

Agent-Based Models, Macroeconomic Scaling Laws and Sentiment Dynamics

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Chapter 1

Introduction

1.1 Agent-Based Model in Social Sciences

The history of agent-based models can be traced back to Von Neumann (1966) machine, a “theoretical machine” capable of *reproduction*. The concept was then widely developed in the field of *biology* (Crist and Haefner (1994), Percus and Torney (1999)), *ecology* (DeAngelis et al. (1994), Grimm (1999)), *epidemiology* (Longini et al. (1986), Eubank et al. (2004)), and etc., thanks to fast-growing computer technology. In contrast to the wide range of computer simulations as a methodology in nature sciences, the introduction of computer usage is a comparatively recent approach in the social sciences (cf. Zeigler (1976)). Schelling (1971) was the first researcher who used agent-based programs to simulate interesting processes people come across in real life¹. Later on this method started to become popular in explaining many other social phenomena, including *opinion formation and voter dynamics* (Tessone and Toral (2005), Lambiotte

¹Thomas Schelling was awarded the Nobel Memorial Prize in Economics in 2005 “for having enhanced our understanding of conflict and cooperation through game-theory analysis” (cf. “The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2005” from Nobelprize.org.). Schelling (1971) explains a dynamic model of segregation, where he shows that if every agent in the model slightly prefers his neighbors to be the same color, it will finally leads to total segregation.

et al. (2007)), *culture and language dynamics* (Dunbar (1993), Steel (2000)), *traffic and crowd flow design* (Sanford (1994), Helbing and Molnar (1995), Hughes (2000)), *diffusion of news and innovations* (Guardiola et al. (2002), Elkink (2006)), *spread of World Wide Web* (Huberman and Adamic (1999), Adamic and Huberman (2002)), *dynamics of financial stock markets* (Lux and Marchesi (2002), Feng and Jo (2003)), and so on. Recent books discussing large-scale agent-based modeling in economics or sociology are Epstein and Axtell (1996), Krugman (1996), Gilbert (1999), Axelrod (1997), Sawyer (2005), Schelling (2006), Miller and Page (2007), Batty (2007), among others.

By definition, *social sciences* comprise academic disciplines concerned with the study of human groups and individuals in social life. If the society can be built into a huge agent-based model, the social scientists seek to understand not only how individuals behave, but also the scaling outcomes of many interacting individuals, i.e., how the aggregate outcome can be more than the summation of all parts.

An agent-based model should be quite suitable for the objective of social scientists. It is a method for studying systems exhibiting the following two properties: (1) the system is composed of interacting agents; and (2) the system exhibits emergent properties, which are properties arising from the interactions of agents that cannot be deduced simply by adding up the properties of these agents. When interactions of agents are dependent on past experience, and especially when agents continually adapt to their experiences, mathematical analysis is typically very limited in its ability to derive dynamic consequences, but agent-based modeling might be more than affordable to complete this task.

The breakthrough in computational modeling in social sciences came with the development of multi-agent systems (MAS). Standard MAS model a collection of autonomous decision-making entities called “agents”. These agents “communicate” via messages passing among them through a defined “environment”. Here “agents” refer to broadly defined entities who are the constituent parts of a computationally constructed environment. Examples of possible “agents” range from passive receivers to sophisticated

rational learners. They can be either *physical entities* (e.g., molecules, cells and atoms in biology or physics), or *individuals* (e.g., consumers, producers), or *social groups* (e.g., firms, families, markets). In the later cases, agents can be composed of other agents, permitting hierarchical construction. In typical agent-based models, each agent individually assesses its situation and behaves strictly according to a set of rules, and its movement has autonomy (i.e., can be fully controlled by itself). Besides, agents are considered to be “intelligent” or partially “*rational*”, with the characteristics of being **proactive** (i.e., being able to influence others), **reactive** (i.e., being influenced by others), and **social** (i.e., in an inter-connected network) (cf. Wooldridge and Jennings (1995)). Meanwhile, agents need to execute various behaviors according to the system’s design, for example, producing, consuming or selling goods. Repetitive interactions among agents are essential, which rely on the power of computers to explore certain dynamics out of pure computational mathematical methods. In the end, the constructed MAS framework should be able to provide insights into both the macro structure and the micro behavioral rules, which includes analysis of strategies that evolve automatically, rather than by human intervention. Agent-based models applied by sociologists usually contain the following features², (the articles in parenthesis are representative works talking about certain respective features):

- *Bottom-up:*

The model is built from bottom-up, which aims at the micro-macro link, with a view to assess the effects of the community as a whole. (Epstein and Axtell (1996), Tesfatsion (1997))

- *Complex system:*

Agents live in a multi-dimensional complex system, with complicated interactions among each other, which evolve over time. (Kirman (1997))

- *Heterogeneous:*

²Some of the features are drawn from Epstein (1999) and Fagiolo et al. (2006).

Agents are heterogeneous, the common method of homogeneous “representative agent” in macroeconomics may lose grounds in agent-based modeling. (Stoker (1993), Kirman (1992))

- *Bounded rationality:*

Two components related to “bounded rationality” are: agents’ bounded information and bounded computing power. In agent-based models, although agents are rational and adaptive, they do not have global information as well as infinite computing power. Typically, they make choices based on local information, and interact only with their neighbors. This character coincides with most decision making processes by human beings in our life. (Makowsky (2006), Simon (1982), Rubinstein (1998))

- *Learning capacity:*

Learning based on certain algorithms is a natural ability for agents. These algorithms can be simple rules of thumb, or sophisticated learning algorithms incorporating ideas from artificial intelligence and cognitive science. (Dosi et al. (2005))

- *Non-analytical:*

The aim of agent-based modeling is *not* to find its analytical solution. In fact, most of the micro and macro variables within agent-based models are governed by complicated stochastic processes which can be *hardly* solved analytically. (Fagiolo et al. (2006))

- *Indirect calibration:*

Agent-based models help to explain social phenomena by replicating some stylized facts using indirect calibration. Validations are done at the aggregate macroeconomic level, mostly with no restriction on parameters. (LeBaron (2006))

- *Generative experiment:*

So far, the object of many agent-based models has been conducting laboratory

experiments to gain knowledge on the underlying causal relationships in real life. This kind of model attempts to explain how deterministic and finite rules interacting with each other to generate indeterministic and infinite behaviors. (Doran (1997))

- *Data-free:*

Agent-based researchers are often interested in explaining the emergence of qualitative aggregate patterns. After knowing that experiments cannot perfectly mimic the real world, they are more interested in generating practically persuasive outcomes (distributions), instead of perfect matching of data. The later is usually considered as a “top-down” classical approach. (Gilbert and Terna (2000))

In a word, a well-designed agent-based model should be able to

“...situate an initial population of autonomous heterogeneous agents in a relevant spatial environment; allow them to interact according to simple local rules, and thereby generate — or “grow” — the macroscopic regularity from the bottom up ...”

— Epstein (1999), p.42

At present, standard tools for interpreting the collective behaviors in agent-based models are still lacking, but some techniques can be nevertheless suggested. For instance: (1) one could study the clusters of agents to see how their behaviors can be interpreted (such as the grouping of agents as different classes as we do in Chap. 3 and subsequent chapters); (2) it is also interesting to check the direction of movement for certain variable (such as the cyclical behavior of the wage-share discussed in Chap. 4); (3) moreover, by keeping the actions of some social groups unchanged, it is possible to analyze the behavioral tendency of a particular group, and test out which group of agents are more likely to affect the aggregate outcome (such as the small firm dynamics discussed in Chap. 5).

1.2 Agent-Based Model and Mainstream Economics

“Mainstream, axiomatic economics is right: the invisible hand is truly invisible. It continues to be out of sight simply because it is of a completely different nature than we were used to think so far or it has never been where it has been looked for . . . ”

— Delli-Gatti et al. (2008), p.4

Generally speaking, the conventional way of building up economic models, carried on by mainstream economists, is based on a particular type of *reductionism* (Delli-Gatti et al. (2005)), which assumes that:

- the functional relationships among the variables are *linear*; and,
- there is *no direct interaction* among the agents.

In other words, by assuming independence, it images the macro phenomena as the summation of micro individual behaviors, ignoring distinctions among them. A trick that many seminal economic models adopt is to suppose that all agents in the model are “*perfectly rational*” holding “*perfect information*”, totally “*independent*”, and the relationships among them can be described by *linear* functions. Following this tradition, it is popular to use a “*representative agent*” to describe the “*typical*” individual of a certain type, such as a “*typical consumer*” or a “*typical firm*” in *general equilibrium models*. Then, economists try to create mathematical models with simultaneous equations to explain economic phenomena by optimizing “*representative agents*”. Since all agents (in the same category) are identical to each other, the aggregate behavior of a certain group can be treated as the total summation (i.e., $\sum_{n=1}^N f(x)$, where $f(x)$ can be a certain function, e.g., production) of each isolated entity. Similarly, the *optimal* aggregate solution can be achieved by simply adding up all *optimizing* agents. In the end, even if the model can not fit the data, it is the fact that the model does, to

some extent, explain “something” plausible, which makes the research exciting. Therefore, the traditional approach provides a reasonable simplification for a huge system. As a matter of fact, most mainstream economists acknowledge that agents should be “heterogenous”. They argue that in reality, even if the system contains *different* “representative agents”, the sum of their choices may still be mathematically equivalent to the decision of one individual or many identical individuals, so that their models are an admissible approximation to reality.

Given all the advantages of classical modeling, non-mainstream economists started to post the following questions: is the dynamics of aggregate behaviors the same as the “representative agent”? is it reliable to treat each individual as identical? is the system optimization reached at the same time when its individual reaches optimum? are the correlations among different entities fully described by those parallel equations?... Some renowned researchers have pointed out the apparent weakness of the “representative agent” approach almost on the foundation of mainstream theory:

“... There are no assumptions on ... isolated individuals which will give us the properties of aggregate behavior which we need to obtain uniqueness and stability. Thus we are reduced to making assumptions at the aggregate level, which cannot be justified, by the usual individualistic assumptions. *This problem is usually avoided in the macroeconomic literature by assuming that the economy behaves like an individual.* Such an assumption cannot be justified in the context of the standard economic model and the way to solve the problem may involve rethinking the very basis on which this model is founded ...”

— Hildenbrand and Kirman (1988), p.239

Following Hildenbrand and Kirman (1988)’s study, Stoker (1993) states that the idea of “representative agent” is at odds with empirical evidence: he reviews the empirical literatures and proposes that heterogeneity matters since there are *systematic* individual

differences in economic behavior, which “account for skewness and clustering of economic distributions”. In addition, it is not possible for a modeling strategy based on the “representative agent” to reproduce the aggregate behavior of heterogeneous economic agents. Axtell (1999) claims in his study of firm dynamics that “given the power law character of actual firms’ size distribution, it would seem that the equilibrium theories of the firm . . . will never be able to grasp this essential empirical regularity”. Delli-Gatti et al. (2005) argue that “the adoption of the scaling perspective in economics implies rejecting the very definition of a representative agent because the dynamics of the system originate from the interaction of heterogeneous agents . . . as a consequence, the occurrence of scaling laws in economics is incompatible with mainstream economics”. Other similar research that questions the aggregate properties of models based on the assumption of a “representative agent”, can be found in Lebwel (1989), Kirman (1992), Carroll (2001). As an alternative, agent-based modeling is suggested by most of them to overcome the existing problems in the classical economic models.

Compared with the classical approach, the agent-based researchers believe that most social phenomena are sometimes too complicated to be analytically solvable via the pure summation of their subsystems, especially when non-linear relationship and *scaling*³ properties are involved. The economy may not “behave like an individual”, scaling emerges because the sub-elements of the system are heterogenous and interacting with each other. Following this line, agent-based research is usually conducted in the following way: At first, the stylized facts are examined. Then the basic rules of the model are defined, in particular how each agent interacts in a huge stochastic system to generate aggregate behaviors. Once a single agent’s action rule is figured out, the multi-agent system can be established. As this kind of models sometimes fits even better to empirical data, with reasonable explanations and predictions, it starts to draw more and more attention from young economic researchers. In the meantime, some

³The law of “scaling” refers to a set of properties associated to complex systems either in their spatial organization or during their temporal development (cf. Pumain (2003)). Such a complex system usually has nonlinear relations among the attributes of their subsystems.

cross-subject areas, such as “Econophysics”, “Neuroeconomics”, have been introduced as up-and-coming fields in graduate studies, offering new insights into human behavior and social phenomena.

The debates among agent-based researchers and mainstream economists will certainly continue. Although the researchers working on agent-based modeling are sceptical on the foundation of the mainstream approach, their standpoint is not well accepted by the mainstream economists either. According to the study of Richiardi and Leombruni (2005), by the year 2004, the number of published articles using agent-based models account for only 0.03% in top economic journals, showing that this area did not win much recognition among economists. Most of counterarguments or criticism are concentrated on saying that simulations “*do not prove anything*”. The major concerns can be boiled down to the doubt that (1) agent-based simulations are difficult to interpret and generalize, and that (2) they can not arrive at a fixed “equilibrium” as a steady state solution as for classical models. Our reasonings are as follows:

Many mainstream economists do not “believe” simulation can be as sound as mathematical models, especially some agent-based models do not even offer a compact set of equations, as well as algebraic solutions, which can be easily generalized or interpreted. We argue that, in fact, there are well-designed sets of intrinsic equations behind the computer programs. This is easy to understand because the computer conducts its calculations in accordance with the rules given by the researcher. What might be a little cumbersome is that, most of the programs for such models are written in a “text-based” algorithmic tool called “pseudo code”⁴, which is a high level version of an algorithm, and may be either deterministic or stochastic. For instance, a typical kind of agent-based models looks like a three-dimensional chessboard, with agents sitting in

⁴Most publications on agent-based models only contain descriptions of the general algorithms instead of detailed programming code. This is because even today programs are often not easily portable from one machine (or one operating system) to another, and there are always a wide variety of programs for choice, e.g., MATLAB, C++, GAUSS, or other special programming tools, such as, StarLogo, Swarm, Lisp-Stat, and so on. All of them are not directly transferrable to each other.

a network and having lattice-like neighbors both in the space and time domain. The location of a particular agent and his responsive and purposeful behavior are encoded in algorithmic form using simulation rules. Once the program starts, the simulation generates a “history” that is able to reveal the dynamic consequences of many designed assumptions, so that watching phenomena emerging from agents’ interactions becomes possible (cf. Parunak et al. (1998)). At the end of the simulation, the resulting distribution should be unique with a statistic equilibrium of the macro dynamics.

In spite of that, most agent-based models may be highly dependent on the structure parameters or initial conditions (Richiardi and Leombruni (2005)). Take the SA model which will be introduced in Chap. 3 for example. Sometimes even a small modification will change the quality of results, or slightly change the final distributional form, which makes generalization of certain results difficult⁵. Since computer calculation capacity gives an upper limit for running simulations, agent-based models should always be executed under reasonable guesses on particular combinations of relevant parameters within a system of limited sample size.

“Equilibrium”, as the opposite to “disequilibrium” (cf. left part of Fig. 1.1), is believed by most economists to be the optimal state of the economy, as well as the principle or ultimate goal of solving many economic models. A popular measurement in standard macroeconomic textbook is that market equilibrium should be a fixed intersection determined when supply meets demand in an “*efficient*” market. If the market reaches equilibrium, it will stay there forever in case no shocks hit the system, such as in the right part of Fig. 1.1. This *efficient* market relies on the *homogeneity*⁶ of individuals and *perfect information*⁷. In such a system, while all agents are homogeneous, equi-

⁵However, it does not mean that any modifications applied to agent-based models are not possible, and actually trial and error is a common way to construct better models. It is believed that, if the design of experiment is sufficiently close to reality, and the sample can be as large as real population, it is totally possible for simulation to replicate the “real world”.

⁶There are some general equilibrium models with heterogeneity, yet only few of them allow direct or indirect interactions among their entities.

⁷Grossman and Stiglitz (1980) point out that the assumption of perfect information runs into

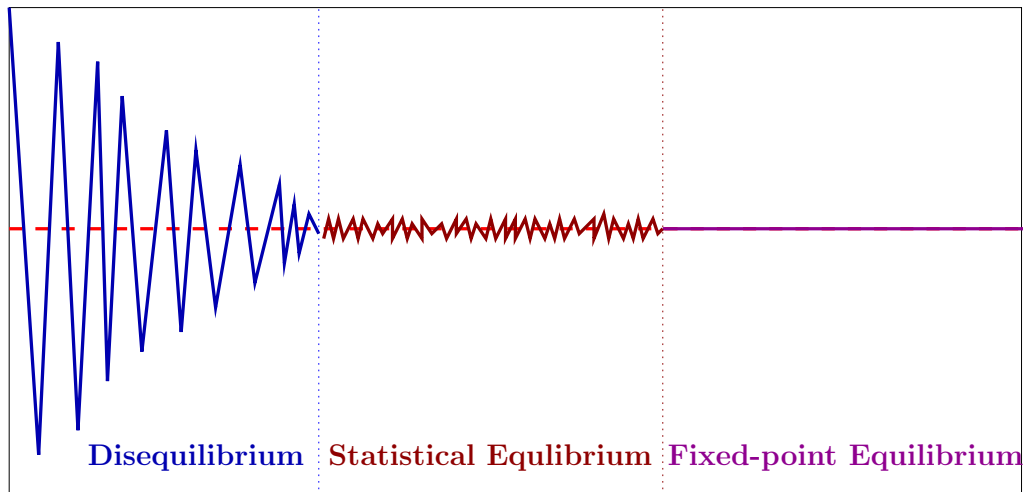


Figure 1.1: Disequilibrium, statistical equilibrium and classical fixed-point equilibrium. The left part is obviously in “disequilibrium”, it is a converging time series where we typically observe smaller and smaller variances. As the variance becomes steady, the regular fluctuation part shows “statistical equilibrium” that agent-based models are going to predict. Finally, the last part of a “fixed-point equilibrium” is only theoretically possible in classical modeling, but will never be realized either in our simulations, nor perhaps in the real world.

librium of the whole system means that each of its components reaches equilibrium as well.

In contrast, the concept of “equilibrium” in agent-based models is quite different. Instead of demanding equilibrium of all sub-elements, it only requires the aggregate distributional form of the system to be “*stable*” or “*quasi-stable*”. This distinct way of looking into “equilibrium” is usually named “*statistical equilibrium*” (cf. middle part of Fig. 1.1), which results from interactions among “slowly changing, age-dependent characteristics of a population which ages and renews itself only gradually . . . (Steindl (1965) (p.142))”. As a consequence, statistical equilibrium emerges as a state of “macroscopic equilibrium . . . maintained by a large number of transitions in opposite logical paradox, and if information is not perfect, the market cannot be efficient. Delli-Gatti et al. (2005) add that, “*market failure leads to agents’ interaction and to coordination failures, emerging properties of aggregate behavior, and to a pathological nature of business fluctuations*”.

directions . . . (Feller (1957), Vol. I, p.395)”. Note that the word “statistical” is emphasized instead of “deterministic”, meaning that our agent-based simulations are based on *randomness* and scaling outcomes, with the whole process growing from diverse causes and incomplete knowledge. We quote Delli-Gatti et al. (2008)’s saying that “direct and/or indirect interaction among heterogeneous agents at the microeconomic level is a sufficient condition for macroeconomic regularities to emerge”. A system is in *statistical equilibrium* when it reaches a stationary distribution, with fixed mean(s) and variance(s). It is not surprising that, no matter how much random noise is added into the system, after all, the agent-based model which is in statistical equilibrium will still achieve a new “stable” statistical equilibrium. In other words, the model will often stay roughly unchanged even if enlarging the system, prolonging simulations or adding up additional disturbances. It is perhaps also interesting to note that, most of the variables in statistical equilibrium will look normally distributed when their behaviors in the time domain are collected and plotted in an aggregate level. This holds almost “vice versa”, which means if we find that a variable follows normal distribution, this variable is more likely to be in “statistical equilibrium”, because: (1) there is a strong tendency for the variable to take a central value; (2) positive and negative deviations from this central value are equally likely; (3) the frequency of deviations falls off rapidly as the deviations become larger. Since a variable that is in statistical equilibrium completely satisfies all these rules, as a result, we may observe many distributions in our daily life actually turn out to be bell-shaped (i.e., the random variable follows Normal or Gaussian distributions in aggregation).

1.3 Contribution of Study

The thesis will be divided into two parts. The first part of study will be focused on a recent agent-based model of social production, whereas in the second part, a simple model of interaction of agents in a particular setting will be introduced and estimated.

The significance of the first part is to provide a social simulation model (inspired by Wright (2005)). This agent-based model produces many distributions, that can help to understand various social phenomena, ranging from firm/individual behavior, to social wealth, or GDP and business cycle, etc. Normally all these topics are studied in an isolated manner⁸, whereas this model connects them within a simple causal network. As far as complexity is concerned, the models discussed in our study may be more complex than replicating a single phenomenon, while those agent-based models which emphasize one target are not able to mimic the whole network of social relations. After reviewing the recent research on agent-based modeling, we feel that a complete model of production relation, which should be able to simulate a large variety of social phenomena, is still lacking.

Apart from “one model for all” character, we will show that the non-vanishing proportion of unemployment, business fluctuations, the existence of wealth inequality, and so on, are actual outcomes of complex interactions of firms and individuals. In classical economic thought, the above mentioned phenomena are all considered as “transient” or “non-stable”. It is believed that social “equilibrium” is reached only when “supply” meets “demand”, where there will be full employment, no growth (or forever growth) and equal wealth distribution ... By creating models of a recurring process of employment, production and consumption, we will show that this seeming instability is a “statistical equilibrium” consequence of numerous interacting individual entities having impacts on each other. In a statistical framework containing collective and cooperative properties of economic units, it is the long-lasting (relatively) stationary *distribution* of aggregate behavior, which should be emphasized. Our later models will demonstrate that, after the economy reaches a statistical equilibrium (in a relatively short period), the number of employers/employees/the unemployed are displayed always as relatively fixed *proportions* (instead of fixed numbers). Meanwhile, the number of firms, the

⁸In order to study firm growth or agents’ wealth, usually separate models should be built responsible for each task, using one model to describe two or more than two distinguished facts is comparatively rare.

growth of the economy, and many other variables are all fluctuating around their mean values, which shows that the system has no tendency to deviate from “equilibrium” over time.

When mentioning unemployment, it will turn out that a relatively small but fixed proportion of unemployment is the result of competition within a complex system of production relations. That is to say, under the current way of social production relation in most developed countries, there will always be a small amount of people who suffer from layoffs. This means the common belief in macroeconomics under a “supply = demand” system, that there is full employment (i.e., the unemployment rate is zero), will never be realized in the modern economy. Consequently, given no outside enforcement from the government in a competitive market, there should be a permanent level of *frictional* unemployment, which consists of workers temporarily searching for new jobs.

Furthermore, our models are able to reproduce the ubiquitous Pareto principle (or “80/20 rule”). The principle was suggested by management thinker Joseph M. Juran. It was named after Italian economist Vilfredo Pareto, who observed that 80% of income in Italy was received by 20% of the population. As a matter of fact, the 80/20 proportion is only a symbolic approximation, which points out that most results in many situations are determined by a small number of causes. The exact ratios in real life may vary from cases to cases. The 80/20 rule is universal around us. For example, one may find 80% of support comes from 20% of his/her friends, 80% of important work is done in 20% of time, 80% of national total income is earned by 20% of population, and many other cases in life. At the first glance, it looks like an “imbalance”, yet it actually can be an outcome in a statistical equilibrium. The interesting findings of the “80/20 rule” in our models include, e.g., most wealth is concentrated in the hands of a small number of rich people; a large majority of workers are hired by a small minority of firms; small firms account for a large percentage of existing firms, but they only contribute to a small percentage of social output, and so on.

Moreover, we have also devoted a detailed discussion to firm dynamics and wealth dynamics in Chap. 5. It is proposed that many emerging distributions can be treated as a combination of individual Gaussian distributions. At an aggregate level, some of them will display multiple modes, and some of them remain uni-modal with certain skewness. Take the income distribution for instance. The common approach is to truncate the data and convert the income graph into a log-log form, then the upper tail and lower tail of the income distribution are studied separately. This approach has apparent shortcomings: by adopting a log scale and cumulative density, the upper tail of income will be enlarged and emphasized, which only includes extremely rich people who constitute less than 20% of the whole population. The lower income class which represents 80% of population, is condensed and limited to a small region, so that the “knee-shape”⁹ of the lower tail gets blurred. During examining the wealth dynamics from our simulations, a multimodal Gaussian distribution has been successfully fitted. It is known that the Gaussian distribution is very common. If one studies certain object which is composed of several sub-groups which is individually Gaussian distributed, these Gaussian distributions may easily overlap each other in the aggregate level, and a multimodal Gaussian will arise. For example, Schilling et al. (2002) explain that the height distribution of humans is bimodal, which can be understood as a mixture outcome of two distributions Z_1 (males) and Z_2 (females) with weight α and β . Nevertheless, even if two Gaussians have different means and variances, their mixture distribution does not necessarily look bimodal if their means are close enough to each other compared with their standard deviations. On the other hand, it may easily become extremely complicated if a unimodal mixed-Gaussian distribution is treated in a normal way, then some distributions will look very odd, and it is almost impossible to find an existing functional form to describe the entire region of the distribution¹⁰.

⁹The overlapping area of lower income and higher income area is usually observed to be a “knee-shape”, showing an uneven transition from the lower income class to the higher income class (cf. Chap. 5).

¹⁰For example, it seems possible to replicate the profit rate distribution in Wright (2005) by a weighted mixture of two Gaussian distributions, whereas originally he uses a 6-parameter functional

We are going to show that, not only the income distribution, but also the wealth distribution (either for the poor or for the rich) can be treated as special combination of Gaussian distributions *without using any truncation*. In our experience, if a multiple-Gaussian distribution helps in explaining certain distribution, usually a mixture of up to three Gaussian distributions is sufficient for a good fit (with R-squared 99.99%).

Besides, despite the complex interactions among agents, the computational capacity required for our models is not too demanding. One typical model contains a main program of less than 100 lines, a standard run of 1,200,000 iterations (i.e., 1,000 Agents \times 100 years \times 12 months) takes only 15 minutes with MATLAB (cf. program codes in the Appendix), less than 1 minute with GAUSS, and 2 seconds with C++. Due to the fact that most of the agent-based models may easily reach the upper boundary of computer capacity, this relatively small demand ensures possibilities of further extensions to the current model. Future research can be easily developed based on this framework, e.g., with currently simple rules regulating firms or individuals, it is expected that more detailed rules can be added into the model, such as rules to regulate the reinvestment of firms, the rationality of individuals, the role of banks and government, and so on.

To sum up, the first part of thesis focuses on setting up a well-developed agent-based model, with the aim of simulating and explaining the “social relations of production”. The baseline model is simple but robust in explaining many empirical distributions, and most of them are not only qualitatively, but also quantitatively in agreement with the documented reality of developed capitalism, such as the power-law firm size distribution, the Laplace firm growth distribution, the Lognormal firm demise distribution, the exponential recession duration distribution, and etc. In addition, the model also touches a wide variety of social phenomena which still lack empirical support, (perhaps) due to the complexity of data collection in real life, such as the GDP growth distribution (of a certain country), the wage-profit rate distribution, the relation between the number of small firms and business cycles, and so on.

form to describe this unimodal distribution.

The second part of the thesis will be devoted to models with “animal spirits” or “sentiment dynamics”. “Animal spirits” is the term John Maynard Keynes used (in Keynes (1936)¹¹) to describe emotions or ideas which influence human behaviors and other social activities. As agent-based models are becoming more and more popular, interest in building the “animal spirits” into agent-based simulation or behavioral models has consequently been quite high as well. Among them, several leading researches are the *ants’ model* by Kirman (1993), the *active walker model* by Helbing et al. (1997), the *compromise model* such as in Weisbuch et al. (2001), the *neighborhood model* as in Galam (2002), the *voter model* as in Liggett (1999). Recently, at least two books (i.e., Pasquinelli (2008), Akerlof and Shiller (2009)) with a focus on “animal spirits” have been published to promote the understanding of the role played by emotions in influencing economic decision making.

In order to better understand the possible influences caused by human sentiment to our economy, in Part II of the thesis, we will build up VAR models and opinion dynamics models including both sentiment and economic reality. First of all, we use consumer confidence, industrial confidence and industrial production indices, to construct a concrete VAR model, this forecast-oriented VAR analysis will provide a grasp of how the trust or confidence of consumers or industry will affect real output. In the later chapter, we will study a simple model of interactions (inspired by Weidlich and Haag (1983), Lux (1995)) that can be estimated. Apart from showing a significant effect of sentiment on output, this model is able to reveal the underlying mechanism of

¹¹The original passage in Keynes (1936) reads “Even apart from the instability due to speculation, there is the instability due to the characteristic of human nature that a large proportion of our positive activities depend on spontaneous optimism rather than mathematical expectations, whether moral or hedonistic or economic. Most, probably, of our decisions to do something positive, the full consequences of which will be drawn out over many days to come, can only be taken as the result of *animal spirits* — a spontaneous urge to action rather than inaction, and not as the outcome of a weighted average of quantitative benefits multiplied by quantitative probabilities.”

periodic switching of the “animal spirits” between two attitudes. We will prove again that there is a strong causal relation among industrial sentiment and the outputs.

1.4 Outline of Study

In general, agent-based models can be understood as a class of *computational models* for simulating the actions and interactions of autonomous agents. This thesis contains studies of two different types of agent-based models: the first one focuses on simulations of social relations, and the second one focuses on simulations of opinion formation. The 7 chapters are structured as follows:

After the *introduction* in Chap. 1, where we discuss the application of agent-based modeling in social sciences and the relevance of our study, in Chap. 2 we will provide a literature review of most distributions and stylized facts that are going to be mentioned as outcomes of our simulation, including firm dynamics, income, business cycle, and so on. Then, the first type of agent-based model of social relation will be described from Chap. 3 to Chap. 5.

Chap. 3 introduces the baseline model from Wright (2005). The idea is to build up a model involving the elementary relationships of modern production. The model is characterized by its originality and creativity. It is simple but straight-forward, with many realistic hypothesis. When compared with many other models which try to replicate empirical facts, this model is more vigorous at using simulations to approximate reality. Nevertheless, when exploring and replicating this model, we find some serious shortcomings which might impede its explanatory power of model. After a number of experiments, we offer some amendment suggestions. In order to support our reasoning, comparisons of simulation results with and without these changes will be provided.

In Chap. 4, we put together all suggestions discussed in Chap. 3, and try to recover the whole series of simulation experiments of Wright (2005) in our new model. The model

will be named “Agent-based Model of Employment, Production and Consumption (EPC)”, to better characterize its basic feature of describing social production relation. The improvement of the new simulation results against the original model will be presented.

In Chap. 5, we extend the EPC model and take endogenous wages into account. The model will be called EPCE (EPC model with endogenous wages) model. This extension offers the possibility of studying certain new characteristics of agents’ behaviors, and we are thus able to explain some interesting economic phenomena at the micro level, which can not be captured in the previous models, such as the role of small firms in the economy, the relationship between firm numbers and business cycles, income inequality, and etc.

In the last two chapters, we will dig into agent-based models of opinion formation. The aim of our study is to find out whether the subjective opinion dynamics may, to some extent, influence objective economic variables. Particularly, we will investigate how consumer sentiment or industrial sentiment can help to explain or predict the movement of industrial production. As a benchmark, in the subsequent agent-based model, Chap. 6 offers a forecast-oriented VAR analysis. Different kinds of VAR models with various lags will be estimated and evaluated with respect to their overall forecast abilities for industrial production.

Taking nonlinearity and interactions among sentiment and industrial production into account, Chap. 7 sets up an agent-based model of opinion formation following the pioneering work of Weidlich and Haag (1983) and Lux (1995), to model the joint dynamics of sentiment and output. A nonlinear Fokker-Planck equation is added to help in explaining the transient density of this stochastic model. Then the model is further extended into two dimensions. We observe a strong causal relation from industrial sentiment to the output.

Finally, in Chap. 8, a short conclusion with the major findings of our studies will be provided.

Part I

An Agent-Based Model of Production and Exchange

Chapter 2

Stylized Facts from Macroeconomics and Industrial Organization

In this chapter, we are going to present those typical stylized facts of macroeconomic data, that can be more-or-less partly recovered later by our models. It is worth mentioning that, many of these stylized facts are quantitatively extremely robust, yet their specific functional characterization is still controversial, and most of them are not applicable to all countries (economies) in the world. Sometimes, our models are able to generate distributions which are in accordance with some empirical studies, but this does not necessarily mean that the distributions of all countries will follow this rule. On the other hand, if our models do not generate the same distributions as certain empirical studies, this might not be the failure of models either. Our aim is to study how agents interact to generate scaling characteristics, instead of proving (e.g.,) whether a distribution is closer to the power law or the Lognormal¹. More specifically, it may

¹If a logarithmic scale is applied, the power law and the Lognormal distribution will display similar upper tails, thus it is sometimes hard to differentiate between them. However, what we emphasize in our study is that some of their characteristics are common, e.g., they both satisfy the “80/20 rule”.

happen that the distribution generated by our model can be fitted well by more than one statistical distributions, depending on different truncation methods applied. We insist that it is not that relevant to decide which one of the distributions is more accurate, but it is more important that we have successfully built up a simplified model to generate various distributional forms which shed some lights on explaining complicated interactions in our society.

2.1 Firms, Employees and the Unemployed

In industrialized economies, the total population can be categorized into employers² (firm owners), employees (workers), as well as the unemployed. Because their group proportions vary across different countries and from time to time, it is difficult to provide an ideal percentage of each class. Nevertheless, it goes without saying that there should be only a minority of firm owners employing a majority of workers, and the remaining small percentage is the unemployed class.

According to the “Survey of Business Owners” by the U.S. Census Bureau³, in the year 2002, there are altogether 5,524,784 firm owners with 110,766,605 employees. This means in the U.S., the number of employees is more than 20 times as high as the number of employers. In spite of this large difference, the exact percentages of employers and workers also vary from sector to sector. For example, in the mining industry, there are 19,324 employers employing 534,478 workers in 2002 — namely the number of workers is nearly 28 times larger than the employers, whereas in the forestry & fishing industry, there are 29,250 employers against 250,838 employees, which means the number of employees is less than 9 times more than the employers

²The “employer” in Social Architecture (SA) Model by Wright (2005) is called “capitalists”, since the model is originally invented within political economics framework.

³Statistics are from “2002 Survey of Business Owners (SBO)” by U.S. Census Bureau, released on 09.14.2006, available at <http://factfinder.census.gov/>. The number of “employers” has already taken “self-employed employed persons” into account.

... As a matter of fact, providing a target percentage of employer-to-employee ratio for each country and each sector is perhaps tedious and an unnecessary task, yet it might be easier to accept that an industrialized economy should have a small number of employers hiring a large majority of workers.

Unemployment is usually measured using the official unemployment rate, which is defined as the percentage of those in the labor force who are available and willing to work but are currently without jobs. Keynesian economists believe that unemployment results from insufficient effective demand for goods and services in the economy (cyclical unemployment), others point out structural problems and inefficiencies inherent in the labor market. Classical or neoclassical economists tend to focus more on rigidities imposed on the labor market from the outside, such as minimum wage laws, tax regulations which may discourage the hiring of workers (classical unemployment). A certain part of unemployment is thought to be due to voluntary choices by the unemployed who might take the time to find a new job (frictional unemployment).

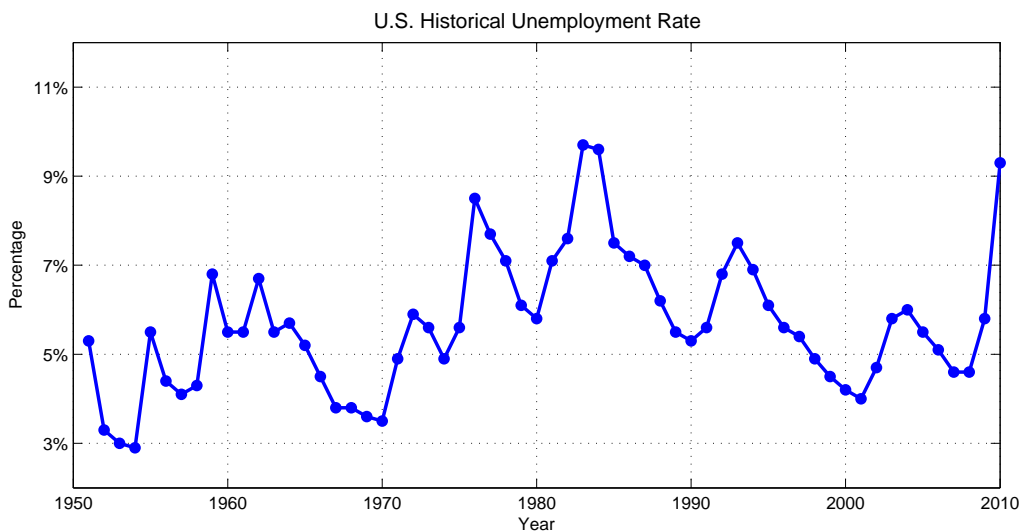


Figure 2.1: Unemployment rate in the U.S. (1950–2010). Source: <http://www.bis.gov/data/home>, by the U.S. Bureau of Labor Statistics: Labor Force Statistics from the Current Population Survey.

Different countries experience different levels of unemployment during different periods

of time. The unemployment rate of Singapore dropped to 1.6%⁴ in 2007. China's officially registered unemployment in urban areas stood at 4.3%, and in some of the big cities the unemployment rate was below 2.3% in 2009; other cities like Hongkong and Macao kept their unemployment rates around 3% at the same year⁵. From historical data, the U.S. has a lower unemployment rate than most European countries, with some variants among them, such as U.K. and Denmark who had lower unemployment rates than Italy and France over many decades (cf. Schmitt et al. (2009)). Fig. 2.1 records the U.S. unemployment rate throughout the past 60 years. It gives us an impression that the U.S. unemployment rate is fluctuating year to year, following a cyclical style⁶. The unemployment rate may drop to as low as 3% or rise to as high as 9.6%. Overall, the extreme cases are relatively rare, the unemployment rate is by and large persistent over the past decades. In fact, there is no country in the world that has ever realized full employment. As 3% unemployment rate is possibly the lowest rates for the U.S., it may be understood as "frictional" unemployment rate, which contains people waiting to be hired or searching for next job opportunities. To some extent, although full employment is not realistic, the lowest unemployment rate in a certain economy can be set as a reliable target unemployment rate for the government, which can be achieved with some efforts.

2.2 Firm Size Distribution

There are many models studying firms, and most of them focus on firm size or growth rate of firms. These models presume certain rules for firm birth, growth and death,

⁴Refer to <http://www.singstat.gov.sg/> by the Singapore government.

⁵Refer to official report "*Labor unions help relieve employment pressure during crisis*" on March 9, 2010, from <http://english.gov.cn/> by Chinese government.

⁶Keynesians argue that this type of cyclical unemployment has similar pattern as the business cycle. The cyclical unemployment rises during economic downturns and falls when the economy booms. Typically, in an economic recession, the demand for goods and services falls, less production is needed and consequently fewer workers are needed, which leads to mass unemployment.

and it is demonstrated through simulation that in the long horizon the firm sizes typically follow a Lognormal distribution. For example, Gibrat (1931) was the first to investigate the so-called “law of proportionate effect”⁷. He proposes to use log annual growth rates $r = \ln(\frac{S_{t+1}}{S_t})$ (where S is firm size) for all firms in finite time. His interesting finding is that, if we assume that all companies are independent, born at roughly the same time with similar initial sizes, and obey an independent and identically distributed (i.i.d.) growth rate r , then the cumulative statical distribution of firm sizes will approach Lognormality. Some extensions to Gibrat (1931)’s model are proposed in Kalecki (1945), Simon (1955), Steindl (1965), Chesher (1979), Cabral and Mata (2003), and others, with similar findings of Lognormally distributed firms sizes. Many empirical papers (e.g., Ijiri and Simon (1977), Silberman (1967), Hall (1987)) look into real firm sizes in practice, and find a clear concave parabolic density in the upper tail of the firm size distribution, which supports Gibrat’s finding. An overview of steady-state firm size distributions resulting from diverse firm dynamics models can be found in Wit (2005).

On the other hand, some empirical studies point out that “the size distribution of firms conforms fairly well to the Lognormal distribution, (but) with possibly some skewness to the right” (cf. Hall (1987), p. 584). Hart and Paris (1956) observe for British data from 1885 to 1950 that firm size distribution can be fitted into Lognormal distribution reasonably well, but the distribution is “somewhat skewed to the right and slightly leptokurtic”, which indicates a deviation of the true distribution from Lognormal. Later on, Quandt (1966) proposes the Pareto distribution⁸ (also known as power law) based on different tests, and rejects Lognormality for Fortune 500 firm sizes based on both the 1955 and 1960 databases. Stanley et al. (1995) study 4,701 publicly

⁷A random process of a variable is said to obey the “law of proportionate effect” if the change of variable at any step of the process is a random proportion of its previous value.

⁸The probability density function (PDF) of Pareto distribution can be written as: $f(x) = C \cdot x^{-(\beta+1)}$, where C is a constant term. The Pareto distribution is in fact a power law distribution written in a special form.

traded manufacturing firms in the Standard & Poor's Compustat (database), and find that the Lognormal distribution fits the data reasonably well except for the upper tail, where deviations from Lognormality are statistically significant. Axtell (2001) argues that firms covered by Compustat are not representative of the overall population of U.S. firms (because only large firms are considered). Instead, he collects data from the U.S. Census Bureau of *all* firms with more than one employee in 1997, and shows that the upper tail of firm size distribution satisfies a special case of a power law, called Zipf's law⁹. Furthermore, investigations carried out by Ramsden and Kiss-Haypál (2000) show that the power law upper tail (with perhaps different exponents) in firm size distributions are also common for more than 20 countries of America, Asia and Europe. Given a number of different findings on the firm size distribution announced by empirical researchers, Sutton (1997) concludes that probably there is no general density function that describes the entire region of the empirical densities well.

So far, most of the artificial models are not able to show that there should be an upper limit for firm size. Gibrat (1931) and its companion models are all simulated for a short time, because in a longer horizon, the biggest firm generated by simulation tends to employ the whole population¹⁰.

Concerning the empirical firm size, Teitelbaum and Axtell (2005) study the U.S. Census Bureau's Statistics of U.S. firms surviving from 1998 to 1999¹¹, they find that the

⁹The power law distribution can be written as $P(x) \propto x^{-(a+1)}$. When plotted in a log-log scale, it displays a straight downward sloping line with slope $-(a+1)$. If $a = 1$, this special case of power law is known as Zipf's law.

¹⁰The EPC model and the EPCE model introduced in Chap. 4 and Chap. 5 are able to demonstrate that firms may not grow continuously. When the system reaches statistical equilibrium, there should be always reasonable proportions of employers, workers and the unemployed. Only one firm employing all working population is not a statistical equilibrium solution.

¹¹In fact, the year 1998 and 1999 are the very peak of so-called "dot-com" boom. Large volumes of venture capital were pouring into high-tech areas, and there emerged a lot of start-up firms which experienced extremely fast booming. Consequently, data collected from these years might not be a good representation of an average year.

modal firm size in the U.S. has only single employee, while the median is as few as 2-3 employees (depending on whether or not self-employed persons are taken into account), the mean is only somewhat higher than 20. A study carried by Headd and Kirchhoff (2007) based on the same dataset reports that firms with less than 100 employees constitute 99% of all firms in the U.S., with the median firm size of only 4 employees and the mean of 23 in recent years; besides, they also discover that most firms start up in the 1-4 employee size class, but might not grow beyond this size class throughout their lives. In general, all these statistics point to the fact that the typical business in the U.S. is extremely small, and there is a large number of small firms coexisting with a small number of relatively large firms.

2.3 Firm Growth Distribution

The study of firm size goes often hand-in-hand with the study of firm growth rate. If there is high relation between firm growth and size, Gibrat (1931)'s famous assumption that "firm growth rate is independent of size" may not be realistic. The relationship between size and growth of firms, has previously been modeled by a number of researchers, such as Sutton (1997), Richiardi (2004), Christopher and Rodrigues (2005), Coad (2007), among others. The main assumption of the above-mentioned models is that firm growth rate decreases with firm size. In other words, smaller firms should enjoy faster growth rates than large firms. This conjecture is, to some extent, in accordance with some survey studies. Alhadeff and Alhadeff (1964) compare growth rates of the largest 200 U.S. banks between 1930 and 1960 and state that smaller banks tend to enjoy fastest growth; later on, Rhoades and Yeats (1974) provide evidences of differing average growth rates for different sizes of banks. Other similar findings outside banking industry include Hart and Paris (1956)'s research on business concentration based on British data, Piergiovanni et al. (2002)'s study on Italian firms in hospitality industry, and so on.

However, the same distributional forms may not hold for all countries or all industrial sectors. Quite opposite to the above findings, Pagano and Schivardi (2001) focus on R&D firms in the European union, and obtain a positive association between average firm size and growth; they explain that firms with larger size are more able to take advantage of the returns on R&D in order to foster productivity growth. Meanwhile, it is also obvious that the growth rates of different firms might be highly correlated with their past performances, and even interact within the same industry. Goddard et al. (2009) research into the growth performance of “Federally Chartered U.S. Credit Unions” and their sizes during 1991 and 1997, they find out that most of larger credit unions grow faster than their smaller counterparts over the sample period; interestingly, they also discover that firms experience growth rate fluctuations regardless of their sizes: a credit union with “above average” growth in one period tends to experience “below average” growth in the next period; nonetheless, large unions seem less susceptible to extreme fluctuations in growth, whereas small credit unions might have more inconsistency in their performances.

In the meantime, there is an abundance of studies on the shape of the distribution of growth rates. Teitelbaum and Axtell (2005) look into the North American Industry Classification Systems’ (NAICS) 20 major industry sectors, and reveal that firms belonging to different industrial sectors may have differing mean growth rates, but the empirical firm growth rates across these sectors have similar distributions and variances, which can be characterized by a tent shaped “*Laplace distribution*”¹². Since the Laplace distribution has much heavier tails than normality, this implies that the U.S.

¹²The Laplace distribution is a continuous probability distribution named after Pierre-Simon Laplace. It is sometimes called “double exponential distribution”, because it can be considered as two exponential distributions. The PDF of Laplace distribution can be written as: $f(x|\mu, b) = \frac{1}{2b} \exp\left(-\frac{|x-\mu|}{b}\right)$, where μ is a *location* parameter and $b > 0$ is a *scale* parameter. At the first glance, this PDF expression looks very close to a normal distribution. However, the normal distribution is expressed in terms of squared differences from the mean μ , whereas the Laplace density is expressed in terms of absolute difference from the mean. Consequently, the Laplace distribution has fatter tails than the normal distribution.

firms experience much stronger fluctuations than in the case of a normally distributed firm growth rate distribution. The same Laplace distributed firm growth rate has been proposed in Stanley et al. (1996)¹³ in which they study all publicly-traded U.S. manufacturing companies between 1975 and 1991, as well as in Bottazzi et al. (2002)¹⁴'s study of Italian manufacturing industry, among others. However, their studies are not completely in line with others. Fu et al. (2005) look into pharmaceutical industry database (PHID) of 21 countries from 1994 to 2004, as well as U.S. publicly-traded firms from 1973 to 2004 of all industries. They discover that the firm growth distribution may be more *leptokurtic* than Laplace, and it can be understood as a combined distribution with exponential (Laplace) in the central part and asymptotically power law tails decaying as $f(x) = x^{-\alpha}$ where $\alpha = 3$. This kind of *exponential power* distribution, also known as *Subbotin* distribution¹⁵ includes both Gaussian distribution and Laplace distributions as special cases. It provides a better fit to some other empirical statistics as well, such as the firm profit rate distribution (cf. Alfarano and Milaković (2008)). Reed (2001) offers a simple but reasonable explanations of why many empirical size distributions in economics and elsewhere exhibit power law behavior in the upper tail. We would like to emphasize that no matter whether realistic growth rate distribution is closer to Laplace or Subbotin distribution, it suggests that fluctuations in the growth rates are very common, and in fact, “such fluctuations are largely respon-

¹³Their other research papers on similar topics can be found in Amaral et al. (1997), Amaral et al. (2001), Buldyrev et al. (2007).

¹⁴Their other research papers on similar topics can be found in Bottazzi et al. (2001), Bottazzi et al. (2003), Bottazzi and Secchi (2003b) and Bottazzi and Secchi (2005).

¹⁵The original reference of Subbotin distribution can be found in Subbotin (1923). Later on, this distribution was refined in Bottazzi and Secchi (2003b). The functional form of the Subbotin family in the symmetric version is given by: $f(x) = \frac{1}{2ab^{\frac{1}{b}}\Gamma(1+\frac{1}{b})}e^{-\frac{1}{b}|\frac{x-\mu}{\alpha}|^b}$, where $a > 0$, $b > 0$ and $\Gamma(\cdot)$ is the Gamma function. The Subbotin distribution is thus characterized by three parameters: a *location* parameter m , a *scale* parameter a and a *shape* parameter b . If $b = 2$, the distribution turns out to be Gaussian, while for $b = 1$ the distribution is Laplace. As b gets smaller, the tails get heavier and the peak of the density becomes more pronounced. Some detailed descriptions of Subbotin distribution can be found in Castaldi and Dosi (2007).

sible for what makes the small business sector so productive, an engine of economic growth” (cf. Teitelbaum and Axtell (2005), p. 1).

Apart from that, many researchers find out that the empirical firm growth rate distribution is actually asymmetric. A recent report by the U.S. Small Business Administration, Office of Advocacy (cf. Perline et al. (2006)) looks into comprehensive data on U.S. firms extracted from the Census database over 1998–2003. They show that the observed business growth rates have a tendency towards slight asymmetry over longer horizons, and negative rates appear more volatile than positive rates. A similar study carried out by the same institution (cf. Headd and Kirchhoff (2007)) highlights several interesting facts that: (1) “growing firms are generally a constant share of the economy with a minor business cycle effect; firms with employment growth outnumber firms with employment decline”; (2) the fast growing firms (defined as having a 50% or more increase in annual employment with at least a 5-employee increase) are just a small share of all firms (around 3% in 1992); (3) about 35% of firms have no employment change from one year to the next, about 25% shrink in employment, while only about 28% of firms will grow in employment each year.

2.4 Firm Entry and Exit

Theoretically, as in many models, firms could live forever (or until the model breaks down). Likewise, firm could grow or shrink as needed in order to stay competitive as designed. In Lucas (1978)’ seminal paper on firm size, he predicts that the world would be taken over by large businesses, which means the existing firms will grow larger and larger with franchise (e.g., monopoly) in certain areas and prevent small firms to enter. This has simply never occurred, and in fact, there are very few firms lasting more than 100 years. The development of technology, the diversification of consumer tastes, the better financing, and other factors allow continuous birth of new firms, and keep forcing uncompetitive firms to exit. Cook and Ormerod (2003) reveal in their U.S. firm

life-cycle study, that about 611,000 firms die each year out of a total 5.73 million from 1989 to 1997, which tells a yearly firm demise rate of approximately 10.7%. The same evidence is also found in Headd and Kirchoff (2007), where they calculate around 11% of firms close down each year in the U.S. from 1992 to 2002.

It is easily imagined that the newly born firms are usually small and more fragile — they account for most of the entries and exits happening everyday in industry, and at the same time, create and destroy most of the job opportunities. This phenomenon has drawn many empirical researchers' attention in the past 30 years. Midrigan (2007) analyzes the U.S. "Survey of Business Administration" data and argues that most firm turnovers happen with small firms, and firms with less than 20 employees together account for 18% of the total U.S. employment; he also points out that the dying firms are concentrated in firms with less than 20 employees. Other similar findings are documented in Hijzen et al. (2007)'s study on U.K. small businesses, Sakai et al. (2005)'s study on Japanese firms, and so on. A very comprehensive review of the research related to firm formation in the 1980s and 1990s can be found in Kirchoff (1994) and Caves (1998), more recent research is summarized in Haltiwanger et al. (2008).

As far as the firm demise or survival rates are concerned, according to Headd and Kirchoff (2007), the survival rate for U.S. firms is only 50% after four years with little fluctuations over time, which is in line with an earlier empirical study by Birch (1987) on American firms' life expectancy. If the firms' survival rates do not fluctuate too much, it seems that the number of firms' entry and exit may remain more-or-less steady throughout years. Cook and Ormerod (2003) mention in their research of U.S. firms from 1989 to 1997 that the number of demises bears little connection with the overall state of the economy, namely, the total number of firm demises varies very little from year to year in economic booms and recessions. For example, in the recession year 1991 the U.S. real GDP fell by 0.5%, and in the boom year 1997 it grew by 4.3%, yet the number of demises was very similar in both years, being 630,000 and 648,000

respectively. This stylized fact has suggested that the firm demise distribution (i.e., the number of yearly firm demises recorded in a histogram) should be relatively stationary throughout the years, which in aggregation, may follow a bell-shape close to normal distribution.

Meanwhile, the study on causal relation between firm size, age and growth rate is also popular, which may date back to Prais (1976) and Evans (1987). More recent efforts in this area include Lotti et al. (2003), Headd and Kirchhoff (2007), Shanmugam and Bhaduri (2002), Rodríguez et al. (2003), Yasuda (2005), Phillips and Kirchhoff (1989), Sanghamitra (1995)'s studies across different industries of both developing and developed economies. Their general findings are: firm size and age have a negative effect on firm growth; a firm's survivability appears to rise with its size and age (until a certain limit), with the exact survival rates and firm ages differing across countries or sectors. Fig. 2.2 presents the nonparametric estimate of firm failure rates ($= 1 - \text{survival rate}$) conditional on the given ages for Spanish manufacturing firms (cf. Farinas and Moreno (2000), p. 11). It displays that the failure rates decline steeply over firms with ages under 20 years old; for firms within the range of 20–40 years old, the exit rate flattens; and finally, for firms over 40 years old, no clear relationship in failure rates and firm ages is obtained.

Although all these empirical firm dynamics are interesting to economists, so far, there is still no unified theory to explain why firms grow, decline or close in such a way, and no theory will tell us why 10% to 12% of firms enter or exit each year, and how some new firms grow while the others shrink or die. In the later chapters, we will use agent-based models to explain what actually occurs to the firm dynamics, and how the continuous entry and exit happens.

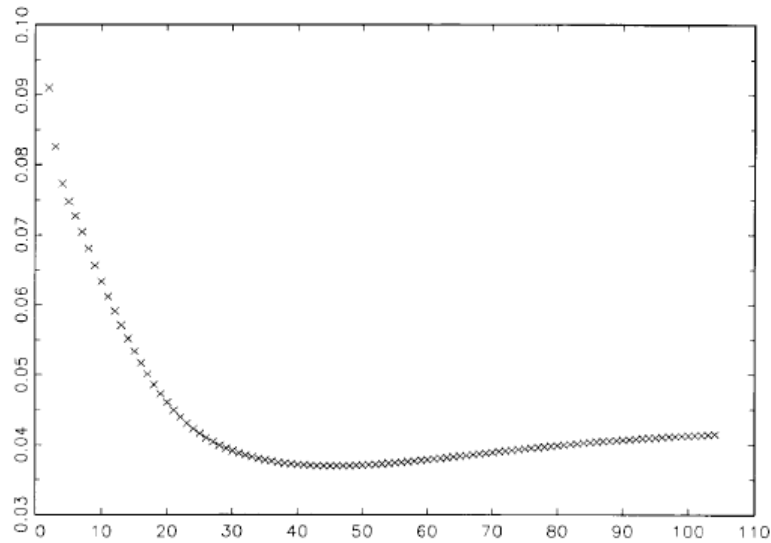


Figure 2.2: Relation of firm age and failure rate. The x-axis represents firms’ age in number of years, and the y-axis represents firms’ failure rate. Source: Farinas and Moreno (2000)’s study on Spanish manufacturing firms (p.11).

2.5 Profit Rate Distribution

Despite the existence of non-profitable organizations around us, a clear goal for most firms is to earn profit in order to grow, to stay competitive, or to survive. There has been a dominant tradition in economic thought that the profit rate tends to be uniform in the long run, which is the result of free and perfect competition. In other words, capital will look for industries where profit rates are higher than average, then more labor will be attracted to these industries, output will be raised and price as well as firms’ profit will be lowered down until the average level is reached. However, this traditional belief of classical competition has raised more and more debates. Ernst Mandel and Alan Freeman’s (cf. Shaikh (1984)) argue that *“the concept of uniform rate is neither necessary nor reasonable for understanding accumulation, price formation or profit formation. Indeed, . . . a uniform rate and free competition are contradictory concepts that cannot be reconciled”*. A similar opinion is expressed by Emmanuel

Farjoun and Moshé Machover (cf. “Laws of Chaos” by Farjoun and Machover (1983)) who stress that: *“it is perfectly well understood by most that in reality rates of profit are never actually uniform. This is always explained by a deviation of reality from the idea of perfect competition”*.

A number of models have been proposed for the profit rate distribution. For example, Farjoun and Machover (1983) propose that the industry profit rate might be approximated by a *gamma distribution*¹⁶; Wright (2005) derives a six-parameter distribution to match the model-generated profit rates, and suggests that “empirical rate-of-profit distribution will be consistent with a parameter-mix of a ratio of normal variates with means and variances that depend on a firm size parameter that is distributed according to a *power law*”, and this conjecture is tested in Wright (2004a). Julian (2007) uses a U.K. company account database to demonstrate that the profit rate distribution is highly right-skewed, with an upper tail close to a power law, that appears longer than the lower tail (in normal scale). Alfarano and Milaković (2008) design a statistical equilibrium model which predicts an exponential power (or Subbotin) distribution of firms’ profit rates, with successful empirical application on the *Forbes Global 2000* list of the worlds’ largest companies. All of these studies tell that, although the empirical distribution of profit rate is still in question, it should be obviously not a uniform distribution. We agree with Alfarano and Milaković (2008) that, in general, *“the average profit rate corresponds to a measure of central tendency, while the complex movement of capital in search of profit rate equalization translate into a measure of dispersion*

¹⁶The Gamma distribution is a right-skewed distribution, it is widely used in engineering, science and business, to model continuous variables that are always positive and have skewed distributions. The PDF of the Gamma distribution is: $f(x|\alpha, \beta) = \frac{\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left(-\frac{x}{\beta}\right)}{\beta\Gamma(\alpha)}$, where $x \geq 0$. $\Gamma(\cdot)$ is the Gamma function, and both the parameters α and β are positive (i.e., $\alpha, \beta > 0$). α is a *shape* parameter: when $\alpha \leq 1$, the Gamma distribution is exponentially shaped and asymptotic to both the vertical and horizontal axes; when $\alpha > 1$, the Gamma distribution assumes a unimodal but skewed shape, and the skewness reduces as the value of α increases. β is a *scale* parameter, having the effect of stretching or compressing the range of Gamma distribution. Finally, a Gamma distribution with $\beta = 1$ is known as standard Gamma distribution.

around the average”.

