

# Essays on spatial econometrics: Hodges-Lehmann estimators and hospital efficiency

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Für meine Frau und unsere kleine Familie

## Vorwort

Die vorliegende Dissertation entstand während meiner Tätigkeit als wissenschaftlicher Mitarbeiter am Institut für Statistik und Ökonometrie sowie am Lehrstuhl für Gründungs- und Innovationsmanagement der Christian-Albrechts-Universität zu Kiel. Mein besonderer Dank gilt meinem Doktorvater Professor Dr. Helmut Herwartz, dessen erstklassige Betreuung entscheidend zum Gelingen dieser Arbeit beigetragen hat. Herrn Professor Dr. Carsten Schultz danke ich herzlich für die Übernahme des Zweitgutachtens.

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Christoph Strumann

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# Chapter 1

## Introduction

Spatial econometrics deals with spatial interaction and spatial structure in regression models. Spatial econometric techniques have been increasingly applied in almost all fields of economics. Anselin (2001) points out two main factors for the growing popularity of spatial econometrics. On the one hand, there is an increasing interest in modeling explicitly the interaction of economic agents with other heterogenous agents. On the other hand, more and more spatial data are available and need to be handled by appropriate techniques.

This dissertation consists of three articles considering spatial econometrics. The first article in Chapter 2 proposes an alternative estimation procedure of spatial error panel regression models, while the other articles in Chapter 3 and 4 apply spatial econometrics for the analysis of hospital efficiency in Germany. To be more precisely, the focus of Chapter 3 is to estimate spatial spillovers of hospital efficiency. In Chapter 4, spatial econometric techniques are applied in order to handle appropriately the spatial dependence in the data as detected in Chapter 3.

In the following a more detailed description of the articles of this cumulative dissertation is given. Moreover, the author's contribution of the articles in Chapter 3 and 4 is described.

### **Chapter 2 - Implementation and performance of Hodges-Lehmann estimators in static panel models with spatially correlated disturbances**

To estimate spatial regression models Maximum Likelihood (ML) estimation is typically applied, although ML spatial parameter estimates are characterized by a substantial downward bias (e.g. Mizruchi and Neuman, 2008, Neuman and Mizruchi, 2010, Farber et al., 2009, Smith, 2009). In Chapter 2, Hodges-Lehmann (HL) type point (Hodges and Lehmann, 1983) and interval estimators for the spatial parameter are proposed. The estimators are based on the inversion of common spatial correlation tests. While the actual coverage of ML interval estimates might violate the nominal target due to the estimation bias, the application of Monte Carlo testing offers exact HL confidence intervals in finite samples under any spatial structure. To identify most appropriate test statistics for the HL procedure size and power features of a variety



of diagnostics for spatial correlation are investigated by means of a simulation study in a first step. Secondly, the empirical performance of HL point and interval estimators are compared with their ML counterparts. Simulation results show that the bias of the HL estimator is markedly smaller than its ML counterpart. In addition, HL confidence intervals are characterized by less size distortions and appear more robust against spatial connectivity in comparison with ML interval estimates.

### **Chapter 3 - On the effect of prospective payment on local hospital competition in Germany**

In recent years, German hospitals have experienced a dramatic change in their incentive structure due to the introduction of prospective reimbursement based on diagnosis related groups (DRG) in 2004. One of the main goals of this financial reform has been to reduce the steady increase of hospital expenditures by encouraging hospitals to raise their efficiency. In contrast to the former per diem payments, under prospective payment it is profitable for hospitals to treat their patients efficiently. As a byproduct hospitals face the incentive to preferably treat cases with high reimbursement rates and a low level of complexity (Böcking et al., 2005) potentially leading to an increased competition for this so called low cost patients. In the study of Chapter 3, the competition between hospitals, quantified as spatial spillover estimates of hospital efficiency, is analyzed for periods before and after the reform. A two-stage efficiency model that allows for spatial interdependence among hospitals is implemented. Hospital efficiency is determined by means of non-parametric and parametric econometric frontier models. A significant increase of negative spatial spillovers of hospital performance is diagnosed, and thus, confirm the expected rise of competition.

While the idea of this study and the framework of the empirical model have been developed jointly with my co-author Helmut Herwartz, I was writing the text of the manuscript with editing help of Helmut Herwartz and carrying out the implementation of the preparation and descriptive analysis of the data, and of the estimation of the econometric model.

The article has been published in *Health Care Management Science*, 2012, Vol. 15, No. 1, pp. 48-62.

### **Chapter 4 - Hospital efficiency under prospective reimbursement schemes: An empirical assessment for the case of Germany**

In Chapter 4, the effect of the DRG reform on overall hospital efficiency is addressed by two complementary testing approaches. On the one hand, a two-stage procedure based on non-parametric efficiency measurement is applied. On the other hand, a stochastic frontier model is employed that allows a one-step estimation of both production frontier parameters and inefficiency effects. Efficiency gains are identified as a consequence of changes in the hospital incentive structure because technological

progress, spatial dependence and hospital heterogeneity are taken into account. Contrary to the goal of prospective payments, the results of both approaches do not reveal any increase in overall efficiency after the DRG reform. Instead, a significant decline in overall hospital efficiency over time is observed.

Similar to the article of the previous chapter, together with my co-author Helmut Herwartz we have developed jointly the idea of this study and the framework of the empirical model. I have written the text of the manuscript with editing help of Helmut Herwartz. Moreover, the implementation of the data analysis and the estimation of the econometric model have been done by myself.

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## References

- Anselin, L. (2001). Spatial econometrics. In Baltagi B.: *A companion to theoretical econometrics*. (Oxford: Blackwell), 310-330.
- Böcking, W., U. Ahrens, W. Kirch and M. Milakovic (2005). First results of the introduction of DRGs in Germany and overview of experience from other DRG countries. *Journal of Public Health* 13(3), 128-137.
- Farber, S., A. Páez and E. Volz (2009). Topology, dependency tests and estimation bias in network autoregressive models. In Páez, A., J. Le Gallo, R. Buliung and S. Dall'Erba: *Advances in Spatial Sciences* (Berlin: Springer Verlag), 29-57.
- Hodges Jr., J.L., and E.L. Lehmann (1983). Hodges-Lehmann estimators. In Johnson, N.L., S. Kotz and C. Read: *Encyclopedia of Statistical Sciences*, vol. 3. (Wiley, New York), 642-645.
- Mizruchi, M.S. and E.J. Neuman (2008). The effect of density on the level of bias in the network autocorrelation model. *Social Networks* 30, 190-200.
- Neuman, E.J. and M.S. Mizruchi (2010). Structure and bias in the network autocorrelation model. *Social Networks* 32(4), 290-300.
- Smith, T.E. (2009). Estimation bias in spatial models with strongly connected weight matrices. *Geographical Analysis* 41(3), 307-332.

## Chapter 2

# Implementation and performance of Hodges-Lehmann estimators in static panel models with spatially correlated disturbances

Christoph Strumann<sup>1</sup>

### Abstract

Several studies point out a substantial downward bias of the Maximum Likelihood (ML) estimator of the spatial correlation parameter under strongly connected spatial structures. This paper proposes Hodges-Lehmann (HL) type point and interval estimators for the spatial parameter in static panel models with spatially autoregressive or moving average disturbances. HL estimators are implemented by means of 'inverting' common diagnostics for spatial correlation. Exact inference is implemented by means of Monte Carlo testing. A simulation study covering models with distinct degrees of spatial connectivity shows that the bias of the HL estimator is markedly smaller than its ML counterpart. In addition, HL confidence intervals are characterized by less size distortions and appear more robust against spatial connectivity in comparison with ML interval estimates.

JEL-Classification: C12, C15, C21, C23

Keywords: Panel data, spatial correlation, specification tests, Monte Carlo test, exact confidence sets, Hodges-Lehmann estimators

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## 2.1 Introduction

Spatial regression models are typically estimated by means of Maximum Likelihood (ML) techniques. However, several studies point out a substantial downward bias of ML estimates of the spatial correlation parameter, which increases with the degree of spatial connectivity (e.g. Mizruchi and Neuman, 2008, Neuman and Mizruchi, 2010, Farber et al., 2009, Smith, 2009). Lee (2004) highlights a potential irregularity of the information matrix under strongly connected spatial structures, which affects the convergence rate of the ML estimator. Bao and Ullah (2007) demonstrate that the bias of the ML estimator is sensitive to the structure of the spatial weights matrix. They suggest for cross sectional spatial lag models a bias corrected estimator, which is effective under weakly connected spatial structures.

As opposed to the spatial lag model, a biased spatial parameter estimate has no impact on the bias of regression coefficient estimates in the spatial error model. However, if the spatial parameter is of particular interest, the respective bias matters. Recent applications of the spatial error model include the analysis of economic convergence (Fingleton and López-Bazo, 2006, Lundberg, 2006, López-Bazo et al., 2004, Villaverde, 2005, Rey and Montouri, 1999), house prices (Baumont, 2009), technology adaptation (Billon et al., 2009), local governments expenditure (Bivand and Szymanski, 2000), and tax mimicking (Bordignon et al., 2003, Revelli, 2002). In these models a downward biased estimator leads to a systematic understatement of spatial responses to local shocks and, thus, weakens the empirical underpinning of economic policy advice.

This paper proposes Hodges-Lehmann (HL) type point (Hodges and Lehmann, 1983) and interval estimators for the spatial autocorrelation or moving average parameter in static panel models with spatially autoregressive (SAR) or moving average (SMA) disturbances. HL estimators are implemented by means of 'inverting' common diagnostics for spatial correlation. The particular value of the spatial parameter that maximizes the test  $p$ -value is interpreted as a point estimate (Coudin and Dufour, 2011). Confidence sets comprise all admissible values of the spatial parameter that do not involve a rejection of the null hypothesis (Dufour, 1990). The inversion of spatial correlation tests may follow three alternative strategies, first order asymptotic approximations, bootstrap approaches and Monte Carlo (MC) techniques (Dufour, 2006). In contrast to asymptotically valid bootstrap techniques (Lin et al., 2011) MC testing can offer exact size control in finite samples under any spatial structure.

As derived from the inversion of spatial diagnostics the performance of HL estimators is most likely to reflect efficiency characteristics of the underlying spatial correlation tests. To identify most appropriate test statistics for the HL procedure size and power features of a variety of diagnostics for spatial correlation are investigated by means of a simulation study in a first step. Secondly, the empirical performance of HL point and interval estimators are compared with their ML counterparts. Noting

that the actual coverage of ML interval estimates might violate the nominal target due to the estimation bias, the potential merits of HL interval estimates are highlighted for small sample scenarios with regular and irregular, correct and misspecified spatial weights matrices. Moreover, a potential finite sample bias reduction of the HL estimator is addressed.

As potential candidates for the construction of HL point estimates and confidence intervals (CIs) three sorts of rival diagnostics are considered, differing with respect to the exploitation of modeling specific assumptions on the connectivity structure. In the first place, Moran's I statistic ( $MI$ ) (Cliff and Ord, 1981) and the Lagrange Multiplier statistic for spatial error autocorrelation ( $LME$ ) (Burrige, 1980) are considered. Several simulation experiments recommend  $MI$  as a powerful test against spatial error correlation (e.g. Florax and Rey, 1995). However, as simulation results in Anselin and Rey (1991) suggest, the power of both tests declines with increasing spatial connectivity. Moreover, Smith (2009) shows that for strongly connected spatial weights matrices realizations of  $MI$  are concentrated close to the mean and, thus, the test could suffer from power weakness. Generally, the application of  $MI$ ,  $LME$  and various other spatial diagnostics requires an a-priori guess about the spatial pattern that underlies the data. Any choice of the so called spatial weights matrix is subjected to the risk of misspecification. To raise the robustness of the HL estimation against the specification of the spatial layout, statistics that do not rely on a spatial weights matrix are also considered in the second place, i.e. the Lagrange Multiplier (LM) statistic proposed by Breusch and Pagan (1980), the  $CD$  test of Pesaran (2004) and two non-parametric tests of Friedman (1937) and Frees (1995). For power improvement Pesaran (2004) suggests a modification of the  $CD$  statistic capturing spatial features of the regression error terms. A similar modification is also achievable for the LM statistic of Breusch and Pagan (1980). In the third place, both modified statistics are also included in the list of potential candidates for HL estimation. While the latter statistics exploit a-priori information about the spatial contiguity structure, the former diagnostics are fully invariant to the spatial weights matrix. On the one hand, one may expect more robust performance of these statistics against potential misspecification of the spatial structure. On the other hand, it is of interest to uncover potential power losses in comparison with more parametric diagnostics,  $MI$  and  $LME$  say.

In Section 2.2 the spatial panel model under the SAR and SMA error distribution is introduced along with the considered tests for spatial correlation. Section 2.3 outlines point and interval estimation of the spatial parameter. Furthermore, the MC test procedure is described. Section 2.4 documents design and outcomes of the simulation study. Section 2.5 concludes.

## 2.2 Model representation and diagnostics

In this section, the spatial panel model with SAR and SMA disturbances is introduced. To concentrate on the relative merits of HL and ML estimation the entire analysis in this paper is focussed on the pooled regression as a rather stylized model in panel data econometrics. However, if the time dimension is sufficiently large to guarantee a consistent estimation of the model parameters, the proposed HL procedure can also be applied to panel models with a more complex structure, e.g. heterogenous intercepts and coefficients. Moreover, test statistics that are alternatively considered for the construction of HL estimators are listed in this section.

### 2.2.1 The spatial panel model

The considered pooled linear regression model reads as

$$y_t = \mathbf{X}_t \beta + e_t, \quad t = 1, \dots, T, \quad (2.1)$$

where  $y_t$  is an  $N \times 1$  vector of observations of the dependent variable in time  $t$ ,  $\mathbf{X}_t$  is an  $N \times K$  matrix of observations of fixed (or exogenous) explanatory variables including a constant,  $\beta$  is a  $K \times 1$  vector of parameters and  $e_t$  is an  $N \times 1$  vector of spatially correlated error terms. The cross sectional and time dimension are denoted by  $N$  and  $T$ , respectively. Two specifications of  $e_t$  are distinguished, the SAR and SMA model, respectively

$$e_t = \rho \mathbf{W} e_t + \epsilon_t \text{ and } e_t = \gamma \mathbf{W} \epsilon_t + \epsilon_t, \quad \epsilon_t = \sigma \xi_t, \quad \mathbb{E}[\xi_t] = 0, \quad \mathbb{E}[\xi_t \xi_t'] = \mathbf{I}_N, \quad \xi_t \stackrel{iid}{\sim} D(0, \mathbf{I}_N),$$

where  $D$  is a known multivariate distribution with zero mean and covariance  $\mathbf{I}_N$ , the identity matrix. The spatial weights matrix  $\mathbf{W}$  is of dimension  $N \times N$  with zero diagonal elements and row normalized constants (such that each row sums to unity),  $\rho$  and  $\gamma$  are the spatial autocorrelation and spatial moving average parameter, respectively. To establish asymptotic normality of the spatial diagnostics introduced below, the following conditions are assumed to hold throughout (Kelejian and Prucha, 2001): (i) the fourth order moments of the idiosyncratic innovations  $\epsilon_{it}$  are finite, (ii) the row and column sums of  $\mathbf{W}$  are uniformly bounded in absolute value as  $N \rightarrow \infty$  and (iii) the matrices  $(\mathbf{I}_N - \rho \mathbf{W})$  (SAR) and  $(\mathbf{I}_N + \gamma \mathbf{W})$  (SMA) are nonsingular for all  $|\rho| < 1$  and  $|\gamma| < 1$ , respectively. Assumption (ii) is satisfied if for a given cross sectional unit the maximum number of neighbors is restricted, or the spatial weights decline as a function of some measure of distance between neighbors (Kelejian and Prucha, 1998). Owing to (iii) the error term of the SAR model can be expressed as

$$e_t = (\mathbf{I}_N - \rho \mathbf{W})^{-1} \epsilon_t, \quad \text{with}$$

$$E[e_t e_t'] = \sigma^2 [\mathbf{I}_N - \rho(\mathbf{W} + \mathbf{W}') + \rho^2 \mathbf{W}' \mathbf{W}]^{-1} = \boldsymbol{\Omega}_{SAR}. \quad (2.2)$$

For  $\rho \neq 0$ ,  $\boldsymbol{\Omega}_{SAR}$  is a non-sparse matrix, implying that a shock in one location is transmitted globally, i.e. to all locations. For the SMA specification one has

$$E[e_t e_t'] = \sigma^2 [\mathbf{I}_N + \gamma(\mathbf{W} + \mathbf{W}') + \gamma^2 \mathbf{W}' \mathbf{W}] = \boldsymbol{\Omega}_{SMA}. \quad (2.3)$$

If  $\mathbf{W}$  is a first order contiguity matrix say,  $\boldsymbol{\Omega}_{SMA}$  formalizes local linkages between first and second order neighbors. With ‘ $\otimes$ ’ denoting the Kronecker product, the pooled SAR and SMA panel regression model can be written as

$$y = \mathbf{X}\beta + e, \quad e = \mathbf{G}_\bullet \epsilon, \quad (2.4)$$

where

$$\mathbf{G}_\rho = [\mathbf{I}_T \otimes (\mathbf{I}_N - \rho \mathbf{W})^{-1}] \quad \text{and} \quad \mathbf{G}_\gamma = [\mathbf{I}_T \otimes (\mathbf{I}_N + \gamma \mathbf{W})], \quad (2.5)$$

respectively. Specifically,  $y = (y_1', \dots, y_T')'$ ,  $e = (e_1', \dots, e_T')'$  and  $\epsilon = (\epsilon_1', \dots, \epsilon_T')'$  are  $(TN \times 1)$  vectors, and  $\mathbf{X} = (\mathbf{X}_1', \dots, \mathbf{X}_T')'$  is a  $(TN \times K)$  matrix independent of  $\epsilon$  with  $\text{rank}(\mathbf{X}) = K$ . In the following  $\theta$  denotes either  $\rho$  (SAR) or  $\gamma$  (SMA) for convenience of notation.

