrow 0} (\exp(-\nu g_\nu(\rho))) \lim_{\rho \rightarrow 0} \left( \frac{1}{2\rho} (-\nu g'_\nu(\rho) g_\nu(\rho) - \nu \rho g''_\nu(\rho) g_\nu(\rho) \right) \quad (7.243)$$

$$\begin{aligned} & -\nu \rho g'_\nu(\rho) g'_\nu(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) - \nu \rho g''_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \\ & + \nu \rho g'_\nu(\rho) g(\rho)^{-2} g'(\rho) g_\rho(\rho) - \nu \rho g'_\nu(\rho) g(\rho)^{-1} g'_\rho(\rho) + g'_\nu(\rho) \\ & + \rho g''_\nu(\rho) - g(\rho)^{-2} g'(\rho) g_\rho(\rho) + 2\rho g(\rho)^{-3} (g'(\rho))^2 g_\rho(\rho) \\ & - \rho g(\rho)^{-2} g''(\rho) g_\rho(\rho) - \rho g(\rho)^{-2} g'(\rho) g'_\rho(\rho) + g(\rho)^{-1} g'_\rho(\rho) \\ & - \rho g(\rho)^{-2} g'(\rho) g'_\rho(\rho) + \rho g(\rho)^{-1} g''_\rho(\rho) \\ & - g'_\nu(\rho) + g(\rho)^{-2} g'(\rho) g_\rho(\rho) - g(\rho)^{-1} g'_\rho(\rho) \end{aligned}$$

$$= \frac{1}{2} \lim_{\rho \rightarrow 0} (\exp(-\nu g_\nu(\rho))) \lim_{\rho \rightarrow 0} \left( \frac{1}{\rho} (-\nu g'_\nu(\rho) g_\nu(\rho) - \nu \rho g''_\nu(\rho) g_\nu(\rho) \right) \quad (7.244)$$

$$\begin{aligned} & -\nu \rho g'_\nu(\rho) g'_\nu(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) - \nu \rho g''_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \\ & + \nu \rho g'_\nu(\rho) g(\rho)^{-2} g'(\rho) g_\rho(\rho) - \nu \rho g'_\nu(\rho) g(\rho)^{-1} g'_\rho(\rho) + \rho g''_\nu(\rho) \\ & + 2\rho g(\rho)^{-3} (g'(\rho))^2 g_\rho(\rho) - \rho g(\rho)^{-2} g''(\rho) g_\rho(\rho) \\ & - 2\rho g(\rho)^{-2} g'(\rho) g'_\rho(\rho) + \rho g(\rho)^{-1} g''_\rho(\rho) \end{aligned}$$

$$= \frac{1}{2} \lim_{\rho \rightarrow 0} (\exp(-\nu g_\nu(\rho))) \quad (7.245)$$

$$\begin{aligned} & \lim_{\rho \rightarrow 0} \left( \frac{1}{\rho} (-\nu g'_\nu(\rho) g_\nu(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho)) \right. \\ & - \nu g''_\nu(\rho) g_\nu(\rho) - \nu g'_\nu(\rho) g'_\nu(\rho) - \nu g''_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \\ & + \nu g'_\nu(\rho) g(\rho)^{-2} g'(\rho) g_\rho(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g'_\rho(\rho) \\ & + g''_\nu(\rho) + 2g(\rho)^{-3} (g'(\rho))^2 g_\rho(\rho) - g(\rho)^{-2} g''(\rho) g_\rho(\rho) \\ & \left. - 2g(\rho)^{-2} g'(\rho) g'_\rho(\rho) + g(\rho)^{-1} g''_\rho(\rho) \right) \end{aligned}$$

$$= \frac{1}{2} \lim_{\rho \rightarrow 0} (\exp(-\nu g_\nu(\rho))) \quad (7.246)$$

$$\begin{aligned} & \left( \lim_{\rho \rightarrow 0} \left( \frac{1}{\rho} (-\nu g'_\nu(\rho) g_\nu(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho)) \right) \right. \\ & + \lim_{\rho \rightarrow 0} \left( -\nu g''_\nu(\rho) g_\nu(\rho) - \nu g'_\nu(\rho) g'_\nu(\rho) - \nu g''_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \right. \\ & + \nu g'_\nu(\rho) g(\rho)^{-2} g'(\rho) g_\rho(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g'_\rho(\rho) + g''_\nu(\rho) \\ & + 2g(\rho)^{-3} (g'(\rho))^2 g_\rho(\rho) - g(\rho)^{-2} g''(\rho) g_\rho(\rho) \\ & \left. \left. - 2g(\rho)^{-2} g'(\rho) g'_\rho(\rho) + g(\rho)^{-1} g''_\rho(\rho) \right) \right) \end{aligned}$$

Before we can apply de l'Hospital's rule again, we have to check if the numerator also converges to zero. We do this by defining a helper function  $k_\rho(\rho)$ , where the numerator converges

to zero if  $k_\rho(\rho)$  converges to zero for  $\rho \rightarrow 0$

$$k_\rho(\rho) = -\nu g'_\nu(\rho) g_\nu(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \quad (7.247)$$

$$k_\rho(0) = \lim_{\rho \rightarrow 0} k_\rho(\rho) \quad (7.248)$$

$$= \lim_{\rho \rightarrow 0} \left( -\nu g'_\nu(\rho) g_\nu(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \right) \quad (7.249)$$

$$= -\nu g'_\nu(0) g_\nu(0) - \nu g'_\nu(0) g(0)^{-1} g_\rho(0) \quad (7.250)$$

$$= -\frac{\nu\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2 (-\delta \ln x_1 - (1-\delta) \ln x_2) \quad (7.251)$$

$$- \frac{\nu\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2 (\delta \ln x_1 + (1-\delta) \ln x_2) \\ = 0 \quad (7.252)$$

As both the numerator and the denominator converge to zero, we can apply de l'Hospital's rule.

$$\lim_{\rho \rightarrow 0} \frac{k_\rho(\rho)}{\rho} = \lim_{\rho \rightarrow 0} k_\rho(\rho) \quad (7.253)$$

$$= \lim_{\rho \rightarrow 0} \left( -\nu g''_\nu(\rho) g_\nu(\rho) - \nu (g'_\nu(\rho))^2 - \nu g''_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \right. \\ \left. + \nu g'_\nu(\rho) g(\rho)^{-2} g'(\rho) g_\rho(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g'_\rho(\rho) \right) \quad (7.254)$$

and hence,

$$f'_\rho(0) = \frac{1}{2} \lim_{\rho \rightarrow 0} (\exp(-\nu g_\nu(\rho))) \quad (7.255)$$

$$\left( \lim_{\rho \rightarrow 0} \left( \frac{1}{\rho} \left( -\nu g'_\nu(\rho) g_\nu(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \right) \right) \right. \\ \left. + \lim_{\rho \rightarrow 0} \left( -\nu g''_\nu(\rho) g_\nu(\rho) - \nu g'_\nu(\rho) g'_\nu(\rho) - \nu g''_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \right. \right. \\ \left. + \nu g'_\nu(\rho) g(\rho)^{-2} g'(\rho) g_\rho(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g'_\rho(\rho) \right. \\ \left. + g''_\nu(\rho) + 2g(\rho)^{-3} (g'(\rho))^2 g_\rho(\rho) - g(\rho)^{-2} g''(\rho) g_\rho(\rho) \right. \\ \left. - 2g(\rho)^{-2} g'(\rho) g'_\rho(\rho) + g(\rho)^{-1} g''_\rho(\rho) \right) \\ = \frac{1}{2} \lim_{\rho \rightarrow 0} (\exp(-\nu g_\nu(\rho))) \quad (7.256) \\ \left( \lim_{\rho \rightarrow 0} \left( -\nu g''_\nu(\rho) g_\nu(\rho) - \nu (g'_\nu(\rho))^2 - \nu g''_\nu(\rho) g(\rho)^{-1} g_\rho(\rho) \right. \right. \\ \left. \left. + \nu g'_\nu(\rho) g(\rho)^{-2} g'(\rho) g_\rho(\rho) - \nu g'_\nu(\rho) g(\rho)^{-1} g'_\rho(\rho) \right) \right)$$

$$\begin{aligned}
 & + \lim_{\rho \rightarrow 0} \left( -\nu g''_{\nu}(\rho) g_{\nu}(\rho) - \nu g'_{\nu}(\rho) g'_{\nu}(\rho) - \nu g''_{\nu}(\rho) g(\rho)^{-1} g_{\rho}(\rho) \right. \\
 & + \nu g'_{\nu}(\rho) g(\rho)^{-2} g'(\rho) g_{\rho}(\rho) - \nu g'_{\nu}(\rho) g(\rho)^{-1} g'_{\rho}(\rho) + g''_{\nu}(\rho) \\
 & + 2g(\rho)^{-3} (g'(\rho))^2 g_{\rho}(\rho) - g(\rho)^{-2} g''(\rho) g_{\rho}(\rho) \\
 & \left. - 2g(\rho)^{-2} g'(\rho) g'_{\rho}(\rho) + g(\rho)^{-1} g''(\rho) \right)
 \end{aligned}$$

$$= \frac{1}{2} \exp(-\nu g_{\nu}(0)) \left( -\nu g''_{\nu}(0) g_{\nu}(0) - \nu (g'_{\nu}(0))^2 \right) \quad (7.257)$$

$$\begin{aligned}
 & -\nu g''_{\nu}(0) g(0)^{-1} g_{\rho}(0) + \nu g'_{\nu}(0) g(0)^{-2} g'(\rho) g_{\rho}(0) \\
 & -\nu g'_{\nu}(0) g(0)^{-1} g'_{\rho}(0) - \nu g''_{\nu}(0) g_{\nu}(0) - \nu g'_{\nu}(0) g'_{\nu}(0) \\
 & -\nu g''_{\nu}(0) g(0)^{-1} g_{\rho}(0) + \nu g'_{\nu}(0) g(0)^{-2} g'(\rho) g_{\rho}(0) \\
 & -\nu g'_{\nu}(0) g(0)^{-1} g'_{\rho}(0) + g''_{\nu}(0) + 2g(0)^{-3} (g'(0))^2 g_{\rho}(0) \\
 & -g(0)^{-2} g''(0) g_{\rho}(0) - 2g(0)^{-2} g'(0) g'_{\rho}(0) + g(0)^{-1} g''(0)
 \end{aligned}$$