The measurement of the firm profit rate distribution is actually very close to firm growth distribution. There are commonly two different ways of measuring firm growth, one is in terms of employee numbers (i.e., the size of firm), and the other is in terms of sales (or revenue) amount. Since profit is usually calculated as “revenue – cost”, it is plausible to believe that there are close relations among firm size, growth and profit rate too. In business, the “optimal” firm size is usually considered as the size where the ratio between profit and capital invested in the firm is maximized¹⁷. Related studies of inter-relationship among size, growth and profit rate can be found in Gort (1962) (in which time series models are set up and estimated), and more recent research with diverse results from different countries or industries such as in Luis (1995), Ballesta and Lema (2003), Liu and Hsu (2006), Ishikawa (2007), among others.

2.6 GDP Growth and Business Cycle

Gross Domestic Product (GDP) is a basic measurement of a country’s economic output. There are three different ways to calculate GDP: the product (or output) approach, the income approach, and the expenditure approach. All three approaches should in principle provide the same result. Among them, the most direct method is the product approach, which sums the outputs of all industries and firms to arrive at the total. The expenditure approach works on the principle that all products must be purchased in the end, thus the total value of products must be equal to consumers’ total expenditures. The income approach is also commonly used, as it is believed that the income of firm owners and their workers must be equal to the value of their products, and the final GDP is calculated by summing up the overall income in a population.

The literatures on the distribution of the growth rate of GDP is quite sparse, neverthe-

¹⁷Because there is a great difficulty in determining the best proportion of reinvested capital as well as profit margin, it is consequently difficult to evaluate the correct size of “optimal” firms.

less, a few efforts have been devoted to understand the inherent characteristics of GDP growth rates across countries or within a country. Fu et al. (2005) study growth rates of GDP in 195 countries from 1960 to 2004 based on the World Bank database, they find out that distribution of aggregate GDP is close to the Laplace distribution. Lee et al. (1998) and Canning et al. (1998) analyze the fluctuations in GDP of 152 countries for the period 1950 to 1992, and reveal that the distribution of annual growth rates decays with “fatter” tails than a Gaussian, in particular, the tails of the distributions scale as a power law. These findings are in surprising accordance with the study of firm growth rates¹⁸.

Another topic closely related to GDP is business cycle. In reality, the time that GDP continuously rises is usually called “economic boom”, and “recession” is named for the opposite, when GDP keeps declining. Burns and Mitchell (1946) estimate that the duration of a business cycle, including booms and recessions, would be minimal 5-7 quarters and maximal 33-35 quarters in the U.S., and 45-47 quarters in U.K., according to data from the National Bureau of Economic Research (NBER) during 1885 and 1931. Everts (2006) uses a different procedure to analyze the duration of business cycle since 1955, he indicates that the mean duration of recessions is around 2.6 to 3.25 quarters, which means the majority of recessions should be less than 1 year; meanwhile, he also argues that the business cycle is highly sensitive to both the course of time and industry sectors; for example, business cycles tend to become longer in the post-war period with longer booms than recessions (which is confirmed by Burns and Mitchell (1946) and Sichel (1991) as well), and the frequency of the cycles is much higher in the agricultural sector than in the service sector.

The debates on the distribution of recession duration have not come into an agreement either. Ormerod and Mounfield (2001) study 17 western economies over the period

¹⁸Note that our later models are all based on a single economy instead of many different economies. Therefore, we are not able to show that GDP across countries may follow a Laplace distribution. We track the GDP growth rates of our typical economy, and find that the GDP growth rate is closer to a Gaussian distribution (cf. Chap. 4.3.5).

1871–1994, and demonstrate that the duration of a recession is power law distributed. Wright (2004b) reinterprets this data and argues that the frequency of recession duration follows an exponential law instead of a power law. The same but extended dataset is later studied by Ausloos et al. (2004). They conclude that it is not entirely clear whether the duration of recession follows a power law or exponential law throughout the whole period, but the duration of booms seems to follow a power law.

2.7 Wage and Profit Share

If GDP is a standard for measuring a country's economic output, profit as a proportion of GDP can be understood as a rough measurement of the share in output that firms get, and the leftover proportion is wage share that workers get. Many countries publish national accounts to show how income is distributed in the economy, namely how various groups in the economy earn their incomes (e.g., corporations earn profits, workers earn wages and salaries, etc.). Foley and Michl (1999) study the income distribution in U.S., U.K. and Japan for a period over 100 years. They show that the profit share (normally lower than wage share) is between 0.25 and 0.4 of GDP, with very rare cases that the profit rate can be as high as 0.5. Kalecki (1954) finds the profit share in the U.S. is on average 0.46 between 1929 and 1941.

In reality, wage and profit share in the economy is not always stationary, it can be very different across countries and industries. For example, the profit share for Japanese firms is known to be much lower than the U.S. firms, and sometimes even as low as 8%. Fig. 2.3 provides adjusted wage shares as proportions of GDP for Germany (FRG), the U.S. and Japan throughout the last four decades, which illustrates some differences in the average wage share among these countries.

Economists believe that the wage share is a (rough) indicator for the distribution of income between capital and labor. In the short term it moves counter-cyclically to the business cycle. Some empirical researchers find evidence of an upward trend in the

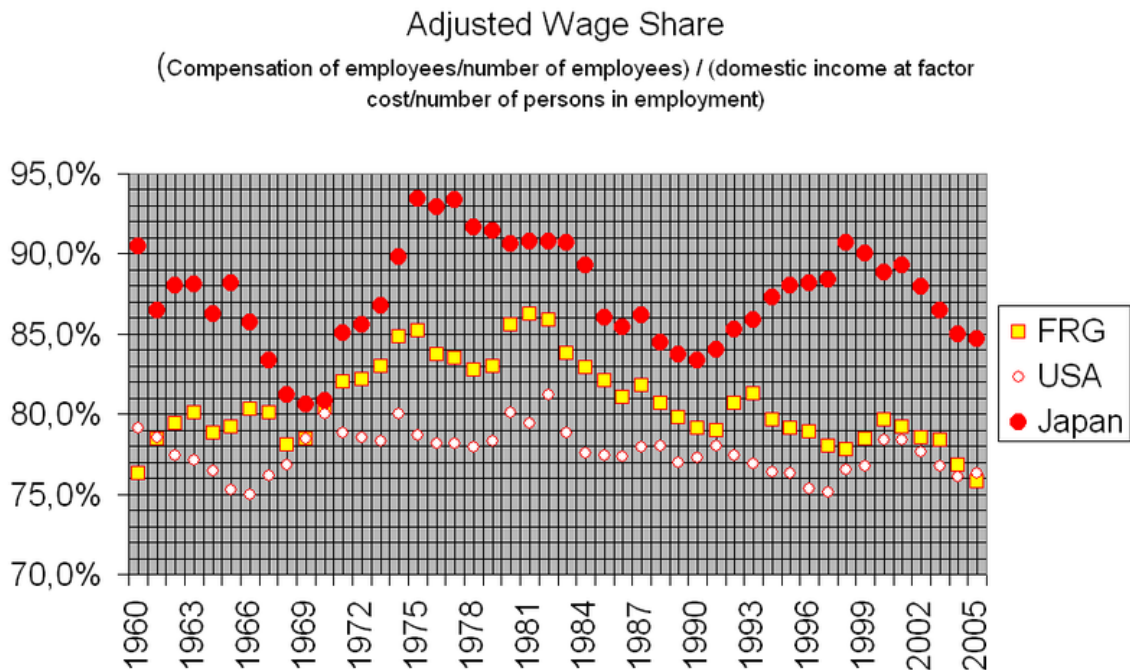


Figure 2.3: Adjusted wage share as proportion of GDP for Germany (FRG), the U.S. and Japan, 1960–2005. Source: Ameco database from European Commission staff offices (Ameco-Datenbank der Dienststellen der EU-Kommission).

profit share since mid 1980s in most developed economies (cf. Ellis and Smith (2007)), which can be partly observed in Fig. 2.3 as well. Russell and Dufourr (2007) study the growing economy of Canada between 1975 and 2005, and indicate that development in economy and productivity is not reflected in real wages for workers. The stagnant average real wages have led to a decreasing share of GDP going to workers, with an increasing share going to corporate profits. In a prolonged dataset, they even observe the profit share following a “U” shape: between 1961 and mid 1970s the profit share dropped from 30% to 25%, but this trend has been reversed in the latest 30 years, and the profit share rises hand-in-hand with the growing economy to as high as 33% in 2005, leading to a downward trend for the wage share. Similar facts are found in many economies, e.g., the U.S., China, Australia, Germany and France¹⁹. It seems

¹⁹Refer to “*Not sharing in the gains*” in “The New York Times”, August 2006, available at www.nytimes.com; “*China: wage share plunges*” in “International News, Green Left Weekly Issue”,

that there is an international claim that economic growth has failed to deliver real benefits to workers since World War II. Studies by the International Monetary Fund (IMF (2007)) say that this trend of increasing profit share mainly comes from a *slightly* increasing trend in skilled sectors, a *steadily* increasing trend in high skilled sectors but a more significant decreasing trend of wage share in unskilled sectors. Possible explanations for the nonstationary empirical trend can be global trade, immigration, or increase in productivity, which might erode workers' bargaining power, so that the wage share is decreasing continuously.

It is still an open question how the distribution of the wage/profit (share) ratios will look like in reality. Farjoun and Machover (1983) claim that “at least one parameter — the ratio of profits to labor costs (= the ratio between a firm's annual gross profit and its annual wage bill) — is much more narrowly distributed, and therefore much closer to uniformity”. The calculation and comparison of the wage/profit share is a relatively difficult task, since different (skilled or unskilled) sectors may have different ratios.

2.8 Wage and Income Distribution

The discussion of which distribution better represents the wage or income distribution has been going on for a long time. Pareto (1897) was perhaps the first to investigate personal income distribution and introduced the well-known Pareto distribution. Later on, another significant contribution to this area is from the philologist George Kingsley Zipf (1949) who introduced the Zipf's law. Since then, there are a lot of debates regarding how well an inverse power law fits income data. Discussions of income dis-

No. 728, October 2007; “*Corporate profit growth — slowing but not collapsing*” in “AMP Capital Investors”, Edition 32, November 2005; “*Wage share variations in France and Germany since 1970: what does really matter?*” by N. Canry and A. Lechevalier, available at http://halshs.archives-ouvertes.fr/docs/00/14/05/29/PDF/Canry_and_Lechevalier.pdf, respectively.

tribution are (1) the Lognormal (or generalized Lognormal) type, as in Montroll and Badger (1974), Brunazzo and Pollastri (1986), West and Deering (1995), Aitchison and Brown (1957), yet the curve fitting is usually unsatisfactory in the tails; (2) power law type, as in Guilmi et al. (2003), Kim and Yoon (2004), Clementi et al. (2006), when truncation of the datasets at the lower tail is applied; or recently (3) exponential Boltzmann-Gibbs law, as in Drăgulescu and Yakovenko (2000), Chatterjee et al. (2005), Chakrabarti et al. (2006).

Truncation is a statistical tool often applied in curve fitting tests. Sometimes, it helps to support the argument of the author. It has been pointed out that many examples used by Pareto, Zipf, as well as some other researchers have truncated datasets. Perline (2005) argues that if one looks more carefully into the lower tail that has been originally excluded, the power law behavior tends to break down at certain point. He categorizes the “power-law-like distributions” mentioned in the literature into three kinds: besides a true power law that completely fits the data, there is also a “*weak power law*” where the power law only fits the upper range of the values, and a “*false power law*”, which refers to some “Lognormal-like” distributions which can convincingly *mimic* a power law only when truncation is applied. His study reminds us that perhaps the true income distribution can not be explained by any of the single distributional forms discussed above. More recent research by Drăgulescu and Yakovenko (2001, 2003), Reed (2003), Silva and Yakovenko (2005), Gallegati and Clementi (2005) have shown that the empirical income distribution might be separated into a lower wage-income region and a higher property-income region, passing from an exponential distribution to a power law distribution.

It should be noted that, different studies usually rely on different datasets for income, hence they will result in differing explanations, e.g., some researchers use wage data from various sources and the others use income data reported by the government. Since we are going to explain how the wage and income are distributed as a simulation result of our agent-based models, it is worthwhile to differentiate between wage and income

strictly. “Wage” is defined as compensation, usually in financial form, received by workers as exchange for their labor. “Income” is defined as the consumption (or saving) opportunity gain by an entity (e.g., household or individual) within a certain period; it is the sum of all wages, salaries, profits, interests payments, rents and other forms of earnings received. As a result, wage is only one part of income, and is not equal to income. For those employees who work for salary only, wage might be their only source of income, yet for employers who own firms and pay the workers for their labor efforts, they might have many other sources of income, such as rents, interests, profits from selling products, and so on. Because wage is narrower than income, the empirical wage distribution might be more narrowly distributed than the income distribution as well. In reality, income data is an important indicator of the economy situation especially in the cross-country comparison, but reliable statistics on income is extremely difficult to collect. This might be due to the fact that the real income from rich people is not easy to collect. As a rough comparison, most of the data describing income distribution use wage data as approximation.

In the past decades, increase in income goes hand-in-hand with technology and productivity growth. It is a pity that the above-mentioned studies cannot explain the ever-changing empirical income distribution, because most of their selected cases are limited to the income distribution based on a specific year (or within a short period). Fig. 2.4 provides the income distribution plots for Japan²⁰ at two selected years 1970 and 1998. Although the right figure looks very close to a Lognormal distribution (note the x-axis is in logarithm), the left figure looks much more complicated with more than one peak, which looks like a multimodal distribution.

As a matter of fact, the multimodal distribution, which is a continuous probability distribution with two or more modes, is not rare in our daily life. For example, a histogram of the heights of students in a high school class would be bimodal when the class contains a mix of boys and girls (Iversen and Gergen (1997), Devore and Peck (1997),

²⁰Provided by Swedish software company “Gapminder”, <http://www.gapminder.org/>

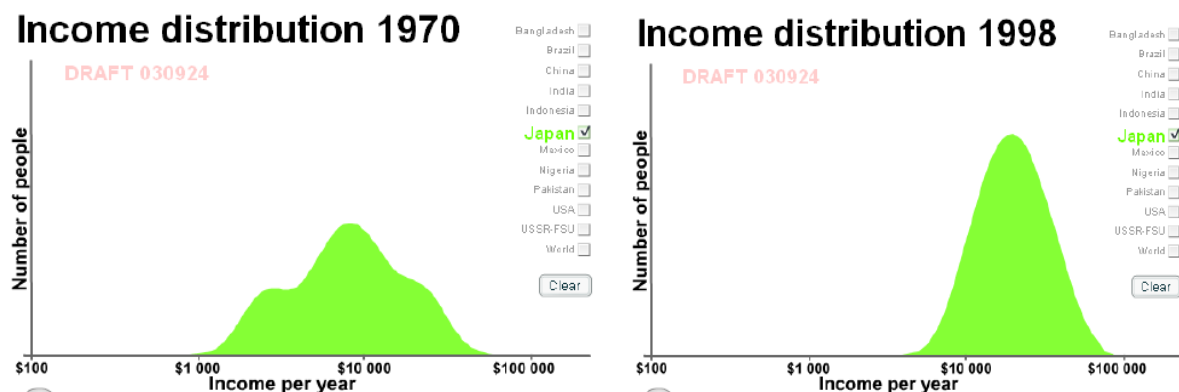


Figure 2.4: Income distribution of Japan in 1970 and 1998.

Source: <http://www.gapminder.org/downloads/income-distribution-2003/>.

Wild and Seber (2000)); however, when the heights of boys and girls are examined separately, both their distributions are actually unimodal. Other phenomena having the same character include the age of incidence of certain diseases, e.g., Hodgkin’s Lymphoma (Stein and Morgan (2003)), the spatial distribution of earthquake dispersion (Zhu and Long (2000)), and many others.

The finding that the entire region of income distribution follows a bimodal or so-called “twin peaks” distribution has been mentioned by Quah (1996, 1997, 2002), Jones (1997), Martin (2006), and others. Martin (2006) examines the World Distribution of Income (WDI) data by integrating individual income distributions for 138 countries between 1970 and 2000, and plots the evolution of country specific income distribution for many countries. Fig. 2.5 provides plots for China, India and the U.S. in the year 1970, 1980, 1990 and 2000. Note that all the countries’ density functions of income “shift” to the right over time, reflecting increases in income. For the case of China, the enlarging variance shows that the rich citizens increase their incomes much faster than the poor, which implies significant raise in *income inequality*. This situation is comparatively better in the U.S. and India. In addition, Martin (2006) also use some other countries’ empirical income figures to show that bimodal or tri-modal is

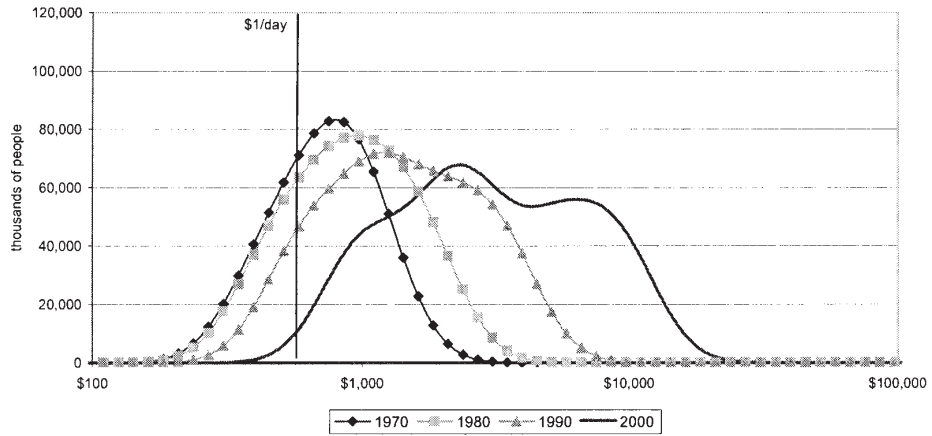


FIGURE IIa
Distribution of Income in China

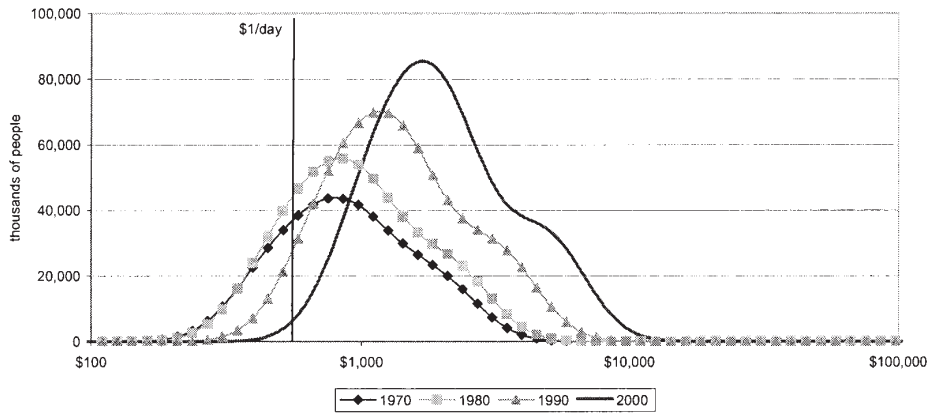


FIGURE IIb
Distribution of Income in India

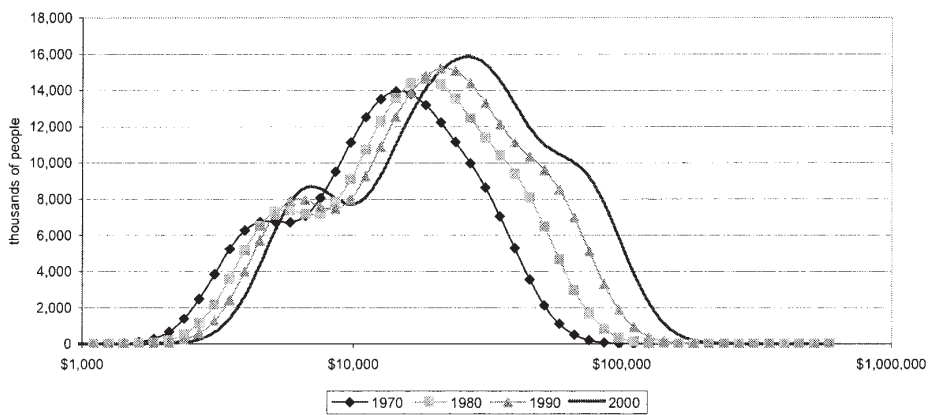


FIGURE IIc
Distribution of Income in the United States

Figure 2.5: Income distribution of China, India and the U.S., source: empirical plots from Martin (2006).

quite common in the income distribution, which reveals a hidden fact that income distribution may be made up of more than one income groups. Nevertheless, in some years, clear multiple modes are perhaps not directly visible. It might be because the means of these underlying distributions are not far away from each other to display additional modes. In Chap. 5, we will use our agent-based model to more-or-less simulate this multi-modal distribution in the aggregate income distribution.

2.9 Summary

This chapter provides a literature review of some empirical distributions, which will be mentioned or recovered later by our agent-based models. In general, it gives an overview of the firm dynamics (firm size, growth, age, entry and exit, profit, etc.), the GDP growth distribution, the income distribution, and other closely related topics. Since our world is full of random effects and is much more complicated than the model, it is not surprising that most of the studies discussed above have not reached an agreement. As far as modeling is concerned, only very few influential factors can be taken into account. Therefore, it is not possible for a model to exactly replicate the empirical distributions. It is our hope that through modeling and simulation, we can have a better understanding of how the interactions among different factors could exhibit scaling behaviors (laws) in the real world.

What's more, through reviewing the literature, it is interesting to find that most of the *size distributions* display power laws in the upper tails. Here “size” can be firm size measured by total employees, sales, assets or capitals, income distribution of different countries, the trading volumes in stock market (e.g., Mu et al. (2008)), the distribution of city size (e.g., Garmestani et al. (2008)), GDP across countries (e.g., Guilmi et al. (2003)), and etc.

Meanwhile, if we use the *logarithm* on the size increment to describe the growth distribution, the aggregate growth rate distribution may be close to a tent-shaped or

bell-shaped distribution (i.e., Subbotin class distribution, such as Laplace, Gaussian, and others). Studies on growth rate distribution range from the firm growth rate distribution in terms of sizes, sales (e.g., Teitelbaum and Axtell (2005)), to the growth rates of industrial production (e.g., Fagiolo et al. (2007)) and outputs (GDP) (e.g., Fagiolo et al. (2008)), among others.

Chapter 3

Reconsideration of the Social Architecture (SA) Model

3.1 Understanding of the SA Model

Our base line model is named “Social Architecture” (SA) model by Wright (2005). Before we extend the model, it is necessary to introduce the idea of the SA model, the author’s general findings, as well as our understanding and motivation for further model replication.

The fundamental relation in modern society is the “relation of production”, which includes manufacturing, sales and consumption. The SA model describes the social relations of production within an advanced capitalist economy, in that *“a small class of capitalists employ a large class of workers, organized within firms of various sizes, to produce goods and services for sale in the marketplace. Under normal circumstances, the capitalists, as owners of firms, collect revenue and workers receive a share of revenue in the form of wages”* (cf. Wright (2005), p.590). The model was invented on the belief that many striking phenomena of capitalism are results of structural features in the social production.

We interpret “social relation of production” as in Fig. 3.1¹, where we have a production cycle and a monetary cycle. It looks straight-forward that the two cycles always go in opposite directions. This infinite process constitutes the operation of our modern economy. The production cycle starts with the individual agent, each one represents a single household, or an self-contained economic entity, such as a firm. The firm owners are responsible for hiring workers and producing goods, and the workers offer labor in exchange for wages. As a result, firms convert workers’ labor into various products, sell them in market and pay the workers. In order to survive, the workers use their wages to consume, so that money goes back to firms for possible further investments, e.g., hire more workers or produce more goods. More specifically, the entire production process is carried out in a way that something enters the process (inputs such as labors), something is transformed, and something utilizable comes out (outputs such as cars). Because all the input and output quantities during this process are heterogeneous, it is equivalent but more convenient to reduce them to a common denominator and express everything as monetary values², so that all materials, labor, resources can be treated on monetary basis. This means when building up economic models, the relations among different production factors can be quantified and reproduced in terms of monetary factors, thus directly modeling the perplexing “production cycle” can be avoided. The SA model and other models described in this study apply this methodology. Although the whole process of production as well as consumption is simplified as the flow of “money” among firms, workers and the market, it actually replicates the inherently much more complicated production cycle of modern economies. For example, “assets” redistribution in the SA model can be represented by exchange of “coins”, which may be cash or physical products in reality. To realize the exchange, the firm owner trades his goods in market and gains “coins” as revenue; since these “coins” can always

¹The figure is inspired by Jürgen Mimkes’s presentation “Differential forms — a new tool in Economics: from biological models to Econophysics”, when he discussed the usage of differential forms in Econophysics, at the Econophysics Colloquium Kiel, August 2008, Germany.

²This “money” in reality, takes the form of value items such as banknotes, financial papers, contracts, leases, and so on.

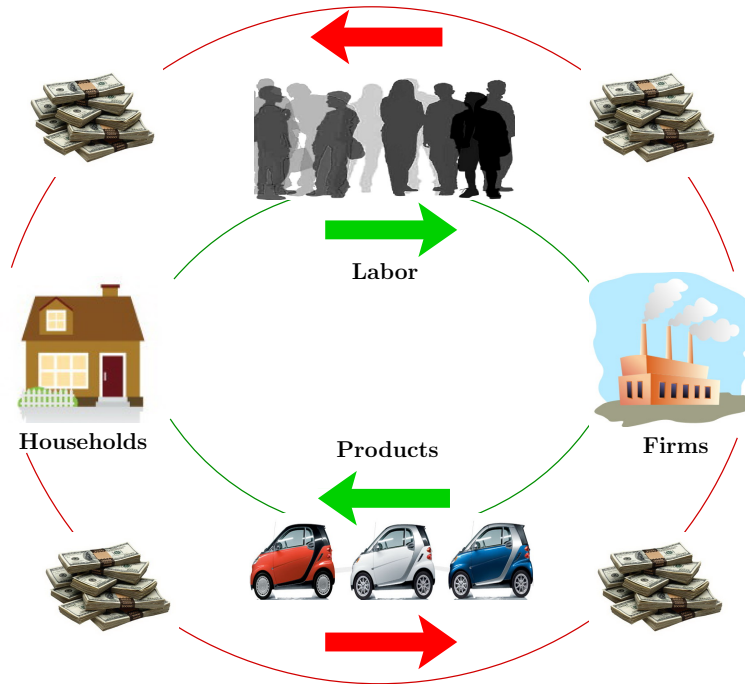


Figure 3.1: Production cycle and monetary cycle

counterbalance the products been traded, the exchange behavior will be observed in models in the way of “having profit” instead of “selling goods”.

Agents’ behavior in the SA model is described in sequence of steps using *pseudo code*. The system consists of three components: a set of rules, a working memory and a rule interpreter. Each rule includes a *condition*, which specifies when the rule should be executed (e.g., “firing”), and an *action* part, which determines what is the consequence of the rule “firing”. After the program starts, agents’ actions are recorded in the working memory, while the rule interpreter takes the job of considering each agent or each rule in turn, and repeats this cycle. As the loop goes on, agents have the potential to learn about their environment and the other agents through adding to their “knowledge” (held in their working memories). The behavioral rules and the rule interpreter, however, remain unchanged.

Apart from that, agents in the SA model exchange states continuously, such as shown in Fig. 3.2. Every single agent in the model must hold one (but no more than one) of

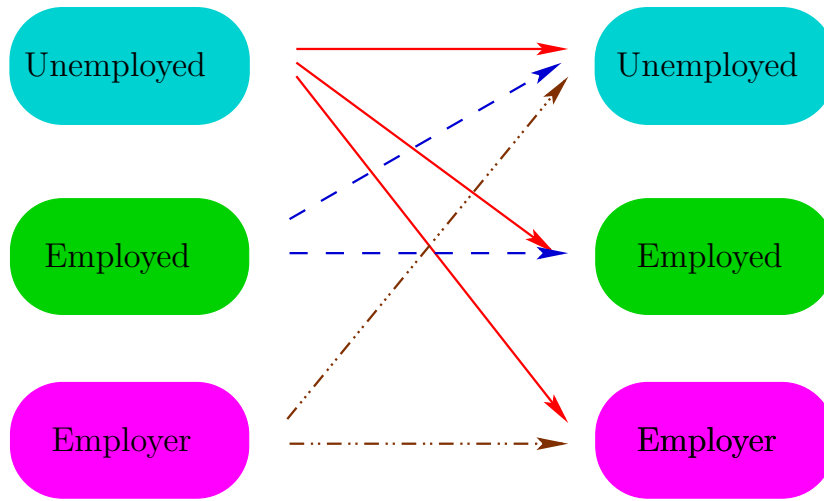


Figure 3.2: Exchange of states in the SA model

the stages — he can be either an unemployed, a worker (i.e., employed) or an employer. As illustrated, it is possible for an unemployed agent to remain unemployed, become employed, or start a business as an employer in the next period, but it is not possible for a worker to become an employer directly (or vice versa) without an intermediate stage that he becomes an unemployed (loses current job) in the first place.

3.2 Simulation of the SA Model

The SA model describes a “closed” economy (i.e., an autarky that is self-sufficient). This economy does not take part in international trade and is not affected by the rest of the world. The system consists of a complex local network of N economic agents, with $i = 1 \dots N$, $N \gg 1$, who may be either an employer (capitalist, denoted by C), an employee (denoted by E) or the unemployed³ (denoted by U). Each actor belongs to only one class, $C \cap E \cap U = \emptyset$, and the total population set is $A = C + E + U$ ($A = N$).

³Although “unemployed” and “non-employed” is not the same in empirical research, we don’t differentiate them in the SA model: an actor who is neither an employer nor an employee, should be in an unemployed stage.

As introduced before, all agents are proactive, reactive and social. They are free to exchange states when certain requirements are met. At the beginning, there are no firms or employment relations: every agent is considered to be unemployed, a household is treated as a single actor. It is also imposed that each firm should be owned by only one actor, and there should be at least one person other than the firm owner employed by a firm, which means co-owners of firms or self-employment is not possible. Consequently, the number of firm owners should be always equal to the number of firms.

Money is used as a counterbalance for exchanging goods and services. The firms produce goods and services, the workers are paid for their labor by salaries, and at the same time, the consumers (including workers themselves) pay for the goods and services in the market. In the whole process, every payment must be counterbalanced. The value of money maximizes simultaneously the utility of all buyers and sellers. What is interesting for us is the amount of money being exchanged, instead of which product to be traded (or in what form), hence only the amount of money Δm that changes hands will be recorded in the model. Unlike reality, where the maximum value of a paper money note is (sometimes) not more than a hundred, and the minimum note is (sometimes) not less than one cent, the “coin” using in our model represents a unique currency, which is assumed to be non-negative (i.e., debt is not permitted) and indefinite divisible, $m_i(t) \in \mathbb{R}^+$. At initialization time $t = 0$, each actor i holds an equal amount of “coins” m_i , thus the total “coins” hold by all actors is $M = \sum_{i=1}^N m_i(t)$. Meanwhile, a market⁴ is created with initial value $V = 0$, where the actors can do fair trade. As time goes on, money coins are free to be exchanged among actors, or flow into the market. The total money in circulation is $W = M + V$. Note that in the original model, since only exchange of money between actors and market happens, and money in circulation is always conserved, consequently, V increases when M decreases, and $W \neq M$ immediately after the model starts.

⁴Wright (2005) defines V as “market value”. In his later paper Wright (2008), he explains V as “effective demand”, which consists of personal and productive consumption as well as investment.

The character of an actor can be fully described by a state variable $S_i(t) = \{m_i(t), e_i(t)\}$, which indicates agent i 's money holding and employment state at the time t . If $e_i(t) = 0$, the actor is not employed, otherwise if $e_i(t) = j$, the employer j hires i as one of his employees. The state evolution from $S(t) \rightarrow S(t + 1)$ is determined by a set of 6 simulation rules, which take place at each unit of time, measured in “months”. In fact, the interval length of time can also be either “years”, “days” or “hours”, and “month” is chosen here typically because most of the employment and payment are happening once per month in real life. As simplification, no matter how many times consumptions are made, we suppose that the sum of money spent within that month is computed at once at the end of the month after a monthly account balance (i.e., workers receive salaries, firm owners pay their workers, etc.).

Here is a short description of the simulation rules⁵:

1. *Actor selection rule:*

Randomly select an actor a from the set A according to (a draw from) uniform distribution.

2. *Hiring rule:*

(a) If actor a is unemployed, then his potential employer can be either an existing capitalist or an unemployed who tends to become an employer in the next period (potential employer set $H = C \cup U$).

(b) Choose an employer c for a according to the probability function of Eq. 3.1, that weights potential employers by their wealth:

$$P(c) = \frac{m_c}{\sum_{x \in H} m_x} \quad (3.1)$$

(c) The wage interval $w = [w_a, w_b]$ is an exogenous parameter, and the average wage is $\bar{w} = (w_a + w_b)/2$. If c 's money holding exceeds the average wage, $m_c > \bar{w}$, then c hires a .

⁵For detailed description of the simulation rules, please refer to Wright (2005).

3. *Expenditure rule:*

Randomly select a consumer actor b from the set $\{A - \{a\}\}$ and an expenditure amount m from b 's money holding $[0, m_b]$ according to uniform distribution. Add m coins to the available market value V (as expenditure by consumer b). Hence, m_b is reduced by m , and the market value V is increased by m .

4. *Revenue rule:*

- (a) If actor a is not unemployed, a can be either an employer or an employee. In both cases, he works for the firm, and gains profit m from the market. The amount m is uniformly selected from the market value $[0, V]$.
- (b) If actor a is an employee, then transfer m coins to the employer. Alternatively, if actor a is an employer, then transfer m coins to himself. All transferred money is counted as firm revenue.

5. *Firing rule:*

- (a) If actor a is an employer, he determines the number of workers to fire according to function 3.2⁶, so that the firm's workforce is reduced to a size that the wage bill is payable.

$$u = \max\left(|W_a| - \left\lceil \frac{m_a}{\bar{w}} \right\rceil, 0\right) \quad (3.2)$$

- (b) Select u actors from the set of employees W_a according to uniform distribution, and fire them.

6. *Wage payment rule:*

For each worker in a 's employee set, pay a certain wage amount w , where w is uniformly selected from a minimum and maximum wage interval $[w_a, w_b]$. If

⁶The employee set W_a shows the number of workers already been hired by employer a . The number of employees that a can hire is determined by his wage bill (m_a) divided by the average wage (\bar{w}). This formula means no worker will be fired if employer a 's ex-ante wage bill is payable from his current money holding.

employer a has insufficient budget to pay the selected wage w , w should be re-selected from the discrete interval $[0, m_a]$ according to a uniform random draw.

To simplify the simulation procedure, the following happens during each month⁷:

- If an actor is unemployed, he may remain unemployed, or get hired by an existing firm, or form a new firm and become firm owner. As an unemployed, he may not have income, but still he can spend some money from his own savings, so that the market value is increased by his expenditure, and his own money holding is lowered down.
- If an actor is an employee (worker), he may maintain the current state, contribute to the firm revenue by his labor, and receive wage as compensation. Otherwise, he may get fired and switch to be an unemployed when his employer does not have sufficient money to pay him. In both cases, the actor needs to make consumption in the market to survive.
- If an actor is an employer, his wealth is increased by the firm's revenue from selling the products (made by workers) as well as effort from himself. At the end of the month, he should pay wages to current workers, decide how many workers to fire or hire, and his own consumption amount. In case the employer fires all his workers, he does not own a firm any more, and enters the unemployed class.

Alternatively, Fig. 3.3 displays a graphic structure of the 6 simulation rules for better understanding. Basically, when an actor a is selected, the computer first checks whether he is unemployed or not. If actor a is unemployed, then he is possible to be selected as a worker by certain firm through the "hiring rule". If actor a is not unemployed,

⁷This is the general idea of the model, note that the original model does not allow the actor a to make consumption within one iteration. Instead, the consumption right is passed down to a uniformly selected actor b .

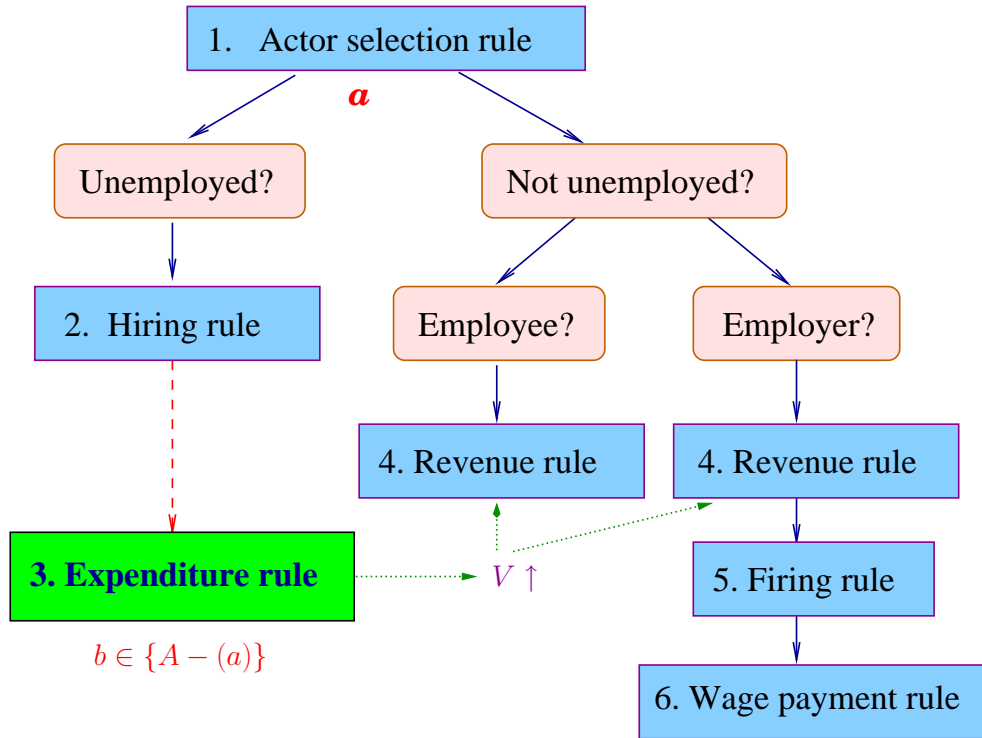


Figure 3.3: Simulation structure of the SA model.

the computer will further check whether he is an employee or an employer: if he is an employee, he works for the firm and earns revenue through the “revenue rule”; if he is an employer, as the owner of firm, he also needs to pay wages to his workers, and decide how many workers to fire according to his current budget. Note that one shortcoming of the original SA model is that, although the “expenditure rule” is positioned as the third rule, it is a rule that a will never perform in the current iteration. Instead, this rule will be executed by another actor called b ($b \in \{A - \{a\}\}$), therefore, it is connected to the other rules using dotted line.

The above given rules are “executed N times (per month) to allow each of N actors an opportunity” to realize all possible rules from 1 to 6. A year counter records every 12th applications of monthly calculation, so that once the intended simulation year is given, the whole program will perform $12 \times \text{years} \times N$ times iterations of the whole process. There are three exogenous parameters: the total money holding by all agents

M , the total number of actors, N , and the wage interval w . Unless specified, a standard simulation with $M=100,000$, $N=1,000$ and $w=[w_a, w_b]=[10, 90]$ is applied.

The result of the simulation is not sensitive to initial conditions. After the program starts, it will rapidly self-organize into a stochastic equilibrium characterized by stationary distributions. The details of the original simulation results can be found in Wright (2005). Since we investigate the possibilities to improve the SA model, some distributions before (replication according to the original rules) and after modifications will be provided in the next section. In short, the general findings according to Wright (2005) are: (1) the class distribution of agents follows a normal distribution; (2) the firm size distribution follows a power law; (3) the firm growth distribution follows a Laplace distribution; (4) the firm demise distribution follows a Lognormal distribution; (5) the log-GDP growth rate distribution follows a Laplace distribution; (6) the duration of recessions follows an exponential law; (7) the wage share is normally higher than the profit share in national income, and their ratios look normally distributed; (8) the lower region of the income distribution is Lognormally distributed, while the higher region is close to a power law; (9) the rate-of-profit distribution is highly right-skewed, and might be dependent on firm size.

3.3 Some Considerations on the SA Model

The purpose of this section is to present our reconsideration and improvement suggestions to the original SA model. We will show some possible modifications or extensions that can be added to the model. After each suggested modification, simulation results after modification will be provided in comparison with the original results reported in Wright (2005). It should be mentioned that the discussion of any modification does not mean that the original simulation results from the SA model were “incorrect”, and it just tells that we are improving the model in a certain way and making it more plausible. It is also probable that combining all these modifications, simulation results

will be “significantly” changed, due to our models’ high sensitivity on simulation rules. The EPC model discussed in the next chapter will incorporate all these improvements into consideration.

Before conducting the following experiments, we replicate the SA model in strict accordance with the original rules. Except for some stochastic differences which come from random selections, we generate most of the distributions very similar to those in Wright (2005), and only few of them (as originally formulated) can not be obtained⁸. All simulation results are neither sensitive to the number of participating actors, nor to the length of the simulation. The convergence period for the model, which explains most of the extreme outliers in some figures, is as short as 7-10 years. Because this period is comparatively short to a 100 year simulation, to have a better overview of the results, all following figures show a complete simulation with transient periods included. Curve fitting applied to some histograms offers a means of quantitative comparison of modified simulation results with the original figures.

3.3.1 On Actor Selection Rule

The uniform distribution is the single distribution adopted many times in the SA model. The idea is based on “Bernoulli’s Principle of Insufficient Reason”, which says that in the absence of better knowledge, all outcomes can be equally likely. This applies in cases that the selection times are *sufficiently large*, then all outcomes within the selection interval are equally likely. However, when applied in our simulation, we find that this theoretically correct method can lead to some computational problems: due to too *limited* times of selection, it is difficult for uniform selection to pick up every number equally. This weakness becomes more severe when applied in the “actor

⁸In our replication, the figures are usually quite similar to those of Wright (2005), but we find some differences in the curve fitting part. For example, it seems that the original firm demise distribution follows a normal distribution instead of a Lognormal, and the original distribution of log-GDP growth rates looks normal instead of Laplace.

selection rule” than the other rules, since it immediately turns out that some actors will be selected many times, while some others will not be selected until the end of an iteration.

The “actor selection rule” says *randomly select an actor a from the set A according to the uniform distribution*. After the program starts, it will be repeated N times to “allow each of the N actors an opportunity” to go through the rest of the simulation. The left figure of Fig. 3.4 explains how the computer makes choice if we only repeat 100 times of selection in the hope that each of 100 actors can be selected once. Similarly, the middle figure provides 1,000 times of selection out of 1,000 actors, which is exactly the case of our standard simulation. Both figures show that when the number of selection times is the same as the number of agents, no matter how many actors are in the model, it is highly possible that some actors will be chosen more often than the others. In fact, the first two figures of Fig. 3.4 also show that some actors are even selected more than 5 times, while others are simply neglected. When implemented in our program, it means a single agent can be fired, hired, or paid several times within a month, and a few others may not be even considered. In contrast, if our selection times can be *sufficiently large*, e.g. in the last figure of Fig. 3.4, where we have 100,000 times selection among 100 integers, the frequency of each number chosen looks more equalized. Unfortunately, this will not happen in the original SA model, because it regulates the number of selection times to be equal to the number of actors.

To overcome this significant shortcoming and give each actor an equal opportunity to be active, our suggestion is to use a simple “for-loop” with “ $a = 1 : N$ ” instead of the original “actor selection rule”. This common “for-loop” guarantees that each actor takes turns to be active one by one, which is exactly what the SA model should realize according to the author’s description of the actor selection rule (cf. Wright (2005), p. 591–592). One may argue that given actors are selected based on natural order, there might be causal relations among the order of the selected actors, namely an actor who is chosen later might have fewer choices than those who are active before him.

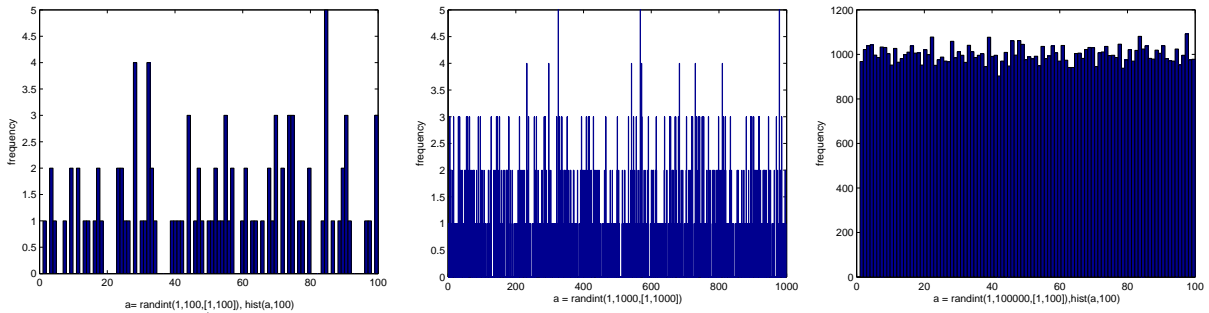


Figure 3.4: Selection errors from the uniform selection rule. Histograms of 100, 1,000 and 100,000 times of uniform selection (left to right). Left: frequencies of each integer number chosen within the interval $[1,100]$ with 100 times selections. Middle: frequencies of each integer number chosen within the interval $[1,1000]$ with 1,000 times selections. Right: frequencies of each integer number chosen within the interval $[1,100]$ with 100,000 times selections.

An ideal but easy way to provide fair selection is to first shuffle (interleave) the order of the actors before each round of iteration. Thus regardless of each actor’s index, his acting order each month is kept random and varied throughout the whole simulation.

To highlight how the “for-loop” applied in the “actor selection rule” will change the model behavior, we offer two examples of simulation results with the original “actor selection rule” and with the “for-loop”. To make it more straight-forward, only the simplest method (i.e., without shuffling orders) is applied, and we temporarily call it “simple for-loop”⁹ as comparison to the original “actor selection rule”.

Fig. 3.5 plot histograms of the class distributions, with each series fitted by a Gaussian distribution (the smooth curve). The left column figures are simulation results with the original “actor selection rule”: they reveal that on average 12.4% of population are employers, 70.4% are workers, and the remaining 17.2% are the unemployed. These values (as well as standard deviations) are very similar to Wright (2005)’s when selection randomness is taken into account. The right column figures show simulation results with the “simple for-loop”: we compute that around 14.9% of the population

⁹A temporary name for this rule. In fact, this may not be designed as a simulation rule, since “for-loop” is just a normal iteration process in programming languages.

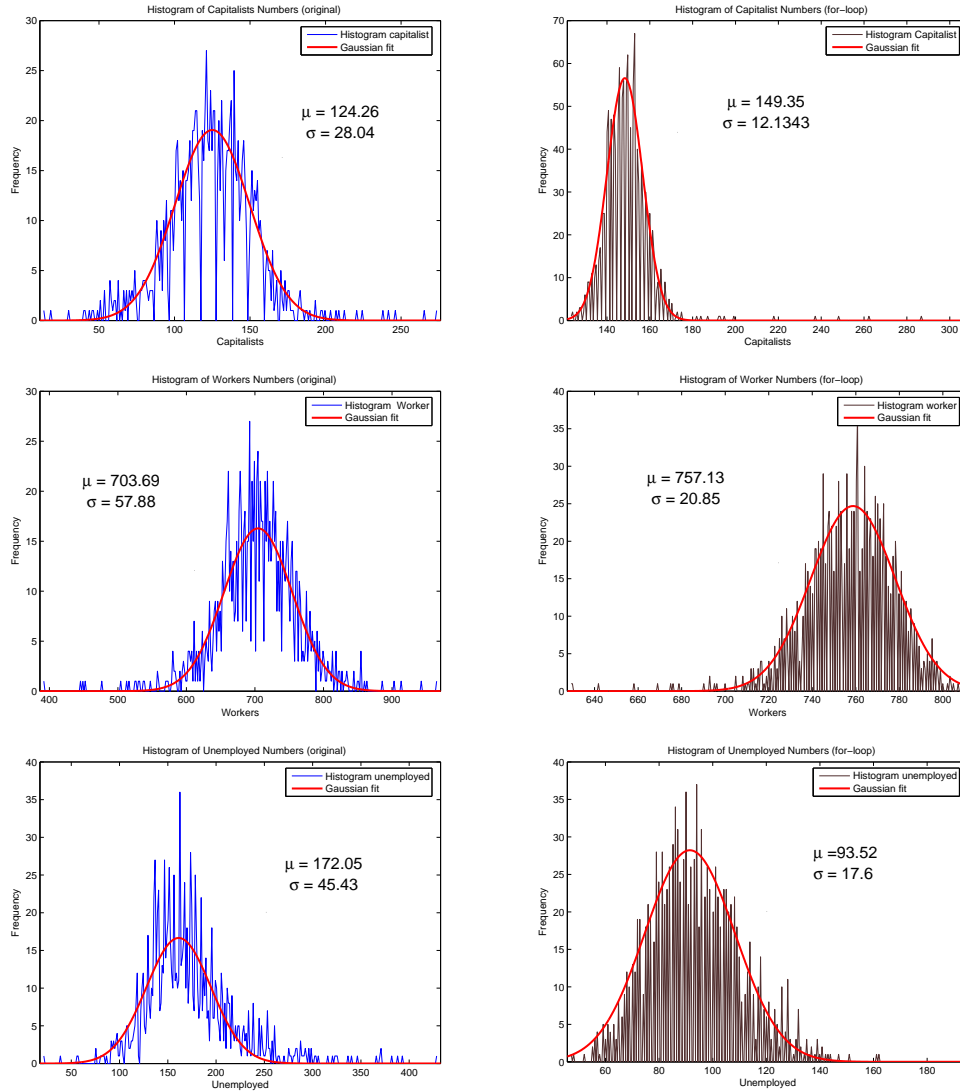


Figure 3.5: Application of the “simple for-loop” in the class distribution of agents. Histograms of number of actors in each economic class with a constant bin size of 1. The figures of the left column are from replications of the original rules (compared with Wright (2005), Fig. 1), whereas the right column figures are simulation using “simple for-loop”, *while remaining all the other rules unchanged*.

are employers, 75.7% are workers, and only around 9.4% are unemployed. Compared with 17%, the 9% unemployment rate, coming from modified model, is perhaps closer to empirical rate of unemployment¹⁰.

If we take a second look at Fig. 3.5, the variances of distributions before and after changes are very different: the standard errors of the later are reduced by more than half, so that we arrive at improved Gaussian fits with less noises (i.e., outliers). For example, the bottom left figure of Fig. 3.5 shows that the original unemployment population ranges from 10 (i.e., almost full employment) to 450 (i.e., half population are unemployed). For comparison, the bottom right figure shows a much narrower Gaussian fit, with minimum unemployment of 55, and maximum around 180, which is explicitly much more plausible. The highest unemployment rate most probably comes from the first several months of the iteration, since at initialization, all actors are in the “unemployed” states.

Fig. 3.6 plot the firm size distribution under a shortened length of 15 years’ simulation, as well as a standard 100 years’ simulation. The original firm size distribution plotted in Wright (2005) (cf. Fig. 2, p.600) applies only 15 years’ simulation. What has drawn our attention is that, if we test the simulation for a longer period, the firm size distribution under the original setting will have ever increasing mean and variance until it reaches its higher bound where the largest firm nearly employs the whole workforce¹¹ (cf. left

¹⁰It might be hard to compare our model predicted unemployment rate with real data. First of all, the real economy also contains other sectors, such as the agriculture sector, which may have different production processes from the industry sector. Secondly, the officially announced “unemployment rate” counts percentage of people in the labor force who do not have jobs and are actively seeking jobs as the unemployed, therefore, it tends to underestimate the “true” unemployment rate which is the sum of both the “unemployed” plus “discouraged workers”, students, homemakers, and so on. In our model, for simplicity, “unemployed” is equivalent to “non-employed”. Those agents who are neither employers nor employees are considered as the “unemployed”, thus the model generated “unemployment rate” should be higher than reality.

¹¹This is similar to Gibrat (1931), where the firm sizes also have increasing mean and variance as time goes by, and the model has no steady state.

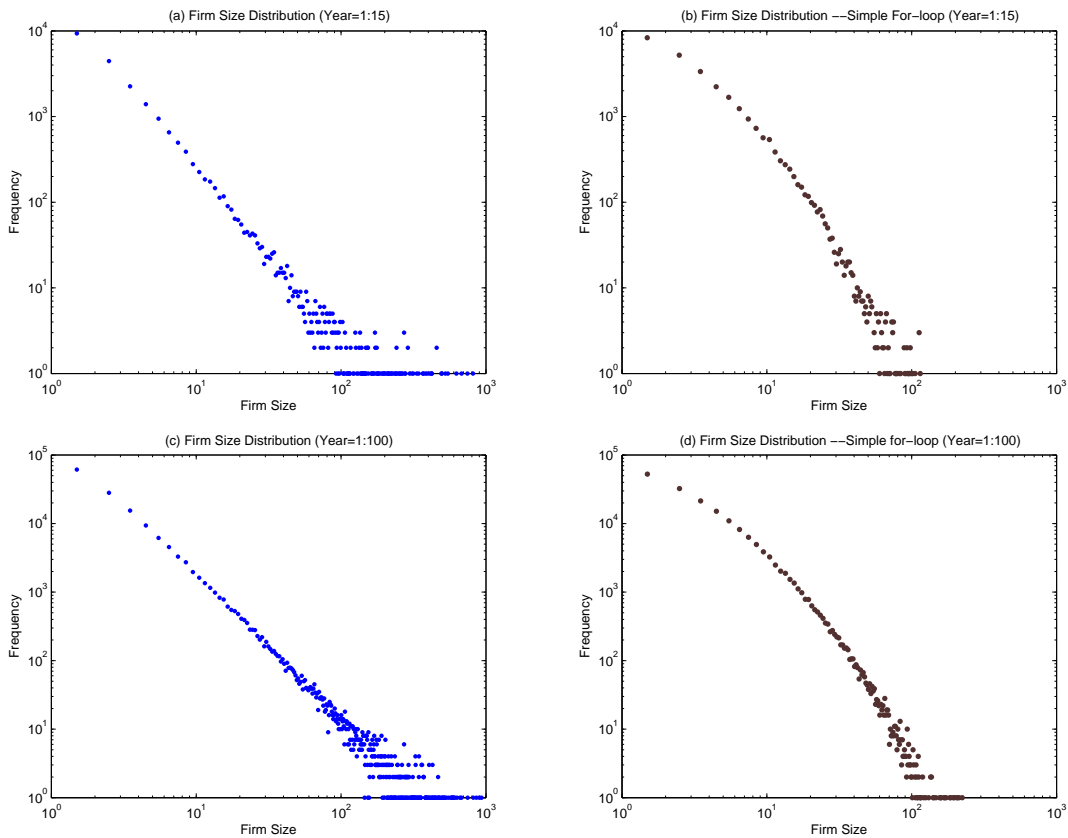


Figure 3.6: Application of the “simple for-loop” in firm size distribution. Histograms of aggregate firm size distribution in log-log scale (Year=1:15 and Year=1:100). Sub-figures in the left column are simulations using the “actor selection rule” with 15 and 100 years’ simulation lengths, and the figures in the right column are the corresponding figures with “simple for-loop”. Note that all figures are with transient periods (around 7-10 years) included, since we need enough data to generate a firm size distribution of 15 years for the top figures. The firm size distributions with or without transient periods for 100 years’ simulation will look visibly little different. Finally, *except for the “actor selection rule”, all the other rules remain unchanged.*

column figures of Fig. 3.6). For comparison, the right column figures also have a more obvious nonlinear curve shape, and the curvature seems to be less noisy in a prolonged simulation.