## 2.2.2 Test statistics

Several alternative diagnostics are employed to test the hypotheses

$$H_0 : \theta = \theta_0 \quad \text{vs.} \quad H_1 : \theta \neq \theta_0. \quad (2.6)$$

In a first step the panel model is respecified as  $y = \mathbf{X}\beta + \mathbf{G}_\bullet \epsilon$  and the estimated residuals are obtained as

$$\hat{\epsilon}_0 = \mathbf{M}_0 y_0^* = \mathbf{M}_0 \epsilon, \quad \text{with} \quad \mathbf{M}_0 = \mathbf{I}_{NT} - \mathbf{X}_0^* (\mathbf{X}_0^{*'} \mathbf{X}_0^*)^{-1} \mathbf{X}_0^{*'}, \quad (2.7)$$

where  $\mathbf{X}_0^* = \mathbf{G}_\bullet^{-1} \mathbf{X}$  and  $y_0^* = \mathbf{G}_\bullet^{-1} y$  are spatially filtered according to  $H_0$ . The test statistics differ with respect to the exploitation of modeling specific assumptions on the connectivity structure implied by  $\mathbf{W}$ .

### Tests for spatial correlation

To date, the most commonly used diagnostic for spatial dependence is the *MI* statistic (Cliff and Ord, 1981). It is given along with its asymptotic distribution by

$$MI = \frac{I - \mu_I}{\sigma_I} \xrightarrow{d} N(0, 1), \quad (2.8)$$

where

$$I = \frac{\hat{\epsilon}'_0(\mathbf{I}_T \otimes \mathbf{W})\hat{\epsilon}_0}{\hat{\epsilon}'_0\hat{\epsilon}_0}, \quad \mu_I = \frac{\text{tr}(\mathcal{M})}{(NT - K)}, \quad \sigma_I = \sqrt{\frac{\text{tr}(\mathcal{M}\mathcal{M}') + \text{tr}(\mathcal{M}^2) + \text{tr}(\mathcal{M})^2}{(TN - K)(TN - K + 2)} - \mu_I^2}$$

and  $\mathcal{M} = \mathbf{M}_0(\mathbf{I}_T \otimes \mathbf{W})$ .

As an alternative to  $MI$  the LM test for spatial error correlation (Burrige, 1980) is considered. Extended to the panel regression model (e.g. Anselin et al., 2008) the test reads as

$$LME = \frac{[\hat{\epsilon}'_0(\mathbf{I}_T \otimes \mathbf{W})\hat{\epsilon}_0 / (\hat{\epsilon}'_0\hat{\epsilon}_0 / NT)]^2}{T \text{tr}(\mathbf{W}^2 + \mathbf{W}'\mathbf{W})} \xrightarrow{d} \chi^2(1). \quad (2.9)$$

### Tests for general contemporaneous correlation

In contrast to  $MI$  and  $LME$  the following tests do not rely on a spatial weights specification. Instead, the statistics summarize general correlation patterns of the residuals in (2.7) by means of the parametric Pearson correlation coefficient

$$\hat{\rho}_{0,ij} = \frac{\sum_{t=1}^T \hat{\epsilon}_{0,it} \hat{\epsilon}_{0,jt}}{\sqrt{\sum_{t=1}^T \hat{\epsilon}_{0,it}^2 \sum_{t=1}^T \hat{\epsilon}_{0,jt}^2}}$$

or the non-parametric Spearman's rank correlation coefficient

$$\hat{\tau}_{0,ij} = \frac{\sum_{t=1}^T (\mathcal{R}_{it} - (T + 1/2))(\mathcal{R}_{jt} - (T + 1/2))}{\sum_{t=1}^T (\mathcal{R}_{it} - (T + 1/2))^2},$$

where  $\mathcal{R}_{it}$  is the rank of the estimated filtered residual  $\hat{\epsilon}_{0,it}$ .

The LM statistic of Breusch and Pagan (1980) is based on the average of squared pair-wise Pearson correlation coefficients

$$BP = T \sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{\rho}_{0,ij}^2 \xrightarrow{d} \chi_{N(N-1)/2}^2. \quad (2.10)$$

For small  $T$  and large  $N$  the test exhibits substantial size distortions (Pesaran, 2004).

As an alternative with reasonable small sample properties Pesaran (2004) proposes a statistic that exploits the average of non-squared pair-wise Pearson correlation co-



efficients<sup>2</sup>

$$CD = \sqrt{\frac{2T}{N(N-1)}} \left( \sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{\varrho}_{0,ij} \right) \xrightarrow{d} N(0, 1). \quad (2.11)$$

The Friedman's (1937) test of cross sectional dependence measures the correlation pattern of estimated residuals by means of the non-parametric Spearman's rank correlation and is given by

$$FT_n = (T-1)((N-1)R_{AVE} + 1) \xrightarrow{d} \chi_{T-1}^2, \quad (2.12)$$

where

$$R_{AVE} = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{\tau}_{0,ij}.$$

The test statistic of Frees (1995) is a quadratic version of the Friedman's (1937) test and reads as

$$FT_q = \frac{N(R_{AVE}^2 - (T-1)^{-1})}{\sigma_F} \xrightarrow{d} N(0, 1), \quad (2.13)$$

where

$$R_{AVE}^2 = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^N \hat{\tau}_{0,ij}^2$$

and

$$\sigma_F = \sqrt{\frac{32}{25} \frac{(T+2)^2}{(T-1)^3(T+1)^2} + \frac{4}{5} \frac{(5T+6)^2(T-3)}{T(T-1)^2(T+1)^2}}.$$

### Tests for local correlation

Unlike in the case of *MI* and *LME*, the power of the tests for general contemporaneous correlation does not increase with  $N$ . For power improvement Pesaran (2004) suggests to exploit the correlation only among contiguous residual processes. The modified *CD* test is

$$CD^W = \sqrt{\frac{2T}{p}} \left( \sum_{i=1}^{N-1} \sum_{j=i+1}^N \mathcal{I}(w_{ij} > 0) \hat{\varrho}_{0,ij} \right) \xrightarrow{d} N(0, 1), \quad (2.14)$$

where  $p$  is the number of non-zero off diagonal elements of  $\mathbf{W}$  and  $\mathcal{I}(\cdot)$  is an indicator function.

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<sup>2</sup>Pesaran et al. (2008) suggest corrections for the mean and variance of the *BP* test to address the size distortions for fixed  $T$  and large  $N$ . In this paper MC testing offers full control over type I error probabilities. Moreover, simulation results of Pesaran et al. (2008) indicate the *BP* statistic as a more powerful diagnostic than their bias-adjusted *BP* test. Therefore, the latter statistic is not considered in this paper.

Similarly to the  $CD^W$  test the  $BP$  statistic can be transformed to test for local correlation as

$$BP^W = T \sum_{i=1}^{N-1} \sum_{j=i+1}^N \mathcal{I}(w_{ij} > 0) \hat{\varrho}_{0,ij}^2 \xrightarrow{d} \chi_{p/2}^2. \quad (2.15)$$

If  $\mathbf{W}$  is a non-sparse matrix,  $CD^W = CD$  and  $BP^W = BP$ .<sup>3</sup>

## 2.3 Test based estimation

In this section HL interval and point estimation are briefly described. To assess the marginal significance of a particular test statistic an analyst can rely on first order asymptotic approximations or MC techniques which are briefly sketched.

### 2.3.1 Interval estimation

To construct confidence sets for  $\theta$ , one proceeds from testing (2.6) for all admissible values  $\theta_0 \in (-1, 1)$ . Under  $H_0 : \theta = \theta_0$  the estimated residuals in (2.7) are spatially uncorrelated. In the alternative case,  $H_1 : \theta \neq \theta_0$ , some spatial correlation remains and should be detected by a correlation test. Given a realization  $\hat{\mathcal{T}}(\theta_0)$  of any statistic  $\mathcal{T}(\theta_0) \in \{MI, LME, BP, CD, FT_n, FT_q, CD^W, BP^W\}$   $H_0$  is rejected if the respective  $p$ -value,  $p(\hat{\mathcal{T}}(\theta_0))$ , is smaller than the nominal level  $\alpha$ . A confidence interval for  $\theta$  with coverage probability  $1 - \alpha$  comprises all admissible values of  $\theta_0$  that do not obtain a rejection of  $H_0$  (Coudin and Dufour, 2011), i.e.

$$CI_{\mathcal{T}}(\alpha) = [CI_{\mathcal{T}}^l(\alpha), CI_{\mathcal{T}}^u(\alpha)] = \{\theta_0 \in S : p(\hat{\mathcal{T}}(\theta_0)) \geq \alpha\}, \quad (2.16)$$

with  $S = \{\theta : |\theta| < 1\}$ . In (2.16) the lower  $CI_{\mathcal{T}}^l(\alpha)$  and upper bound  $CI_{\mathcal{T}}^u(\alpha)$  are, respectively,

$$CI_{\mathcal{T}}^l(\alpha) = \min_{\theta_0} \{\theta_0 \in S : p(\hat{\mathcal{T}}(\theta_0)) \geq \alpha\}$$

and

$$CI_{\mathcal{T}}^u(\alpha) = \max_{\theta_0} \{\theta_0 \in S : p(\hat{\mathcal{T}}(\theta_0)) \geq \alpha\}.$$

The determination of interval estimates in (2.16) requires (some approximation of) the distribution of  $\mathcal{T}$ . As an immediate approach, one might rely on asymptotic  $p$ -values. Based on exact  $p$ -values the actual coverage probability of the HL interval estimate in (2.16) is  $1 - \alpha$  (Dufour, 1990).

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<sup>3</sup>Further test statistics can also be considered as potential candidates for HL estimation: the multivariate independence test of Tsay (2004), and the statistics proposed by Dufour and Khalaf (2002), i.e. extensions of the exact independence test of Harvey and Phillips (1982). However, these tests require a sufficiently large time dimension ( $T > N$ ). In order to avoid dimensional restrictions, these diagnostics are not considered for HL estimation here.

### 2.3.2 Test based point estimation

In analogy to interval estimation the HL point estimator, denoted  $\hat{\theta}_{\mathcal{T}}$ , is the particular  $\theta_0$  attaining the highest confidence in form of the maximum  $p$ -value of  $\hat{\mathcal{T}}(\theta_0)$  (Hodges and Lehmann, 1983, Coudin and Dufour, 2011), i.e.

$$\hat{\theta}_{\mathcal{T}} = \max_{\theta_0} \left\{ p \left( \hat{\mathcal{T}}(\theta_0) \right) \right\}. \quad (2.17)$$

Since HL estimators relate to some underlying diagnostic in the following these estimators are referred as  $\text{HL}(\mathcal{T})$ , such that, for instance,  $\text{HL}(MI)$  is short for HL estimators based on  $MI$ . If the test statistic  $\mathcal{T}$  has reasonable power against the alternative hypothesis,  $H_1 : \theta \neq \theta_0$ ,  $\hat{\theta}_{\mathcal{T}}$  is unique (Dufour, 1990). Martellosio (2010) shows that for some combinations of  $\mathbf{X}$  and  $\mathbf{W}$  the power of any invariant spatial error correlation test vanishes as  $\theta \rightarrow 1$ . Then, the maximum  $p$ -value,  $p \left( \hat{\mathcal{T}}(\theta_0) \right) = 1$ , could be obtained for more than one candidate  $\theta_0$ . To detect a potential power trap Martellosio (2010) recommends to apply alternative tests as robustness checks.

### 2.3.3 Monte Carlo test

If the distribution of a test statistic  $\hat{\mathcal{T}}(\theta_0)$  does not depend on nuisance parameters, it is straightforward to obtain exact  $p$ -values by means of simulating the theoretical null distribution.<sup>4</sup> To show pivotalness of all considered diagnostics  $\mathcal{T} \in \{MI, LME, BP, CD, FT_n, FT_q, CD^W, BP^W\}$  it is convenient to rewrite the estimated spatially filtered residuals (2.7) as

$$\hat{\epsilon}_0 = \mathbf{M}_0 \epsilon = \sigma \mathbf{M}_0 \xi.$$

Under  $H_0$ , correct choice of  $\mathbf{W}$  and fixed  $\mathbf{M}_0$  the distribution of  $\mathcal{T}$  is free of nuisance parameters, since it does not depend on  $\sigma^2$  or  $\beta$ . To obtain MC realizations  $\mathcal{T}_r(\theta_0)$ ,  $r = 1, \dots, R$ , spatially filtered residuals  $\hat{\xi}_{0r} = \mathbf{M}_0 \xi_r$ ,  $\xi_r \stackrel{iid}{\sim} D(0, \mathbf{I}_N)$ , enter the test statistic. Then, the MC  $p$ -value is estimated as

$$p_R \left( \hat{\mathcal{T}}(\theta_0) \right) = \frac{\sum_{r=1}^R \mathcal{I} \left( \mathcal{T}_r^2(\theta_0) \geq \hat{\mathcal{T}}^2(\theta_0) \right) + 1}{R + 1}.$$

As shown by Dufour (2006) the MC implied significance level equals the nominal level  $\alpha$  if the number of MC replications  $R$  is chosen such that  $\alpha(R+1)$  is an integer. While alternative choices of  $R$  offer the same reliability of the test under  $H_0$ , the power of an MC test can be improved when opting for a larger  $R$ ,  $R = 999$  if  $\alpha = 0.05$ , say. Moreover, owing to the inverse relationship between a test statistic  $\hat{\mathcal{T}}(\theta_0)$  and

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<sup>4</sup>Notably, the exact distribution of  $MI$  can also be derived by numerical integration (Bivand et al, 2009). In comparison, the MC approach is immediate to implement for all considered diagnostics, and, thus, facilitates the evaluation of simulation results over a set of rival diagnostics  $\mathcal{T} \in \{MI, LME, BP, CD, FT_n, FT_q, CD^W, BP^W\}$ .

the respective  $p$ -value, minimizing the former is equivalent to maximizing the latter (Coudin and Dufour, 2011). As a consequence, applying asymptotic or MC based  $p$ -values obtains identical HL point estimates.

Regarding the scope of MC approaches in spatial panel models two remarks are in order. Firstly, MC testing builds upon sampling from the known distribution  $D$ . In empirical practice a misspecification of the underlying distribution is eventually detrimental for the performance of MC testing. Thus, careful diagnostic checking of the distributional assumptions made for (estimated) residual processes should accompany the application of MC based critical values. Secondly, the described MC procedure is exact, since spatial correlation is restricted to the residual processes. Within the spatial lag model typical diagnostics, e.g. the LM lag test in Anselin et al. (2008), lack pivotalness since the conditional mean coefficients impact on the asymptotic distribution (Martellosio, 2010). For MC testing in presence of nuisance parameters the reader may consult Dufour (2006).

## 2.4 Simulation study

To shed light on the relative merits of MC testing and HL estimation under a variety of correlation structures, a simulation study including misspecification of the spatial weights matrix is conducted. At first, the size and power features of the test statistics listed in Section 2.2.2 are analyzed. The simulations will be informative with respect to potential size distortions of the asymptotic tests in finite samples and under strongly connected spatial weights matrices. With regard to the implementation of HL estimators it is particularly important to uncover powerful tests, since the power of a test governs HL estimation accuracy and the precision of HL interval estimates. Secondly, HL point and interval estimators are compared with their ML counterparts commonly employed in spatial modeling. To economize on space the latter analysis is concentrated on a set of 3 alternative test approaches, which have shown most accurate performance in terms of power ( $MI$ ) or robustness under diverse (misspecified) spatial structures ( $CD$ ,  $CD^W$ ).

