

High-Frequency Analysis and Moment-Matching Estimation of the Baseline New-Keynesian Model

Inaugural-Dissertation
zur Erlangung des akademischen Grades eines
Doktors der Wirtschafts- und Sozialwissenschaften
der Wirtschafts- und Sozialwissenschaftlichen Fakultät
der Christian-Albrechts-Universität zu Kiel

vorgelegt von

Diplom-Volkswirt Stephen Sacht
aus Eckernförde, geb. 24.06.1982

Kiel, Februar 2014

Gedruckt mit Genehmigung der Wirtschafts- und Sozialwissenschaftlichen Fakultät der
Christian-Albrechts-Universität zu Kiel

Dekan: Professor Horst Raff, Ph.D.

Erstberichterstattender: Professor Dr. Hans-Werner Wohltmann
Zweitberichterstattender: Professor Dr. Maik Wolters

Tag der Abgabe der Arbeit: 25. November 2013
Tag der mündlichen Prüfung: 29. Januar 2014

Publication Overview

The main body of this thesis consists on the following publications:

Chapter 2 represents an independent and single-handed research paper. The latter is entitled *Analysis of Various Shocks within the High-Frequency Versions of the Baseline New-Keynesian Model* and has been published as the Economics Working Paper No. 2014-02, Department of Economics, University of Kiel, January 2014. It can be downloaded via <http://ideas.repec.org/p/zbw/cauewp/201402.html>

Chapter 3 represents an independent and single-handed research paper. The latter is entitled *Optimal Monetary Policy Responses and Welfare Analysis within the High-Frequency Framework* and has been published as the Economics Working Paper No. 2014-03, Department of Economics, University of Kiel, January 2014. It can be downloaded via <http://ideas.repec.org/p/zbw/cauewp/201403.html>

Chapter 4 represents a research paper, which had been written under collaboration with Steffen Ahrens. The paper is entitled *Estimating a High-Frequency Phillips Curve* and has been published in *Empirical Economics*, Vol. 46(2), pp. 607-628, March 2014. My own substantial contribution relies on the entire theoretical derivation of this kind of New-Keynesian Phillips Curve in higher frequencies, writing and literature research. The paper can be downloaded via <http://link.springer.com/article/10.1007/s00181-013-0684-7> (DOI: 10.1007/s00181-013-0684-7). Hereby note that the ‘/’ in front of the ‘%’ must be erased.

Chapter 5 represents a research paper, which had been written under collaboration with Reiner Franke and Tae-Seok Jang. The paper is entitled *Moment Matching versus Bayesian Estimation: Backward-Looking Behaviour in the New-Keynesian Baseline Model* and has been published as the Economics Working Paper No. 2012-08, Department of Economics, University of Kiel, September 2012. My own substantial contribution relies on the entire Bayesian estimations (including data processing), writing and literature research. Furthermore, for this thesis, the Chapter has been extended by the description of the Bayesian approach (subsection 5.3) and the discussion with respect to sufficiency and transparency of the moment-matching approach (subsection 5.4). Both subsections had been written completely by my own. The original paper can be downloaded via <http://ideas.repec.org/p/zbw/cauewp/201208.html>

Chapter 6 represents an independent and single-handed research paper. The latter is entitled *Identification of Prior Information via Moment-Matching* and has been published as the Economics Working Paper No. 2014-04, Department of Economics, University of Kiel, January 2014. It can be downloaded via <http://ideas.repec.org/p/zbw/cauewp/201404.html>

Chapter 7 represents a research paper, which had been written under collaboration with Tae-Seok Jang. The paper is entitled *The Rational Expectation Hypothesis versus Animal Spirits within the New-Keynesian Framework: Empirical Evidence from Moment-Matching*

and has been published as the Kiel Working Paper No. 1798 entitled “Identification Of Animal Spirits In A Bounded Rationality Model: An Application To The Euro Area”, Kiel Institute for the World Economy, October 2012. My own substantial contribution relies on the estimation of both model specifications (partly and especially with respect to the underlying source code), writing (especially with respect to the description of the model specifications, the economic interpretation of the results and the comparison between the moment-matching and alternative estimation approaches in the case of non-linear models) as well as literature research. The original paper can be downloaded via <http://ideas.repec.org/p/zbw/cauewp/201212.html>

Contents

List of Figures	V
List of Tables	VIII
List of Acronyms	IX
Acknowledgements	XI
1 Introduction	1
1.1 Topics and Literature Review	1
1.1.1 High-Frequency Analysis of the Baseline New-Keynesian Model	3
1.1.2 Moment-Matching Estimation of the Baseline New-Keynesian Model	4
1.2 Outline of Part One	7
1.3 Outline of Part Two	12
Part I: High-Frequency Analysis within the New-Keynesian Framework	16
2 Analysis of Various Shocks within the High-Frequency Versions of the Baseline New-Keynesian Model	17
2.1 Introduction	17
2.2 The Concept of a High-Frequency Economy	19
2.3 The Impacts of Frequency-Dependent Persistence	25
2.4 Numerical Analysis of the Baseline NKM	29
2.4.1 Nominal Interest Rate Shock	31
2.4.2 Technology Shock	33
2.4.3 Cost-Push Shock	34
2.5 Conclusion	35
2.6 Appendix A	36
2.6.1 A1: Microfoundations of the Baseline NKM in the Case $0 < h \leq 1$	36
2.6.2 A2: Solution of the Shock Process in the Case $0 < h \leq 1$	43
2.6.3 A3: Solution of the Baseline NKM in the Case $0 < h \leq 1$	44
2.6.4 A4: Impulse Response Functions (TR) in the Case $0 < h \leq 1$	47

3	Optimal Monetary Policy Responses and Welfare Analysis within the High-Frequency New-Keynesian Framework	59
3.1	Introduction	59
3.2	Optimal Monetary Policy Responses in an h -Economy	62
3.2.1	The Design of Optimal Monetary Policy	62
3.2.2	Analytical Results	63
3.3	Numerical Results: Impulse Response Functions	67
3.4	Numerical Results: Welfare Analysis	69
3.5	Conclusion	73
3.6	Appendix B	75
3.6.1	B1: Solution of the OMP Regime Discretion in the Case $0 < h \leq 1$	75
3.6.2	B2: Solution of the OMP Regime Commitment in the Case $0 < h \leq 1$	77
3.6.3	B3: Impulse Response Functions (D,C) in the Case $0 < h \leq 1$	80
3.6.4	B4: Development in the Loss Relations D/C	86
4	Estimation of a High-Frequency New-Keynesian Phillips Curve	89
Part II: Moment-Matching Estimation of the Baseline New-Keynesian Model		90
5	Moment-Matching versus Bayesian Estimation: Backward-Looking Behaviour in the Baseline New-Keynesian Model	91
5.1	Introduction	91
5.2	The Moment-Matching Estimation Approach	93
5.3	The Bayesian Approach	96
5.4	A Critical Discussion on Sufficiency and Transparency	101
5.5	The Baseline NKM in Gap Notation	103
5.6	The Great Inflation Period	105
5.6.1	On the Model's Goodness of Fit	105
5.6.2	Price Indexation versus Shock Persistence	110
5.6.3	Admitting Stronger Backward-Looking Behaviour	111
5.6.4	Which are the Central Moments for the Results?	114
5.6.5	Evaluation of the Estimated Parameters	115
5.7	The Great Moderation Period	118
5.8	Conclusion	123
5.9	Appendix D	125
5.9.1	D1: Prior Densities of the Bayesian Reference Estimations	125
5.9.2	D2: Posterior Predictive Check for GM	126
6	Identification of Prior Information via Moment-Matching	127
6.1	Introduction	127

6.2	Micro- versus Macropriors	128
6.2.1	Micro-priors: The Impact of Transformation	128
6.2.2	Macropriors: An Investigation via MoMBay	132
6.3	Comparison with other Empirical Studies	135
6.4	Conclusion	139
6.5	Appendix E	140
6.5.1	E1: Prior Densities of the MoMBay Estimations	140
6.5.2	E2: Graphical Results – Prior versus Posterior Densities	141
7	The Rational Expectation Hypothesis versus Animal Spirits within the New-Keynesian Framework: Empirical Evidence from Moment-Matching	156
7.1	Introduction	156
7.2	The Model: Rational Expectations versus Bounded Rationality	159
7.3	The Simulated Method of Moments Approach	163
7.4	On the Validity of the Moment-Matching Approach	166
7.5	Empirical Application to the Euro Area	168
7.5.1	Data	168
7.5.2	Basic Results	169
7.5.3	Economic Interpretation of the Results	170
7.5.4	Comparison with other Empirical Studies	176
7.6	Conclusion	177
7.7	Appendix F	178
7.7.1	F1: Solution of the Baseline NKM under RE and BoR	178
7.7.2	F2: Delta Method and Confidence Interval for Auto-/Cross-Covariances	180
8	Summary and Outlook	182
	Bibliography	188
	Curriculum Vitae	205
	Eidesstattliche Erklärung (Certificate of Authorship/Originality)	209