A closer look shows that, although Wright (2005)’s 15-year firm size distribution is said to follow Zipf’s law, it applies only with data truncation, which excludes too large ($x \geq 600$) and too small firms ($x \leq 10$), as shown in the left hand of Fig. 3.7. Since

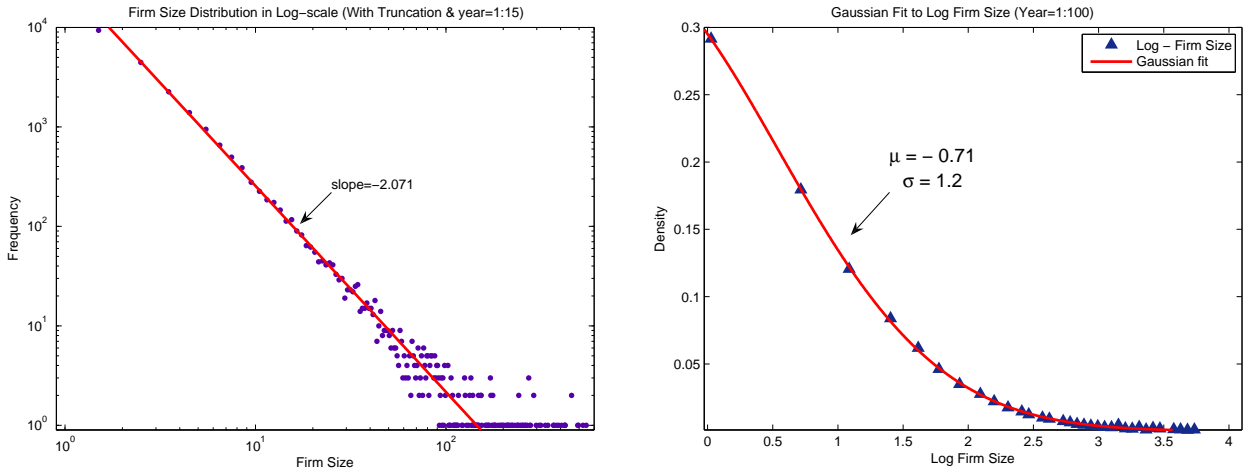


Figure 3.7: Application of the “simple for-loop” and curve fitting to the firm size distribution. Left: histogram of the firm sizes measured by the number of employees in log-log scale under the original rules (Year=1:15, replication of Wright (2005) Fig. 2). The truncation excludes $x \leq 10, x \geq 600, y \leq 10, y \geq 10,000$; it is then fitted into a power law distribution: $P(x) \propto x^{-(a+1)}$ with slope -2.071 , and $a \approx 1$ is known as Zipf’s law. Right: firm size distribution with the “simple for-loop” in linear scale (Year=1:100); the smooth curve is a Gaussian fit, which indicates the original firm size distribution closely follows a Lognormal.

a large majority of firms are small firms with no more than several employees, the left-side truncation is somewhat cumbersome. Meanwhile, longer period simulation (cf. figure (c) of Fig. 3.6) shows that there are a lot of large firms hiring more than 100 employees, in particular, the largest firm employs more than 700 workers out of a total of 1,000 agents, which contradicts Axtell (2001)’s empirical results for U.S. firms, that the largest firms may not employ more than one-tenth of the total workforce. However, no matter how long the simulation lasts, the largest firm generated under “simple for-loop” has no more than 105 employees, which seems more reasonable than the original cases of more than 700 employees.

In addition, the left figure in Fig. 3.7 is Wright (2005)’s curve fitting experiment for 15 years’ firm size distribution, whereas the right figure of Fig. 3.7 plots the logarithm of the firm size simulated under “simple for-loop”. The smooth line is a curve fitting which indicates that the firm size distribution under the “simple for-loop” follows

Lognormality instead of a power law, in accordance with the findings in Gibrat (1931), Chesher (1979), Sutton (1997), Cabral and Mata (2003), Allanson (1992), and others. When comparing the right figure of Fig. 3.7 with the right column figures in Fig. 3.6, we see that the log-log scale plots only emphasize the upper tail, whereas the normal scale provides a better fit of the entire region of the firm size distribution, especially the high frequency lower region. Note that the Lognormal distribution is skewed and bell-shaped, the extreme case of a monotonically decreasing Lognormal distribution happens when we have a small mean and variance as in this plot.

So far, we have shown two examples of how a slight modification in the “actor selection rule” can improve the model. In the meantime, there are also minor changes in the other distributions, such as the firm demise distribution, the fluctuations in wage and profit share, the GDP growth distribution, the profit-rate distribution, the income distribution, and others. In order to save space, these distributions before and after modifications will not be provided here. Following this chapter, the EPC model including all suggestions in this chapter will show a complete realization of the above-mentioned distributions.

3.3.2 On Wage Payment Rule

The “wage payment rule” is the only rule which uses the exogenous wage interval $[w_a, w_b]$. The wage interval is set to be $[w_a, w_b] = [10, 90]$, to better reveal the “large” wage differential as designed by Wright (2005). Workers are getting their monthly salaries according to uniform random draws from the wage interval $[w_a, w_b]$, and this is the only way for them to earn more “money”. As a matter of fact, “money” is the most important determinant for possible changes to the actor’s current state in our model, e.g., it decides whether an unemployed agent can be rich enough to become an employer. While replicating the model, we are asking two questions: (1) does the wage differential really matter? (2) is the uniform wage payment “realistic”?

Let's first look into the first question related to the wage differential.

In order to “generate data in good overall agreement with the empirical data surveyed”, Wright (2005) takes a reasonable wage interval $w=[10,90]$ to address the large wage differential found in many capitalistic economies. He also addresses that “the total number of coins, M , and the total number of actors, N , on condition that $M \gg N$, appear to act as scaling parameters and do not affect the relative dynamics, **unlike the wage interval parameters...**”, which explains that the varying of wage parameters will “qualitatively” change aggregate dynamics of the model.

Our first argument regarding the “wage payment rule” is that: *the total wage bill which an employer should pay to his employees will not be changed, regardless of which wage differential is arranged.* This comes from the special character of the uniform distribution: when the mean of variables is not changed, the sum of total remains unchanged. In other words, if the mean of the wage differential is the same, and the number of workers to be paid is fixed, the employer's wage bill will not be changed either¹². Particularly, in the “firing rule” (before the “wage payment rule”), the program will check whether a firm owner has sufficient money to pay his workers if each of them requires only the average wage (i.e., the number of workers actor a can hire is determined by m_a/\bar{w}), consequently, no matter how large the wage differential is, it has no influence to the “firing rule”. Since the employer's total wage bill is the only thing that could change his state, we believe that the SA model's class distribution, firm size, firm demise, wage and profit share will not be changed, even if the wage interval is shifted.

We test this idea by using different wage interval, e.g. $[40,60]$, $[45,55]$ or assign 50 coins wage for each worker, with all original game rules unchanged. The final plots look quite similar except for some random effects. It proves that the SA model is neither

¹²Mathematically, if the number of uniform selection times is fixed, no matter how the boundary is shrinking, the summation $(\sum(w_a + w_b)/2 \times N)$ will result in no difference, unless the mean value is different.

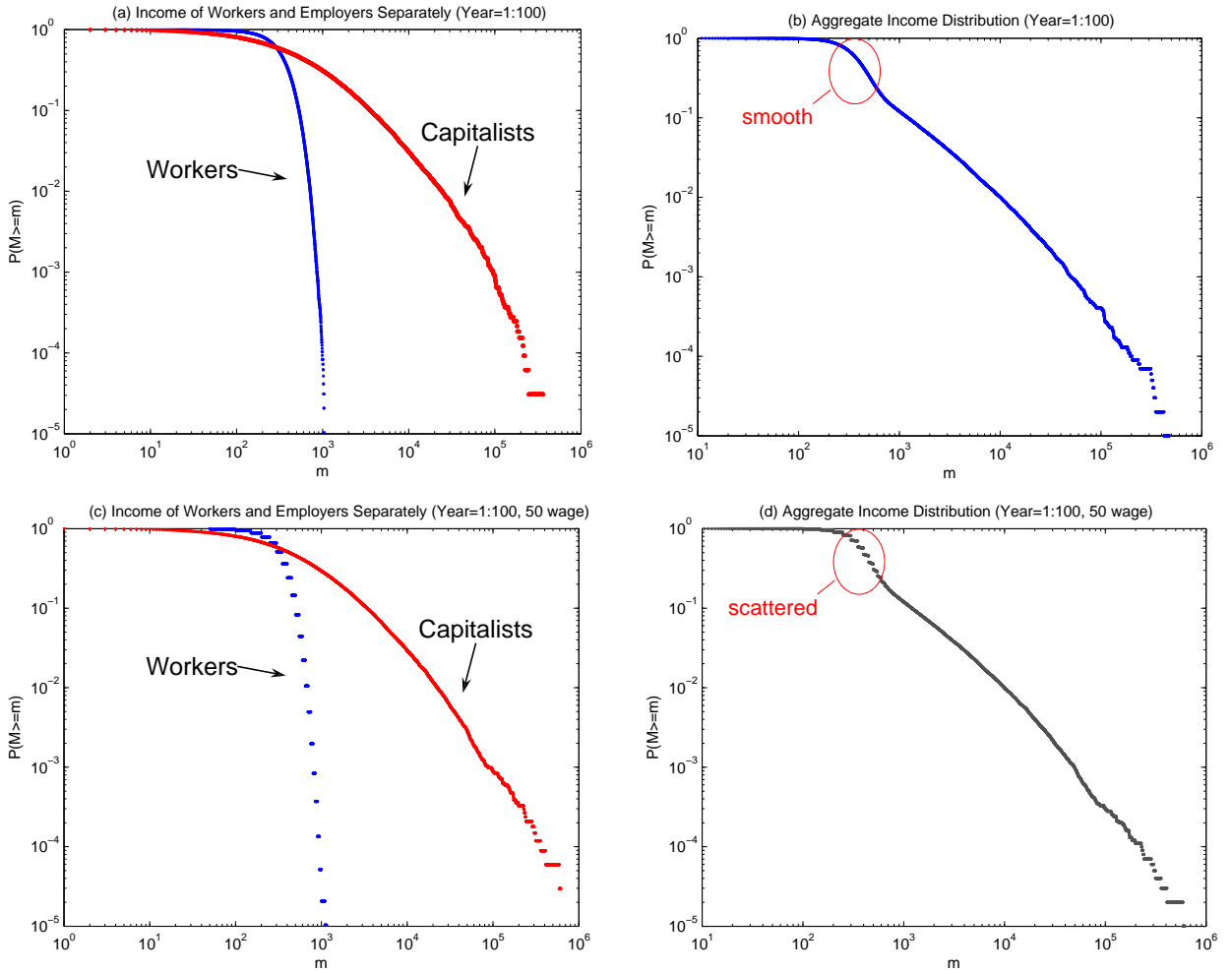


Figure 3.8: Income distribution under different wage payment conditions. Figure (a) (b): CCDF of separate income by workers and employers and an aggregate plot, according to the original SA model (compared with Wright (2005) Fig. 8 and Fig.9). Figure (c) (d): CCDF of separate income by workers and employers and an aggregate plot, when every worker receives the same wage amount of 50 coins (Year=1:100). *The bottom figures are from simulations without any other modification to the model, except removing the wage differential.*

quantitatively nor qualitatively sensitive to changes in the wage interval.

Apart from that, we would like to mention a slight difference in the income¹³ distri-

¹³Note that “income” for an employer in the SA model is actually the revenue which he earns through the “revenue rule”, without deducting the wage bill paid to workers. This might not be so convincing, and we will re-define income as “revenue - cost (wage bill paid)” in Chap. 5.

bution if we eliminate the wage differential and pay 50 coins salary to each worker. Fig. 3.8 shows the income distribution of workers and employers in complementary CDF (CCDF) in log-log scale. Figure (a) and (b) in the first row are replications of the original model, and the figure (c) and (d) at bottom are simulations without differentiation of wages (i.e., each worker receives 50 coins as wage). The left column figures (a) and (c) display the income by workers and employers separately, and the right column figures (b) and (d) are aggregate plots of their income in CCDF. It seems that the wage income for workers are more narrowly distributed (cf. the almost vertical line in figure (a) and (c) of Fig. 3.8), whereas the revenue income for employers has a long and stretching tail. Particularly, in figure (a), we observe a smoother curve, showing that workers are receiving all possible wages below an approximate maximum of $1080 (= 90 \times 12)$ coins. In contrast, in figure (c), the tail of the workers' wages becomes more scattered. This happens because without a “for-loop” preventing multiple choices of the same actor, an actor can be paid x times a month with the wage amount $50x$, but other wage amounts are not achievable. However, if we include the “simple for-loop” for actor selection as discussed in the previous section, we will not see so many scattered points: the possible wages for workers will be reduced to $50 \times 1 \rightarrow 50 \times 12$, depending on how long a worker stays in the firm¹⁴.

Moreover, when plotting the income distribution at the aggregate level (cf. figure (b) and (d) of Fig. 3.8), we can easily observe a “knee shape” in the lower tail. Our reasoning is that, the “knee shape” will always exist because incomes from the two classes are distinct: the wages as the only source of income for workers are naturally bounded and cluster around a certain mean value, while the employers' incomes are much wider, including selling products or other sources, and those incomes from employers constitute the upper tail of the income distribution. According to Wright (2005)'s study,

¹⁴As mentioned, we only report the figures before and after *one* small modifications. Therefore, income figures under the “simple for-loop” as suggested in the last section, is not provided here. How the overall modification will change the model behavior will be reported in the simulation results for the EPC model in the next chapter.

the lower tail of income distribution of the SA model is said to be Lognormal, and the upper tail is close to a power law, but our study shows a different result in the lower tail of income distribution.

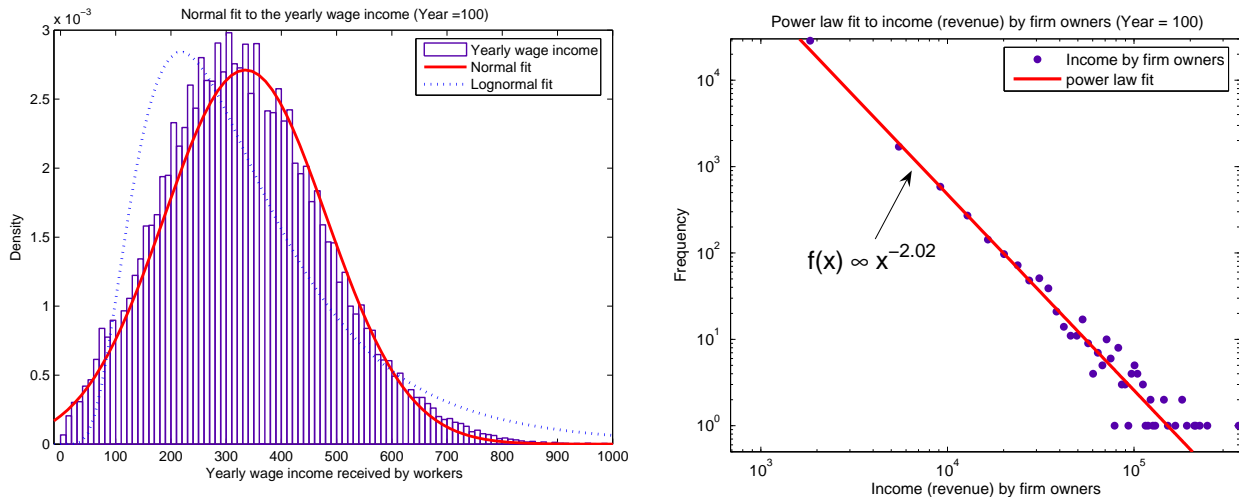


Figure 3.9: Histogram of yearly income distribution (Year=1:100). (Left: lower-income distribution measured by the wages paid to the workers, which can be fitted into a Gaussian (smooth curve, $R\text{-sq}= 98.9\%$, $R\text{-sq}(\text{adj})= 98.8\%$); The dotted line is a relatively poor fit to the Lognormal distribution, suggested in Wright (2005)). Right: higher-income distribution measured by property income from firm owners (in log-log scale), the smooth line is a curve fitting to the power law (Zipf’s law, $R\text{-sq}= 100\%$, $R\text{-sq}(\text{adj})= 100\%$). *The figures are from simulations without any other modification to the model, except our mentioned one.*

Instead of plotting in CCDF, the left figure in Fig. 3.9 records the PDF of yearly wages paid to workers. The dotted line is a Lognormal fit to the wage distribution suggested by Wright (2005), but the fitting is relatively poor when compared with a Gaussian (the smooth line with fitting reported using $R\text{-sq}$ values¹⁵). This is not surprising considering the “Central Limit Theorem”, which says that *the sum of many independent and identically (i.i.d.) distributed random variables will be approximately normally distributed, if the random variables have a finite variance.* To apply in the

¹⁵“R-sq” stands for “R-squared”. It is a statistical measurement of how well a regression line approximates real data points. The formula for R-squared is $R(X,Y) = Cov(X,Y)/\sigma_X\sigma_Y$, and a R-squared of 1.0 (100%) indicates a perfect fit.

“wage payment rule”, it means that as long as the workers’ wages are i.i.d., their yearly wage distribution will always be Gaussian, no matter how the monthly wage payment rule is changed (i.e., no matter whether the wage is uniformly selected or normally selected, etc.). For comparison, the right figure of Fig. 3.9 plots the histogram of monthly income by firm owners in a log-log scale, which can be fitted by a Zipf’s law, in accordance with many empirical findings (e.g., Axtell (2001), Fujiwara et al. (2004)). Since the workers’ wage distribution and the employers’ income distribution are very different, when they are plotted into the same figure in CCDF, there exists an overlapping transition area from the higher-wage workers to the lower-income employers. We agree that in an aggregate plot, the lower income region might still look different from a normal distribution. However, from the modeling perspective, it seems that the lower part of income distribution will look more skewed to the left (since overlapping might happen more frequently for the higher-income workers and the lower-income employers), instead of skewed to the right, as a Lognormal distribution (cf. the blue dotted line in the left figure of Fig. 3.9 is a Lognormal curve fitting to the workers’ wage distribution).

Our second question to the SA model’s “wage payment rule” is that: is uniform distributed wage payment “realistic”?

Fig. 3.10 depicts the histogram of wages that the employers pay to their workers in a short 10 years’ simulation (left figure) and a standard one of 100 years (right figure), respectively. During the simulation, all wages that employers pay to their workers are recorded. It shows that despite a sudden drop at the very low wage region ($[0, 10]$), the remaining part inside the “reasonable” wage interval ($[10, 90]$) still looks unlikely in reality: imagine if all wages are equally possible, then the number of workers who receive the highest wages is the same as the number of workers receive the low or middle level wages¹⁶. This contradicts reality that in industrialized economies, modern

¹⁶It is also odd that no matter whether 10 years or 100 years simulation is run, the histogram of wage payment is somewhat downward sloping from the lower to the higher values. This phenomenon

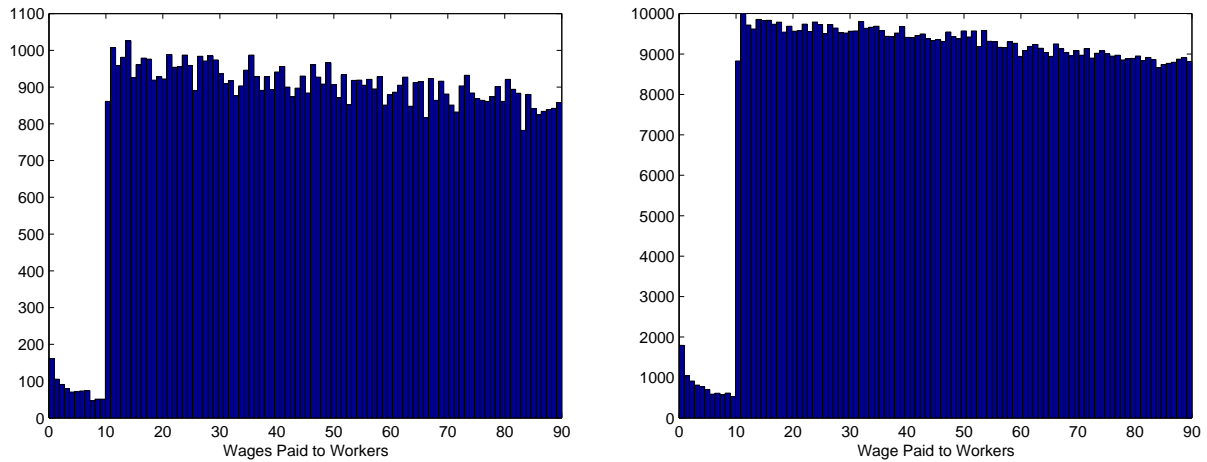


Figure 3.10: The distribution generated by the uniform wage selection rule of the SA model. The left figure is the aggregate histogram of workers’ wages selected using the “uniform wage selection rule” at a simulation length of 10 years. The right figure is the corresponding figure generated under 100 years’ simulation. Note that there is an obvious downward slope from the lower wage to the higher wage region.

businesses distribute salary according to an employee’s working experience, education, age, as well as other qualifications. Hence, it might be more reasonable to assume a small number of workers receiving the highest wages or the lowest wages, and the remaining majority receive the middle wages.

Besides, if we track a worker’s wages during the time he stays at one firm, the amount of wage he receives every month is completely random, ranging from 0 to 90. This means within a particular month, he may get a payment as high as 90, but the next month his wage may possibly drop to as low as less than 10 (or even 0). While in reality the wage is usually fixed at certain level for some periods according to an employee’s qualification, the big fluctuations in a worker’s wage payment generated by the original SA model will continue throughout his whole employment, which looks not very convincing.

possibly comes from the regulations in the “wage payment rule” that some employers may not afford the first wage selected, and resort to a second selection, which leads to an increase in the frequency of the smaller wage amount.

We suggest a slight modification to the original payment setting: choose the first wage randomly, then let the worker's wage remain at the same level as long as he stays at one firm; if at a certain month, the employer fails to pay the worker his "guaranteed" monthly wage, the firm owner will consider firing this too "expensive" labor or going bankrupt himself. What's more, if working is the only way to earn money, nobody will work for free, so paying 0 coins to workers for their work (as in the original setting) is perhaps not too realistic. It may be more reasonable to impose a lowest wage payment of 10 coins per month, similar as in many countries there is a minimum wage set by law.

This modification of regulating a minimum wage threshold will not change workers' monthly wage distribution significantly when implemented alone. If the starting wages of workers are uniformly selected, the resulting wage distribution will still look like Fig. 3.10. To overcome this shortcoming, we propose a way of Gaussian-like selection within a fixed interval, which may be closer to reality. It highlights the fact that only a minority of workers receive extreme wages, and a majority of them receive average wages.

The PDF of Gaussian can be written as:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \quad \int_{-\infty}^{+\infty} f(x)dx = 1 \quad (3.3)$$

In order to adopt a Gaussian distribution, we need to know not only its arithmetic mean (μ), but also the standard deviation (σ). The mean can be easily calculated as the average value of the highest and the lowest wage, so that $\mu = 50$ ($w_a = 10, w_b = 90, \bar{w} = (w_a + w_b)/2 = 50$). One difficulty that we encounter when using Gaussian selection in a bounded area is how to find a reliable and reasonable σ . Mathematically, the PDF of Gaussian has no close boundary, and integral as Eq. 3.3 is not analytically solvable. However, if we presume 95% of the selections should fall into the interval between $[10, 90]$, the area under the PDF should be near to 0.95 (instead of 1), then

$\sigma = 20.5$ can be calculated from the numerical approximation.

$$\begin{aligned} \text{if } [w_a, w_b] = [10, 90] \quad \text{and} \quad \mu = 50, \\ \int_{10}^{90} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-50)^2}{2\sigma^2}\right) dx \approx 0.95 \\ \implies \sigma \approx 20.5 \end{aligned} \tag{3.4}$$

The Gaussian distribution with $\mu = 50, \sigma = 20.5$ is plotted in Fig. 3.11, which shows 95% of the selections are between $[10,90]$. In the computer programming, given σ and μ , we may construct a Gaussian-like selection by the following process:

- (1) Select a random number according to a Gaussian distribution with given mean and standard deviation. In our example, 95% of selections will be valid and fall between the boundary $[10, 90]$.
- (2) The number being selected is taken into account only when it lies between 10 and 90. When the number selected is outside the boundaries (i.e., $w < 10$ or $w > 90$ with 5% possibility), we simply redraw numbers until the selected number is indeed between the boundaries.

The newly generated Gaussian-like distribution (cf. histograms in bar plot as in the left figure of Fig. 3.11) will lie slightly above the original one, but with the area within the boundary $[10, 90]$ still equal to 1. The right figure of Fig. 3.11 offers a better view of the original Gaussian fit and a new Gaussian fit to the bounded data, where the areas under the smooth line and dotted line should be equal.

When compared with the original wage payment rule, this Gaussian-like wage payment method seems simpler and closer to reality, which means paying a worker less than the minimum wage amount is no longer possible. If the firm owner fails to pay the lowest wage (i.e., 10 coins), he himself goes bankrupt, and enters the unemployment class with all his employees.

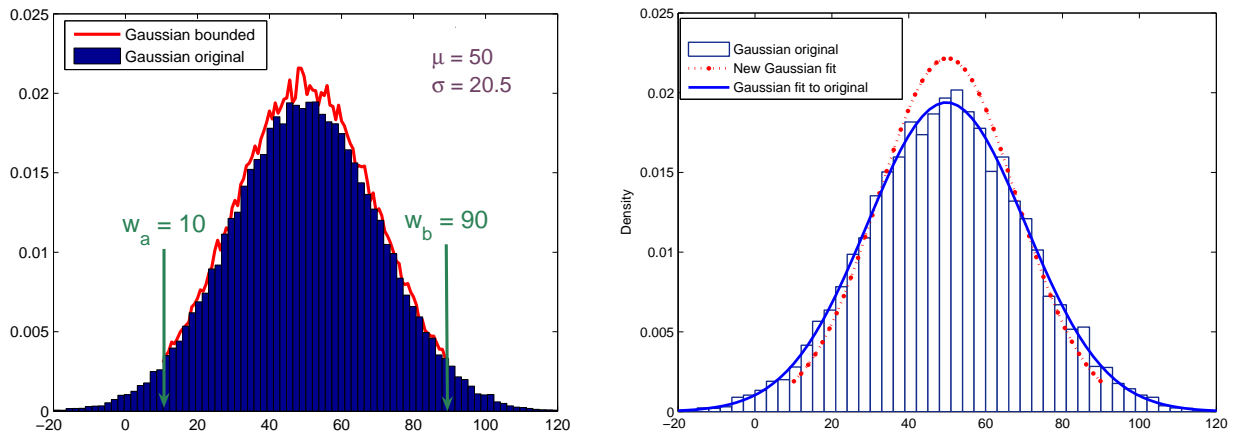


Figure 3.11: Gaussian-like selection with comparison to the original Gaussian. In the left figure, the histograms represent the original Gaussian distribution, and the smooth curve plots a Gaussian-like histogram with boundaries $[10, 90]$, and the area of newly generated bell-shape should be the same as the original Gaussian. In the right figure, the smooth curve is the Gaussian fit to the original histogram, and the dotted line is the newly generated Gaussian fit, corresponding to the smooth curve histogram in the left side.

The variance represents the wage differential: the larger the variance, the larger the wage differential, the flatter the wage distribution. For example, it can be seen from Fig. 2.4 and Fig. 2.5, that China, India and the U.S. have much larger wage (income) differentials than Japan in the late of 1990s, and there is a significant increase in income inequality for China in the past three decades. It is believed that if the boundary value can be achieved with very low probability, the variance should not be too small. However, when a relative large variance is applied, the computer will end up selecting too many times invalid numbers (in such cases, redrawing numbers should be repeated), and this might significantly increase the calculation time and slower down the program.

In order to reduce the calculation time, a simpler substitute is the tent-shaped *triangular* distribution, which can be used as an approximation to bell-shaped distributions¹⁷. In this case, the variance is no longer needed, and what we need to know is only the

¹⁷This method is recommended by Ian Wright, when he commented on the previous version of the manuscript.

boundary values and a mode, which are all directly available from the model’s assumption. We provide in Fig. 3.12 an example of a triangular wage selection within the boundary $[10, 90]$. If we have 10,000 times selections, no matter which mode we have, the height of triangle can be directly calculated by $10000 \times 2/(b-a) = 20000/(90-10) = 250$ for the property of triangle. Another advantage is that the triangular distribution can be also asymmetric (while the Gaussian should be symmetric) — it can be “skewed” to the left or to the right, depending on which mode (most frequent wage) is set. In addition, this easy triangular wage selection rule does not have invalid cases, and therefore, it runs very fast in a computer simulation.

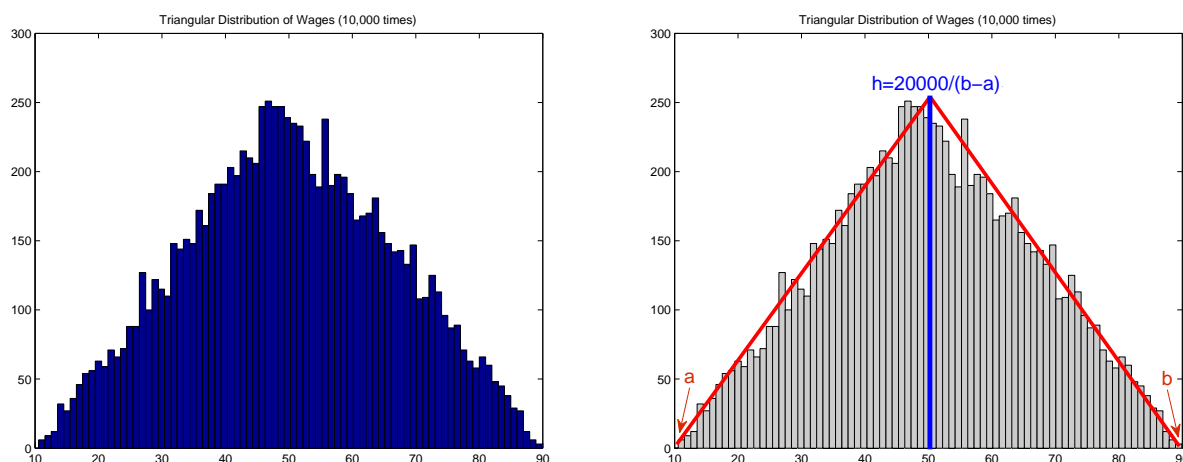


Figure 3.12: The triangular wage selection. The histograms are generated by 10,000 times selections of wages in the interval $[w_a, w_b] = [10, 90]$. The left figure is the original histogram, and the right figure plots a symmetric case of triangular distribution, where the mode equals the mean at the middle. Note that under asymmetric case, which is very possible in the wage distribution, the height of the distribution will still be $h = 10,000 \times 2/(b-a)$, for the property of triangle.

In the end, perhaps it is still necessary to mention that since the wage selection is always i.i.d., no matter which form of distribution it actually takes, the figures plotting the aggregate monthly performance, such as class distribution, firm size distribution, rate of profit distribution, etc., may not be changed significantly when we switch from one wage payment rule to the other. However, we insist that a good model should not

only explain the observed statistical behavior (macro-output), but also be built upon reasonable underlying rules that produce these statistics (micro-foundation). The modification in the “wage payment rule” directly influences the majority of workers’ money holding. Another extension of the wage payment rule will be provided in Chap. 5.

3.3.3 On Expenditure Rule

Our third consideration is on the “expenditure rule”, which says: “*uniformly select a consumer actor b from the set $\{A - \{a\}\}$ and an expenditure amount m from b ’s money holding $[0, m_b]$. Hence, m_b is reduced by m , and the market value V is increased by m* ”. This rule indicates that although actor a is selected to go through the simulation, he does not have the right to make consumption, and whether he is able to consume in the next periods depends on the possibility of being chosen the following months. In other words, some of the agents will be prohibited to be active during the whole simulation, while some other agents will perform the iteration more than the requirement (i.e., making consumptions several times a month). The significant shortcoming of uniformly selection has been discussed in Chap. 3.3.1. As suggested, an easy method to prevent this trouble is to use “for-loop”, which practically guarantees that each actor has a chance to make consumption every month, so that extreme expenditure amount coming from multiple consumptions by a single agent can be avoided.

Furthermore, the “expenditure rule” is originally designed to be the third rule (before the “revenue rule” and much before the “wage payment rule”) of the model. The reason to put it in this position is because the “revenue rule” says a firm needs to earn profit from the market, but the market value at initialization is 0. Therefore, it requires someone to spend money in the marketplace, especially for the first month, when the first firm wants to trade in the market. On the other hand, when the “expenditure rule” is positioned before the “wage payment rule”, logically it means the actors’ expenditures are based on their money holdings at the beginning of month rather than at the end of month after receiving salaries. This kind of calculation will lower the overall level

of expenditure. Particularly in our test simulation, putting the “expenditure rule” in the middle will cost 7 coins less of average consumption than putting it at the end of program; meanwhile, since consumption is the only source for market value increment, the firms’ total revenues are comparatively lower as well.

We therefore suggest to put the “expenditure rule” behind the other simulation rules, which means all active actors are subject to make consumption payable from their money holdings at the end of month. Note that even if we post the “expenditure rule” in the end, the overall distribution of monthly expenditure will not change too much in the shape, but there is a minor change at its position (i.e., the mean of the distribution). Moreover, to overcome the problem of zero initial market value, we assign $V = 100$ instead of $V = 0$, so that the first-come firms may already profit from trading. These small modifications do not qualitatively change the model behaviors, but will make the simulation rules more reasonable.

Finally, as previously discussed, the uniformly selected expenditure seems not optimal too. In reality, most people are forward-looking. They tend to save some money while spending, and the rich people might spend more than the poor, if they have more money at hand. Therefore, it is suggested that a Gaussian-like selection or a triangular selection method can be applied, meaning on average an actor will only spend half of his money buying living articles or making investment while saving the other half. A detailed presentation of how the model behaves including our suggestions will be provided in the next chapter.

3.3.4 Growth in the SA Model

As mentioned in Chap. 2.6, taking the sum of firm revenue/income is one of the ways to calculate GDP. Although in the SA model goods and their productions are not directly visible, the capital gains from trading goods can still be observed from firms’ monthly incomes, thus GDP can be simplified as coming from the overall revenues of firms. The

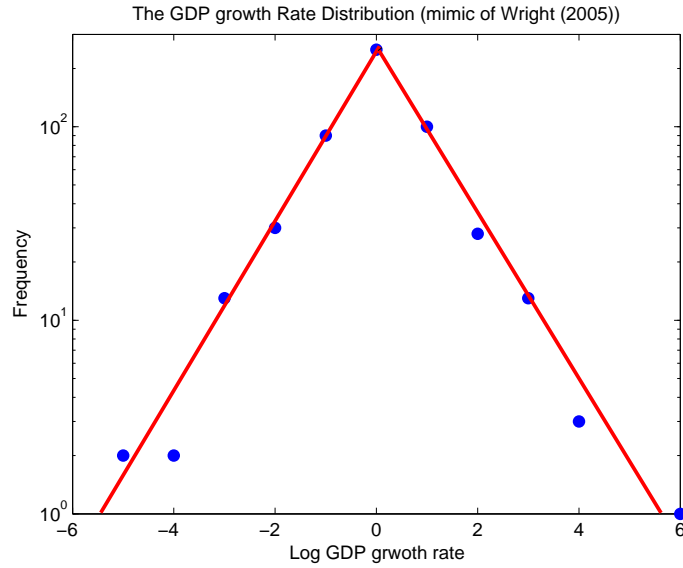


Figure 3.13: Log-GDP growth rate distribution of the SA model. The figure is from Wright (2005) p.604. According to the original setting, the GDP growth rate is defined by R_t/R_{t-1} , where R is the total revenues of the firms. Likewise, the log-GDP growth rate is measured by $\log(R_t/R_{t-1})$. The tick label in the x-axis perhaps stand for percentages of the GDP growth rates.

regulation for the GDP growth rate reads: “*at the end of each year, calculate the total revenue by all firms during the year, R_t , the (log) GDP growth rate is then R_t/R_{t-1} ”.*

Wright (2005) (p. 604) plots a Laplace distributed log-GDP distribution as shown in Fig. 3.13.

Unfortunately, we could not replicate the same form of the log-GDP growth rate distribution with the original formulation of the model. In our replication, following the author’s description, we arrive at figure (a) of Fig. 3.14. The log-GDP growth rate is calculated as the division of yearly GDP, $\log(R_t/R_{t-1})$, reported on the x-axis¹⁸. It seems that our largest log growth rates are given by -0.2 and $+0.2$. At first glance, we guess the x-axis in Fig. 3.13 (according to Wright (2005)’s Fig. 5) records the numbers in percentage (i.e., the real number should be divided by 100), but then the num-

¹⁸Approximately, the x-axis can be treated as *net* growth rate, because when the growth rate R_t/R_{t-1} is just a little over 1, we see $\log(1.01) \approx 0.01$, $\log(1.02) \approx 0.02$, $\log(1.05) \approx 0.05$, and so on. However, this does not apply for large numbers, which are significantly larger than 1.

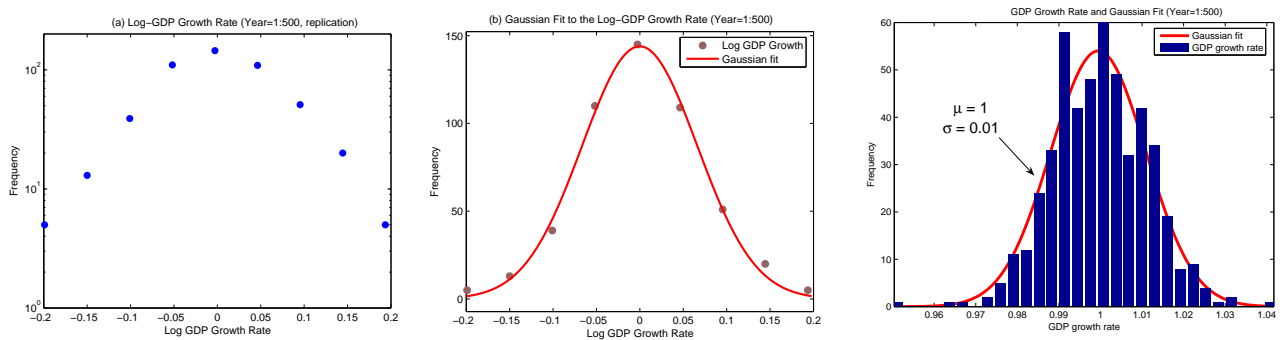


Figure 3.14: Figure (a): replication of the log-GDP growth rate with log-y scale, according to the original SA settings. Figure (b): the Gaussian fit to the log-GDP growth rate in figure (a) with linear-y scale (R-sq=99.43%, R-sq(adj)=99.27%). Figure (c): the original GDP growth rate distribution can be fitted into a Gaussian distribution (R-sq=98.86%, R-sq(adj)=98.79%) (Year=1:500). The last two figures reveal that the original GDP growth rate can be fitted both by the Gaussian and the Lognormal distribution. In fact, under a small variance and symmetric case, the Gaussian and the Lognormal distribution look very similar to each other (except the differences in the tick labels).

bers are still much higher than our replication outcomes in figure (b) of Fig. 3.14, which converts the log-y scale into natural scale. Curve-fitting shows that a Gaussian distribution fits the log-GDP growth rate very well, which means the original GDP growth rate distribution follows a Lognormal distribution. Meanwhile, because taking logarithm will not significantly change the positions of extremely small numbers, the original GDP growth rate should be fitted by a normal distribution as well, as illustrated in figure (c)¹⁹.

Our second point is whether the SA model can indeed characterize the empirical economic growth. In reality, GDP is generally growing with more and more advanced technology and industrialization, and the production capacity of today is surely much higher than a century ago. From the figures of the GDP growth rate distribution, it

¹⁹As mentioned in the last footnote, since the GDP growth rates in our model are only small percentages, and $\log(1+x) \approx x$, when x is very small, the original GDP growth rate distribution should have the same shape as the log GDP growth rate distribution. Figure (c) of Fig. 3.14 provides an example of 500 years' GDP growth rate distribution without taking logarithm (please note the numbers on x-axis of the two figures).

seems that GDP in the SA model has to grow, yet closer look shows that the model is in fact “stationary”: no matter how many years of simulation we are running, the total money in circulation will retain at 100,000 coins. Meanwhile, the market value does not really grow, and actually looks completely random. For example, at some months, we record a market value as high as 300 coins, but it could sharply drop down in the following months.

In order to make the model “non-stationary”, we suggest to let the market value grow with the GDP growth rate²⁰: whenever there is a recession or a boom, the market value will also decrease or increase at the end of corresponding years. Since the first year’s GDP growth rate is not available, we assume that there is no growth in the first year, and the market value only starts to change from the end of the second year. This slight modification lowers the probability of getting no market value as in the original design, and the range of market value will be significantly reduced and concentrated.

3.4 Summary

As the use of computer simulation skyrocketed, economists and sociologists start to use computer to characterize social and economic activities. The agent-based model is one of the most important computer-based methodologies. It starts with assumptions about agents and their interactions, and then uses computer simulation to generate “histories” that can reveal the dynamic consequences of these assumptions. Generally, agent-based modeling is able to investigate how large-scale effects arise from micro-processes of interactions among many agents. Here the “agents” can be not only human actors (such as consumers, sellers, or workers), but also social groups (such as families, firms or government agencies).

²⁰The best way is perhaps adding a role for a central bank, but then we would need to decide how much money the bank issues every year, how to distribute money to firms or actors, what is interest rate, and so on, which will make our model much more complicated.

The SA model, similar to the other agent-based models, provides both macro structure and micro interpretation. It might be the first model which includes the whole process of modern production into one simulation, instead of focusing on a stylized simplification of an individual branch. The contribution of the SA model is that: it generates many reasonable distributions, from firm dynamics to the whole economy, such as the power-law firm size distribution (e.g., Axtell (2001), Fujiwara et al. (2004)), the Laplace firm and the GDP growth distribution (e.g., Stanley et al. (1996), Lee et al. (1998), Bottazzi and Secchi (2005)), the Lognormal firm demises distribution (e.g., Cook and Ormerod (2003)), the exponential recession duration distribution (e.g., Wright (2004b), Ausloos et al. (2004)), the Lognormal-Pareto income distribution (e.g., Levy and Solomon (1997), Silva and Yakovenko (2005)), etc., with most of them quite close to empirical findings. Normally, these distributions are studied in an isolated manner, they are named “stylized facts”, whereas this model connects them within a single causal network. From this point of view, the SA model is an innovation. During our replication, we find that it is possible to make the model more plausible through some modifications in the game rules, which will lead to a qualitative improvement of many simulation results.

To be specific, our changes include: (1) the “actor selection rule” has been replaced by a “for-loop”; (2) in the “wage payment rule”, the wage interval has been modified using a Gaussian-like distribution or a triangular distribution; (3) the “expenditure rule” has been re-ordered and further modified to ensure everyone’s equal consumption chance; (4) if the market value could change with GDP, the total money in circulation will be changed accordingly, and the SA model becomes “non-stationary”. Note that all the figures provided after modification only consider *one* of these mentioned amendments, while keeping all the other rules unchanged; all parameters and statistics shown in this study are just to show the similarities between model-generated distributions and the real ones (some of them are presented in Chap. 2), or indicate how close we move to more realistic distributions. It should be emphasized that it is the *similarities in*

the distribution forms which are important, instead of *parameter values* or *summary statistics*.

The SA model actually describes the simplest relations of production, in particular, it simplifies the advanced production procedure to “**E**mployment, **P**roduction and **C**onsumption”. In the next chapter, an agent-based model combining and summing up all modifications mentioned in this chapter will be introduced. To differentiate it from the original SA model, it will be called the “EPC” model instead.

Chapter 4

Agent-based Model of Employment, Production and Consumption (EPC)

4.1 EPC Model and Improvement

The agent-based model of Employment, Production and Consumption (EPC) introduced in this chapter is based on the “Social Architecture” (SA) model introduced in Chap. 3. The SA model is robust in generating many empirical distributions within a single causal framework. However, there are some serious shortcomings in the original SA model. For example, if the simulation is prolonged for longer horizons, although we could not find a strong evidence of non-stationarity, it seems that some statistics fluctuate too much (e.g., one person becomes extremely rich). Another example is that, although GDP growth is observed, money in circulation always remains unchanged, which means the model is in fact a stationary economy. Given these unsatisfactory findings, is it possible to improve the SA model?

We have used Chap. 3 to explain one by one why some suggestions of modifications are

useful. In this chapter, all previously mentioned considerations will be considered into the current model. In general, the main modifications of the EPC model compared with the SA model are:

1. The original 6 rules have been reduced to 5 rules: the “actor selection rule” has been replaced with a normal “for-loop” iteration¹, in order to make sure that each actor is economically active once every period.
2. Since each actor at initialization has the same status, a random order generator is used to shuffle the original natural order of active actors, so that the new order of agents becomes completely random within every iteration.
3. In the SA model, not only wages and revenues, but also expenditures are uniformly selected, which means the possibility of selecting any amount is practically equal, whereas in reality this would appear unrealistic. In the EPC model, a triangular distribution is applied in all these places to highlight different probabilities of choice. In other words, most agents make similar selections (e.g., spend half of their money) according to the triangular distribution rules, and those who are making extreme selections (e.g., spend all money or do not spend at all) are in the minority.
4. According to Wright (2005), if the employer has an insufficient budget, workers may get very low or even no salary for their work resulting from the uniform selection procedure. In the EPC model, a more reasonable rule is adopted, which requires (e.g., in reality by law) the firm owner to pay the amount of wage randomly selected from the minimum and maximum wages $[10, 90]$ (according to the triangular distribution); if the employer fails to pay the wage chosen, he should pay the amount between the minimum wage and his pocket money (i.e., his

¹The “for-loop” actually takes place everywhere in our program, thus it will not be counted as a rule of the game. The original uniform selection rule might cause a multiple selection problem, as discussed in Chap. 3.

money holding); when he owns less than 10 coins, he pays all of his money to the employee, then the firm goes bankrupt and this employer enters the unemployed class with his employees.

5. In the SA model, consumption for a certain actor in his active period is actually not allowed, whereas multiple consumptions for another actor may take place quite often. In contrast, the EPC model ensures that each actor takes turns to be active once per period, including spending money.
6. Compared with the SA model, the “expenditure rule” has been positioned at the end of the iteration, instead of at the beginning, which means actors are consuming after receiving revenues or wages every month. The original setting of the SA model might prohibit the actors from consumption, since actors own less money before gaining profit or being paid.
7. The total money in circulation in the SA model is an invariant quantity. However, the EPC model enables market value to change hand-in-hand with the GDP growth rate, so that the market always reflects social production: a booming market is mainly resulting from a high GDP growth period, and vice versa. As the total money in circulation is the sum of money holdings from all actors and market value, the overall money in the society is consequently also varying each year, then our model is no longer stationary.
8. There is no market value at initialization in the SA model, which means no market is existing at the beginning of simulation. This might result that the first firm cannot do any trade in market to earn profit. However, in the EPC model, the market value at initialization is assumed to be 100 coins, so that the first-come firms can already profit from the market through making trade.

4.2 Simulation Rules of the EPC Model

The EPC model is designed to have N economic actors, with $i = 1 \dots N$. At initialization ($t=0$), each actor holds an equal amount of money m , measured in “coins”; there are still no firms or employment relations, everyone is in a state of “unemployed”; a household is treated as a single actor, and a firm can be owned by only one actor; meanwhile, a market is created with an initial value V , as a prerequisite for companies to trade their goods; the total money in circulation in this economy is the sum of money holdings by all actors and the market value (i.e., $M + V = \sum_{i=1}^N m_i(t) + V$).

The actor i in the economy can be either a firm owner (employer, denoted by E), an employee (worker, denoted by W) or a non-employed (unemployed², denoted by U), thus the total population set is $A = E + W + U$. The character of a certain actor can be specified by a state variable $S_i(t) = \{m_i(t), e_i(t)\}$, which gives i 's money holding $m_i(t)$ and employment state $e_i(t)$ at a certain time t . If $e_i(t) = 0$, the actor is not employed, otherwise if $e_i(t) = j$, the employer j hires i as one of his employees. The state evolution from $S(t) \rightarrow S(t+1)$ is determined by a set of 5 simulation rules, which take place at each unit of time, measured in months.

There are 5 basic simulation rules, and each actor takes turns to go through all simulation rules once per month. To avoid possible artificial effects from the order of selection, we shuffle (interleave) the N economic actors at the beginning of *each month* before implementing the “for-loop”. This is mathematically equivalent to a random selection among N actors, then the following simulation rules are applied to the selective actor a when applicable. The detailed program code in MATLAB is provided in Appendix B.

1. *Hiring rule:*

Randomly select an actor a , by first shuffling N numbers and then choose one by

²Although “unemployed” and “non-employed” is not the same in empirical research, we do not differentiate them in the EPC model: an actor who is neither employer nor employee, should be unemployed.

one. If the actor a is unemployed³, his potential employer can be either an existing firm owner or an unemployed who will become an employer in the same period (potential employer set $P = E \cup U$). Choose an employer c for a according to the probability function (4.1), that weights potential employers by their wealth:

$$P(c) = \frac{m_c}{\sum_{x \in H} m_x} \quad (4.1)$$

The wage interval $w = [w_a, w_b]$ is an exogenous parameter. It shows the minimum and maximum possible wages, and the average wage is $\bar{w} = (w_a + w_b)/2$. If c 's money holding exceeds the average wage $m_c > \bar{w}$, then c hires a .

2. *Revenue rule:*

The revenue rule enables firms to earn revenue from market by selling their products. If the actor a is not unemployed, a can be either a firm owner or a worker. In both cases, they work for the firm and gain profit m from the market. The amount m is a number which is randomly selected from the current market value $[0, V]$ according to a triangular distribution⁴. If actor a is an employee, then transfer m coins to the employer. Alternatively, if actor a is an employer, the m coins are given to himself. All transferred money is counted as firm revenue (the total amount of yearly firm revenue will be later recorded as yearly GDP).

3. *Firing rule:*

If actor a is a firm owner, he determines the number of workers to fire through uniformly selecting u actors from the employee set W_a , according to Eq. 4.2, so

³All agents' starting stages are unemployed, so all of them go through the "hiring rule" during the first month. In the first round, some of them become firm owners and the other remain unemployed. Starting from the second month, the agent who is already employer or employee, does not need to execute the "hiring rule".

⁴The triangular distribution is used here to prevent a certain firm from earning almost all the money from the market at once, so that nothing is left for the other firms. This is very likely to happen in the original SA model, where uniform selection is applied in this rule.

that the firm's workforce is reduced to a size such that his wage bill is payable.

$$u = \max\left(W_a - \left\lceil \frac{m_a}{\bar{w}} \right\rceil, 0\right) \quad (4.2)$$

In this function, W_a shows the number of workers currently hired by employer a . The number of employees that a can hire is determined by his total wage bill, m_a (i.e., possible wage bill he can afford), if an average wage (\bar{w}) is paid to each worker. Accordingly, the employer should fire some employees if he runs out of money. If his budget is sufficient, no workers will be fired, and new employment is possible from the "hiring rule" (in the next iteration).

4. *Payment rule:*

For each worker in a 's employee set, pay a certain wage amount w , where w is randomly selected from an exogenous minimum and maximum wage interval $[w_a, w_b]$ according to the triangular distribution⁵. If employer a has an insufficient budget to pay the selected wage w , then w is reselected between the minimum wage and a 's money holding $[w_a, m_a]$; if the employer's money holding is less than the minimum wage level (i.e., $m_a < w_a$), he pays all the money that he owns to the worker, and goes bankrupt with his employees.

5. *Expenditure rule:*

Actors make consumption in the market at the end of each month⁶. Randomly select an expenditure amount m from a 's money holding $[0, m_a]$ according to triangular distribution. Hence, m_a is reduced by m , and the market value V is increased by m .

⁵The triangular distribution applied here implies that the employer will pay the majority of workers the average wage \bar{w} , and pay only few of them (e.g., managers or gate-keepers) extremely high or low wages.

⁶In reality, people have multiple times of consumption every month depending on their budgets. Here in the model, we assume that they sum up their spending at the end of a month, and this amount may dependent on their total money holding after receiving wages (or gaining profit for firm owners).

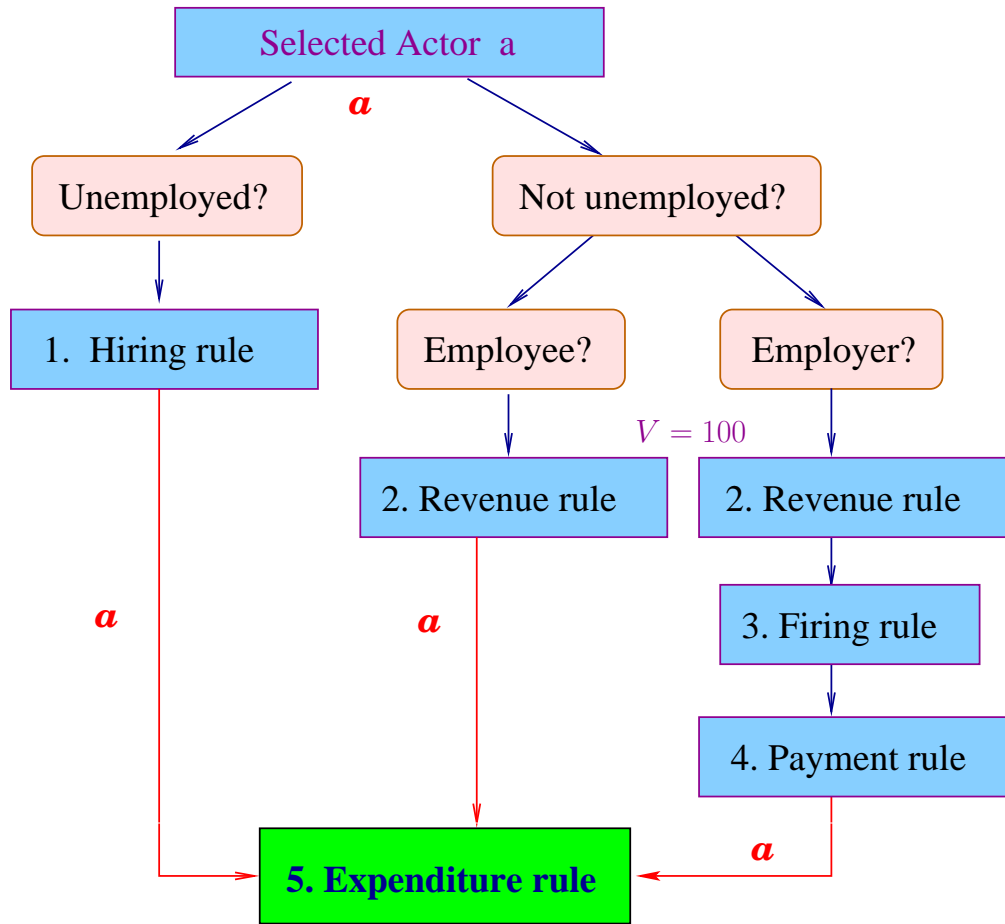


Figure 4.1: Simulation Structure of the EPC Model

Fig. 4.1 provides an overview of simulation rules that a selected actor a will go through. The “expenditure rule”, which was the third rule in the SA model has now been positioned in the last place, the market has been assigned $V = 100$ coins as initial value, which guarantees that trading occurs at the first round of iteration. The selected actor a is able to go through all five rules including the “expenditure rule”, while in the original SA model this activity was not allowed. The whole iteration should be executed N times per month so that each actor (after permutation) has a chance to be active once. Before each iteration starts, the computer breaks the natural sequence of agents and puts them into a new random order, then the iteration will be implemented

to every actor one by one. When actor a is selected, the computer will first judge whether he is unemployed or not. If he is unemployed, he might be hired through the “hiring rule”. If he is not unemployed, the computer will further decide whether he is an employee or an employer; as an employee, he works for the firm, and contributes to the revenue of the firm; as an employer, he not only works for the firm, but also needs to decide the wage payments for workers and how many workers to fire or hire. Finally, no matter whether actor a works or not, at the end of the corresponding iteration, his monthly consumption is calculated.

Every 12 replications of all N actors will be recorded as one “year”. The total amount of yearly firm revenue is counted as GDP by the end of 12 iterations. The yearly GDP growth rate is measured by GDP_t/GDP_{t-1} . Since there is no comparison for starting GDP, the first year’s GDP growth rate is assumed to be 1. The change of GDP growth rate will be applied to the last recorded market value V , so that the market value will also grow or decline with yearly GDP. The four exogenous parameters are: the money holding of all actors, M , the total number of actors, N , the wage interval, w , and the market value, V . A standard simulation with $M=100,000$, $N=1,000$, $w = [w_a, w_b]=[10, 90]$ and $V = 100$ will be used in the following simulations.

4.3 Simulation Results

4.3.1 Firms and the Unemployed

Class size measure: After each month, count the number of employers, workers and the unemployed.

Fig. 4.2 plots histograms and time series of the number of employers, employees and the unemployed. The top three figures (figure (a) (b) (c)) are histograms of each class of actor numbers. Data are collected for a standard 100-year simulation at a

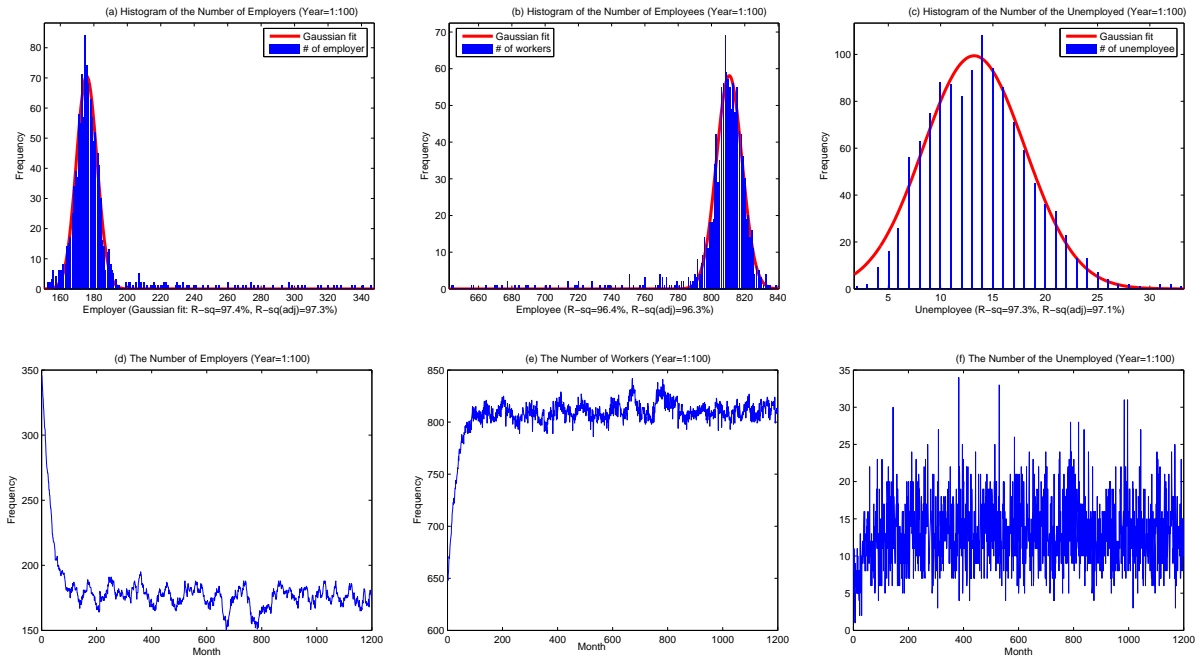


Figure 4.2: Figure (a) (b) (c): histograms of the number of actors each month as either an employer, an employee or an unemployed with a constant bin size of 1. Figure (d) (e) (f): time series of the numbers of employers, employees and the unemployed throughout the simulation.

constant bin size of 1. All three figures display good agreement with a Gaussian distribution, with variables clustering around the means. It shows that on average 18.05% of the population are firm owners, 80.65% are workers, and the remaining 1.31% are unemployed⁷. Although statistics about class percentage are hard to find in reality, the simulated unemployment rate seems quite close to empirical results, i.e., according to our previous discussion in Chap. 2.1, the common unemployment rate in most of the development countries is around or under 5%.