In the following the data generating models (DGMs) are described in detail. Moreover, the considered loss functionals are listed and a few remarks are added on the provision of simulation results before it comes to a discussion of the relative performance of rival approaches to spatial diagnosis and estimation.

## 2.4.1 Simulation design

### Data generating models

Vector valued spatial data are generated according to

$$y_t = \iota_N \beta_1 + x_t \beta_2 + e_t, \quad t = 1, \dots, T,$$

with  $\beta_1 = \beta_2 = 1$  and  $\iota_N$  is a  $N \times 1$  vector of ones. The  $N$ -dimensional vector of explanatory variables  $x_t$  is drawn once from the uniform distribution  $U[0, 10]$  and kept constant for all replications of an experiment. SAR and SMA dependence is introduced, respectively, by means of disturbance vectors

$$e_t = (\mathbf{I}_N - \theta \mathbf{W})^{-1} \epsilon_t \quad \text{and} \quad e_t = (\mathbf{I}_N + \theta \mathbf{W}) \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, \mathbf{I}_N).$$

In a first set of experiments four distinct sample sizes are considered: a 'small' sample ( $T = 10, N = 10$ ), experiments with an increased time ( $T = 25, N = 10$ ) or cross sectional ( $T = 10, N = 25$ ) dimension, and a 'large' sample scenario ( $T = 25, N = 25$ ). These relatively small sample sizes are sensible to highlight the bias of the ML estimator and could motivate a need for exact inference or HL estimation. Notably, such sample dimensions are typical for macroeconomic panel models, where both the number of available time periods and cross sectional units (e.g. G7, EU or OECD members) are small to medium.

Data are generated for 9 distinct values of  $\theta$ . To avoid trivial power estimates, the power analysis is concentrated on parameter settings  $\theta \in \{-0.20, -0.15, \dots, 0.2\}$ . The empirical performance of alternative estimators is analyzed over a wider range of the parameter space,  $\theta \in \{-0.8, -0.6, \dots, 0.8\}$ . As it turns out, the choice of any nominal test level  $\alpha \in \{0.01, 0.025, 0.05, 0.1\}$  obtains performance patterns of test statistics or HL estimators which are qualitatively equivalent. Therefore, simulation results are only provided for  $\alpha = 0.05$ . The number of MC replications is chosen as  $R = 999$  to foster the discriminatory content of MC testing under the alternative hypothesis. Assuming knowledge of the true error distribution MC innovations are drawn from the multivariate Gaussian.

As outlined so far the simulation experiments cover the ideal (or artificial) case where MC testing is implemented by means of the true underlying conditional distribution. In practice, MC testing could suffer from selecting a false innovation distribution. For such instances, potential losses of HL estimators implemented by means of MC based or asymptotic  $p$ -values are of natural interest. Moreover, sample dimensions considered so far are generally 'medium sized' such that it is unclear if potential merits of HL estimation vanish in larger samples. To investigate asymptotic properties of rival test and estimation approaches, 'large' sample experiments with  $(N, T) \in \{(10, 50), (10, 100), (50, 10), (100, 10)\}$  are designed on the one hand. On the other hand, scenarios are considered where true model innovations  $\epsilon_{it}$  are skewed and

MC testing is falsely based on the Gaussian distribution. To be precise, iid innovations are determined as  $\epsilon_{it} = (\nu_{it} - q)/\sqrt{2q}$ , where  $\nu_{it}$  is  $\chi^2$ -distributed with  $q$  degrees of freedom. Considering distinct degrees of nonnormality  $q$  is alternatively chosen as  $q = 5, 15$ . To uncover the impacts of skewness in small and large sample cases the respective sample dimensions are  $(N, T) \in \{(10, 25), (10, 100), (25, 10), (100, 10)\}$ . Keeping the documentation and discussion of the sensitivity study short in space, respective simulations are concentrated on a selection of tests ( $MI$ ,  $CD$ ,  $CD^W$ ) and DGMs with SAR disturbances.

### Spatial connectivity patterns

With regard to the row normalized weights matrix  $\mathbf{W}$  regular and irregular lattices are distinguished. Moreover, distinct degrees of connectivity are implemented to investigate the sensitivity of spatial diagnostics and estimates with regard to the spatial specification. A regular  $\mathbf{W}$  matrix is given by a ‘circular’ state in the form of a ‘ $J$  ahead and  $J$  behind’ weights matrix. To distinguish alternative degrees of connectivity, the integer  $J$  is chosen such that  $c = 2J/N \approx 0.2, 0.4, 0.6$ .<sup>5</sup> Further, ‘distance’ weights that build upon the NUTS 3 code of European regions (Eurostat: Regions, 1999) are used to implement irregular lattices. For this purpose the German financial center Frankfurt (code 312) jointly with its  $N - 1$  closest neighbors are considered. As detailed in Bröcker et al. (2002) economic distance is approximated by means of the travel time by private car (in minutes) between the economic centers of the cross sectional entities  $i$  and  $j$ , denoted  $\zeta_{ij}$ . Prior to row normalization the  $ij$ -th element of  $\mathbf{W}$  is determined as  $w_{ij}^* = \exp\{-a\zeta_{ij}\}$ . The distance decay parameter,  $a \in \{0.01, 0.05, 0.1\}$ , governs the degree of connectivity. The smaller is  $a$ , the slower is (prior to normalization) the decline of spatial weights across units and the higher is the connectivity.

### Specification of spatial weights

To implement spatial estimators or diagnostics an analyst has to select a particular weights matrix. In the simulation study a set of experiments, denoted  $W_c$ , is characterized by coincidence of the applied weights matrix and its counterpart underlying the data generation. In many empirical studies the choice of the weights matrix relies upon ad-hoc assumptions about the underlying spatial pattern and might also depend on data availability (Bhattacharjee and Jensen-Butler, 2005). Thus, misspecification

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<sup>5</sup>For this specification of  $\mathbf{W}$  the maximum number of neighbors for a given cross sectional unit is not restricted as  $N \rightarrow \infty$ , since  $J$  increases with  $N$ . As a consequence no additional information is gained from individual observations as  $N$  increases (Smith, 2009). Thus, the bias of the ML estimator might not vanish asymptotically and the asymptotic distributions of the considered spatial diagnostics might not be established. However, the main focus of this paper is rather on the estimation bias of  $\hat{\theta}$  than on asymptotic test and estimation properties. Moreover, even in this case the MC approach offers full control over the stochastic model features.

of the spatial weights matrix is inevitable and an important issue in spatial econometrics. Accordingly, two cases of misspecification are considered. In both instances, the true circular and distance weights matrix are specified with  $c \approx 0.40$  and  $a = 0.05$ , respectively, while the applied spatial weights matrix mimics scenarios of under- or over-specifying the actual connectivity. For the circular matrix, a choice of  $c \approx 0.20$  corresponds to an underspecified weights matrix, i.e. the true number of neighbors exceeds the number of neighbors that is formalized by the applied weighting scheme. For  $a = 0.1$  the decay of the distance based weights is faster as it is the case for the true DGM. Such scenarios of underspecification are denoted by  $W_u$ . Accordingly, the case of overspecification, denoted  $W_o$ , is implemented with  $c \approx 0.60$  and  $a = 0.01$ .

### Comments on optimization

All simulations are performed with MATLAB. The HL (restricted ML) estimator of the spatial parameter is obtained by maximizing the asymptotic  $p$ -value (concentrated log-likelihood function) over  $\theta \in \{-1, 1\}$ . Upper and lower bounds of HL CIs with nominal coverage probability  $(1 - \alpha)$  are obtained as parameter values  $\theta_0 > \hat{\theta}$  and  $\theta_0 < \hat{\theta}$  obtaining a  $p$ -value of  $\alpha$ , respectively. If the intended  $p$ -value is not found in the interval  $[\hat{\theta}, 1)$  ( $(-1, \hat{\theta}]$ ), the upper (lower) bound of the CI is set to the upper (lower) bound of the parameter space.

### Provision of results

Most simulation experiments are implemented for 4 sample sizes, 2 families of spatial weights matrices, 3 degrees of spatial connectivity, 2 spatial error processes and 9 distinct values of  $\theta$ . For space considerations simulation results are not provided in full detail but, rather, in a selective and condensed manner.

**Loss measures** To evaluate the finite sample properties of alternative test statistics  $\mathcal{T}(\theta_0) \in \{MI, LME, BP, CD, FT_n, FT_q, CD^W, BP^W\}$  their rejection frequencies are determined under  $H_0 : \theta = 0$  (size) and  $H_1 : \theta \in \{-0.20, -0.15, \dots, 0.2\}, \theta \neq 0$  (power). As performance measures for alternative approaches to point estimation (ML and HL based on MC critical values) two common loss functions are considered, the bias and the RMSE. Following Kelejian and Prucha (1999) the bias of an estimator  $\hat{\theta}^\bullet$  is estimated as the difference between the median and the true parameter

$$bias_i^\bullet = (\text{med}(\hat{\theta}_i^\bullet) - \theta), \quad (2.18)$$

where ' $\bullet$ ' is shorthand for a particular modeling approach, the index  $i$  refers to a simulation experiment, and  $\text{med}(\hat{\theta}_i^\bullet)$  is just the 50% quantile of simulated estimators over the set of the replications. Further, Kelejian and Prucha (1999) suggest an RMSE

estimator

$$RMSE_i^\bullet = \left( (bias_i^\bullet)^2 + (IQ_i^\bullet/1.35)^2 \right)^{1/2}, \quad (2.19)$$

where  $IQ_i^\bullet$  is the interquartile range, i.e. the difference between the 75% and 25% quantile of simulated quantities  $\hat{\theta}_i^\bullet$ . Notably, if the distribution of  $\hat{\theta}_i^\bullet$  is Gaussian the estimator in (2.19) coincides asymptotically with the standard RMSE. As a particular advantage of the statistic in (2.19) one might consider its robustness to outliers that eventually occur when simulating scenarios of spatial misspecification. However, standard bias and RMSE estimates have been also applied without detecting qualitative differences of simulation outcomes.

The assessment of the performance of competing interval estimators (ML and HL estimators based on asymptotic or MC critical values) is relied on two criteria: the empirical coverage frequencies of nominal 95% interval estimates and their average length. With  $CI_i^{l,\bullet}$  and  $CI_i^{u,\bullet}$  denoting modeling specific lower and upper CI bounds, respectively, these loss measures are determined as

$$cov_i^\bullet = \text{prob} \left[ CI_i^{l,\bullet} \leq \theta \leq CI_i^{u,\bullet} \right] \text{ and } len_i^\bullet = E[CI_i^{u,\bullet} - CI_i^{l,\bullet}], \quad (2.20)$$

where both 'prob' and 'E' operate over the set of simulated quantities. Notably, over a set of simulation experiments absolute coverage distortions,  $|cov_i^\bullet - 0.95|$ , are aggregated to guard against a canceling of simulation outcomes indicating overly liberal and conservative interval estimates. The average length of the CIs is considered as a measure of precision. Determining the length of interval estimates, their bounds are eventually truncated to be less than unity in absolute value.

**Tables and figures** Size estimates of competing correlation tests are documented in Table 2.2. Since the  $CD^W$  and  $BP^W$  test are identical to  $CD$  and  $BP$  under the distance weights specification, respectively, and the latter, as well as  $FT_n$  and  $FT_q$  are invariant to the spatial weights matrix, size estimates under the distance matrix are only reported for  $MI$  and  $LME$ . Moreover, for SAR and SMA models with  $T = 10$ ,  $N = 10$  estimates of power are depicted as a function of the spatial parameter  $\theta$  in Figure 2.1 (SAR disturbances) and Figure 2.2 (SMA). Similar plots are provided for bias and CI coverage in Figure 2.3 (SAR) and Figure 2.4 (SMA). These figures show results for underlying circular (left hand side panels) and distance weights (right hand side) matrix with  $c = 0.40$  and  $a = 0.05$  ( $W_c$ ), respectively, and two scenarios of spatial misspecification ( $W_u$ ,  $W_o$ ). Although being restricted to the case  $T = 10$ ,  $N = 10$  the displayed results do not differ qualitatively over the other three combinations of sample dimensions. However, this rather small sample size highlights the discrepancies in power of rival test statistics most clearly and demonstrates the need of HL estimation due to the bias and coverage distortion of the ML estimator. Further simulation



results are documented in Table 2.3 (size adjusted power<sup>6</sup>), Table 2.5 (bias/RMSE) and Table 2.7 (CI coverage/length) in the form of performance statistics averaged over  $\theta \in \{-0.20, -0.15, \dots, 0.2\}$  (power analysis) or  $\theta \in \{-0.8, -0.6, \dots, 0.8\}$  (point and interval estimation). To facilitate the interpretation of results the entries in these tables are given in normalized form where *MI* is typically the benchmark approach. Finally, at an even higher level of condensation results from surface response (SR) analysis are documented in Table 2.4 (power) and Table 2.6 (point and interval estimation). Next some implementation issues are discussed for the SR regressions.

**Surface response analysis** Similar to Anselin and Moreno (2003), Egger et al. (2009) or Das et al. (2003) the outcome of simulation experiments is related with econometric tools (e.g. HL vs. ML estimators), specifications (e.g. SAR vs. SMA) or modeling decisions (e.g. on the connectivity of spatial weight matrices). SR designs allow a rather condensed representation of simulation outcomes in response to characteristics of the underlying DGM and modeling decisions taken by the econometrician. Several metric variables enter the right hand side of the SR design, namely  $1/T_i$ ,  $1/N_i$  and  $\theta_i^2$ . Relating the simulation outcome to  $\theta_i^2$  allows nonlinear performance in response to the strength of spatial dependence. Further explanatory variables, listed and defined in Table 2.1, are binary and suitable to indicate the marginal response of loss measures to estimators and characteristics of the underlying DGM. Moreover, the binary measures allow to uncover potential interaction effects (e.g. of applying particular diagnostics in case of spatial over- ( $W_o$ ) or underspecification ( $W_u$ )). Each SR regression design also includes an intercept term such that performance characteristics are determined with regard to some benchmark modeling approach.

The dependent variables of the SR regressions are in the following denoted by  $z_i$  and correspond to the natural logarithm of the loss measures mentioned above. The log transformation is applied to ensure that  $z_i$  is reasonably scaled (Anselin and Moreno, 2003). With regard to each loss statistic two SR regressions are estimated summarizing the simulation results obtained under circular and distance weights matrices. The following performance measures are subjected to SR regressions:

- power  
 $z_i = \ln(erf_i^\bullet)$ , where  $erf_i^\bullet$  is the empirical rejection frequency characterizing the  $i$ -th simulation experiment (and modeling approach ' $\bullet$ '). For the power analysis the benchmark approach is *MI*. The number of simulation experiments evaluated with regard to power characteristics is 4096 (circular  $W$ ) and 2816 (distance  $W$ ).<sup>7</sup>

<sup>6</sup>Rejection frequencies are adjusted by tuning the critical values of the tests such that empirical rejection frequencies are exactly 5% under  $H_0$ .

<sup>7</sup> $4$  (sample sizes)  $\times 2$  (SAR and SMA)  $\times 8$  (choices of  $\theta$  under  $H_1 : \theta \neq 0$ )  $\times 2$  (MC based and asymptotic critical values)  $\times 3$  (degrees of spatial connectivity)  $\times$  test statistics (circular  $W$ : 8 and distance  $W$ : 6)  $+ 2$  (scenarios of misspecification)  $\times$  (spatial) test statistics (circular  $W$ : 4

Table 2.1: Definitions of explanatory SR dummy variables

$\theta_i^-$	is unity if an MC experiment is performed with a negative spatial correlation parameter, $\theta < 0$ .
$SMA_i$	is unity if an MC experiment is performed with SMA disturbances
$CON_i^m$	is unity if the applied spatial weights matrix is correct specified and has medium connectivity (circular: $c \approx 0.4$ , distance: $a = 0.05$ )
$CON_i^h$	is unity if the applied spatial weights matrix is correct specified and has high connectivity (circular: $c \approx 0.6$ , distance: $a = 0.01$ )
$W_{oi}$	is unity if the applied spatial weights matrix amounts to an overspecification of the true connectivity structure
$W_{ui}$	is unity if the applied spatial weights matrix amounts to an underspecification of the true connectivity structure
$ASY_i$	is unity if a test statistic or an HL estimator is implemented by means of asymptotic critical values
$MC_i$	is unity if a test statistic or an HL estimator is implemented by means of MC based critical values
$MI_i$	is unity if the $MI$ test is applied
$LME_i$	is unity if the $LME$ test is applied
$CD_i$	is unity if the $CD$ test is applied
$BP_i$	is unity if the $BP$ test is applied
$FT_{ni}$	is unity if the $FT_n$ test is applied
$FT_{qi}$	is unity if the $FT_q$ test is applied
$CD_i^W$	is unity if the $CD^W$ test is applied
$BP_i^W$	is unity if the $BP^W$ test is applied

The table lists the binary variables entering the SR analysis of simulation outcomes. Generally, these variables are zero except for those states described in the right hand side panel of the table. SR regressions further condition on an intercept,  $\theta_i^2$ ,  $1/N_i$  and  $1/T_i$  and the interaction of binary measures. The particular SR regression designs that have been estimated by ordinary least squares are implicitly given by column 1, 2, 5 and 6 (the first 3 columns) of Table 2.4 (Table 2.6).