$$= \frac{1}{2} \exp(-\nu g_{\nu}(0)) \left( -\nu g''_{\nu}(0) g_{\nu}(0) - \nu (g'_{\nu}(0))^2 - \nu g''_{\nu}(0) g_{\rho}(0) \right) \quad (7.258)$$

$$\begin{aligned}
 & + \nu g'_{\nu}(0) g'(0) g_{\rho}(0) - \nu g'_{\nu}(0) g'_{\rho}(0) \\
 & -\nu g''_{\nu}(0) g_{\nu}(0) - \nu g'_{\nu}(0) g'_{\nu}(0) - \nu g''_{\nu}(0) g_{\rho}(0) + \nu g'_{\nu}(0) g'(0) g_{\rho}(0) \\
 & -\nu g'_{\nu}(0) g'_{\rho}(0) + g''_{\nu}(0) + 2(g'(0))^2 g_{\rho}(0) - g''(0) g_{\rho}(0) \\
 & -2g'(0) g'_{\rho}(0) + g''(0)
 \end{aligned}$$

$$= \frac{1}{2} \exp(-\nu g_{\nu}(0)) \left( -2\nu g''_{\nu}(0) g_{\nu}(0) - 2\nu (g'_{\nu}(0))^2 - 2\nu g''_{\nu}(0) g_{\rho}(0) \right) \quad (7.259)$$

$$\begin{aligned}
 & + 2\nu g'_{\nu}(0) g'(0) g_{\rho}(0) - 2\nu g'_{\nu}(0) g'_{\rho}(0) \\
 & + g''_{\nu}(0) + 2(g'(0))^2 g_{\rho}(0) - g''(0) g_{\rho}(0) \\
 & -2g'(0) g'_{\rho}(0) + g''(0)
 \end{aligned}$$

$$= \frac{1}{2} \exp(-\nu g_{\nu}(0)) (g''_{\nu}(0) (-2\nu g_{\nu}(0) - 2\nu g_{\rho}(0) + 1)) \quad (7.260)$$

$$\begin{aligned}
 & + \nu g'_{\nu}(0) \left( -2g'_{\nu}(0) + 2g'(0) g_{\rho}(0) - 2g'_{\rho}(0) \right) \\
 & + 2(g'(0))^2 g_{\rho}(0) - g''(0) g_{\rho}(0) \\
 & -2g'(0) g'_{\rho}(0) + g''(0)
 \end{aligned}$$

$$= \frac{1}{2} \exp(-\nu (-\delta \ln x_1 - (1 - \delta) \ln x_2)) \quad (7.261)$$

$$\left( \left( -\frac{1}{3} (1 - 2\delta) \delta (1 - \delta) (\ln x_1 - \ln x_2)^3 \right) \right)$$

$$(-2\nu (-\delta \ln x_1 - (1 - \delta) \ln x_2) - 2\nu (\delta \ln x_1 + (1 - \delta) \ln x_2) + 1)$$

$$\begin{aligned}
 & +\nu \frac{\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2 \left( -2 \frac{\delta(1-\delta)}{2} (\ln x_1 - \ln x_2)^2 \right. \\
 & +2(-\delta \ln x_1 - (1-\delta) \ln x_2)(\delta \ln x_1 + (1-\delta) \ln x_2) \\
 & \left. -2(-\delta (\ln x_1)^2 - (1-\delta) (\ln x_2)^2) \right) \\
 & +2(-\delta \ln x_1 - (1-\delta) \ln x_2)^2 (\delta \ln x_1 + (1-\delta) \ln x_2) \\
 & -\left( \delta (\ln x_1)^2 + (1-\delta) (\ln x_2)^2 \right) (\delta \ln x_1 + (1-\delta) \ln x_2) \\
 & -2(-\delta \ln x_1 - (1-\delta) \ln x_2) \left( -\delta (\ln x_1)^2 - (1-\delta) (\ln x_2)^2 \right) \\
 & +\delta (\ln x_1)^3 + (1-\delta) (\ln x_2)^3) \\
 = & \frac{1}{2} x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( -\frac{1}{3} (1-2\delta) \delta (1-\delta) (\ln x_1 - \ln x_2)^3 \right. \tag{7.262} \\
 & (2\nu\delta \ln x_1 + 2\nu(1-\delta) \ln x_2 - 2\nu\delta \ln x_1 - 2\nu(1-\delta) \ln x_2 + 1) \\
 & +\frac{1}{2} \nu\delta (1-\delta) \left( (\ln x_1)^2 - 2\ln x_1 \ln x_2 + (\ln x_2)^2 \right) \\
 & \left( -\delta(1-\delta) (\ln x_1)^2 + 2\delta(1-\delta) \ln x_1 \ln x_2 - \delta(1-\delta) (\ln x_2)^2 \right. \\
 & -2\delta^2 (\ln x_1)^2 - 4\delta(1-\delta) \ln x_1 \ln x_2 - 2(1-\delta)^2 (\ln x_2)^2 \\
 & \left. +2\delta (\ln x_1)^2 + 2(1-\delta) (\ln x_2)^2 \right) \\
 & +2\delta^3 (\ln x_1)^3 + 6\delta^2(1-\delta) (\ln x_1)^2 \ln x_2 + 6\delta(1-\delta)^2 \ln x_1 (\ln x_2)^2 \\
 & +2(1-\delta)^3 (\ln x_2)^3 - \delta^2 (\ln x_1)^3 - \delta(1-\delta) (\ln x_1)^2 \ln x_2 \\
 & -\delta(1-\delta) \ln x_1 (\ln x_2)^2 - (1-\delta)^2 (\ln x_2)^3 - 2\delta^2 (\ln x_1)^3 \\
 & -2\delta(1-\delta) \ln x_1 (\ln x_2)^2 - 2\delta(1-\delta) (\ln x_1)^2 \ln x_2 - 2(1-\delta)^2 (\ln x_2)^3 \\
 & \left. +\delta (\ln x_1)^3 + (1-\delta) (\ln x_2)^3 \right) \\
 = & \frac{1}{2} x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( -\frac{1}{3} (1-2\delta) \delta (1-\delta) (\ln x_1 - \ln x_2)^3 \right. \tag{7.263} \\
 & +\left( \frac{1}{2} \nu\delta (1-\delta) (\ln x_1)^2 - \nu\delta (1-\delta) \ln x_1 \ln x_2 + \frac{1}{2} \nu\delta (1-\delta) (\ln x_2)^2 \right) \\
 & \left( (-\delta(1-\delta) - 2\delta^2 + 2\delta) (\ln x_1)^2 \right. \\
 & + (2\delta(1-\delta) - 4\delta(1-\delta)) \ln x_1 \ln x_2 \\
 & \left. + (-\delta(1-\delta) - 2(1-\delta)^2 + 2(1-\delta)) (\ln x_2)^2 \right) \\
 & + (2\delta^3 - \delta^2 - 2\delta^2 + \delta) (\ln x_1)^3 \\
 & + (6\delta^2(1-\delta) - \delta(1-\delta) - 2\delta(1-\delta)) (\ln x_1)^2 \ln x_2 \\
 & + (6\delta(1-\delta)^2 - \delta(1-\delta) - 2\delta(1-\delta)) \ln x_1 (\ln x_2)^2
 \end{aligned}$$

$$\begin{aligned}
 & + \left( 2(1-\delta)^3 - (1-\delta)^2 - 2(1-\delta)^2 + (1-\delta) \right) (\ln x_2)^3 \\
 = & \frac{1}{2} x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( -\frac{1}{3} (1-2\delta) \delta (1-\delta) (\ln x_1 - \ln x_2)^3 \right. \\
 & + \left( \frac{1}{2} \nu\delta (1-\delta) (\ln x_1)^2 - \nu\delta (1-\delta) \ln x_1 \ln x_2 + \frac{1}{2} \nu\delta (1-\delta) (\ln x_2)^2 \right) \\
 & \left( (-\delta + \delta^2 - 2\delta^2 + 2\delta) (\ln x_1)^2 \right. \\
 & - 2\delta (1-\delta) \ln x_1 \ln x_2 \\
 & + (-\delta + \delta^2 - 2 + 4\delta - 2\delta^2 + 2 - 2\delta) (\ln x_2)^2 \\
 & + (2\delta^3 - 3\delta^2 + \delta) (\ln x_1)^3 \\
 & + (6\delta^2 - 6\delta^3 - \delta + \delta^2 - 2\delta + 2\delta^2) (\ln x_1)^2 \ln x_2 \\
 & + (6\delta - 12\delta^2 + 6\delta^3 - \delta + \delta^2 - 2\delta + 2\delta^2) \ln x_1 (\ln x_2)^2 \\
 & \left. + (2 - 6\delta + 6\delta^2 - 2\delta^3 - 1 + 2\delta - \delta^2 - 2 + 4\delta - 2\delta^2 + 1 - \delta) (\ln x_2)^3 \right)
 \end{aligned} \tag{7.264}$$

$$\begin{aligned}
 = & \frac{1}{2} x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( -\frac{1}{3} (1-2\delta) \delta (1-\delta) (\ln x_1 - \ln x_2)^3 \right. \\
 & + \left( \frac{1}{2} \nu\delta (1-\delta) (\ln x_1)^2 - \nu\delta (1-\delta) \ln x_1 \ln x_2 + \frac{1}{2} \nu\delta (1-\delta) (\ln x_2)^2 \right) \\
 & \left( (-\delta^2 + \delta) (\ln x_1)^2 - 2\delta (1-\delta) \ln x_1 \ln x_2 + (-\delta^2 + \delta) (\ln x_2)^2 \right) \\
 & + (2\delta^3 - 3\delta^2 + \delta) (\ln x_1)^3 + (-6\delta^3 + 9\delta^2 - 3\delta) (\ln x_1)^2 \ln x_2 \\
 & + (6\delta^3 - 9\delta^2 + 3\delta) \ln x_1 (\ln x_2)^2 + (-2\delta^3 + 3\delta^2 - \delta) (\ln x_2)^3
 \end{aligned} \tag{7.265}$$