The bottom three figures (d), (e) and (f) in Fig. 4.2 show the number of employers, employees and the unemployed throughout the years. At the beginning of the simula-

⁷Note that unemployment rate in the SA model is around 17%, which is much higher than reality. This rate is reduced to around 10% if the “for-loop” is used in the “actor selection rule” (cf. Fig. 3.5). The further reduction in the unemployment rate of the EPC model comes from a better logical formulation of the whole simulation rules. When multiple selections are prevented, less extreme cases (e.g., firing or hiring) take place, so that more workers are able to find jobs.

tion, all actors are treated as unemployed, but since everyone is given an equal amount of 100 coins, almost all agents are able to become either employed or an employer to hire others, thus the unemployment rate was kept very low at the start of the simulation. Later on, some employers may not afford the wage payments and either become unemployed or workers themselves. A clear decrease in the number of employers with an obvious increase in the workers and the unemployed numbers can be observed. The convergence behavior will last until the proportions of these three classes reach a stationarity distribution after approximately 10 years (120 months), which means all data collected after this transient period should be trusted for further analysis. Meanwhile, this transient part of the simulation also explains the extreme values in figure (a) and (b), which result in long flat tails of these plots. In fact, if the first 10 years of simulation are truncated, the same figures will exhibit even better Gaussian fit with a lower variance (cf. figure (a) of Fig. 4.3). In order to represent the ergodic distribution of our Markov process, *all of the following simulations will exclude this transient period.*

Moreover, since each firm is assumed to be owned by a single firm owner, the number of firm owners is just equal to the number of firms calculated. Figure (d) of Fig. 4.2 provides a general view of the change in the firm numbers. At the start of the simulation, there are more than 300 firms, then the number of firms gradually drops to 175 within 10 years; for the rest of the simulation, there will be continuous entry and exit of firms, causing fluctuations in firm numbers around the mean (or mode), which shows that the distributions of the number of firms in the EPC model is stationary, which is in accordance with empirical facts given in Chap. 2.4.

4.3.2 Firm Size Distribution

Firm size measure: The firm size is calculated as the number of employees in each firm. At the end of each month, count the number of employees of all firms.

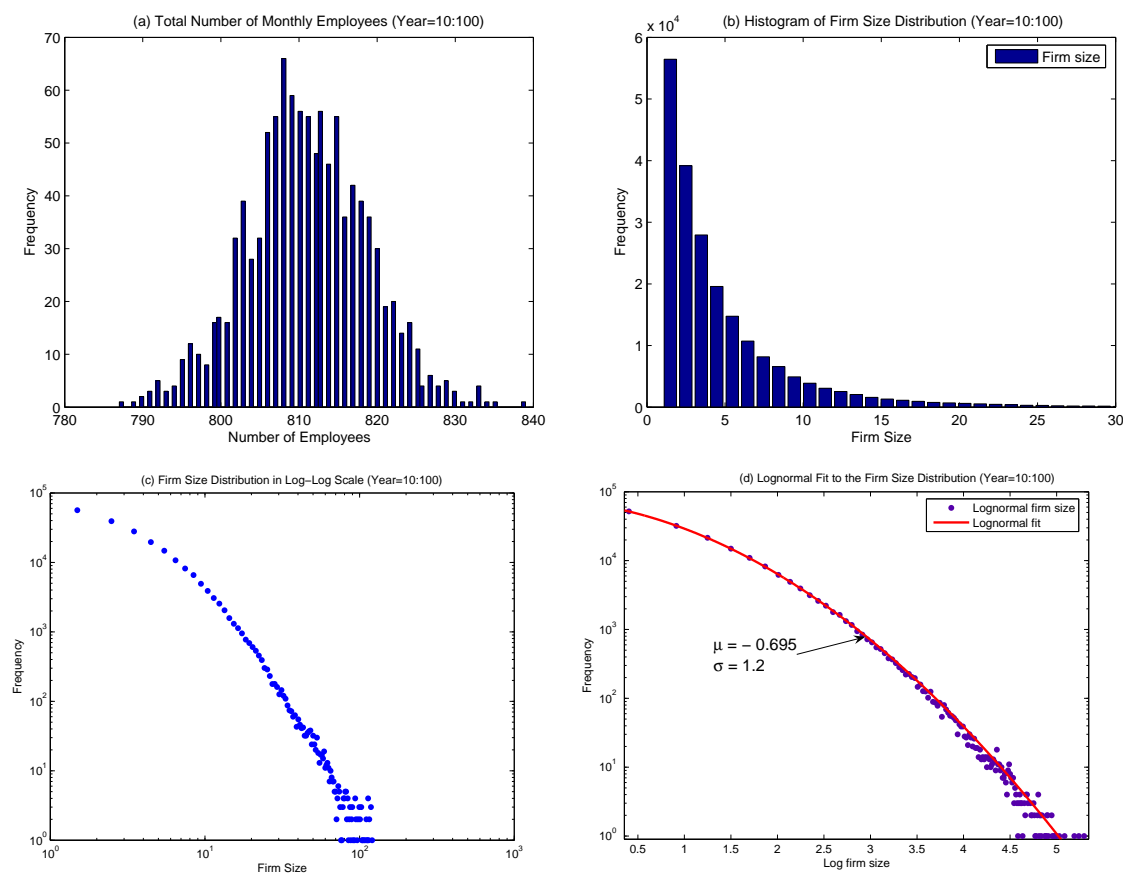


Figure 4.3: Firm size distribution. Figure (a): histogram of the total number of employees at the aggregate level; this histogram is a truncation of figure (b) in Fig. 4.2; the unstable data from the first 10 years of the simulation are omitted, thus this histogram records the last 90 years' simulation. Figure (b): histogram of firm size distribution; firm sizes that are larger than 30 with comparably very low frequency are omitted. Figure (c): firm sizes measured by the number of employees in the log-log scale (Year=10:100). Figure (d): curve fitting to figure (c), which shows that the firm size distribution can be fitted well by a Lognormal distribution.

Most of the older statistic models of firm dynamics assume that firms grow, decline or exit with a probability that may be a function of firm size, and not all models evolve to a steady state (e.g., the models of Gibrat (1931), Kalecki (1945), Simon (1955), Steindl (1965), Levy and Solomon (1997) generally have no steady state). Some other studies assume that firm sizes and their growth rates are independent, but that there is a minimum size for a firm (e.g., Levy and Solomon (1997)). Apart from that, some

models presume that the industry size and the number of firms are kept constant throughout the simulation, i.e., by fixing the number of firms, they assume that the number of entry and exit firms are always equal during the simulation (e.g., Kalecki (1945), Simon (1960)).

Compared with these models, the EPC model simulates firm dynamics from bottom up. Instead of assuming that the growth rate and firm size are negatively connected, it is assumed that the firms' behavior is determined by the money holding of the firm owners, hence rich firms tend to hire more employees than poor firms. Meanwhile, entering and exiting industry, in the EPC model, is governed by the employment dynamics. Since the minimum firm size is set to 1, a firm comes into being if an actor hires another, and a firm dies if a firm owner cannot afford the wage costs and fires all his employees.

Fig. 4.3 reveals some facts of firm size distribution in a 100 years' simulation. Figure (a) of Fig. 4.3 depicts the number of workers (employees), which is the same figure as Fig. 4.2 (b) after removing the first 10 years' transient part. It shows that the total number of employees (or "*labor*") per month does not fluctuate much from a mean of 810, and can be well fitted by a Gaussian distribution.

Instead of looking at the number of workers in figure (a) of Fig. 4.3, from a different point of view, figure (b) displays a monthly histogram of employee numbers (i.e., firm size). Generally, most firms are small, the mean firm size is 5 with median size of 4, while the mode is just the minimum possible firm size — 1, which takes almost 27% of the total frequency. Besides, only very rarely firms have more than 30 employees; for a better observation of the clustering character of small firms, these large firms are truncated in figure (b). Figure (c) shows a log-log plot of the firm size distribution, where an obvious curve shape can be observed: it seems that the number of large firms is much lower than small firms. As a matter of fact, in our study of firm size, the largest firm size in record is 121 employees out of 830 (around 14% of total employee number). According to Axtell (2001), the largest U.S. firm in 1997 had 10^6 employee out of a

total workforce 10^7 , meaning that the largest firm size should not exceed one-tenth of the total workforce. Hence, the result from the EPC model looks closer to reality. Figure (d) conducts curve fitting to figure (c). It seems that the firm size distribution can be fitted well by a Lognormal distribution (R-sq = 99.9%, R-sq(adj) = 99.8%), which is in harmony with the findings of Gibrat (1931)⁸, and other researchers, such as Chesher (1979), Sutton (1997), Cabral and Mata (2003) (cf. discussions in Chap. 2.2).

Although Gibrat (1931)'s model is very famous among studies of firm dynamics, it has some pronounced shortcomings. For example, the firm size distribution tends to slowly move to the right, with ever increasing mean and variance as time goes by. This means the size of the biggest firm becomes larger and larger, consequently, the model will have no statistical equilibrium distribution. In contrast, we have tested the EPC model for both 500 and 1,000 years, and the results of the firm size distribution are visibly stable: the largest firm size will not deviate much from 120, which perhaps indicates only slight growth in large firms, in other words, the large firms tend to grow much slower than small firms. This is also one of the stylized facts that have been mentioned by many empirical researchers, as discussed in Chap. 2.3.

Finally, in our literature review of Chap. 2.2, some researchers found that the firm size distribution (in log-log scale) satisfies a power law rather than a Lognormal distribution, at least for the upper tail (e.g. in Axtell (2001), Silva and Yakovenko (2005)). If we look at figure (c) of Fig. 4.3, the EPC model can also mimic a power law firm size distribution when the lower tail is truncated. It shows again that the power law and the Lognormal distribution can be very similar in the upper tail when a log-log scale is applied. Note that we are more interested in the scaling behaviors generated by the interactions among agents, instead of showing which one of the distributional forms can

⁸Gibrat (1931) proposes a simple model to study the firm size distribution, namely, using a log annual growth rate $r = \ln(\frac{S_{t+1}}{S_t})$ to measure the firm growths in aggregation, where S is the firm size. His interesting finding is that, if we assume all companies are born at roughly the same time and of similar initial sizes, in case they face the same i.i.d. growth rate r (independent of size), the resulting firm size distribution is Lognormality.

better represent the firm size distribution. It is possible that with further modifications to the current model, the firm size distribution may deviate from Lognormality, and approach a power law more closely. This can be seen at the EPCE (EPC model with *endogenous* wages) model introduced in the following chapter.

4.3.3 Firm Growth Distribution

Unlike many traditional models assuming that firm size and firm growth rate are independent (e.g., in Gibrat (1931), Kalecki (1945), Simon (1955), Steindl (1965)), most of the empirical studies mentioned in Chap. 2.3 (e.g., Stanley et al. (1996), Bottazzi et al. (2002)) suggest a company's current growth should be highly correlated with its past performances, and might also influence the near future. This finding is captured in the EPC model as well, since the growth of a firm mainly results from the money holding from its firm owner. The capacity to enlarge one firm either in size or in sales will consequently depend on the firm's past revenue and the employer's wage budget.

Firm growth measure: After each month, calculate the current size, s_t , for a firm that traded during this month. Size can be measured both in terms of number of employees and total revenues within that month, the growth rate is therefore s_t/s_{t-1} ⁹.

The EPC model generates log annual growth rates of firms consistent with the Laplace distribution, no matter whether it is measured in terms of employee numbers (top of Fig. 4.4) or total revenues (bottom of Fig. 4.4)¹⁰. A random variable obeys a

⁹The firm growth rates are recorded at an aggregate level in log-form. Only the firms who are continuously trading in 2 successive months are counted.

¹⁰Note the differences in the x-axis of Fig. 4.4 and Fig. 3.14 (from Chap. 3.3.4). The variation of firm size can be very large, e.g., an increase of 100 times firm size will end up around 4.6 on the x-axis ($\log(100) = 4.6$), whereas the largest GDP increment may not be more than 25% (as in our replication of the SA model).

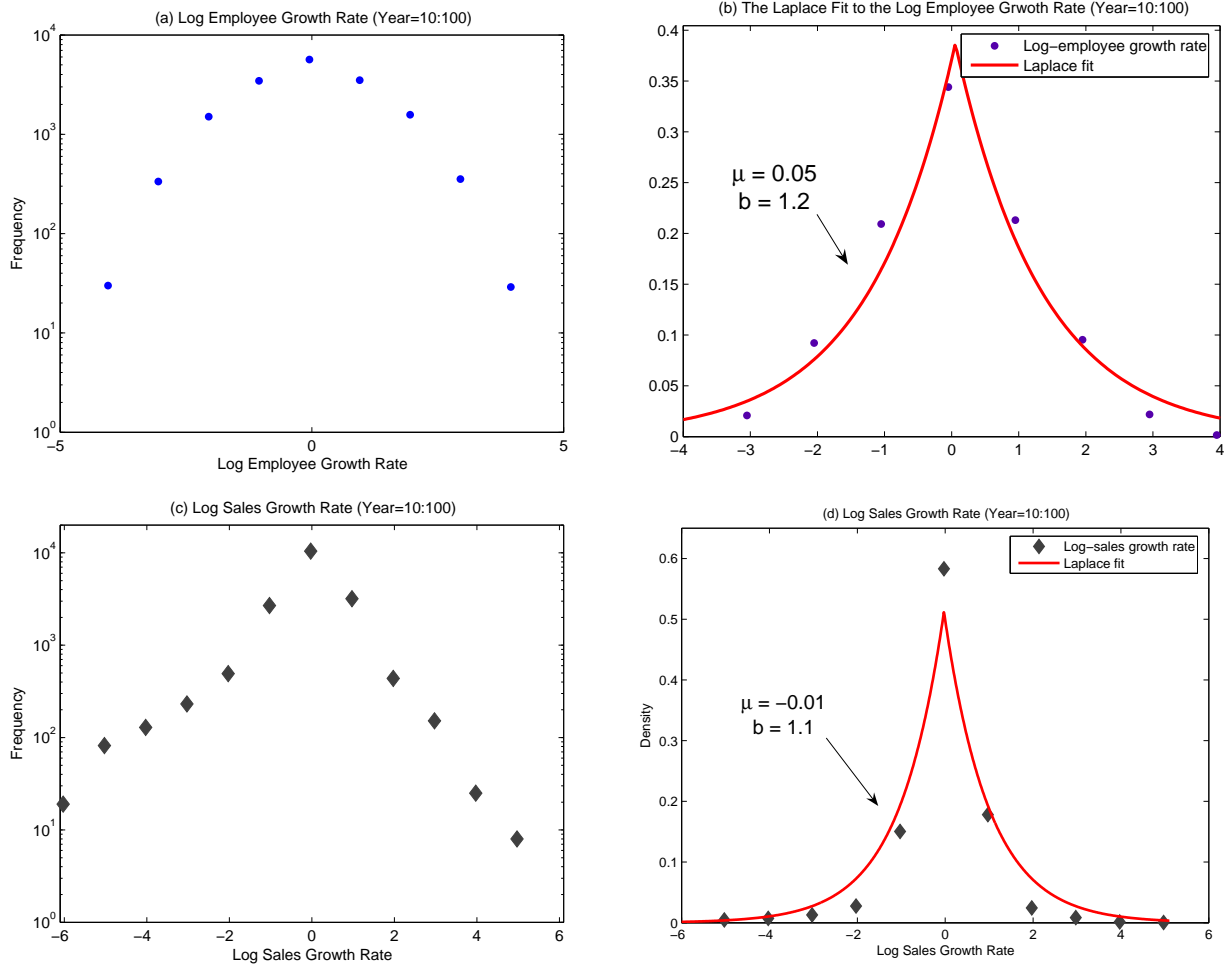


Figure 4.4: The Laplace fit to the firm growth rate distribution. Figure (a) is the histogram of the firm growth rate distribution measured by employee numbers and plotted in a log-y scale; Figure (b) is the PDF of the firm growth rate distribution measured by employee numbers, and the smooth curve represents a Laplace curve fitting (OLS regression: R-sq=97.5%, R-sq(adj)=97.3%); Figure (c) plots the histogram of the firm growth rate distribution measured by sales; Figure (d) is the PDF of firm growth rate distribution measured by sales, and the smooth curve is a Laplace curve fitting (OLS regression: R-sq=96.8%, R-sq(adj)=96.5%).

Laplace(μ, b) distribution if its PDF is:

$$f(x|\mu, b) = \frac{1}{2b} \exp\left(-\frac{|x - \mu|}{b}\right) \quad (4.3)$$

The Laplace distribution is a special case of the Subbotin distribution introduced in Chap. 2.3. Here, μ is a *location* parameter and $b > 0$ is a *scale* parameter. If $\mu = 0$

and $b = 1$, Eq. 4.3 is exactly an exponential distribution scaled by $1/2$. Figure (d) of Fig. 4.4 is close to this case.

From the shape of the firm growth distribution, it is easily observed that given small firms dominating the simulation process, most of the probability mass in the tails of Laplace distribution are contributed by small firms experiencing either rapid growth or rapid decline. Namely, the variances of firm growth rates depend negatively on sizes. Furthermore, when looking closer, especially in figure (a) and (c) of Fig. 4.4, there is a slight asymmetry in the growth rate distribution. It seems that the firms' expansions and contractions are less peaked and wider than a Gaussian, but asymmetrically, while the negative rates appear more volatile than the positive rates. Interestingly, this stylized fact is also reported by Perline et al. (2006) who study the U.S. firms (cf. Chap. 2.3), and similar simulation results can be found in Wright (2005) as well.

4.3.4 Firm Demise Distribution

Firm demise measure: After each month, count the number of firm demises that occurred during the month. A firm demise occurs when a firm fires all its employees.

The firm demise distributions shown in Fig. 4.5 illustrate that the firm demise is generally Gaussian distributed, both in a 100-year or 500-year simulations¹¹. On average around 2-3 firms die each month, an extreme case in a trial 100-year run is that in a certain month, a maximum number of 7 firms die, whereas this record is increased to 10 firms if we run a prolonged simulation, e.g., for 500 years.

¹¹The curve fitting from Fig. 4.5 is conducted using the MATLAB "Curve Fitting Toolbox", which supports more than 100 regression models. In our *histogram* curve fitting, truncation of data is not necessary. However, in some cases, one may need to include starting conditions to find the global optimum.

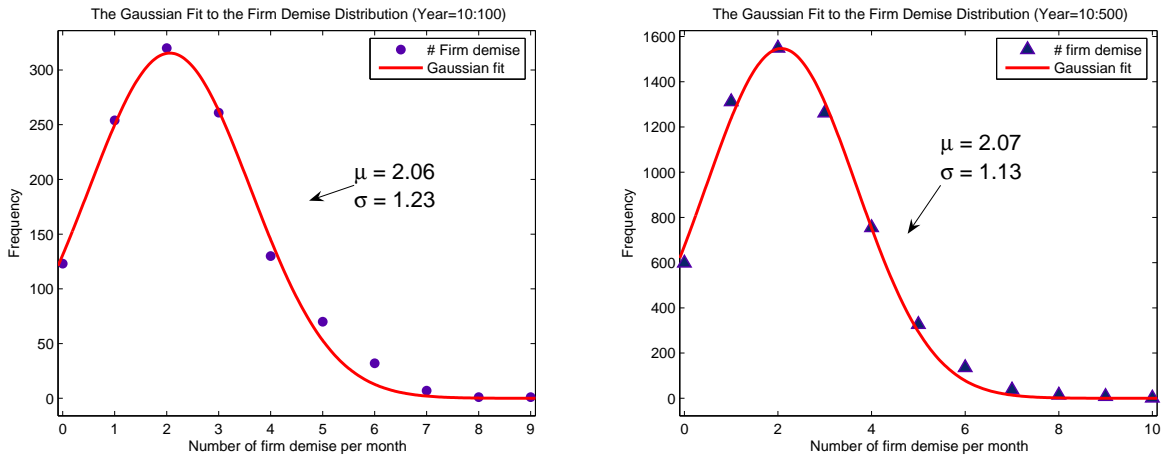


Figure 4.5: The Gaussian fit to the firm demise distribution. The smooth lines are OLS regressions of the histograms of the firm demise distribution. Both distributions can be fitted well into Gaussian (Left: Year=10:100, R-sq=99.2%, R-sq(adj)=98.9%; Right: Year=10:500, R-sq=99.5%, R-sq(adj)=99.3%).

If dividing the monthly number of demise firms by the number of existing firms, it shows that roughly 2% of firms die per month, which is much lower than 16% predicted by Wright (2005) in the SA model. The significant drop in firm demise rates comes from a different distribution of wage payments, so that the firm owners become less endangered by bankruptcy after paying the wage bills.

According to a study of the U.S. economy during 1989–1997 by Cook and Ormerod (2003), around 611,000 firms died each year out of a total 5.73 million, which gives a yearly firm demise rate of approximately 10%. This percentage, at the first glance, looks much larger than our model-generated firm demise rate. However, note that in the empirical calculation, the yearly firm demise *rate* should *not* be the summation of 12 monthly firm demise rates¹². From figure (a) and (d) in Fig. 4.2, it is known that the number of firms is characterized by stationary fluctuations after the transient

¹²Note that the number of yearly demises is the summation of the number of monthly demise firms. However, the yearly firm demise *rate* should not be the summation of 12 monthly firm demise rates. Theoretically, if the monthly entry and exit rates remain roughly constant, and if the total number of firms is not varying too much, the yearly firm demise rate is close to monthly firm demise rate.

period (i.e., ca. 10 years). If we use the same calculation method as Cook and Ormerod (2003), and count the actual firm exits per year, the average firm demise rate is close to 10% as well. Nevertheless, it should be kept in mind that, since entry and exit of firms happen at the same time, the number of total firms will stay roughly unchanged.

Our simulation is also in line with Midrigan (2007), who finds that in the U.S., most firm demises are happening with small firms: firms with less than 20 employees together account for 18% of the total employment, and dying firms are concentrated in firms with less than 20 employees (cf. Chap. 2.4). These findings are similar in the EPC model, since we have seen in the growth rate distribution of Fig. 4.4 that large firms are less volatile than small firms, and small firms may experience more entry and exit.

4.3.5 GDP Growth Rate Distribution

GDP growth rate measure: At the end of each year, calculate the total revenues by all firms during the year, X_t , GDP growth rate is then X_t/X_{t-1} .

For simplicity, the first year GDP growth rate is assumed to be 1.

In our simplified social model without government and international trade, GDP can be approximated by the total yearly revenues of firms. From the “Central Limit Theorem”, we know that the sum of a large number of independent and identically distributed (i.i.d.) random variables is close to a Gaussian distribution given a finite variance. Consequently, if the simulation rule says all revenues are randomly picked from a triangular distribution (or any other distribution), then the revenues should be i.i.d. distributed, and the sum of them will naturally result in a Gaussian. This is also conceded by Wright (2005) (cf. p. 612 of the “rate-of-profit distribution”). Fig. 4.6 plots the histogram of the GDP growth rate distribution based on a 500 years’ simulation. The resulting distribution seems to fit a Gaussian distribution better than a Laplace

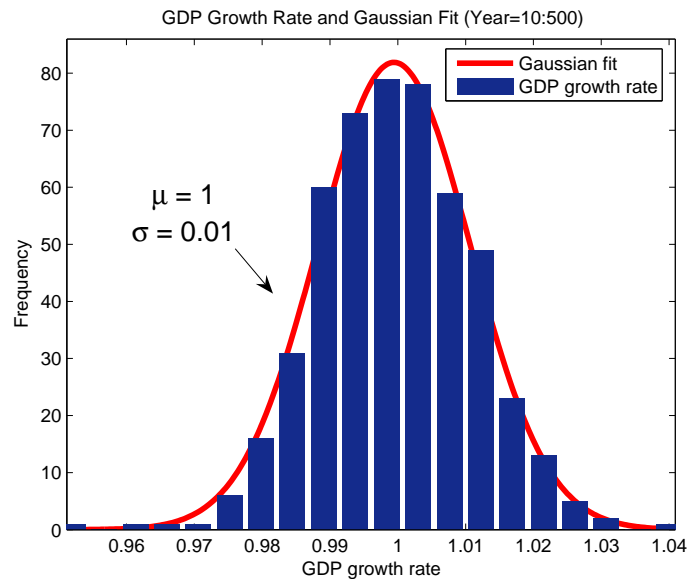


Figure 4.6: Gaussian fit to the GDP growth rate distribution (Year=10:500, R-sq=99.5%, R-sq(adj)=99.3%). Note the difference in the x-axis of this figure and Fig. 3.14: here we display the natural rate of GDP growth, while Fig. 3.14 uses logarithms.

distribution (cf. Fig. 3.13), which we have mentioned in Chap. 3.3.4¹³. Unfortunately, due to the limitation of the i.i.d. revenue collecting rule, the EPC model is unable to reproduce the empirical Subbotin or Laplace distribution of GDP, suggested by e.g., Lee et al. (1998), Castaldi and Dosi (2007), Canning et al. (1998) (cf. the discussion in Chap. 2.6). We conjecture that if there were no wars, diseases or serious corruption that stop economies from growing, GDP growth rates might be usually positive (thus the whole distribution will look a little skewed to the left, with most of density at the right side), unlike in the EPC model's prediction that negative growth happens with almost the same frequency as positive growth.

¹³We have addressed the question of the Laplace GDP growth rate distribution simulated by Wright (2005) in Chap. 3.3.4. From our replication of his model, it appears that the distribution seems not fitted very well by a Laplace distribution, but seems closer to a Gaussian distribution.

4.3.6 Duration of Recession

Recession duration measure: A recession begins when GDP declines, i.e., $X_t/X_{t-1} < 1$ and ends when GDP recovers, i.e., $X_{t+k}/X_{t+k-1} \geq 1$, and the duration of recession is k years.

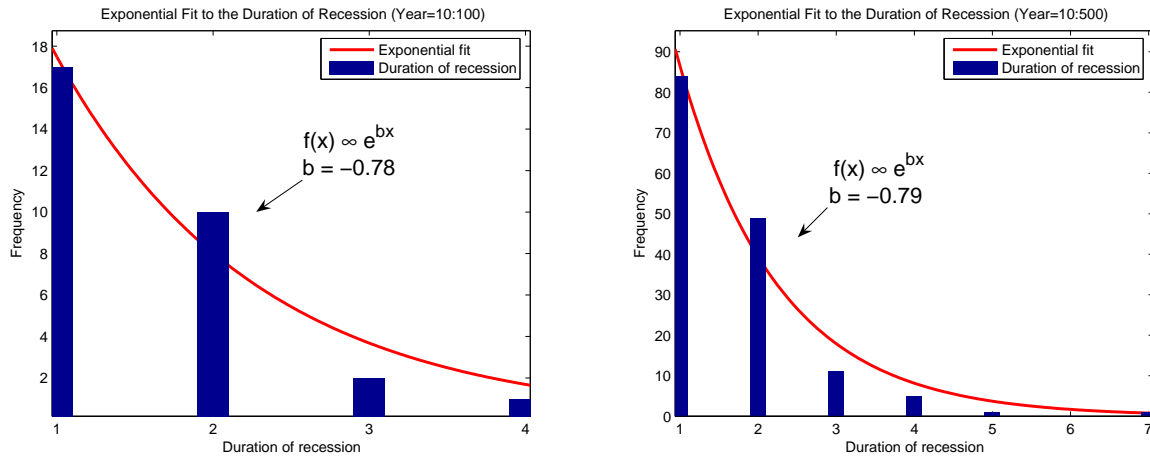


Figure 4.7: Exponential fit to the duration of recession. The smooth line is an OLS regression for the data, and both distributions can be fitted well by an exponential function $f(x) \propto e^{bx}$ where $b \approx -0.8$ for both simulation lengths. (Left: Year=10:100, R-sq=95.6%, R-sq(adj)=93.4%; Right: Year=10:500, R-sq=99.1%, R-sq(adj)=98.4%).

Fig. 4.7 displays the distribution of recession durations simulated by the EPC model, and the smooth curve is an exponential curve fitting. Since a standard simulation only provides 100 data for GDP, which are not enough for better curve fitting, we therefore prolong the simulation for 500 years and receive a better exponential fit (right of Fig. 4.7). A closer look shows that the mean duration is around 1.7 years, and the mode is 1 year, which is quite close to empirical findings by Everts (2006), Burns and Mitchell (1946) and Sichel (1991), as mentioned in Chap. 2.6. In a 100 year simulation, the duration of recession ranges from 1 to 4 years, only when the model is run for 500 years, we will reach a maximum recession duration of 7 years. Although there is still debate on whether a power law or an exponential law is more appropriate

to describe the distribution of recession durations, we see that the EPC model predicts an exponential law for the duration of recession in line with Wright (2004b).

4.3.7 Income Shares and Profit Shares

Income shares measure: Given yearly GDP, X , which is equal to the total firm revenue, calculate the total wage bill, W , paid to workers, thus wage share is $X_w = \frac{W}{X}$, and profit share is $X_p = 1 - X_w$.

Fig. 4.8 plots the yearly income share and profit share of GDP simulated by the EPC model. The top two figures (a) (b) are results of 100 years' simulation, and the bottom two figures (c) (d) are results of 500 years' simulation. Figures (a) and (c) in the left column show that the income and profit share in GDP of the economy is visibly stationary; the profit share is around 0.3, which is lower than the wage share of 0.7. This goes hand in hand with some empirical findings discussed in Chap. 2.7, such as Kalecki (1954), or Foley and Michl (1999). A similar wage-profit plot is also provided by Wright (2005) (p. 606, Fig. 7), but the original time series fluctuate significantly, and the proportion of profit share against wage share is around 0.45 : 0.55. In comparison, the results generated by the EPC model are more reasonable, with a more stationary wage and profit share. Unfortunately, since the EPC model is stationary and oversimplified from reality, it cannot reproduce the fact of a rising profit share which has happened in the recent decades (cf. discussions in Chap. 2.7).

Concerning the ratios of wage/profit share distribution, we have addressed in Chap. 2.7 that, from a time series properties, if both time series are generally stationary, their ratios should also be stationary, which might look like a bell-shaped distribution having various variances in an aggregate manner. The simulation results by the EPC model display that both the wage share and the profit share are fluctuating very little about their means, hence their ratios should be fitted well by Gaussians with small variances, as exhibited in figure (b) (d) of Fig. 4.8. Note that the Gaussian curve fitting is better

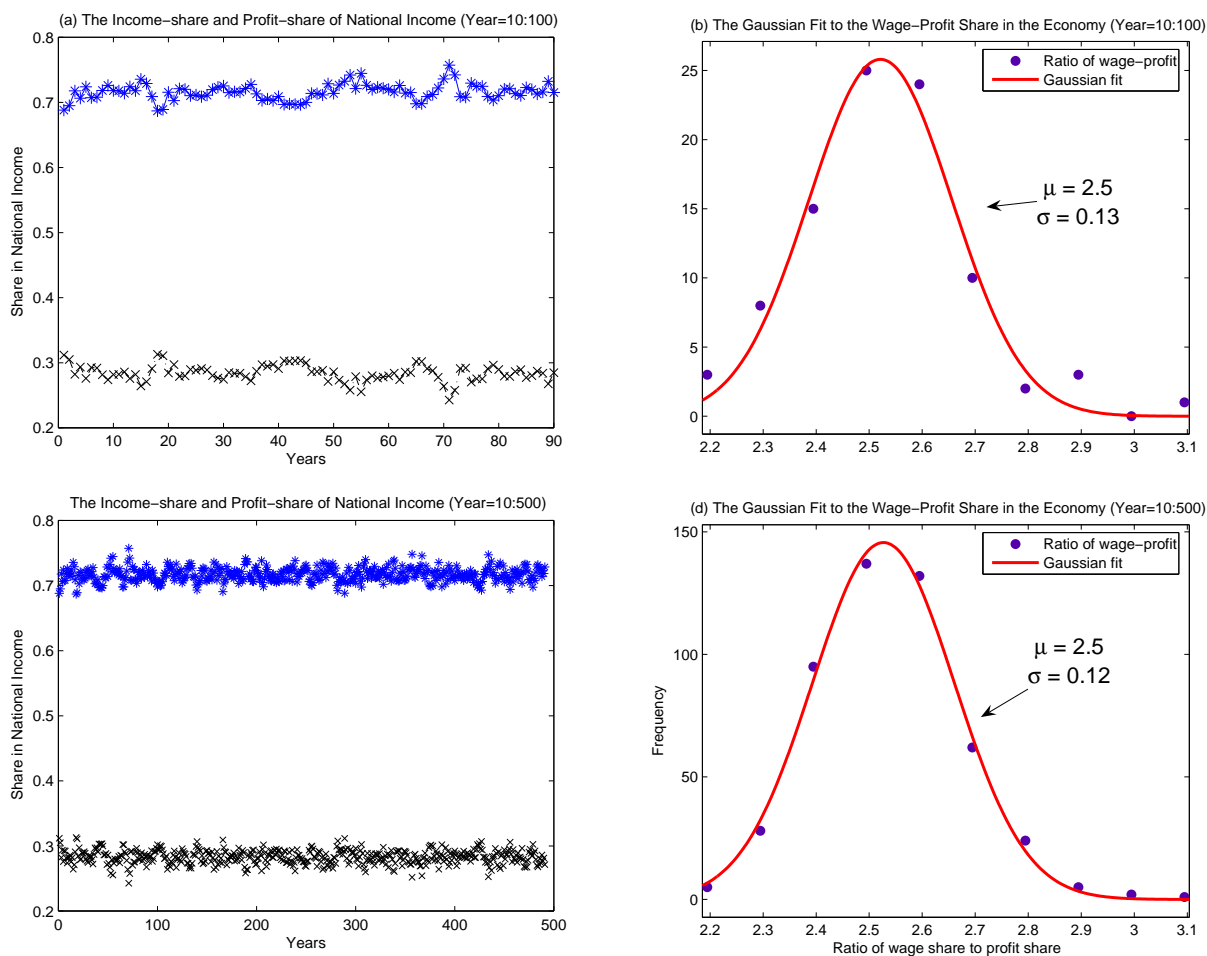


Figure 4.8: The income share and profit share of GDP. Figure (a) (b) is 100 years' simulation without transient period, the smooth curve in figure (b) is a Gaussian fit with R-sq=97.2%, R-sq(adj)=96.5%; Figure (c) (d) is 500 years' simulation without transient period, the smooth curve in figure (d) is a Gaussian fit with R-sq=99.5%, R-sq(adj)=99.4%.

with a longer (i.e., 500 years') simulation, which gives the same mean but even smaller variance.

4.3.8 Profit Rate Distribution

Profit rate measure: After each month, calculate the profit rate for each firm throughout this month. The profit rate, p_i , of firm i is defined as:

$$p_i = 100 \left(\frac{r_i}{w_i} - 1 \right) \quad (4.4)$$

where r_i is the total revenue received by firm owner i , and w_i is his total wage bill during the month.

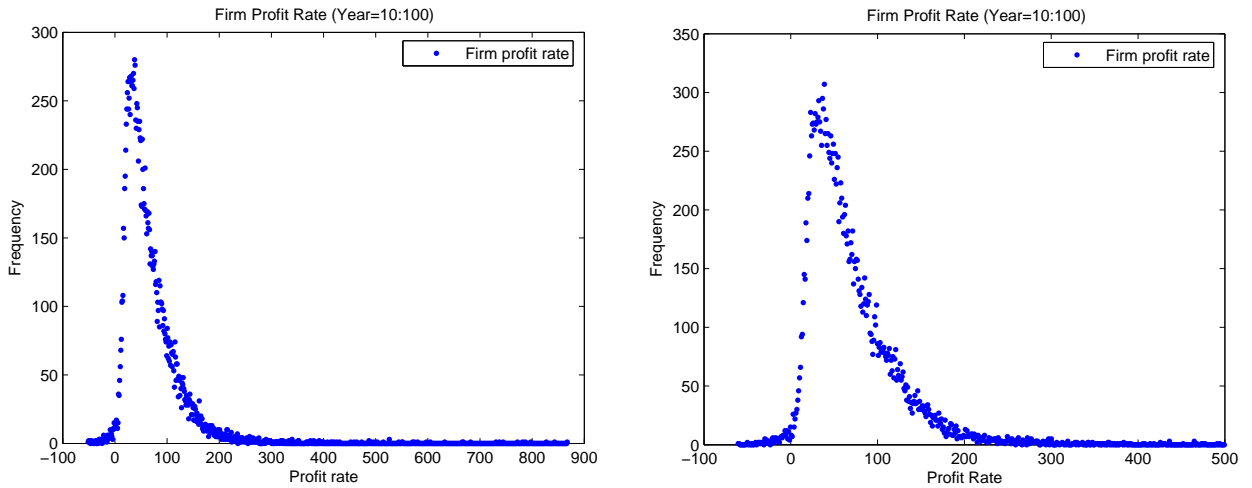


Figure 4.9: Firm profit rate distribution (Year=10:100). (Left: 100 years' simulation with minimum = -51.2, maximum = 867.5, mode = 44.4; Right: truncation for values larger than 500, the accumulated firm profit-rate distribution is apparently right-skewed.

Fig. 4.9 illustrates that the firm profit rate distribution is apparently right-skewed, especially in the right figure where we have truncated those extreme monthly profit rates of more than 500. This right-skewed profit rate distribution has also been discovered in related researches, e.g., Wright (2004a) and Julian (2007), as mentioned in Chap. 2.5. When compared with Wright (2005)'s profit-rate figures, where he uses both firm-weighted and capital-weighted profit rate to reduce distributional noise, Fig. 4.9, although without any such adjustment, is much less noisy and consistent. This might be because the EPC model adopts a triangular distribution in many places instead of

uniform draws. Consequently, the collection of revenues, the distribution of wages as well as the expenditures (consumption + reinvestment¹⁴) from employers become more moderate, so that the model results in less noisy profit rate distribution than the original SA model. Meanwhile, Wright (2005) provides a six-parameter functional form to fit the model-generated profit rate distribution, with the mean and variance depending on the *power law* distributed firm size. We have not applied this method, since our firm size distribution looks more Lognormally distributed (instead of power law). However, we agree with his opinion that a firm's profit rate may be closely correlated with its size. According to our simulation rules, the profit rate is calculated from the division of the total firm income by the wage bill, and either the income (total revenues earned by both firm owners and his employees) or the wage bill (sum of all wage payments by firm owners) should be correlated with firm size (i.e., employee numbers). The amount of investment or profit may be higher for large companies than their small competitors.

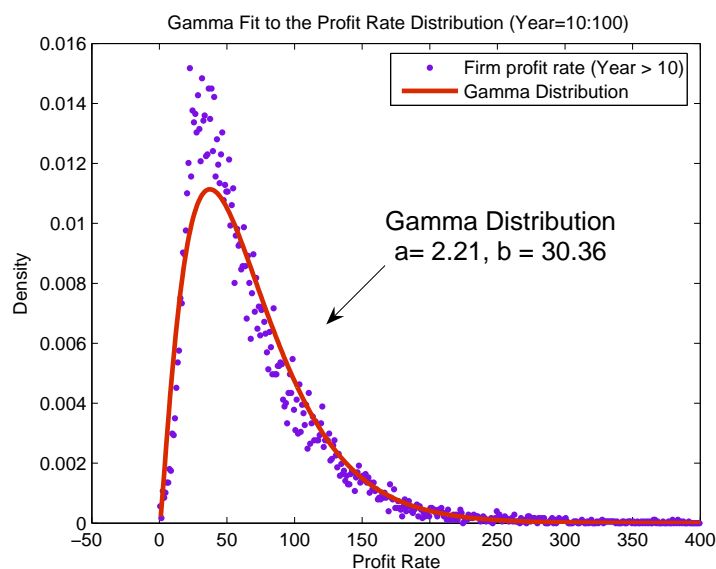


Figure 4.10: Gamma fit to the firm profit rate distribution. The negative profit rates are truncated, and only the profit rates recorded after transient period are plotted (i.e., year > 10).

¹⁴The rest of income after deducting a firm owner's wage bill and consumption goes to his money holding, which directly decides whether he is able to hire more workers in the following month.

We show that the profit rate distribution may be fitted well by a *Gamma distribution*, similar as the industry profit rate distribution proposed by Farjoun and Machover (1983) (cf. our discussions on the Gamma distribution and industry profit rate in Chap. 2.5). Since the Gamma distribution requires positive variables, the negative profit rates in Fig. 4.9 must be truncated prior to estimation. The PDF of the Gamma distribution is:

$$f(x|\alpha, \beta) = \frac{\left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left(-\frac{x}{\beta}\right)}{\beta\Gamma(\alpha)}, \quad \alpha, \beta > 0 \quad (4.5)$$

where α is a *shape* parameter. When $\alpha > 1$, the Gamma distribution is unimodal but has a skewed shape; the larger α , the less the skewness. β is a *scale* parameter, representing the range of the Gamma distribution; the larger β , the more stretched is the distribution.

The curve fitting in Fig. 4.10 is done based on the maximum likelihood method. It shows that the Gamma distribution fits both tails of the profit rate distribution quite well, if negative values are neglected. Most of the profit rate density is concentrated at the right long tail of the distribution, displaying a clear right-skewed feature.

4.3.9 Income Distribution

Income measure: After each year, calculate the total income received by each actor during that year. Income for workers is just the wage amount, and the employer's income¹⁵ is the total amount of revenues he received within that year.

Fig. 4.11 displays the income distribution simulated by the EPC model. Figure (a) shows yearly income for workers. Instead of Lognormality suggested by Wright (2005)

¹⁵Note that “income” for employer measured here is just his total amount of revenues collected through the “revenue rule”, following the definition of Wright (2005). In Chap. 5.4, we argue that income for an employer should be in fact the amount obtained after subtracting the wage bill from his total revenue. In the later case, an employer' income can also be negative.

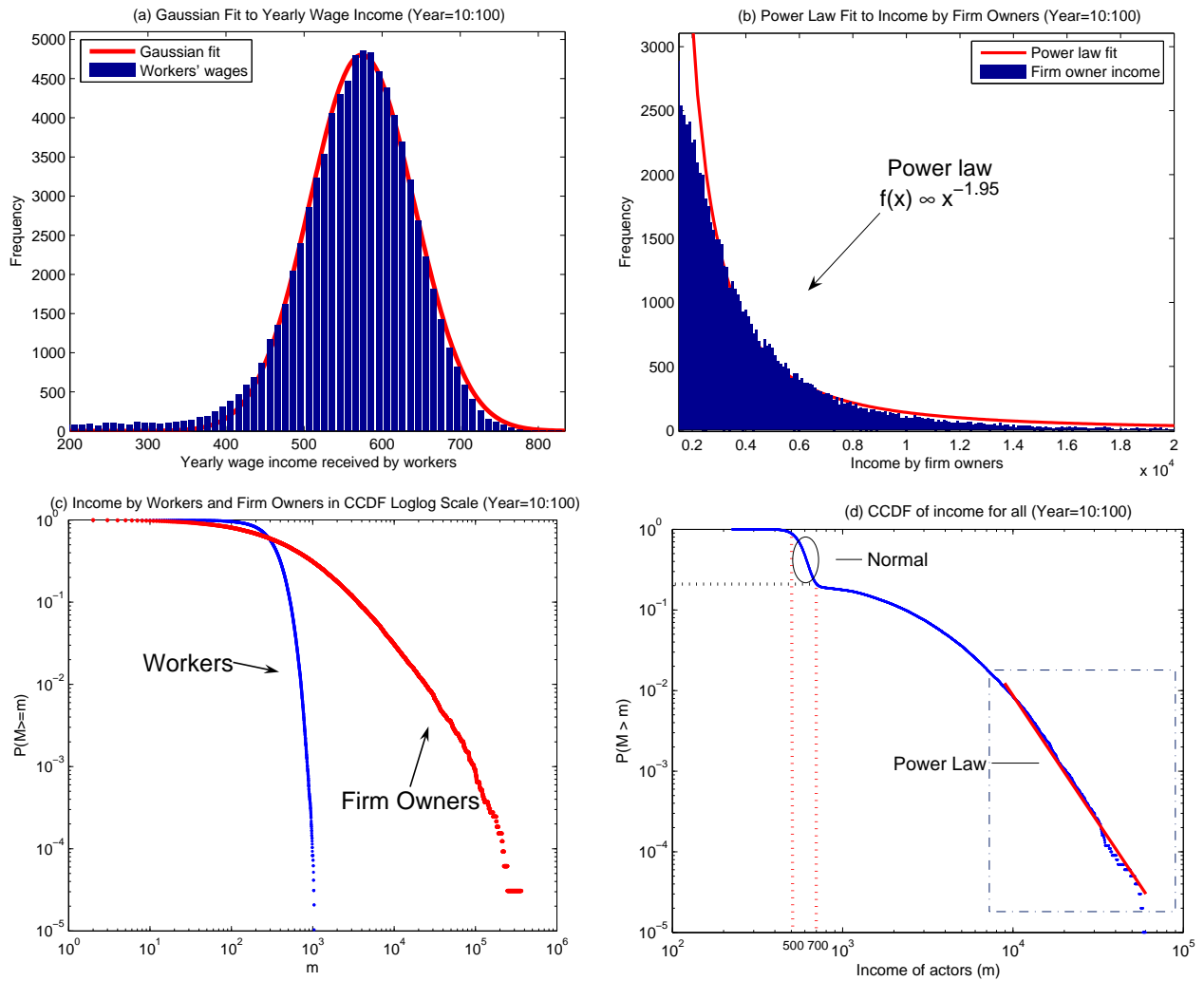


Figure 4.11: Histogram of yearly income distribution (Year=10:100). Figure (a): lower-income distribution measured by wages paid to workers; the smooth curve is a Gaussian fit (R-sq= 99.3%, R-sq(adj)= 99.1%). Figure (b): higher-income distribution measured by the revenue incomes from the firm owners; the smooth curve is a power law (Zipf distribution) fit after truncation for income ≤ 3500 (R-sq= 98.7%, R-sq(adj)= 98.4%). Figure (c): income distribution from the workers and the firm owners respectively, plotted as CCDF in log-log scale. Figure (d): a complete income distribution as CCDF in log-log scale.

(cf. p. 608, Fig. 8), it seems that workers' income distribution is rather Gaussian following the Central Limit Theorem (CLT), and their yearly wages are generally stationary and clustering around a certain mean value. Figure (b) records yearly income for firm

owners; if the noisy lower part can be neglected, the upper tail of the income distribution can be fitted by Zipf's law. The agents who stay in this region are those who run businesses, and normally they have other sources of incomes.

If we put workers' wage income and employers' revenue income into the same plot, figure (c) of Fig. 4.11 displays the CCDF of the overall income distribution in log-log scale. It seems that the workers' income is almost a vertical line (which shows clustering of wages), but the firm owners' income has a long tail, indicating a much wider income range, and some of them are even extremely rich. Finally, it might not be surprising that when plotting figure (c) in an aggregate style as in figure (d), there will be a "knee" shape characterizing the transition area of lower-income to higher-income by an exponential (normal) law to a power law. This emerging power law (or Zipf's law) in the upper tail of the income distribution is in accordance with the findings of researchers such as Drăgulescu and Yakovenko (2000), Chatterjee et al. (2005), Chakrabarti et al. (2006), and others (cf. discussions in Chap. 2.8).

4.4 Summary

We introduce in this chapter an agent-based model of Employment, Production and Consumption (EPC), which simulates and explains a wide variety of phenomena emerging from the social relation of production, inspired by the work of Wright (2005). The model consists of agents playing different roles in the economy, they can be either employers, employees or the unemployed. Firms are the main organizations to produce goods, whereas a market, as a pool of different kinds of goods, is a place for the firms to trade their products, and for the agents to consume. The monetary cycle goes hand-in-hand with the production cycle: the owners of firms (employers) gain profit from the market, they distribute salaries to employees, so that the employees can spend money in the marketplace.

The model generates many statistical distributions, most of them are in accordance

with empirical stylized facts. To sum up, the most important distributions generated from the model are the following:

- 1 . The EPC model generates different classes of people in the society, among them are a majority of workers, a comparatively small number of employers, and a seemingly frictional unemployed class. Since each group's proportion remains roughly unchanged, the distributions of their aggregate numbers look Gaussian, (with much smaller variances when compared with the corresponding figures in the SA model).
- 2 . The firm size distribution of the EPC model follows Lognormality instead of a power law. This is perhaps the result from a more logical formulation in the simulation rules. A closer look shows that, if uniform selection rule is applied, the firm size distribution will follow a power law, such as in Wright (2005) and Wright (2008). However, if we ensure random selection for each agent (i.e., each agent should be selected only once depending on a random order), an apparent curvature will be observed in the firm size distribution, such as in the slight modification to the SA model (mentioned in Chap. 3.3.1), the EPC model, and the EPC model with endogenous wages (which will be discussed in the next chapter).
- 3 . The firm growth distribution follows a special kind of Subbotin distribution called Laplace distribution. If we measure the growth by sales, the firms' expansions and contractions occur asymmetrically, which is also observed in empirical researches.
- 4 . The firm demise distribution follows a Gaussian distribution, instead of Lognormality proposed by the original SA model. The underlying property under any Gaussian distribution is that, the variables that come into our statistics always fluctuate regularly around the mean, showing "statistical equilibrium".

- 5 . The annual GDP growth rate distribution follows a Gaussian according to the Central Limit Theorem (CLT). However, the model generates the positive growth rates symmetric to the negative growth rates, which might not be realistic. Unfortunately, related literature on the GDP growth rate distribution (for a particular country¹⁶) is rather sparse, hence the real GDP growth rate distribution is still under question. This is because most of the real world data for specific countries is usually too short to capture the real growth rate distribution, some of them is even truncated for the economic depression or the breakout of war. As a result, although there are some studies based on the aggregate GDP growth rates of certain years across all countries, a track of the GDP growth rate for one country throughout the years is rarely done.
- 6 . The distribution of recession durations follows an exponential law instead of a power law. Although there are still furious debates on which distribution is more appropriate to portray the realistic recession duration, the EPC model predicts a reasonable recession duration interval which is closer to empirical findings.
- 7 . Our generated time series of wage-shares, profit-shares and their proportions, have much less variations than the original SA model, and get closer to the empirical statistics. As both profit and wage share are generally stationary time series, their ratios should be also distributed around a long-term stable mean, showing “statistical equilibrium” and a Gaussian bell-shape in aggregation.
- 8 . Concerning the profit rate distribution, the EPC model does not follow the 6-parameter function designed by Wright (2005). Instead, our simulation outcomes show that the profit rate distribution can be well fitted by a right-skewed Gamma distribution, similar as many empirical findings.

¹⁶Note that we have addressed in Chap. 2.6 about the research by Lee et al. (1998) and Canning et al. (1998). They collect GDP of 152 countries for the period 1950 to 1992, and show that the distributions of annual GDP growth rates have “fatter” tails than Gaussian. However, similar research on GDP for a particular country throughout the years has been relatively sparse.

9 . Finally, income distribution explains in an easy manner why we often observe a “knee” shape in the CDF of overall income in many developed countries. From the modeling perspective, it is already clear that the workers’ salaries and firm owners’ incomes are very differently distributed, therefore, the overlapping area of aggregate income may not look smooth¹⁷.

Interestingly but not so surprisingly, an important implication of this computational social model is that, some of the features of our economic reality, such as income inequality, unemployment problem as well as regular recessions, are inevitable consequences of the social relations of production under driven of labor rather than so-called “capitalistic exploitation”. Hence, essential properties of capitalism might not be accidental or transitory as believed by Karl Marx and others.

¹⁷“Income distribution” mentioned here is not the same as the “wealth distribution”, and the difference will be interpreted in the next chapter.

Chapter 5

EPC Model with Endogenous Wages (EPCE)

5.1 Inclusion of Endogenous Wages

From the previously discussed SA model and the EPC model, we have already learned how a better micro formulation of the simulation rules helps to improve the macro quality of our output. In other words, if the model is well designed, it should be able to reproduce many empirical distributions. As a matter of fact, both the SA model and the EPC model are based on monetary exchange: the change of status of agents, the employment relation, investments as well as consumptions, are all depending on the money holdings of the agents. In this respect, the “payment rule” plays a key role in the whole model, since it is the only element that determines money holdings of the workers. Therefore, any change in this rule will directly affect the simulation at the microscopic level.

The idea of including endogenous wages had been suggested by Ian Wright, when he commented on the previous version of the EPC model. Later on, he introduced a “closed social architecture model” in Wright (2008) where endogenous wages were

adopted¹, and the whole series of distributions mentioned in the SA model was revisited, with all results in accordance with the original SA model. However, Wright (2008)'s model still includes the shortcomings of the SA model discussed in Chap. 3 (e.g., the way of actor selection, investment and consumption is still based on the uniform selection rule): it does not take into account economic growth, and is hence still static and “closed” (as it is called “*closed* social architecture model”); in addition, if we follow the individual behavior of either one firm or one agent, the fluctuation of its activity is usually too large. This means although there is stationarity of the *aggregate* quantities, the individual behavior might not be as stationary as we would expect.

The model to be introduced in this chapter will be called EPCE model (EPC model with endogenous wage) based on the framework of the EPC model. Apart from the money holdings (m_i) and the employment states (e_i), the wage expectation (w_i) becomes an *endogenous* and important descriptive variable for an individual. If we make the wage variable endogenous, it turns out to be a factor in the causal system whose value is determined or influenced by the other variables, such as the worker's previous wage payments, the budget constraint of his employer, etc., which makes the model more flexible. What's more, unlike the SA or the EPC model², where the wage payment is completely random, agents in the EPCE model keep track of their recent history, e.g., one may require generally the same wage level throughout his employment at one firm; even if he gets fired, his wage expectation for the next employment will not drop immediately. This kind of “memory” does not exist in the previous models.

In the EPCE model, we are going to discuss some distributions or model-generated

¹In the “closed social architecture model” by Wright (2008), the identifier for one agent is $S = \{m_i, e_i, w_i, \eta_i\}$, representing money holding, employment state, wage level and wage expectation of agent i . In the EPCE model, the identifier has been reduced to $S = \{m_i, e_i, w_i\}$.

²Note that in the SA model (where workers' wages are uniformly distributed) or even EPC model (where workers' wages are triangularly distributed), there are no correlations between a worker's monthly wage payments, namely the same worker's wage may fluctuate from very high to very low in the next month.

interesting facts, which have not been mentioned in the studies of Chap. 4 or in Wright (2005, 2008). A significant advantage of the EPCE model is that it allows for a close look into how a single firm or agent develops throughout the simulation, while this is not accessible in the previous models. By examining the firm behavior at the micro level, some additional statistical properties can be interpreted. For instance, the stylized fact of the “firm life expectancy distribution” will be analyzed; the relationship between “economic growth and small firms” can be interpreted; the income and wealth dynamics can be studied in great detail. Note that in the EPCE model, as the “payment rule” changes, the wage boundary no longer exists, thus the “firing rule” will also be changed correspondingly, so that it no longer depends on the average wage payment. The distributions mentioned in the previous chapters can be easily recovered in this model too. However, to save space, the same distributions will not be mentioned here again. Before talking about the simulation results, let’s first discuss the setting of the model.

5.2 Simulation Rules of the EPCE Model

The EPCE Model is based on the EPC model adding endogenous wages, therefore, its modeling platform is similar. It contains four exogenous parameters: (1) $N = 1,000$ economic actors, with $i = 1 \dots N$; (2) the total money held by an actor at initialization is $M = \sum_{i=1}^N m_i(t) = 1000$, thus each actor i at time $t=0$ holds an equal amount of money $m = 10$, measured in “coins”, and this money endowment is assumed to be infinitely divisible; (3) the initial wage expectations of all actors are also equal, so that $w_i = 10$; (4) A market with initial value $V = 100$ is created as starting value. The simulation rules are designed on a monthly basis. At the starting time, there are still no firms or employment relations, hence everyone is treated as unemployed, and each actor takes turns to go through monthly simulation rules. A year counter records every 12 applications of monthly rules, and a standard simulation runs for 100 years.

Likewise, the actor i in the economy can be either a firm owner (E), a worker (W), or

an unemployed (U). Each actor should belong to one, but not more than one class, $E \cap W \cap U = \emptyset$, and the total population set is $A = E + W + U$. The character of a certain actor can be specified by a state variable $S_i(t) = \{m_i(t), e_i(t), w_i(t)\}$. $m_i(t) \geq 0$ gives i 's money holding at time t . $e_i(t)$ gives i 's employment state: if $e_i(t) = 0$, the actor is not employed, otherwise if $e_i(t) = j$, then the employer j hires i as one of his employees. $w_i(t) \geq 0$ is the wage expectation of a certain actor: if the actor is employed, w_i is also his *negotiated wage*, which needs to be settled at the beginning of each new employment, and is expected to increase step by step as long as he stays at the same firm and the employer has sufficient wage budget to pay him³; if the actor is not yet employed, w_i is used to record his last wage payment, and he will lower down the wage expectation in case he could not find the next job very soon. Besides, as a starting rule, for a firm owner himself who never experiences unemployment or for an unemployed who never experiences employment, their wage expectation will stay at 10, although they never get paid⁴. The state evolution from $S(t) \rightarrow S(t+1)$ is determined by a set of simulation rules as follows, which take place at each unit of time, measured in months:

1. *Hiring rule:*

Randomly select an actor a from the set A at the beginning of each month⁵.

- (a) If actor a is unemployed, then his potential employer can be either an existing firm owner or an unemployed who becomes an employer in the same period (potential employer set $H = E \cup U$). Choose an employer h for a

³This regulation guarantees stable wage payments for workers, while in the SA or the EPC model, wage payments are completely random.