- bias

$z_i = \ln \left( |bias_i^\bullet| \sqrt{T_i N_i} \right)$ , where  $bias_i^\bullet$  is determined for an estimator ' $\bullet$ ' within the  $i$ -th simulation experiment.

- RMSE

$z_i = \ln \left( RMSE_i^\bullet \sqrt{T_i N_i} \right)$ , where  $RMSE_i^\bullet$  is the RMSE obtained from an estimator ' $\bullet$ ' within the  $i$ -th simulation experiment. For SR regressions describing the pattern of bias and RMSE statistics the benchmark approach is the ML estimator. The number of observations is 1440 (circular weights matrix) or 1080

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and distance  $W$ : 2)).

(distance) observations.<sup>8</sup>

- CI coverage  
 $z_i = \ln \left( |cov_i^\bullet - 0.95| \sqrt{T_i N_i} \right)$ , where  $cov_i^\bullet$  is the empirical coverage of CIs determined within the  $i$ -th experiment by means of an estimator  $\bullet$ . The SR benchmark approach is the ML interval estimator. The number of observations entering the SR regressions is 2520 (1800) for DGMs with circular (distance) weights matrices.<sup>9</sup>
- CI length  
 $z_i = \ln \left( |len_i^\bullet| \sqrt{T_i N_i} \right)$ , where  $len_i^\bullet$  is the average CI length. With regard to the benchmark approach and the number of observations the SR regressions for interval precision are in full analogy to coverage analysis.

## 2.4.2 Small sample properties of correlation tests

### Size properties

Empirical size estimates in Table 2.2 characterize the asymptotic tests for unspecified cross sectional correlation ( $BP$ ,  $CD$ ,  $FT_n$  and  $FT_q$ ) and the  $BP^W$  statistic as conservative since its actual rejection frequencies are mostly significantly below the nominal level. In contrast, under the circular weights matrix the empirical size of asymptotic inference by means of the  $CD^W$  and  $MI$  test is close to the nominal level for almost all specifications. For  $T = 10$  and  $c \geq 0.40$  the asymptotic  $LME$  test shows empirical significance levels falling below the nominal counterpart. Turning to the distance matrix,  $MI$  and  $LME$  yield empirical rejection frequencies which are mostly significantly below the nominal target. In summary, common approaches to the diagnosis of spatial and general residual correlation are overly conservative in small samples and under strongly connected spatial units. Thus, MC testing appears as a viable alternative for empirical practice as it offers full control over type I error probabilities.

### Power features

The Figures 2.1 and 2.2 show that unadjusted power estimates of the MC tests are asymmetrically U-shaped with somewhat weakened power over states of negative spatial correlation  $\theta \in \{-0.2, \dots, -0.05\}$ . Notably, all displayed power curves show slightly higher detection rates for global spatial correlation (SAR) in comparison with local patterns (SMA). However, in relative terms there are no qualitative differences

<sup>8</sup>4 (sample sizes)  $\times$  (3 (degrees of spatial connectivity) + 2 (scenarios of misspecification))  $\times$  2 (SAR and SMA)  $\times$  9 (values of  $\theta$ )  $\times$  estimators (circular  $W$ : 4 and distance  $W$ : 3).

<sup>9</sup>4 (sample sizes)  $\times$  (3 (degrees of spatial connectivity) + 2 (scenarios of misspecification))  $\times$  2 (SAR and SMA)  $\times$  9 (values of  $\theta$ )  $\times$  estimators (circular  $W$ : ML, 3 MC and 3 asymptotic HL and distance  $W$ : ML, 2 MC and 2 asymptotic HL).

Table 2.2: Empirical size of tests on spatial correlation (in %)

$N, T$	$BP$		$CD$		$FT_n$		$FT_q$		$c$		$BP^W$		$CD^W$		$MI$		$LME$		$MI$		$LME$		
	asy	MC	asy	MC	asy	MC	asy	MC			asy	MC	asy	MC	asy	MC	asy	MC	asy	MC	asy	MC	
10, 10	<b>3.74</b>	5.15	<b>3.09</b>	5.16	<b>4.54</b>	5.31	<b>5.67</b>	4.85	.20		<b>2.80</b>	4.91	5.02	5.00	5.05	5.20	5.14	5.20	.10	<b>4.30</b>	5.04	<b>3.36</b>	4.97
	-	-	-	-	-	-	-	-	.40		<b>2.95</b>	4.93	<b>4.36</b>	4.64	<b>4.54</b>	4.70	<b>4.47</b>	4.80	.05	<b>4.24</b>	4.95	<b>2.84</b>	5.03
	-	-	-	-	-	-	-	-	.60		<b>3.29</b>	5.13	4.57	4.96	4.67	4.86	<b>4.36</b>	5.19	.01	<b>4.15</b>	4.98	<b>2.70</b>	4.92
10, 25	<b>3.95</b>	4.76	<b>4.16</b>	5.06	<b>4.11</b>	4.80	<b>5.92</b>	4.81	.20		<b>3.95</b>	4.96	4.82	4.81	5.10	5.11	4.96	5.04	.10	4.73	5.04	<b>4.32</b>	4.95
	-	-	-	-	-	-	-	-	.40		<b>4.16</b>	5.09	5.21	5.29	5.12	5.22	4.95	4.99	.05	4.59	5.00	<b>4.18</b>	4.89
	-	-	-	-	-	-	-	-	.60		<b>3.94</b>	4.82	5.01	5.05	5.24	5.25	4.86	5.23	.01	4.58	4.99	<b>4.02</b>	4.91
25, 10	<b>3.53</b>	4.86	<b>2.97</b>	5.24	4.79	5.05	5.27	5.09	.16		<b>2.92</b>	5.04	4.98	5.09	4.90	5.06	4.67	4.97	.10	4.77	4.92	<b>4.44</b>	4.89
	-	-	-	-	-	-	-	-	.40		<b>3.29</b>	5.18	4.57	5.11	4.75	4.85	<b>4.23</b>	4.91	.05	<b>4.39</b>	4.92	<b>3.66</b>	4.99
	-	-	-	-	-	-	-	-	.56		<b>3.44</b>	5.06	<b>4.33</b>	4.97	4.83	5.06	<b>4.12</b>	4.96	.01	<b>4.25</b>	4.88	<b>2.99</b>	5.17
25, 25	4.60	5.12	<b>3.92</b>	4.78	4.70	5.08	<b>5.49</b>	5.15	.16		4.69	<b>5.61</b>	4.91	5.06	4.88	5.09	4.91	4.97	.10	4.94	5.01	4.85	5.03
	-	-	-	-	-	-	-	-	.40		<b>4.30</b>	5.16	4.83	4.93	4.82	4.83	4.61	4.95	.05	<b>4.54</b>	4.68	<b>4.23</b>	4.75
	-	-	-	-	-	-	-	-	.56		<b>4.47</b>	5.31	4.75	4.97	4.83	4.95	4.70	4.90	.01	<b>4.39</b>	4.87	<b>3.86</b>	4.76

Rejection frequencies of asymptotic (asy) and MC correlation tests,  $\mathcal{T}(\theta_0) \in \{MI, LME, BP, CD, FT_n, FT_q, CD^W, BP^W\}$ , under  $H_0 : \theta = 0$ . Simulation results are distinguished for circular and distance spatial weights matrices  $W$  and respective degrees of connectivity,  $c$  and  $a$ , respectively. The number of MC replications is  $R = 999$ . 10000 replications are performed for each experiment, the nominal significance level is  $\alpha = 0.05$ . Rejection frequencies in bold face are outside a 95% confidence interval determined around the nominal significance level, i.e.  $[\alpha \pm 2\sqrt{\alpha(1-\alpha)}/10000 = [4.56\%; 5.44\%]$ .

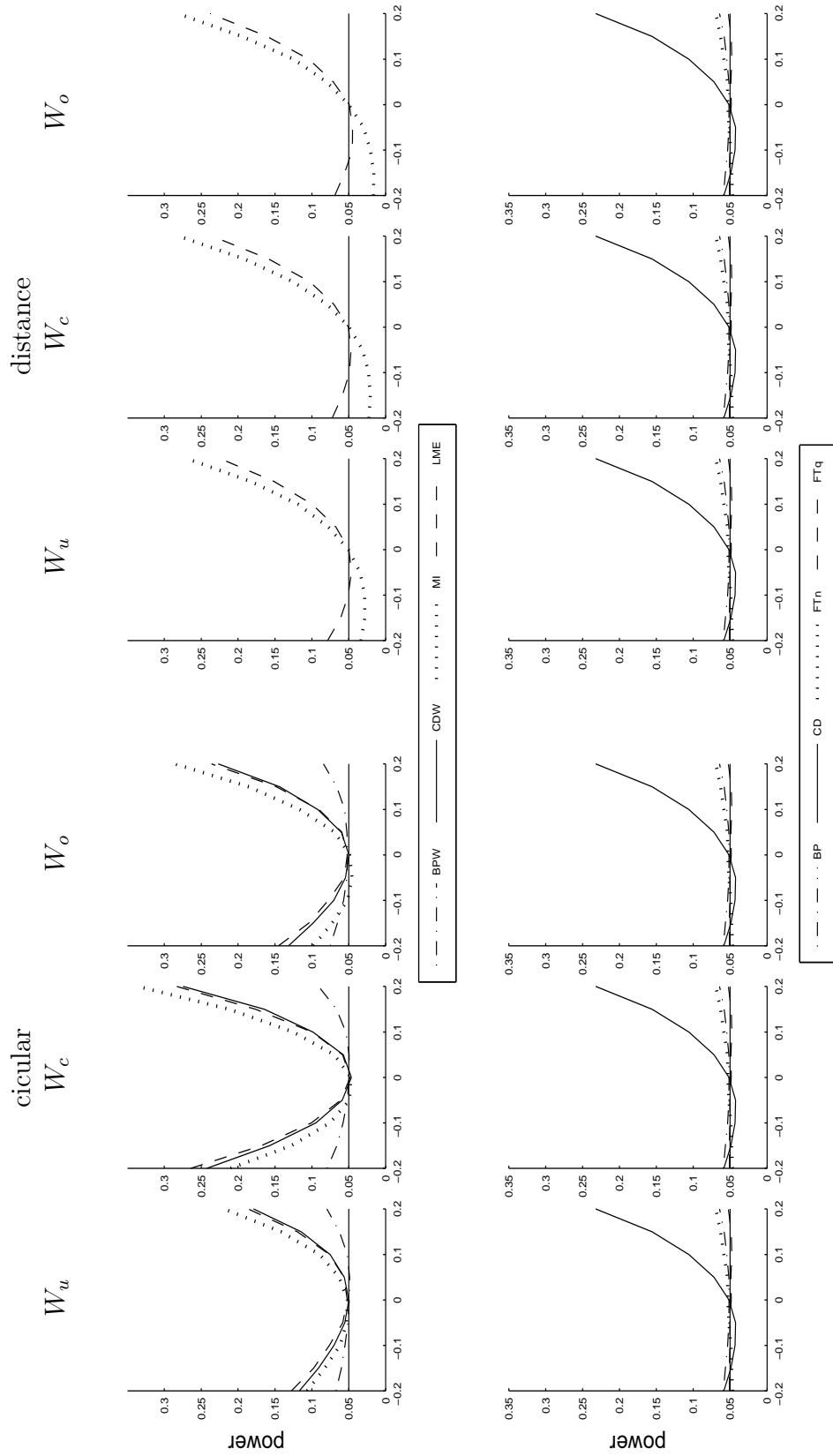


Figure 2.1: Power features under SAR disturbances

Simulation results are shown as a function of the spatial parameter  $\theta$  over the support  $-0.2 \leq \theta \leq 0.2$ . The spatial weights matrix of the underlying DGMs is specified with  $c = 0.40$  (circular) and  $a = 0.05$  (distance) weights.  $W_c$ ,  $W_u$  and  $W_o$  refer to scenarios where an analyst employs the correct spatial weighting, underspecifies and overspecifies the true design, respectively. For further notes see also Tables 2.2 and 2.3.

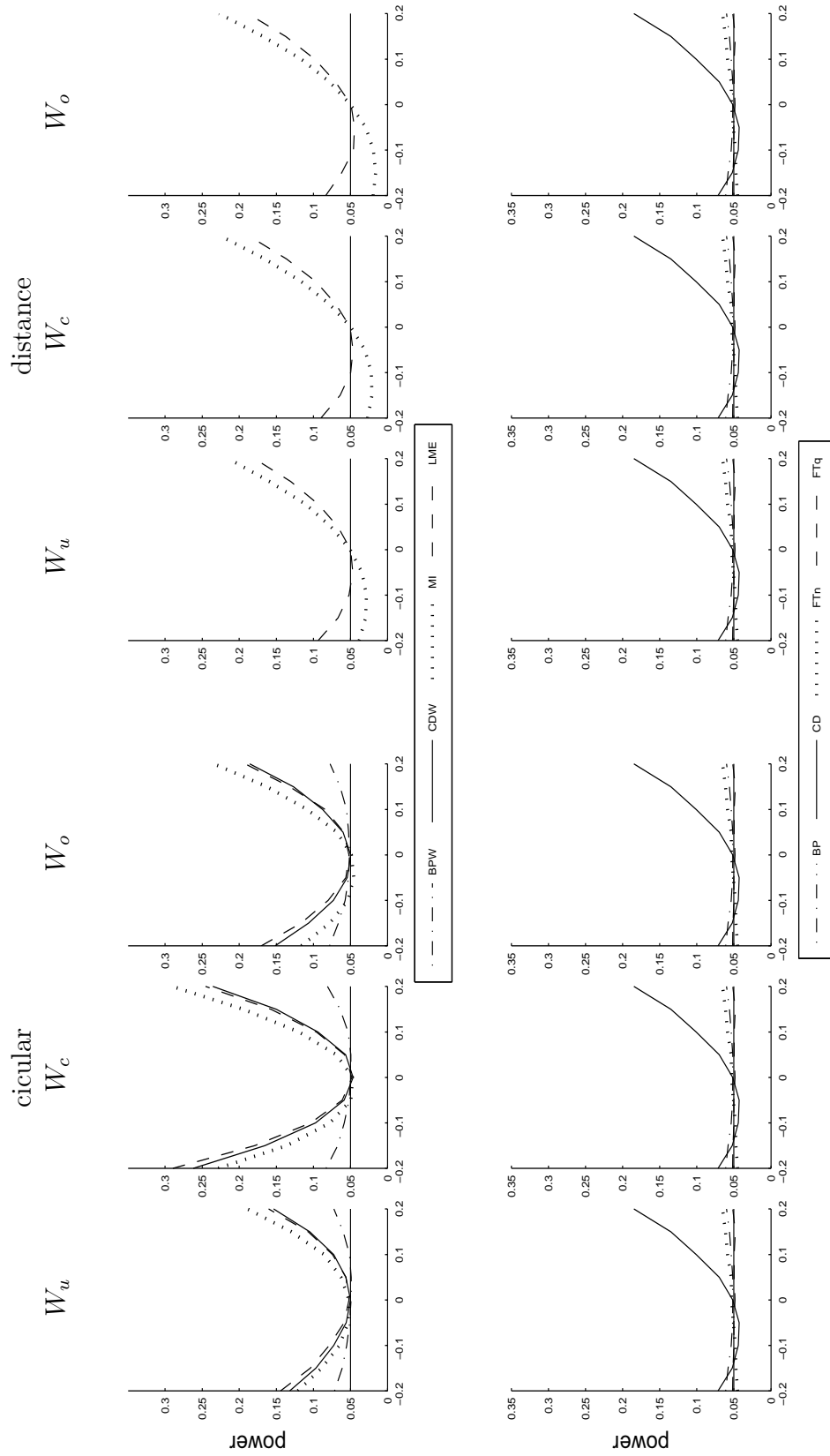


Figure 2.2: Power features under SMA disturbances

For further notes see Figure 2.1.

between the power estimates of rival inferential procedures. For the (likely more common) case of testing against positive spatial correlation  $MI$  turns out as the most powerful variant of MC testing. In the case of negative spatial dependence the  $LME$  test obtains highest power estimates under a circular and distance spatial weights matrix. Focussing on directly linked processes is effective to improve the power of the  $CD$  test. For the  $BP$  test the corresponding power gains of  $BP^W$  are substantially smaller. Surprisingly, the non-parametric tests for contemporaneous correlation,  $FT_n$  and  $FT_q$ , suffer from substantial power weakness. Only for  $\theta \geq 0.15$  the empirical rejection frequencies exceed the nominal level. These results hold for all choices of the spatial weights matrix including scenarios of under- or overspecification. Adopting an overly large ( $W_o$ ) or small ( $W_u$ ) number of neighbors under circular spatial states reduces the power of all diagnostics relying on the spatial weights matrix. The power loss is stronger for  $W_u$ . In contrast, misspecification of the decay parameter in the distance model does only weakly impact on the power of the tests in both scenarios  $W_o$  or  $W_u$ .