List of Figures

2.1	Frequency of price adjustments in the quarterly NKM versus an h-economy ($h = 1/3$).....	28
2.2	IRFs in case of a nominal interest rate shock ($\rho^i = \chi = \alpha = 0$)	47
2.3	IRFs in case of a nominal interest rate shock ($\rho^i = 0, \chi = \alpha = 0.5$)	48
2.4	IRFs in case of a nominal interest rate shock ($\rho^i = 0.5, \chi = \alpha = 0$)	49
2.5	IRFs in case of a nominal interest rate shock ($\rho^i = \chi = \alpha = 0.5$).....	50
2.6	IRFs in case of a technology shock ($\rho^y = \chi = \alpha = 0$)	51
2.7	IRFs in case of a technology shock ($\rho^y = 0, \chi = \alpha = 0.5$)	52
2.8	IRFs in case of a technology shock ($\rho^y = 0.5, \chi = \alpha = 0$)	53
2.9	IRFs in case of a technology shock ($\rho^y = \chi = \alpha = 0.5$)	54
2.10	IRFs in case of a cost-push shock ($\rho^\pi = \chi = \alpha = 0$)	55
2.11	IRFs in case of a cost-push shock ($\rho^\pi = 0, \chi = \alpha = 0.5$)	56
2.12	IRFs in case of a cost-push shock ($\rho^\pi = 0.5, \chi = \alpha = 0$)	57
2.13	IRFs in case of a cost-push shock ($\rho^\pi = \chi = \alpha = 0.5$)	58
3.1	Development of the weights $\psi(h)$ and $\xi(h)$	66
3.2	IRFs in case of a cost-push shock under the Regime Discretion ($\rho^\pi = 0$)	80
3.3	IRFs in case of a cost-push shock under the Regime Discretion ($\rho^\pi = 0.5$)	81
3.4	IRFs in case of a cost-push shock under the Regime Commitment ($\rho^\pi = 0$) ...	82
3.5	IRFs in case of a cost-push shock under the Regime Commitment ($\rho^\pi = 0.5$) .	83
3.6	Daily ($h = 1/90$) IRFs in the case of a cost-push shock under the Regimes TR, D and C ($\rho^\pi = 0$)	84
3.7	Daily ($h = 1/90$) IRFs in the case of a cost-push shock under the Regimes TR, D and C ($\rho^\pi = 0.5$)	85

3.8	Development and maximum of the loss relation D/C ($\rho^\pi = 0, \rho^\pi = 0.5, 0 \leq \rho^\pi < 1$)	86
3.9	Development of the loss relation D/C ($\rho^\pi = 0, \rho^\pi = 0.5, 0.5 \leq \theta \leq 0.9$)	87
3.10	Maximum loss relation D/C ($0 \leq \rho^\pi < 1, 0.5 \leq \theta \leq 0.9$)	88
5.1	Estimated versus empirical covariance profiles (GI)	107
5.2	Posterior predictive check for estimation BR (GI)	109
5.3	MM estimation of the model under exogenous variations of α (GI)	110
5.4	Covariance profiles of model F from Table 5.2	113
5.5	Selected frequency distributions of the bootstrap re-estimations of model A (GI)	117
5.6	Estimated versus empirical covariance profiles (GM)	121
5.7	MM estimation of the model under exogenous variations of α (GM)	122
5.8	Posterior predictive check for estimation BR (GM)	126
6.1	Posterior density estimates for BR (GI)	142
6.2	Posterior density estimates for T1 (GI)	143
6.3	Posterior density estimates for T2 (GI)	144
6.4	Posterior density estimates for V0 (GI)	145
6.5	Posterior density estimates for V1 (GI)	146
6.6	Posterior density estimates for V2 (GI)	147
6.7	Posterior density estimates for V3 (GI)	148
6.8	Posterior density estimates for BR (GM)	149
6.9	Posterior density estimates for T1 (GM)	150
6.10	Posterior density estimates for T2 (GM)	151
6.11	Posterior density estimates for V0 (GM)	152
6.12	Posterior density estimates for V1 (GM)	153
6.13	Posterior density estimates for V2 (GM)	154
6.14	Posterior density estimates for V3 (GM)	155

7.1	Dynamics in the output and inflation gap (RE versus BoR model).....	173
7.2	Model covariance profiles in the Euro Area	174

List of Tables

2.1	Numerical parameter scenario	30
3.1	Selected values of the loss relations for $\rho^\pi = 0$	70
3.2	Selected values of the loss relations for $\rho^\pi = 0.5$	71
5.1	Parameter estimates for GI	106
5.2	Estimations when the constraints on α and χ are dropped (GI)	112
5.3	Selected estimations with a small set of moments	114
5.4	Main correlations between the re-estimated parameters of model A in GI and GM	118
5.5	Parameter estimates for GM	119
5.6	Prior densities of the BR estimations in Tables 5.1 and 5.5	125
6.1	Bayesian estimation: sensitivity analysis regarding the prior information (GI) ..	131
6.2	Bayesian estimation: sensitivity analysis regarding the prior information (GM) ..	133
6.3	Construction of macropriors	137
6.4	Prior densities of the MoMBay estimations (GI)	140
6.5	Prior densities of the MoMBay estimations (GM)	140
7.1	Estimation results (RE versus BoR model)	171

List of Acronyms

BoR	Bounded Rationality (model)
BR	Bayesian Reference (estimation)
CPI	Consumer Price Index
ECB	European Central Bank
FCP	Frequency(-dependent) Contraction Persistence
FED	(US) Federal Reserve Bank
FHP	Frequency(-dependent) History (dependence) Persistence
FIP	Frequency(-dependent) Inherited Persistence
GDP	Gross Domestic Product
GI	Great Inflation (period)
GM	Great Moderation (period)
GMM	Generalized Method of Moments
HICP	Harmonised Index of Consumer Prices
II	Indirect Inference
IRF(s)	Impulse Response Function(s)
MCI	Missed the Confidence Intervals
MiDas	Mixed Data sampling (regression)
MLE	Maximum Likelihood Estimation
MM	Moment-Matching
MoMBay	Moment-Matching based Bayesian (estimation)
NKM	New-Keynesian Model

NKPC	New-Keynesian Phillips Curve
NLLS	Non-Linear Least Squares
OMP	Optimal Monetary Policy
PPI	Producer Price Index
RBC	Real Business Cycle (model)
RE	Rational Expectations (model)
SMM	Simulated Method of Moments
VAR	Vector Autoregression (model)

Acknowledgements

I am especially indebted to Hans-Werner Wohltmann for his guidance and support as my first Ph.D. supervisor and principal. I am very grateful for his collaboration with respect to our joint (non-)academic work at the Chair of Macroeconomics. Moreover, I thank him for providing the resources to present my research at several international conferences and workshops in Europe as well as in the United States and Canada. I would also like to thank Maik Wolters for being my second Ph.D. supervisor and his fruitful discussions on topics related to this thesis.

I am incredibly indebted to Reiner Franke, who spend a lot of time on my minor and major economic problems as well as non-economic issues. I received invaluable words of advice and suggestions from him.

Furthermore, I would also like to thank my other co-authors Tae-Seok Jang and Steffen Ahrens for their support and successful collaboration. Additionally, I am thankful to my colleagues at the Chair of Macroeconomics and the Kiel Institute for the World Economy and there, in particular, the members of the research area on ‘Macroeconomic Policy under Market Imperfections’.

Most importantly, I also like to thank my family for their extensive emotional support.

This thesis is dedicated to my beloved son *Erik*.

Kiel, February 2014

Stephen Sacht

1 Introduction

1.1 Topics and Literature Review

In this thesis we contribute to the existing literature on the theory and estimation of *dynamic stochastic general equilibrium* (DSGE) models by modifying the core structure of this model class with respect to the timing interval. Furthermore, we introduce an alternative approach to estimating DSGE models. Our focus is on the baseline *New-Keynesian model* (NKM; also known as the model of the New Neoclassical Synthesis), which has been considered to be the workhorse model in macroeconomics since the beginning of the 21st century.

Nowadays DSGE models are widely used to analyze the dynamic behavior of economic key variables like the variation in the inflation rate and the output gap as a result of global disturbances. This class of models exhibits a dynamical system of difference equations specified by a lead (and lag) structure. Since this dynamic system focuses on the dependency of different markets in the economy, the DSGE approach stands in contradiction to the partial equilibrium theory. Since a DSGE is fully microfounded it meets the famous Lucas (1976) critique, whereby agents' expectations on economic indicators are influenced by (non-)policy disturbances rather than being static.¹ In this respect, the rational expectations hypothesis serves as a core assumption in the DSGE framework.² The convenient mathematical fundament of the DSGE framework along with the assumption of rational expectations lead to a reliable manageability in terms of solving, simulating and estimating these models.

Pioneered by the seminal work of Kydland and Prescott (1982), the *real business cycle* (RBC) model is commonly known as one of the first frameworks for analysing (temporary) macroeconomic disturbances based on a DSGE structure. This model in its baseline representation is characterized by perfect competition, fully flexible prices and wages and the efficiency of the equilibrium allocation. The set of equations of a RBC model describes the dynamics of the system based on intertemporal consumption behavior, the adjustment in the capital stock and movements in the real wage. Due to the complete flexibility of nominal variables, monetary policy is neutral since prices and wages fully adjust to shocks in the same proportion. According to the RBC (in conjuncture with the neoclassical) theory, the business cycle is mainly driven by technology shocks while monetary policy interventions have no real effects. On the contrary, studying the dynamics caused by technology shocks over the business cycle helps to identify the merit of fiscal policy measures in terms of stabilisation.