$$\begin{aligned}
 = & \frac{1}{2} x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( -\frac{1}{3} (1-2\delta) \delta (1-\delta) (\ln x_1)^3 \right. \\
 & + (1-2\delta) \delta (1-\delta) (\ln x_1)^2 \ln x_2 - (1-2\delta) \delta (1-\delta) \ln x_1 (\ln x_2)^2 \\
 & + \frac{1}{3} (1-2\delta) \delta (1-\delta) (\ln x_2)^3 \\
 & + \left( \frac{1}{2} \nu\delta (1-\delta) (\ln x_1)^2 - \nu\delta (1-\delta) \ln x_1 \ln x_2 + \frac{1}{2} \nu\delta (1-\delta) (\ln x_2)^2 \right) \\
 & \left( \delta (1-\delta) (\ln x_1)^2 - 2\delta (1-\delta) \ln x_1 \ln x_2 + \delta (1-\delta) (\ln x_2)^2 \right) \\
 & + \delta (1-\delta) (1-2\delta) (\ln x_1)^3 - 3\delta (1-\delta) (1-2\delta) (\ln x_1)^2 \ln x_2 \\
 & + 3\delta (1-\delta) (1-2\delta) \ln x_1 (\ln x_2)^2 - \delta (1-\delta) (1-2\delta) (\ln x_2)^3
 \end{aligned} \tag{7.266}$$

$$\begin{aligned}
 &= \frac{1}{2} x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( \frac{1}{2} \nu \delta^2 (1-\delta)^2 (\ln x_1)^4 - \nu \delta^2 (1-\delta)^2 (\ln x_1)^3 \ln x_2 \right. \\
 &\quad + \frac{1}{2} \nu \delta^2 (1-\delta)^2 (\ln x_1)^2 (\ln x_2)^2 - \nu \delta^2 (1-\delta)^2 (\ln x_1)^3 \ln x_2 \\
 &\quad + 2\nu \delta^2 (1-\delta)^2 (\ln x_1)^2 (\ln x_2)^2 - \nu \delta^2 (1-\delta)^2 \ln x_1 (\ln x_2)^3 \\
 &\quad + \frac{1}{2} \nu \delta^2 (1-\delta)^2 (\ln x_1)^2 (\ln x_2)^2 - \nu \delta^2 (1-\delta)^2 \ln x_1 (\ln x_2)^3 \\
 &\quad + \frac{1}{2} \nu \delta^2 (1-\delta)^2 (\ln x_2)^4 \\
 &\quad + \left( -\frac{1}{3} (1-2\delta) \delta (1-\delta) + \delta (1-\delta) (1-2\delta) \right) (\ln x_1)^3 \\
 &\quad + ((1-2\delta) \delta (1-\delta) - 3\delta (1-\delta) (1-2\delta)) (\ln x_1)^2 \ln x_2 \\
 &\quad + (- (1-2\delta) \delta (1-\delta) + 3\delta (1-\delta) (1-2\delta)) \ln x_1 (\ln x_2)^2 \\
 &\quad \left. + \left( \frac{1}{3} (1-2\delta) \delta (1-\delta) - \delta (1-\delta) (1-2\delta) \right) (\ln x_2)^3 \right) \quad (7.267)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( \frac{1}{2} \nu \delta^2 (1-\delta)^2 (\ln x_1)^4 - 2\nu \delta^2 (1-\delta)^2 (\ln x_1)^3 \ln x_2 \right. \\
 &\quad + 3\nu \delta^2 (1-\delta)^2 (\ln x_1)^2 (\ln x_2)^2 \\
 &\quad - 2\nu \delta^2 (1-\delta)^2 \ln x_1 (\ln x_2)^3 + \frac{1}{2} \nu \delta^2 (1-\delta)^2 (\ln x_2)^4 \\
 &\quad + \frac{2}{3} \delta (1-\delta) (1-2\delta) (\ln x_1)^3 - 2\delta (1-\delta) (1-2\delta) (\ln x_1)^2 \ln x_2 \\
 &\quad \left. + 2\delta (1-\delta) (1-2\delta) \ln x_1 (\ln x_2)^2 - \frac{2}{3} \delta (1-\delta) (1-2\delta) (\ln x_2)^3 \right) \quad (7.268)
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{2} x_1^{\nu\delta} x_2^{\nu(1-\delta)} \left( \frac{1}{2} \nu \delta^2 (1-\delta)^2 \left( (\ln x_1)^4 - 4 (\ln x_1)^3 \ln x_2 \right) \right. \\
 &\quad + 6 (\ln x_1)^2 (\ln x_2)^2 - 4 \ln x_1 (\ln x_2)^3 + (\ln x_2)^4 \\
 &\quad + \frac{2}{3} \delta (1-\delta) (1-2\delta) \\
 &\quad \left. \left( (\ln x_1)^3 - 3 (\ln x_1)^2 \ln x_2 + 3 \ln x_1 (\ln x_2)^2 - (\ln x_2)^3 \right) \right) \quad (7.269)
 \end{aligned}$$

$$\begin{aligned}
 &= \delta (1-\delta) x_1^{\nu\delta} x_2^{\nu(1-\delta)} \\
 &\quad \left( \frac{1}{3} (1-2\delta) (\ln x_1 - \ln x_2)^3 + \frac{1}{4} \nu \delta (1-\delta) (\ln x_1 - \ln x_2)^4 \right) \quad (7.270)
 \end{aligned}$$

Hence, we can approximate  $\partial y / \partial \rho$  by a second-order Taylor series approximation:

$$\frac{\partial y}{\partial \rho} \approx \gamma \nu \left( f_\rho(0) + \rho f'_\rho(0) \right) \quad (7.271)$$

$$= -\frac{1}{2}\gamma\nu\delta(1-\delta)x_1^{\nu\delta}x_2^{\nu(1-\delta)}(\ln x_1 - \ln x_2)^2 \quad (7.272)$$

$$+ \gamma\nu\rho\delta(1-\delta)x_1^{\nu\delta}x_2^{\nu(1-\delta)}\left(\frac{2}{3}(1-2\delta)(\ln x_1 - \ln x_2)^3 + \frac{1}{2}\nu\delta(1-\delta)(\ln x_1 - \ln x_2)^4\right)$$

$$= \gamma\nu\delta(1-\delta)x_1^{\nu\delta}x_2^{\nu(1-\delta)}\left(-\frac{1}{2}(\ln x_1 - \ln x_2)^2 \quad (7.273)$$

$$+ \frac{1}{3}\rho(1-2\delta)(\ln x_1 - \ln x_2)^3 + \frac{1}{4}\rho\nu\delta(1-\delta)(\ln x_1 - \ln x_2)^4\right)$$



---

## 7.B Script for Monte Carlo Simulation

---

```
# load the micEconCES package
library( micEconCES )

# seed for random number generation
set.seed( 1234 )

# number of replications
nRep <- 1000

# number of observations
nObs <- 100

# rho
rho <- 1/3

# variance of the error term
uStdDev <- 1.5

# create data set with explanatory variables
cesData <- data.frame( x1 = rchisq( nObs, 10 ), x2 = rchisq( nObs, 10 ) )

# names of explanatory variables
xxNames <- c( "x1", "x2" )

# variable returns to scale
vrs <- TRUE # FALSE #

# coefficients
cesCoef <- c( gamma = 1, delta = 0.6, rho = rho, nu = 1.1 )[ 1:( 3 + vrs ) ]

# calculate deterministic endogenous variable
cesData$yd <- cesCalc( xNames = xxNames, data = cesData, coef = cesCoef )

# estimation methods
allMethods <- c( "Kmenta", "nls", "LM", "CG", "Newton", "BFGS",
  "Nelder-Mead", "SANN", "DE", "L-BFGS-B", "PORT" )

# objects to store the results
estCoef <- array( NA,
  dim = c( nRep, length( cesCoef ), length( allMethods ) ),
  dimnames = list( 1:nRep, names( cesCoef ), allMethods ) )
```

```

convergence <- estCoef[ , 1, ]
rss <- estCoef[ , 1, ]
rSquared <- estCoef[ , 1, ]
iterations <- estCoef[ , 1, ]

## start the monte carlo experiment
for( i in 1:nRep ) {
  cat( i, ":", sep = " " )
  ptm <- proc.time()

  # adding noise to the endogenous variable
  repeat{
    cesData$ys <- cesData$yd + rnorm( nObs, sd = uStdDev )
    if( min( cesData$ys ) > 0 ) {
      break
    } else {
      cat( "#" )
    }
  }
}

# estimate the model using different estimation methods
for( method in allMethods ) {
  extraArgs <- list()
  if( method == "nls" ) {
    extraArgs <- list(
      control = nls.control( warnOnly = TRUE ) )
  } else if( method == "LM" ) {
    extraArgs <- list(
      control = nls.lm.control( maxiter = 250 ) )
  } else if( method == "Newton" ) {
    extraArgs <- list( iterlim = 250 )
  } else if( method %in% c( "BFGS", "L-BFGS-B" ) ) {
    extraArgs <- list( control = list( maxit = 250 ) )
  } else if( method == "CG" ) {
    extraArgs <- list(
      control = list( maxit = 1000, reltol = 1e-4, type = 2 ) )
  } else if( method == "SANN" ) {
    extraArgs <- list( control = list( maxit = 50000 ) )
  } else if( method == "DE" ) {
    extraArgs <- list(
      control = DEoptim.control( trace = FALSE, itermax = 1000 ) )
  }
  allArgs <- c( list( yName = "ys", xNames = xxNames, data = cesData,

```

```

        method = method, vrs = vrs ), extraArgs )
cesResult <- try( do.call( "cesEst", allArgs ) )
if( class( cesResult )[1] != "try-error" ) {
  # store the estimated coefficients
  estCoef[ i, , method ] <- coef( cesResult )
  # store if the estimation has converged
  if( !is.null( cesResult$convergence ) ) {
    convergence[ i, method ] <- cesResult$convergence
  }
  # sum of squared residuals
  rss[ i, method ] <- cesResult$rss
  # R-squared values
  rSquared[ i, method ] <- summary( cesResult )$r.squared
  # number of iterations
  if( !is.null( cesResult$iter ) ) {
    iterations[ i, method ] <- sum( cesResult$iter )
  }
}
}

ptmNew <- proc.time()
cat( ptmNew - ptm, "\n" )
ptm <- ptmNew
}

##### calculate summary results #####
# differences between the estimated and the true coefficients
diffCoef <- estCoef - aperm(
  array( cesCoef, dim = c( length( cesCoef ), length( allMethods ), nRep ) ),
  c( 3, 1, 2 ) )

# elasticities of substitution and difference between estimates and true value
estSigma <- 1 / ( 1 + estCoef[ ,"rho", ] )
diffSigma <- estSigma - 1 / ( 1 + rho )

# function to calculate summary results of the Monte Carlo simulation
# depending on the selection of replications
calcMcResults <- function( repSelect ) {

  result <- list()

  # biases of the estimated coefficients and elasticity of substitution
  result$bias <- colMeans( diffCoef[ repSelect, , ] )