Size adjusted power estimates averaged over  $\theta \in \{-0.20, -0.15, \dots, 0.2\}$  for all levels of contiguity and sample sizes (Table 2.3) underline the strong and robust power features of  $MI$ . Under lowest spatial connectivity ( $c = 0.2$ ) power estimates documented for  $MI$  (and thus for  $LME$  and  $CD^W$ ) underpin its consistency over both sample dimensions  $N$  and  $T$ . With increased connectivity ( $c = 0.4, c = 0.6$ ) the power gains are, on average, rather small when comparing simulation results for  $N = 10, T = 10$  and  $N = 25, T = 10$ . In response to an increase of the time dimension, consistency of the latter tests is more evident irrespective of the degree of connectivity ( $c, a$ ). While the  $LME$  and  $CD^W$  test are characterized by power estimates that are throughout close to those of  $MI$  ( $> 90\%$  of  $MI$  rejection frequencies) the other diagnostics ( $BP, CD, FT_n, FT_q$  and  $BP^W$ ) suffer from massive power losses, especially under weak spatial connectivity ( $c \approx 0.2, a = 0.1$ ). Average power estimates for  $MI$  (and thus for all tests except for  $CD, FT_n$  and  $FT_q$ ) decrease with spatial connectivity. Under the circular weights matrix average rejection frequencies documented for  $MI$  are approximately twice as large for  $c \approx 0.2$  in comparison with  $c \approx 0.6$ . In contrast, the power of the  $CD$  test and the non-parametric statistics is unaffected by the spatial connectivity.

SR results in Table 2.4 do not indicate power losses as a consequence of MC testing since the effects of interacting the test dummy variables with MC inference are either insignificant ( $MI$ ) or similar to the interaction-effects with asymptotic inference. Generally, the SR analysis underpins the power weakness that characterizes the  $BP^W$  test and the statistics that do not rely on  $\mathbf{W}$ . However, under distance based spatial structures the  $CD$  test do not offer a power loss in comparison with  $MI$ . Interestingly, the SR coefficient estimates for the  $CD$  test interacted with  $MC_i$  and  $ASY_i$  are larger in absolute value as those for  $MI, LME$  and  $CD^W$  interacted with an overspecification of the spatial weights matrix. Thus, on average, the latter

Table 2.3: Absolute and relative size adjusted power estimates

$N, T$	$c$	circular $W$										distance $W$				
		$BP$	$CD$	$FT_n$	$FT_q$	$BP^W$	$CD^W$	$MI$	$LME$	$a$	$BP$	$CD$	$FT_n$	$FT_q$	$MI$	$LME$
		spatially autoregressive disturbances (SAR)														
10, 10	.20	0.283	0.340	0.195	0.205	0.419	0.943	25.59	0.990	.10	0.539	0.916	0.509	0.516	10.08	1.015
	.40	0.374	0.571	0.326	0.336	0.414	0.953	15.84	0.981	.05	0.559	0.954	0.536	0.538	9.67	1.025
	.60	0.457	0.751	0.421	0.432	0.473	0.932	12.20	0.954	.01	0.568	0.970	0.546	0.547	9.51	1.045
10, 25	.20	0.269	0.327	0.108	0.105	0.504	0.994	49.91	1.003	.10	0.372	0.925	0.287	0.289	18.18	1.028
	.40	0.278	0.536	0.170	0.168	0.337	0.982	30.87	1.022	.05	0.385	0.979	0.299	0.303	17.18	1.030
	.60	0.326	0.737	0.235	0.228	0.346	0.991	22.66	1.004	.01	0.388	0.998	0.306	0.314	16.85	1.034
25, 10	.16	0.198	0.289	0.166	0.163	0.256	0.937	31.02	1.013	.10	0.343	0.594	0.342	0.329	15.67	1.018
	.40	0.336	0.571	0.329	0.301	0.351	0.915	16.12	0.993	.05	0.484	0.867	0.498	0.469	10.76	0.998
	.56	0.424	0.742	0.426	0.401	0.434	0.992	12.47	1.019	.01	0.519	0.933	0.537	0.503	9.96	0.965
25, 25	.16	0.147	0.289	0.094	0.090	0.265	0.984	55.96	1.007	.10	0.200	0.545	0.171	0.167	30.44	1.007
	.40	0.192	0.516	0.165	0.158	0.214	0.974	31.93	1.002	.05	0.282	0.839	0.266	0.258	19.80	1.004
	.56	0.234	0.675	0.218	0.204	0.244	0.975	24.55	1.015	.01	0.331	1.012	0.323	0.310	16.41	1.052
		spatial moving average disturbances (SMA)														
10, 10	.20	0.285	0.334	0.199	0.211	0.420	0.942	24.91	0.989	.10	0.596	0.935	0.551	0.572	9.04	1.062
	.40	0.384	0.549	0.334	0.347	0.424	0.965	15.29	0.998	.05	0.629	0.989	0.591	0.612	8.56	1.079
	.60	0.484	0.735	0.442	0.453	0.497	0.941	11.48	0.966	.01	0.639	1.009	0.605	0.613	8.40	1.096
10, 25	.20	0.262	0.323	0.107	0.108	0.497	0.996	49.33	1.005	.10	0.380	0.936	0.298	0.308	17.11	1.050
	.40	0.275	0.526	0.171	0.170	0.330	0.982	30.40	1.021	.05	0.396	1.001	0.320	0.331	16.01	1.059
	.60	0.327	0.736	0.238	0.239	0.347	1.001	21.75	1.014	.01	0.401	1.019	0.320	0.333	15.71	1.058
25, 10	.16	0.199	0.275	0.169	0.163	0.255	0.942	30.49	1.015	.10	0.358	0.567	0.351	0.343	14.91	1.034
	.40	0.352	0.548	0.343	0.321	0.366	0.920	15.32	1.004	.05	0.533	0.876	0.543	0.517	9.68	1.038
	.56	0.455	0.723	0.453	0.425	0.465	1.019	11.60	1.053	.01	0.594	0.970	0.610	0.567	8.69	1.004
25, 25	.16	0.142	0.278	0.092	0.088	0.253	0.982	55.82	1.005	.10	0.201	0.522	0.174	0.174	29.82	1.008
	.40	0.189	0.494	0.164	0.160	0.209	0.972	31.53	1.002	.05	0.294	0.836	0.279	0.275	18.68	1.022
	.56	0.237	0.653	0.221	0.212	0.246	0.980	23.89	1.023	.01	0.355	1.035	0.350	0.333	15.08	1.083

The table documents size adjusted power estimates for testing  $H_0 : \theta = 0$  over  $\theta \in \{-0.2, -0.15, -0.1, -0.05, 0.05, 0.1, 0.15, 0.2\}$ . With regard to  $MI$  absolute rejection frequencies ( $\times 100$ ) are shown, results for the remaining diagnostics are given in relation to  $MI$ . For further notes see Table 2.2.



Table 2.4: Surface response analysis (power)

Variable		$W^c$	$W^d$	Variable		$W^c$	$W^d$
$const$		-1.337**	-1.780**				
$1/N_i$		0.708**	-0.868**	$1/T_i$		-6.860**	-6.355**
$\theta_i^2$		25.385**	18.558**	$\theta_i^-$		-0.221**	-0.552**
$CON_i^m$		-0.288**	-0.092**	$CON_i^h$		-0.404**	-0.142**
<i>tests for spatial correlation</i>							
$MI_i$	$ASY_i$	-	-	$LME_i$	$ASY_i$	0.028	0.174**
$MI_i$	$MC_i$	-0.002	-0.001	$LME_i$	$MC_i$	0.024	0.178**
$MI_i$	$SMA_i$	-0.008	-0.017	$LME_i$	$SMA_i$	-0.011	-0.011
$MI_i$	$W_{ui}$	-0.790**	-0.085*	$LME_i$	$W_{ui}$	-0.799**	-0.166**
$MI_i$	$W_{oi}$	-0.622**	-0.277**	$LME_i$	$W_{oi}$	-0.591**	-0.189**
<i>tests for general contemporaneous correlation</i>							
$BP_i$	$ASY_i$	-1.083**	-0.580**	$CD_i$	$ASY_i$	-0.687**	-0.026
$BP_i$	$MC_i$	-1.083**	-0.577**	$CD_i$	$MC_i$	-0.681**	-0.018
$BP_i$	$SMA_i$	-0.014	-0.013	$CD_i$	$SMA_i$	-0.009	-0.011
$FT_{ni}$	$ASY_i$	-1.353**	-0.684**	$FT_{qi}$	$ASY_i$	-1.325**	-0.671**
$FT_{ni}$	$MC_i$	-1.341**	-0.674**	$FT_{qi}$	$MC_i$	-1.324**	-0.665**
$FT_{ni}$	$SMA_i$	-0.002	-0.001	$FT_{qi}$	$SMA_i$	-0.014	-0.015
<i>tests for local correlation</i>							
$BP_i^W$	$ASY_i$	-0.914**	-	$CD_i^W$	$ASY_i$	-0.011	-
$BP_i^W$	$MC_i$	-0.916**	-	$CD_i^W$	$MC_i$	-0.010	-
$BP_i^W$	$SMA_i$	-0.014	-	$CD_i^W$	$SMA_i$	-0.011	-
$BP_i^W$	$W_{ui}$	-0.441**	-	$CD_i^W$	$W_{ui}$	-0.781**	-
$BP_i^W$	$W_{oi}$	-0.398**	-	$CD_i^W$	$W_{oi}$	-0.583**	-
$R^2$		0.771	0.632	observations		4096	2816

The table documents SR parameter estimates with distinct significance levels: \*\* 5%; \* 10%.  $R^2$  is the degree of explanation. Columns  $W^c$  and  $W^d$  refer regressions for simulation experiments under circular and distance spatial weights matrices, respectively. Binary variables are defined in Table 2.1. For further notes see Tables 2.2 and 2.3.

statistics appear even more powerful under an overspecified spatial structure than the robust  $CD$  test. Furthermore, SR results highlight the overall adverse impact of strengthened spatial connectivity (coefficient estimates for binary variables  $CON^m$  and  $CON^h$ ) on the power of the diagnostics. Finally, potentially adverse effects of under- or overspecifying the connectivity structure are remarkably similar for  $MI$ ,  $LME$  and the modified  $CD$  statistic.

In summary, the simulation results support  $MI$  as a powerful test. Though being robust under misspecification of the spatial layout, the tests for general contemporaneous correlation suffer from massive power losses under weak spatial connectivity. The  $CD$  test achieves highest power features within this group of tests. Focussing on directly linked neighbors, the  $CD^W$  statistic obtains an average empirical power close

to that of  $MI$ .

To economize on space the following analysis of the empirical performance of HL point and interval estimators is concentrated on a smaller set of alternative test approaches. The empirical performance of the considered correlation tests suggests to choose as potential candidates for HL estimation diagnostics with a rather accurate performance in terms of power ( $MI$ ) or robustness under diverse (misspecified) spatial structures ( $CD$ ). As a third alternative the  $CD^W$  test is considered, gathering both, a (relatively) robust and powerful performance.

### 2.4.3 Point estimation

As mentioned above, the maximization of the asymptotic and the MC based  $p$ -value to obtain HL point estimates is equivalent. Thus, each test  $\mathcal{T}(\theta_0) \in \{MI, CD, CD^W\}$  obtains one HL point estimate. In the upper panels of Figures 2.3 and 2.4 bias estimates are displayed. The HL( $MI$ ) estimator yields the smallest bias of all approaches over the entire parameter space considered for  $\theta$ . In particular, the bias of the HL( $MI$ ) estimator is markedly smaller than respective ML statistics. While under correct model specification it is uniformly negative, the bias might eventually become positive under misspecification of the spatial weights and scenarios of negative correlation. Under correct specification of the spatial weights matrix bias estimates for the HL( $MI$ ) estimator are markedly smaller (i.e. by a factor of  $\approx 1/10$ ) under a circular ( $c = 0.4$ ) in comparison with a distance ( $a = 0.05$ ) weights matrix.

The average performance measures in Table 2.5 confirm that HL( $MI$ ) estimates are most effective with regard to bias loss. The bias of the ML and HL( $CD^W$ ) estimators is generally more than twice as large in comparison with the HL( $MI$ ) counterpart. In small samples ( $N = T = 10$ ), experiments can be observed where the ML bias is by a factor of more than 7 larger in comparison with results documented for the HL( $MI$ ) estimator. ML biases are, on average, by factors 5.64 (SAR, circular  $W$ ), 4.51 (SMA, circular  $W$ ), 3.45 (SAR, distance  $W$ ) and 2.75 (SMA, distance  $W$ ) larger than losses featuring the HL( $MI$ ) approach. Comparing ML and HL( $CD^W$ ) estimates, the former appear to have a small overall lead which holds, in particular, if the cross section is 'large' ( $N = 25$ ). In this case the asymptotic distribution of  $CD^W$  is characterized by a relatively large number of degrees of freedom which might adversely impact on its discriminatory content. For the same reason, under circular states the HL( $CD$ ) estimator suffers from huge average biases, such that it is not recommendable if an analyst pays particular attention to this loss criterion.