¹The expression microfoundation refers to the underlying intertemporal optimization applied by agents over an infinite time horizon, e.g. the utility and profit maximization of the representative household and firm, respectively. As a result, the associated reaction functions define the system of equations within a DSGE model. Note that throughout this thesis we make use of the singular and plural expressions for the agents being considered, e.g. we talk about the 'representative firm' or mention (just) 'firms', which can be seen as being identical within our analysis due to the assumption of homogeneity across agents.

²According to the rational expectations hypothesis, agents capitalize all information available in an optimal way, i.e. they are fully informed about the structure of the economy (described by the underlying model) and the functional form (the first- and second moments) of exogenous shock processes. The latter incorporate stochastic impulses, which represent the only uncertain components applied on the dynamic system.

The NKM accounts for the stylized facts on the existence of nominal rigidities, e.g. staggered price- and wage-setting mechanisms or incomplete markets like observed monopolistic competition on the supply side. The *baseline* NKM consists of two first order difference equations and a static instrument rule of the monetary authority.³ The assumption of nominal rigidities leads to the formulation of a *New-Keynesian Phillips Curve* (NKPC), which describes the dynamics of inflation due to a stochastic price-setting scheme introduced by Calvo (1983). As a result the representative firm faces a specific probability to adjust its price in the current period. It follows that the NKPC becomes forward-looking by considering future movements in inflation. Since the optimal price of the monopolist is set as a mark-up over real marginal costs, the output gap serves as a driving force of inflation under consideration of firm's production function. The output gap itself is determined by an Euler equation, which incorporates intertemporal consumption smoothing by considering the real interest rate. The latter one is defined as difference between the nominal interest rate and the future level of inflation according to the relation formulated by Fisher (1930).

The model is closed by introducing a nominal interest rate rule inspired by Taylor (1993), who empirically shows that the US Federal Reserve Bank (FED) reacts (contemporaneously) to movements in the output gap and the inflation rate in order to stabilize the economy. Over the last decade, a strand of the New-Keynesian literature consists on the study of *optimal* monetary policy responses in the (baseline) NKM. Here, the central bank seeks to minimize the deviations of the inflation rate and the output gap (as well as the nominal interest rate) from its steady states values or, simply, the volatility in these economic indicators. The optimization approach leads to the determination of so-called targeting rules, which describe the optimal response of the central bank to various shocks based on the relationship between the inflation rate and the output gap. The structure of these targeting rules differs depending on whether the central bank is able to influence the expectation channel of monetary policy or not.

In this thesis we contribute to the existing literature on DSGE by introducing two (theoretical and empirical) frameworks for the modelling and estimation of NKMs. First, we break with the convention on the timing of agents' decisions over the business cycle. This gives new insights in the analysis of (optimal) monetary policy using DSGE models in general and the use of high-frequency relative to low-frequency data for a better consulting of decision makers. Second, by presenting a new estimation method based on specific characteristics of macroeconomic time series, we show a way that makes the estimation criterion transparent and, hence, simplifies communicating the estimation results to policy makers and the general public. Both investigations are entirely new in the literature and strengthen the understanding of the baseline NKM along the theoretical and empirical dimension. This is of serious concern, especially in times of world-wide economic turbulences, which manifests in the financial and sovereign debt crisis as observed since 2008 in the US and the Euro Area. In the following we discuss our frameworks in greater detail.

³Goodfriend and King (1997) are the first who discuss a model of the New Neoclassical Synthesis. For an extensive discussion on the baseline NKM and its extensions see e.g. Woodford (2003), Galí (2008) and Walsh (2010) among others.

1.1.1 High-Frequency Analysis of the Baseline New-Keynesian Model

In *Part One* of this thesis we question the convention that NKMs are usually formulated in quarterly magnitudes. This assumption stems mainly from the fact that the underlying time series are given on a low frequency. The frequency is defined as the number of transactions undertaken by agents within a specific interval, i.e. over the length of the period. For example the dynamics in the output gap and the inflation rate are interpreted in quarterly magnitudes — where a quarter serves as the benchmark period length. Instead, we allow for the transition of the period length in the baseline NKM from a lower to a higher frequency. We call a NKM with a period length less than a quarter, a high-frequency or simply an *h-economy*. A reduction of the period length is tantamount to more frequent decisions and transactions within a given calendar time unit, i.e. relative to a benchmark period length defined (here) by a quarter. Here the polar cases are determined by the decisions taken either on a discrete (in quarterly magnitudes) or a continuous time (approximated by daily magnitudes) interval. We show that by deviating from the general assumption of a quarter as the underlying period length, fundamental insights on the modelling and empirical evaluation of macroeconomic behavior can be obtained. As far as we know our theoretical results are entirely new in the macroeconomics literature. Furthermore, these results stand in contradiction to the so-called Foley’s precept, where “no substantive prediction or explanation in a well-defined macroeconomic period model should depend on the real time length of the period” (Foley (1975, p. 310)).

In our view, thinking about the timing in DSGE models is important. An economist might ask how realistic the quarterly change in consumption will be rather than a more frequent decision making of households. This is even more of serious concern as we consider intertemporal consumption smoothing, i.e. that the level of consumption depends on the real interest rate. Kendrick and Amman (2011) emphasize that changes on the nominal interest rate in the US are undertaken by the Open Market Committee of the FED on a monthly frequency. In contradiction, during the first term of the Obama administration, the US Congress decided on fiscal stimulus packages annually. Obviously, the consideration of high-frequency decision making is obviously needed when integrating financial markets in macroeconomic models (cf. Lengnick and Wohltmann (2012) for investigations within a so-called agent-based computational framework).

Seminal empirical work with the focus on the timing interval is done by Aadland (2001) and Christiano (1985). In their studies the authors consider the period length explicitly. Aadland (2001) contrasts a weekly version of a standard RBC model to its monthly and quarterly counterparts. Therefore, he applies specific transformation rules (under consideration of the lag operator) on the data in weekly magnitudes, in order to make them comparable to data on a lower frequency through time aggregation. Based on second moment estimates, the author shows that the weekly version of the model exhibits a better fit relative to the quarterly model. Furthermore, the RBC model in weekly magnitudes helps to explain the relative volatility of hours worked to output and the correlation between the real wage and hours worked — both observations are known as the labor-market puzzles connected to the (quarterly) RBC framework. Christiano (1985) estimates the timing interval in the Taylor model of staggered wage contracts. In particular, and in contradiction to Calvo (1983), Taylor (1980) introduces a price setting scheme based on overlapping wage contracts. In this model the workers negotiate on their contracts, where the latter will expired after the defined

period of time. The author augments the model by allowing the agents to negotiate wage contracts on a higher frequency relative to the given life span of the contract. By applying the maximum likelihood estimation technique, it is shown that the model's fit will improve in conjunction with an estimated decline in the period length.

1.1.2 Moment-Matching Estimation of the Baseline New-Keynesian Model

Over the last decade *Bayesian* estimation became the most popular method for the statistical evaluation of NKMs, replacing the pure maximum likelihood method and the Generalized Method of Moments (GMM; Hansen (1982)). According to Griffoli (2011, p. 80) Bayesian estimation is "somewhere in between calibration and maximum likelihood estimation". This can be explained by the fact that within the Bayesian framework the likelihood function is reweighed by a prior density. The latter captures the pre-sample beliefs about an a priori distribution of the parameter values, which are going to be estimated (Fernández-Villaverde (2010, p. 9)). The goodness of fit of the model is evaluated via the logarithmic marginal likelihood. One of the advantages of the Bayesian approach is that the moments and the distributions of the parameters of interest can be easily computed with beneficial software packages like MATLAB DYNARE (Adjemian et al. (2011)). An introduction to Bayesian techniques can be found in e.g. Smets and Wouters (2003, 2007), An and Schorfheide (2007) and Fernández-Villaverde (2010).

In *Part Two* of this thesis we introduce an alternative approach for estimating macroeconomic models (in quarterly magnitudes). This approach is based on minimum distance procedures and here, in particular, the *method of moments* (MM; where we refer more precisely to as *moment-matching*). This technique concentrates on a number of statistics, also called moments, that summarize salient features of the dynamic system. MM seeks to identify numerical parameter values such that, as measured by a suitable criterion (or loss) function, these model-generated (simulated) moments come as close as possible to their empirical counterparts. Besides possibly the mean values of some of the variables, the moments that are most often referred to are either impulse-response functions (IRFs) or autocovariance functions (of vectors of variables), which convey similar information if the same shocks are underlying. In this thesis our focus is on *second-order moments*, i.e. the auto- and cross-covariances of the variables/observables.