```

```

# all.equal( bias, colMeans( estCoef ) - matrix( cesCoef, nrow = length( cesCoef ), ncol = length(
result$bias <- rbind( result$bias,
  sigma = colMeans( diffSigma[ repSelect, ] ) )

# median deviation of estimated coef. and elast. of subst. from their true values
result$devMed <- colMedians( diffCoef[ repSelect, , ] )
result$devMed <- rbind( result$devMed,
  sigma = colMedians( diffSigma[ repSelect, ] ) )

# root mean squared errors of the estimated coefficients and elasticity of substitution
result$rmse <- sqrt( colSums( diffCoef[ repSelect, , ]^2 ) / nRep )
result$rmse <- rbind( result$rmse,
  sigma = colSums( diffSigma[ repSelect, ]^2 ) / nRep )

# mean absolute deviations
result$mad <- colMeans( abs( diffCoef[ repSelect, , ] ) )
result$mad <- rbind( result$mad,
  sigma = colMeans( abs( diffSigma[ repSelect, ] ) ) )

# median absolute deviations
result$adMed <- colMedians( abs( diffCoef[ repSelect, , ] ) )
result$adMed <- rbind( result$adMed,
  sigma = colMedians( abs( diffSigma[ repSelect, ] ) ) )

# mean RSS
result$rssMean <- colMeans( rss[ repSelect, ] )

# mean R-squared values
result$rSquaredMean <- colMeans( rSquared[ repSelect, ] )

return( result )
}

# summary results of *all* replications
resultAll <- calcMcResults( 1:nRep )

# summary results of replications without errors (in any method)
resultNoErr <- calcMcResults( rowSums( is.na( rss ) ) == 0 )

# summary results of replications without errors or non-convergence (in any method)
resultConv <- calcMcResults(
  rowSums( is.na( rss ) | ( !convergence & !is.na( convergence ) ) ) == 0 )

```

```
##### create tables for the paper #####
# general results
tabGeneral <- data.frame( nNoConv =
  colSums( is.na( rss ) | ( !convergence & !is.na( convergence ) ) ) )
tabGeneral$nConv <-
  colSums( !is.na( rss ) & ( convergence | is.na( convergence ) ) )
tabGeneral$rssAll <- resultAll$rssMean
tabGeneral$rssConv <- resultConv$rssMean

##### write tables to disk #####
library( xtable )
# general results
xTabGeneral <- xtable( tabGeneral, digits = c( rep( 0, 3 ), rep( 7, 2 ) ),
  align = c( "l", rep( "r", 4 ) ) )
print( xTabGeneral, file = "../tables/mcGeneral.tex", floating = FALSE )

# bias
xBias <- xtable( t( resultAll$bias ), digits = rep( 5, 6 ),
  align = c( "l", rep( "r", 5 ) ) )
print( xBias, file = "../tables/mcBias.tex", floating = FALSE )

# root mean square error
xRmse <- xtable( t( resultAll$rmse ), digits = rep( 5, 6 ),
  align = c( "l", rep( "r", 5 ) ) )
print( xRmse, file = "../tables/mcRmse.tex", floating = FALSE )
```

---

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# Kapitel 8

## Erläuterung der Methoden

### Influence of Social Networks on Governance Structure

Die Methoden werden ausführlich im Beitrag erläutert.

### Networks and Transaction Costs

Der Ablauf der Schätzung ist ausführlich im Beitrag erklärt. Da im Rahmen der Schätzung ein lineares Gleichungssystem als ein „Seemingly Unrelated Regression“ (SUR) Modell geschätzt wird, soll diese Methode hier kurz vorgestellt werden.

Der von ZELLNER (1962) entwickelte Ansatz ist ein „Feasible Generalised Least Square“ (FGLS) Schätzer für die simultane ökonometrische Schätzung von mehreren Schätzgleichungen, deren Fehlerterme korreliert sind. Diese Probleme treten gerade in ökonomischen Modellen häufig auf und ZELLNER (1962) konnte zeigen, dass durch die Anwendung eines SUR Modells die Effizienz der Schätzer gegenüber der Schätzung von Einzelgleichungen verbessert wird. Der FGLS Schätzer entwickelt sich wie folgt:

Wir gehen von einem Gleichungssystem mit  $G$  Gleichungen aus, bei dem die  $i$ te Gleichung folgende Form hat:

$$y_i = \mathbf{X}_i \beta_i + u_i, \quad i = 1, 2, \dots, G, \quad (8.1)$$

wobei  $y_i$  der Vektor der abhängigen Variable,  $\mathbf{X}_i$  die Matrix der exogenen Variablen,  $\beta_i$  der Koeffizientenvektor und  $u_i$  der Vektor der Störterme der  $i$ te Gleichung ist.

Das „gestapelte“ System kann dann folgendermaßen geschrieben werden

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_G \end{bmatrix} = \begin{bmatrix} \mathbf{X}_1 & 0 & \cdots & 0 \\ 0 & \mathbf{X}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{X}_G \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_G \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_G \end{bmatrix} \quad (8.2)$$

oder einfach als

$$y = \mathbf{X}\beta + u. \quad (8.3)$$

Beim SUR Schätzer wird angenommen, dass die Störterme verschiedener Beobachtungen nicht korreliert sind, aber die Korrelation von Fehlertermen derselben Beobachtung zwischen verschiedenen Gleichungen wird explizit modelliert. Unter diesen Annahmen gehen wir von folgender Verteilung des Residuen aus.

$$u \sim \mathcal{N}(0, \mathbf{W}) \quad (8.4)$$

mit

$$\mathbf{W} = \begin{pmatrix} \sigma_{11}\mathbf{I}_N & \dots & \sigma_{1G}\mathbf{I}_N \\ \vdots & \ddots & \vdots \\ \sigma_{G1}\mathbf{I}_N & \dots & \sigma_{GG}\mathbf{I}_N \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \dots & \sigma_{1G} \\ \vdots & \ddots & \vdots \\ \sigma_{G1} & \dots & \sigma_{GG} \end{pmatrix} \otimes \mathbf{I}_N = \mathbf{\Sigma} \otimes \mathbf{I}_N, \quad (8.5)$$

wobei  $N$  die Anzahl der Beobachtungen pro Gleichung,  $\mathbf{I}_N$  eine Einheitsmatrix der Dimension  $N$ , und  $\otimes$  das „Kronecker-Produkt“ ist. Der daraus abgeleitete FGLS Schätzer  $\hat{\beta}$  für den unbekanntes Koeffizientenvektor  $\beta$  nutzt die Kovarianzmatrix  $\mathbf{W}$ , um der Korrelation der Störterme zwischen den einzelnen Gleichungen sowie der unterschiedlichen Varianz der Störterme der einzelnen Gleichungen Rechnung zu tragen.

$$\hat{\beta} = (\mathbf{X}^\top \mathbf{W}^{-1} \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{W}^{-1} y \quad (8.6)$$

mit

$$\text{VAR}(\hat{\beta}) = \mathbf{E}[(\hat{\beta} - \beta)(\hat{\beta} - \beta)^\top] = (\mathbf{X}^\top \mathbf{W}^{-1} \mathbf{X})^{-1}. \quad (8.7)$$

Da die wahre Kovarianzmatrix der Residuen  $\mathbf{\Sigma}$  unbekannt ist, wird diese durch eine aus den Residuen einer OLS-Schätzung geschätzte Kovarianzmatrix  $\mathbf{S}$  ersetzt, dessen Elemente folgendermaßen berechnet werden:

$$s_{ij} = \frac{\epsilon_i^\top \epsilon_j}{N} \quad \forall i, j = 1, \dots, G, \quad (8.8)$$

wobei  $\epsilon_i$  die Residuen einer OLS Schätzung der  $i$ ten Gleichung sind. Somit kann  $\mathbf{W}$  folgendermaßen approximiert werden:

$$\widehat{\mathbf{W}} = \begin{pmatrix} s_{11} & \dots & s_{1G} \\ \vdots & \ddots & \vdots \\ s_{G1} & \dots & s_{GG} \end{pmatrix} \otimes \mathbf{I}_N. \quad (8.9)$$

Die Schätzung des FGLS Schätzers erfolgt in einem zweistufigen Verfahren:

1. Schätzen des Modells als OLS und Berechnen der Gewichtungsmatrix  $\widehat{\mathbf{W}}$  aus der Kovarianzmatrix der OLS Schätzung
2. Schätzen des FGLS Schätzers

$$\hat{\beta} = (\mathbf{X}^\top \widehat{\mathbf{W}}^{-1} \mathbf{X})^{-1} \mathbf{X}^\top \widehat{\mathbf{W}}^{-1} y \quad (8.10)$$

$$\widehat{\text{VAR}}(\hat{\beta}) = (\mathbf{X}^\top \widehat{\mathbf{W}}^{-1} \mathbf{X})^{-1} \quad (8.11)$$

### Multiple Output Stochastic Frontiers: Two Approaches

Die Output Distanz-Funktion sowie die „Stochastic Ray“ Frontierfunktion werden ausführlich im Beitrag erläutert. Auch die Durchführung der Monte Carlo Simulation wird eingehend beschrieben. Es verbleibt eine kurze Erläuterung des Grundprinzips einer Monte Carlo (MC) Simulation.

Monte Carlo Methoden haben ein weites Anwendungsgebiet, das weit über den Bereich der Ökonomie hinausgeht. In der Ökonometrie wird die Monte Carlo Methode häufig verwendet, um Eigenschaften — i.d.R. erster und zweiter Moment — von Schätzern zu ermitteln oder die Eigenschaften mehrerer Schätzmethode zu vergleichen. Im vorliegenden Fall werden mit einer Monte Carlo Simulation zwei Methoden zur Schätzung von Multi-Output Produktionsfunktionen verglichen.

Jeder Schätzer basiert auf Zufallsvariablen und muss daher selbst als eine Zufallsvariable betrachtet werden. Interessant ist dabei die Frage, wie gut die Verteilungseigenschaften des Schätzers den wahren Wert approximieren. Eine weitere Anwendung der MC Simulation ist die Analyse, in wie weit die Eigenschaften des Schätzers auf veränderte Rahmenbedingungen (z.B. Anzahl der Beobachtungen, Parameterbereiche, Varianz des Fehlerterms) reagieren (GREENE, 2008, p. 584).