SR results for the bias loss are documented in the fourth and fifth column of Table 2.6. For both families of spatial weights matrices the HL( $MI$ ) estimator outperforms the benchmark ML approach which is indicated by a significantly negative coefficient estimated for the interacted  $MI_i, MC_i$  dummy variables. As a reflection of power features discussed before, overall, the bias is higher over states of medium

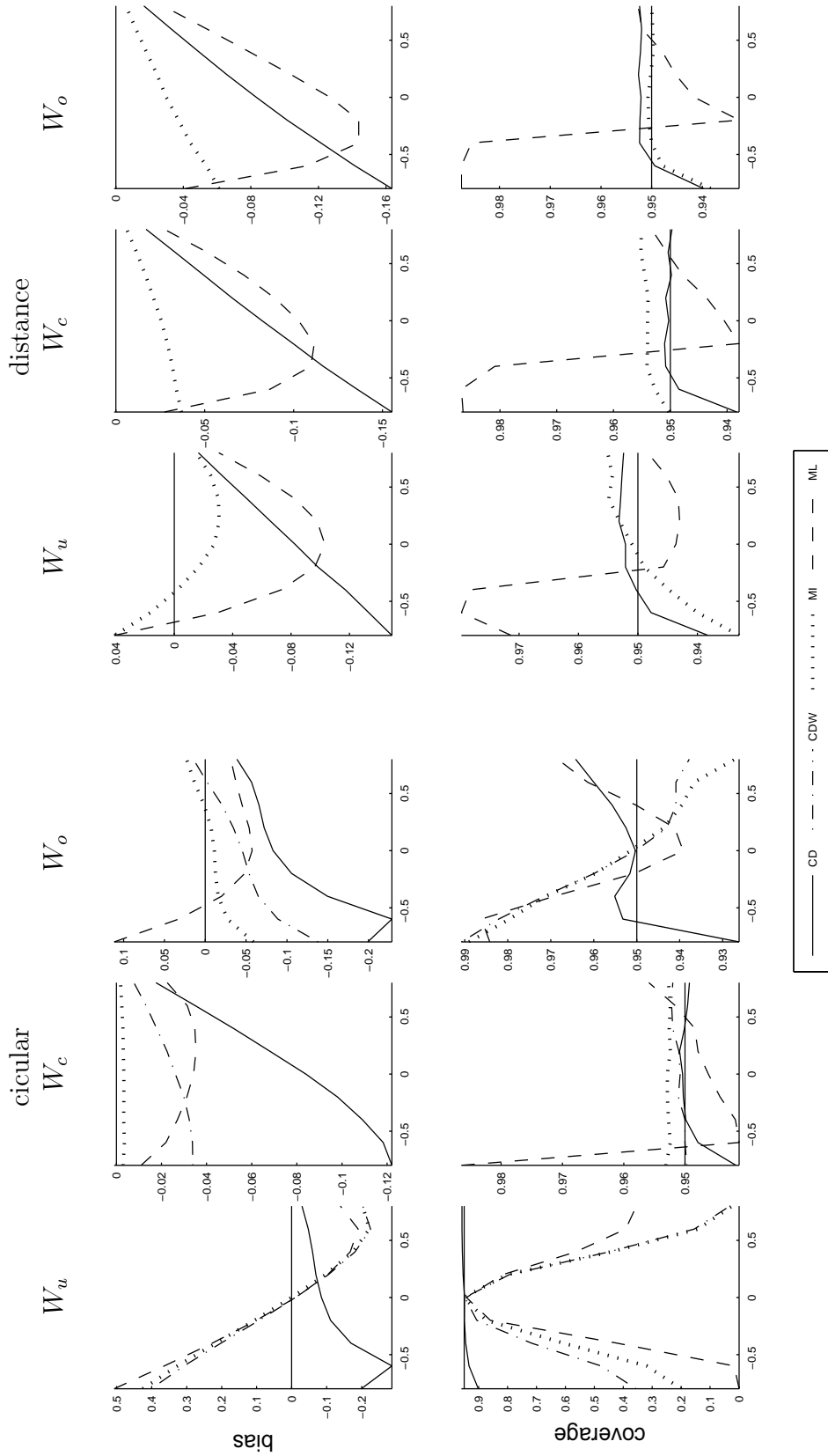


Figure 2.3: Estimation properties under SAR disturbances

Simulation results are shown as a function of the spatial parameter  $\theta$  over the support  $-0.8 \leq \theta \leq 0.8$ . For further notes see Figure 2.1.

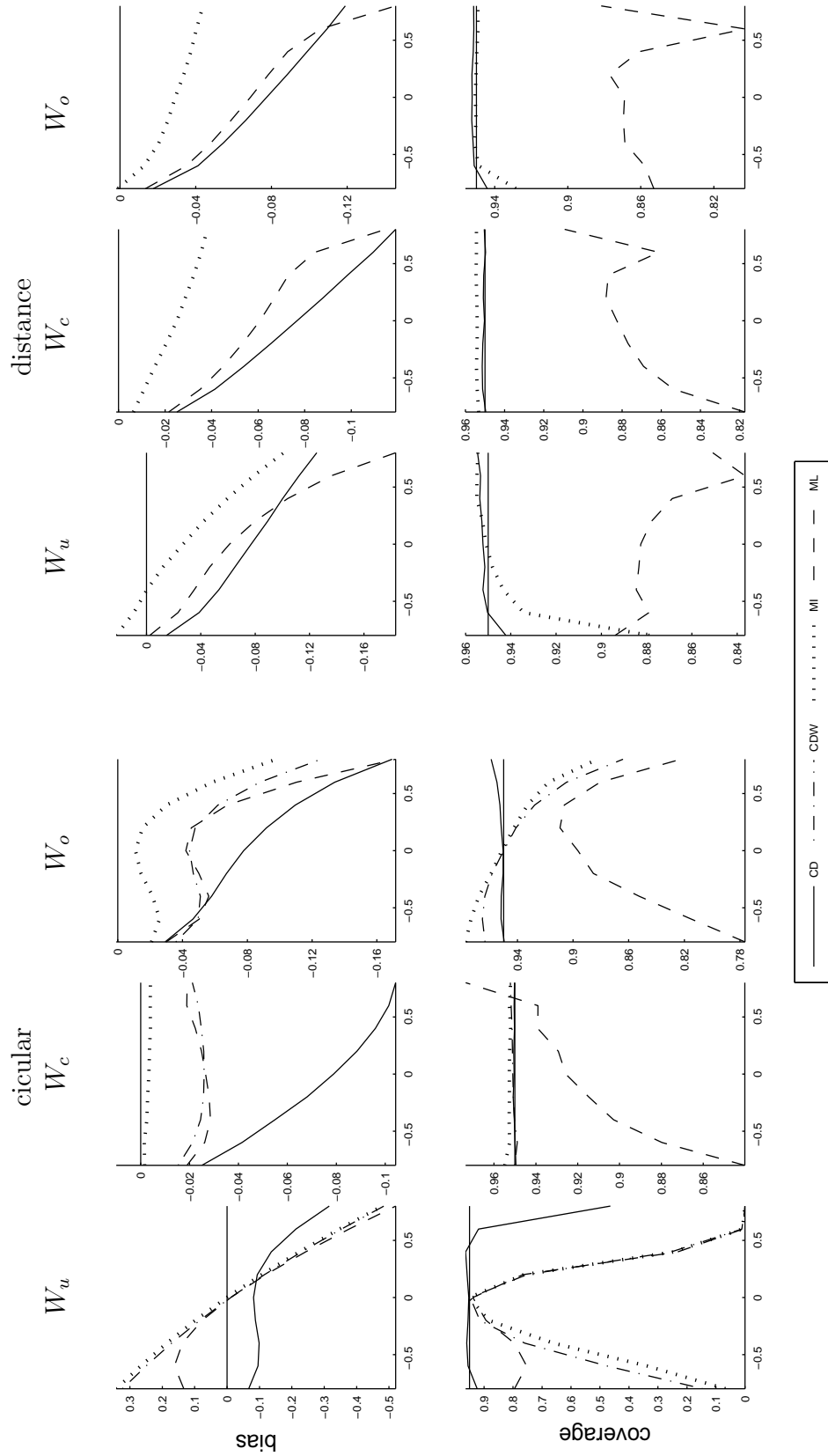


Figure 2.4: Estimation properties under SMA disturbances

For further notes see Figure 2.1.

Table 2.5: Performance of ML and HL point estimates (bias & RMSE)

$N, T$	$c$	circular $W$						distance $W$								
		SAR			SMA			SAR			SMA					
		$CD$	$CD^W$	$MI$	$ML$	$CD$	$CD^W$	$MI$	$ML$	$a$	$CD$	$MI$	$ML$	$CD$	$MI$	$ML$
		Average bias														
10, 10	.20	57.58	6.34	0.12	7.71	56.64	6.32	0.12	7.63	.10	3.55	2.36	3.19	3.18	2.37	2.60
	.40	26.40	8.15	0.30	9.37	24.02	7.59	0.30	7.78	.05	3.02	2.80	3.10	2.74	2.77	2.55
	.60	8.16	4.18	1.00	4.29	7.36	3.89	1.02	3.56	.01	2.88	2.96	3.07	2.64	2.89	2.56
10, 25	.20	23.13	2.53	0.10	3.45	23.30	2.67	0.10	3.12	.10	4.26	0.68	4.33	4.02	0.67	3.17
	.40	23.12	8.08	0.12	8.38	22.90	7.88	0.11	6.35	.05	3.40	0.86	4.01	3.30	0.82	3.09
	.60	8.52	4.58	0.33	4.78	7.94	4.30	0.34	3.56	.01	2.90	1.01	3.57	2.74	0.99	2.70
25, 10	.16	71.52	7.14	0.12	8.51	60.47	6.20	0.14	5.85	.10	13.92	0.65	4.75	13.19	0.63	3.97
	.40	12.05	3.70	0.78	4.13	10.70	3.50	0.80	3.17	.05	3.73	2.51	2.78	3.42	2.48	2.36
	.56	7.73	3.41	1.22	3.72	7.09	3.30	1.21	3.04	.01	2.56	3.71	2.61	2.42	3.55	2.29
25, 25	.16	32.20	4.32	0.10	4.89	33.77	4.33	0.09	3.78	.10	10.16	0.33	3.90	10.01	0.32	2.95
	.40	10.50	3.49	0.32	4.29	10.04	3.38	0.32	3.05	.05	3.58	0.96	3.08	3.48	0.94	2.39
	.56	7.44	3.58	0.45	4.41	7.05	3.47	0.46	3.22	.01	2.45	1.40	2.92	2.38	1.37	2.32
		Average RMSE														
10, 10	.20	2.78	1.05	8.12	0.97	2.68	1.05	8.14	0.99	.10	1.20	20.58	1.03	1.14	20.32	1.03
	.40	1.85	1.07	12.92	0.98	1.75	1.05	12.91	1.01	.05	1.12	21.93	1.07	1.08	21.53	1.02
	.60	1.51	1.11	16.13	0.99	1.42	1.07	16.09	1.02	.01	1.11	22.27	1.07	1.06	21.91	1.02
10, 25	.20	2.29	1.02	5.13	0.95	2.25	1.02	5.13	0.97	.10	1.13	12.41	1.02	1.09	12.24	1.00
	.40	1.66	1.03	8.03	0.98	1.59	1.02	8.04	0.99	.05	1.07	13.15	1.03	1.04	12.93	1.00
	.60	1.40	1.05	9.87	0.99	1.33	1.03	9.86	0.99	.01	1.05	13.38	1.03	1.03	13.09	1.00
25, 10	.16	3.20	1.06	8.01	0.97	3.00	1.06	8.02	0.99	.10	1.88	13.58	0.98	1.78	13.53	1.00
	.40	1.87	1.08	13.85	1.00	1.78	1.05	13.71	1.00	.05	1.29	20.06	1.01	1.23	19.80	1.02
	.56	1.57	1.07	16.55	1.00	1.49	1.05	16.35	1.01	.01	1.12	23.19	1.07	1.07	22.75	1.02
25, 25	.16	2.78	1.01	5.08	0.98	2.65	1.01	5.09	0.99	.10	1.71	8.55	0.97	1.64	8.53	0.98
	.40	1.73	1.04	8.48	0.99	1.66	1.03	8.45	1.00	.05	1.20	12.31	1.01	1.17	12.11	1.00
	.56	1.45	1.04	10.18	0.99	1.40	1.03	10.11	0.99	.01	1.05	14.23	1.02	1.03	13.91	1.00

The table provides average simulation results over parameter settings  $\theta \in \{-0.8, -0.6, \dots, 0.8\}$ . For  $MI$  absolute averages ( $\times 100$ ) are shown, while remaining statistics are in relation to this benchmark. For further notes see Tables 2.2 and 2.3.

Table 2.6: Surface response analysis (point and interval estimation)

Variable	bias		RMSE		CI coverage		CI length	
	$W^c$	$W^d$	$W^c$	$W^d$	$W^c$	$W^d$	$W^c$	$W^d$
$const$	-2.126**	-0.456**	0.704**	1.311**	-2.773**	-2.083**	2.029**	2.498**
$1/N_i$	-10.347**	-9.536**	-8.253**	-7.610**	-7.114**	-13.645**	-7.531**	-6.777**
$1/T_i$	4.488**	7.200**	-0.071	0.841	-3.759**	1.200	1.112**	1.800**
$\theta_i^2$	0.239**	-0.496**	-0.429**	-0.728**	1.587**	1.178**	-0.897**	-1.098**
$\theta_i^-$	0.182**	0.007	0.032	0.006	0.271**	0.315**	0.131**	0.172**
$CON_i^m$	0.909**	0.472**	0.366**	0.234**	0.389**	-0.011	0.332**	0.186**
$CON_i^h$	1.297**	0.538**	0.503**	0.216**	0.010	0.112	0.465**	0.172**
$CD_i ASY_i$	-	-	-	-	1.196**	0.810**	0.627**	0.264**
$CD_i ASY_i SMA_i$	-	-	-	-	0.111	0.099	0.044	0.077*
$CD_i ASY_i W_{ui}$	-	-	-	-	0.514**	-0.031	0.252**	0.125**
$CD_i ASY_i W_{oi}$	-	-	-	-	0.250	-0.066	0.287**	0.111*
$CD_i MC_i$	1.246**	0.054	0.628**	0.159**	-1.627**	-2.025**	0.533**	0.163**
$CD_i MC_i SMA_i$	0.011	-0.013	-0.059	-0.058	-0.195*	-0.067	0.032	0.067
$CD_i MC_i W_{ui}$	1.329**	0.300**	0.401**	0.144*	2.307**	1.212**	0.252**	0.129**
$CD_i MC_i W_{oi}$	1.019**	0.286**	0.325**	0.166**	1.023**	1.108**	0.290**	0.122**
$CD_i^W ASY_i$	-	-	-	-	-0.164	-	0.083**	-
$CD_i^W ASY_i SMA_i$	-	-	-	-	-0.010	-	0.006	-
$CD_i^W ASY_i W_{ui}$	-	-	-	-	4.755**	-	0.070	-
$CD_i^W ASY_i W_{oi}$	-	-	-	-	1.531**	-	0.575**	-
$CD_i^W MC_i$	-0.141	-	0.080	-	-0.802**	-	0.080**	-
$CD_i^W MC_i SMA_i$	0.169*	-	0.000	-	-0.140	-	0.008	-
$CD_i^W MC_i W_{ui}$	3.352**	-	1.156**	-	5.421**	-	0.083*	-
$CD_i^W MC_i W_{oi}$	1.606**	-	0.611**	-	2.107**	-	0.566**	-
$MI_i ASY_i$	-	-	-	-	-0.895**	-0.350**	0.053	0.106**
$MI_i ASY_i SMA_i$	-	-	-	-	-0.015	0.105	0.011	0.079*
$MI_i ASY_i W_{ui}$	-	-	-	-	5.490**	1.774**	0.064	0.009
$MI_i ASY_i W_{oi}$	-	-	-	-	2.298**	0.248	0.579**	0.237**
$MI_i MC_i$	-1.611**	-1.288**	0.033	-0.032	-1.095**	-1.492**	0.055	0.083*
$MI_i MC_i SMA_i$	0.163*	0.001	0.006	0.006	-0.061	0.083	0.014	0.080*
$MI_i MC_i W_{ui}$	4.608**	1.590**	1.197**	0.318**	5.710**	2.795**	0.070	0.026
$MI_i MC_i W_{oi}$	2.411**	0.630**	0.599**	0.265**	2.515**	0.757**	0.578**	0.229**
$ML_i SMA_i$	-0.012	-0.297**	0.035	-0.022	1.366**	1.953**	-0.008	-0.025
$ML_i W_{ui}$	3.242**	0.813**	1.160**	0.258**	4.004**	0.947**	0.126**	0.062
$ML_i W_{oi}$	1.726**	0.440**	0.581**	0.276**	1.216**	0.357**	0.521**	0.204**
$R^2$	0.739	0.454	0.496	0.241	0.761	0.598	0.645	0.446
observations	1440	1080	1440	1080	2520	1800	2520	1800

The table documents SR parameter estimates with distinct significance levels: \*\* 5%; \* 10%.  $R^2$  is the degree of explanation. The respective explanatory variables (interaction effects) are provided in the 3 left hand side columns. Columns  $W^c$  and  $W^d$  refer regressions for simulation experiments under circular and distance spatial weights matrices, respectively. Binary variables are defined in Table 2.1. For further notes see Tables 2.2, 2.3, 2.5 and 2.7.

( $CON^m$ ) and strong ( $CON^h$ ) spatial connectivity. Similarly, an underestimation of the spatial connectivity structure is more detrimental for bias loss in comparison with scenarios where an analyst overestimates the actual degree of connectivity.

Noting that the HL( $MI$ ) estimator offers marked bias reductions, it is of interest if this gain has to be paid with increased estimation uncertainty. Average RMSE statistics in Table 2.5 and SR results for the RMSE loss in column 6 and 7 of Table 2.6 show that the former gain does not go along with adverse second order characteristics. The ML and HL( $MI$ ) estimators are characterized by similar estimation accuracy. In summary, with particular regard to the estimation bias, HL( $MI$ ) and to some weaker extent HL( $CD^W$ ) are viable alternatives to the ML estimator. Though being robust under misspecification of the spatial layout, HL( $CD$ ) estimates suffer from power weakness of the underlying diagnostic.