For clarification, in the following we describe the differences in the estimation approaches of MM, GMM and the Bayesian approach. For a general textbook presentation we refer to Kennedy (2007, p. 46 and pp. 151) as well as DeJong and Dave (2007, Chapter 7). The fundament of Bayesian estimation is the Maximum Likelihood Estimation (MLE) approach, where the associated likelihood function is sufficient in respect to the description of the joint distribution of the estimated parameter values.⁴ Bayesian estimation is nested into MLE if the prior information of the parameters to be estimated is non-informative, i.e. that uniform distributions of the parameters are assumed a priori. MLE (and, of course, the Bayesian approach) stands for a *full* information procedure, where the likelihood function incorporates the full range of statistical implications (expressed by the moments) associated with the underlying statistical model.

In contradiction, GMM (and MM) represents a *limited* information procedure, where a pre-

⁴'Sufficiency' means that no additional information from the same sample size is provided by other statistics (Fisher (1922) and Mittelhammer (1999, in his section 7.4)).

selected collection of moments is considered. The term ‘Generalized’ indicates that various estimation techniques (e.g. Ordinary Least Squares, Non-Linear Least Squares or even MLE) are nested into GMM under consideration of the associated *moment conditions*.⁵ The latter describes that the expectation value of the product of the model parameters and the *empirical* data – in particular, the covariance between the explanatory variables and the error terms – should be zero at the true value of the parameters. The GMM method then minimizes a certain norm of the sample averages of the moment conditions based on a criterion function. In comparison to MLE and the Bayesian approach, advantages and drawbacks of GMM can be identified. First, in cases where the distribution function of the data generating process (with respect to e.g. the underlying shocks) is unknown, MLE can not be applied. Since the distribution functions are not part of the moment conditions, no such assumption is needed in the case of GMM, i.e. it does not suffer from model misspecification. Second, GMM entails a loss of efficiency if a specific collection of moments is considered only. This holds especially for a small sample size. Third, GMM can be applied without immense computational power, even if the underlying statistical model exhibits non-linearities.

The MM approach is obviously closely related to GMM. However, there exist major and minor differences between both frameworks. Most importantly, although both approaches share an equivalent criterion function, the MM technique takes into account the *simulated* and *empirical* data available (like the MLE one). In particular, while the GMM approach relies on the specific moments given by the prediction errors of the empirical time series only, MM provides summary statistics like e.g. autocovariances (second-order moments) of the model-generated and empirical time series.⁶ In a nutshell, GMM seeks to answer the question which kind of *prediction errors* are generated by the model, while MM is concerned with the *properties* of the model. With respect to the latter, different moments like IRFs or specific characteristics of the variables’ distributions (e.g. its tails) or the time series of the model variables can be considered for estimation. Furthermore, it must be mentioned that in comparison to GMM, in the MM approach, the weighting matrix associated (with the criterion function) is *independent* of the parameters of the model. In contrast, within GMM, the weighting matrix is computed under consideration of the moment conditions and, therefore, incorporates the vector of model parameters explicitly. This leads to a recursive updating mechanism of the weighting matrix under consideration of a starting vector (Hansen (1982, p. 1038)). Furthermore, in the presence of endogenous variables included among the elements of the explanatory variables (which is indeed the case in Chapter 4), so-called instruments (i.e. the variables and their respective lags) must be considered in the moment conditions. These instruments have to be correlated with the explanatory variables of the system but being orthogonal to the error terms. This orthogonal condition is especially required in cases, where the amount of moment conditions is less than the amount of parameters to be estimated. Instead, the use of instruments is not required within MM estimation technique. Finally, the GMM approach is mainly applied if the dependent variable is latent, i.e. unobservable.

It must be emphasized that the expression MM turns into the *simulated method of mo-*

⁵The connection between MLE and GMM is given by the fact, that the moments (also the second-order ones) are the first-order conditions for maximizing the likelihood function in the former case. In particular, Kennedy (2007, p. 152) states that “choosing between MLE and GMM rests on the choice of moments”.

⁶This statement indicates also the main difference between the Bayesian and the MM approach. Within the former, the distributions of the parameters are required based on a *statistical* model only, while within MM, the simulated moments are computed based on a *structural* economic model. Nevertheless, both approaches can be combined in order to identify prior information via MM, which then can be used within Bayesian estimation. An introduction to this technique is given in Chapter 6 of this thesis.

ments (SMM), in cases where the computation of the moments of interest can not be done analytically, e.g. when non-linear models are estimated. Within this thesis, we estimate a *non-linear bounded rationality* model and compare the point estimates for the parameters together with the goodness of fit of this model to a rational expectations one. Hereby, the bounded rationality model exhibits an expectation formation process regarding the output gap and the inflation gap under consideration of discrete choice theory.⁷ Despite of the higher computational power when SMM is applied, the procedure is less resource demanding like e.g. the particle filter method within the Bayesian approach.⁸ Especially, as long as this filter method is applied for evaluating the likelihood function, the estimation can be subjected to e.g. an increase in approximation errors of the non-linear model.

In our study we seek to match the model-generated second moments of economic variables to their empirical counterparts. Besides the relatively low computational cost, a main advantage of this estimation approach is its *transparency*. MM allows the researcher to concentrate on what he or she considers to be the most important stylized facts of the economy, but it also requires him or her to make them perfectly explicit. While in the end the choice of moments is a matter of judgement, it is a useful and informative decision to make since a model, at whatever level of complexity, cannot possibly reproduce all of the empirical regularities that we observe. In addition, the MM approach provides an intuitive notion of the goodness of fit of a model, which may be checked by visual inspection of the covariance structure or more formally by an econometric assessment of the minimized value of a loss function.

In fact, the MM estimation approach is closely related to a research agenda called *indirect inference* (II). This simulation-based estimation method comes up with the same advantages like MM, namely the estimation of economic models which are hard to evaluate. Reasons for this can be found in the non-linear structure of models, the lack of macroeconomic data and/or the existence of latent (unobservable) variables (Smith (2008)). In general, II and MM might be successful in the estimation of models, where the associated likelihood function is analytically intractable or too difficult to evaluate. As technical fundament, both approaches share a criterion function, where the distance between model-based (via simulation) and empirical moments are matched under consideration of a covariance matrix - however, it must be stated that this criterion function is based on a (quasi) likelihood function in the case of II (Gallant and Tauchen (2010)).⁹ Indeed, from a macro perspective using such kind of function refers directly to the GMM estimation approach. However it is well known, that the latter is most likely applied within single-equation rather than system-of-equation estimation and is highly sensitive to the instruments, e.g. the lag-structure in the data used. Furthermore, the II approach does not estimate the model directly as the MM approach does. Instead, an auxiliary model like a vector autoregression (VAR) model is applied on the underlying economic model. Given the simulated moments based on the auxiliary model, within the II approach these moments are matched to the empirical ones. This procedure indeed helps to evaluate parameters, which are hard to estimate in the case where e.g. the economic model is highly non-linear. Pioneer work in this field has been

⁷Henceforth, we mentioned (S)MM as we refer to an investigation, which is undertaken via the simulated together with the, let's say, original MM approach within one Chapter.

⁸The particle filter is a tool for the numerical approximation of the likelihood function in case of non-linear model dynamics. In contraction, the famous Kalman filter is applied in linear cases. Both methods consider latent (unobservable) variables. We refer to DeJong and Dave (2007) for an extensive discussion.

⁹A quasi-likelihood function shows similar properties to the standard log-likelihood function except for the specification of the distribution of the parameters. Instead a quasi-likelihood function consists on a relationship between the variance and the mean of the observations only (see Wedderburn (1974) for a detailed description of this approach).

undertaken by Smith (1993) as well as Gallant and Tauchen (1996, 2010). Prominent studies like the ones of Christiano et al. (2005) and Del Negro and Schorfheide (2008) deal with the estimation of IRFs as being the moments to match. Nevertheless, the question arises which kind of auxiliary model to choose in order to generate significant and plausible parameter estimates. A further concern in this respect is about the appropriate structure of the VAR model, namely the number of lags considered.

Furthermore, applying the Bayesian approach to DSGE models requires the choice of appropriate prior distributions associated with the underlying economic interpretation of the structural parameters. It is still an open question what criteria are suited best in order to identify the most accurate prior information. In the majority of studies each parameter of interest is assumed to be independently distributed a priori. The associated sufficient statistic of these so-called *micropriors* are taken from microeconomic evidence. In this respect, the Bayesian estimation must be designed to cope with the shape of the prior distribution, which is often unspecified, i.e. ‘uninformative’ priors; as a result, the estimated posterior becomes quite similar to the prior distribution. We show in this thesis that the MM approach serves as a promising procedure for the *identification of prior information*. Therefore, the evolution of macroeconomic variables is based on econometric time series analysis e.g. the moments of observable variables is considered. Taking this information into account will lead to the formulation of so-called *macropriors*, which can be then used as appropriate prior information within the Bayesian approach. We introduce a two-stage estimation procedure, where the prior information (mainly the prior mean values) is evaluated via MM and used within Bayesian estimation. In addition, we compare the corresponding goodness of fit of the model to the ones, where we allow for a standard prior information taken from the literature and a transformation of the structural form of specific parameters. Our main results indicate, that the prior information evaluated by the *Moment-Matching based Bayesian* (MoMBay) estimation approach accounts for a best fit of the model to the data compared to other types of prior information.