⁴Since the employer is the leading person of a firm, we assume that he does not really pay a wage to himself. Instead, after paying all his workers, he decides how to spend the leftover of his money, either for reinvestment (i.e., his wealth in the following months directly decides whether he can hire more workers) or his own expenditures.

⁵This is done by first random permutation of all actors, and then choosing the actor according to the shuffled ordering.

according to the probability function 5.1, that weights potential employers by their wealth:

$$P(h) = \frac{m_h}{\sum_{x \in H} m_x} \quad (5.1)$$

- (b) The worker's negotiated wage "w" should be settled the moment he gets employed by a certain firm. The employer h selects the negotiated wage, w , from the possible employee a 's wage expectation interval $[w_a, 1.5w_a]$ ⁶ according to uniform distribution. To avoid too small a wage expectation, here we define the workers' minimum wage expectation should be $w_a \geq 1$ (for $w_a < 1$, adjust negotiated wage to $w_a = 1$). If the employer can afford the negotiated wage ($m_h \geq w$, which is higher than the minimum wage $w \geq w_a$), h hires a : set a 's employee index to $e_a = h$, a 's new wage expectation has increased to the current negotiated wage level $w'_a = w$, and the number of employees for employer h has been increased by 1.
- (c) If the negotiated wage is higher than the employer can afford, $m_h < w$, actor a remains unemployed, $e_a = 0$. However, after this unsuccessful experience, actor a starts to think that he might have set the wage expectation too high, therefore, he sets a new wage expectation, which can be 50% lower than the original wage expectation: select a new wage expectation for the next job search $w'_a \in [0.5w_a, w_a]$ according to a random draw from a uniform distribution. The new wage expectation should be at least the minimum wage ($w'_a \geq 1$).

2. Revenue rule:

⁶Each actor a may set individual wage expectation, but he also has an acceptable minimum wage expectation w_a , as well as a higher wage expectation $1.5w_a$, which is 50% more than his lowest acceptable wage. If the negotiated wage has been accepted at the date of employment, the employee's wage will start at this level and increase by 10% from year to year as long as he stays at the same firm.

- (a) If actor a is not unemployed, a can be either an employer or an employee, and in both cases, he gains revenues m from the market. The amount m is selected from the current market value $[0, V]$ according to a triangular distribution.
- (b) If actor a is an employee, then transfer m coins to the employer. Alternatively, if actor a is an employer, then transfer m coins to himself. All transferred money is counted as firm revenue, and is finally added to the employer's money holding.

3. *Payment and firing rule:*

- (a) If the selected actor a is an employer, he needs to pay wages to his employees, according to their negotiated wage w_i when a certain employee i is hired (through the “hiring rule”). If the employee stays at the same firm, his wage (together with his wage expectation) should increase by 10% on a yearly basis. To avoid unrealistic wage increments, we pose as an upper boundary that the maximum wage expectation should not be more than 5 times the negotiated wage when this employee was hired ($1 \leq w_i \leq 5w_i$).
- (b) The wage payment w_i should be the amount that employer a can afford. Therefore, if $m_a > w_i$, add w_i to i 's money holding m_i , and the amount of employer's cash reserve m_a is reduced by w_i .
- (c) If the employer a has insufficient budget to pay the employee i , $m_a < w_i$, the employer pays half of this money to this employee, but needs to fire him. Add $0.5m_a$ to i 's money holding m_i , so that the employer's cash reserve is reduced to $0.5m_a$, the number of a 's employees is reduced by 1. The fired employee goes to the unemployed class with his wage expectation being equal to the last wage payment, thus $e_i = 0$ and $w_i = 0.5m_a$. When the last employee has been fired, the firm goes bankrupt, and the employer a himself enters the unemployed class too ($e_a = 0$), while his wage expectation remains at the initial amount the moment he became an employer.

4. *Expenditure rule:*

At each iteration, the actor a is able to buy consumption goods in the marketplace. We assume that no matter how many times he consumes, his total expenditure is calculated at once at the end of each iteration (i.e., month). Randomly select an expenditure amount m from a 's money holding, $m \in [0, m_a]$, according to a triangular distribution. Add m coins to the available market value V , hence, m_a is reduced by m , and the market value V is increased by m .

At the end of each year, count the total amount of firm revenues as GDP of that year. The growth rate of GDP is defined by $g_t = \frac{\text{GDP}_t}{\text{GDP}_{t-1}}$ (the first year GDP growth rate is set to be 1, which stands for no growth). The market value V is varying hand-in-hand with GDP, and at the beginning of the next year it becomes $V_{t+1} = V_t \cdot g_t$, which allows for non-stationary behavior of the monetary circle of our economy (i.e., the sum of the money holdings of all actors and the market value V is continuously changing).

5.3 Firm Dynamics

Except for quantitative differences, most simulation outcomes from the EPCE model, such as the social class distribution, the fluctuations of wage and profit share in GDP, etc., are generally similar as in the EPC model. However, since the EPCE model is more detailed in regulating both the firm's and individual's behavior, it allows us to study some properties of firms, which have not been accessible in the previous models.

What is the major property of our model generated firm dynamics? Fig. 5.1 answers the questions by plotting a typical firm generated by the model which lasts for 60 years. Fig. 5.1 (a) shows the change of firm size (employee numbers) and Fig. 5.1 (b) shows the firm's growth rate (in terms of firm size (or employee number) at the end of each year) throughout its life. It is clear that although this firm is indeed "long-lasting", it is not always a "large" one and its growth rate is fluctuating. Within the

model framework, the growth rate of any firm, no matter whether it is big or small, long-lasting or short-lived, has no *obvious* trend⁷. Moreover, in reality, large firms usually last longer. In our model, however, it is not possible to differentiate a “large” or a “small” firm by its length of surviving years, we call a firm “large” only when it maintains a “large” number of employees.

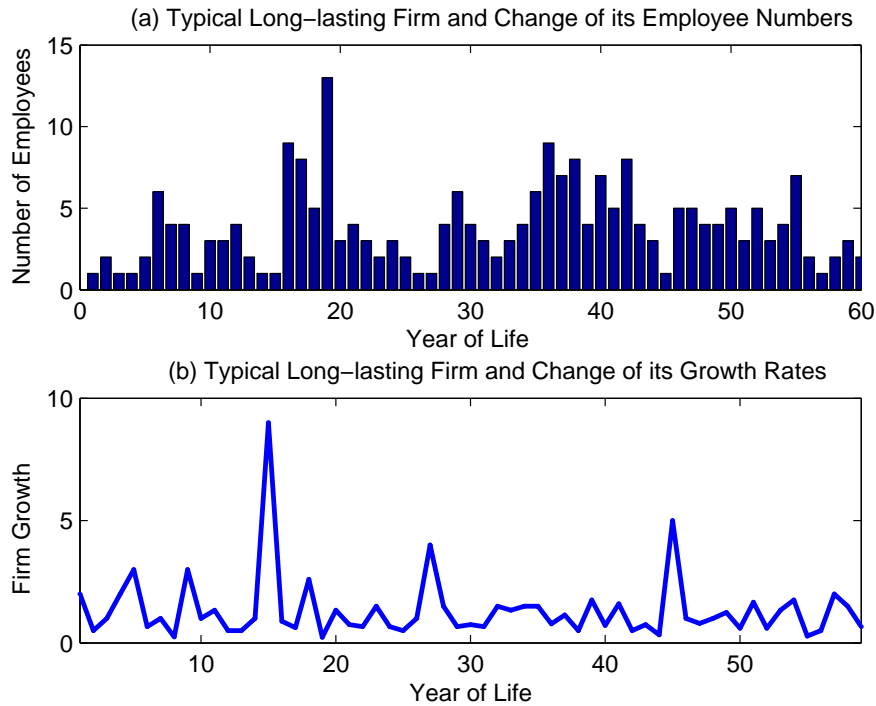


Figure 5.1: Firm size and growth of a typical long-lasting firm. Figure (a): a firm which lasts for 60 years and its employee numbers during the whole life expectancy; Figure (b): the firm growth rates calculated by division of yearly employee numbers shown in figure (a).

Fig. 5.1 displays a representative long-lasting firm. This kind of firms are in the minority and most firms are short-lived. Meanwhile, it does not necessarily mean that firm size has no correlation with firm growth. Later we will show our interesting findings that, firm size or growth rate is not completely random, and large firms tend to grow more slowly than small firms. In a “prosperous” year it is more possible for a firm to

⁷This is in line with the literature research in Chap. 2.4 that the firm survival rates do not fluctuate too much, and that the entry and exit of firms remain roughly steady across years

hire more employees and become “larger”.

5.3.1 Firm Life Expectancy

Firm life expectancy measure: At the end of each month/year, check the number of employees in each firm which remains in business for the entire period, if the employee number is more than (or equal to) 1, the firm life is prolonged for another month/year. The firms’ life expectancy starts when the firm hires its first employee, and ends when the firm fires the last employee.

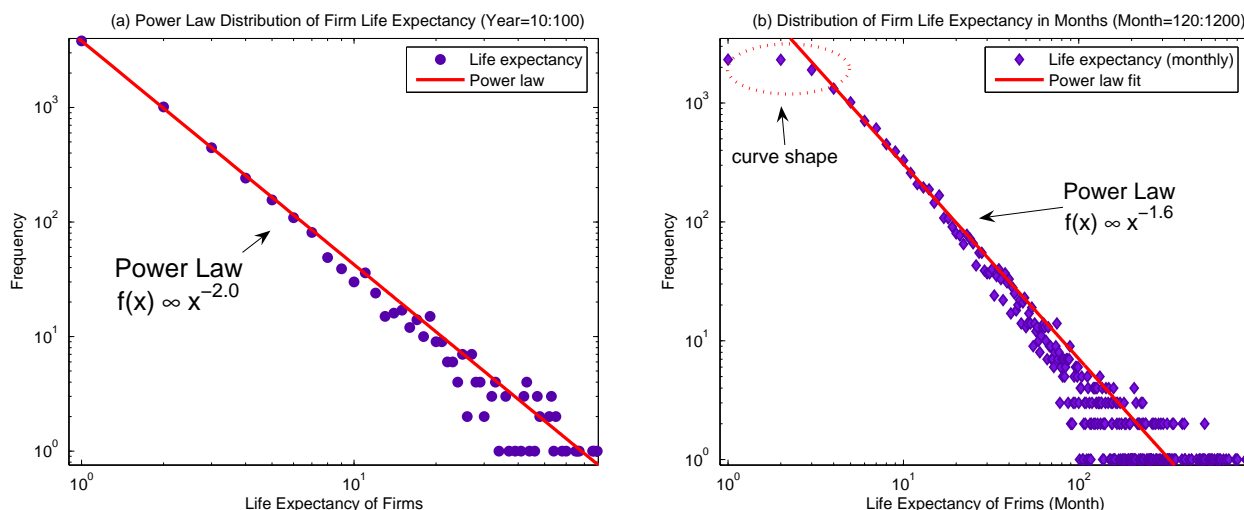


Figure 5.2: Life expectancy of the firms. Figure (a): histogram of the yearly life expectancy of the firms plotted on a log-log scale, the smooth curve is a power law fit (R-sq= 99.99%, R-sq(adj) = 99.99%) (Year=10:100, the transient period is omitted); Figure (b): histogram of the firm life expectancy calculated in months, the smooth curve is a power law curve fit after truncating for the firms which survive for less than three months (R-sq= 99.57%, R-sq(adj) = 99.47%) (Month=120:1200, transient period is omitted).

Mathematically, a quantity x obeys a power law if it is drawn from a probability distribution $f(x) \propto x^{-\alpha}$, where α is a constant parameter known as the *exponent* or *scaling parameter*. A special case when $\alpha = 2$ is known as Zipf’s law. Fig. 5.2 records

simulation outcomes of firms' life expectancy (or firm age) distribution. The distribution seems well represented by a power law. This pattern is commonly called "80/20 rule"⁸, and has long been familiar with those studying distributions in *income* (Pareto (1897), few rich control most of social wealth), *word frequencies in text* (Zipf (1932), some words such as "the", "that" are much more frequently used than the others), *city sizes* (Zipf (1949), a few large cities have far more population than the average), *page numbers and links of websites on the internet* (Huberman and Adamic (1999), a small percentage of websites are much frequently viewed or linked), and so on.

It seems that the curve fitting of firm life expectancy based on yearly calculation, as shown in figure (a) of Fig. 5.2, displays a satisfactory Zipf's law. However, in practice, only very few empirical phenomena obey a power law for the entire region, it is more often that the power law applies only for values larger than a certain minimum (i.e., $x \geq x_{min}$), which means only the upper part of distribution follows a power law. For example, figure (b) of Fig. 5.2 calculates firm life expectancy in months, however, the complete range does not fit entirely into a power law distribution. The first three or four values are obviously outliers for the power law fitting (i.e., the dotted cycle area shows a curved shape). After truncating for the firms which last less than (or equal to) three months, the power law fitting is immediately improved⁹.

So far, we have not found any empirical research targeting the study of firm life expectancy. However, there are a lot of studies focusing on demise or exit of firms, since more related datasets are available¹⁰. By looking into the high exit rate of firms

⁸Also named "Pareto principle" discovered by the Italian economist Vilfredo Pareto, who observed that 80% of income in Italy was received by 20% of the Italian population. The conjecture is that most of results in many situations are determined by a small number of causes. Pareto demonstrated in his research that this "rule" holds roughly true to an 80/20 ratio, but in many cases the ratio can be a lot higher.

⁹The power law fitting to figure (b) of Fig. 5.2 without truncating very short-lived (lasts less than 3 months) firms is roughly R-sq= 85%, the curve fitting after truncation improves to R-sq= 99%.

¹⁰For example, the *U.S. Small Business Administration* provides data accumulated by *Census Bureau and Internal Revenue Service*, reporting the number of firms and establishments each year, along

(cf. Chap. 4.3.4), it is obvious that the “80/20” rule holds, and only a minority of firms have long life expectancies, whereas most of the rest enter and exit within a short time. A recent study by Morris (2009) emphasizes that small firms are more likely to die than the large firms. Typically, from the year 2003 to 2004, 15% of the small firms (with fewer than 5 employees) exit, but only 3.1% of the large firms (with 100–499 employees) die, this number is further reduced to 1.78% for mega firms (with over 500 employees). A related analysis of the firm demise rate distribution (which follows Gaussian) will be discussed in Chap. 5.3.4.

5.3.2 Firm Size Distribution

Fig. 5.3 displays the firm size distribution with respect to employee numbers. Although in the EPC model the firm size distribution can be fitted by a Lognormal distribution, it seems that in the current model, the Lognormal distribution can not fit both the lower tail and upper tail well (such as in figure (a) (b) of Fig. 5.3), and a power law can not fulfill this task for the entire distribution region as well (due to significant curvature of the upper tail in Fig. 5.3 (b)).

However, Fig. 5.3 (b) actually looks quite close to many empirical plots of the firm size distributions, which in fact, look partly Lognormal, and partly power law before truncation, and cannot be entirely described by either of them. This interesting fact has been mentioned by Stanley et al. (1995) for American firms and Ishikawa (2008, 2009) for Japanese firms. Usually, on a log-log plot, the Lognormal distribution is seemingly curved, and a power law looks like a straight line. A closer look into Fig. 5.3 (b) also shows a modest curvature, especially in the middle region. We also demonstrate in figure (c) and figure (d) that a power law firm size supported by Axtell (2001) only applies after a significant truncation for large (> 20) and small firms (< 3). We agree with Sutton (1997) that probably there is no general density function that could

with the number of firms exiting the industry. The data is available since 1988.

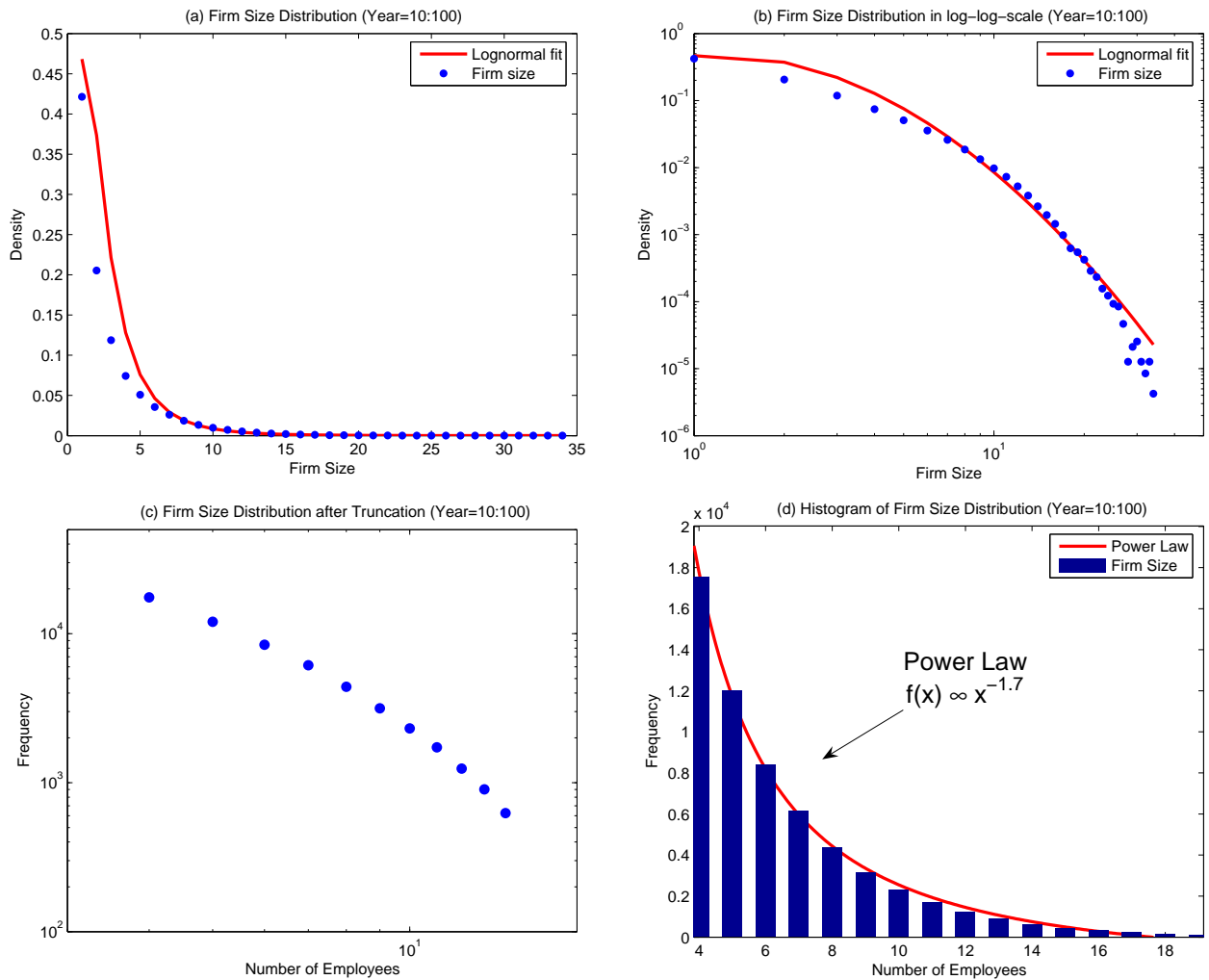


Figure 5.3: Figure (a): PDF of the firm size distribution after truncating the transient period of first 10 years, and the smooth curve is a corresponding Lognormal fit using the sample mean and variance; Figure (b): the same plot in log-log scale as figure (a); Figure (c): firm size distribution for firms with employee numbers larger than 3 and less than 15 plotted in log-log scale; Figure (d): firm size distribution after truncating very large (> 20) and small firms (< 3), the smooth curve is a power law fit (R-sq= 99.83%, R-sq(adj)= 99.81%). (Year=10:100).

successfully describe the empirical densities of the firm size distribution.

Disregarding the discussion of whether a Lognormal distribution or a power law fits the firm size data better, the simulated distribution of firm size by the EPCE model is quite different from previous models. The maximum monthly firm size is 34, and the

minimum firm size is 1 which is the minimum requirement for setting up a firm. As comparison, in the original SA model, the largest firm size has exceeded 600 employees even in a 15 years' short simulation; whereas in the EPC model, a test simulation indicates that the largest firm has become much smaller having 121 employees. It happens that if one firm holds a large number of employees, then the other firms are "forced" to be small since the total population in the model is fixed.

Our conclusion is in accordance with Simon and Bonini (1958)'s finding that: the overall distribution of firm size is highly skewed, with a small number of large firms coexisting with a large number of small firms in the economy; the exact form of such skewed distribution seems to lie somewhat between a Lognormal and Pareto law.

5.3.3 Firm Size and Growth Rate

While Gibrat (1931) assumes that the firm growth rates are independent of firm size, more recent empirical research finds that the growth rates fall with firm size (e.g., in Prais (1976), Evans (1987), Lotti et al. (2003)). The present model assumes that the firm size is dependent on the firm owner's budget and growth is dependent on the firm's revenues.

Figure (a) of Fig. 5.4 displays the histogram of the log growth rate, measured by employee numbers, which is similar to the results from the EPC model (cf. Fig. 4.4) and looks Laplace distributed (i.e., a two-sided exponential distribution). Apparently, most of the growth rates are relatively small, and large increases or decreases rarely take place. Figure (b) of Fig. 5.4 plots the relationship between firm sizes and growth rates, which exhibits a cluster at the lower region. It seems that fast-growing firms are all small firms (with less than 5 employees), and the larger the firm, the slower the growth rate. In other words, it is more probable for a small firm of 5 employees to hire 5 new workers, as it is for a size 30 firm to hire another 30 new workers.

Most of the literature studying firm dynamics in the area of "Industrial Organization" is

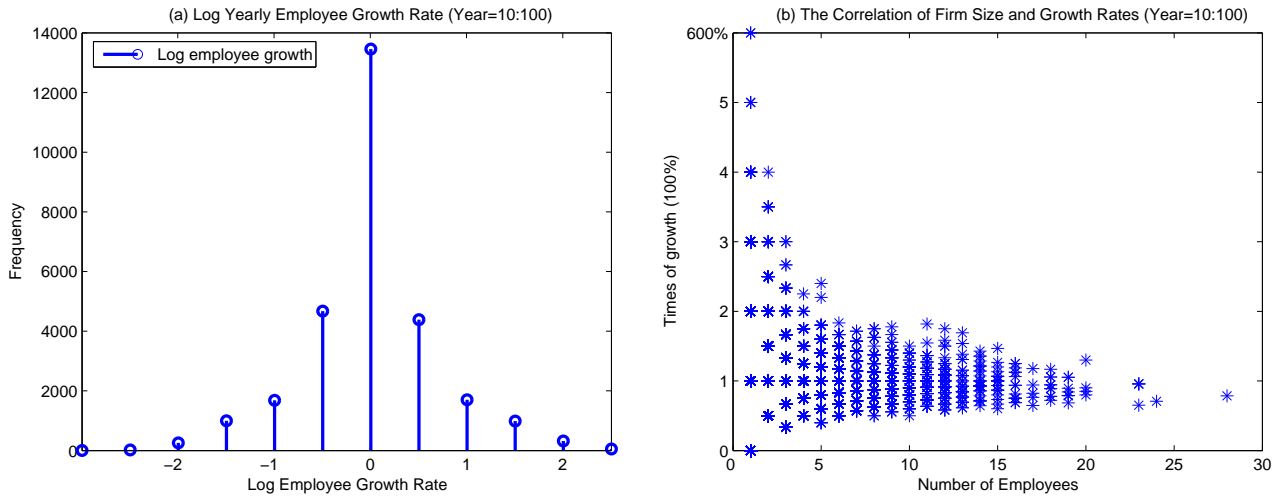


Figure 5.4: Relationship between firm sizes and growth rates. Figure (a): histogram of the firm growth rate distribution; Figure (b): relationship between firm sizes and firm growth rates (Year=10:100).

based on the assumption that firm growth rates are independently *normally* distributed (cf. random growth hypothesis by e.g., Caves (1998), Lipczynski et al. (2005)). As both the EPC and the EPCE model suggest a Laplace distribution, which distribution is in fact closer to the empirical firm growth rate distribution, and what is the major difference between the Laplace and the normal distribution?

We randomly pick up 500 realizations from Fig. 5.4 (a), and plot Fig. 5.5 (b), which shows the time series for the *Laplace*-shaped log employee growth rate. Then we use the same mean and variance of the Laplace distribution to generate a corresponding *Gaussian* time series shown in Fig. 5.5 (c). When comparing these two figures, although they have the same first and second moment, in figure (b) large deviation away from $[-1.5, 1.5]$ can be observed, with the majority of values clustering around the mean. On the contrary, the probability of extreme values in figure (c) is very rare, with almost all data fluctuating within the boundary $[-1.5, 1.5]$, and a comparably much less centered distribution than figure (b). Figure (a) plots the histogram of both time series into the same figure, the tent-shaped Laplace distribution possesses more extreme data than the bell-shaped Gaussian, especially in the center and in the tail. To apply it

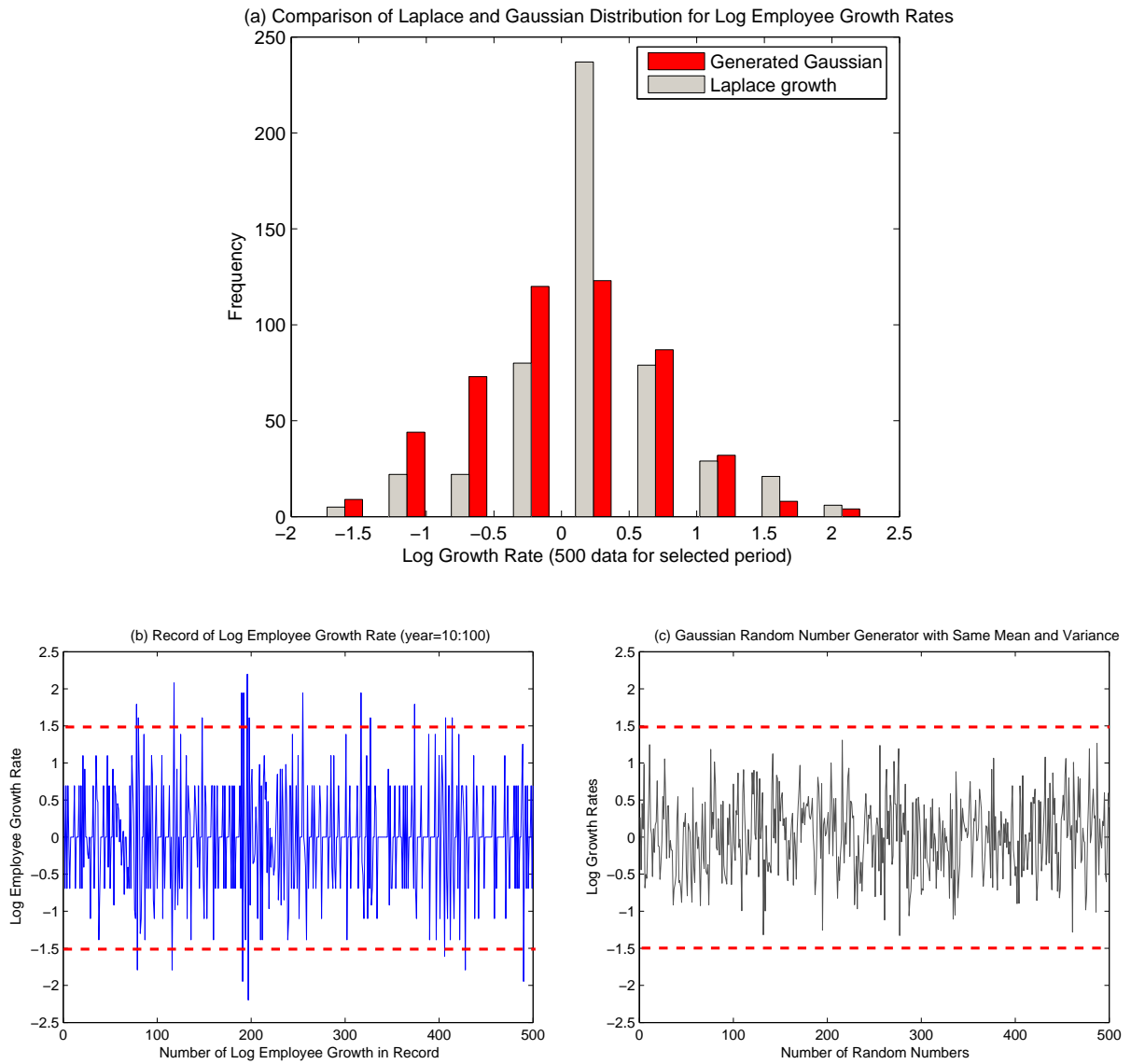


Figure 5.5: Figure (a): comparison of the Laplace and the Gaussian distribution for the log employee growth rates; the left (grey) columns are (500 realizations of) the original log employee growth rates generated from model, the right (red) columns are generated corresponding Gaussian distributed growth rates with the same mean and variance as the Laplace distribution; Figure (b): time series of the Laplace distributed data (i.e., 500 realizations of the log growth rates); Figure (c): time series of the Gaussian distributed data with the same mean and variance.

in empirical study of employee growth rates, it seems that the Laplace distribution is more capable of explaining why a large number of firms either experience very strong

n	$Pr_{Normal}(r \geq n\sigma)$	$Pr_{Laplace}(r \geq n\sigma)$	$Pr_{Laplace}/Pr_{Normal}$
2	2.3×10^{-2}	6.7×10^{-2}	2.97
3	1.3×10^{-3}	2.5×10^{-2}	18.4
4	3.2×10^{-5}	6.7×10^{-3}	289
5	2.9×10^{-7}	3.4×10^{-3}	1.18×10^4
6	9.9×10^{-10}	1.2×10^{-3}	1.25×10^6
7	1.3×10^{-12}	4.6×10^{-4}	3.56×10^8
8	6.2×10^{-16}	1.7×10^{-4}	2.7×10^{11}

Table 5.1: Tail probabilities of Laplace and normal Distribution

growth fluctuations or no growth at all, and in particular, why the fast-growing firms are actually mostly small firms whereas large firms usually enjoy slow or modest growth rates.

To further compare the Laplace distribution and the Gaussian (at the same mean and variance), we quote part of Teitelbaum and Axtell (2005)'s interesting table studying the tail behavior of both distributions in Table 5.1. It illustrates that compared with Laplace, the normal distribution has much less frequent tail events, e.g., the chances of extreme events (i.e., events happen at the distance of more than 4σ from the mean) in the Laplace distribution is hundreds of times larger than the corresponding normal distribution.

Our simulation of a Laplace distributed firm growth rate distribution is consistent with Stanley et al. (1996), Bottazzi and Secchi (2003a), who study the whole database of the U.S. firms. It has been shown that the Laplace distribution also holds within industry sectors. For example, Demirel and Mazzucato (2008) look into 323 pharmaceutical firms quoted in *S&P* 500 index during the year 1950 and 2003, and find out “the firm growth distribution is not a normal distribution but a more peaked distribution with thicker tails than the normal distribution”. It is suggested by Teitelbaum and Axtell (2005) that in some special industries, the growth rate distribution is even more heavily

tailed than the Laplace distribution, rather than tending towards normality; besides, since small firms are quite vulnerable and experience severe growth fluctuations, “government policies whose goal is to shield firms from fluctuations may be ineffective, however, policies designed to help firms ride out difficult times may go a long way in prompting firm survivals”.

5.3.4 Small Firms and the Economy

As we have mentioned, extremely large firms such as Microsoft, Walmart, GE are very exceptional cases across industries. They have monopoly power in their respective areas, and their growth histories are not easily replicable. Therefore, these extremely successful firms are not the representatives of the whole business sector, and are often discarded by researchers when studying the firm dynamics. As a matter of fact, the majority of firms are normally young and small, and more small businesses can be observed in the economic boom years than in the depression. Many researchers (such as Acs (1999), Acs et al. (1999), Teitelbaum and Axtell (2005), Lotti et al. (2003), among others) believe that the number of small firms is an indicator for the current economic situation.

In order to see how the number of small firms and big firms vary with the economic situation, Fig. 5.6 presents bivariate scatter plots¹¹ of the relationship between firm numbers and GDP. Since extremely large firms do not exist in our model, our definition of the “large” firm is limited to the *relatively* big firms. Particularly, a firm with less than 15 workers is defined as a “*small*” firm, and a firm with more than 15 employees

¹¹Bivariate scatter plot graphs the degree and pattern of relationship between two variables under measurement. In these scatter plots, the straight line shows the correlation coefficient estimated by a simple linear regression, but the points do not follow this straight line perfectly, and the distance between the points and the regression line is called “scatter”. A large amount of scatters indicate weak relationship, whereas fewer scatters represent strong relationship. If all points fall directly onto the straight line, we have a perfect linear relationship between these two variables.

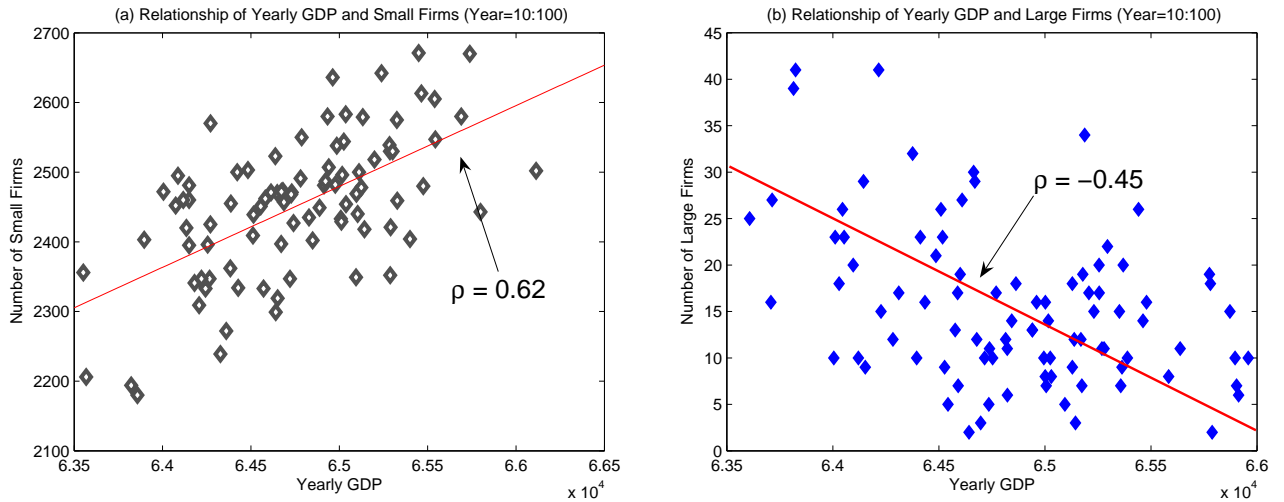


Figure 5.6: Figure (a): bivariate scatter plot of yearly GDP and small (with employee numbers between $[1, 15]$) firm numbers; the smooth curve shows a linear regression, and there is a positive relationship between these two variables ($\rho = 0.62$). Figure (b): bivariate scatter plot of yearly GDP and large (with employee numbers larger than 15) firm numbers; the smooth curve shows a linear regression, there is a negative relationship between these two variables ($\rho = -0.45$). (Year=10:100).

belongs to a “large” firm. Figure (a) of Fig. 5.6 plots relationship between the numbers of small firms and corresponding yearly GDP. The straight line is a simple linear regression, and the correlation coefficient estimated is around 0.62, showing a strong positive relation between them. This indicates that small firms play an important role in the economy, and when the economy is prosperous (i.e., with high GDP), there are more small firms existing in the market too. As comparison, Figure (b) plots the relationship between the numbers of large firms and GDP. The straight line shows even a negative relationship with correlation coefficient $\rho = -0.45$, which suggests that even in good years, the number of large firms does not necessarily increase.

Our simulation results are more-or-less in line with reality, as business cycle influences the growth of small firms more than the large firms (cf. Gertler and Gilchrist (1994), Quiros and Timmermann (2000), Pindado et al. (2006), and others). This is because small firms tend to be more risk-seeking and more productive, and growth of the economy will easily encourage more small firms to start businesses and share profits.

Nevertheless, small firms are comparably more fragile. Once a depression comes, they are usually the first to be kicked out of the industry. A good example is the U.S. industry: in the 1990s, the U.S. economy had been growing hand-in-hand with an exploding number of small startup firms, yet starting from the end of 2008, the (world) economy declined with a large number of small firms closing down at the first stage of the crisis. Meanwhile, the fact of frequent entrance and exit of small firms in the business cycles also coincides with the highly peaked and heavily tailed Laplace firm growth rate distribution, which can be explained by small firms experiencing much stronger fluctuations than they would encounter if their growth rates were Gaussian. It seems that one way to avoid too many fluctuations is to let the smaller firms enter the group of large firms. Therefore, it is perhaps more effective if our government could support the small firms to grow, rather than encouraging more start-ups, especially in recession times.

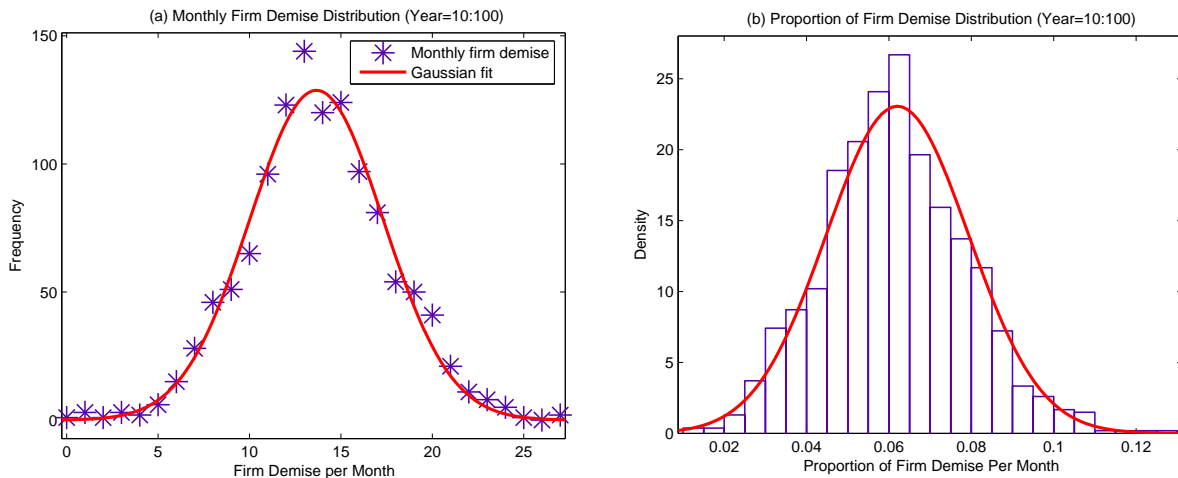


Figure 5.7: Figure (a): histogram of monthly firm demise distribution and Gaussian curve fitting (R-sq= 98.05%, R-sq(*adj*) = 97.9%); Figure (b): proportion of firm demise distribution and Gaussian curve fitting (R-sq= 99.35%, R-sq(*adj*) = 99.17%) (Year=10:100).

Finally, we provide the firm demise distribution in Fig. 5.7, which is in accordance with the study by Cook and Ormerod (2003). It shows that the number of firm demises has very little variation throughout the whole simulation. Figure (a) records the number of

demises on a monthly bases, and Figure (b) calculates the proportion of firm demise in the overall existing firms. It tells that on average there is 13 firms exiting the industry per month, which is around 6% of the total number of firms¹². As fluctuations happen more frequently with small firms, they tend to contribute most to the frequency of firm demise.

5.4 Income and Wealth Dynamics

The improvement in the micro formulation of the EPCE model also enables one to have a deeper look into the distribution of income and wealth. Strictly speaking, income is only part of wealth. For workers, wages are their only source of income, whereas the employers have broader sources of income. In our simulation, the leftover of revenues after paying wage bills, are all calculated as income for employers, which can include revenue income, material wealth, capital assets, and so on.

Much of the literature (e.g., since Pareto (1897), Zipf (1949), followed by Drăgulescu and Yakovenko (2000), among others) argue that the upper tail of the income distribution is power law after truncating the lower part. Others (e.g., Montroll and Badger (1974), West and Deering (1995), among others) find that the income distribution may be better fitted by a Lognormal type, but this fitting is perhaps unsatisfactory in the upper tail (cf. discussions in Chap. 2.8). However, perhaps neither of them is able to explain the whole story well. In this section, after splitting the income distribution into two parts, i.e., the income for workers and employers respectively, we propose that the whole region of income distribution might be described by a multimodal distribution aggregating several normal distributions, which represent income of different groups.

¹²Note that this result should be in line with the EPC model, and actually the original SA model as well. Wright (2005) suggests that the firm demise distribution may follow a Lognormal distribution, but our replication shows that the firm demise distribution following either the original or the modified rules is fitted better by a normal distribution.

This feature is overlooked by many economists. The popular way of studying income is to convert the normal scale into the log-scale, so that the upper tail is many times enlarged against the lower end. We will see that the mixed normal distribution explains the entire region of income distribution quite well. Although sometimes multiple modes are not directly visible, this is just because the means of separate distributions are not far away from each other to display an additional mode, yet their overlapping tails can still be well described by multiple normal distributions. The simplest case of a multimodal distribution is presented in Schilling et al. (2002), showing that a mixture of two equally weighted normal distributions with standard deviation σ is bimodal if and only if the difference between the means of the two distributions is greater than 2σ . As a matter of fact, multimodal distributions exist everywhere in our daily lives, and some of the typical multimodal-distributed stylized facts have been discussed in Chap. 2.8.

5.4.1 Wages for Workers

Wage is a compensation, usually in terms of salary, received by workers in exchange for their labor. Empirically, it is the most easily achievable data among wage, income and wealth, because it is carefully recorded in every firm's accounting book. As a result, most of the income data publicly available are in fact the wage data reported by the government's tax office. What's more, since wage is just the compensation for (hourly or daily) labor input, it should not be much different across firms, consequently, using only wage data to model income (or wealth) may largely underestimate the gap between the rich and the poor.

In the previously discussed EPC model, the yearly wage for workers is found to be Gaussian distributed: as all the wages are selected within a certain boundary, according to the CLT, the sum of many *i.i.d.* random variables will be normally distributed (cf. Fig. 4.11 (a) in Chap. 4.3.9). However, in the EPCE model, the wage limit is

released, the worker's wage expectation increases when he remains employed, and decreases when he could not find the next job. For example, if a worker stays in the same company for more than 10 years, his wage expectation will increase to as high as 3 times his starting wage expectation¹³.

Fig. 5.8 shows the aggregate wage distribution throughout the years. Figure (a) displays a clear long and flat right tail, obviously the wage distribution is not Gaussian due to the asymmetry in the distributional form. If the same figure is plotted in log-log scale, as in figure (b), we will see a straight line in the upper level of the wage distribution, indicating a power law¹⁴. However, this power law only applies for a yearly wage larger than 100, and figure (a) indicates that the majority of the yearly income is in fact around 50 with very few workers having wages larger than 100, therefore, the power law in figure (b) actually only explains one-third of the whole wage distribution.

On the other hand, Fig. 5.8 (a) at the first glance, may look Lognormal, thus we plot in figure (c) the log of the wage distribution. If the wage distribution follows Lognormality, the log of wages will follow the normal distribution. Unfortunately, the log of wages in figure (c) still look asymmetric with a flat left tail, and the dashed line (Gaussian fit) reveals that the original wage distribution can not be Lognormal as well. In the end, we try to use a multimodal normal distribution to explain the skewness and fat tail found in the data. In figure (d), the dashed red line displays a single Gaussian fitting ($f(x) \sim N(59.1, 37.5^2)$) to the worker's yearly wage distribution, and obviously both the lower and upper tails are not well fitted. In contrast, the blue smooth line, which

¹³As most of firms do not last very long, it rarely happens in our model that an actor will stay in one firm for more than 10 years. Meanwhile, in the "payment rule", we have set an upper boundary, that the worker's wage expectation should not be more than 5 times his initial wage expectation when he was hired. In addition, the firm owner will fire this person the moment he can not afford to pay his wage expectation.

¹⁴Note that in this figure, the wage amount smaller than 20 is truncated, so that the upper tail behavior can be emphasized, this is exactly how most of the income plots are dealing with data, and the final plots will be usually in a log-log scale.

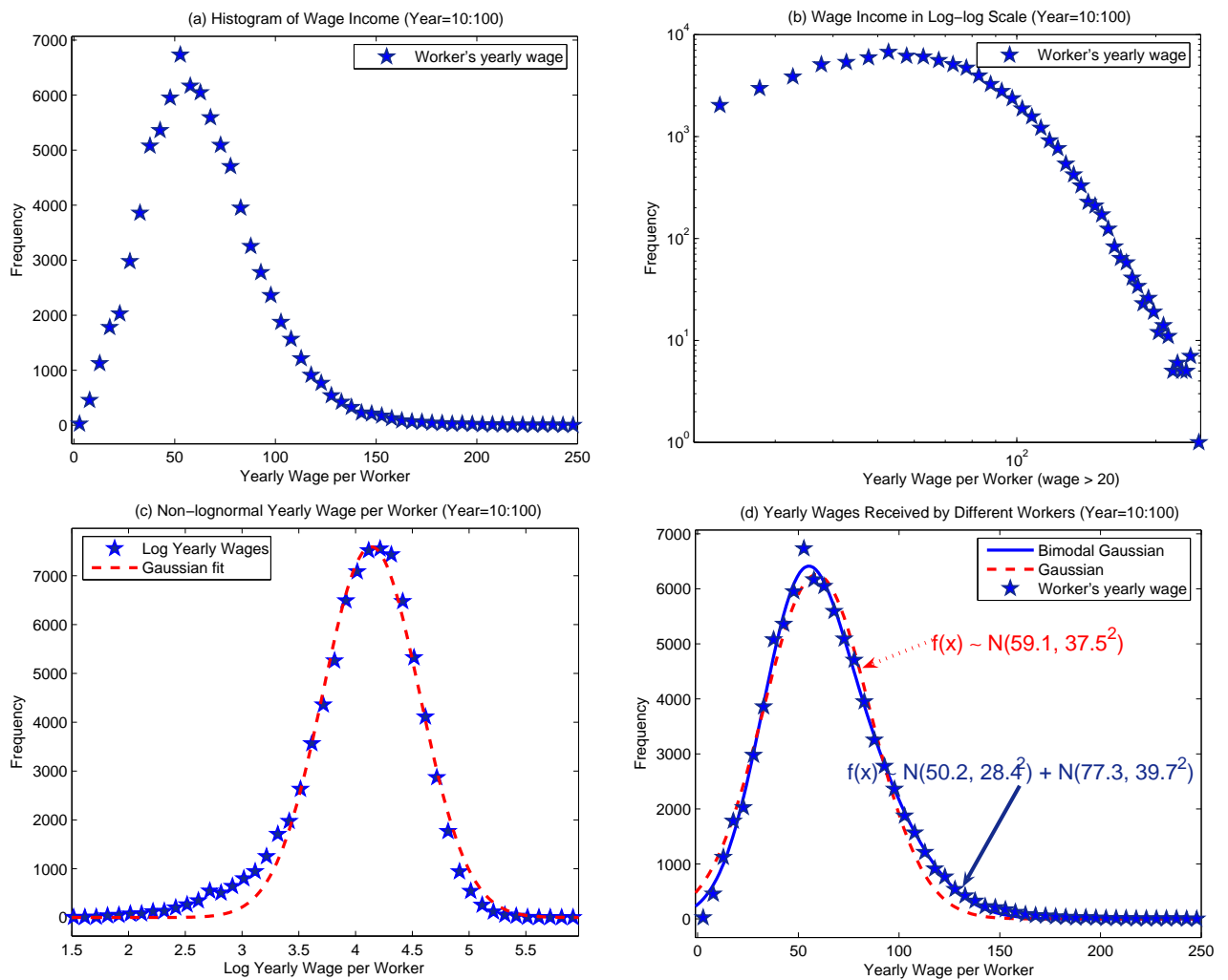


Figure 5.8: Figure (a): histogram of the yearly wage incomes from workers; Figure (b): the same figure as in (a), but on a log-log scale, wage amount smaller than 20 is truncated for a better view of the upper tail; Figure (c): the dashed line is Gaussian fit to the log of the yearly wages (R-sq= 91.55%, R-sq(adj) = 90.89%); Figure (d): bimodal Gaussian fit to the yearly wage distribution (R-sq= 99.34%, R-sq(adj) = 99.29%). (Year=10:100).

represents a mixture of Gaussians ($f(x) \sim N(50.2, 28.4^2) + N(77.3, 39.7^2)$) has provided a satisfactory fit for the entire region of the wage distribution including both tails. As can be seen, the difference in the means of these two different-weighted Gaussian distributions is too small compared with their standard deviations to produce bimodality,

so that the distribution itself, in appearance, is still unimodal¹⁵. A clear bimodality can be observed in the income distribution for employers, as will be demonstrated in the next section.

5.4.2 Income for Employers

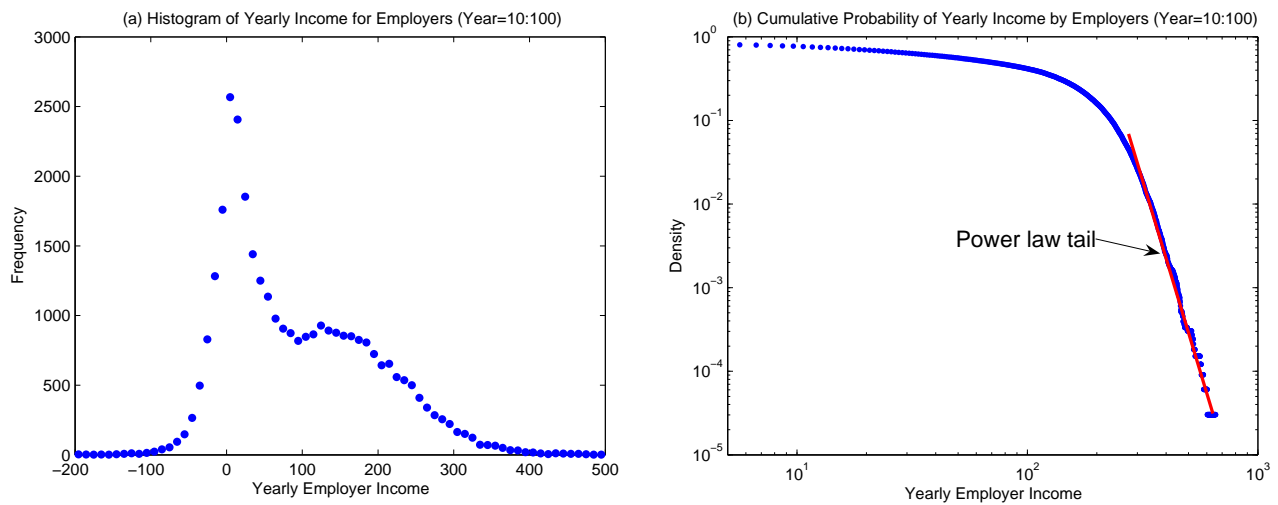


Figure 5.9: Figure (a): histogram of yearly income for employers; Figure (b): cumulative probability of yearly income for the employers, the smooth straight curve is a power law fit (Year=10:100).

Income¹⁶, by definition, refers to consumption opportunity gain by an entity within a specified time frame, which is generally expressed in monetary terms (cf. Case and Fair (2007), p. 54). Unlike wage, which is always positive as a compensation for labor, negative income can be as common as negative growth for firms. In other words, if

¹⁵Note that the weight of each separate Gaussian distribution is different, i.e., $a_1 \neq a_2$, (for this histogram plot, $a_1 = 7,495$, and $a_2 = 5,919$). For simplicity, we write the expression $f(x) = a_1 N(50.2, 28.4^2) + a_2 N(77.3, 39.7^2)$ in the reduced form: $f(x) \sim N(50.2, 28.4^2) + N(77.3, 39.7^2)$.

¹⁶In the SA model and the EPC model (as a modified version of the SA model), income for employers is calculated as the total amount of firm revenues. This definition may not be completely satisfactory, since in the model, the monetary gains of the employer should exclude the cost of labor. Therefore, letting income for employers to be the gains after subtracting the wage bill may be more accurate and plausible.

the firm is enlarging (i.e., employing more workers), its income is usually positive, however, if the firm is shrinking (i.e., firing the workers), it is a sign that its income might become negative. Note that a firm having continuous negative incomes does not necessarily mean that it will soon close down. If the firm owner is rich enough, he should be able to afford the negative profit for some time, fire some workers and try to recover. In some cases, if the reserved money holding of this employer is large enough to pay for his workers, the employer does not need to fire any worker at all even though his monthly income gets negative.

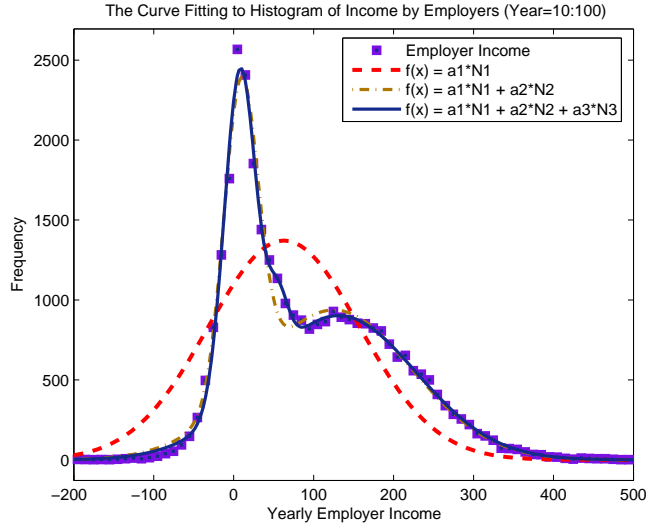
As we have mentioned, the most popular way of studying income distribution is limited to the upper tail of income, usually plotted in log-log scale to demonstrate a straight line (cf. Clementi and Gallegati (2004), Silva and Yakovenko (2005), Chatterjee et al. (2005), among others). This method not only truncates the major part of the income distribution, which is the high frequency part including the middle and lower income classes, but also automatically drops all negative values when the logarithmic transformation is applied. Figure (a) of Fig. 5.9 illustrates the full range of possible income in our simulations, and we observe that there are many observations around zero with the percentage of negative values around 10% of all incomes in record. Nevertheless, most of the incomes are positive, in order for the majority of firms to survive. Fig. 5.9 (b) converts Fig. 5.9 (a) into a log-log plot, where a typical power law is observed at the upper tail after a “knee” shape around the middle part. It is clear that the power law explains the income distribution for extremely rich employers, yet it only constitutes 10% of the entire employers’ income distribution, and the lower part (as well as the not displayed negative income part) which account for 90% of the income can not be described by the same power law. In addition, Fig. 5.9 (a) also shows that the resulting income distribution actually displays an additional mode around 150, making the histogram look like a bimodal distribution. This fact can not be seen in Fig. 5.9 (b), which looks quite close to some empirical income plots for Germany, Italy, U.K., the U.S., discussed in Clementi and Gallegati (2004), Yakovenko and Silva (2005), among

others.

Knowing that the upper tail of income may be more-or-less close to a power law, is there a distribution capable of explaining the entire range of the income distribution? Figure 5.10 exhibits our model generated employers' income distribution without any truncation of the original data (the same as Fig. 5.9 (a)), and the shaded scattered squares are the histogram data. We have tried an unimodal Gaussian, bi-Gaussian and tri-Gaussian distribution to fit the data. a_1 , a_2 and a_3 in Fig. 5.10 provide the estimated weights for each individual Gaussian distribution. The property of each curve fitting is listed in the bottom table¹⁷.

The unimodal Gaussian generally does not fit the data well and shows extremely high RMSE (Root Mean Square Error). The mode of this single Gaussian is equal to the mean of all income data, which seems far away from the true mode (the highest and second highest frequency of the histogram). More importantly, the original plot itself looks asymmetric while the Gaussian distribution displays a symmetric bell shape. A mixture of two Gaussian distributions has done a much better fitting job in all stages except for the central area. A significant improvement is that it delivers exactly the same shape as the original histogram with two different modes. If we check the curve fitting property of the bi-Gaussian, both R-squared and adjust R-squared announce 98% fit, and the RMSE has dropped considerably. It is interesting to extend the bi-Gaussian to a tri-Gaussian. We find that the tri-Gaussian distribution provides an excellent 99.99% fit. This means adding additional weighted Gaussians is not necessary any more. The blue smooth line (representing the tri-Gaussian fit) in Fig. 5.10 locates itself exactly at all points of the histogram, from center to both tails with RMSE

¹⁷This exercise is carried out using MATLAB's "Curve Fitting Toolbox". It estimates not only the Gaussian statistics, but also the parameters assigned to each single Gaussian distribution, which can be understood as weights for each individual distribution. For example, here in the bi-Gaussian case, we have $a_1 = 3,758$, $a_2 = 1,868$, and in the tri-Gaussian case, we have $a_1 = 2,128$, $a_2 = 2,187$, $a_3 = 1,775$. Note that this is a histogram plot, and if we draw the same figure in PDF format, the weights become very small.



	Fitting Function	R-square	Adj.R-sq.	RMSE
Uni-Gaussian	$f(x) \sim N(62.8, 133.9^2)$	0.732	0.726	278.72
Bi-Gaussian	$f(x) \sim N(9.2, 30.1^2) + N(125, 138.4^2)$	0.987	0.985	64.10
Tri-Gaussian	$f(x) \sim N(7.9, 27.7^2) + N(54.9, 18.1^2) + N(131.1, 137.5^2)$	0.999	0.998	6.15

Figure 5.10: Curve fitting to the multimodal income distribution of employers, with single Gaussian, bi-Gaussian, and tri-Gaussian distributions. (Year=10:100). We use a_1 , a_2 , a_3 to indicate different weights assigned to different Gaussian distributions, N_1 , N_2 , N_3 refer to the different Gaussian distributions in the mixture model, and the weights are omitted in the table.

dropping down to a very low level. The fitting function from the last line of table shows a summation of three weighted Gaussians. It is obvious that the means of these three Gaussians are differing enough, so that we could observe more than one mode. Nevertheless, since the variance of the second Gaussian is not large enough compared with the first and the last Gaussian, we can only observe a slight convexity in between the two modes, instead of three modes.