Die Idee der Monte Carlo Simulation wird hier am Beispiel des ersten Moments der Verteilung eines Schätzers demonstriert:

Sei  $\hat{\beta}_1, \dots, \hat{\beta}_n$  eine iid Zufallsvariable mit dem Erwartungswert  $\mu < \infty$  und der Varianz  $\sigma^2 < \infty$ . Dann lässt sich der Mittelwert der Stichprobe darstellen als

$$\bar{\hat{\beta}}_n = \frac{1}{n} \sum_{i=1}^n \hat{\beta}_i \quad (8.12)$$

Das Gesetz der großen Zahlen besagt, dass

$$\bar{\hat{\beta}}_n \xrightarrow[a.s.]{} \mu, \quad \text{wenn } n \rightarrow \infty \quad (8.13)$$

und aus dem zentralen Grenzwertsatz folgt

$$\sqrt{n} \left( \bar{\hat{\beta}}_n - \mu \right) \xrightarrow[d]{} N(0, \sigma^2), \quad \text{wenn } n \rightarrow \infty \quad (8.14)$$

(MITTELHAMMER, 1999)

Beide Sätze macht sich die Monte Carlo Simulation zunutze. Bei einer Monte Carlo Analyse von Regressionsmodellen wird ausgehend von einem realen oder generierten Datensatz  $R$  mal eine neue endogene Variable generiert und der Schätzer  $\hat{\beta}$  berechnet. Anders als bei einer erhobenen Stichprobe, aus der nur ein Schätzer  $\hat{\beta}$  berechnet werden kann, können bei einer Monte Carlo Simulation durch die wiederholte Durchführung beliebig viele Schätzungen vorgenommen werden. Der ermittelte erste und zweite Moment der Verteilung von  $\hat{\beta}$  kann dann dazu herangezogen werden, um zu analysieren, ob der Schätzer verzerrt ist bzw. wie nahe die einzelnen Schätzer an dem wahren Wert  $\beta$  liegen. Der Algorithmus läuft dabei wie folgt ab:

Algorithmus [Monte Carlo]

1. Definition der  $n$  Kovariaten (Realdaten oder generierte Daten) und der „wahren“ Parameter.
2. Ziehen von  $n$  Pseudo-Zufallszahlen für den (die) Fehlerterm(e) aus der angenommenen Dichtefunktion.
3. Berechnen der abhängigen Variable aus (1) und (2).
4. Schätzen des Modells zur Ermittlung der Parameter.
5. Wiederholen von Schritt (2) - (4)  $R$  mal.
6. Vergleich der geschätzten Parameter mit den „wahren“ Parametern.

Da der datengenerierende Prozess vom Programmierer vorgegeben wird ist die Generalisierung der Ergebnisse limitiert. Daher sollten erstens realistische Kovariate angenommen werden, zweitens Parameterwerte aus Bereichen gewählt werden, die sich an Schätzungen von Realdaten orientieren, sowie drittens realistische Verteilungsannahmen sowohl im Bereich der generierten Kovariaten, als auch des (der) Fehlerterms(e) angenommen werden.

Darüber hinaus erhöht sich der Informationsgehalt der Ergebnisse, wenn die Rahmendaten des datengenerierenden Prozesses, wie bereits Eingangs geschrieben, variiert werden.

### Measuring the Influence of Information Networks on Transaction Costs Using a Non-parametric Regression Technique

Die ökonometrische Analyse dieses Beitrags wendet ein nicht-parametrisches Regressionsverfahren an. Im Folgenden soll dieses Verfahren kurz beschrieben werden.<sup>1</sup> Es ist dabei zu beachten, dass nicht-parametrische Verfahren einen großen Bereich in der Ökonometrie einnehmen. Es ist daher im Rahmen dieses Beitrags nur möglich, einen kurzen Einblick in diese Thematik zu geben.

Die nicht-parametrische Regression verwendet Methoden aus der nicht-parametrischen Schätzung von Dichtefunktionen. Das zugrunde liegende Prinzip basiert auf der Annahme, dass es folgende Beziehung zwischen einer oder mehreren erklärenden Variablen  $x$  und der abhängigen Variable  $y$  gibt:

$$Y_i = g(X_i) + u_i, \quad i = 1, \dots, n, \quad (8.15)$$

wobei  $Y_i$  die  $i$ te Beobachtung der abhängigen Variable,  $X_i$  der Vektor der  $i$ ten Beobachtungen der  $q$  erklärenden Variablen und  $u_i \sim N(0, \sigma^2)$  das  $i$ te Element eines iid Fehlerterms ist. Die zu schätzende Funktion  $g(x)$  ist unbekannt und wird mit Hilfe von Kernelfunktionen approximiert, wobei  $g(x) = E[Y_i | X_i = x]$ , d.h.  $g(x)$  gibt den bedingten Erwartungswert von  $Y_i$  gegeben  $X_i = x$  an. Es kann gezeigt werden, dass die mit nicht-parametrischer Regression geschätzte Funktion  $g(x) = E[Y|x]$  den mittleren quadrierten Fehler (MSE) minimiert. Durch die Berechnung der Dichtefunktionen  $f_{y,x}(x, y)$  und  $f(x)$  lassen sich schließlich die Schätzer für  $g(x) = E[Y|x]$  ermitteln.

Um  $f_{y,x}(y, x)$  und  $f(x)$  ermitteln zu können, nutzt die nicht-parametrische Regression eine oder mehrere Kernelfunktionen. Ein essentielle Einflussgröße ist die Bandbreite. Vereinfacht dargestellt, lässt sich das Verfahren wie folgt beschreiben. Um obige Dichtefunktionen zu ermitteln, führt die nicht-parametrische Regression an jedem Datenpunkt eine Berechnung des gewichteten Mittelwertes („local constant“), eine gewichtete lineare Regression („local linear“) oder eine gewichtete Polynomregression („local polynomial“) durch, wobei die Datenpunkte um den zentralen Datenpunkt mit einem Gewichtungsfaktor in die Berechnung eingehen. Dieser Gewichtungsfaktor fällt mit zunehmender Entfernung des Datenpunktes vom zentralen Datenpunkt ab. Die Gewichtungsfaktoren werden einerseits durch die Kernelfunktion

<sup>1</sup>Die Beschreibung des nicht-parametrischen Regressionsverfahrens in diesem Abschnitt basiert auf LI and RACINE (2007).

und andererseits durch die Bandbreite bestimmt, wobei grob definiert die Kernelfunktion die „Form“ der Gewichtungsfunktion und die Bandbreite die „Varianz“ bestimmt.

Damit die Kernelfunktion ein konsistenter Schätzer für  $f_{y,x}(y, x)$  und  $f(x)$  ist, muss sie folgende Bedingungen erfüllen:

$$(i) \quad \int k(v)dv = 1 \quad (8.16)$$

$$(ii) \quad k(v) = -k(v) \quad (8.17)$$

$$(iii) \quad \int v^2k(v)dv = \kappa > 0 \quad (8.18)$$

mit  $k(v)$  der Kernelfunktion und  $\kappa = \int v^2k(v)dv$ . Um geeignete Schätzer für  $g(x) = E[Y|x]$  zu generieren, müssen die Kernelfunktion  $k(v)$  und die Bandbreite  $h$  so gewählt werden, dass sowohl die Verzerrung  $\text{BIAS}(\hat{f}(x)) = E[\hat{f}(x)] - f(x)$  als auch die Varianz  $\text{VAR}(\hat{f}(x)) = \text{VAR}\left[\frac{1}{nh}\sum_i k\left(\frac{X_i - x}{h}\right)\right]$  minimiert werden, und somit auch der MSE minimiert wird:

$$\text{MSE}(\hat{f}(x)) \equiv E\left\{[\hat{f}(x) - f(x)]^2\right\} \equiv \text{VAR}(\hat{f}(x)) + [\text{BIAS}(\hat{f}(x))]^2. \quad (8.19)$$

In unserer Analyse verwenden wir den „local linear“ Schätzer. Daher wird sich die weitere Beschreibung des Verfahrens auf diesen Schätzer konzentrieren<sup>2</sup>. Der „local linear“ Schätzer basiert auf dem Minimierungsproblem

$$\min \sum_i (Y_i - \beta_0 - (X_i - x)^\top \beta_1)^2 K\left(\frac{X_i - x}{h}\right) \quad (8.20)$$

mit  $\beta_0$  dem Schätzer für  $g(x)$  äquivalent zu den Schätzern einer OLS,  $\beta_1$  dem Schätzer für  $g_x(x) = \partial g(x)/\partial x$ , dem Steigungsparameter, und  $K\left(\frac{X_i - x}{h}\right) = k\left(\frac{X_{i1} - x_1}{h_1}\right) \times \dots \times k\left(\frac{X_{iq} - x_q}{h_q}\right)$ . Sei  $\beta = (\beta_0(x), \beta_1(x)^\top)^\top$  der Parametervektor,  $\mathcal{Y}$  ein Vektor der endogenen Variable mit  $n$  Elementen  $Y_i$ ,  $\mathcal{X}$  eine  $n \times (1 + q)$  Matrix der Kovariaten und  $\mathcal{K}$  eine  $n \times n$  Matrix mit den Diagonalelementen  $K\left(\frac{X_i - x}{h}\right)$ , dann folgt in Anlehnung an den GLS Schätzer aus dem obigen Minimierungsproblem

$$\begin{aligned} \hat{\beta}(x) &= (\mathcal{X}'\mathcal{K}(x)\mathcal{X})^{-1} \mathcal{X}'\mathcal{K}(x)\mathcal{Y} \\ &= \left[ \sum_i K\left(\frac{X_i - x}{h}\right) \begin{pmatrix} 1 \\ X_i - x \end{pmatrix} (1, (X_i - x)^\top) \right]^{-1} \end{aligned} \quad (8.21)$$

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<sup>2</sup>Für eine detaillierte Beschreibung des Verfahrens, sowie des „local constant“ und „local polynomial“ Schätzers wird auf LI and RACINE (2007) verwiesen.



$$\times \sum_i K\left(\frac{X_i - x}{h}\right) \begin{pmatrix} 1 \\ X_i - x \end{pmatrix} Y_i.$$

Wobei gezeigt werden kann, dass  $\hat{\beta}(x)$  unter folgenden Bedingungen ein konsistenter Schätzer ist

- i.  $\{X_i, Y_i\}_{i=1}^n$  sind iid.,  $g(x)$ ,  $f(x)$  und  $\sigma^2 = E[u_i^2|x]$  sind zweifach differenzierbar.
- ii.  $K$  ist eine beschränkte Kernelfunktion
- iii. Geht  $n \rightarrow \infty$  dann folgt  $h \rightarrow 0$ .