To sum up, based on our observations in Part Two, the MM approach can be seen as a good choice i) to widen the scope for the estimation of (non-linear bounded rational) NKMs and ii) for identifying (macro)prior information using for Bayesian estimation due to the features of transparency and muted manageability (relative to Bayesian estimation).

1.2 Outline of Part One

In *Chapter 2*, we allow the period length to vary over a time horizon. We investigate analytically and numerically dynamics of the variables in the baseline NKM in its forward-looking and hybrid representation as response to various shocks. Throughout Part One of this thesis, these shocks are assumed to be *deterministic* ones. The hybrid version is characterized by first-order leads and lags in the Euler consumption equation and the NKPC. By considering qualitative and quantitative similarity we investigate the change in the shape and the dissimilarity in IRFs of the economic variables with respect to variations in the period length.

We show that the effectiveness of monetary policy responses to a nominal interest rate, a technology and a cost-push shock is dampened in transition from a lower to a higher frequency. In particular, the pass-through of changes in the output gap into inflation rate dynamics, measured by the slope of the NKPC, is reduced. Furthermore, the volatility in the

economic indicators increases as the period length declines. In order to explain these results, we shed a light on the expressions which account for the intrinsic, extrinsic and inherited persistency in the model. Since some parameters in the model turn out to be *frequency-dependent*, i.e. they are directly influenced by the change in the period length, we find that overall inertia is increasing due to an additional persistence effect in the exogenous process. With respect to the impact effects and the adjustment processes of the variables after the shock(s) occur(s), monetary policy becomes less effective to stabilize the variables due to the impact of the change on the frequency of decision making on the price setting scheme of the firms. In general, the qualitative and quantitative dissimilarities (measured by IRFs) can be explained by the increase in the transactions and reallocation processes of agents on a higher relative to a lower frequency.

Our work is closely related to Franke and Sacht (2014), who study also qualitative and quantitative dissimilarities in the IRFs due to a nominal interest rate shock in the baseline (hybrid) NKM. In addition, their work is based on an intensive analytical investigation of the model's dynamic properties. In particular, they derive certain expressions in order to explain the impact effects of the variables in the polar case of a continuous-time limit. Furthermore, they check also for determinacy in an h -economy framework. Finally, a robustness check with respect to the variation in the Calvo parameter of price stickiness is applied. We contribute to their work by extending the analysis by considering a technology and a cost-push shock. Furthermore, we show that the microfoundation of the NKM remains unaffected if the frequency-dependent model variables and parameters are augmented by a specific parameter, which denotes the period length, as stated (but not shown) in Franke and Sacht (2014). Finally, we fill the gap in their paper by giving an explicit economic rationale for the observed qualitative and quantitative dissimilarities.

Besides our contribution and the work of Franke and Sacht (2014), studies on the theoretical modification of DSGE models with respect to the frequency of decision making are rare. This holds especially for the NKM. Only a few studies focus on determinacy. Flaschel et al. (2008) show that determinacy of a NKM model with sticky wages and prices remains unaffected by the change in the frequency. On the contrary, Anagnostopoulos and Giannitsarou (2010a,b) contribute to the economic discussion of why determinacy may depend on the length of the adjustment period in both a NKM and RBC model. Determinacy is also analyzed in Benhabib and Farmer (1994) who consider high-frequency versions of a RBC model (with capital adjustment). Hintermaier (2005) discusses a set of numerical parameters and shows that in the model of Benhabib and Farmer (1994), quarterly magnitudes are determinate but not their counterparts on a weekly (or higher) frequency. By leaving out a further investigation, Hintermaier (2005) just calls this observation a 'paradox' and states that "[t]he length of a period turns out to be a fundamental parameter that research in this field should be aware of" (p. 290). However, recent work, which applies tests on qualitative and quantitative similarities in the NKM are missing in the literature so far. As far as we know the only empirical studies, which consider the period length explicitly, are Aadland (2001) and Christiano (1985).

In *Chapter 3* we discuss optimal monetary policy in the purely forward-looking high-frequency version of the baseline NKM under a standard parameter scenario taken from the literature. While the assumption of nominal rigidities stands in contraction to the RBC framework, there is evidence for the non-neutrality of monetary policy due to price (and wage) stickiness. In particular, empirical studies show that central bank market operations have indeed an impact on the business cycle (cf. Chari et al. (2000) and Christiano et al. (2005) among

others). Furthermore, there exists a large amount of contributions in the literature which sheds a light on the optimal response of the monetary and fiscal authorities to exogenous disturbances. In particular the central bank and/or the government seeks to minimize primarily the volatility in the output gap and the inflation rate. Technically speaking optimal control theory applies.¹⁰

In the so-called *Discretion* regime the central bank takes agents' expectations as given and is forced to re-optimize every period. On the contrary, in the so-called *Commitment* regime the monetary authority is able to make use of the expectations channel of monetary policy due to its high reputation in the private sector.¹¹ Monetary policies under Discretion and Commitment differ in terms of the welfare loss (evaluated by the central bank's loss function). In the baseline NKM, Commitment policy leads to smooth adjustment in the dynamics of the economic indicators, which generate a lower welfare loss relative to discretionary policy (cf. Galí (2008) and Walsh (2010) for a discussion of this issue). As a consequence monetary policy analysis within the NKM call for a systematic and trusted intervention. For an overview of optimal monetary (and fiscal) policy analysis, the interested reader may refer to Erceg et al. (2000) as well as Schmitt-Grohé and Uribe (2004, 2007) among others.

In this Chapter, we analyze theoretically (by applying optimal control theory) and numerically (by computing IRFs) the dynamic behavior and economic welfare in case of a cost-push shock. This is done by considering an ad-hoc loss function under the policy regimes Discretion and Commitment for different frequencies. Our results show that the welfare loss increases with a reduction in the period length since the monetary authority faces a more persistent adjustment of the variables to a shock relative to the baseline NKM. In a case of a cost-push shock, the higher the frequency the more the central bank's influence on stabilizing the economy is dampened. This is more severe as an inflation rate/output gap trade-off in this case of a cost-push shock emerges, where in this stagflation scenario the central bank is unable to stabilize both variables at the same time. Our intensive discussion in this Chapter is grounded on the economic rationale stemming from the impact of the frequency-dependent persistence parameters of the model. We apply also a welfare and sensitivity analysis, where we show how the related loss function under both regimes vary with the changes in the period length, the extrinsic persistence parameter in the shock process, the weight on output stabilisation in the loss function and the degree of price stickiness.

Our work is closely related to Sacht and Wohltmann (2013). In their work they focus on the derivation of the analytical expressions for the loss functions under Discretion, Commitment and an Optimal Simple Rule in case of a deterministic *and* stochastic shock process. The

¹⁰In general, the associated (welfare) loss function consists in an ad-hoc manner of the squared deviations of the output gap and the inflation rate from their corresponding (constant) steady-state values. An example is given by Winkler and Wohltmann (2008), who discuss the impact of anticipated shocks on economic dynamics. Recent studies apply a second-order Taylor approximation on agents utility. Hence, the loss function becomes microfounded (cf. Schmitt-Grohé and Uribe (2004)). Furthermore, as an alternative procedure the optimization problem of the central bank (or the government) is given by the maximization of household's utility with respect to the system of equations. This so-called Ramsey approach is often used in the literature nowadays and was introduced by Kahn et al. (2003). For an example of an application we refer directly to Schmitt-Grohé and Uribe (2007).

¹¹It is well-known that the solution of dynamic optimal control problems with forward-looking rational agents is dynamic inconsistent or strategic incoherent (see, for example, Wohltmann and Krömer (1989) as well as McCallum (2005) among others). In order to overcome the problem of the dynamic inconsistency of optimal policy under Commitment, Woodford (1999) proposes the concept of *timeless perspective* policy-making. The concept has achieved a lot of attention and is widely considered as a good approximation of the globally optimal policy under Commitment – see also Dennis and Söderström (2006), McCallum and Nelson (2005) and McCallum (2005), who extensively discuss the timeless perspective policy approach.

latter is described by a Taylor rule, where the optimal values of the corresponding weights on the inflation rate and the output gap are given by the outcome of the minimization of the loss function. As a main result, the authors show that the analytical solution of the loss functions for all policy regimes are mainly based on the corresponding unstable and stable eigenvalues of the dynamic system, respectively. While our study focuses mainly on the numerical analysis (in case of a deterministic shock process only) along with an illustration of the economic rationale, needed to explain these results, it can be stated that together with the work of Sacht and Wohltmann (2013), both studies represent an entirely new contribution to the literature.