5.4.3 Aggregate Income Distribution

Fig. 5.11 depicts the overall income distribution when both the employees' and the employers' incomes are taken into account. The aggregate income again looks unimodal as shown in Fig. 5.11 (a), and when plotted in log-log scale, it has a power law upper tail. Obviously, the log-log plot hides many facts of the true distribution, especially the lower tail and negative part of the values. In particular, it hides the fact that, behind the “knee” shape, a certain part of the distribution, when plotting in histogram or probability density function, may look multimodal.

The overall income distribution does not show more than one mode, just because the differences in the means of all subgroups are not large enough (compared with their standard deviations) to display bimodality. This is also a common phenomenon in reality. Devore and Peck (1997) study the heights of college students, and find that both male and female height distribution is unimodal; however, the histogram of heights of all students will display one peak at a typical male height of 70 inches, and another peak at a typical female height of about 65 inches. Another study by Schilling et al.

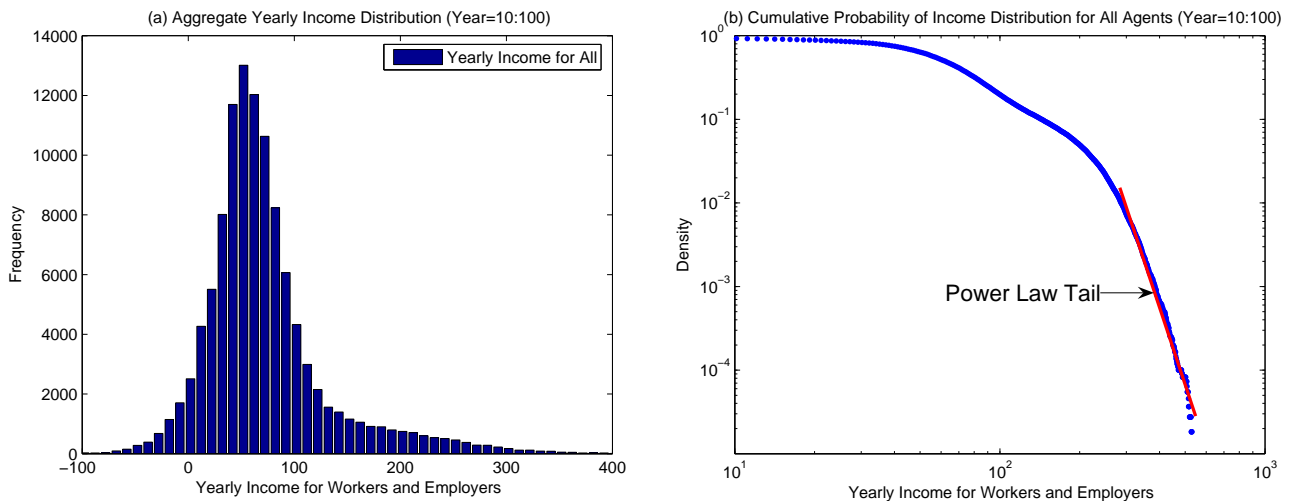
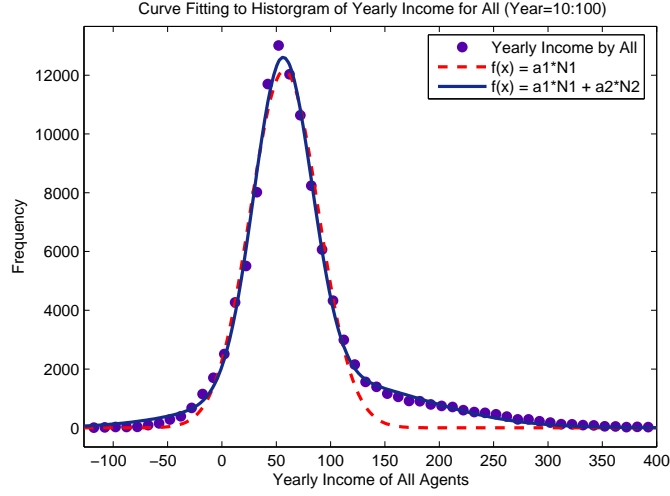


Figure 5.11: Figure (a): histogram of the yearly income for all agents; Figure (b): cumulative probability of the yearly income (after truncating the negative values), and the smooth line is a power law curve fitting. (Year=10:100).

(2002) uses the government data from National Health and Nutrition Examination Survey (NHANE) (conducted in 1988–1994 by the United States National Center for Health Statistics) to construct the approximate theoretical density function for heights of population in the 20-29 age bracket. They show that both height data for male and female population follow normality separately, with $f_{male} \sim N(69.3, 2.92^2)$ and $f_{female} \sim N(64.1, 2.75^2)$, and the mixture distribution of the heights will look very similar to figure (a) of Fig. 5.11: it has only one mode, is slightly asymmetric, and is neither normal nor Lognormal¹⁸. From these examples, we learn that there might be actually more stories hidden behind an existing distribution, hence it is worthwhile to know how the separate subgroups behave to generate the aggregate outcome. A general curve fitting using some popular distributional forms to any empirical plot or an odd distribution can easily run into trouble and is not always satisfactory.

Fig. 5.12 provides the curve fitting results for the aggregate income distribution of all agents, as well as the fitted functions listed in the attached table. Although the emerging distribution looks unimodal, it is not a symmetric Gaussian: a single Gaussian function does not fit very well the right tail of the distribution, and it has relatively large RMSE. In contrast, the bi-Gaussian function does a better job at fitting both tails with much lower RMSE and improves R-squared value to 99.88%. Because the means of two Gaussian distributions are not too distinct from each other (compared with their standard deviations), the final distribution does not exhibit bimodality.

¹⁸Schilling et al. (2002) show that: “if f_1 and f_2 be normal densities with respective means μ_1 and μ_2 , and common variance σ^2 , and let f be the mixture density $\frac{1}{2}f_1 + \frac{1}{2}f_2$, then f is unimodal if and only if $|\mu_2 - \mu_1| \leq 2\sigma$ ”. For a mixture of Gaussians which have different weights, different means and variances, the case can be more complicated, and it usually requires that the difference between two means is much larger than both standard deviations in order to generate a bimodal distribution.



	Fitting Function	R-square	Adj.R-sq	RMSE
Uni-Gaussian	$f(x) \sim N(57.4, 44.4^2)$	0.977	0.969	429.87
Bi-Gaussian	$f(x) \sim N(55.9, 37.6^2) + N(96.8, 124.1^2)$	0.998	0.997	26.8

Figure 5.12: Curve fitting to the multimodal income distribution of all agents, with a single Gaussian and a bi-Gaussian. (Year=10:100). We use a_1 , a_2 to indicate different weights assigned by different Gaussian distributions, and N_1 , N_2 to record different Gaussian distributions in short. Here, $a_1 = 11,190$, $a_2 = 1,585$ for the bimodal case.

5.4.4 Wealth Distribution

Wealth is defined as “assets - liabilities” that an individual holds, while income is an “*inflow*” of items of economic value. If the *expense* of an individual can be known as an “*outflow*” of items of economic value, the relation among wealth, income and expense can be written as: change of wealth = income – expense.

Although income is considered to be part of wealth, the distribution of income is substantially different from the distribution of wealth. As a matter of fact, empirical wealth data is not easy to obtained and hardly reliable. For example, it is more difficult to measure real estate, permanent assets and other capital assets, although they are significant parts of people’s wealth (especially for the rich). We address that in our study, income and wealth is not equivalent either: *income can be negative but wealth*

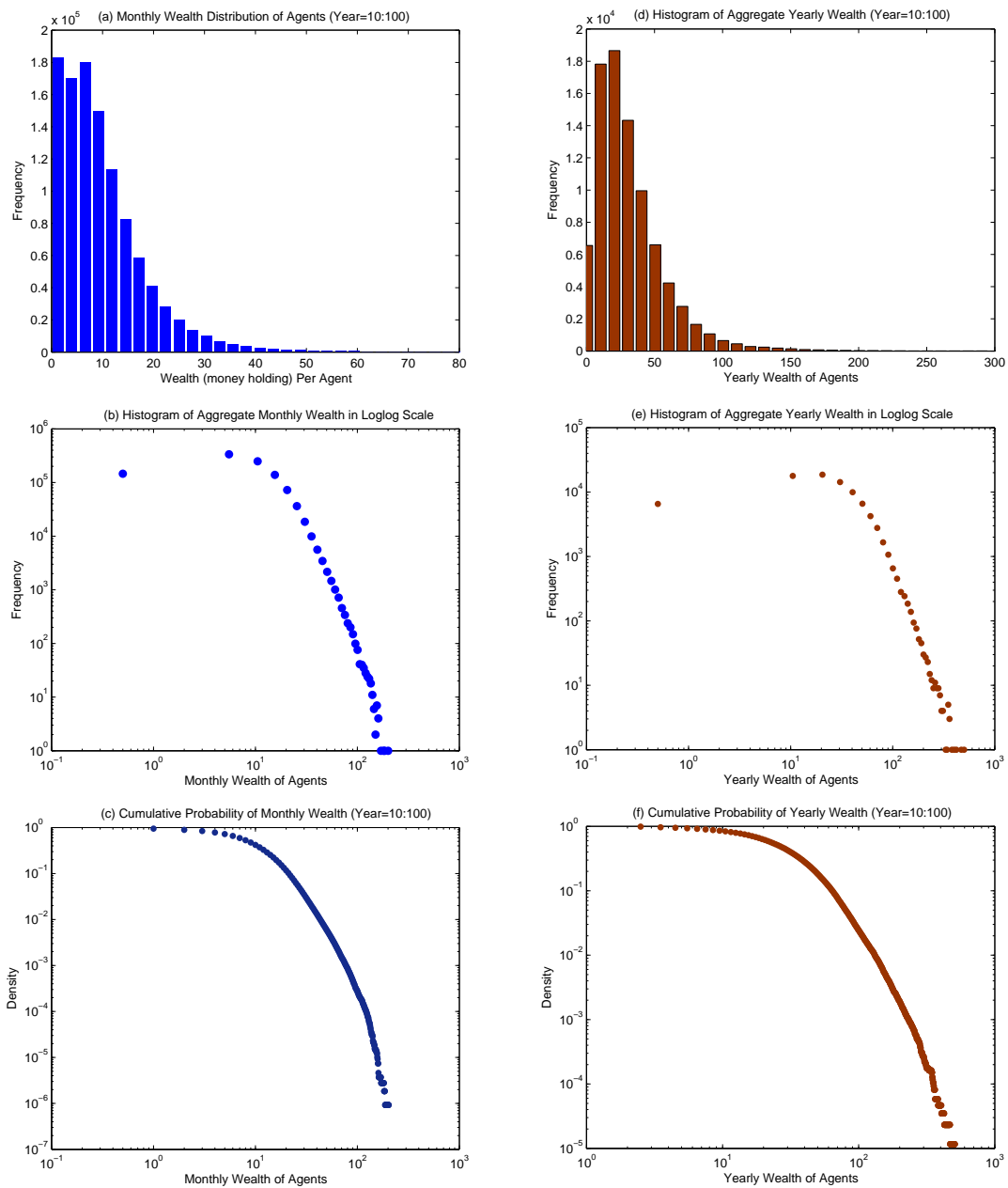


Figure 5.13: Monthly and yearly wealth distribution in the EPCE model. Figure (a): histogram of the monthly wealth distribution; Figure (b): histogram of the monthly wealth distribution in log-log scale; Figure (c): CCDF of the monthly wealth distribution; Figure (d): histogram of the yearly wealth distribution; Figure (e): histogram of the yearly wealth distribution in log-log scale; Figure (f): CCDF of the yearly wealth distribution (Year=10:100).

can not be; income is considered as the monetary gain of a actor within a certain period, and wealth is the overall money holding (including savings) of the actor at a particular time.

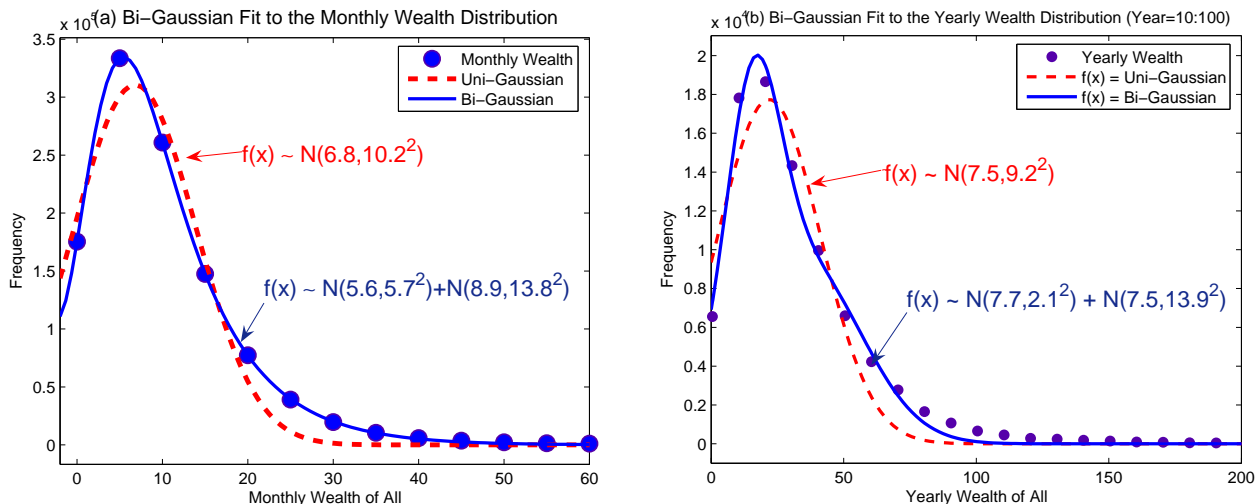


Figure 5.14: Figure (a): uni-Gaussian and bi-Gaussian fit to the monthly wealth distribution; Figure (b): uni-Gaussian and bi-Gaussian for the yearly wealth distribution. (Year=10:100).

Monthly Wealth Distribution

Fitting Function	R-square	Adj.R-sq.	RMSE
$f(x) \sim N(6.8, 10.2^2)$	0.967	0.962	920.3
$f(x) \sim N(5.6, 5.7^2) + N(8.9, 13.8^2)$	0.999	0.998	70.1

Yearly Wealth Distribution

$f(x) \sim N(7.5, 9.2^2)$	0.962	0.961	835.6
$f(x) \sim N(7.7, 2.1^2) + N(7.5, 13.9^2)$	0.998	0.997	97.8

Table 5.2: Fit properties of single and bi-Gaussian to the monthly and yearly wealth distributions. The weights are omitted in the table. RMSE represents “Root Mean Square Error”. The weights for the first bi-Gaussian is $a_1 = 190,200$ and $a_2 = 156,800$; for the second bi-Gaussian is $a_1 = 39,820$ and $a_2 = 15,940$

Fig. 5.13 displays the (cumulative) wealth distribution of all actors, in the sense of their money balances (i.e., money holdings). No matter whether it is counted monthly

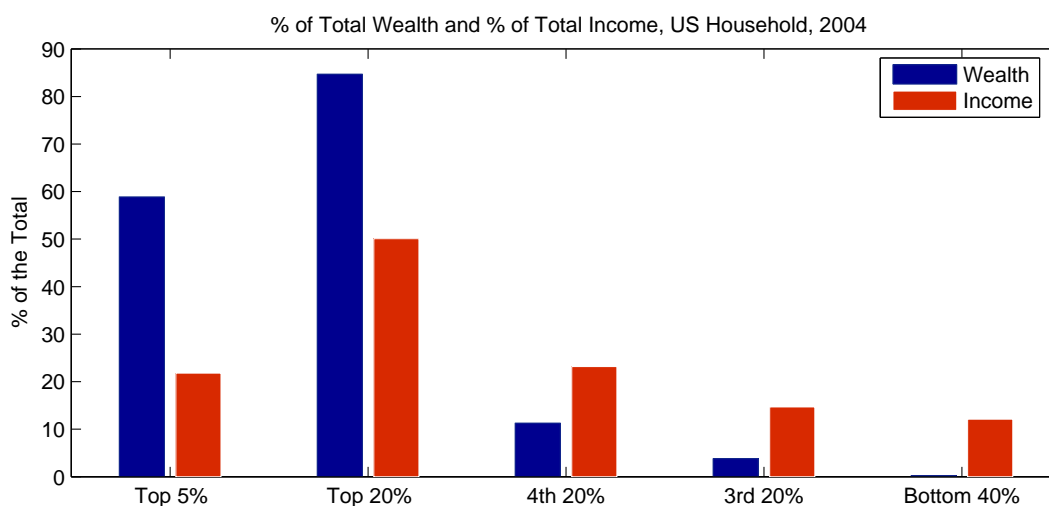
or yearly, when plotting the histogram with a log-log scale, as in figure (b) and (e) (i.e., the second row of Fig. 5.13), the upper tail of the wealth distribution is a straight line very close to a power law, after truncating the agents with wealth less than 10 coins. The same straight line power law behavior can be observed in figure (c) and (f) (i.e., the last row of Fig. 5.13), when we plot the wealth distribution in a cumulative probability form.

Similar to the income distribution plots, the wealth distribution for either separate groups or the aggregate wealth distribution can be fitted well by a multiple-Gaussian distribution. For example, Fig. 5.14 and Table 5.2 give the bi-Gaussian fit to the monthly and yearly wealth distributions, corresponding to figure (a) and (d) of Fig. 5.13. Both the monthly and yearly plots reveal that a bi-Gaussian distribution measures the data more appropriately than a single Gaussian function. In order to have a better image of the majority in the lower part of the wealth distribution, the horizontal axis has been shortened, but the curve fitting is applied without any truncation of the original datasets. From both cases, we observe only one mode of the distribution, simply because the difference between the means of sub-Gaussian function is too small compared to their standard deviations. In spite of that, the bi-Gaussian curve fitting still does a good job in explaining not only the peak, but also the tail behavior of the overall wealth distribution.

5.4.5 Income and Wealth Inequality

Inequality is a hot topic today. According to one report by the Canadian Center for Policy Alternatives (cf. Morel (2006)), even in the economic expansion period (1998–2004), 65% of Canadians felt they were not benefiting from the economic growth, which tells that perhaps a great share of benefits from the economic growth is going to the most richest people. Similar studies related to the expansion of inequality among or within nations can be found in Davies and Shorrocks (2000), Babeau and Sbrano (2003), Dell et al. (2005), and others.

Most of inequality studies are concentrated on income differences, because income data is easier accessible. As a matter of fact, the wealth inequality is much more severe across the world. Davies et al. (2007) carefully collect wealth data for many developed and developing countries, such as Sweden, the U.S., OECD, China, India, for the year 2000, they find out that “*the world distribution of wealth is much more unequal than that of income*”, especially when focusing on the bottom 60% of households. Another study by Wolff (2007) on the U.S. wealth data from the “Survey of Consumer Finances” suggests that, wealth inequality, like income inequality, has been increasing in the past years; most of the wealth is concentrated in the hands of a small number of families. In particular, the top 10% of the population own nearly 70% of the total wealth, and the bottom 40% of the population own perhaps less than 1% of the nation’s wealth.

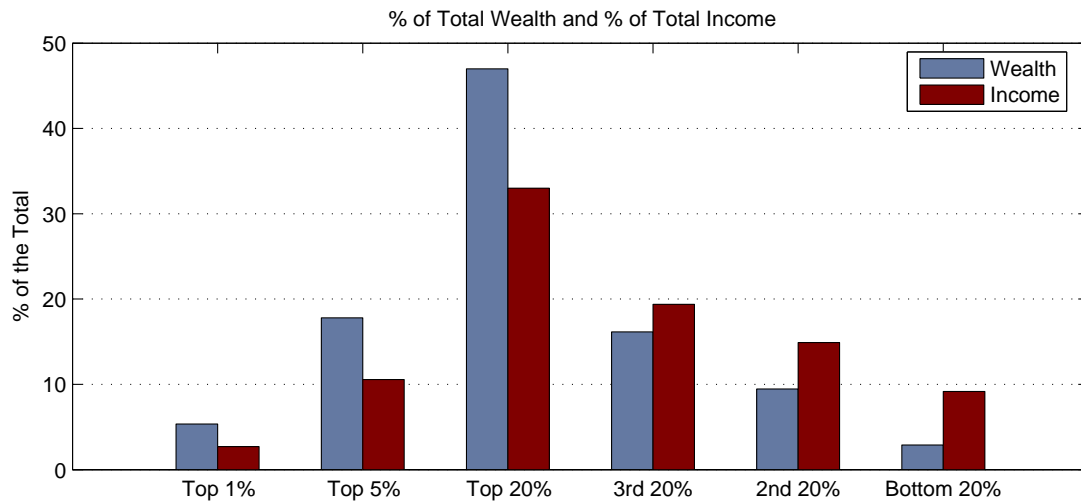


	Top 5%	Top 20%	3rd 20%	2nd 20%	Bottom 40%
Wealth	58.9	84.7	11.3	3.8	0.2
Income	21.8	50.1	23.2	14.7	12.1

Table 5.3: Comparison of the Income and Wealth Inequality of the U.S. Households in 2004.

Table 5.3 replicates the wealth and income distribution of the U.S. households in the year 2004. The source of data is the *Survey of Consumer Finances* sponsored by the Federal Reserve Board, and this replication follows Hodgson (2008)’s methodology. In

each percentage population group, the left column represents wealth, and the right column represents income. It is obvious that the distribution of wealth is much more unequal than the distribution of income, especially when focusing on the bottom 60% of households. It is reported that in the U.S., the bottom 60% of households possess only 4% of the nation’s wealth while they earn 26.8% of all income, according to Table 5.3.



	Top 1%	Top 5%	Top 20%	3rd 20%	2nd 10%	Bottom 20%
Wealth	5.34	17.80	46.98	16.15	9.45	2.88
Income	2.71	10.56	33.00	19.39	14.9	9.16

Table 5.4: Comparison of income and wealth inequality in the EPCE model

We also test whether the EPCE model is able to replicate the fact of wealth and income inequality in Table 5.3. Although Table 5.4 has less concentration of wealth/income than the empirical survey of Table 5.3, it coincides with the empirical findings that a minority of the population controls most of the social wealth, making the bottom 60% of the population much poorer. If we compare the height of columns of wealth and income, we immediately see larger differences among columns of wealth than that of income, which indicates that the distribution of wealth is much more unequal than the distribution of income.

As both income and wealth differences can be used to study the striking gap between

the rich and the poor, which one of them should be emphasized? Considering the accessibility of dataset, many researchers may still prefer income. However, it seems that Alan Greenspan, the chairman of the Federal Reserve Bank prefers wealth, because “*ultimately, we are interested in the question of relative standards of living and economic well-being. Thus, we need also to examine trends in the distribution of wealth, which, more fundamentally than earnings or income, represents a measure of the ability of households to consume . . .*” (cf. Greenspan (1998)).

5.5 Summary and Discussion

5.5.1 Summary

The basis of any economic activity in modern societies is the interactions among individuals, which result in redistribution of their assets (measured in terms of monetary values). In our multi-agent system, an individual is either viewed as a single person, or a self-contained economic entity, such as a firm. Allocation of “assets” is represented by exchange of “coins”, which can be analogous to cash or physical products in real life. The producers produce goods and sell them in the market, and the workers are paid for their labor by wages which are used for consumption. The simple hypothesis under this exchange system is that: *every payment must be counterbalanced by another*, which is observed as redistribution of “money” in the simulation. The EPCE model has its strength in the micro design. In particular, it contains an elaborate “wage payment and firing rule”, taking into account the agent’s memory of the past and expectation for the future.

For the “firm dynamics”, a short summary of our findings is:

- (1) The life expectancy of the firms follows a power law. It shows that the majority of firms are short-lived, whereas only a minority of firms will maintain the business for a long period.

- (2) The exact form of firm size distribution may lie between Lognormal and power law. This highly skewed distribution provides the fact that small firms are in the majority and large firms are relatively rare. The number of firms drops exponentially as the firm size increases. Since firms usually start with a small size, we may derive that many start-up companies close down within their starting years, but once the firm passes through the most “difficult” time and grows bigger, there will be less firms competing with them in the same size category.
- (3) From the relationship between firm sizes and growth rates, we observe that small firms are not only fast growing, but also fast declining. Usually large firms have smaller fluctuations in growth rates.
- (4) Although small firms are relatively weak, they are in the majority and their overall performance is like a thermometer of the economy. We observe more small firms in the booming years than in the depression, which tells us that small firms may have an important contribution to the economic growth, whereas there are no apparent correlation detected between the economy and large firms’ performance.

For the “income and wealth dynamics”, we point out that the usual “money plot” in a cumulative probability or a log-log form, may be misleading to some extent, because too much truncation needs to be adopted to reach a power law in the upper tail. If we realize the fact that most of the population receives lower income (or wealth), a heavy truncation will lead us to neglect the majority of the distribution, and just focus on perhaps less than 20% of the “true” income (wealth) distribution, which provides just a limited image (i.e., within a small region) of the real distribution. In order to have a comprehensive view of the true income distribution, the income of workers and employers is studied first in an isolated manner, and then in an aggregate format. We use a mixture of Gaussian distributions to recover the income distribution for workers and employers, and obtain quite good fits without any truncation of data. Although this fitting may include more parameters¹⁹, it uses a simple way to describe seemingly

¹⁹In fact, the number of parameters is 2 for single Gaussian (μ and σ) and 4 for bi-Gaussian (if

very complicated income distributions and perhaps some other distributions too. It also explains the hidden mystery behind the “knee” shape of income distribution, which may just be the overlapping area among two or more normally distributed subgroups.

5.5.2 Discussion

Agent-based modeling came into being as an alternative to traditional methodology in the past 20 years. Quite in contrast with the classical method, most of the agent-based models are almost equation-free, with many “realistic” built-in assumptions, so that the final simulation is able to replicate or explain some real world phenomena. When discussing this thesis with the colleagues working on the mainstream economic fields, we are often asked to explain “how do the descriptive rules work?”, “why does the inequality emerge out of an equality initialization?”, “without using a ‘representative agent’, how many people can be built into your model?” ... Some of their questions are difficult to answer shortly, but there is one thing that is sure, among all college students who are trained in economics, very few of them have ever learned simulation methodology as a third way of doing research in standard economic courses, or as a complementary tool to the traditional analytic methods. Even if some students have learned about using programs to build up simple models, their experiences of creating a large system based on descriptive rules evolving from complex network are quite limited. As a consequence, most of the agent-based social simulations have been done by physicists, engineers, and other scholars, who have been trained to use this technique to do laboratory experiments in natural sciences, such as thermodynamics. It should be admitted that, although agent-based social simulation is becoming popular, it has not been accepted by most of the “mainstream” economists yet. We would like to contribute to the “non-mainstream” economic study by advertising our social models in this thesis.

weights are not counted). In our experience, a bi-Gaussian distribution already allows for a good fit in most cases.

The fact that a single model can be useful in understanding the dynamics of interactions among adaptive agents, is not due to the accuracy of the model in representing the details of all situations. Instead, it is due to its generalization capacity that an extremely simple model captures the fundamental features of many interactions, and is able to generate many macroscopic regularities from microscopic behavior, involving intricate interactions among simple micro-economic entities. The application of agent-based models is highly dependent on the improvement in computer capacity and the efficiency of programming languages. The purpose of this dissertation is to present a discussion that a certain kind of agent-based model can generate a range of social phenomena, and many of them may be quite close to reality. Instead of focusing on a single solution resulting from “utility maximization”, we emphasize more the aspect of “statistical equilibrium”.

Below are some interesting findings from our study:

- The 80/20 rule is ubiquitous in nature, instead of being a “transient” or “temporary” phenomena. It happens not only for firms, but also for individuals in the social production process. This rule can be also observed in our simulation, for example: there is always a minority of large firms coexisting with a majority of small firms; a large number of poor and middle class agents together with very few rich agents determine the shape of the income distribution; the general life expectancies of firms are short with only very few exceptions; the majority of small firms influence the economy . . . Moreover, if we keep improving the microscopic design of the model, we will be able to find some other facts that are in accordance with 80/20 rule, such as: many markets have historically been dominated by a small number of best-selling products; 80% of value creation for one firm comes from 20% of products; most firms seek to grow but will only risk 20% of revenue for further investment; most of the individuals do not spend all their money but save around 20%, and so on.
- The power law with varying exponents is very common in reality. It is usually

observed as a straight line or a “long tail” in log-log scale, which proves again the pervasive 80/20 rule, and the logarithm scale has helped to emphasize the extreme behavior happening in the upper 20% part. Since the 80/20 rule is so common, it is possible that the upper tail behavior of many “size” distributions will also follow a power law (when truncation of data is applied), such as the firm size distribution, the firm profit rate distribution, wealth and income distribution, etc. Furthermore, we find that the higher the exponent $-\alpha$ (i.e., the smaller the absolute value), the flatter the tails, and the higher the inequality among the entities.

- Concerning the business cycle phenomenon, classical economic text books propose that there is a fixed equilibrium state for an economy, which is time independent and can be easily described as a crossing point where supply equals demand in a competitive market. In our agent-based simulation, we find that the economy actually never stays at a fixed equilibrium stage and it will be continuously fluctuating around a certain “mean” equilibrium. In other words, we argue that the business cycle is neither “transient” nor “temporary”, it is a natural outcome showing that the system is in a “statistical equilibrium”. As a matter of fact, business cycles can start from “imperfect information” of both firms and consumers located in the “local” networks; since their information is never “perfect” and their network is never “global”, consequently, the supply is not always equal to the demand, the business cycle occurs as an outcome of continuous adjustment. The resulting fluctuations might always be around a stable mean, displaying the character of a “statistical equilibrium”.
- The Gaussian distribution is considered to be the most common distribution in life. Therefore, our reasonable conjecture is that certain kinds of mixture-Gaussian distributions may also be widespread. It is demonstrated in Chap. 5 that some seemingly complicated distributions, such as the income and wealth distribution, can be well described by a mixture of Gaussians with different means

and variances. This possibility is neglected by most researchers, since many believe that mixtures of Gaussians only generate multimodal distribution, while this is not always the case if the means of these sub-Gaussians stay close enough (compared with their standard deviations) to each other. Similarly, (although we don't have space to explain in this study), the "leptokurtosis" of financial time series in aggregation can be represented by a mixture of two Gaussians with the same mean but different variances. Some other distributions, e.g., the profit rate distribution, the Maxwell-Boltzmann distribution of energy might be also reproduced as a mixture of Gaussians with different means and variances. We agree that this way we are facing more parameters to be estimated, however, if one faces a realistic distribution which is difficult to explain, why not splitting the whole distribution into two or more subgroups and study them in isolation? Remember that our nature contains a lot of mathematical "beauty". Many things that we find in reality are in "statistical equilibrium", which should often look Gaussian in aggregation, just with various means and variances.

Part II

Agent-Based Modeling of Sentiment Dynamics

Chapter 6

Business and Consumer Sentiment and Industrial Output — A VAR Analysis

6.1 Introduction

There is nothing more interesting than what is happening in human brains. Different kinds of opinions come into our minds as a result of emotion or interpretation of *facts* from time to time. It has been a long time that people recognize that the accumulation of a variety of attitudes or sentiments may affect economic activities. For example, in a bullish stock market, most investors expect upward price movement, and on the contrary, if the market sentiment is bearish, most investors expect downward price movement.

Consumer or producer sentiment are the most well-known among all market sentiment topics, and they are usually monitored by various surveys using statistical methods. There are a large number of papers studying the predictive power of consumer or producer sentiment. Taylor and McNabb (2007) find evidence of causality between the

indicators of business confidence and GDP, and show that the confidence indicators would appear to have good predictive power in identifying turning points in European business cycles. Gelper et al. (2007) investigate the U.S. consumer sentiment index and report a significant causality from the consumer sentiment to the future consumption both for the short term and longer horizons. Similar survey-based sentiment research are in Zullo (1991), Vuchelen (1995), Lemmens et al. (2005), among others.

Does sentiment always have a significant impact on economic activities? Brown and Cliff (2004) study that the U.S. stock market data since 1965, and find that the stock returns do Granger-cause sentiment, while sentiment is not a significant predictor of returns. Kling and Gao (2008) investigate the Shanghai stock market and reports causation from returns on sentiment but not *vice versa*. However, their studies are questioned by some other researchers. Verma and Verma (2007) find significant predictive power from sentiment on near-term returns. Apart from that, Schmeling (2007) performs sentiment-based trading experiments and finds significant predictability in long horizons. More recently, using sentiment data and returns for the German stock market, Lux (2009b, 2010) report a significant causal influence from sentiment on returns.

In this chapter, we follow the companion studies Lux (2009b, 2010) to study the interactions between sentiment dynamics and industrial output with the vector autoregressive (VAR) approach, focusing on the possible influences of sentiments on final output. Several econometric tools are applied and addressed, including forecast oriented estimation of the number of dynamic factors and tests for the factor restriction imposed on the VAR. Later on in Chap. 7, we will contrast the VAR results to those of an agent-based model of opinion dynamics, following the companion research work by Lux (2009a).

6.2 Data Overview

Our empirical analysis takes two German opinion indices and the industrial production index into consideration. These datasets are presented in Fig. 6.1. The figure contrasts

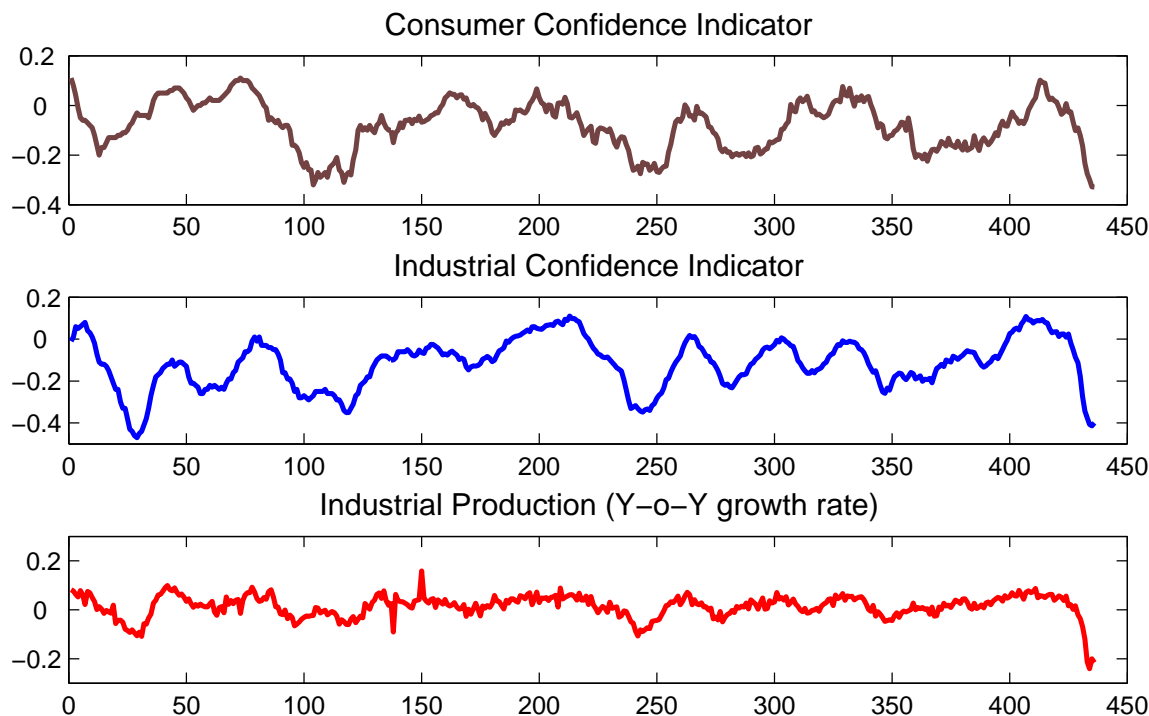


Figure 6.1: Consumer Confidence Indicator (CCI), Industrial Confidence Indicator (ICI) and Industrial Production (IP) indices. The time horizon records monthly (seasonally adjusted) data from January 1973 to April 2009.

three monthly time series for the German economy throughout the past four decades, namely the Consumer Confidence Indicator (CCI), Industry Confidence Indicator (ICI) and Industrial Production (IP). All datasets start from January 1973 until April 2009, with 436 *seasonally adjusted* observations each. The first two historical series (CCI and ICI) are collected and released by the European Commission Directorate-General for Economic and Financial Affairs¹ in the form of a *diffusion* index. Obviously, the positive (negative) values reveal whether optimistic (pessimistic) opinions are in the

¹The expanded name for CCI is “Consumer Confidence Indicator – Germany” with mnemonic code “BDEUSCCIQ”, and the expanded name for ICI is “Industrial Confidence Indicator – Germany” with code “BDEUSICIQ”. The value of these indices come from business surveys. At first, the answers obtained from the survey are aggregated in the form of “balances”, then the indices are constructed as the differences between the percentages of respondents giving positive and negative replies.

majority among respondents to the questionnaires (on their attitudes towards industrial output). The farther the index is away from zero, the more pronounced the positive (negative) opinion for the German industry. The monthly IP data is compiled by Thomson Financial and National Source² on a “year-on-year” base. In the following study, we will test whether there is any relationship among the sentiment of firms, consumers and the industrial outputs.

Table 6.1: Summary Statistics

Panel A: Full sample (436 observations)						
	Mean	S.D.	Skewness	Kurtosis	ρ_1	ADF
CCI	-0.079	0.098	-0.344	2.433	0.971	-3.207*
ICI	-0.116	0.121	-0.439	2.856	0.986	-4.932**
IP	0.014	0.047	-1.450	7.816	0.860	-3.501**
Panel B: In-sample (300 observations)						
CCI	-0.081	0.101	-0.335	2.324	0.976	-2.928*
ICI	-0.128	0.124	-0.361	2.616	0.987	-4.206**
IP	0.013	0.044	-0.486	3.243	0.819	-3.795**

Notes: Table 6.1 provides summary statistics for CCI, ICI and IP. ρ_1 is the autocorrelation coefficient at lag 1 (with non-zero mean). The ADF test statistics have been computed with 4 lags and included a constant term, the one-sided 5% and 1% critical values are -2.871 (-2.870) and -3.458 (-3.440), for the full-sample (in-sample) data respectively. It turns out that all time series are generally stationary. * and ** denote significant parameters at the 5% and 1% significance level, respectively.

Table 6.1 provides the summary statistics for our data. Panel A presents the full sample with 436 data, and in Panel B, we have truncated the original full sample and taken out their first 300 values to represent the in-sample data for further analysis.

²The expanded name for IP is “Industrial Production Including Construction” with mnemonic code “BDIPTOT%G”, and the monthly data are collected on the “year-on-year” percentage changes (YOY%).

First of all, the mean values for CCI and ICI are clearly negative, while for IP it is slightly positive, therefore we will include a non-zero mean in constructing the later tests. Second, the statistics of higher moments and standard deviations confirm our impression that fluctuations in CCI and ICI are more volatile than IP, for both full sample and reduced in-sample. Third, it seems that all three time series have strong autocorrelation with their past values, showing long-term memory regardless of their lengths. Finally, the ADF test statistics have been computed with 4 lags³, the H_0 hypothesis of a unit root is rejected at the 5% critical value for all full sample and in-sample datasets, which proves that our time series are generally stationary.

6.3 Specification and Testing of a VAR Model

The VAR approach is widely used for analyzing the dynamic impact of random disturbances on a system of variables. It models every endogenous variable as a function of the lagged values of all variables in the system. To explore the dynamic relationships among time series of CCI, ICI and IP, a tri-variate VAR model will be constructed⁴, with vector $y_t = (y_{1t}, y_{2t}, y_{3t})'$ denoting the triple observations on CCI, ICI and IP at time t . The reduced form of the tri-variate VAR model can be written as:

$$y_t = \nu + \mathbf{A}_1 y_{t-1} + \cdots + \mathbf{A}_p y_{t-p} + u_t, \quad t \in \mathbb{Z} \quad (6.1)$$

where

- y_t is a 3×1 random vector

³An important practical issue for the implementation of ADF test is the specification of lag length p . If p is too small, the remaining serial correlations in errors will bias the test; if p is too large, then the power of the test will suffer. Monte Carlo experiments (e.g., in Kilian (2001)) suggest it is better to err on the side of including more lags, therefore, a lag of 4 instead of 1 is selected for high autocorrelation in our time series (one could also select the lag length via information criteria).

⁴Detailed description of the methodology can be found in Lütkepohl (2005), and a similar approach is also applied in e.g., Brown and Cliff (2004), Kling and Gao (2008).

- A_i are 3×3 fixed coefficient matrices
- ν is a 3×1 fixed vector of intercept terms
- u_t is a 3×1 white noise process with
 - $Eu_t = 0$
 - $Eu_t u_t' = \Sigma_u$ (nonsingular)
 - $Eu_t u_s' = 0$, if $s \neq t$

Determining the lag length of a VAR(p) process of y_t , is to find an exact lag order p such that $A_i = 0$ for all $i > p$ in the model. In other words, we need to find the index of the most lagged value of y_t that contributes to the current value. The common practice is to choose a lag-length arbitrarily allowing just enough lags, and make sure that the residuals are white noise when maintaining the precision of estimates. There are some well-known procedures used to determine an appropriate lag length such as the Akaike Information Criterion (AIC, Akaike (1974)), the Hannan-Quinn Information Criterion (HQ, Hannan and Quinn (1979)), and the Bayesian Information Criterion (BIC, also known as Schwartz Information Criterion (SIC, Schwarz (1978))). In short, information criteria (IC) compare the gains of having a more generous lag structure in the regression, with the cost represented by the loss of degrees of freedom. The test statistics can be generally expressed as: $IC = -(\log\text{-likelihood of parameters}) + (\text{penalty term for number of parameters})^5$. Among these three information criteria, BIC and HQ tend to select the correct order for sufficiently large sample size, while AIC may suffer from inconsistency, and it tends to asymptotically overestimate the true lag order with positive probability⁶. Therefore, in the literature, corrected forms of AIC

⁵Lütkepohl (1991) (Chap. 4) presents a number of results regarding consistent lag order selection in the VAR models.

⁶The AIC methodology attempts to find the model that best explains the data with a minimum of free parameters, and the model with smaller AIC is preferred. For simplicity, AIC can be written as: $AIC = -2\ln(L_n(k))/n + 2k$, where $L_n(k)$ is the likelihood function and k is the degrees of freedom in

have been introduced, e.g., Hurvich and Tsai (1989)'s AICC method includes additional penalty term. Based on their methodology, Huang (2008) uses a simple example to prove that for small sample size, AIC is able to select the correct order, but not for large samples. However, the original version of AIC is still the most popular method used by researchers, Liew (2004) claims that AIC is superior than other information criteria for limited sample sizes.

Table 6.2: Lag Selection of Tri-variate VAR Model

Information criteria			
Lags	AIC	HQ	BIC
0	11.256	11.267	11.284
1	4.559	4.604	4.671
2	4.181	4.259	4.378
3	4.046	4.157	4.328
4	4.010	4.155	4.378
5	4.038	4.217	4.491
6	4.045	4.258	4.584
7	4.043	4.290	4.668
8	4.062	4.343	4.773
9	4.092	4.407	4.890
10	4.132	4.482	5.017

Table 6.2 provides the lag selection preferred by various information criteria. It seems that both AIC and HQ opt for a minimum of 4 lags to be included, while BIC alone the model. Clearly, AIC does not depend on the sample size. Meanwhile, it is usually claimed that AIC is too liberal and comes with the danger of overfitting (resulting in a choice of too many lags). BIC and SIC take the sample size into account, and the later methods have a higher penalty for k , especially when the sample size is large. This overcomes the overfitting tendency of AIC (cf. Chatterjee and Hadi (2006)).

favors a more parsimonious model of 3 lags. By reading this table, it is very difficult to decide immediately which lag length should be adopted. Although AIC might have the problem of inconsistency, the consistency problem might not be so important for a preparatory selection of the model, particularly for a moderate sample size like ours. Since different information criteria emphasize different aspects of the data generation process and may all provide useful information for further analysis, we test our model based on all favored lags instead of relying on a single procedure, then different statistical tools will be used to aid the choice of appropriate lag-length. In the following steps, we will check the model adequacy for both VAR(3) and VAR(4) systems.

Parameter Estimation for VAR(3) Model

$$\begin{aligned}
 \begin{bmatrix} \text{CCI} \\ \text{ICI} \\ \text{IP} \end{bmatrix} &= \begin{bmatrix} 1.06^{**} & 0.27^{**} & 0.01 \\ 0.05 & 1.29^{**} & 0.08^* \\ -0.00 & 0.26^{**} & 0.32^{**} \end{bmatrix} \begin{bmatrix} \text{CCI}_{-1} \\ \text{ICI}_{-1} \\ \text{IP}_{-1} \end{bmatrix} + \begin{bmatrix} -0.04 & -0.30^{**} & 0.05 \\ -0.10 & -0.06 & 0.03 \\ 0.08 & 0.13 & 0.19^{**} \end{bmatrix} \begin{bmatrix} \text{CCI}_{-2} \\ \text{ICI}_{-2} \\ \text{IP}_{-2} \end{bmatrix} \\
 &+ \begin{bmatrix} -0.06 & 0.01 & -0.01 \\ 0.05 & -0.27^{**} & -0.07 \\ -0.04 & -0.34^{**} & 0.14^{**} \end{bmatrix} \begin{bmatrix} \text{CCI}_{-3} \\ \text{ICI}_{-3} \\ \text{IP}_{-3} \end{bmatrix} + \nu + \mathbf{u}_t \tag{6.2}
 \end{aligned}$$

Parameter Estimation for VAR(4) Model

$$\begin{aligned}
 \begin{bmatrix} \text{CCI} \\ \text{ICI} \\ \text{IP} \end{bmatrix} &= \begin{bmatrix} 1.05^{**} & 0.32^{**} & -0.01 \\ 0.05 & 1.24^{**} & 0.06 \\ -0.01 & 0.25^{**} & 0.31^{**} \end{bmatrix} \begin{bmatrix} \text{CCI}_{-1} \\ \text{ICI}_{-1} \\ \text{IP}_{-1} \end{bmatrix} + \begin{bmatrix} -0.03 & -0.34^{**} & 0.05 \\ -0.12^* & -0.07 & 0.05 \\ 0.07 & 0.11 & 0.20^{**} \end{bmatrix} \begin{bmatrix} \text{CCI}_{-2} \\ \text{ICI}_{-2} \\ \text{IP}_{-2} \end{bmatrix} \\
 &+ \begin{bmatrix} -0.02 & -0.02 & -0.03 \\ 0.07 & -0.01 & -0.03 \\ -0.02 & -0.23^* & 0.16^{**} \end{bmatrix} \begin{bmatrix} \text{CCI}_{-3} \\ \text{ICI}_{-3} \\ \text{IP}_{-3} \end{bmatrix} + \begin{bmatrix} -0.03 & 0.02 & 0.05 \\ 0.01 & -0.19^{**} & -0.09^{**} \\ -0.00 & -0.08 & -0.04 \end{bmatrix} \begin{bmatrix} \text{CCI}_{-4} \\ \text{ICI}_{-4} \\ \text{IP}_{-4} \end{bmatrix} + \dots \tag{6.3}
 \end{aligned}$$

Eq. 6.2 and Eq. 6.3 display parameter estimation results for both VAR(3) and VAR(4) models, using *maximum likelihood* method. * and ** denote the significant coefficients at 95% and 99% confidence interval judged by the t-statistics, respectively. It shows

Table 6.3: Covariance Matrix of VAR Models

Covariance Matrix of VAR(3)				Covariance Matrix of VAR(4)			
dep. var	CCI	ICI	IP	dep. var	CCI	ICI	IP
CCI	0.021**	-	-	CCI	0.020**	-	-
ICI	0.002**	0.016**	-	ICI	0.002**	0.015**	-
IP	0.001	0.004**	0.021**	IP	0.001	0.003**	0.021**

Table 6.4: Diagnostic Checking of Estimated Unrestricted VAR Models

Var(3)
Portmanteau test for autocorrelation: 157.256 (0.000)
<i>Test for Nonnormality:</i>
Skewness Statistics: 4.060 (0.255)
Kurtosis Statistics: 391.882 (0.000)
Skewness + Kurtosis Statistics: 395.942 (0.000)
Var(4)
Portmanteau test for autocorrelation: 146.595 (0.000)
<i>Test for Nonnormality:</i>
Skewness Statistics: 2.947 (0.399)
Kurtosis Statistics: 350.626 (0.000)
Skewness + Kurtosis Statistics: 353.573 (0.000)

Notes: Portmanteau test for residual autocorrelation with 12 lags are computed (probability in parenthesis).

that all three variables in our model have strong dependence on their past values. We observe a strong causal influence of ICI on both CCI and IP and this influence lasts for more than 1 lag. On the other hand, CCI seems to be caused uni-directionally by ICI, but uncorrelated with IP. Besides, ICI and IP seem to have causal relations to each other, particularly, ICI appears to be marginally affected by IP in the short run (lag 1)

for the VAR(3) model, but in the long run (lag 4) for the VAR(4) model. Meanwhile, the influence of ICI on IP lasts at least for 3 lags. When comparing the estimation results of both the VAR(3) and VAR(4) models, the estimation results for CCI and IP dynamics are quite similar. The only difference is that in the VAR(4) model, the influence from IP on ICI vanishes in the first entry, but moves to the fourth entry. Apart from that, in the second entry, ICI looks marginally affected by CCI, which does not exist in the VAR(3) model. Quite surprisingly, both of our models identify CCI as an exogenous variable whose dynamics have no explicit cause on IP (for both models) and ICI (for the VAR(3) model). In other words, ICI alone should be exploitable to help predicting future industrial output movements.

After parameter estimation, Table 6.3 illustrates the estimated covariance matrices for both VAR(3) and VAR(4) models. The values indicated in the left corner of matrices show significant correlations in the innovations of ICI and IP, as well as significant correlation between both sentiment CCI and ICI. In contrast, CCI and IP appear not to be instantaneously correlated.

It is perhaps worth pointing out that both the VAR(3) or VAR(4) processes estimated above are *stable*, satisfying the *stability condition* in Lütkepohl (1991) (p. 11), that the moduli of eigenvalue for the *reverse characteristic polynomial* “ $\det(I_K - A_1z - \dots - A_pZ^p)$ ” should be outside the complex unit circle. In particular, for the VAR(3) model, the moduli of the eigenvalues of the reverse characteristic polynomial are: $|z| = \{2.1269, 2.1269, 2.1315, 4.5890, 3.1409, 1.0666, 1.1243, 1.1243, 1.3064\}$, and for the VAR(4) model, we have $|z| = \{1.8477, 1.8477, 2.0515, 2.0515, 3.8683, 3.8683, 2.7899, 1.1211, 1.1211, 1.0569, 17.0259, 1.1129\}$. Obviously, all moduli are greater than 1 (i.e., outside the unit circle) in absolute values, which prove that both processes are stable.

Knowing that a VAR(p) process can be rewritten in a *moving average* representation, the procedure of deciding the order of VAR system may also be interpreted as methods for determining a filter that transforms the given data into white noise series. Correctly specified VAR(p) model should have residuals that are nearly random. However, if

forecasting is the objective of our analysis, it may not be of prime importance whether the residuals are really white noise as long as the model forecasts well (Lütkepohl (1991), Chap. 4). To capture the model's property at large, Table 6.4 reports the diagnostic checking results of the selected VAR(3) and VAR(4) models.

The Portmanteau test⁷ jointly tests the significance of all error autocorrelations up to a set order h ($h=12$ in our tests). It is displayed in Table 6.4 that there is temporal dependence in the white noise term u_t for both VAR models. Also displayed in Table 6.4 are results of the non-normality test. Lütkepohl (1991) suggests using a multivariate generalization of the Jarque-Bera test (Jarque and Bera (1987)) to test for multivariate normality of u_t . The idea underlying this non-normality test⁸ is to transform the residual vector such that its components are independent and then check the compatibility of the third and fourth moments with those of a normal distribution. Our tests for nonnormality are based on skewness and kurtosis of standardized residuals, and statistics indicate that both residuals of these unrestricted models suffer from significant kurtosis, but do not display excess skewness.

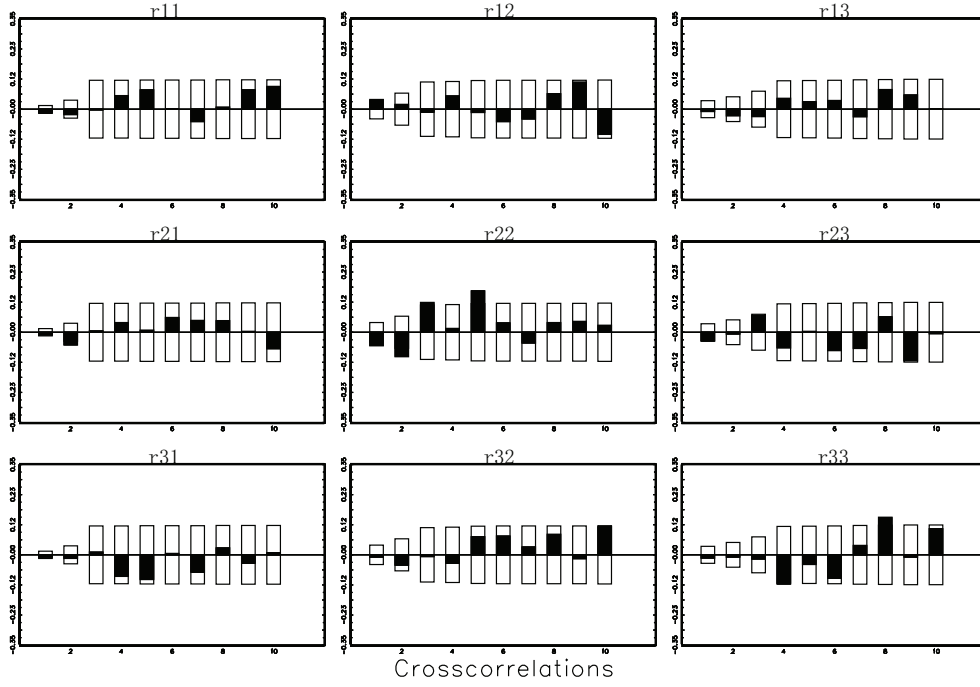
Fig. 6.2 displays the residual autocorrelation and cross-correlation of our multivariate VAR(3) and VAR(4) systems, respectively. *Twice* their asymptotic standard errors (approximately 95% confidence bounds) around zero⁹ are depicted in transparent format. For both VAR(3) and VAR(4) models, we observe strong autocorrelations of ICI and IP residuals to their own past, especially for IP in the long term. Moreover, the residual cross-correlations between CCI and IP are not significant, which is in harmony with our previous finding on the covariance matrices shown in Table 6.3. Comparing

⁷Define R_i as autocorrelation matrix for lag i among errors, the Portmanteau test evaluates: $H_0 : \mathbf{R}_h = (R_1, \dots, R_h) = 0$ vs. $H_1 : \mathbf{R}_h \neq 0$.

⁸The underlying H_0 hypothesis of the non-normality test is that the error term u_t follows a normal distribution, namely, if the u_t are indeed white noise, there will be no significant deviation from the skewness or kurtosis of the normal distribution.

⁹For a rough check of 5% level significance of autocorrelations at higher lags, we use $\pm 2\sqrt{T}$ -bounds which are convenient from computational purpose.

Crosscorrelations



Crosscorrelations

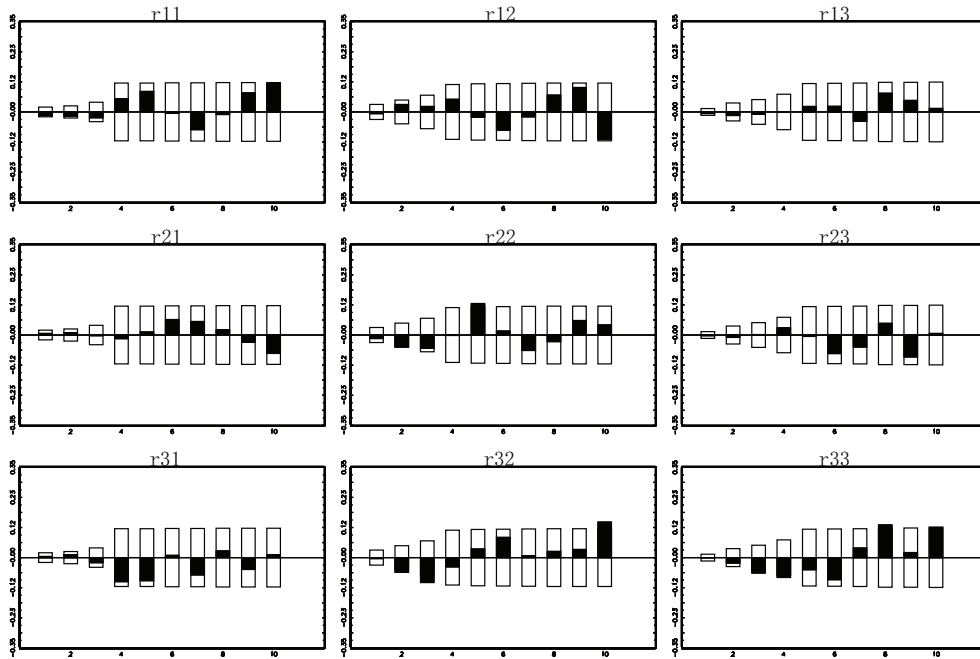


Figure 6.2: Residual Autocorrelations of VAR(3) and VAR(4) Models

both models, VAR(3) finds residual cross-correlations in CCI and ICI at low lags, which is not exhibited in the VAR(4). Last but not least, both VAR(3) and VAR(4) report cross-correlations in ICI and IP at low lags.