Neben der Wahl der Kernelfunktion ist auch die Wahl der Bandbreite wichtig, um geeignete Schätzer für  $g(x)$  zu finden. Im allgemeinen besteht bei zu großen Bandbreiten die Gefahr, dass der Schätzer verzerrt (undersmoothed) ist, aber bei zu kleinen Bandbreiten besteht die Gefahr, dass der Schätzer ineffizient (oversmoothed) ist. Während ältere Arbeiten häufig die Bandbreiten per Faustregel definierten, lässt ein neueres Verfahren, Cross Validation, es zu, die „optimalen“ Bandbreiten in Abhängigkeit von der Dichte der Beobachtungen und dem Grade der Nicht-Linearität zwischen dem jeweiligen Regressor und der endogenen Variable zu finden. Dieses Verfahren führt damit zu einer optimalen Anpassungsfähigkeit des Schätzverfahrens an die Daten und minimiert das Problem zu großer Anpassung (oversmoothing) oder zu großer Abweichung (undersmoothing) der Schätzfunktion.

Auch wenn neue Bootstrappingverfahren es nun möglich machen, genaue Signifikanzniveaus für den Einfluss der Regressoren zu bestimmen, so liefern auch die Bandbreiten bereits Aufschluss über die Eigenschaften der Beziehung zwischen Regressoren und endogener Variable. Im Falle stetiger Variablen sind sehr große Bandbreiten oftmals ein Indikator für einen linearen Zusammenhang zwischen abhängiger und erklärender Variable. Dies folgt aus der Eigenschaft des „local linear“ Schätzers, dessen Verzerrung bei jeder Bandbreite  $h$  0 wird, wenn  $g(x)$  linear in  $x$  ist. Bei einem linearen Zusammenhang ist selbst bei einer unendlichen Bandbreite  $h = \infty$  der Schätzer vollkommen unverzerrt, d.h. der nicht-parametrische Schätzer ist identisch mit dem Schätzer eines OLS Modells.

Für kategoriale Variablen gilt diese Regel nicht. Hier muss als erstes zwischen geordneten und ungeordneten kategorialen Variablen unterschieden werden. Für beide Variablentypen gilt ein oberer und unterer Grenzwert. Der untere Grenzwert liegt für beide Variablentypen bei 0. Eine Bandbreite von 0 bedeutet, dass kein Zusammenhang zwischen den einzelnen Kategorien bestehen und die Funktion sich damit auf eine Indikatorfunktion reduziert. Die Obergrenze liegt für geordnete kategoriale Variablen bei 1 und für ungeordnete kategoriale

Variablen bei  $(d - 1)/d$ , mit  $d$  der Anzahl an Kategorien. Erreicht die Bandbreite diese Obergrenze, so ist dies ein Indikator dafür, dass die Variable insignifikant ist.

Wie bereits eingangs geschrieben, ist es nicht möglich, das Thema der nicht-parametrischen Regression in diesem Rahmen erschöpfend zu diskutieren. Daher kann dieser Beitrag nur einen kleinen Einblick in die Funktionsweise der nicht-parametrischen Regression geben. Für eine weiterführende Einführung in die nicht-parametrische Regressionstechnik sei auf die Bücher von LI and RACINE (2007) und DULLER (2008) verwiesen. Für die Schätzung einer nicht-parametrischen Regression in R sei auf HAYFIELD and RACINE (2008) verwiesen.

### **Estimating the CES Function in R: Package micEconCES**

Die Methoden werden ausführlich im Beitrag erläutert.

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# Kapitel 9

## Schlussbetrachtung

Die vorgestellten Beiträge untersuchen den Wirkungszusammenhang zwischen verschiedenen Aspekten von Transaktionskosten und sozialen Netzen. Da der Forschungsansatz in der hier vorgestellten Form — besonders im Bereich der Agrarökonomie — als Neuerung betrachtet werden muss, ist der Rückgriff auf frühere Arbeiten stark limitiert. Unter diesem Aspekt stellt sich die interessante Frage, in welcher Richtung dieser Ansatz zukünftig weiterverfolgt werden kann und welche Forschungsfragen sich daraus eröffnen.

Die Messung von Netzwerkeffekten ist mit einer Reihe von Schwierigkeiten konfrontiert, u.a. der Herausforderung, Netze sauber zu erheben, aber auch der Tatsache, dass Netzwerkeffekte in der Regel gering sind und damit sehr hohe Anforderungen sowohl an die Qualität der Daten als auch an die Sauberkeit der ökonometrischen Schätzung haben. Das Risiko, durch weißes Rauschen oder handwerkliche Fehler überlagert oder verzerrt zu werden, ist bei Netzwerkeffekten groß. Vor diesem Hintergrund möchte ich im Folgenden die Beiträge und die darin gewählten Methoden kritisch bewerten. Des weiteren möchte ich auch auf kritische Aspekte der beiden methodischen Beiträge eingehen und diese näher diskutieren.

### **Influence of Social Networks on Governance Structure**

Dieser Beitrag untersucht den Einfluss sozialer Netze auf die Wahl der „Governance Structure“ (im weiteren „Regime“ genannt). Obwohl Netze einen messbaren Einfluss auf die Wahl des Regimes zeigen, sind die Ergebnisse in mehrfacher Hinsicht problematisch.

Die größte Schwierigkeit in der Schätzung von Netzwerkeffekten liegt darin, dass i.d.R. keine Aussage über die Ursächlichkeit des Ergebnisses getroffen werden kann. Der in diesem Beispiel gemessenen positive Einfluss des Handelsnetzes auf die Wahl des Marktregimes könnte durchaus auch umgekehrt interpretiert werden: Betriebe, die viel am Markt handeln, weisen ein dichtes Händlernetz auf. Bisherige Ansätze, die Netze als rein exogen annehmen, greifen in dieser Hinsicht zu kurz. Besonders in der dynamischen Betrachtung dieses Wirkungszusammenhangs sind starke und reziproke Einflüsse sozialer Netze auf die Wahl des

Regimes und umgekehrt zu erwarten. Das Problem der Kausalität in den Wirkungszusammenhängen zwischen sozialen Netzen und ökonomischen Größen erfordert in erster Linie eine theoretische Basis, die über eine einfache Exogenitätsannahme hinausgeht, und stellt damit eine Herausforderung für zukünftige Forschungsarbeiten dar.

Ein weiterer Aspekt, der einen Einfluss auf die Wahl des Regimes hat, jedoch im Artikel nicht berücksichtigt wird, ist der Effekt von Getreide- und Futtermittelpreisen auf die Wahl des Regimes. Leider standen den Autoren nur Querschnittsdaten und sehr begrenzte Informationen über Preise zur Verfügung, so dass eine adäquate Berücksichtigung dieses Aspekts im Rahmen der vorliegenden Untersuchung nicht möglich war. Da stark davon auszugehen ist, dass Preisentwicklungen zumindest kurzfristig einen erheblichen Einfluss auf die Wahl des Regimes haben, sind zur sauberen Trennung der Effekte von Preisen und sozialen Netzen Panel-Daten unerlässlich. Auch dieser Punkt sollte in zukünftigen Arbeiten aufgegriffen werden.

Als letzter Kritikpunkt ist zu nennen, dass informelle Kontrakte, d.h. mündliche Absprachen, nicht sauber erfasst wurden und evtl. in das Marktregime eingingen. Bei strenger Interpretation würde dieser Effekt das Ergebnis verzerren. Dem ist allerdings entgegen zuhalten, dass es gerade diese Transaktionen sind, die auf einer großen Vertrauensbasis beruhen, d.h. die Effekte, die wir in unserer Hypothese thematisieren. Langfristige Handelsbeziehungen, die von gegenseitigem Vertrauen geprägt sind, sind auch auf Märkten nicht unüblich und befinden sich damit in der Grauzone zwischen einem reinen Spotmarkt und einem formalen Kontrakt. Eine saubere Lösung wäre die von MÉNARD (2004) vorgeschlagene Einführung eines weiteren Regimes, der Hybridform, die alle Transaktionen dieser Grauzone umfasst.

Für zukünftige Forschungsarbeiten wäre es interessant, diesen Ansatz auf Güter auszuweiten, die in höherem Maße als Kraftfuttermittel von Transaktionskosten betroffen sind. Dazu gehören alle Güter, die in ihrer Art eine stärkere Heterogenität, eine größere Spezialisierung und/oder in hohem Maße versteckte Qualitätsmerkmale aufweisen.

### **Multiple Output Stochastic Frontiers: Two Approaches**

Die vorliegende Arbeit vergleicht mit Hilfe einer Monte-Carlo Simulation die Qualitäten der Schätzer der Output Distanz-Funktion und der „Stochastic Ray“ Frontier-Funktion. Auch wenn die in diesem Beitrag verwendete Methode im Gegensatz zu früheren Arbeiten eine bessere Beurteilung der Qualität der Schätzer beider Funktionen erlaubt, gibt es eine Reihe von Verbesserungen und Erweiterungen, die für eine endgültige Beurteilung der Schätzer hilfreich wären.

Die bisherige Untersuchung ist — in Anlehnung an das Modell im dritten Beitrag (Kapitel vier) — auf den Zwei-Output Fall limitiert. Da beide Schätzfunktionen ausdrücklich für den n-Output Fall entwickelt wurden, sollte eine Erweiterung der Simulation auf den n-Output Fall erfolgen.

Methodisch ist bei dem beschriebenen Verfahren zu kritisieren, dass die Generierung des Verteilungsschlüssels aus der Dirichlet Verteilung unabhängig von dem Verhältnis der Inputfaktoren geschieht. Dies ist eine realitätsferne Annahme, die dahingehend korrigiert werden sollte, dass die Generierung des Verteilungsschlüssels aus einer bedingten Verteilungsfunktion erfolgt, die das Verhältnis der Inputfaktoren berücksichtigt. Damit soll vermieden werden, dass bei gegebenen Inputkombinationen unrealistische Outputverhältnisse generiert werden.