We contribute to the overwhelming number of studies on optimal monetary policy in the (baseline) NKM like e.g. Woodford (2003), Winkler and Wohltmann (2006) and Walsh (2010) among others. However, the only studies, which deal with the connection of (optimal) monetary policy to the frequency, are concerned with estimation only. For example, Faust et al. (2004) estimate the effect of monetary policy announcements on the expected future values of the nominal interest rate and the price level. This is done via the matching of IRFs. In their work they use high-frequency data on the prices of Fed Funds future contracts in a vector autoregression (VAR) model. The authors report evidence for a small impact of monetary policy shocks on output. Rosa (2013) applies ordinary least squares under consideration of high-frequency data on energy prices. He shows that asset prices respond on a high-frequency due to sudden monetary policy announcements. Finally, Assenmacher-Wesche and Gerlach (2008) estimate a two-pillar NKPC of the European central bank (ECB), where they account for the ECB's simultaneous analysis of monetary and economic indicators. In their study they investigate the impact of high frequency (e.g. exchange rates, import and fresh food prices and value-added taxes) as well as low-frequency (money and real output growth and the rate of velocity) components on the inflation process in the Euro Area. The authors check on cointegration of the inflation rate relative to these components under consideration of spectral regression techniques in order to account for the different time horizons. They find that low-frequency movements of money and real output growth as well as high-frequency fluctuations of the output gap account for the variation in inflation over the period from 1970 to 2004. However, these studies are not closely related to our theoretical approach presented in this thesis. Instead, our contribution reveals the need for a more elaborate theoretical analysis on the connection of optimal monetary policy and the frequency of decision making.

In *Chapter 4*, we apply a single-equation estimation via GMM. We derive an open-economy high-frequency NKPC and confront it with daily inflation data in order to evaluate the value of the Calvo parameter of price stickiness. The latter measures the probability that the price of a representative firms remains unchanged within a given period. Since this random variable follows a Poisson process, the higher the values of the Calvo parameter the more the price stickiness prevails on average over a specific amount of periods. This is tantamount to a sticky price environment in general, while in contradiction, a low value of the Calvo parameter indicates flexible prices. In the polar case of zero probability of not changing the price, the latter is by definition purely flexible. This has an important implication for modelling economic dynamics either in a RBC or NKM.

By considering a quarterly time series for estimation, one must be aware of the fact that structural breaks might occur. This can lead to a bias in the estimation results. A prominent example is given by Argentine inflation dynamics. The reasons are twofold. First, Argentina is a country that suffers firm structural breaks due to external and financial crisis and the

corresponding policy reaction to these crisis in the end of the 20th century. Second, official statistics on inflation have to be challenged since the dynamics are not accordingly reported by the Argentine government (Cavallo (2012b)).

It is the aim of this Chapter to focus on a short time period in the inflation dynamics in order to reveal the true degree of price stickiness in Argentina. In a first step we estimate our open-economy high-frequency version of the forward-looking NKPC via GMM using daily inflation data from 2007Q4 to 2010Q4. In order to do so, we eliminate all variables expressed in quarterly magnitudes. In the end, the NKPC consists of economic indicators only, where data is given on a daily basis like the nominal exchange rate and the domestic and foreign nominal interest rate.¹² Then we develop a transition formula in order to extract the value of the frequency-dependent parameter from a lower frequency (a quarter) out of a higher frequency (a day). In conclusion, under consideration of high-frequency data we are able to make inference on the structural parameters of a DSGE model in quarterly magnitudes. We report evidence that, by applying our method to Argentine data, the daily Calvo parameter indicates averagely fixed prices of approximately two to three months. These values are well in line with micro evidence for Argentina as shown by Cavallo (2012a, p. 30). Most importantly, an average price stickiness of nearly one quarter means that on a quarterly frequency a flexible price model has to be applied to analyze the effects of policy measures, a situation, which - under standard assumptions - renders the analysis of monetary policy redundant. In the same vein, to analyze monetary and fiscal policy in a sticky price framework, a monthly model like the equivalent (augmented) variation of the standard baseline NKM discussed in Chapter 2 seems more appropriate. Our results turn out to be robust to alternative calibrations of the model. In the end we offer a way to expose the dynamics on a quarterly basis given by the underlying transformation approach presented in this Chapter. On the second stage this pseudo-quarterly values might be useful for the analysis of monetary and fiscal policy in a low-frequency environment. In general, allowing for higher-than-usual frequencies, when applying our approach presented here, strongly mitigates the problems of small-sample bias and structural breaks.¹³

Moreover, there is an increasing interest in the literature on the high-frequency behavior of price changes. In a scanner data study for British supermarkets Ellis (2009, pp. 10-11) shows that the frequency of price changes is considerably higher in high-frequency studies compared to the traditional monthly or quarterly consumer price index analysis undertaken by statistical agencies. Furthermore, Ellis (2009, p. 11) shows that lower frequency data tend to overstate the true price stickiness. Abe and Tonogi (2010, pp. 725-726)) strongly support this conclusion for the Japanese market. Additionally, Kehoe and Midrigan (2007, p. 9) find very short average price stickiness spells for suburban Chicago, and Cavallo (2012a, p.

¹²The reason for this transformation is given by the fact that neither the real marginal costs, the domestic and the foreign output gaps nor the labor share of income stand for appropriate proxies since all are also not available in daily magnitudes. In fact, these driving forces of inflation can be seen as latent variables, which are hardly computable and predictable in an econometric point of view. However, it does not mean that e.g. a daily output gap is a non-reasonable construct per se. Moreover, it is a problem of measuring such a construct and hence, the availability of the data only. For example, it is intuitive to think about a definition of the gross domestic product (GDP), which says that GDP is the measure for all products and services produced within a *day* rather than a quarter.

¹³It has been shown by Fuhrer et al. (1995, p. 143), however, that GMM suffers from a small sample bias with the consequence that this method demands a critical amount of observations to achieve reliable estimates. Lindé (2005, p. 1140) argues that it takes approximately 1,000 observations for GMM to converge to the true values when estimating a NKPC. In order to obtain that many observations in a quarterly setup - with only four observations per year - a time span of 250 years is necessary. By using around 830 daily observations in our empirical work we are able to avoid this kind of sample bias.

30) finds this for Argentina, Brazil, Chile, and Colombia. Related to the current sovereign debt crises, van Roye (forthcoming) derives two financial market stress indicators for the Euro Area and Germany, which measures the volatility in stock markets, using a dynamic factor model. These indicators consist on various financial market variables (like different interest rate spreads and indices), which are given on monthly and daily frequencies. Under consideration of a specific aggregation scheme, the author shows that about 30 (15) percent of Euro Area (German) GDP growth is accounted by financial stress.

1.3 Outline of Part Two

In *Chapter 5* of this thesis we estimate the baseline NKM in its hybrid representation via the MM and Bayesian approach, respectively. Note that throughout *Part Two* we do not vary the period length but consider the model(s) in quarterly magnitudes only. We seek to identify empirically the degree of backward-looking behaviour in consumption and price-setting. The corresponding parameters display external habit formation in consumption and price indexation.¹⁴ In addition, we allow for an autocorrelated exogenous shock process in the NKPC. The data sample is split into the Great Inflation (GI) and Great Moderation (GM) period in order to account for structural breaks according to the change in the monetary policy strategy in the beginning of the 1980s. In particular, US time series for output, inflation and the nominal interest rate are considered in *gap* form, where we account explicitly for a time-varying trend in inflation and the natural rate of interest. The corresponding gaps are simply given by taking the difference of the actual value for the output, the inflation and the interest rate from their trends (i.e. time-varying steady state values), respectively, where the latter is computed by applying the Hodrick-Prescott filter (Hodrick and Prescott (1997)).

Estimating macroeconomic DSGE models with gap data is rare in the literature. However, the results of many theoretical studies show that assuming a constant trend, like a zero-inflation steady state, leads to misleading results. For example, Ascari and Ropele (2009) observe that the dynamic properties (i.e. mainly the stability of the system) depend on the variation in trend inflation. Cogley and Sbordone (2008) also provide evidence for the explanation of inflation persistence by considering a time-varying trend in inflation. In the same vein, we can abandon the assumption of a constant natural rate of interest as being empirically unrealistic. Furthermore, inflation and money growth are likely to be non-stationary. If that is the case, an estimation methodology such as the Bayesian approach will lead to biased estimates. Taken this into account, in this Chapter we consider the gaps rather than the levels in order to ensure the stationarity of the times series.

By applying the Bayesian estimation technique, we find a value for the price stickiness parameter close to zero in both periods, while regarding habit formation just the opposite holds. Also, substantial autocorrelation in the inflation shock process is observed. These results are commonly known in the literature under consideration of the variables given in levels instead of gaps, cf. Smets and Wouters (2003, 2007), Adolfson et al. (2007), Benati and Surico (2009), Fève et al. (2009), Cogley et al. (2010) among others. Inflation persistence is here brought about by serial correlation in the shock process, besides the inherited

¹⁴External habit formation indicates that the decision on current consumption is made relative to the previous level of aggregate consumption (Abel (1990)), while with price indexation, the representative firm sets its price under consideration of the previous price level in order to avoid rapid and large differences in the movements of this nominal variable over time (Christiano et al. (2005)).

persistence in the output gap.

Our MM estimations may add new insights into this discussion. In fact, we find strong evidence for exactly the contrary. While the degree of price indexation is very close to unity in the GM period, it is estimated at its maximal value of unity in the GI sample. Moreover, in both cases the supply side shocks are white noise and inherited persistence is weak. We even show that the goodness of fit of the model will improve as we allow the degree of price stickiness and habit formation being unbounded at the unit interval. All qualitative results are supported by (moment-specific) p -values above the 5% significance level.