#### 2.4.4 Interval estimation

Coverage estimates for the ML and MC based approaches to interval estimation are displayed in the lower panels of Figures 2.3 and 2.4. By construction, the empirical coverage of HL interval estimates does not differ significantly from the nominal 95% level under correct spatial model implementation. With regard to ML interval estimation it appears that, in particular, SMA disturbances invoke some risk to end up with overly liberal or falsely centered CIs even if the spatial structure presumed by an analyst coincides with the underlying model ( $W_c$ ). Again the case of underspecifying the spatial structure ( $W_u$ ) appears most detrimental for interval estimation, since it is characterized by massive coverage distortions at least under circular spatial weighting. For such scenarios HL( $CD$ ) intervals still offer accurate coverage probabilities empirically.

Estimates of average absolute coverage distortions in Table 2.7 confirm that ML performs markedly worse than HL( $MI$ ) implemented by means of MC critical values. For instance, regarding DGMs with distance weights matrices and SAR disturbances, average ML coverage errors exceed the benchmark statistics by factors of 1.6 ( $a = 0.10, N = 25, T = 10$ ) to 26 ( $a = 0.05, N = 10, T = 25$ ). Notably, such large relative measures reflect that in nominal terms the benchmark approach shows only slight deviations from the nominal coverage. For some instances ( $N = T = 25$ ) conditioning the HL( $MI$ ) estimator on asymptotic critical values obtains even smaller average absolute coverage loss in comparison with MC based implementations. The gathering of critical values either from MC simulations or the asymptotic distribution matters, in particular, for HL( $CD$ ) and HL( $CD^W$ ). For these estimators average coverage distortions are mostly (much) larger if critical values are taken from the asymptotic distribution. The latter merit of MC based HL estimation, however, has to be taken with care since the simulation of critical values is done by means of the true underlying error term distribution. To achieve such efficiency gains in practice, careful residual

Table 2.7: Performance of ML and HL interval estimates

$N, T$	circular $W$								distance $W$					
	$c$	$CD$		$CD^W$		$MI$		$ML$	$a$	$CD$		$MI$		$ML$
		<i>asy</i>	<i>MC</i>	<i>asy</i>	<i>MC</i>	<i>asy</i>	<i>MC</i>			<i>asy</i>	<i>MC</i>	<i>asy</i>	<i>MC</i>	
SAR: Average coverage distortions														
10,10	.20	19.19	2.03	4.41	2.99	1.24	0.10	5.04	.10	5.24	0.50	2.27	0.38	4.09
	.40	7.20	0.55	0.83	0.39	1.18	0.27	3.08	.05	4.91	0.47	2.22	0.40	4.09
	.60	19.62	1.83	4.67	0.58	1.40	0.10	11.06	.01	7.44	0.72	3.37	0.27	6.21
10,25	.20	1.70	0.13	0.72	0.97	0.57	0.43	0.92	.10	16.87	1.68	5.47	0.04	17.63
	.40	5.86	0.58	1.86	1.31	0.95	0.12	2.31	.05	18.40	1.89	8.89	0.04	26.71
	.60	5.65	0.53	2.05	1.10	1.34	0.13	4.97	.01	12.88	1.30	6.86	0.06	19.68
25,10	.16	25.05	2.36	3.12	2.69	0.92	0.09	4.01	.10	7.02	1.05	0.93	0.31	1.63
	.40	15.40	2.08	4.77	2.47	2.10	0.14	5.20	.05	40.90	6.44	7.02	0.05	18.13
	.56	24.15	3.33	7.40	0.78	3.11	0.09	10.70	.01	6.60	0.94	1.84	0.33	4.98
25,25	.16	9.36	0.61	1.64	2.45	1.16	0.08	2.20	.10	3.45	0.20	1.00	0.23	2.03
	.40	1.63	0.14	1.45	1.01	1.00	0.46	1.47	.05	6.41	0.32	1.54	0.12	5.84
	.56	8.01	0.40	2.81	0.56	0.42	0.09	7.21	.01	3.28	0.14	0.92	0.23	4.00
SMA: Average coverage distortions														
10,10	.20	19.54	0.38	4.11	2.65	1.23	0.10	23.97	.10	4.95	0.18	2.03	0.41	18.88
	.40	7.46	0.14	0.85	0.39	1.27	0.27	14.30	.05	5.78	0.12	2.56	0.36	25.51
	.60	15.53	0.30	3.45	0.48	1.50	0.13	42.00	.01	9.22	0.20	4.06	0.22	43.25
10,25	.20	2.35	0.40	0.75	0.81	0.72	0.34	3.66	.10	19.64	3.11	7.56	0.04	70.08
	.40	5.93	0.83	1.89	1.18	1.23	0.13	7.79	.05	13.96	1.84	6.33	0.06	55.25
	.60	7.50	1.01	2.38	1.25	1.39	0.11	21.70	.01	7.21	1.11	3.41	0.11	28.80
25,10	.16	29.07	1.42	3.32	3.32	1.06	0.08	16.61	.10	7.33	2.00	0.90	0.29	12.77
	.40	15.27	0.71	4.75	2.30	2.16	0.15	22.87	.05	82.96	7.35	14.52	0.03	297.87
	.56	24.24	1.17	7.44	0.58	2.94	0.09	53.61	.01	21.64	1.01	6.13	0.10	96.93
25,25	.16	8.70	0.59	1.26	2.08	0.90	0.09	7.11	.10	3.37	0.30	1.00	0.24	7.66
	.40	1.84	0.13	1.50	1.07	1.04	0.43	3.28	.05	5.62	0.40	1.27	0.14	22.83
	.56	11.97	0.92	3.80	1.18	0.70	0.07	36.92	.01	4.46	0.33	1.48	0.18	23.71
SAR: Average CI length														
10,10	.20	2.69	2.36	1.05	1.06	1.00	32.56	0.96	.10	1.20	1.03	1.03	80.35	0.87
	.40	1.80	1.57	1.04	1.04	1.00	50.80	0.95	.05	1.10	0.95	1.05	85.42	0.85
	.60	1.44	1.25	1.04	1.02	1.00	64.09	0.92	.01	1.08	0.94	1.06	86.71	0.85
10,25	.20	2.32	2.25	1.00	1.02	0.99	20.22	0.96	.10	1.09	1.05	1.01	47.11	0.97
	.40	1.58	1.53	1.02	1.02	1.00	31.41	0.97	.05	1.04	1.00	1.01	49.47	0.96
	.60	1.30	1.26	1.02	1.01	1.00	38.84	0.96	.01	1.02	0.99	1.02	50.15	0.96
25,10	.16	3.13	2.64	1.05	1.05	1.00	31.51	0.98	.10	2.09	1.70	1.00	53.33	0.94
	.40	1.91	1.60	1.05	1.03	1.00	52.11	0.96	.05	1.39	1.13	1.02	77.25	0.88
	.56	1.59	1.34	1.05	1.02	1.01	62.49	0.94	.01	1.10	0.92	1.06	91.46	0.82
25,25	.16	2.62	2.52	1.02	1.02	1.00	19.82	0.99	.10	1.63	1.57	1.00	32.99	0.97
	.40	1.62	1.56	1.02	1.01	1.00	32.76	0.98	.05	1.17	1.12	1.01	46.18	0.96
	.56	1.37	1.32	1.02	1.01	1.00	38.83	0.97	.01	1.03	0.99	1.02	51.99	0.96

The table provides average simulation results over parameter settings  $\theta \in \{-0.8, -0.6, \dots, 0.8\}$ . Absolute performance statistics ( $\times 100$ ) are shown for HL( $MI$ ) implemented by means of MC based critical values. All remaining results are given in relation to this benchmark. For further notes see Tables 2.2, 2.3 and 2.5.



diagnosis is required including a check of the conditional distribution of the data. Therefore, one might be more interested in a comparison of ML intervals with HL counterparts derived from the asymptotic distribution. Doing so, it turns out that the latter have uniformly a clear lead over the former if the spatial weights matrix is of the circular type.

Since HL estimation can offer substantial coverage improvements in relation to ML, it is immediate to investigate if this advantage comes at a cost of (markedly) less precise estimates. As documented in the lower panel of Table 2.7 ML interval estimates are uniformly shorter than the HL( $MI$ ) benchmark with relative lengths between 0.82 ( $a = 0.01, N = 25, T = 10$ ) and 0.99 ( $c \approx 0.2, N = T = 25$ ). Noting an overall only slightly weakened precision of HL( $MI$ ) CIs, an analyst might prefer the benchmark approach when 'weighting' over both loss functions, coverage error and precision of interval estimates. Taking estimation bias into account, the more precise ML interval estimates do not seem to offer an attractive tradeoff, since they provide confidence in the wrong value. Further results in Table 2.7 indicate that the precision of HL( $MI$ ) intervals is almost unaffected if they are based on asymptotic critical values. Moreover, HL( $CD^W$ ) intervals are of comparable precision relative to the HL( $MI$ ) benchmark if the spatial weighting is of circular form.

SR results for absolute coverage errors and CI lengths are documented in the columns 8 to 11 of Table 2.6. The response functions confirm that for both spatial weights matrices the coverage distortions of all HL approaches to interval estimation based on MC critical values are significantly lower in comparison with the ML benchmark. Interestingly, the effects of under- or overestimating a models connectivity are similar for HL interval estimation implemented with MC based or asymptotic critical values, although the latter device features a slight lead over the former. Again the case of underestimating the spatial connectivity is more detrimental for coverage performance as an overly general spatial specification. Explaining the average length of interval estimates by means of modeling characteristics underpins that HL( $MI$ ) and HL( $CD^W$ ) are moderately inferior to ML, while HL( $CD$ ) estimates are overly wide on average.

In summary, the construction of exact (MC based) or asymptotic HL interval estimators helps to overcome coverage biases of ML interval estimates. Noting the finite sample biases of ML estimates of the spatial parameter such distortions might be driven by false centering of CIs. Misspecification of the spatial layout appears similarly detrimental for all competing approaches to interval estimation with MC testing showing more pronounced adverse effects. Noting that HL interval estimators are only slightly wider than their ML counterparts, HL( $MI$ ) and HL( $CD^W$ ) interval estimates have a clear lead over the ML estimation.

### 2.4.5 Sensitivity analysis

As documented in Table 2.8 the sensitivity analysis largely confirms the relative merits of MC testing and HL estimation. Over both 'directions' (large samples and skewed innovations) RMSE estimates are similar for HL(*MI*) and ML estimation, while the latter shows bias estimates being 2 to 12 times larger in comparison with the former. Average ML coverage errors are mostly larger than the benchmark statistics. As one might expect, under centered and standardized  $\chi^2$ -distributed innovations the bias of the HL(*MI*) estimator shrinks for larger sample sizes and less pronounced skewness. Interestingly, HL interval estimates based on MC critical values are not characterized by markedly larger coverage distortions in comparison with their asymptotic counterparts. In summary, the estimation bias and CI coverage error reduction of HL inference are also effective in larger sample dimensions and under skewed innovations.

## 2.5 Conclusions

In this paper, Hodges-Lehmann (HL) point and interval estimators for the spatial parameter in static panel models with spatially autoregressive or moving average error terms are proposed. Since such estimators are obtained from the inversion of spatial correlation diagnostics, several implementation issues as, for instance, the choice of spatial correlation tests or the gathering of critical values are discussed. By means of a simulation study the finite sample characteristics of HL estimation are described and contrasted against their commonly used ML counterparts.

The simulation results reveal a likelihood of distorted type one error probabilities of common correlation tests in finite samples, especially under strongly connected spatial units. MC testing is suggested for the diagnosis of spatial correlation. Moran's *I* (*MI*) obtains higher power in comparison with rival diagnostics. While it is characterized by similar RMSE loss, the HL point estimator based on the inversion of *MI* outperforms the ML estimator in terms of estimation bias. Moreover, opposite to ML interval estimates, the coverage of MC interval estimates appears largely unaffected by variations of the spatial parameter.

As a main caveat of MC based critical values one could regard the requirement of a fully correct stochastic model used for simulation purposes. Although, simulation results reveal a robust performance of MC testing under misspecification of the conditional distribution, the approach is at risk to deliver false critical values in such situations. Thus, an analyst should accompany MC testing with thorough residual diagnosis, including a test of a model's conditional distribution. As an alternative to exact MC based testing, however, HL estimation can be based on asymptotic *p*-values which also improves upon the bias of ML estimation. Moreover, one might derive HL estimators from parametric or even non-parametric bootstrap procedures as proposed by Lin et al. (2011) for the case of *MI*. The potential merits of resampling seems to

Table 2.8: Sensitivity Analysis

$N, T$	c	$\epsilon_{it} \sim \mathcal{N}(0, 1)$						$N, T$	c	$\epsilon_{it} = (\nu_{it} - 5)/\sqrt{10}, \nu_{it} \sim \chi^2(5)$						$\epsilon_{it} = (\nu_{it} - 15)/\sqrt{30}, \nu_{it} \sim \chi^2(15)$						
		CD		CDW		MI				ML		CDW		MI		ML		CDW		MI		ML
		asy	MC	asy	MC	asy	MC			asy	MC	asy	MC	asy	MC	asy	MC	asy	MC	asy	MC	
Size estimates under $H_0 : \theta = 0$																						
10, 50	.40	4.77	4.99	4.90	4.79	4.97	4.82	-	10, 25	.40	5.10	4.99	4.98	4.80	-	5.15	5.10	5.10	5.10	5.06	-	
10, 100	.40	5.02	5.25	5.23	5.14	5.23	5.14	-	10, 100	.40	4.64	4.68	4.71	4.75	-	5.44	5.42	<b>5.48</b>	5.41	5.41	-	
50, 10	.08	<b>3.04</b>	5.26	4.77	4.65	4.91	4.78	-	25, 10	.16	4.87	4.99	4.89	4.80	-	4.90	4.73	4.90	5.00	5.00	-	
100, 10	.04	<b>2.67</b>	4.73	5.11	5.08	5.26	5.16	-	100, 10	.04	5.26	5.05	4.56	4.59	-	5.26	5.14	5.26	5.15	5.15	-	
Average bias over parameter settings $\theta \in \{-0.8, -0.6, \dots, 0.8\}$																						
10, 50	.40	15.08	5.10	5.10	5.10	0.09	4.97	4.97	10, 25	.40	4.85	0.19	4.87	4.87	4.87	5.62	0.17	0.17	0.17	5.19	5.19	
10, 100	.40	6.36	2.53	2.53	2.53	0.11	2.69	2.69	10, 100	.40	2.09	0.16	2.01	2.01	2.01	8.35	0.02	0.02	0.02	9.39	9.39	
50, 10	.08	70.59	4.32	4.32	4.32	0.12	3.93	3.93	25, 10	.16	1.56	0.32	3.30	3.30	3.30	5.34	0.14	0.14	0.14	6.19	6.19	
100, 10	.04	89.16	2.46	2.46	2.46	0.10	3.04	3.04	100, 10	.04	1.76	0.13	2.64	2.64	2.64	3.52	0.02	0.02	0.02	12.32	12.32	
Average RMSE over parameter settings $\theta \in \{-0.8, -0.6, \dots, 0.8\}$																						
10, 50	.40	1.63	1.00	1.00	1.00	5.54	0.96	0.96	10, 25	.40	1.03	7.93	0.98	0.98	0.98	1.03	7.97	7.97	7.97	0.98	0.98	
10, 100	.40	1.60	1.01	1.01	1.01	3.94	0.96	0.96	10, 100	.40	1.01	3.85	0.97	0.97	0.97	1.00	3.93	3.93	3.93	0.96	0.96	
50, 10	.08	4.60	1.06	1.06	1.06	5.56	0.97	0.97	25, 10	.16	1.04	8.08	0.97	0.97	0.97	1.05	8.05	8.05	8.05	0.98	0.98	
100, 10	.04	6.57	1.05	1.05	1.05	3.98	0.98	0.98	100, 10	.04	1.01	3.94	0.97	0.97	0.97	1.02	4.01	4.01	4.01	0.97	0.97	
Average coverage distortions over parameter settings $\theta \in \{-0.8, -0.6, \dots, 0.8\}$																						
10, 50	.40	5.88	1.47	2.76	3.29	0.85	0.04	9.35	10, 25	.40	0.88	0.37	0.32	0.14	3.34	0.62	0.61	0.57	0.17	2.85	2.85	
10, 100	.40	0.30	1.39	1.16	1.29	1.27	0.19	1.54	10, 100	.40	1.74	1.29	1.62	0.21	1.06	0.88	1.03	1.01	0.51	0.90	0.90	
50, 10	.08	9.82	2.83	1.09	1.49	0.52	0.20	0.73	25, 10	.16	1.01	0.37	1.18	0.11	2.60	1.88	3.76	2.35	0.04	10.62	10.62	
100, 10	.04	11.67	1.25	0.55	0.59	1.30	0.20	1.66	100, 10	.04	0.65	0.44	1.13	0.38	0.77	0.88	0.82	0.94	0.27	1.51	1.51	
Average CI length over parameter settings $\theta \in \{-0.8, -0.6, \dots, 0.8\}$																						
10, 50	.40	1.56	1.54	1.01	1.01	22.09	0.99	0.99	10, 25	.40	1.00	1.00	1.00	31.39	0.97	1.01	1.01	1.00	31.40	0.98	0.98	
10, 100	.40	1.55	1.54	1.00	1.00	15.56	0.99	0.99	10, 100	.40	1.00	1.00	1.00	15.56	0.99	1.00	1.00	1.00	15.55	0.99	0.99	
50, 10	.08	4.54	3.78	1.05	1.05	22.18	0.99	0.99	25, 10	.16	1.03	1.03	1.00	31.52	0.98	1.04	1.04	1.00	31.52	0.98	0.98	
100, 10	.04	6.53	5.41	1.05	1.05	15.63	0.99	0.99	100, 10	.04	1.02	1.03	1.00	15.63	1.00	1.04	1.04	1.00	15.62	1.00	1.00	

The table provides simulation results under a circular spatial weights matrix with  $J = 2$  and SAR errors. Notably keeping  $J$  fixed implies that the connectivity in circular states vanishes as  $N \rightarrow \infty$  ensuring that the number of neighbors attached to cross sectional entities is limited (see assumption (ii) in Section 2.2.1). Power results are fully in line with those reported in Table 2.3 and, thus, not shown. For space considerations and due to its inferior performance results for  $CD$  under nonnormality are not provided. Further notes are made in Tables 2.2, 2.3, 2.5 and 2.7.

be an interesting future direction of research on the implementation of HL estimators.