Schlussendlich sollte zur besseren Generalisierbarkeit der Ergebnisse das Modellszenario variiert werden. Dabei sind zu nennen:

- eine variierende Anzahl an Beobachtungen
- Variation in der durchschnittlichen Ineffizienz
- Variation in der Varianz des generellen Fehlerterms
- unterschiedliche Korrelationen zwischen den Kovariaten
- sowie Variation in den „wahren“ Parameterwerten

Auch interessant für zukünftige Arbeiten dürfte die Reaktion der beiden Schätzfunktionen auf Ausreißer sein. In diesem Zusammenhang stellt sich die interessante Frage, inwieweit beide Funktionen besser als die Data Envelopment Analysis (DEA) auf Ausreißer reagieren.

Ein weiterer wichtiger Forschungsbereich ist die Endogenität von Regressoren, die bei beiden Funktionen in Form eines erklärenden Terms zum Outputverhältnis auftritt. Sowohl die Output Distanz-Funktion als auch die „Stochastic Ray“ Frontier-Funktion haben erklärende Variablen, die aus Outputmengen sowohl in Form von einzelnen Outputs als auch in Form der Euklidischen Distanz berechnet werden. Diese Outputmengen sind eindeutig nicht exogen, weshalb beide Schätzfunktionen ein immanentes ökonomisches Problem haben. Weder die Output Distanz-Funktion noch die „Stochastic Ray“ Frontier-Funktion stellt hier eine befriedigende Lösung dar. Eine Analyse des Einflusses des Endogenitätsproblems auf die Qualität der Schätzer sowie die Entwicklung alternative Ansätze sind interessante Forschungsfragen für zukünftige Arbeiten.

## Networks and Transaction Costs

Der hier vorgestellte Ansatz zur Messung des Einflusses sozialer Netze auf die Höhe von Transaktionskosten muss — sofern den Autoren bekannt — als ein neuer Ansatz gewertet werden. Andere Arbeiten, die in einem vergleichbaren Maß Transaktionskostentheorie über mikroökonomische Produktionstheorie mit sozialer Netzwerktheorie verknüpfen, sind nicht bekannt. Es ist daher nicht überraschend, dass der Ansatz noch nicht völlig ausgereift ist und mit einigen Kinderkrankheiten kämpft.

Der erste Kritikpunkt gilt der unbefriedigenden Messung des Einflusses anderer Größen als der Transaktionskosten auf die Divergenz zwischen geschätzter Technologie und beobachteten Preisen. Die bisher gewählte Lösung einer Abfrage der Einschätzung der Landwirte kann als erster Ansatz gesehen werden, ist jedoch zu *ad hoc* für eine optimale Lösung. Dieses Problem muss im Rahmen weiterer Forschungsarbeiten gelöst werden.

Zwei weitere Probleme betreffen die Durchführung der Schätzung anhand der konkreten Datenlage. Das Modell stellt sehr hohe Anforderungen an die Datenqualität, da die zu messenden Effekte klein sind und dementsprechend sensitiv auf weißes Rauschen reagieren. Leider entsprechen die verfügbaren Daten nur teilweise diesen Ansprüchen, so dass der ursprünglich große Datensatz stark reduziert wurde. Es ist anzunehmen, dass die Qualität der Schätzer sich bei Erhöhung der Anzahl an Beobachtungen noch verbessern wird. Hinzu kommt die hohe Korrelation der Regressoren, die starken Einfluss auf die Signifikanz und Sensitivität der Parameter bzgl. der Modellspezifikation hat. Auch dieses Problem ließe sich am besten durch eine Erhöhung der Datenmenge verringern.

Abschließend lässt sich sagen, dass das vorgestellte Vorgehen einen innovativen und interessanten Ansatz darstellt. Zukünftige Arbeiten sollten hauptsächlich auf die Lösung der angesprochenen statistischen Probleme fokussiert sein.

## Measuring the Influence of Social Networks on Transaction Costs Using a Non-parametric Regression Technique

Der vorgestellte Beitrag analysiert den Einfluss sozialer Netze auf die Produktivität von landwirtschaftlichen Betrieben. Obwohl signifikant positive Einflüsse sozialer Netze auf die Produktivität gemessen werden können, ist die Interpretation der Ergebnisse problematisch. Der Grund liegt einerseits in den teils widersprüchlichen oder uneindeutigen Ergebnissen aus der Literatur, andererseits in der Komplexität des Wirkungszusammenhangs zwischen sozialen Netzen und Transaktionskosten.

Transaktionskosten können in verschiedenen Formen, z.B. technische und institutionelle Transaktionskosten, und auf verschiedenen Stufen einer Transaktion entstehen, z.B. Suchko-

sten, Verhandlungskosten oder Kontrollkosten. Leider ist die Wirkung sozialer Netze auf die verschiedenen Formen der Transaktionskosten nicht einheitlich, sondern steht sich in manchen Fällen diametral entgegen. Als Beispiel sei hier die Wirkung der Dichte eines Netzes auf Such- und Kontrollkosten genannt. Während in Folge BURT (1992) große dünne Netze vorteilhaft für einen Informationsaustausch im Sinne eines Suchprozesses sind, zeigt BUSKENS (1999), dass soziale Kontrolle besonders durch einen engen Zusammenhalt der Akteure eines Netzes generiert wird.

Der bisherige Aufbau unseres Schätzmodells löst dieses Problem nur unbefriedigend. Um differenziertere Aussagen über den Einfluss sozialer Netze auf die unterschiedlichen Arten von Transaktionskosten treffen zu können, sollten verschiedene Typen sozialer Netze in die Schätzung mit aufgenommen werden. Obwohl den Autoren diese Daten zur Verfügung standen, wurde vom Einsatz selbiger bisher abgesehen. Der Grund liegt in der extrem hohen Korrelation zwischen den Netzwerkparametern der unterschiedlichen Netze. Diese hohe Korrelation ist besonders im Rahmen einer nicht-parametrischen Regressionsanalyse problematisch (LI and RACINE, 2007).

Für die weitere Arbeit an diesem Modell, steht daher die Lösung dieses Problems an erster Stelle.

### **Estimating the CES Function in R**

Der vorgestellte Beitrag demonstriert die Anwendung des Zusatzpakets **micEconCES** für die Statistikumgebung „R“ und vergleicht die darin verwendeten Methoden. Obwohl die Monte Carlo Simulation für alle Methoden gute bis befriedigende Ergebnisse liefert, sollten die Methoden an weiteren datengenerierenden Prozessen, auch unter weniger idealen Bedingungen getestet werden, um abschließend beurteilen zu können, welche Methoden sich für bestimmte empirische Problemstellungen besonders eignen.

Des weiteren bleibt anzumerken, dass alle Methoden bis hierhin nur auf den klassischen zwei Input Fall reduziert sind. Allerdings lassen alle Methoden, inklusive der Kmenta Approximation (HOFF, 2004) — zumindest theoretisch — auch den n-Input Fall zu. Eine interessante Fragestellung stellt sich, wie praxistauglich die nicht-linearen Optimierungsverfahren und die „Grid Search“ Methode im n-Input Fall sind. Die Untersuchung dieser Möglichkeit ist eine Aufgabe für die weitergehende Forschung in diesem Bereich.

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# Kapitel 10

## Summary

This research was conducted within the framework of the EU project “Advanced Eval” (contract No 022708, 6th framework). The aim of the project was to develop and improve methods for ex ante and ex post evaluation of rural development policies. All articles presented here fall under the category of ex ante policy evaluation, and investigate a sub area of the project, the main focus of which is to analyse the microeconomic behaviour of farms, in particular, the empirical relationship between social networks and transaction costs on agricultural product markets. The empirical results, as well as methodological contributions, enter to some extent further steps in the project, modelling a regional rural economy. The empirical results are included on the micro level of the simulation, while the methodology helps to form a link between the micro and macro level in the sense of Coleman’s micro-macro model (COLEMAN, 1990).

To model empirically the influence of social networks on transaction costs, we unite elements from sociology with microeconomic production theory. This approach is—to the author’s knowledge—a novelty, as until now, the literature has been limited on the theoretical aspects of this topic (for example BATENBURG et al. (2003)) and the few empirical studies which address this subject (for example BUSKENS (1999); ROOKS et al. (2000)) do not include economic theory and so must be assigned to sociology or business economics.

The presented contributions investigate the influence of social networks on different aspects of transaction costs. Furthermore, they present empirical methods that contribute to solving the above mentioned research questions. The first article is in the tradition of the empirical transaction costs literature and analyses the influence of social networks on the choice of governance structure. Therefore, the focus is first and foremost on the empirical analysis. The second article develops a theoretical model which unites sociological network theory and microeconomic production theory. Furthermore, we develop an empirical model to estimate the impact of social networks on the level of transaction costs in agricultural product markets. Anyway, the main focus of the article is on the empirical analysis. Following article two, the third article addresses a methodological question and investigates the quality of two different

approaches to estimating a multiple output stochastic frontier. The fourth and fifth articles theoretically and empirically analyse the influence of social networks on a farm's productivity. Finally, the last article is clearly a methodological contribution. It presents and compares several economic methods to estimating the CES production or utility function, and demonstrates the practical application.

### **Influence of Social Networks on Governance Structure**

Following classical empirical analysis in transaction costs theory (RINDFLEISCH and HEIDE, 1997; BOERNER and MACHER, 2001), we examine the choice of governance structure for a specific type of transaction. The presented article analyses the acquisition of concentrated feeding stuffs for cattle, pigs, and poultry by *a)* own production, *b)* long and short term contracts, or *c)* the market. Contrary to earlier studies, our analysis includes social networks as an explanatory variable for the choice of governance structure.

Following scholars such as for example BUSKENS (1998, 1999); RAUB and WEESIE (1990); HENNING (2005) we derive the assumption that social networks have an influence on the level of trust between trading partners. Therefore, farms show different preferences with respect to the choice of governance structure. The derived hypothesis presumes that farms associated with beneficial network structures, i.e. network structures that increase the level of trust in trading partners, show a tendency to source their concentrated feed from the market rather than by own production.

As the endogenous variable—the percentage of concentrated feed purchased by one of the three regimes—is a fractional variable, applying a simple OLS would be an inadequate solution. An additional difficulty is the huge quotient of farms that limited to one kind of regime. Consequently, the amount of observations with either 0 or unity is considerable. We solve this problem by applying a General Linear Model (GLM) with a logit link function. The logit link function maps the interval  $[0, 1]$  of fractional response date  $y$  on the real line.