While the general argument for likelihood functions which are based on sufficient statistics is striking, MM indeed can be seen as a reliable alternative approach: the choice of what kind of moments are used within the estimation is linked to a specific economic reasoning. In particular, within MM the focus is on the moments of interest connected to the business cycle. This stands in contradiction to the Bayesian approach, where the whole frequency domain is taken into account for optimization. According to this, both approaches impose different prior information (likelihood evaluation versus moment selection) on the data, while the MM approach is more narrow with respect to the data considered. The results presented in this Chapter indicate that the evidence on backward-looking behaviour in the (baseline) NKM is ambiguous, which has strong implications for policy analysis and macroeconomic consulting work. Applying the MM approach helps to reveal this issue. However, a more critical discussion on sufficiency with respect to the MM approach is also provided in this Chapter.

In *Chapter 6* we discuss the identification of prior information via MM. In general, by consideration of the prior information, the evaluation of the modes within Bayesian estimation is restricted to a subregion of the parameter space, where the distribution of the parameters is set by the researcher. However, as we already stated, the estimation outcome is sensitive to the choice of prior distributions associated with the underlying economic interpretation of the model's structural parameters. In this Chapter we consider the parameter estimates from Bayesian estimation documented in Chapter 5 and apply a sensitivity analysis regarding the prior information. We seek to answer the question to what extent the choice of micro- or macropriors has an influence on the estimation outcomes. In particular, how sensitive will the estimation results be to a change in the functional form of the micropriors? Is the moment-matching approach able to identify sufficient macropriors, i.e. prior information which leads to higher values of the marginal likelihood as in the case of micropriors? Then we compare the corresponding outcome (in terms of the goodness of fit) to the one when the standard prior information, which is in line with the recent literature and used in Chapter 5, is taken into account.

We account mainly for two things. First, we investigate the impact of the transformation of those model parameters which are bounded to the unit interval. The idea is to allow for a more diffuse prior distribution of the parameters of interest through transformation. The latter expresses the mapping of structural parameters into an alternative representation in order to allow for a change in the interval the parameter is defined on (e.g. being bounded versus unbounded on a unit interval). Hence, we compare the estimation outcome under three alternative sets of micropriors, i.e. which are considered within the Bayesian estimation from Chapter 5 and two specifications, which differ with respect to the transformation of the price indexation parameter.

Second, we shed a light on the investigation of a combination of both MM and Bayesian

techniques. Since parameter estimates from our MM account for the second moments of the underlying time-series they can be identified as macropriors for Bayesian estimation. In this respect, we introduce the two-stage MoMBay approach: we take the point estimates from MM estimates for the GI and GM period, respectively and consider them as prior mean values of the parameters within Bayesian estimation. For a robustness check, we vary the priors regarding the standard deviations of all the persistence parameters in the model in order to allow for more diffuse priors.

We show that while micropriors are often used in the literature, applying macropriors evaluated via MoMBay leads to a *better* fit of the structural model to the data. This holds for the GI and GM period. The goodness of fit is measured by the value of the loss function and the logarithmic marginal likelihood associated with the MM and Bayesian approach. According to the estimation results, both measures are in favor of considering the standard prior information from the literature relative to the one based on transformed parameters. However, using prior mean values evaluated via MoMBay leads to a distinct fit relative to the standard prior information. The reason for this is that the MoMBay approach reduces the parameter space where appropriate starting values (given by the prior mean values) for the optimization within the Bayesian approach are taken from. Hence, the MoMBay approach can be seen as an alternative procedure for the estimation and evaluation of macroeconomic models. We show that this indeed leads to a change in the estimates of some parameters when we compare the results using micro- to macropriors used. As a general observation, however, the results from Chapter 5 are confirmed when applying the MM-based procedure. This means that there is evidence for rather intrinsic (degree of price stickiness) than extrinsic (autocorrelation in the shock process) persistence — an observation which, again, stands in contradiction to the results documented in the recent literature. Similar observations are made also by Del Negro and Schorfheide (2008) as well as Lombardi and Nicoletti (2012) who also found differences in the posterior values under consideration of macropriors based on alternative methods.

The advantages of the MM approach rely — besides how transparently the results are computed and documented — in its detailed diagnosis of merits and demerits of models, whose moments are hard to match like non-linear DSGE models. A good example for the reliability of the SMM approach with respect to non-linearity is shown in *Chapter 7*. Here, we apply the SMM approach on the baseline NKM, where the expectation formation processes regarding the output gap and the inflation gap are based on discrete choice theory. This model specification is directly taken from De Grauwe (2011). Households' decision on future consumption depends on the subjective view, how the business cycle will evolve over time, i.e. if a boom or bust will occur. Hence agents' beliefs display waves of optimism and pessimism — so called *animal spirits* (Keynes (1936)). The discrete choice mechanism describes also the decision, which kind of inflation expectation process the agents finally choose in every period. In this respect, they can either believe in the announced inflation target by the central bank to become true or form expectations in a static way, i.e. assume that the previous inflation gap being the realized future value.

In this Chapter we show that (based on Euro Area data) the SMM approach provides reasonable estimation results on the bounded rationality parameters. The latter are those parameters associated to the non-rational decision rules, used by the agents to form output and inflation gap expectations. Furthermore, we contrast the bounded rationality version of the standard NKM to the same model assuming rational expectations. In particular, we report three main results. First, over the whole time interval the agents had expected

moderate deviations of the output gap from its steady state value. Second, in the absence of rational behavior we find strong evidence for a static expectation formation process regarding the inflation gap. Both observations explain a high degree of persistence in the output gap and the inflation gap. From this, we found that animal spirits strengthen the optimists' belief about the future output gap to diverge in the historical Euro Area data. Finally, we provide evidence, that the hybrid bounded rationality model shows an equal fit to data as the rational expectations one. In other words the value of the criterion function as a measure for the goodness of fit is slightly lower for the hybrid bounded rationality model where animal spirits are assumed. In this respect, the Bayesian analysis is not a panacea for a bounded rationality model, since prior information is not available at least for the behavioral parameters associated to animal spirits. It follows from this, that the SMM approach serves as an appropriate procedure to estimate (highly) non-linear DSGE model.

In the last Chapter, our main results are summarized and an outlook for future research is given.

Part I: High-Frequency Analysis within
the New-Keynesian Framework

2 Analysis of Various Shocks within the High-Frequency Versions of the Baseline New-Keynesian Model

2.1 Introduction

In this Chapter we are concerned with an elementary methodological issue in the macroeconomic modelling of DSGE models. These models are usually formulated in discrete time. As an approximation to reality, they assume that decisions are taken discontinuously and that all transactions of a certain class occur in the same synchronized rhythm. There is, however, no argument that would allow one to identify a uniform *natural decision period*. If the models, especially in numerical studies, invoke a definite period, which is mostly a quarter, the choice is entirely determined by convention or the frequency of the available data.

However, over the last years researchers seek to develop new procedures in order to collect and analyze high-frequency data. Various research projects have benefited from rapid advances in information and communication technologies during the last two decades. As a prominent example, the Billion Prices Project at the MIT Sloan provides daily inflation data based on scraped price indices, i.e. indices calculated from prices which are available online. Throughout this thesis (namely in Chapter 4), we show that such kind of data can be used for the estimation of a NKPC since the risk of a small sample bias and structural breaks are reduced. The former is based on the fact that just a small amount of observations are available on a quarterly (or even annual) frequency. In order to circumvent this problem a long time span of several months (or years) must be considered with a high probability of structural breaks included. For instance, when estimating a model based on US data, the sample is split into two sub-samples, i.e. the GI and GM period, respectively. This can be motivated by a rapid decline in the volatility of inflation at the beginning of the 1980's. As we show later in Chapter 4, under consideration of daily data over an interval of roughly two years, we are able to provide significant parameter estimates for the degree of price stickiness (in Argentina).

As information on high frequent adjustments in the economic indicators become available, this calls for a modification of the DSGE model with respect to the period length. In particular, a reduction of the period length is tantamount to more frequent decisions and transactions within a given calendar time unit. A model version with a shorter period than in the original formulation is thus a high-frequency economy. If the period of the benchmark model serves as the time unit, which is fixed, and a high-frequency economy is constituted by a period of length $0 < h < 1$, the latter will also be designated as an h -economy. In this Chapter we are going to analyze various shocks in a baseline three-equations NKM under explicit consideration of the period length. In particular, we compare IRFs based on these shocks across different frequencies of decision making. Furthermore, we give an explanation

for how the transmission channel of monetary policy is affected in transition from one h-economy to the other. In order to transform the model into its high-frequency equivalent we apply specific transition rules on the frequency-dependent variables and parameters.