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## 2.6 References

- Anselin, L. and S. Rey (1991). Properties of tests for spatial dependence in linear regression models. *Geographical Analysis* 23, 112-131.
- Anselin, L. and R. Moreno (2003). Properties of tests for spatial error components. *Regional Science and Urban Economics* 33(5), 595-618.
- Anselin, L., J. Le Gallo, and H. Jayet (2008). Spatial panel econometrics. In Matyas, L. and P. Sevestre: *The econometrics of panel data, fundamentals and recent developments in theory and practice* (3rd Edition ed.) (Berlin Heidelberg: Springer - Verlag), 625-660.
- Bao, Y. and A. Ullah (2007). Finite sample properties of maximum likelihood estimator. *Journal of Econometrics* 137, 396-413.
- Baumont C. (2009). Spatial effects of urban public policies on housing values. *Papers in Regional Science* 88, 301-326.
- Billon M., R. Ezcurra and F. Lera-López (2009). Power calculations for global and local Moran's. *Growth and Change* 40(1), 54-84.
- Bivand, R., W.G. Müller, and M. Reder (2009). Power calculations for global and local Moran's. *Computational Statistics & Data Analysis* 53(8), 2859-2872.
- Bivand, R. and S. Szymanski (2000). Modelling the spatial impact of the introduction of Compulsory Competitive Tendering. *Regional Science and Urban Economics* 30(2), 203-219.
- Bordignon, M., F. Cerniglia and F. Revelli (2003). In search of yardstick competition: a spatial analysis of Italian municipality property tax setting. *Journal of Urban Economics* 54(2), 199-217.
- Breusch, T.S. and A.R. Pagan (1980). The Lagrange Multiplier test and its applications to model specification in econometrics. *Review of Economic Studies* 47(1), 239-254.

- Bröcker, J., U. Kretschmer, C. Schürmann, K. Spiekermann, D. Stelder and M. Wegener (2002). The IASON common spatial database. IASON Deliverable 3, Institute for Regional Research, Christian Albrechts University Kiel, Institute for Spatial Planning, University Dortmund.
- Burridge, P. (1980). On the Cliff-Ord-Test for spatial autocorrelation. *Journal of the Royal Statistical Society B* 42, 107-108.
- Cliff, A. and J. Ord (1981). Spatial process: Models and applications (London: Pion).
- Coudin E. and Dufour J.-M. (2011). Robust sign-based and Hodges-Lehmann estimators in linear median regressions with heterogenous serially dependent errors. CIRANO Scientific Publication No. 2011s-24.
- Das, D., H.H. Kelejian and I.R. Prucha (2003). Finite sample properties of estimators of spatial autoregressive models with autoregressive disturbances. *Papers in Regional Science* 82(1), 1-26.
- Dufour, J.-M. (2006). Monte Carlo tests with nuisance parameters: A general approach to finite-sample inference and nonstandard asymptotics. *Journal of Econometrics* 133(2), 443-477.
- Dufour, J.-M. (1990). Exact tests and confidence sets in linear regressions with autocorrelated errors. *Econometrica* 58(2), 475-94.
- Dufour, J.-M. and L. Khalaf (2002). Exact tests for contemporaneous correlation of disturbances in seemingly unrelated regressions. *Journal of Econometrics* 106, 143-170.
- Egger, P., M. Larch, M. Pfaffermayr and J. Walde (2009). Small sample properties of maximum likelihood versus generalized method of moments based tests for spatially autocorrelated errors. *Regional Science and Urban Economics* 39(6), 670-678.
- Eurostat: Regions (1999). Nomenclature of territorial units for statistics-NUTS. Office for Official Publications of the European Communities, Luxembourg.
- Farber, S., A. Páez and E. Volz (2009). Topology, dependency tests and estimation bias in network autoregressive models. In Páez, A., J. Le Gallo, R. Buliung and S. Dall'Erba: *Advances in Spatial Sciences* (Berlin: Springer Verlag), 29-57.
- Fingleton, B. and E. López-Bazo (2006). Empirical growth models with spatial effects. *Papers in Regional Science* 85(2), 177-198.
- Florax, R. and S. Rey (1995). The impacts of misspecified spatial interaction in linear regression models. In Anselin, L. and R.J.G.M. Florax: *New Directions in spatial econometrics* (Berlin: Springer Verlag), 111-135.
- Frees, E.W. (1995). Assessing cross-sectional correlation in panel data. *Journal of*

*Econometrics* 69, 393-414.

Friedman, M. (1937). The use of ranks to avoid the assumption of normality implicit in the analysis of variance. *Journal of the American Statistical Association* 32, 675-701.

Harvey, A.C. and G.D.A. Phillips (1982). Testing for contemporaneous correlation of disturbances in systems of regression equations. *Bulletin of Economic Research* 34(2), 79-81.

Hodges Jr., J.L., and E.L. Lehmann (1983). Hodges-Lehmann estimators. In Johnson, N.L., S. Kotz and C. Read: *Encyclopedia of Statistical Sciences*, vol. 3. (Wiley, New York), 642-645.

Kelejian and I.R. Prucha (2001). On the asymptotic distribution of the Moran I test statistic with applications. *Journal of Econometrics* 104(2), 219-257.

Kelejian, H.H. and I.R. Prucha (1999). A generalized moments estimator for the autoregressive parameter in a spatial model. *International Economic Review* 40(2), 97-130.

Kelejian and I.R. Prucha (1998). A generalized spatial two-stage least squares procedure for estimating a spatial autoregressive model with autoregressive disturbances. *The Journal of Real Estate Finance and Economics* 17(1), 99-121.

Lee, L.-F. (2004). Asymptotic distributions of Quasi-Maximum Likelihood estimators for spatial autoregressive models. *Econometrica* 72(6), 1899-1925.

Lin, K.-P., Z.-H. Long and B. Ou (2011). The size and power of bootstrap tests for spatial dependence in a linear regression model. *Computational Economics* 38(2), 153-171.

López-Bazo, E., E. Vayá and M. Artís (2004). Externalities and growth: Evidence from european regions. *Journal of Regional Science* 44(1), 43-73.

Lundberg J. (2006). Using spatial econometrics to analyse local growth in Sweden. *Regional Studies* 40(3), 303-316.

Martellosio, F. (2010). Power properties of invariant tests for spatial autocorrelation in linear regression. *Econometric Theory* 26, 152-186.

Mizruchi, M.S. and E.J. Neuman (2008). The effect of density on the level of bias in the network autocorrelation model. *Social Networks* 30, 190-200.

Neuman, E.J. and M.S. Mizruchi (2010). Structure and bias in the network autocorrelation model. *Social Networks* 32(4), 290-300.

Pesaran, M.H. (2004). General diagnostic tests for cross section dependence in panels. *Cambridge Working Papers in Economics* No. 1229, Faculty of Economics University

of Cambridge.

Pesaran M.H., A. Ullah and T. Yamagata (2008). A bias-adjusted LM test of error cross-section independence. *The Econometrics Journal* 11(1), 105-127.

Revelli, F. (2002). Testing the tax mimicking versus expenditure spill-over hypotheses using English data. *Applied Economics* 34(14), 1723-31.

Rey, S.J. and B.D. Montouri (1999). US regional income convergence: A spatial econometric perspective. *Regional Studies* 33(2), 143-156.

Smith, T.E. (2009). Estimation bias in spatial models with strongly connected weight matrices. *Geographical Analysis* 41(3), 307-332.

Tsay, W.-J. (2004). Testing for contemporaneous correlation of disturbances in seemingly unrelated regressions with serial dependence. *Economics Letters* 83, 69-76.

Villaverde, J. (2005). Provincial convergence in Spain: a spatial econometric approach. *Applied Economics Letters* 12(11), 697-700.

## Chapter 3

# On the effect of prospective payment on local hospital competition in Germany

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### Abstract

The introduction of prospective hospital reimbursement based on diagnosis related groups (DRG) in 2004 has been a conspicuous attempt to increase hospital efficiency in the German health sector. As a consequence of the reform a rise of competition for (low cost) patients could be expected. In this paper the competition between hospitals, quantified as spatial spillover estimates of hospital efficiency, is analyzed for periods before and after the reform. We implement a two-stage efficiency model that allows for spatial interdependence among hospitals. Hospital efficiency is determined by means of non-parametric and parametric econometric frontier models. We diagnose a significant increase of negative spatial spillovers characterizing hospital performance in Germany, and thus, confirm the expected rise of competition.

JEL-Classification: C21, D61, I11, I18

Keywords: Hospital efficiency, stochastic frontier analysis, data envelopment analysis, spatial analysis, diagnosis related groups

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## Chapter 4

# Hospital efficiency under prospective reimbursement schemes: An empirical assessment for the case of Germany

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### Abstract

The introduction of prospective hospital reimbursement based on diagnosis related groups (DRG) has been a conspicuous attempt to decelerate the steady increase of hospital expenditures in the German health sector. In this work, the effect of the financial reform on hospital efficiency is subjected to empirical testing by means of two complementary testing approaches. On the one hand, we apply a two-stage procedure based on non-parametric efficiency measurement. On the other hand, a stochastic frontier model is employed that allows a one-step estimation of both production frontier parameters and inefficiency effects. To identify efficiency gains as a consequence of changes in the hospital incentive structure, we account for technological progress, spatial dependence and hospital heterogeneity. The results of both approaches do not reveal any increase in overall efficiency after the DRG reform. In contrast, a significant decline in overall hospital efficiency over time is observed.

JEL-Classification: C21, D61, I11, I18

Keywords: Hospital efficiency, stochastic frontier analysis, data envelopment analysis, spatial analysis, diagnosis related groups

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# Chapter 5

## Concluding Remarks

This dissertation consists of three articles considering spatial econometrics. While the first article in Chapter 2 is dealing with methodological aspects of the estimation of spatial error models, the articles in the Chapters 3 and 4 apply spatial econometric techniques to either estimate spatial spillovers of hospital efficiency or handle appropriately the spatial dependence in the hospital data.

In Chapter 2, Hodges-Lehmann (HL) type point and interval estimators for the spatial parameter in static panel models with spatially autoregressive or moving average disturbances are proposed. HL estimators are implemented by means of 'inverting' common diagnostics for spatial correlation. Exact inference is implemented by means of Monte Carlo (MC) testing. A simulation study shows that HL confidence intervals are characterized by less size distortions and appear more robust against spatial connectivity in comparison with Maximum Likelihood (ML) interval estimates. Moreover, the bias of the HL estimator is markedly smaller than its ML counterpart. As opposed to the spatial lag model, a reduced bias of the spatial parameter estimate has no impact on the bias of regression coefficient estimates in the considered spatial error model. The implementation of HL estimation on the spatial lag model can be regarded as an interesting future direction of research. As mentioned in the paper within the spatial lag model typical diagnostics, e.g. the Lagrange Multiplier lag test in Anselin et al. (2008), lack pivotalness. Alternatively to MC testing, HL estimation can be based on asymptotic  $p$ -values or critical values determined by parametric or even non-parametric bootstrap procedures.

In Chapter 3, the effect of prospective payment system based on diagnosis related groups (DRG) on local competition of hospitals is analyzed. The robustness of the empirical findings is addressed by comparative applications of the non-parametric *Data Envelopment Analysis* (DEA) and parametric *Stochastic Frontier Analysis* (SFA). The results reveal an increase in the magnitude of negative spatial spillovers after the DRG reform. This is in line with an expected rise of competition for low cost patients invoked by the announcement or advent of the new financing system. For future research it might be important to analyze the effects of the increased competition on the treatment of patients and the overall hospital efficiency. If the competitive behavior in the hospital market encourage hospitals to select the most lucrative patients, an

## *Chapter 5 - Concluding Remarks*

equal access to inpatient care is no longer guaranteed. Moreover, if higher competition does not lead to a better hospital performance (as theoretically expected in a well functioning market), the justification of the DRG reform should be reconsidered.

The study of Chapter 4 might obtain some insights for the latter issue. In this work, the effect of prospective payment on overall hospital efficiency is subjected to empirical testing. To identify efficiency gains as a consequence of changes in the hospital incentive structure, technological progress, spatial dependence and hospital heterogeneity are taken into account. Similar to the previous study in Chapter 3, two complementary testing approaches are applied, i.e. a two-stage procedure based on non-parametric efficiency measurement and a parametric stochastic frontier model that allows a one-step estimation of both production frontier parameters and inefficiency effects. The results of both approaches do not reveal any increase in overall efficiency after the DRG reform. In contrast, a significant decline in overall hospital efficiency over time is observed. This might suggest that the increased competition has not a positive effect on hospital efficiency. However, there might be various alternative reasons to explain the decline of hospital performance, e.g. a dominance of opportunistic practices. The identification of these practices and their effects on hospital efficiency, the treatment of patients and the quality of care could deliver important guidelines for future political reforms targeting a reduction of inefficiencies, and an equal and qualitative treatment of patients in the hospital sector.

From an econometric perspective some remarks regarding the adopted efficiency approaches are in order. Firstly, the two-stage approach of SFA efficiency scores applied in the study of Chapter 3 is generally not recommended because of an inherent bias problem on both stages (Kim and Schmidt, 2008, Wang and Schmidt, 2002). Secondly, the applied second-stage treatment of DEA efficiency scores (in Chapter 3) and its Malmquist index decomposition (Chapter 4) might be inefficient (Simar and Wilson, 2007). Finally, the one-step treatment of the SFA model in Chapter 4 incorporates spatial dependence in form of spatial clusters by specifying region-specific random effects. However, the SFA model does not account for two distinct channels of spatial dependence simultaneously as it is observed in the two-stage models with spatial error terms and lag dependence (SARAR). Hence, it would be a valuable direction of future research to focus on the development of SARAR stochastic frontier models. However, for the present studies in Chapter 3 and Chapter 4 the clear results regarding the increase in the magnitude of negative spatial spillovers and the decline in overall hospital efficiency over time suggest that the inherent problems of the employed approaches do not have a significant influence on the empirical outcome of these studies.

## References

- Kim, M. and P. Schmidt (2008). Valid tests of whether technical inefficiency depends on firm characteristics. *Journal of Econometrics* 144(2), 409-427.
- Simar, L. and P. W. Wilson (2007). Estimation and inference in two-stage, semi-parametric models of production processes. *Journal of Econometrics* 136(1), 31-64.
- Wang, H.-J., and P. Schmidt (2002). One-Step and Two-Step estimation of the effects of exogenous variables on technical efficiency levels. *Journal of Productivity Analysis* 18(2), 129-144.

# Eidesstattliche Erklärung

Ich erkläre hiermit an Eides Statt, dass ich, abgesehen von den in der Einleitung präzisierten Kooperationen mit Herrn Herwartz meine Doktorarbeit “Essays on spatial econometrics: Hodges-Lehmann estimators and hospital efficiency” selbständig und ohne fremde Hilfe angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehrenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen nach den mir angegebenen Richtlinien zitiert habe.

Kiel, den 23.05.2013

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(Christoph Strumann)

# Lebenslauf

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