Following MCCULLAGH and NELDER (1983) we estimate the model as Quasi-Maximum-Likelihood (QMLE). We regress the fractional response variable  $y$  on animal heads per hectare, land, quality of land, and the network parameters “outdegree” and “density” of four different network types (trade network, information network, social network, and cooperation network).

Surprisingly, our data shows no significant influence of animal heads per hectare, land, or quality of land on the choice of regime. However, we find measurable effects of the trade and information network on the regime choice. The results confirm our hypothesis. A dense trade network significantly increases the amount of concentrated feed purchased on

the market, while on the other hand a widespread information network increases the amount of concentrated feed obtained through own production. We find no significant influence of networks on the contract regime.

### **Multiple Output Stochastic Frontiers: Two Approaches**

The presented analysis was developed in the context of the second article. In the context of the analysis of the impact of social networks on the level of transaction costs, we estimate a multiple output production frontier. In the meantime, there are two prevalent approaches to estimating a multiple output production frontier:

1. estimating an Output Distance function, based on SHEPHARD (1953, 1970)
2. or estimating a Stochastic Ray Production Frontier, more recently developed by (LÖTHGREN, 2000).

Although scholars have compared both approaches (FUSEKIS, 2002; ZHANG and GARVEY, 2008), these analyses have only been done with real world data. Therefore, it is not possible to derive information about the quality of the two functions' estimates. We want to approach this research question by applying a Monte Carlo simulation. Following COELLI and PERELMAN (1999) we mainly focus on the quality of the inefficiency measure of the two functions, as a direct comparison of the functions' parameters is only possible to a limited extent. The reason is that both function use different measures to model the output ratio. These measures display a considerable difference in the variance, which would—given identical parameter values—result in distinct differences in the variance of the fitted output values. To avoid this problem we are forced to choose parameters that adjust the differences in the variance, at the expenses of the comparability of the estimated parameter values.

An additional difficulty arises out of the unknown output ratio in the multiple output setting. We solve this problem by generating an additional pseudo-random variable out of the Dirichlet distribution. With the aid of this additional random variable we are able to generate distribution key for the output ratio. Although our analysis so far reduces to the two output case, the described procedure can be easily extended to the n-output case.

To run the Monte Carlo simulation, we generate 200 observations. The “true” parameters range in a domain that was set in dependence on real data estimations from literature and own analyses. Following the approach from BATTESE and COELLI (1995), we generate the inefficiency term from a half normal distribution. The variance is chosen to result in an average efficiency of approximately 0.83. The general error term is follows a normal distri-

bution with an expected value of 0 and a variance  $\sigma^2 = 0.1$ . The simulation is run over 1000 iterations.

The results show—for the given scenario—only minimal differences in the quality of the estimates. However, the quality of the single estimates is very heterogeneous and should be further investigated. The estimation of the inefficiencies, on the other hand, delivers very reliable results for both functions. Further analyses are required to examine whether this result persists under less optimal settings.

## Networks and Transaction Costs

Based on the work of BUSKENS (1999) and HENNING (2005) we first show theoretically how the structure of social networks manifests at the level of transaction costs.

We use a dataset of Polish farm accounts and egocentered network data to analyse the influence on farm individual transaction costs for different markets, i.e. the markets for crop and animal products, as well as three input markets.

The theoretical model is derived from classical microeconomic production theory. The basic idea is that non observable transaction costs lead to a deviation between the observable and the optimal production decision according to observable price data. The underlying assumption is that the farm manager in fact knows the true but unobservable shadow price and bases his production decision on this information instead of the observable price. As a consequence, observable output prices are higher and observable input prices lower than the shadow price, so that the “optimal” production decision reflects the hypothetical situation without transaction costs, while the observed production decision reflects the true situation with transaction costs. As we again form the assumption that farm individual transaction costs are affected by the structure of social networks, i.e. the shadow price is dependent on the social networks’ structure, we presume that it is possible to approximate transaction costs by social network parameters. Based on the derived assumptions, we want to explain the divergence between observed and optimal production decisions by the inclusion of social network parameters.

The estimation model is a two step procedure. For the first step, we estimate a production function in the form of a stochastic frontier. As we want to calculate transaction costs for crop and animal products we apply the stochastic Ray production function (see also article 2) to estimate a multiple output production function. We estimate the marginal product for the three inputs and the marginal growth of total output after the increase of a single output commodity. For the second step, we estimate a linear equation system where we estimate the influence of transaction costs, divided into proportional and non proportional transaction

costs, on the divergence between observable and optimal production decisions. As we don't know the true transaction costs, we use network parameters as an approximation.

The results show a weak to significant influence of social networks on proportional and non proportional Transaction costs. Furthermore, the results prove that both proportional and non proportional transaction costs range in reasonable limits.

### **Measuring the Influence of Social Networks on Transaction Costs Using a Non-parametric Regression Technique**

The presented article analyses the influence of social networks on the productivity of agricultural holdings. The underlying assumption is that holdings that face high transaction costs on agricultural in- and output markets show a lower productivity. The line of argumentation is as follows: in the presence of transaction costs, a farm has to sacrifice resources to obtain and sell agricultural products, which would otherwise also enter into the technical production process, for example working hours, fuel, etc.. As standard production data generally only offers aggregated input quantities, estimating a production function in the presence of transaction costs leads to overrated input quantities. In other words, in the presence of transaction costs, one does not estimate the pure technology, but a technology including transaction costs.

An agricultural holding that has to sacrifice resources to cover transaction costs will, in relation to a farm with less transaction costs, show a comparable higher use of inputs to produce the same amount of output. Hence, it will appear less efficient. Thus, under the presence of individual transaction costs and if just standard production data are available, transaction costs translate into lower productivity.

As previously discussed, it is possible to theoretically derive a coherence between the structure of social networks and the level of transaction costs. Therefore, we conclude that an interdependency between social networks and transaction costs and an interdependency between transaction costs and productivity in consequence leads to a measurable influence of social networks on productivity. We estimate with this our theoretically derived assumption in reduced form. We regress the aggregated output value on the inputs labour, land, capital, as well as variable inputs, and on the farm characteristics and network parameters of the farm's and farm-household's network, respectively.

As we have no information about the functional dependency between social networks and productivity, we avoid defining a functional form by applying a non-parametric regression technique. We estimate a local-linear cross-validation model. For the continuous variables, we use the Epanechnikov kernel, whilst for ordered categorical variables, we use the kernel

developed by WANG and VAN RYZIN (1981), and for unordered categorical variables, we apply the kernel presented by AITCHISON and AITKEN (1976). The estimation is performed in R with the help of the **NP** package (HAYFIELD and RACINE, 2008). To derive significance levels for the coefficients, we apply a bootstrapping procedure developed by RACINE et al. (2006) and RACINE (1997). As output and input quantities are logarithmised. we can interpret the gradients of the function as partial production elasticities.

Results show that the output quantity is monotone increasing in inputs. As for the influence of the social networks, we find a weakly significant influence from outdegree of the farm network and a definite influence of the density parameter of the farm-household network. All four network parameters (outdegree and density for the farm and farm-household network, respectively) show, on average, a considerable positive influence on productivity.

### **Measuring the Influence of Social Networks on Transaction Costs Using a Non-parametric Regression Technique**

This contribution is an extract from the fourth article presented here. The emphasis is on the methodological description of the non-parametric regression, using the example of the estimation model described in chapter four. The article thoroughly describes the applied methodologies of the non-parametric regression applied in the fourth contribution. In particular, the assignment of models with mixed type data (in the presented case: continuous, ordered, and unordered categorical variables) and the possibilities to interpret bandwidths and to derive significance level for mixed type data are discussed.

### **Estimating the CES Function in R**

The CES function was originally developed by ARROW et al. (1961). and it has gained popularity in macroeconomics and growth theory as an alternative to the Cobb-Douglas function. In contrast, it has not been very popular in microeconomics. Within the framework of the Ad-Eval project, the CES function is used to link the micro and macro levels of a micro-macro model (COLEMAN, 1990). While it is usual to calibrate the parameters of the CES in simulations on the macro level (for example in a CGE), this option is not possible with micro data.

Anyway, the estimation of the parameters of a CES function is not plain sailing:

1. Contrary to other non-linear functions it is not possible to linearise the CES

2. The target function of the CES has a frequent tendency to flat surfaces around the minimum. This is a troublesome feature for many optimisation algorithms, causing problems in convergence.

The **micEconCES** package takes this problem into account and provides several methods for the estimation of the CES function. Among others:

- The Kmenta approximation (KMENTA, 1967), a second order Taylor approximation of the CES function. To estimate the Kmenta approximation, we apply the method developed by HOFF (2004).
- The Levenberg-Marquardt algorithm (MARQUARDT, 1963), as a classical non linear estimation procedure of the CES function.
- Several local, global, and evolutionary optimisation algorithms to directly estimate the non linear CES function, a.o. Nelder-Mead (NELDER and MEAD, 1965), BFGS (BROYDEN, 1970), Simulated Annealing (KIRKPATRICK et al., 1983), or Differential Evolution (STORN and PRICE, 1997).
- The function `nls` to directly estimate the non linear CES function.
- As well as a grid search procedure over the substitution parameter  $\rho$  that is a robust estimate in cases of extreme non-linearity of the target function. The estimated parameters of the grid search can enter the `nls` function as starting values to improve the quality of the estimates.

Furthermore, the user has a wide range of possibilities to adjust the estimation function to her needs, as she can, among other things, decide on starting values, convergence criteria, or number of iterations.

We demonstrate the use of the methods by means of a generated data set. Furthermore, we compare the methods in a Monte Carlo simulation. For the given data generating process, all methods return satisfying results, although the estimates of the Kmenta approximation display the largest divergence from the original parameters. Our results confirm earlier studies (e.g. THURSBY (1980)) with respect to the problematic estimate of the substitution parameter  $\rho$ . In the same way, the results of our Monte Carlo simulation show considerable divergence of the estimate from the original parameter value. But our results also show that the calculated elasticity of substitution  $\sigma$  returns reliable results so that one should not overstate this problem.

Although the results were generated under controlled lab conditions of a simulation and their transferability to real world data might be limited, the **micEconCES** package provides the user with a broad range of solutions to estimate the CES function. Therefore, it should be possible to find an adequate solution to most data constellations.

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