Since our major task of estimating the tri-variate system is to identify the relationships among industrial output and sentiment indices (from either consumers or industry), in order to test for their causal relations, we report in Table 6.5 the Wald test for Granger (1969) causality as well as instantaneous causality among the innovations of the variables. First of all, we observe from Panel A that the joint sentiments do Granger-cause IP, but not vice versa, and their innovations are closely related. Then in Panel B, it is shown that there is no Granger causation between CCI and IP in either direction, as well as no correlation among their innovations. Finally, in Panel C, we find that ICI has a strong causal influence on IP, but not the other way round. In addition, there is also strong correlation among their innovations, which explains the joint sentiments' behaviors in Panel A. To sum up this table, the only significant causality pair that we could observe from the system is ICI Granger-causes IP. In other words, ICI can help improving the prediction for IP, while the consumer sentiment CCI seems to be exogenous. Its values do not affect the other variables significantly. These findings are all in accordance with our parameter estimation in Eq. 6.2 and Eq. 6.3, and the covariance matrices in Table 6.3.

6.4 Forecast Performance

6.4.1 Forecast for Tri-variate System

In general, time series are sets of ordered observations on certain quantitative characteristics of phenomena at equally spaced time points, and one of the main targets of time series analysis is to forecast future values of the series. Forecasting is the act of predicting the future, it is the estimation of the expected value of a dependent variable

Table 6.5: Causality Tests of the Unrestricted VAR Models

Panel A	CCI/ICI → IP	IP → CCI/ICI	Inst.
Var(3)			
Wald	11.308	1.174	10.204
p-value	0.000	0.320	0.006
Var(4)			
Wald	8.453	1.307	7.953
p-value	0.000	0.239	0.019
<hr/>			
Panel B	CCI → IP	IP → CCI	Inst.
Var(3)			
Wald	1.706	0.370	0.568
p-value	0.166	0.775	0.451
Var(4)			
Wald	1.354	0.521	0.367
p-value	0.250	0.721	0.545
<hr/>			
Panel C	ICI → IP	IP → ICI	Inst.
Var(3)			
Wald	18.359	2.072	9.636
p-value	0.000	0.104	0.002
Var(4)			
Wald	13.811	1.985	7.587
p-value	0.000	0.097	0.005

Note: Two types of causality tests, i.e., tests for Granger causality and tests for instantaneous causality, are implemented. The variables to the left of the arrows are the causal variables, while to the right of the arrows are the affected variables (or dependent variables). The null hypothesis (H_0) is tested against the alternative (H_1) that the causal variable(s) do affect the affected variable(s). A small p-value (computed from the underlying χ^2 -square distribution under H_0) reveals the fact that all coefficients of the first variable(s) at all lags are jointly insignificant in the equation(s) of the second variable(s). The test for instantaneous causality tests the hypothesis of no correlation among the innovations of output and sentiment variable(s). 172

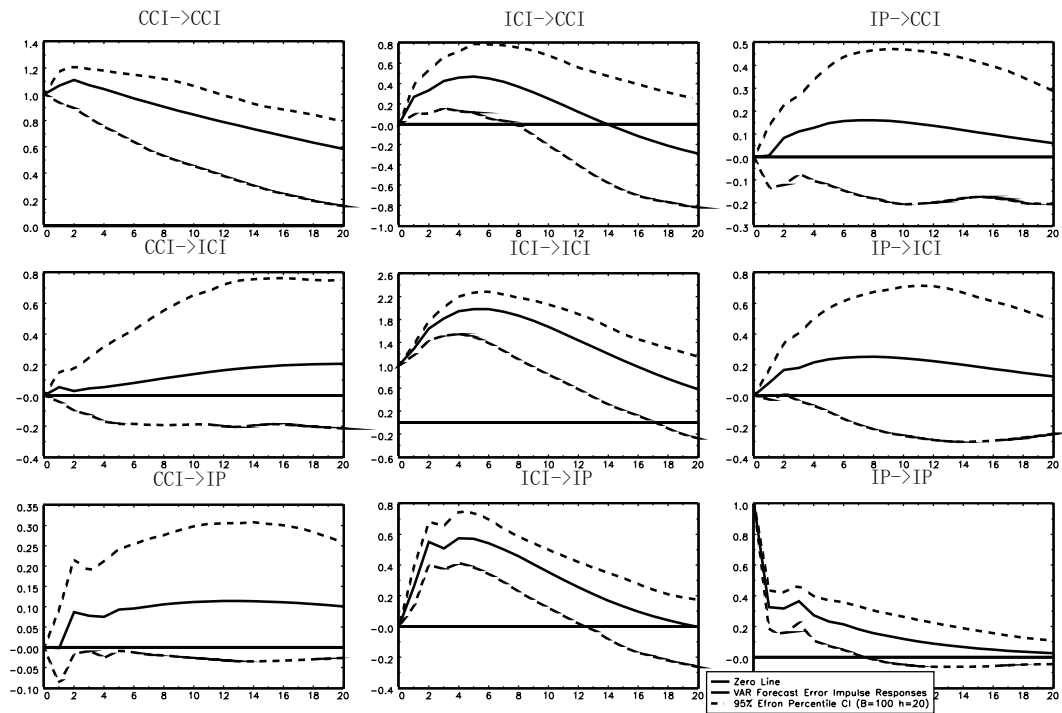
for observations that are not part of the sample dataset. Since it is not clear which system among VAR(3) and VAR(4) is better to help forecasting IP, we start out with inspecting the impulse response functions for both systems.

The impulse response analysis quantifies the reaction of every single variable in the model on an exogenous shock from the other variable (or itself). If one variable is related to the other variable, the shock from the former will have a positive or negative effect on the later, but this impulse dies away in finite time due to the stability of the system. Hence, impulse response analysis is a tool for inspecting inter-relations among model variables. Fig. 6.3 depicts dynamic responses of the system variables to a positive unit shock (forecast error) from one of the other variables. The diagonal figures show the responses of each of the three variables to a unit shock from itself, and the clear convergent trends prove that all three time series are stationary. The left corner picture of the off-diagonal might be most interesting for us. It seems that the external shock from CCI has less significant impact on ICI and IP than the shock from ICI to IP. On the contrary, the effect from ICI on IP looks immediate and strong, which reveals their close relationship; the peak effects are between 2 to 4 months, and then taper off slowly. Note that the convergence period of impulse response takes longer, and shows more variation for VAR(4) model. It seems that a more parsimonious VAR(3) model is robust enough to characterize our system.

In the next step, we move to forecast error decomposition analysis. The process determines how much of the forecast error variances of each variable can be explained by exogenous shocks from other variables. The forecast error variance decomposition of IP from the unrestricted VAR(3) model¹⁰ is provided in Table 6.6. It is shown in Panel A that the major part of the forecast error variances of industrial production are accounted for by either innovations of IP itself, or from the innovations of ICI. Since there are strong correlations in the innovations of IP and ICI discovered before, we

¹⁰Variance decomposition for VAR(4) model is omitted, since there are only marginal differences from the VAR(3) model.

VAR(3) Forecast Error Impulse Responses



VAR(4) Forecast Error Impulse Responses

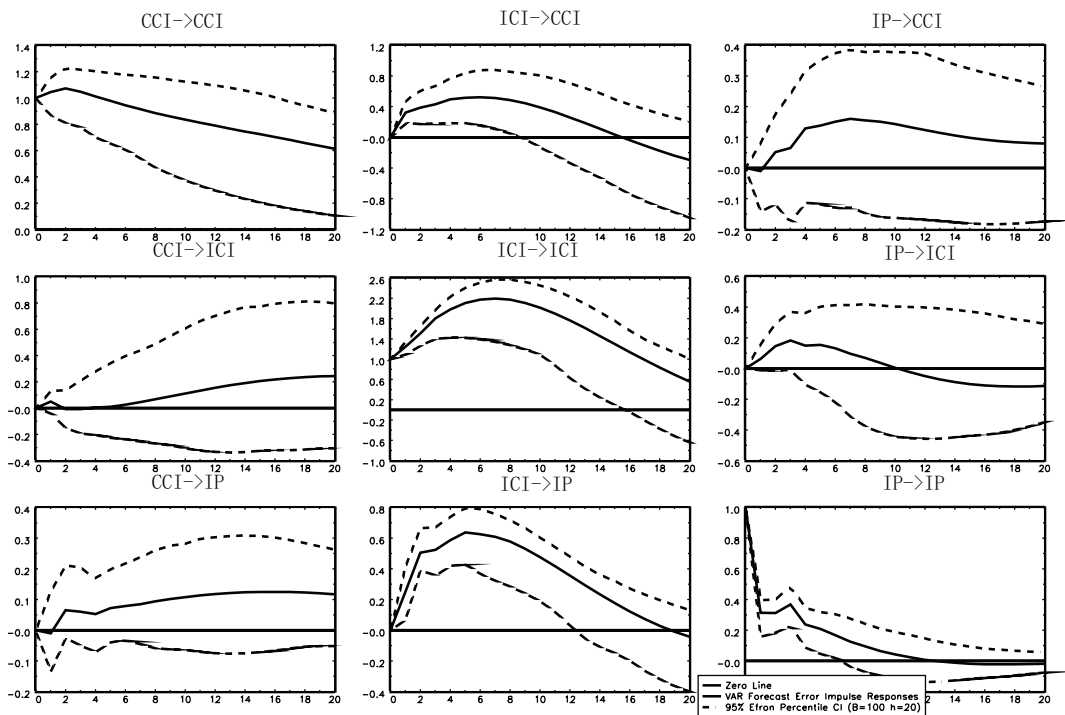


Figure 6.3: Forecast Error Impulse Responses of VAR(3) and VAR(4) Models

Table 6.6: Error Variance Decompositions of Industrial Production

Panel A: VAR(3) model, unrestricted			
periods	CCI innovation	ICI innovation	IP innovations
1	0.00	0.02	0.98
2	0.01	0.08	0.91
3	0.03	0.23	0.74
4	0.04	0.31	0.65
5	0.04	0.38	0.57
6	0.05	0.43	0.52
7	0.05	0.47	0.48
8	0.06	0.49	0.45
Panel B: VAR(3) model, unrestricted, different order			
periods	IP innovations	CCI innovations	ICI innovations
1	1.00	0.00	0.00
2	0.95	0.00	0.04
3	0.81	0.02	0.17
4	0.74	0.02	0.23
5	0.68	0.03	0.30
6	0.63	0.03	0.34
7	0.59	0.04	0.37
8	0.57	0.04	0.40
Panel C: VAR(3) model, restricted			
periods	CCI innovations	ICI innovations	IP innovations
1	0.00	0.02	0.98
2	0.01	0.10	0.89
3	0.01	0.24	0.75
4	0.01	0.32	0.67
5	0.01	0.39	0.60
6	0.01	0.44	0.55
7	0.01	0.47	0.51
8	0.01	0.50	0.49

Note: The order of the variables in the respective VARs corresponds to the ordering of the forecast error components.

Table 6.7: Diagnostic Checking of Estimated Restricted VAR Models

Restricted Var(3)
Portmanteau test for autocorrelation: 150.844 (0.000)
<i>Test for Nonnormality:</i>
Skewness Statistics: 5.946 (0.114)
Kurtosis Statistics: 376.118 (0.000)
Skewness + Kurtosis Statistics: 382.063 (0.000)
Restricted Var(4)
Portmanteau test for autocorrelation: 146.835 (0.001)
<i>Test for Nonnormality:</i>
Skewness Statistics: 5.294 (0.152)
Kurtosis Statistics: 363.591 (0.000)
Skewness + Kurtosis Statistics: 368.885 (0.000)

Notes: Portmanteau test for residual autocorrelation with 12 lags are computed (probability in parenthesis).

change the order of the VAR(3) system in Panel B, where CCI enters just behind IP, and we observe that CCI's contribution to error variances is very tiny, in comparison with a more sizable influence from ICI. As a matter of fact, for long term forecasts in our unrestricted VAR(3) model, IP together with ICI accounts for nearly 95% of the forecast error variances. To reduce estimation noise, we have also conducted innovation accounting for restricted models in Panel C, where the original insignificant parameters are replaced by zero constraints. The exercise gives a comparable proportion of error variances to both ICI and IP, while leaving CCI only a 1% contribution for even longer horizons. This finding is in agreement with our previous causality analysis of Table 6.5, that ICI Granger-causes IP, whereas CCI is almost independent of IP.

Following general-to-specific philosophy, because Eq. 6.2 has too many insignificant parameters which could add extra noise to our VAR system (especially when forecasting

is the aim). In the later forecast experiments, we will focus on the restricted model. Table 6.7 reports the diagnostic checking for the restricted version of the VAR(3) and VAR(4) models. It shows that both unrestricted and restricted models have almost exactly the same pattern of qualitative results. In addition, we have also compared the covariance matrices and Granger causality test of the restricted model with the unrestricted version, which all result in tiny differences as well. Consequently, we may safely use the restricted version of the model, instead of using the original unrestricted version with more variables for further forecasting exercises.

To see in how far sentiment helps to predict industrial production, in the following we conduct an explicit out-of-sample forecasting exercise. Forecast accuracy will be evaluated via the *mean squared error* (MSE), which is a frequently-used way to quantify the difference between an estimator and the true value actually observed. It treats positive and negative forecast errors symmetrically, and the common principle is to find an estimator with minimum MSE. Here we apply the tests for equal forecast accuracy of Diebold and Mariano (1995). If the model has substantial predictive power, we would expect the MSE to be lower than that of an alternative model. This test is applied to both restricted VAR(3) and VAR(4) models, estimated both by the maximum likelihood (ML) method and least square (LS) method.

Table 6.8 shows the results of our forecasting exercise for industrial production (single output as well as cumulative outputs) using time horizons of up to 8 periods ahead. All mean-squared errors in the table are standardized by dividing by the MSE of a random walk with drift as a benchmark and the adjusted *Diebold-Mariano (DM-) Test*¹¹ is

¹¹Diebold and Mariano were interested in a situation where a “cheap” benchmark forecast is compared to a sophisticated forecast. A forecaster may prefer the cheap forecast up to a point where the sophisticated forecast shows its relative merits “significantly”. They assume that the precision of forecast is basically measured by $\mathbf{E}g(x_t - \hat{x}_{t-1})$ and $\mathbf{E}g(x_t - \tilde{x}_{t-1})$ for two different forecasts \hat{x} and \tilde{x} , and a loss function $g(\cdot)$. Under the hypothesis that the difference is zero, it can be shown that the test statistic $\mathbf{S}_1 = \bar{d} / \sqrt{m^{-1}2\pi\hat{f}_{t-1}}$ converges to a standard normal distribution as $m \rightarrow \infty$. Here, \bar{d} denotes the sample average of $d_t = g(x_t - \hat{x}_{t-1}) - g(x_t - \tilde{x}_{t-1})$. The element $\hat{f}_d(0)$ is a scale

reported. It turns out that forecasts based on either the ML method or LS method display almost the same pattern of quantitative outcomes. Nevertheless, in general, the forecast performance of constrained estimation via the LS method (cf. Panel B of Table 6.8) does a slightly better job than estimation via the ML method (cf. Panel A of Table 6.8), especially for the short and medium term forecasts. From both panels, the DM-test statistics prefer the VAR(3) model, which seems to better capture the systematic effects than a richer VAR(4), for both single output and cumulative outputs. Meanwhile, although the LS estimates do a better job for cumulative forecasts, the ML method results in better forecasts for single period forecasts in a long term. Taking our previous findings in Fig. 6.3 and Table 6.6 into consideration, we find that the VAR(3) model can reproduce our system dynamics more consistently, and it is able to cover not only medium-term but also long-term feedback effects among sentiments and industrial output.

6.4.2 Forecast for Bivariate System

The main motivation of estimating the tri-variate system in our study is to find whether the sentiment from consumer or industry could help to explain the quantitative dynamics of output IP. Another way to better observe their influences on IP is to create separately two bivariate VAR systems. Eq. 6.4 shows the LS estimation results for restricted VAR(3) model, CCI & IP and ICI & IP respectively, favored by (difference factor defined as the spectral density estimate of d_t at frequency 0. For the case of *nested* models, the standard DM test is very likely to be undersized, and the correct limiting distribution is not normal any more. Clark and West (2006, 2007) show that most bias can be corrected by a simple adjustment in the statistic, and the adjusted test with Gaussian critical values has a size close to, but a little less than the normal one (cf. Buseti et al. (2009)). In our study, the adjusted Diebold-Mariano test statistics with Newey-West autocorrelation and heteroscedasticity consistent estimator of the standard deviation with automatic lag selection by Andrew's method are computed (cf. Clark and McCracken (2001), Clark and West (2007)).

Table 6.8: MSE of Out-of-Sample Forecasts for Tri-variate Models

A: Forecasts of Model Based on constrained ML				
Forecasts of single output				
horizon	VAR(3)	DM _{adj}	VAR(4)	DM _{adj}
1	0.124	1.812*	0.121	1.805*
2	0.180	1.879*	0.171	1.861*
3	0.294	1.966*	0.274	1.940*
4	0.424	2.138*	0.401	2.101*
5	0.557	2.385**	0.529	2.306*
6	0.686	2.636**	0.657	2.550**
7	0.790	2.720**	0.765	2.716**
8	0.857	2.642**	0.835	2.750**
Forecasts of cumulative outputs				
1	0.124	1.812*	0.121	1.805*
2	0.134	1.944*	0.128	1.930*
3	0.178	2.158*	0.168	2.136*
4	0.239	2.519**	0.226	2.488**
5	0.308	3.132**	0.293	3.073**
6	0.388	3.747**	0.370	3.680**
7	0.446	3.485**	0.429	3.552**
8	0.482	3.296**	0.469	3.410**

B: Forecasts of Model Based on constrained LS				
Forecasts of single output				
horizon	VAR(3)	DM _{adj}	VAR(4)	DM _{adj}
1	0.124	1.825*	0.181	1.863*
2	0.187	1.905*	0.302	2.065*
3	0.299	2.013*	0.450	2.407**
4	0.426	2.212*	0.599	2.840**
5	0.553	2.519**	0.722	3.156**
6	0.683	2.846**	0.827	2.733**
7	0.779	2.959**	0.910	1.842*
8	0.839	2.935**	0.973	1.234
Forecasts of cumulative outputs				
1	0.124	1.825*	0.181	1.863*
2	0.137	1.968*	0.228	2.095*
3	0.182	2.195*	0.308	2.526**
4	0.240	2.568**	0.400	3.213**
5	0.305	3.203**	0.474	3.767**
6	0.383	3.814**	0.536	3.280**
7	0.435	3.478**	0.582	2.768**
8	0.466	3.244**	0.617	2.964**

Note: The estimation results of in-sample restricted models have been applied in this forecast experiment. The first entry gives the relative MSE of the forecasts from either VAR(3) or VAR(4) (i.e., original MSE divided by MSE of random walk model with drift). DM_{adj} is the adjusted Diebold-Mariano test statistic for equal predictive accuracy of nested models. * and ** identify cases of significantly better predictive accuracy of the pertinent VAR forecasts compared to those of a random walk with drift using one-sided 5% and 1% tests respectively.

ent) information criteria¹². Both systems (at various lags) are stable according to the eigenvalues of their characteristic equations (cf. Lütkepohl (1991)¹³.

$$\begin{aligned}
 \begin{bmatrix} \text{CCI} \\ \text{IP} \end{bmatrix} &= \begin{bmatrix} 1.09^{**} & 0 \\ 0 & 0.49^{**} \end{bmatrix} \begin{bmatrix} \text{CCI}_{-1} \\ \text{IP}_{-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0.25^{**} \end{bmatrix} \begin{bmatrix} \text{CCI}_{-2} \\ \text{IP}_{-2} \end{bmatrix} + \begin{bmatrix} -0.12^{**} & 0 \\ 0.02 & 0.11^{**} \end{bmatrix} \begin{bmatrix} \text{CCI}_{-3} \\ \text{IP}_{-3} \end{bmatrix} \dots \\
 \begin{bmatrix} \text{ICI} \\ \text{IP} \end{bmatrix} &= \begin{bmatrix} 1.27^{**} & 0.07^* \\ 0.33^{**} & 0.34^{**} \end{bmatrix} \begin{bmatrix} \text{ICI}_{-1} \\ \text{IP}_{-1} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0.20^{**} \end{bmatrix} \begin{bmatrix} \text{ICI}_{-2} \\ \text{IP}_{-2} \end{bmatrix} + \begin{bmatrix} -0.31^{**} & 0 \\ -0.28^{**} & 0.17^{**} \end{bmatrix} \begin{bmatrix} \text{ICI}_{-3} \\ \text{IP}_{-3} \end{bmatrix} \dots
 \end{aligned} \tag{6.4}$$

The first equation of Eq. 6.4 shows the bivariate restricted VAR(3) model for CCI & IP. The off-diagonal estimates of all 3 lags are either zeros or insignificantly small values, which show that CCI has no correlation with IP, and this finding is in accordance with our tri-variate VAR study. In contrast, the interactions between ICI and IP are very strong in lag 1, and the effect of ICI on IP lasts for 3 lags. If we compare the significant estimates of these equations with Eq. 6.2, they look very similar. Considering that CCI is relatively independent in the system, it seems that the tri-variate model can be safely reduced to a bivariate ICI & IP model.

Table 6.9 lists the DM-test results for forecasting both bivariate systems. Since CCI has already been shown to be independent, Panel A can be equally treated as the forecast

¹²Different criteria are in favor of different lag orders. For example, for the bivariate CCI & IP model, AIC would favor a lag selection of 3, whereas BIC and HQ favor a lag selection of 2; for the bivariate ICI & IP model, AIC favors 7 lags, while BIC favors lag 3 and HQ favors lag 4 selection. To save space, here we choose only VAR(3) for both models.

¹³According to Lütkepohl (1991), the VAR process satisfies stability condition if the moduli of all eigenvalues of the reverse characteristic polynomial are outside the complex unit circle. Here, for bivariate CCI & IP VAR(2) model, the moduli for the corresponding eigenvalues are: $|z| = \{1.02, 5.92, 1.09, 2.71\}$, for VAR(3), $|z| = \{1.02, 2.42, 3.44, 2.67, 2.67, 1.07\}$; for bivariate ICI & IP VAR(3) model, the moduli are $|z| = \{2.15, 2.15, 1.09, 1.09, 2.37, 1.40\}$, and for VAR(4), $|z| = \{1.89, 1.89, 1.85, 1.85, 1.11, 1.11, 1.09\}$.

results for IP VAR(2) and VAR(3) (univariate) models. It seems that a VAR(2) model for IP is good enough if forecasting is our main objective. To allow autocorrelation in its error terms, we will also estimate an ARMA model for IP in the next section. Panel B of Table 6.9 shows the DM-test for the forecast experiments of selected restricted VAR(3) and VAR(4) specifications for bivariate models of ICI & IP. We can easily compare it with panel B of Table 6.8, it is certain that both forecast results are very close to each other. Although the tri-variate model looks slightly better, the parsimonious bivariate ICI & IP models also do a good job in forecasting.

In addition, the forecast figures depicted in Fig. 6.4 provide another possibility to show the equivalence of the tri-variate system and the bivariate ICI & IP system. We illustrate figures of forecasting performances for restricted bivariate models CCI & IP and ICI & IP, as well as the restricted tri-variate model. All selected models are based on VAR(3) estimation, since different lags only result in tiny visible differences. In these figures, the dashed lines depict the out-of-sample data, which we show as benchmarks for comparison. The smooth lines capture the predicted industrial outputs based on a particular multivariate VAR model. The following facts are interesting:

- (1) The restricted CCI & IP model is close to an IP VAR(3) model, since CCI has no significant causal relationship to IP.
- (2) When comparing the CCI & IP model with the bivariate restricted model ICI & IP, the later is more successful in reproducing the real data, especially in the longer term.
- (3) The industrial output boom shortly before the crisis and the downward trend in the economic crisis, are more-or-less followed with a lag by both bivariate ICI & IP and tri-variate models.
- (4) There is little difference in forecast performance between the tri-variate system and the bivariate ICI & IP system, although the former might be slightly better for the near term, the later looks more efficient for the long term. These two

systems' predictability measured by the DM-test statistics is provided in Panel B of Table 6.8 and Table 6.9, respectively.

Table 6.9: MSE of Out-of-Sample Forecasts for Restricted Bivariate Models

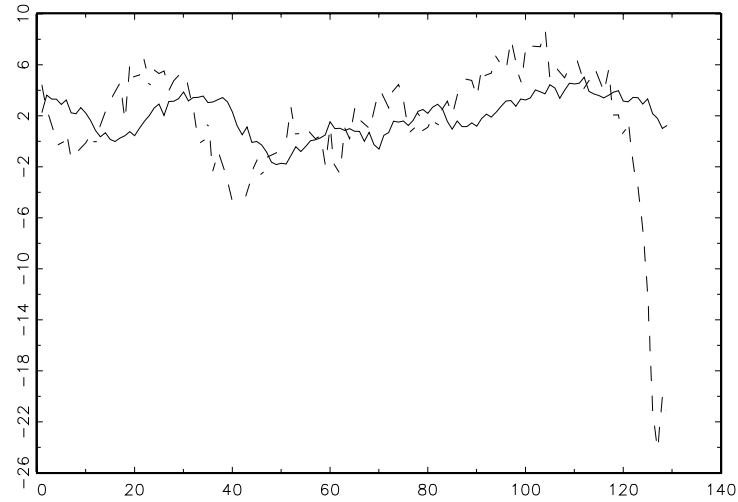
A: Forecasts via constrained LS, for (CCI +)IP				
Forecasts of single output				
horizon	VAR(2)	DM _{adj}	VAR(3)	DM _{adj}
1	0.167	1.757*	0.175	1.791*
2	0.284	1.892*	0.296	1.949*
3	0.454	2.230*	0.450	2.269*
4	0.619	2.692**	0.620	2.762**
5	0.746	3.124**	0.745	3.144**
6	0.833	3.014**	0.837	2.893**
7	0.901	2.266*	0.910	2.091*
8	0.957	1.322	0.973	1.204
Forecasts of cumulative outputs				
1	0.167	1.757*	0.175	1.791*
2	0.212	1.939*	0.224	1.994*
3	0.299	2.338**	0.308	2.409**
4	0.405	3.068**	0.412	3.154**
5	0.486	3.792**	0.491	3.819**
6	0.543	3.751**	0.549	3.589**
7	0.587	3.083**	0.594	2.958**
8	0.623	2.742**	0.633	2.759**

B: Forecasts via constrained LS, for ICI + IP				
Forecasts of single output				
horizon	VAR(3)	DM _{adj}	VAR(4)	DM _{adj}
1	0.125	1.881*	0.181	1.926*
2	0.187	1.965*	0.302	2.148*
3	0.299	2.079*	0.450	2.521**
4	0.425	2.287*	0.598	2.968**
5	0.552	2.607**	0.720	3.217**
6	0.681	2.936**	0.825	2.630**
7	0.776	3.002**	0.906	1.785*
8	0.834	2.978**	0.968	1.238
Forecasts of cumulative outputs				
1	0.125	1.881*	0.181	1.926*
2	0.137	2.033*	0.228	2.178*
3	0.182	2.274*	0.308	2.642**
4	0.239	2.666**	0.400	3.342**
5	0.304	3.320**	0.474	3.773**
6	0.382	3.839**	0.535	3.105**
7	0.434	3.362**	0.580	2.741**
8	0.465	3.135**	0.615	2.879**

Note: DM_{adj} denotes adjusted Diebold-Mariano test statistics for equal predictive accuracy of nested models. * and ** indicate significance at 5% and 1% of the DM-test for hypothesis of no significant difference in MSE with respect to the benchmark. Since it is indicated by Eq. 6.4 that CCI and IP is not correlated, Panel A can be understood as VAR forecasts for IP alone.

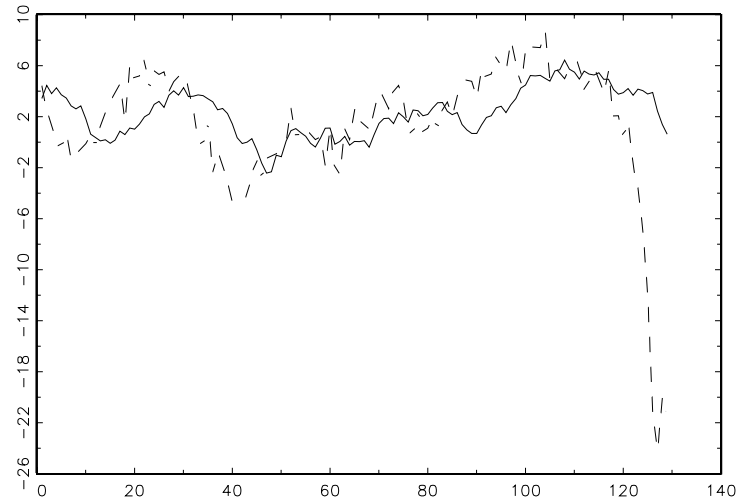
GAUSS Thu Oct 08 12:03:35 2009

Forecast CCI & IP VAR(3)



GAUSS Thu Oct 08 12:11:15 2009

Forecast ICI & IP VAR(3)



GAUSS Thu Oct 08 12:12:52 2009

Forecast CCI & ICI & IP VAR(3)

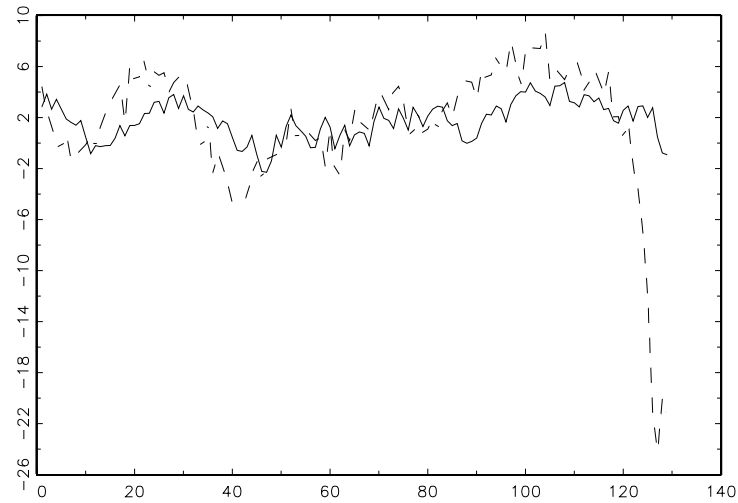


Figure 6.4: Forecast Performance of Bivariate and Tri-variate Models

6.4.3 Forecast for ARMA IP Model

From the last two sections, we have already learned that our tri-variate system can be reduced to a bivariate system of ICI & IP. Nevertheless, we are still curious how the IP time series alone predicts the future, so that we can better understand to which extent the sentiment index ICI could help to explain the future behavior of IP dynamics.

It is known that a forecast is of greater practical value if measurement of uncertainty can also be included. Poddig and Huber (1999) find that *autoregressive moving average* (ARMA) model provides a more accurate forecasting tool for predicting turning points (if there is structural change) than a VAR model. After observing some autocorrelations in IP from Fig. 6.2, we pick up the ARMA model for a deeper analysis of IP, assuming autocorrelations in its error terms.

The ARMA model can be considered as an extension to our standard finite order VAR model defined in Eq. 6.1:

$$y_t = \nu + \mathbf{A}_1 y_{t-1} + \cdots + \mathbf{A}_p y_{t-p} + u_t, \quad t \in \mathbb{Z}$$

By allowing the error terms u_t to be autocorrelated rather than white noise, we will arrive at a finite order *moving average* (MA) representation for u_t :

$$u_t = \varepsilon_t + \mathbf{B}_1 \varepsilon_{t-1} + \cdots + \mathbf{B}_q \varepsilon_{t-q}, \quad (6.5)$$

as in the standard VAR model, ε_t is zero mean white noise with nonsingular covariance matrix Σ_ε .

Following the same procedure as in the VAR analysis, we first conduct a general ARMA order selection using different information criteria. Both ARMA models with and without constant term are tested. For an ARMA model with constant, AIC favors AMRA(1,3), BIC favors ARMA(2,0), and HQ favors ARMA(1,2), but none of their constant terms are significant. As an alternative, for an ARMA model without constant, AIC favors ARMA(4,1), whereas both BIC and HQ opt for ARMA(1,1). Given

that AIC is sometimes inconsistent and tends to overestimate the actual order (as discussed in Chap. 6.3), we focus on the ARMA(1,1) model, which is favored by the other two information criteria. Eq. 6.6 shows the estimation result for the ARMA(1,1) model for IP using LS estimation, the t-ratios reported in brackets are all significant.

$$\begin{aligned} \text{IP}_t = & 0.939 \text{ IP}_{t-1} + 0.361 \varepsilon_{t-1} + \varepsilon_t & (6.6) \\ \text{t-ratio} & (42.2) & (5.85) \end{aligned}$$

In order to check stationarity of the system, we can write the ARMA(1,1) in the form of lag operator:

$$(1 - \phi_1 L) \text{IP}_t = (1 + \theta_1 L) \varepsilon_t \quad (6.7)$$

This simple difference equation is stable provided that the root of “ $1 - \phi_1 z = 0$ ” lies outside the unit circle. As the estimated ϕ_1 is less than 1, this means the only unit root should be outside the unit circle too, and our ARMA(1,1) process is stationary. As a double check, Fig. 6.5 illustrates the residual autocorrelation and partial-autocorrelation, it seems that except for the third lag, there is little autocorrelation in the short horizon, which proves adequacy of the model.

In order to compare the forecast performance of ARMA model with previous models, Table 6.10 provides DM-tests results measuring the MSE of out-of-sample forecasts for ARMA(1,1). It shows that the ARMA-IP model does a good job in forecasting, and it even outperforms the (restricted) VAR(2) or VAR(3) IP model (cf. Panel A of Table 6.9) in the medium and long run, especially for the cumulative output forecasts. However, when comparing the ARMA system with the bivariate ICI & IP system, the later one still provides a significant improvement, for both single output and cumulative output. The last two columns of Table 6.10 show the DM-test statistics of predictive accuracy of the bivariate ICI & IP VAR(3) and VAR(4) models (cf. Panel B of Table 6.9) against the ARMA(1,1) model, which show that although the VAR(4) model predicts relatively worse when compared with the ARMA(1,1) forecasts, the VAR(3) model is

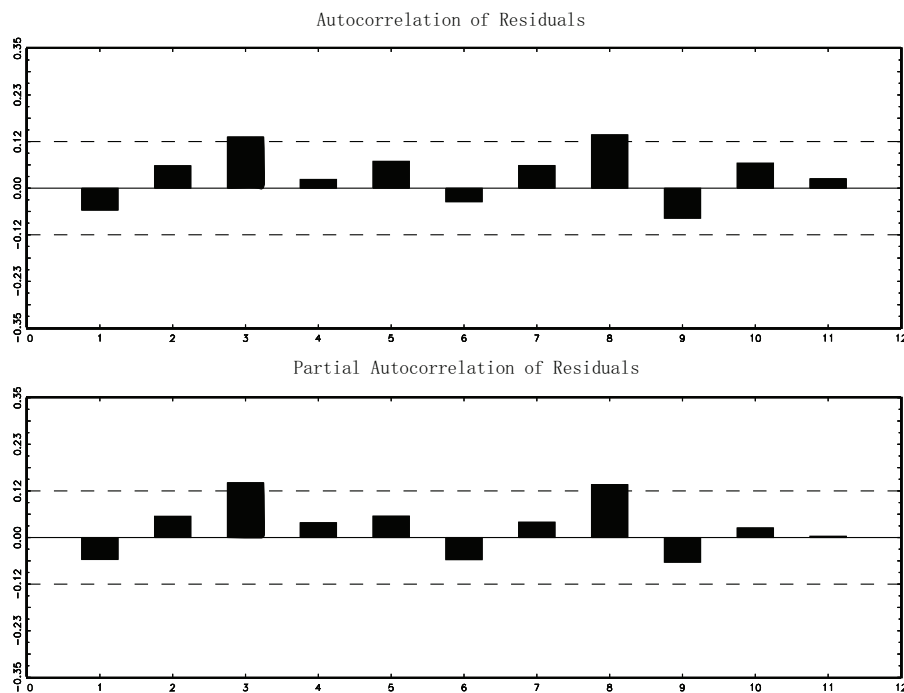


Figure 6.5: Residual Autocorrelations and Partial Autocorrelations of IP ARMA(1,1) Model

still significantly better at predicting both in the short run and in relative long run, as all statistics are significant at all lag lengths. In other words, ICI does help us to better predict the future behavior of IP.

To sum up, it is usually believed that sentiment can influence economic activity. When the word “sentiment” is mentioned, perhaps we are more likely to refer to “consumer sentiment”, since their purchasing willingness will directly influence the “market”, and hence production. This chapter studies two major sentiment indices, which may have close connection to industrial output, namely the consumer sentiment (CCI) and industrial sentiment (ICI). Quite surprisingly, after investigating the tri-variate system, and further the bivariate restricted system, we arrive at the conclusion that consumer sentiment does *not* really help in explaining the dynamics of IP at all, which is in striking contrast to the common conjecture. Meanwhile, the successful recovery of predictabil-

Table 6.10: MSE of Out-of-Sample Forecasts for IP ARMA(1,1) Model

Model forecasts estimated via LS method				
Forecasts of single output				
horizon	ARMA(1,1)	DM _{adj}	DM(3)	DM(4)
1	0.142	1.704*	4.984**	0.020
2	0.242	1.831*	3.688**	-0.227
3	0.369	2.121*	2.985**	-0.467
4	0.538	2.565**	3.880**	-0.256
5	0.699	3.056**	4.348**	0.394
6	0.825	3.193**	4.022**	1.131
7	0.935	2.800**	4.243**	2.136*
8	1.055	1.619	4.599**	3.200**
Forecasts of cumulative outputs				
1	0.142	1.704*	4.984**	0.020
2	0.175	1.855*	4.850**	-0.106
3	0.237	2.200*	4.134**	-0.347
4	0.339	2.894**	4.195**	-0.284
5	0.438	3.713**	4.294**	0.149
6	0.520	3.982**	5.269**	0.709
7	0.591	3.673**	4.999**	1.845*
8	0.670	2.896**	4.635**	2.793**

Note: * and ** indicate significance at 5% and 1% of Diebold-Mariano test for the null hypothesis of no significant difference in MSE with respect to the benchmark. DM_{adj} identify cases of significantly better predictive accuracy of ARMA(1,1) forecasts compared with random walk with drift; DM(3) and DM(4) identify cases of significantly better predictive accuracy of VAR(3) and VAR(4) forecasts (cf. Panel B of Table 6.9) compared with the ARMA(1,1) model.

ity by the bivariate ICI & IP VAR system, proves a very close correlation between industry sentiment and production, namely, part of IP dynamics can be explained by the development of ICI.

Chapter 7

Modeling the Joint Dynamics of Sentiment and Output

7.1 A Stochastic Model for Opinion Formation

There are numerous examples of equilibrium and non-equilibrium systems that exhibit transitions between phases and characteristic spatial-temporal patterns. For instance, transitions between solid, liquid and gaseous states of matter; transitions between ferromagnetic and paramagnetic phases of magnetic materials at the *Curie point* (cf. Heusler et al. (1903)), and others. In human societies, there are also transitions between fashion trends every now and then. Humans, animals and even insects change gaits at critical values of locomotion speeds. An interesting example can be found in Kirman (1993)'s *ants' model*. It offers an explanation of ants' behavior which puzzled entomologists for a long time, that when facing two identical food sources, the ants are observed to concentrate more on one of them, but after a period they will abruptly turn attention to the other food source. The same "herding" behavior can be also observed in humans when they choose between two similar but close-by restaurants, etc. Current interests in the problem of how two competing opinions evolve and how

consensus emerges over time include not only the *ants' model*, but also the *active walker model* (Helbing et al. (1997)), the *compromise model* (Weisbuch et al. (2001), Naim et al. (2003)), the *neighborhood model* (Galam (2002), Tessone et al. (2004)), the *voter model* (Liggett (1999), Lambiotte and Redner (2008)), and many others.

A common phenomenon underlying a system which exhibits phase transition is that they are all many-body systems, which represent an ensemble of many subsystems. Our lives and natural surroundings, such as neural networks or societies are full of many-body systems. Due to self-organization, many-body systems can exhibit properties that do not exist on the level of their subsystems, so the whole system will display a collective or cooperative behavior. Consequently, a complete description of this huge system requires taking the interactions among all subsystems into account. The non-linear Fokker-Planck equation offers a possibility to quantify equilibrium and non-equilibrium phase transitions in terms of subsystem probability dependency (cf. Frank (2005)).

On top of classical time series analysis of our sentiment indices and industrial output, in this chapter, we move to a stochastic model for opinion formation, which might better capture the inter-relation, or phase transition properties of the whole system through digging into the interaction of its subsystems.

7.1.1 Model Framework

Our baseline model can be traced back to Weidlich and Haag (1983), and similar models of opinion formation in financial market have been later studied by Lux (1995, 1997, 1998, 2009b), Alfarano et al. (2008), Ghonghadze and Lux (2009), among others. The model describes a binary choice problem and captures the transition process of opinion formation between two groups of agents. For simplicity, we begin with assuming that the agents are all *homogeneous*, having the same individual probabilities of reactions and interactions in the opinion formation process. They hold either a “positive

(optimistic)” or “negative (pessimistic)” opinion denoted by “+” and “-”. The *socio-configuration* $\{n_t^+, n_t^-\}$ ¹ at given time t consists of the numbers n^+ and n^- of people having opinion “+” and “-”, respectively; the total number of agents is fixed to $2N$; here, N is multiplied by 2 to make sure that the total number of agents participating is an even number, for calculation simplicity. The socio-configuration is determined by another variable n_t based on the difference of the numbers of agents with opposite opinions. Therefore, we define:

$$n := \frac{1}{2}(n^+ - n^-) \quad (7.1)$$

since $n^+ + n^- = 2N$,

$$n^+ = N + n \quad \text{and} \quad n^- = N - n \quad (7.2)$$

where $-N \leq n \leq N$ and $0 \leq n^+, n^- \leq 2N$. Meanwhile, we define another variable called “*opinion index*” x (or x_t) to characterize the *aggregate* or *average opinion* as,

$$x := \frac{n}{N} = \frac{n^+ - n^-}{2N}, \quad \text{with} \quad -1 \leq x \leq 1 \quad (7.3)$$

Since all agents are homogeneous, having the same weights in the population, $x = 0$ can be interpreted as a situation with a balanced distribution of opinions, while $x > 0$ (or $x < 0$) represent the cases that the optimistic (or pessimistic) agents are in the majority. In the next step, define

$$p(n; t) := p[n^+, n^-; t] \quad (7.4)$$

which denotes the probability that one society has the socio-configuration $\{n^+, n^-\}$ at time t , then the condition

$$\sum_{n=-N}^N p(n; t) = 1 \quad (7.5)$$

¹In most of the later expressions, we omit the notation t for simplicity, which only tells that the calculation of these expressions happen at the same time. This applies to other expressions as well.

holds for all t . In a continuous time setting, only the following changes of the socio-configuration happen at an infinitesimal interval of time:

$$\begin{aligned} \{n^+, n^-\} &\rightarrow \{n^+ + 1, n^- - 1\}, \quad \text{or} \\ \{n^+, n^-\} &\rightarrow \{n^+ - 1, n^- + 1\} \end{aligned} \quad (7.6)$$

To save space, these expressions will be reduced to $n \rightarrow (n+1)$ and $n \rightarrow (n-1)$ respectively, meaning that one more agent in the optimistic (pessimistic) group comes from the pessimistic (optimistic) group, who has just changed his opinion. Furthermore, it should be noted that as the agent's opinion may be easily influenced by others, he might change opinion suddenly. Therefore, we assume that the changes of belief happen in continuous time, with a *Poisson* process describing the opinion switches from the optimistic group to the pessimistic group, or *vice versa*. The pertinent transition rates can be written as:

$$\begin{aligned} w_{\uparrow}(n) &:= w(n \rightarrow n+1), \quad \text{or} \\ w_{\downarrow}(n) &:= w(n \rightarrow n-1) \end{aligned} \quad (7.7)$$

Particularly, $w(n \rightarrow n') = 0$ for $n' \neq n \pm 1$, which means there should be no more than one agent change his opinion per unit time. Weidlich and Haag (1983) have defined the so-called “general *Master equation*” to describe the exact transitions of probabilities between different states i and j (for $i, j \in [-N, N]$):

$$\frac{\mathbf{d}p(n; t)}{\mathbf{d}t} = \sum_j \left[w(i \rightarrow j) p(j; t) - w(j \rightarrow i) p(i; t) \right] \quad (7.8)$$

Under the definition from Eq. 7.7, the *equation of motion* for $p(n; t)$ in our model can be transformed into the following function:

$$\begin{aligned} \frac{\mathbf{d}p(n; t)}{\mathbf{d}t} &= w_{\uparrow}(n-1) p(n-1; t) - w_{\uparrow}(n) p(n; t) \\ &\quad + w_{\downarrow}(n+1) p(n+1; t) - w_{\downarrow}(n) p(n; t) \end{aligned} \quad (7.9)$$

If the exact description of the socio-configuration in terms of discrete integer number n , can be approximately described in terms of continuous variables, and if $p(n; t)$, $w_{\uparrow}(n)$,

$w_{\downarrow}(n)$ can be treated as continuous functions of n , using a *Taylor series expansion*, the Master equation of Eq. 7.9 can be approximated by a *Fokker-Planck equation*², which is a special form of *partial difference equation* (PDE):

$$\begin{aligned} \frac{\partial p(n; t)}{\partial t} = & -\frac{\partial}{\partial n} \left\{ [w_{\uparrow}(n) - w_{\downarrow}(n)] p(n; t) \right\} \\ & + \frac{1}{2} \frac{\partial^2}{\partial n^2} \left\{ [w_{\uparrow}(n) + w_{\downarrow}(n)] p(n; t) \right\} \end{aligned} \quad (7.10)$$

Given the definition of the *opinion index* $x = n/N$, the probability distribution function (PDF) for this opinion index can be written as:

$$P(x; t) = N \cdot p(n; t) = N \cdot p(Nx; t) \quad (7.11)$$

which is normalized by

$$\int_{-1}^{+1} P(x; t) dx \approx \sum_{x=-1}^{+1} P(x; t) \Delta x = \sum_{n=-N}^{+N} p(n; t) = 1 \quad (7.12)$$

Applying Eq. 7.11 to Eq. 7.9, we arrive at an equivalent Master equation for the opinion index:

$$\begin{aligned} \frac{dp(x; t)}{dt} = & w_{\uparrow} \left(x - \frac{1}{N}\right) p \left(x - \frac{1}{N}; t\right) - w_{\uparrow}(x) p(x; t) \\ & + w_{\downarrow} \left(x + \frac{1}{N}\right) p \left(x + \frac{1}{N}; t\right) - w_{\downarrow}(x) p(x; t) \end{aligned} \quad (7.13)$$

²The *Fokker-Planck equation* (FPE) was originally designed to describe the time evolution of the probability density function of the position of a particle in a fluid. Later on, it has been widely used in physics, chemistry and other fields in the natural sciences, as a parabolic partial differential equation for the transition density. It is named after Adriaan Fokker (1914) and Max Planck (1917). The former investigated *Brownian motion* in a radiation field and the latter attempted to build up a complete theory of fluctuations based on it. The Soviet Russian mathematician Andrey Nikolaevich Kolmogorov contributed in developing its rigorous basis mathematically in his work Kolmogorov (1931), and therefore it is also known as the *Kolmogorov's equation*. The same methodology of using Fokker-Planck equation for estimation of opinion formation has also been adopted in Weidlich and Haag (1983), Lux (1997, 2009b), Ghonghadze and Lux (2009). A detailed explanation of the Fokker-Planck equation can be found in Risken (1989), Carmichael (1999), Gardiner (2004), Frank (2005), and so on.

Following the same procedure as in Eq. 7.10, the standard form of Fokker-Planck equation for opinion index is obtained:

$$\frac{\partial P(x; t)}{\partial t} = -\frac{\partial}{\partial x} \left\{ A(x)P(x; t) \right\} + \frac{1}{2} \frac{\partial^2}{\partial x^2} \left\{ B(x)P(x; t) \right\} \quad (7.14)$$

with *drift* $A(x)$ and *diffusion* $B(x)$, where

$$\begin{aligned} A(x) &= \frac{1}{N} [w_{\uparrow}(x) - w_{\downarrow}(x)], \quad \text{and} \\ B(x) &= \frac{1}{N^2} [w_{\uparrow}(x) + w_{\downarrow}(x)] \end{aligned} \quad (7.15)$$

In addition, note that Eq. 7.14 can be rewritten in the following form:

$$\frac{\partial P}{\partial t} = -\frac{\partial}{\partial x} \underbrace{\left\{ \frac{1}{2} B(x) \frac{\partial P}{\partial x} + \left[-A(x) + \frac{1}{2} \frac{\partial B(x)}{\partial x} \right] P \right\}}_{F(x)} \quad (7.16)$$

The term denoted by $F(x)$ in the big brackets is often named “*flux*”, which is defined as the rate at which the density is passing the point x at respective time point t . Thus, the original Fokker-Planck parabolic PDE in Eq. 7.14 can be expressed as a *hyperbolic flux conservation equation*:

$$\frac{\partial P(x; t)}{\partial t} = -\frac{\partial F(x, t)}{\partial x} \quad (7.17)$$

Suppose, furthermore, that the state space of our problem is bounded within $[a, b]$, and the process starts at $x = X_h$ at time t_h . If measurement error is negligible, the initial condition turns out to be:

$$P(x, t_h) = \delta(x - X_h) \quad (7.18)$$

where δ is the *Dirac delta function*, and the boundary conditions required to conserve the unit density within the interval $[a, b]$ are given by:

$$\begin{aligned} \lim_{x \rightarrow a^+} \left(\frac{1}{2} \frac{\partial [B(x)P]}{\partial x} - A(x)P \right) &= 0, \quad \text{and} \\ \lim_{x \rightarrow b^-} \left(\frac{1}{2} \frac{\partial [B(x)P]}{\partial x} - A(x)P \right) &= 0 \end{aligned} \quad (7.19)$$

The boundary conditions state that the probability density calculated by the Fokker-Planck equation is a conserved quantity as the process evolves. Unfortunately, a closed-form solution for Eq. 7.16 is not analytically tractable, we therefore resort to numerical estimation using a *finite difference method*. We will apply the Crank-Nicolson Scheme for the one-dimensional problem, and an Alternative Direction Implicit Scheme will be used for the two-dimensional problem.

7.1.2 Explicit Form of the Model

There are various forms for the transition probabilities w_{\uparrow} and w_{\downarrow} . In order to obtain explicit results for the model, an explicit definition for w_{\uparrow} and w_{\downarrow} should be chosen. Here we use a *theoretical* representation, that is considered to be flexible enough to describe manifold possibilities. This specification has been used in Weidlich and Haag (1983), Lux (1995, 1997, 2009a,b), Ghonghadze and Lux (2009), among others. To be in line with our previous VAR analysis, we further denote w_{\uparrow}^C (or w_{\downarrow}^C) and w_{\uparrow}^I (or w_{\downarrow}^I) as the *transition rates* for *consumer confidence index* (CCI) and *industry confidence index* (ICI), respectively. The transition rates for CCI are assumed to be of exponential form:

$$\begin{aligned} w_{\uparrow}^C &= \nu_C \exp\{U_C\}, \quad \text{or} \\ w_{\downarrow}^C &= \nu_C \exp\{-U_C\} \end{aligned} \tag{7.20}$$

Similarly, the transition rates for ICI are:

$$\begin{aligned} w_{\uparrow}^I &= \nu_I \exp\{U_I\}, \quad \text{or} \\ w_{\downarrow}^I &= \nu_I \exp\{-U_I\} \end{aligned} \tag{7.21}$$

where ν is often called the “*flexibility parameter*” that determines the time scale in which opinion changes occur. The expression U covers those variables which may directly influence the transition rates, and can be labeled as a “*forcing function*” for

transition, for example, here we follow the definition of Lux (2009a), and define:

$$\begin{aligned} U_C &= \alpha_0 + \alpha_1 x_C + \alpha_2 x_I + \alpha_3 y, \quad \text{and} \\ U_I &= \beta_0 + \beta_1 x_I + \beta_2 x_C + \beta_3 y \end{aligned} \quad (7.22)$$

In Eq. 7.22, the function of U allows for cross dependencies among consumer sentiment x_C , industry sentiment x_I , as well as dependency on industry output “ y ”. α_0 and β_0 are *preference parameters*, positive (negative) α_0 (or β_0) increases the probability that an individual changes to “+” (“-”) opinion; α_1 and β_1 are often called *herding parameters*, positive α_1 (or β_1) increases the transition probability in favor of the majority opinion and reduces the transition probability in the direction of the minority opinion. According to previous experiments in Alfarano et al. (2008), Lux (2009b), Ghonghadze and Lux (2009), for a single opinion process, the aggregate outcome of the process depends crucially on α_1 (or β_1). It can be also labeled as the intensity of *herding* or interaction: when α_1 (or β_1) is below 1, the stationary distribution has a unique maximum, while it becomes bi-modal under stronger interaction ($\alpha_1 > 1$, or $\beta_1 > 1$). The later case allows for build-up and breakdown of a strong optimistic or pessimistic majority. Besides, α_2 and β_2 , α_3 and β_3 indicate the adjusted parameters for cross dependency on sentiment, and possible causal effects of industrial output, respectively. If we denote the parameter vector, $\theta_C = (v_C, \alpha_0, \alpha_1, \alpha_2)'$ and $\theta_I = (v_I, \beta_0, \beta_1, \beta_2)'$, as the parameter set to be estimated, then the *drift* and *diffusion* coefficients defined by Eq. 7.15 turn out to be:

$$\begin{aligned} A(x_t; \theta_i) &= v_i(1 - x_t^i) \exp(U_i) - v_i(1 + x_t^i) \exp(-U_i), \quad \text{and} \\ B(x_t; \theta_i) &= [v_i(1 - x_t^i) \exp(U_i) + v_i(1 + x_t^i) \exp(-U_i)]/N \end{aligned} \quad (7.23)$$

with $i = C, I$, corresponding to either CCI or ICI indices. $(1 - x)$ and $(1 + x)$ denote the fractions of currently optimistic and pessimistic agents who would potentially change their opinions (note that if these two groups of people are in balance, x is equal to 0).

Our aim is to estimate the parameter set θ , given a sample of $n + 1$ observations (n “space” dimensions plus time) x_0, \dots, x_n at the respective time t_0, \dots, t_n . We adopt

the *maximum likelihood* (ML) method to estimate the unknown parameter set θ , by finding the value of θ that maximizes $\mathcal{L}(\theta)$ (cf. John (1997)):

$$\hat{\theta} = \arg \max_{\theta} \mathcal{L}(\theta) \quad (7.24)$$

The ML estimate of θ can be also understood as generated by minimizing the negative log-likelihood function of the observed sample, with respect to θ :

$$-\log \mathcal{L}(\theta) = -\log P_0(X_0|\theta) - \sum_{h=0}^{n-1} \log P(X_{h+1}|X_h; \theta) \quad (7.25)$$

where $P_0(X_0|\theta)$ is the probability density for the initial state, the conditional probability $P(X_{h+1}|X_h; \theta)$ can be calculated by numerical iteration of the Fokker-Planck equation over a unit time interval given X_h as a previous observation, using *finite difference methods*. The likelihood of initial condition x_0 , should be in principle computed on the basis of the limiting distribution of P_0 . Since its influence over the whole estimation is negligible, we will simply discard this observation in practice. The remaining entries are conditional probabilities evaluated numerically with the above approach.

7.2 Model Estimation in One Dimension

7.2.1 Crank-Nicolson Finite Difference Scheme

In mathematics, *finite difference schemes* (FDS) are numerical methods for approximating the solutions to *differential equations* by replacing derivative expressions with approximately equivalent *difference quotients*³ using *finite difference equations* (FDE). There are three types of FDE,

$$\text{forward difference} : p(x+h) - p(x)$$

$$\text{backward difference} : p(x) - p(x-h)$$

$$\text{central difference} : p(x + \frac{1}{2}h) - p(x - \frac{1}{2}h)$$

³If the finite difference is defined in the form $p(x+b) - p(x+a)$, and is divided by $b-a$, then one gets the *difference quotients*.

Common FDS for PDE include the so-called Crank-Nicolson, Du Fort-Frankel, Laasonen methods, among others. Following Lux (2009a,b), as the Crank-Nicolson method is considered to be numerically robust and stable (Strikwerda (1989)), our Fokker-Planck equation will be solved numerically by the Crank-Nicolson finite difference method, which was developed by John Crank and Phyllis Nicolson in the mid-20th century (Crank and Nicolson (1947)⁴). This approach to estimation is proven to be consistent, asymptotically normal and asymptotically equivalent to full ML estimates (cf. Poulsen (1999) for univariate diffusion processes). The finite difference procedure is based on discretization⁵. First of all, uniform partition is applied to the space domain using grid points $x_n = x_0 + nh$; $n = 0, 1, \dots, N_x$ and similarly in the time domain: $t_m = mk$; $m = 0, 1, \dots, N_t$. Then, P_n^m represents the discrete evaluation of the transient density at space and time coordinates (x_n, t_m) , $P_n^m := p(x_n, t_m)$.

As pointed out by Wilmott et al. (1995), a *fully* central difference is seldom used in practice because it easily ends up with bad numerical results. More often, a *half* central difference is used instead to approximate the partial derivative. The basic idea of Crank-Nicolson's method is to evaluate the value of the midpoint as the average of values at the grid points one half step before and one half step after in time. Before we describe the algorithm, it would be necessary to define the following half-step values:

$$x_{n+1/2} = \frac{x_n + x_{n+1}}{2} \quad (7.26)$$

$$\Delta x_{n+1/2} = x_{n+1} - x_n \quad (7.27)$$

$$\begin{aligned} \Delta x_n &= x_{n+1/2} - x_{n-1/2} = \frac{x_n + x_{n+1}}{2} - \frac{x_n + x_{n-1}}{2} \\ &= \frac{x_{n+1} - x_{n-1}}{2} \end{aligned} \quad (7.28)$$

⁴They use an implicit finite difference method (later called "Crank-Nicolson finite difference scheme") to approximate the solution of a non-linear differential system which arises in the problems of heat flow.

⁵Discretization is a procedure that replaces the continuous problem in certain domain by grids of discrete location. Since the finite difference procedure is to compute approximate PDE solutions only at finite points, grids of space and time interval should be defined beforehand to solve the problem (cf. Wang (2007)).

similarly, in the time domain,

$$t_{m+1/2} = \frac{t_m + t_{m+1}}{2} \quad (7.29)$$

$$\Delta t_{m+1/2} = t_{m+1} - t_m \quad (7.30)$$

$$\begin{aligned} \Delta t_m &= t_{m+1/2} - t_{m-1/2} = \frac{t_m + t_{m+1}}{2} - \frac{t_m + t_{m-1}}{2} \\ &= \frac{t_{m+1} - t_{m-1}}{2}. \end{aligned} \quad (7.31)$$

Since the drift, diffusion term, as well as the transient density depend explicitly on x , we have:

$$A_{n+1/2} = \frac{A_n + A_{n+1}}{2} \quad (7.32)$$

$$B_{n+1/2} = \frac{B_n + B_{n+1}}{2} \quad (7.33)$$

$$P_{n+1/2}^m = \frac{P_n^m + P_{n+1}^m}{2} \quad (7.34)$$

$$P_n^{m+1/2} = \frac{P_n^m + P_n^{m+1}}{2} \quad (7.35)$$

The relation between the flux term $F(x; t)$ and $P(x; t)$ is linear, therefore, the half-step value of the flux $F_n^{m+1/2}$ is given by:

$$F_n^{m+1/2} = \frac{F_n^m + F_n^{m+1}}{2} \quad (7.36)$$

It should be mentioned again that, in the case of uniform grids, all spacial and time intervals are of identical length of h and k respectively:

$$\Delta x = x_{n+1} - x_n = x_{n+1/2} - x_{n-1/2} = h \quad (7.37)$$

$$\Delta t = t_{m+1} - t_m = t_{m+1/2} - t_{m-1/2} = k \quad (7.38)$$

The Crank-Nicolson method is essentially an arithmetic average of an *explicit* method (or *forward difference* method) and an *implicit* method (or *backward difference* method). For our PDE Eq. 7.17, by approximating the flux at intermediate points $(m + 1/2)k$, we have the following finite difference approximation:

$$\frac{P_n^{m+1} - P_n^m}{k} = -\frac{F_{n+1/2}^{m+1/2} - F_{n-1/2}^{m+1/2}}{h} \quad (7.39)$$

Since $t^{m\pm 1/2}$ are points that are not on the grid, we may overcome this difficulty by the half-step values defined before, then the right side of Eq. 7.39 can be rewritten as:

$$\begin{aligned}\frac{F_{n+1/2}^{m+1/2} - F_{n-1/2}^{m+1/2}}{h} &= \frac{1}{h} \left[\frac{F_{n+1/2}^m + F_{n+1/2}^{m+1}}{2} - \frac{F_{n-1/2}^m + F_{n-1/2}^{m+1}}{2} \right] \\ &= \frac{1}{h} \left[\frac{F_{n+1/2}^m - F_{n-1/2}^m}{2} + \frac{F_{n+1/2}^{m+1} - F_{n-1/2}^{m+1}}{2} \right]\end{aligned}\quad (7.40)$$

So far, the half-step values for the flux term $F_{n\pm 1/2}^m$ and $F_{n\pm 1/2}^{m+1}$ are still unknown. However, recall Eq. 7.16 that the flux is defined as:

$$F(x, t) = \frac{1}{2}B(x)\frac{\partial P}{\partial x} + \left[-A(x) + \frac{1}{2}\frac{\partial B(x)}{\partial x} \right]P \quad (7.41)$$

which means the flux $F(x, t)$ is a function of the drift term $A(x)$, the diffusion term $B(x)$, the unknown density $P(x, t)$ and two partial derivatives $\partial P/\partial x$ and $\partial B/\partial x$. Hence we are able to use the half-step values to construct a numerical expression for $F_{n\pm 1/2}^{m(+1)}$:

$$F_{n+1/2}^m = \frac{1}{2}B_{n+1/2}P'_{n+1/2}^m + (-A_{n+1/2} + \frac{1}{2}B'_{n+1/2})P_{n+1/2}^m \quad (7.42)$$

$$F_{n-1/2}^m = \frac{1}{2}B_{n-1/2}P'_{n-1/2}^m + (-A_{n-1/2} + \frac{1}{2}B'_{n-1/2})P_{n-1/2}^m \quad (7.43)$$

$$F_{n+1/2}^{m+1} = \frac{1}{2}B_{n+1/2}P'_{n+1/2}^{m+1} + (-A_{n+1/2} + \frac{1}{2}B'_{n+1/2})P_{n+1/2}^{m+1} \quad (7.44)$$

$$F_{n-1/2}^{m+1} = \frac{1}{2}B_{n-1/2}P'_{n-1/2}^{m+1} + (-A_{n-1/2} + \frac{1}{2}B'_{n-1/2})P_{n-1/2}^{m+1} \quad (7.45)$$

Similarly, when the *difference quotient* is applied, the derivative of $P_{n\pm 1/2}^{m(+1)}$ can be approximately calculated as:

$$P'_{n\pm 1/2}^m = \pm \frac{P_{n\pm 1}^m - P_n^m}{\Delta x_{n\pm 1/2}} \quad (7.46)$$

$$P'_{n\pm 1/2}^{m+1} = \pm \frac{P_{n\pm 1}^{m+1} - P_n^{m+1}}{\Delta x_{n\pm 1/2}} \quad (7.47)$$

$$B'_{n\pm 1/2} = \pm \frac{B_{n\pm 1} - B_n}{\Delta x_{n\pm 1/2}} \quad (7.48)$$

Using half-step methods from Eq. 7.26 to Eq. 7.36 and substituting Eq. 7.40 by the corresponding values defined in Eq. 7.42 to Eq. 7.48, we can reformulate Eq. 7.40 into

a two level, six-point Crank-Nicolson scheme of the form:

$$a_n P_{n-1}^{m+1} + b_n P_n^{m+1} + c_n P_{n+1}^{m+1} = \bar{a}_n P_{n-1}^m + \bar{b}_n P_n^m + \bar{c}_n P_{n+1}^m, \quad (7.49)$$

for $n = 0, 1, \dots, N$, where

$$\begin{aligned} a_n &= -\frac{1}{2} \frac{\Delta t_{m+1/2}}{\Delta x_n} \left[\frac{1/2 \cdot B_{n-1/2}}{\Delta x_{n-1/2}} - \frac{-A_{n-1/2} + 1/2 \cdot B'_{n-1/2}}{2} \right] \\ \bar{a}_n &= \frac{1}{2} \frac{\Delta t_{m+1/2}}{\Delta x_n} \left[\frac{1/2 \cdot B_{n-1/2}}{\Delta x_{n-1/2}} - \frac{-A_{n-1/2} + 1/2 \cdot B'_{n-1/2}}{2} \right] = -a_n \\ b_n &= 1 + \frac{1}{2} \frac{\Delta t_{m+1/2}}{\Delta x_n} \left[\frac{1/2 \cdot B_{n-1/2}}{\Delta x_{n-1/2}} + \frac{-A_{n-1/2} + 1/2 \cdot B'_{n-1/2}}{2} \right. \\ &\quad \left. + \frac{1/2 \cdot B_{n+1/2}}{\Delta x_{n+1/2}} + \frac{-A_{n+1/2} + 1/2 \cdot B'_{n+1/2}}{2} \right] \\ \bar{b}_n &= 1 - \frac{1}{2} \frac{\Delta t_{m+1/2}}{\Delta x_n} \left[\frac{1/2 \cdot B_{n-1/2}}{\Delta x_{n-1/2}} + \frac{-A_{n-1/2} + 1/2 \cdot B'_{n-1/2}}{2} \right. \\ &\quad \left. + \frac{1/2 \cdot B_{n+1/2}}{\Delta x_{n+1/2}} + \frac{-A_{n+1/2} + 1/2 \cdot B'_{n+1/2}}{2} \right] \\ c_n &= -\frac{1}{2} \frac{\Delta t_{m+1/2}}{\Delta x_n} \left[\frac{1/2 \cdot B_{n+1/2}}{\Delta x_{n+1/2}} + \frac{-A_{n+1/2} + 1/2 \cdot B'_{n+1/2}}{2} \right] \\ \bar{c}_n &= \frac{1}{2} \frac{\Delta t_{m+1/2}}{\Delta x_n} \left[\frac{1/2 \cdot B_{n+1/2}}{\Delta x_{n+1/2}} + \frac{-A_{n+1/2} + 1/2 \cdot B'_{n+1/2}}{2} \right] = -c_n \end{aligned}$$

One motivation of ordering Eq. 7.49 is that it tends to average the effect of the variable diffusivity over the entire region (cf. Douglas (1962b)). For simplicity, Eq. 7.49 can be also rewritten in a computationally efficient linear system of equations in a matrix form:

$$R \mathcal{P}^{m+1} = \mathcal{Q}^m \quad (7.50)$$

where the coefficient matrices R and \mathcal{Q} are defined as follows:

$$R = \begin{pmatrix} b_0 & c_0 & 0 & \cdots & \cdots & 0 \\ a_1 & b_1 & c_1 & 0 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & a_{n-1} & b_{n-1} & c_{n-1} \\ 0 & \cdots & \cdots & 0 & a_n & b_n \end{pmatrix}, \quad \mathcal{P}^{m+1} = \begin{pmatrix} P_0^{m+1} \\ P_1^{m+1} \\ \vdots \\ P_{n-1}^{m+1} \\ P_n^{m+1} \end{pmatrix}$$

$$\mathcal{Q}^m = \begin{pmatrix} \bar{b}_0 & \bar{c}_0 & 0 & \cdots & \cdots & 0 \\ \bar{a}_1 & \bar{b}_1 & \bar{c}_1 & 0 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \bar{a}_{n-1} & \bar{b}_{n-1} & \bar{c}_{n-1} \\ 0 & \cdots & \cdots & 0 & \bar{a}_n & \bar{b}_n \end{pmatrix} \times \begin{pmatrix} P_0^m \\ P_1^m \\ \vdots \\ P_{n-1}^m \\ P_n^m \end{pmatrix}$$

Note that R and the first element of \mathcal{Q} have the property that they are tri-diagonal, which means only the diagonal, super-diagonal and sub-diagonal elements are non-zero. The transition density $p(x, t)$ can be generated through time from a given initial condition by repeated solution of the linear system of equations in Eq. 7.50 using the *Cholesky Method*⁶. Furthermore, the Crank-Nicolson scheme is known to have local truncation error which is proportional to square of both space step size and time step size, $O(\Delta x^2) + O(\Delta t^2)$ (cf. Douglas (1961b)).

As a matter of fact, in one dimension, using the method shown in Eq. 7.49 is computationally very efficient. However, in two or more dimensions this is no longer true. Although this method is still stable without restriction on the time step, the extra

⁶Some call it *LU*-method. The *LU factorization* is usually applied for Cholesky decomposition of a matrix. It writes a matrix as the product of a lower triangular matrix and an upper triangular matrix. This decomposition is used in numerical analysis to solve systems of linear equations. For more details on solving the tri-diagonal system of the Crank-Nicolson Scheme, refer to Wang (2007) Appendix A. 3.

computational burden involved turns out to be very considerable: as the matrices of the system do not have tri-diagonal form anymore, solving this equation becomes very time consuming. Therefore, we shall look for other numerical schemes for parabolic equations in more than one dimension cases, which will be discussed in the later sections.

7.2.2 Estimation Results

The main objective of this study is to estimate the parameters of our opinion model in either one dimension or (later) two dimensions. Since this stochastic model measures the aggregate behavior of a number of respondents, in order to have a characterization of the outcome, it requires tracking the transient density $P(x; t)$ as in the Fokker-Planck equation 7.14, conditional on some initial value. Unfortunately, with highly non-linear drift and diffusion terms of the system, no closed-form solution to Eq. 7.14 is available, we therefore, rely on numerical approximation of the Fokker-Planck equation using the Crank-Nicolson FDS in empirical application.

The structures of our models follow the companion study by Lux (2009a), which examines the causation from sentiment indices on stock returns in financial markets. As a matter of fact, some recent literature has documented causality among sentiment and returns as well. For instance, Brown and Cliff (2004) report Granger-causality from returns on sentiment, but not vice versa; Kling and Gao (2008) find similar causal structure for Shanghai stock market; Lux (2010) shows causality from sentiment on returns of German stock market but not the other way round; our previous VAR analysis reveals a clear causation from industrial sentiment to the output, however, the sentiment from the consumers is relatively independent of the industrial output.

Apart from that, previous experience from Lux (2009a,b) and Ghonghadze and Lux (2009) indicates that it is convenient to consider different versions of the basic opinion dynamic models. The following are selected sets of forcing function U and parameter

vectors to be estimated:

- Agent-based Model of CCI:

$$\text{Model I: } U_C = \alpha_0 + \alpha_1 x_C + \alpha_2 x_I + \alpha_3 y; \quad \theta = (\nu_C, \alpha_0, \alpha_1, \alpha_2, \alpha_3, N_C)'$$

$$\text{Model II: } U_C = \alpha_0 + \alpha_1 x_C + \alpha_2 x_I; \quad \theta = (\nu_C, \alpha_0, \alpha_1, \alpha_2)'$$

$$\text{Model III: } U_C = \alpha_0 + \alpha_1 x_C; \quad \theta = (\nu_C, \alpha_0, \alpha_1)'$$

- Agent-based Model of ICI:

$$\text{Model I: } U_I = \beta_0 + \beta_1 x_I + \beta_2 x_C + \beta_3 y; \quad \theta = (\nu_I, \beta_0, \beta_1, \beta_2, \beta_3, N_I)'$$

$$\text{Model II: } U_I = \beta_1 x_I; \quad \theta = (\nu_I, \beta_1, N_I)'$$

$$\text{Model III: } U_I = \beta_1 x_I; \quad \theta = (\nu_I, \beta_1)'$$

Model I is considered as the baseline model of our estimation experiment as defined in Eq. 7.22. We use the U-functions in Eq. 7.22, which consider not only self-dependency (α_1, β_1), but also cross-dependency on the other indices (α_2, β_2), as well as additional possible influence from industrial output on confidence indices (α_3, β_3). The last two influences are added, since we have observed significant causal relations among our indices in the previous VAR analysis. In the next step, since some of the estimated parameters are not significant (or there are cases that the standard errors of estimates are not computed), we follow the standard procedure and leave out these parameters step by step to see how the overall result improves with less parameters.

[Table 7.1 about here]

The estimation results are shown in Table 7.1. All experiments are carried out with a width $\Delta x = 0.01$ in the space direction and $\Delta t = 1/6$ in the time direction. Panel A shows the estimation results for CCI indices. The baseline Model I includes not only

self-dependency α_1 , but also the the cross-dependency on ICI indices α_2 and IP indices α_3 . Estimation shows that the flexibility parameter ν_C is very low, which means the opinion switching occurs at a relatively low frequency. Despite the impression of a dominance of negative assessment over the whole sample period, the bias term α_0 points slightly to “positive”. The “herding intensity” parameter α_1 is larger than unity, indicating CCI indices are characterized by strong interaction in itself. According to the model property, which have been thoroughly investigated in the literature (cf. Weidlich and Haag (1983), Lux (2009a,b)), the finite difference approximation of transient density for such model will display a typical bimodal form. The parameter α_2 explains that the change of the ICI opinion will have a positive effect to the entire opinion formation process, in the mean time, the feedback effect from IP seems not significant. The estimated “effective” numbers of agents for CCI opinion indices turn out to be around $N_C \approx 63$, meaning that a model with $2 \times N$ independent agents would get closest to the dynamic structure of this dataset. In the next step, we fix the number of agents to $N_C = 63$, drop the insignificant parameter α_3 , and repeat the estimation in Model II. It is observed that the likelihood function falls down with the “herding intensity” parameter α_1 slightly increased, and α_2 becomes insignificant, indicating low causal relation from ICI in the current opinion formation process. Model III further discards the insignificant parameter α_2 , it is displayed that except for α_1 , all other parameters are barely affected. In general, the loglikelihood function favors the more complete Model I, while both AIC and BIC information criteria favor the more parsimonious Model III, which takes only the self-interaction of the index itself into account.

Panel B of Table 7.1 shows the estimation results for the ICI indices. When comparing Model I with the previous CCI estimation, we observe a modest rise in both ν_I and β_1 , which means ICI indices have a higher switching tendency (or flexibility) and stronger intensity of interaction. Like with the CCI indices, $\beta_1 > 1$ means the transient density of this model will display bimodality in its opinion formation process. The bias term β_0

as well as β_2 and β_3 are not very significant, indicating ICI is not easily affected by other variables. The estimated effective number of participants is $N_I \approx 138$, which turns to be significant and much higher than the CCI estimation. After eliminating all insignificant parameters in Model I, Model II provides much better estimation outcomes, and we have the loglikelihood function slightly increased (in absolute value), while all remaining parameters stay significant with only marginal adjustments. In Model III, we make the number of agents to be endogenous and fixed at $N_I = 138$. Simulation again confirms our conjecture that without the bias term, the estimation remains its accuracy, and the loglikelihood function is barely changed. In the end, in accordance with the CCI estimation, the loglikelihood function and the other two information criteria choose different models: the former favors the complete Model I, while the later prefer the more parsimonious one.

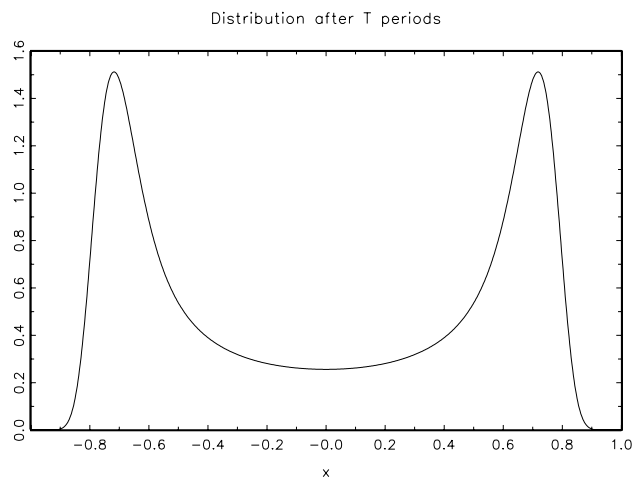


Figure 7.1: Finite difference approximation of the transient distribution for ICI index after T periods. In this exercise, $\nu_I = 0.02$, $\alpha_0 = 0$, $\alpha_1 = 1.4$, $N_I = 138$ and a relatively large T ($T = 200$) is applied. The initial condition x_0 has been approximated by a Normal distribution with small standard deviation and mean x_0 .

Figure 7.1 displays the stationary distribution of the transient density for ICI index. The simulation is done with the estimates: $\nu_I = 0.02$, $\alpha_0 = 0$, $\alpha_1 = 1.4$, $N_I = 138$, and $h = 0.0025$, $k = 0.01$ for the discretization in "space" and "time" direction. A

relative long T ($T = 200$) is applied for the time horizon of the numerical integration, and the space grid -1 to 1 is extended in accordance with the possible range of x defined in Eq. 7.3. For simplicity, we use the first-order Euler approximation using the known drift and diffusion functions for the initialization of the approximation, so that initial state $x_0 = 0$ has been approximated by a Normal distribution with density $\Phi^N(x_0 + A(x)k, B(x)k)$ evaluated at grid points $-1 + nh$; $n = 0, 1, \dots, N$ in the x direction for the first time increment k . The final distribution shows non-negligible probabilities along the whole range with two peaks at around -0.7 and 0.7 . As the exercise has excluded the insignificant bias term α_0 , we arrive at a symmetric bimodal distribution.

Following Lux (2009a,b), we estimate IP dynamics in a form of diffusion process:

$$dy_t = (\gamma_0 + \gamma_1 x_C + \gamma_2 x_I) dt + \sigma_y d\mathbb{W} \quad (7.51)$$

where \mathbb{W} is the *Wiener process*⁷. Note that IP indices are assumed to be related with both the consumer sentiment indices CCI and the industry sentiment indices ICI. When $\gamma_1 = \gamma_2 = 0$, this process becomes a standard Brownian motion with drift.

The estimated parameter set for the baseline diffusion model IP is $\theta = (\gamma_0, \gamma_1, \gamma_2, \sigma_y)'$, and the estimation results are reported in Panel C of Table 7.1. In the more complete Model I, we observe that except γ_1 , all parameter estimates are highly significant. This result is in harmony with the previous finding that CCI indices are relatively independent with the IP indices. In the next step, we drop out the insignificant parameter and repeat the estimation in Model II, so that we have the following parameter set: $\theta = (\gamma_0, \gamma_2, \sigma_y)'$. It shows that the rest of parameters are still significant and only slightly affected, in particular, a strong positive γ_2 indicates a possible causal relation between ICI and IP. This is again fully consistent with the results of our previous VAR analysis that the industrial sentiment should influence the output to some extent.

⁷Wiener process is also called *Brownian motion*, and it is a time-continuous process which is normally distributed, with mean $\mu_{\mathbb{W}} = 0$, and variance $\sigma_{\mathbb{W}}^2 = t$.

7.3 Model Estimation in Two Dimensions

7.3.1 Alternative Direction Implicit Scheme in 2-D

In this section, we proceed to estimate bivariate models for CCI & IP and ICI & IP. As we already learned, for the one dimensional problem, the Crank-Nicolson method is an unconditionally stable, implicit numerical scheme with second-order accuracy in both time and space. However, when applying it to two-dimensional problems, the traditional Crank-Nicolson method becomes quite costly as the resulting matrix in Eq. 7.50 is block tri-diagonal. To overcome this problem, the *Alternative Direction Implicit* (ADI) method can be used to solve the Fokker-Planck equation for diffusion equations in two or more dimensions.

The ADI scheme is a finite difference method designed for parabolic and elliptic partial difference equations, it was first suggested by companion papers, i.e., Peaceman and Rachford (1955), Douglas and Peaceman (1955), Douglas (1955), for solving the heat equation in two dimensions. Since then, the ADI method has been applied to various physical problems⁸, such as in Vries (1984), Chin et al. (1984), Jarzebsky and Thullie (1986), Abarbanel et al. (1986), among others. Later on, Douglas and his colleagues show how it relates to the Crank-Nicolson method and backward difference equations in Douglas (1961a, 1962a), Douglas and Gunn (1963). A general formulation for ADI methods for parabolic and hyperbolic problems can be found in Douglas and Gunn (1964), Varga (1962) Chap. 7, or Strikwerda (1989), Chap. 7.3.

The idea behind ADI method is to reduce the two-dimensional problem to two one-dimensional problems, then the original FDE is split into two, one with the first space dimension taken implicitly and the next with the second space dimension taken implicitly. By simplifying the two-dimensional problem, the systems of equations involved are

⁸Most physical problems leading to parabolic and elliptic differential equations give rise to more general equations than the one that we present in this discussion, which can be found in, e.g., Douglas (1962b), Douglas and Gunn (1964), among others.

again symmetric, tri-diagonal and cheap to be solved by the Choleski decomposition method. In other words, when applying the ADI method to the iterative solution of difference equations for bivariate problems, the equations that have to be solved in every iteration have a simpler structure and thus require much less arithmetic operations than other iterative implicit methods. Apart from computational efficiency, Douglas (1955) proves how the ADI method relates to Crank-Nicolson difference equation, and for some particular cases, the ADI process can be considered as a perturbation of Crank-Nicolson difference equation, having second-order accuracy both in space and time (Douglas and Kim (1999)).

The bivariate Fokker-Planck equation can be written similarly as the expression in Eq. 7.17 of one-dimensional system:

$$\frac{\partial P(x_1, x_2; t)}{\partial t} = \frac{-\partial F(x_1, x_2, t)}{\partial x_1} + \frac{-\partial F(x_1, x_2, t)}{\partial x_2} \quad (7.52)$$

where x_1 and x_2 are two space variables with the grids: $x_{1,n} = x_{1,0} + n \cdot h_1$, $n = 0, 1, \dots, N_{x_1}$ and $x_{2,l} = x_{2,0} + l \cdot h_2$, $l = 0, 1, \dots, N_{x_2}$. We evaluate first implicitly the x_1 derivative and then also implicitly the x_2 derivative with a tri-diagonal scheme:

$$\frac{P_n^{m+1/2} - P_n^m}{\Delta t_m} = \frac{F_{n+1/2}^{m+1/2} - F_{n-1/2}^{m+1/2}}{\Delta x_n}. \quad (7.53)$$

The flux of Eq. 7.52 becomes:

$$\begin{aligned} F_r(x_1, x_2, t) &= \sum_s B_{rs}(x_1, x_2, t) \frac{\partial P(x_1, x_2, t)}{\partial x_s} \\ &+ \left[A_r(x_1, x_2) + \sum_s \frac{\partial B_{rs}(x_1, x_2)}{\partial x_s} \right] P(x_1, x_2; t) \quad (7.54) \\ B_{rs} &= 0 \quad \text{for } r \neq s \end{aligned}$$

The off-diagonal terms of $B_{12}(\cdot)$ and $B_{21}(\cdot)$ are set to zero for simplicity, since it would be very difficult to define explicitly the dependency in innovations between opinions from industry and from the consumer, it is reasonable to treat them as coming from independent surveys. The other reason to maintain the assumption of independent

Markov process for the two data-generating opinion processes is computational convenience: as the strong off-diagonal influence will compromise the positivity of the solutions, the accuracy of simple ADI schemes can be only achieved in the absence of cross-derivatives. The following version of the ADI scheme is based on Peaceman and Rachford (1955), for 2-D cases without cross-dependency among the variables⁹.

Denoting the finite difference approximation of the transient density in 2-D by $P_{n,l}^m$, i.e., $P(x_1, x_2; t)$ conditional on $x_{1,n}$, $x_{2,l}$ and t , we first evaluate the derivative with respect to x_1 and x_2 implicitly using a time step $k/2$:

$$\frac{P_{n,l}^{m+1/2} - P_{n,l}^m}{k/2} = \frac{1}{h_1} (F_{1,n+1/2,l}^{m+1/2} - F_{1,n-1/2,l}^{m+1/2}) + \frac{1}{h_2} (F_{2,n,l+1/2}^m - F_{2,n,l-1/2}^m) \quad (7.55)$$

The function for the next half step can be written as:

$$\frac{P_{n,l}^{m+1} - P_{n,l}^{m+1/2}}{k/2} = \frac{1}{h_1} (F_{1,n+1/2,l}^{m+1} - F_{1,n-1/2,l}^{m+1}) + \frac{1}{h_2} (F_{2,n,l+1/2}^{m+1/2} - F_{2,n,l-1/2}^{m+1/2}) \quad (7.56)$$

Both half-steps of Eq. 7.55 and Eq. 7.56 will lead to a tri-diagonal system of equations similar to Eq. 7.50. In other words, if we subdivide the update procedure into two sub-steps, at each sub-step only one tri-diagonal matrix needs to be solved for one component, so that the 2-dimensional problem becomes no longer expensive. Boundary conditions can be imposed by setting $F_{1,-1/2,l}^m = F_{1,N_x+1/2,l}^m = 0$, for all m , l , and $F_{2,n,-1/2}^m = F_{2,n,N_y+1/2}^m = 0$, for all m , n .

Meanwhile, like the Crank-Nicolson scheme, the ADI scheme is unconditionally stable too. It is also second-order accurate in time and space (i.e., Δt , Δx_1 and Δx_2) as long as cross-derivatives are all equal to zero and there is no dependency between two space variables (cf. Mohanty and Jain (2001), among others).

7.3.2 Estimation Results

In two dimensional estimation exercises, our interest is to see how robust the parameter estimates are, under inclusion of a second simultaneous dynamic process. We will study

⁹Refer to Strikwerda (1989) Chap. 7, or Thomas (1995) Chap. 4 for detailed descriptions.

each pair of the variables separately, so that Panel A of Table 7.2 reports the bivariate model of CCI & IP, and Panel B records simultaneous dynamics for ICI & IP model. Below is a short review of model settings and parameter vectors to be estimated:

[Table 7.2 about here]

- Bivariate Model CCI & IP:

$$\begin{aligned} \text{Model I : } U_C &= \alpha_0 + \alpha_1 x_C + \alpha_2 y; & \mathbf{dy}_t &= (\gamma_0 + \gamma_1 x_C) \mathbf{dt} + \sigma_y \mathbf{dW}; \\ &\Rightarrow \theta &= (\nu_C, \alpha_0, \alpha_1, \alpha_2, N_C, \gamma_0, \gamma_1, \sigma_y)' \end{aligned}$$

$$\begin{aligned} \text{Model II : } U_C &= \alpha_0 + \alpha_1 x_C; & \mathbf{dy}_t &= (\gamma_0 + \gamma_1 x_C) \mathbf{dt} + \sigma_y \mathbf{dW}; \\ &\Rightarrow \theta &= (\nu_C, \alpha_0, \alpha_1, N_C, \gamma_0, \gamma_1, \sigma_y)' \end{aligned}$$

$$\begin{aligned} \text{Model III : } U_C &= \alpha_0 + \alpha_1 x_C; & \mathbf{dy}_t &= (\gamma_0 + \gamma_1 x_C) \mathbf{dt} + \sigma_y \mathbf{dW}; \\ &\Rightarrow \theta &= (\nu_C, \alpha_0, \alpha_1, \gamma_0, \gamma_1, \sigma_y)', \quad N_C = 64 \end{aligned}$$

- Bivariate Model ICI & IP:

$$\begin{aligned} \text{Model I : } U_I &= \beta_0 + \beta_1 x_I + \beta_2 y; & \mathbf{dy}_t &= (\gamma_0 + \gamma_2 x_I) \mathbf{dt} + \sigma_y \mathbf{dW}; \\ &\Rightarrow \theta &= (\nu_I, \beta_0, \beta_1, \beta_2, N_I, \gamma_0, \gamma_2, \sigma_y)' \end{aligned}$$

$$\begin{aligned} \text{Model II : } U_I &= \beta_0 + \beta_1 x_I + \beta_2 y; & \mathbf{dy}_t &= (\gamma_0 + \gamma_2 x_I) \mathbf{dt} + \sigma_y \mathbf{dW}; \\ &\Rightarrow \theta &= (\nu_I, \beta_0, \beta_1, \beta_2, \gamma_0, \gamma_2, \sigma_y)'; \quad N_I = 141 \end{aligned}$$

$$\begin{aligned} \text{Model III : } U_I &= \beta_1 x_I; & \mathbf{dy}_t &= (\gamma_0 + \gamma_2 x_I) \mathbf{dt} + \sigma_y \mathbf{dW}; \\ &\Rightarrow \theta &= (\nu_I, \beta_1, \gamma_0, \gamma_2, \sigma_y)', \quad N_I = 141 \end{aligned}$$

Table 7.2 records the estimation results for the two-dimensional CCI & IP models and ICI & IP models, respectively. Panel A considers the model for CCI indices together with the diffusion model for IP indices. Simulations are carried out with width $\Delta x = 0.01$ in the space domain and $\Delta t = 1/6$ in the time domain. For the baseline CCI & IP model, we estimate a model with unrestricted number of agents. It is observed

that when compared with the one-dimensional cases, we arrive at a little larger bias term α_0 and a little lower “herding” parameter α_1 . The cross dependency parameter α_2 is quite small and insignificant, indicating that the influence from IP on CCI is hardly detectable under current settings, and the effective number of agents is around $N_C \approx 69$. Meanwhile, the main estimates for the diffusion IP process γ_0 and γ_1 become much smaller than in the 1D case with σ slightly increased, and the insignificant parameter γ_1 only turns to be marginally significant, but remains very small. In Model II, discarding the insignificant parameter α_2 , we see that the bias term α_0 becomes smaller while the “herding” parameter α_1 gets a little higher, and the estimates in the diffusion part have a larger variation when compared with the previous model. The loglikelihood function is only slightly affected, with the estimated number of participants reduces to $N_C \approx 64$, close to the 1D case. Finally in Model III, we set $N_C = 64$, which will lower down the number of free parameters. The exercise confirms our Model II estimates with all parameters significant and even a little improved loglikelihood function. In overall, among the three models, the loglikelihood function chooses the complete model, while the AIC and BIC information criteria prefer the model in a reduced form.

Panel B of Table 7.2 shows the estimation results for bivariate ICI & IP models. For the baseline ICI & IP model, the standard errors are not computed since the covariances of the parameters fail to invert, which might be due to a multi-collinearity problem. In Model II, we set $N_I = 141$ according to the Model I estimation. As a result, all parameters vary a little bit, and the maximum likelihood gets immediately higher (in absolute value). The adaptation parameter β_1 of the ICI dynamics is much higher than unity, corresponding to a bimodal equilibrium distribution of the transient density. In the mean time, there is certain kind of interaction among ICI and IP detected, but not very significant. The estimated effective number of agent is very close to the 1D cases, and the estimates for the diffusion IP process are not far from their 1D counterparts as well. Further discarding the insignificant bias β_0 and β_2 and re-estimating the model in Model III leads to slight increase in the “herding intensity” β_1 as well as the

loglikelihood function (in absolute value), while the other parameters are less affected. Like in the previous bivariate models, the loglikelihood favors a more complete Model II, but both information criteria favor the more parsimonious Model III.

To sum up, in spite of a long interest in business climate and economic sentiment, the empirical research on testing behavioral models for survey data is surprisingly scarce. This chapter explores the explanatory power of a behavioral model of opinion formation, based on the canonical model by Weidlich and Haag (1983), and follows the methodology introduced in the pioneering paper Lux (2009a). The models are estimated using a numerical maximum likelihood approach. It has been shown by Lux (2009b) that the Monto Carlo simulation performs relative well at a medium sample size like ours. We use the sentiment indices for German industrial output from both consumer (CCI) and industry (ICI) collected by the European Commission, and study their dynamics together with industrial output (IP). It turns out that sentiment appears to have significant explanatory power for the ups and downs of the industrial output under investigation. Our aim is to contribute to a behavioral theory of sentiment formation, by providing an explicit model of how the opinion interaction effects (through surveys) may help to explain certain economic activity. The overall results in this chapter are also in accordance with the VAR studies in the previous chapter. Surprisingly, consumer sentiment seems to be relatively independent, it does not affect either industry sentiment nor industrial output. In contrast, industrial sentiment ICI exhibits strong bimodality which reflects high interactions among its respondents.

Regarding to future research, there are a few interesting ways to explore our behavioral model further. In particular, one might be interested in the predictability between VAR models and opinion formation models. Besides, extending the current model into three-dimensional format in order to check whether the 3D models outperform the 2D cases (or standard VAR models) is also a fascinating topic. A successful application can be found in Lux (2009a). Other extensions or modifications can be adopted in the forcing

function U , such as introducing a “momentum” effect, as discussed in Lux (2009b) and Ghonghadze and Lux (2009). In addition, this methodology can also be easily applied to similar datasets for other countries, or other interaction patterns. Some comparative projects have been carried out by other researchers of our group¹⁰.

¹⁰Available under http://www.bwl.uni-kiel.de/vwlinstitute/gwif/team_lux.php?lang=de.

Table 7.1: Parameter Estimates of Univariate Models

A: Social interaction model of CCI									
	ν_C	α_0	α_1	α_2	α_3	N_C	logL	AIC	BIC
Model I	0.017 (0.007)	0.014 (0.007)	1.026 (0.386.)	0.322 (0.166)	-1.232 (0.719)	62.684 (21.089)	-696.655	1405.310	1427.412
Model II	0.016 (0.005)	0.013 (0.006)	1.042 (0.409)	0.338 (0.184)	<u>63</u>	<u>63</u>	-697.661	1403.322	1418.083
Model III	0.016 (0.005)	0.013 (0.006)	1.051 (0.435)		<u>63</u>	<u>63</u>	-698.118	1402.236	1413.317
B: Social interaction model of ICI									
	ν_I	β_0	β_1	β_2	β_3	N_I	logL	AIC	BIC
Model I	0.021 (0.011)	0.093 (0.082)	1.479 (0.406)	-2.966 (2.173)	-3.822 (2.248)	138.220 (61.331)	-585.273	1182.546	1204.648
Model II	0.020 (0.002)		1.485 (0.382)			138.057 (59.747)	-587.454	1180.908	1191.989
Model III	0.020 (0.002)		1.496 (0.343)			<u>138</u>	-587.335	1178.670	1186.064
C: Diffusion model of IP									
	γ_0	γ_1	γ_2	σ			logL	AIC	BIC
Model I	-0.219 (0.062)	-0.039 (0.108)	0.441 (0.155)	0.003 (0.001)			-265.396	538.792	553.553
Model II	-0.215 (0.056)		0.432 (0.148)	0.003 (0.001)			-267.235	540.470	551.551

Note: Results in Panel A and Panel B have been estimated via numerical integration of the transient density with $\Delta x = 0.01$ and $\Delta t = 1/6$. For the diffusion model in Panel C, the exact solutions for the transient density are displayed. The underlined parameters are fixed prior to the estimations, in order to reduce the number of free parameters.

Table 7.2: Parameter Estimates of Bivariate Models

A: Social interaction model CCI+IP											
	ν_C	α_0	α_1	α_2	N_C	γ_0	γ_1	σ	logL	AIC	BIC
Model I	0.020 (0.007)	0.019 (0.009)	1.017 (0.453)	0.435 (0.377)	69.754 (25.496)	-0.138 (0.057)	0.027 (0.013)	0.004 (0.002)	-1105.324	2226.648	2256.062
Model II	0.019 (0.007)	0.017 (0.008)	1.019 (0.392)		64.351 (21.337)	-0.121 (0.058)	0.018 (0.009)	0.004 (0.002)	-1105.405	2224.810	2250.571
Model III	0.019 (0.007)	0.017 (0.008)	1.020 (0.395)		<u>64</u> (0.061)	-0.125 (0.061)	0.015 (0.007)	0.004 (0.002)	-1105.401	2222.802	2244.903

B: Social interaction model ICI+IP											
	ν_I	β_0	β_1	β_2	N_I	γ_0	γ_2	σ	logL	AIC	BIC
Model I	0.025 (.)	0.062 (.)	1.568 (.)	-3.305 (.)	141.023 (.)	-0.211 (.)	0.452 (.)	0.003 (.)	-1008.155	2032.310	2061.724
Model II	0.024 (0.004)	0.065 (0.058)	1.566 (0.411)	-3.308 (1.926)	<u>141</u>	-0.213 (0.077)	0.457 (0.189)	0.003 (0.001)	-1007.883	2229.766	2255.527
Model III	0.026 (0.005)		1.569 (0.415)		<u>141</u>	-0.216 (0.079)	-0.465 (0.201)	0.003 (0.001)	-1008.210	2026.420	2044.855

Note: The models are estimated via numerical integration of the transitional density using ADI algorithm. The discretization of the finite difference schemes uses steps of $\Delta x = 0.01$ and $\Delta t = 1/6$. The discretization of IP space dimension is chosen in a way to generate the same number of grid points as in the first dimension, which means $N_C = N_{IP}$ or $N_I = N_{IP}$. “(.)” shows the cases when standard errors are not generated because covariances of the parameter estimates fail to invert.

Chapter 8

Conclusion

Social relations are hard to model, since the relationship between individual behavior and macroscopic regularities can be quite involved. However, even if in principle everything can be described by equations, the complexity of a system may easily raise exponentially as the complexity of behavior increases. Sometimes, even several steps beyond existing simplified macroeconomic models, the job of solving a system with complex and stochastic individual behavior using systems of parallel equations becomes almost intractable. Thanks to the development of computer science, adopting fast-speed computer simulations, agent-based modeling becomes a flexible and powerful technique in dealing with some real-world problems.

Agent-based modeling has been proved already to be quite successful in exploring market and industry dynamics. In some ways, it complements traditional analytic methods. Agent-based modeling not only enables us to characterize the equilibria of certain economic system, but also permits the generalization of a system. Moreover, agent-based models are able to explain the emergence of higher order patterns based on intelligent “learning” individuals and scaling laws, which might result in more complex and robust outcomes.

The purpose of this thesis was to show that a certain kind of agent-based model can

generate a series of social phenomena, and many of them may be quite close to reality. Instead of emphasizing the microscopic aspect of “utility maximization”, we emphasize more that of a “statistical equilibrium”, a stage that economy will generally fluctuate around. Through agent-based simulations, some seemingly puzzling stylized facts in macroeconomics can be replicated without too much effort.

The first part of the thesis has introduced an agent-based model describing the social production process, which consists of employment, production and consumption. The biggest advantage of this model is that: it generates many macroscopic regularities from microscopic behavior within a single framework, involving intricate interactions among simple micro-economic entities. In particular, the model is capable of explaining stylized facts, such as:

- (1) Frictional unemployment population contains agents who have just been fired and start searching for the next job. In reality, the unemployment ratio varies from country to country. However, as frictional unemployment always exists, the government could treat the lowest unemployment rate in history as a reference baseline for a country’s target unemployment rate. In other words, the remaining exceeding rates from this lowest unemployment rate (i.e., frictional unemployment rate) can be treated as unemployment that occurred due to macroeconomic factors.
- (2) Many of the “size distribution” will display power laws in the upper tails. The “size” in our simulation ranges from firm size measured by total employees or sales, the life expectancies of firms, the duration of recessions, the wealth or income of firm owners, and so on.
- (3) Growth is usually measured by the *logarithm* on the size increment. We observe that the growth rate distribution is close to the Subbotin class distribution (i.e., a bell-shaped distribution such as Gaussian or a tent-shaped distribution such as Laplace). Typical examples include the firm growth distribution, the GDP

growth distribution, among others.

- (4) The 80/20 rule (or the “Pareto principle”) is ubiquitous in nature. As a matter of fact, the existence of not only the power law and Lognormal, but also the exponential law proves the 80/20 rule. Through simulations, we show that a majority of small firms are coexisting with a minority of large firms; a large amount of wealth is concentrated at the hands of a few very rich agents; most of the entries and exits happen with small firms; and etc.
- (5) “Statistical equilibrium” emphasizes the “stability” of a system. Agent-based modeling argues that fixed-point equilibria are unrealistic descriptions of macroscopic properties of economic systems. Interestingly, if we analyze the distribution of our key variate, the resulting distribution will often (but not always) be a Gaussian. We show that the number of workers/unemployees, the number of firm demises, the wage/profit share in GDP, satisfy this condition.
- (6) Since the Gaussian distribution is quite common in life, a mixture of Gaussian distributions may be also widespread. For example, the income distribution can be treated as an aggregate of income from different groups of agents, who may have a large income differential. If the income of each subgroup is Gaussian-like distributed, the overall income distribution can be understood as aggregation of multiple Gaussians which may overlap with each other and display either multimodal or unimodal shape.

The second part of the thesis looks into models of “sentiment dynamics”, both with a traditional quantitative approach and with an agent-based approach. Under an “efficient market” hypothesis, sentiment or mood should not affect economic activity. However, there is long lasting discussion that people’s “animal spirits” will influence business or investment fluctuations. We explore the German monthly survey data from consumer confidence index (CCI) and industrial confidence index (ICI) in order to study the possible causal relationship between the sentiment and subsequent industrial output

changes. Chap. 6 builds up a simultaneous system of a tri-variate vector autoregression (VAR). Depending on the chosen VAR specification, industrial output is found to follow a feedback process caused by at least one of the sentiment indices. The causality tests show that the industrial sentiment ICI has direct and significant influence on industrial output, whereas the consumer sentiment CCI is found to be relatively independent. The out-of-sample forecasting experiments based on the VAR models (incorporating ICI) indicate significant predictive power for both the near-term and long-term industrial output development.

The observed strong influence from sentiment on industrial output data motivates us to adopt an agent-based model of dynamic opinion or expectation formation processes with social interactions. We create a simple stochastic framework of opinion formation by a group of agents who face a binary decision following the pioneering work from Weidlich and Haag (1983) and Lux (1995, 1997, 1998). The model parameters are estimated using maximum likelihood and numerical solutions of the transient probability density function for the resulting stochastic process. We have also extended the univariate approach into two dimensions following the methodology introduced in Lux (2009b). During the simulation, we find some evidence of social interactions in both the consumer sentiment indices CCI and the industrial sentiment indices ICI. However, our results are generally in harmony with the previous VAR tests that CCI indices are relatively independent, and the ICI indices are closely related to the IP indices.

Appendix A

MATLAB Code for the SA Model

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Core code for the SA model (Replication of Wright(2005)) %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% by Lin Lin, July 2008

clear all;
close all;

% Parameter specification
N      = 1000;           % number of agents in the economy
M      = 100000;        % total capital/money stock by the agents
w_a    = 10;            % minimum wage
w_b    = 90;            % maximum wage
w_avg  = (w_a + w_b) / 2; % average wage
V      = 0;             % initial market value is 0
nYear  = 10;           % number of years

% Initialization
e = zeros(N,1);          % employer index
m = (M/N) * ones(N,1);  % equal money holdings
employeeSet(1:N,1:N-1) = zeros(N,N-1); % employee list
```

```

employeeNum(1:N,1)    = zeros(N,1);    % number of employees

cnum = [];            % array of capitalists amount
wnum = [];            % array of workers amount
unum = [];            % array of unemployed amount
emNumMonth = [];      % calculate the number of employee at the end of each month

for yidx = 1:nYear,
    for midx = 1:12,
        for repidx = 1:N,

            %%%%%%% Selection rule %%%%%%%

            a = randint(1,1,[1,N]);    % index of active actor

            %%%%%%% Hiring rule %%%%%%%

            if e(a) == 0 && employeeNum(a) == 0,    % check if actor a is unemployed

                % form the set of potential employers
                H = [];
                for i = 1:N,
                    if e(i) == 0 && i ~= a,
                        H = [H,i];
                    end
                end

                % select an employer, weights potential employers by their wealth
                rNum = rand;            % uniformly distributed random number
                nPem = length(H);       % amount of potential employers
                sPem = sum(m(H));       % total capital of potential employers
                thrs = 0;               % threshold for selection
                for i = 1:nPem,
                    thrs = thrs + m(H(i))/sPem; % select employer
                    if rNum < thrs,

```



```

        tidx = H(i);
        break;
    end
end

% check if it employs
if m(tidx) > w_avg,
    e(a) = tidx;
    employeeNum(tidx) = employeeNum(tidx) + 1;
    employeeSet(tidx,employeeNum(tidx)) = a;
end

end

%%%%%%%% Expenditure rule %%%%%%%%%

B = [1:a-1,a+1:N]';
b = B(randint(1,1,[1,N-1]));
expAmount = randint(1,1,[0,m(b)]);
V = V + expAmount;
m(b) = m(b) - expAmount;

%%%%%%%% Revenue rule %%%%%%%%%

if e(a) ~= 0 || employeeNum(a) ~= 0,

    rev = randint(1,1,[0,V]);
    V = V - rev;

    if e(a) ~= 0,
        m(e(a)) = m(e(a)) + rev;
    end
    if employeeNum(a) ~= 0,
        m(a) = m(a) + rev;
    end
end

```

```
end
```

```
%%%%%%%% Firing rule %%%%%%%%%
```

```
if employeeNum(a) ~= 0,  
    u = max(ceil(employeeNum(a) - m(a)/w_avg), 0);  
    for uu = 1:u,  
        uidx = randint(1,1,[1,employeeNum(a)]);  
        aidx = employeeSet(a,uidx);  
        e(aidx) = 0;  
        if uidx == employeeNum(a),  
            employeeSet(a,employeeNum(a)) = 0;  
        else  
            employeeSet(a,uidx:employeeNum(a)-1) =  
                employeeSet(a,uidx+1:employeeNum(a));  
            employeeSet(a,employeeNum(a)) = 0;  
        end  
        employeeNum(a) = employeeNum(a) - 1;  
    end  
end
```

```
%%%%%%%% Payment rule %%%%%%%%%
```

```
if employeeNum(a) ~= 0,  
    for i = 1:employeeNum(a),  
        numCoin = randint(1,1,[w_a,w_b]);  
        if m(a) >= numCoin,  
            m(employeeSet(a,i)) = m(employeeSet(a,i)) + numCoin;  
            m(a) = m(a) - numCoin;  
        else  
            numCoin = randint(1,1,[0,m(a)]);  
            m(employeeSet(a,i)) = m(employeeSet(a,i)) + numCoin;  
            m(a) = m(a) - numCoin;  
        end  
    end  
end
```

```

        end
    end
end

end

% Above is the main program code, you can run the program and plot figures.
% For example, uncheck the following to see the amount of each class
% cnum = [cnum,length(find(employeeNum > 0))];
% wnum = [wnum,length(find(e>0))];
% unum = [unum,N - cnum - wnum];

end
disp(['Year ',num2str(yidx),' finished.']); % display yearly calculation

end

```


Appendix B

MATLAB Code for the EPC Model

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% MatLab Program Code for the EPC Model %%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% by Lin Lin, Oct. 2008

clear all;
close all;

% Parameter specification
N      = 1000;           % number of agents in the economy
M      = 100000;        % total capital/money stock by the agents
w_a    = 10;            % minimum wage
w_b    = 90;            % maximum wage
w_avg  = (w_a + w_b) / 2; % average wage
V      = 100;           % initial market value
nYear  = 100;           % number of years

% initialization
e = zeros(N,1);          % employer index
m = (M/N) * ones(N,1);  % equal money holdings
employeeSet(1:N,1:N-1) = zeros(N,N-1); % employee list
```

```

employeeNum(1:N,1)      = zeros(N,1);    % number of employees

cnum = []; % array of capitalists amount
wnum = []; % array of workers amount
unum = []; % array of unemployers amount
enum = []; % array of employee amount -- firm size
money_M = []; % collect the money holding of each actor at the end of each month
money_Y = []; % collect the money holding of each actor at the end of each year
sales_Y = []; % array of total revenue collected by firms each year
coins_Y = zeros(1,nYear); % array of total money per year received by the workers
income_Y = zeros(N,nYear); % yearly income of each actor
income_Y_W = zeros(N,nYear); % yearly income of each worker
income_Y_C = zeros(N,nYear); % yearly income of each capitalist
wage_Y_C = zeros(N,nYear); % yearly wage paid by each capitalist
wage = []; % record the wage amount paid to all workers
profit_rate = []; % record the profit rate of all firms

for yidx = 1:nYear,

    sales_M = zeros(N,12); % record the monthly sales of firms
    coins_M = zeros(1,12); % total money per month received by the workers
    gdp_growth = zeros(1,1);

    for midx = 1:12,

        aa = randperm(N);
        for x = 1:N,
            a = aa(x);

            %%%%% Hiring rule %%%%%
            if e(a) == 0 && employeeNum(a) == 0, % check if actor a is unemployed

                if e(a) == 0 && employeeNum(a) == 0, % check if actor a is unemployed
                    H = []; % form the set of potential employers
                    for i = 1:N,

```

```

        if e(i) == 0 && i ~= a,
            H = [H,i];
        end
    end
end

% select an employer, weights potential employers by their wealth
rNum = rand;          % uniformly distributed random number
nPem = length(H);    % amount of potential employers
sPem = sum(m(H));     % total capital of potential employers
thrs = 0;            % threshold for selection
for i = 1:nPem,
    thrs = thrs + m(H(i))/sPem; % select employer
    if rNum < thrs,
        tidx = H(i); % the chosen employer is indexed as tidx
        break;
    end
end

% check if it employs
if m(tidx) > w_avg, % if m_tidx exceeds average wage, he hires a
    e(a) = tidx; % tidx hires employee a
    employeeNum(tidx) = employeeNum(tidx) + 1;
    employeeSet(tidx,employeeNum(tidx)) = a;
end
end

%%%%%% revenue rule %%%%%%

% the precondition is that a is not unemployed
if e(a) ~= 0 || employeeNum(a) ~= 0,

    rev = trirnd(0,V/2,V,1); % choose revenue from triangular distr.
    V = V - rev;           % the market value is reduced

    % if actor a is employee, then transfer m=rev coins to the employer

```

```

if e(a) ~= 0,
    m(e(a)) = m(e(a)) + rev;
    sales_M(e(a),midx) = sales_M(e(a),midx) + rev;
end

% if actor a is an employer, transfer m=rev coins to actor a
if employeeNum(a) ~= 0,
    m(a) = m(a) + rev;
    sales_M(a,midx) = sales_M(a,midx) + rev;
end
end

%%%%%%%% Firing rule %%%%%%%%%%

if employeeNum(a) ~= 0,
    u = max(employeeNum(a) - floor(m(a)/w_avg), 0);
    for uu = 1:u,
        uidx = randint(1,1,[1,employeeNum(a)]);
        aidx = employeeSet(a,uidx);
        e(aidx) = 0;
        if uidx == employeeNum(a),
            employeeSet(a,employeeNum(a)) = 0;
        else
            employeeSet(a,uidx:employeeNum(a)-1) = employeeSet(a,uidx+1:employeeNum(a));
            employeeSet(a,employeeNum(a)) = 0;
        end
        employeeNum(a) = employeeNum(a) - 1;
    end
end

%%%%%%%% Payment rule %%%%%%%%%%

if employeeNum(a) ~= 0,
    for i = 1:employeeNum(a),
        numCoin = trirnd(10,50,90,1);

```



```

        if m(a) >= numCoin,
            m(employeeSet(a,i)) = m(employeeSet(a,i)) + numCoin;
            m(a) = m(a) - numCoin;
        elseif 10 < m(a) < numCoin,
            numCoin = trirnd(10, (m(a)+10)/2, m(a), 1);
            m(employeeSet(a,i)) = m(employeeSet(a,i)) + numCoin;
            m(a) = m(a) - numCoin;
        elseif m(a) <= 10,
            numCoin = m(a);
            m(employeeSet(a,i)) = m(employeeSet(a,i)) + numCoin;
            m(a) = 0;
        end
    end
end

%%%%%%%% Expenditure rule %%%%%%%%%

expAmount = trirnd(0,m(a)/2,m(a),1); % select according to tri. distr.
V = V + expAmount; % add the available market value V
m(a) = m(a) - expAmount; % money holding of a is reduced by m
end

%%%%%%%% MONTHLY STATISTICS %%%%%%%%%
% monthly data can be recorded here, for example: amount of each class
cnum = [cnum,length(find(employeeNum > 0))];
wnum = [wnum,length(find(e>0))];
unum = [unum,N - cnum - wnum];
end

%%%%%%%% YEARLY STATISTICS %%%%%%%%%
% yearly data can be recorded here, for example: GDP = Yearly sales
sales_Y = [sales_Y,sum(sales_M,2)];
sumSales_M = [];
sumSales_M = [sumSales_M, sales_M];
sumSales_Y = sum(sales_Y); % GDP in a year

```

```

% calculate GDP growth by yearly the revenue growth
if yidx == 1,          % first year GDP growth is 1
    gdp_growth =1;
else
    gdp_growth = sumSales_Y(:,end)./sumSales_Y(:,end-1);
end

%let market value to grow at the rate of GDP growth rate
V = V * gdp_growth;

disp(['Year ',num2str(yidx),' finished.']);
save(['sim-result-N',int2str(N),'-M',int2str(M),'-T',int2str(nYear),'EPC']);
end

```

Appendix C

List of Abbreviations

ABM	Agent-Based Model
AIC	Akaike Information Criterion
ADF	Augmented Dickey-Fuller (test)
ADI	Alternative Direction Implicit (scheme)
AR	Autoregressive (method/model)
ARMA	Autoregressive Moving Average (model)
BIC	Bayesian Information Criterion
CCDF	Complementary Cumulative Distribution Function
CCI	Consumer Confidence Indicator
CDF	Cumulative Distribution Function
CLT	Central Limit Theorem
DM	Diebold-Mariano Test
EPC	Employment, Production and Consumption (model)
EPCE	Employment, Production and Consumption Model with Endogenous Wages (model)
FDE	Finite Difference Equation
FDS	Finite Difference Schemes
FP	Fokker-Planck (equation)
FRG	Federal Republic of Germany

GDP	Gross Domestic Products
HQ	Hannan-Quinn (information criterion)
ICI	Industry Confidence Indicator
IP	Industrial Production
LTV	Labor Theories of Value
LS	Least Squares
logL	Loglikelihood (result)
MAS	Multi-Agent Systems
ML	Maximum Likelihood
MLEs	Maximum Likelihood Estimates
MSE	Mean Squared Error
NAICS	North American Industry Classification Systems
NBER	National Bureau of Economic Research
NHANE	National Health and Nutrition Examination Survey
OECD	Organization for Economic Co-operation and Development
OLS	Ordinary Least Squares
OU	Ornstein-Uhlenbeck (process)
PHID	Pharmaceutical Industry Database
PDE	Partial Difference Equation
PDF	Probability Density Function
RMSE	Root Mean Squared Error
SBO	Survey of Business Owners
SA	Social Architecture (model)
VAR	Vector Autoregressive (method/model)
WDI	World Distribution of Income

Appendix D

List of Symbols

C	capitalist
E	employer
N	number of agents
O	firm owner
U	the unemployed or forcing function
V	the market value
W	worker or the total wage bill
\mathbb{W}	Wiener process
\mathbb{Z}	positive integer
\mathbf{A}_i	coefficient matrices of lag i
\mathbf{B}_q	coefficient matrices of lag q
X_w	the wage share
X_p	the profit share
X_t	the yearly GDP at time t
\mathbb{W}	the Brownian motion
$S(t)$	the state evolution rule at time t
e_i	the employer of actor i
m_i	the money holding of actor i

m_a	the minimum wage boundary
m_b	the maximum wage boundary
w_i	monthly wage bill of firm i
w_a	the wage expectation of actor a
r_i	monthly revenue of firm i
p_i	monthly profit rate of firm i
g_t	the growth rate of GDP at year t
n	socio-configuration
x	opinion index
u	white noise process
s	the firm size at time t
w	(expected/negotiated) wage or transition rates
ε	white noise process
ν	vector of intercept term or flexibility parameter
κ	speed of reversion
\sum_{ε}	covariance matrix of ε
θ	parameter set to be estimated
σ	standard deviation
$A(x)$	the drift term
$B(x)$	the diffusion term
$P(x)$	the unknown probability density
$F(x)$	the flux
Δx	the divided width in the space direction
Δt	the divided width in the time direction
\bar{x}	long-term equilibrium
R-sq	value of R-squared
R-sq(adj)	value of adjusted R-squared

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Affirmation

I hereby affirm that I have completed my doctoral thesis entitled “Agent-Based Models, Macroeconomic Scaling Laws and Sentiment Dynamics” entirely on my own and unassisted, and that I have specially marked all of the quotes I have used from other authors as well as those passages in my work that are extremely close to the thoughts presented by other authors, and listed the sources in accordance with the regulations I have been given.

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Publications

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