Modeling the impact of different period lengths on the dynamics of the current workhorse-model used for monetary and fiscal policy evaluation, the NKM with sticky prices (and wages), has been done by Flaschel et al. (2008) and Anagnostopoulos and Giannitsarou (2010b) in the first place. These studies are mainly concerned with analyzing the impact of the change in the period length on the determinacy of the model. While Flaschel et al. (2008) indicate that determinacy turns out to pose no problem in a NKM, Anagnostopoulos and Giannitsarou (2010a, pp. 11ff) show that determinacy may depend on the length of the adjustment period (jointly with a tax rate on labour). This is confirmed for high-frequency versions of RBC models (Benhabib and Farmer (1994)). Hintermaier (2005) discusses a set of numerical parameters such that, for instance, the quarterly economy is determinate but not its weekly version. Posch et al. (2011) present a numerical solution technique based on Chebychev polynomials in order solve a NKM formulated entirely in continuous time. Besides theoretical work, empirical investigations on the role of the frequency in DSGE models is even more rare. For example, one might think of a calibration or estimation procedure, which takes explicit account of different decision intervals on the part of the economic agents and tries to let the data decide on an appropriate period. We only know of two examples of such work in the literature, namely Christiano (1985) and Aadland (2001).

Our work is quite closely related to Sacht and Wohltmann (2013) as well as Franke and Sacht (2014). The latter show that by diverging from the standard assumption of the baseline period length to be one quarter this leads to distinct changes in the dynamic properties of the model. In particular, the authors state that while determinacy of the model remains unaffected, the IRFs can differ in a quantitative and qualitative significant way just by increasing the frequency of decision making (i.e. assuming a monthly, weekly or daily length of the period). While Franke and Sacht (2014) also provide an elaborate analytical discussion of the polar case of a decision interval given in continuous-time (where h is approximately zero), in this Chapter we are concerned with the investigation of the IRFs based on different values of h only. Therefore, we also include a nominal interest rate shock in our study but also consider a technology as well as a cost-push shock. Most importantly, we fill the gap in the paper by Franke and Sacht (2014) by giving an explicit (economic) explanation for the observed changes in the IRFs across frequencies.

Sacht and Wohltmann (2013) study analytically and numerically the high-frequency optimal monetary policy responses to a (deterministic and stochastic) cost-push shock. They show that the welfare loss changes (in relation) across different optimal monetary policy regimes under variation of the period length. In particular, the monetary authority faces a decrease in their effectiveness to dampen the increase in the inflation rate. Overall, the welfare loss increases as the period length decreases together with an amplification of the policy trade-off in case of a cost-push, i.e. a more pronounced stagflation scenario occurs. While in their elaborate analytical work mainly the derivation of the loss functions under different monetary policy regimes and frequencies are presented, we contribute to this study by an intensive investigation of the corresponding IRFs in the next Chapter. Here, we also seek to identify the economic rationale which leads to the results. Furthermore, we provide a sensitivity analysis with respect to the degree of price stickiness, the inflation targeting regime and the persistence in the autocorrelated shock process.

The remainder of this Chapter is structured as follows. In the next section we discuss the

concept of a h -economy and provide transition rules for the frequency-dependent variables and parameters. In section 2.3 we provide economic explanations for the upcoming results based on the change in the (frequency-dependent) intrinsic, extrinsic and inherited persistence in the model. Section 2.4 contains the numerical analysis of a nominal interest rate, technology and cost-push shock based on IRFs under consideration of different frequencies. Section 2.5 concludes. The microfoundations of the h -economy model, the solution of the shock process on a higher frequency, the representation of the reduced-form solutions of the model variants as well as all Figures can be found in Appendix A.

2.2 The Concept of a High-Frequency Economy

In a NKM the underlying period length is assumed to be a quarter. Here, we stick to this convention by defining a quarter as the benchmark period length, although the latter is arbitrary and can be e.g. a month or a year instead. In particular, in order to compare two economies with different frequencies of the synchronized actions, we fix the time unit as a quarter and generally allow the agents to make their decisions and carry out the corresponding transactions every *subperiods*, i.e. h quarters. The period length relative to a quarter is then defined by

$$0 < h = \frac{1}{f} < 1 \quad (2.1)$$

where f denotes the *frequency* of decision making. We assume that the frequency changes homogeneously. This means that all decisions and transactions of the representative household, firm and the central bank occur in the same, synchronized rhythm, e.g. on a quarterly ($f = 1$), monthly ($f = 3$), weekly ($f = 12$) or even daily ($f = 90$) frequency. As mentioned above, such a high-frequency economy is simply called an *h-economy*.

As a general assumption we claim that the transactions in quarterly magnitudes are spread symmetrically over the corresponding subperiods in an h -economy. For illustration, a nominal interest rate of, for example, 1.50% per quarter means that, in an h -economy, hundred dollars earn $h \cdot 1.50$ dollars over the period $[t, t + h)$. The time preference rate of the representative household is to be interpreted analogously.¹ This procedure can not only be found in Flaschel et. al (2008), Anagnostopoulos and Giannitsarou (2010a,b), Sacht and Wohltmann (2013) and Franke and Sacht (2014) with respect to standard NKMs, but also in the modelling of search and matching processes (Rogerson et al. (2005, p. 963)).

In order to transform a model under consideration of a benchmark period length into an h -economy, Franke and Sacht (2014) formulate specific transition rules. These rules can be summarized in the following three steps:

- (1) To start with, the *baseline* model with $h = 1$ must be formulated. Hence, given a discrete time specification, h is then considered explicitly. As a main assumption, we claim that the functional form of the model, i.e. the system equations, remain unchanged across different frequencies. As an example, the dynamic IS equation in a NKM describes the consumption decision of a representative household based on consumption smoothing. This holds independently on a low- or high-frequency as we show in Appendix A1.

¹While this treatment introduces a compound interest effect, it is confirmed that this effect is certainly not strong enough to explain the differences over the first few quarters.

- (2) For the transformation into an h -economy framework, the *frequency-dependent* components of the model have to be suitably adjusted. In particular, the frequency-dependent variables and parameters are going to be changed under consideration of h . Regarding the parameters, e.g. the discount rates and probabilities, those differ across all sub-periods relative to the benchmark period length as $0 < h < 1$ holds, while fractions remain unaffected. With respect to the variables, growth rates exhibit a certain time dimension and therefore have to be divided by h , while contemporaneous adjustment rates have no time dimension. Throughout this Chapter we will discuss these issues in greater detail.
- (3) For a direct comparison of several h -economies, the stock variables are uniformly expressed as *normalised*, e.g. quarterized (given the baseline model under $h = 1$ is formulated in quarterly) magnitudes. Given that the modified h -economy specification of the model after step (2) is applied, this means, that all frequency-dependent variables are divided by h in order to ensure conformity across different period lengths. In this respect, note that a variable without time-dimension like the output gap are *not* quarterized. Alternatively, normalization (quarterization) can be seen as nothing else than a specific aggregation technique applied on the high-frequency stock variables relative to the benchmark period length. In particular, different kinds of aggregate schemes exist in (econometric) literature. In our study we apply the so-called *systematic/skip sampling aggregation* scheme based on the deviation of the corresponding variables by h . Once again, throughout this Chapter we will discuss this issue in greater detail.

In this Chapter we consider the following standard NKM in its hybrid representation. In the *first step* we adopt the specifications presented in Smets and Wouters (2003), Christiano et. al (2005) and Galí (2008, his Chapter 3) in order to formulate the model, while the underlying period length is given by a *quarter* ($h = 1$):

$$y_t = \frac{1}{1 + \chi} E_t y_{t+1} + \frac{\chi}{1 + \chi} y_{t-1} - \delta_1 (i_t - E_t \pi_{t+1} - \bar{i}) + \delta_2 v_t^y \quad (2.2)$$

$$\pi_t = \frac{\beta}{1 + \alpha\beta} E_t \pi_{t+1} + \frac{\alpha}{1 + \alpha\beta} \pi_{t-1} + \kappa y_t + v_t^\pi \quad (2.3)$$

$$i_t = \bar{i} + \phi_\pi \pi_t + \phi_y y_t + v_t^i \quad (2.4)$$

$$v_t^z = \rho^z v_{t-1}^z + \varepsilon_t^z, \quad z = \{y, \pi, i\} \quad (2.5)$$

with

$$\delta_1 = \frac{1 - \chi}{\sigma(1 + \chi)} \quad (2.6)$$

$$\delta_2 = \frac{(1 - \chi)(1 + \eta)(\rho^y - 1)}{(1 + \chi)(\sigma + \eta)} \quad (2.7)$$

$$\beta = \frac{1}{1 + \nu} \quad (2.8)$$

$$\kappa = \frac{(1 - \theta)(1 - \theta\beta)}{\theta(1 + \beta\alpha)} (\sigma + \eta). \quad (2.9)$$

E_t denotes the mathematical expectation operator conditional on information up to period t . Equation (2.2) describes a hybrid dynamic IS curve and results from the standard utility maximization approach of a representative household. In this case the current output

gap depends negatively on the real interest rate ($i_t - E_t \pi_{t+1}$), i.e. it is stemming from intertemporal optimization of consumption and saving, which then results in consumption smoothing. The constant natural rate of interest is denoted by \bar{i} . The parameters $\sigma \geq 0$ and $\eta \geq 0$ denote the inverse intertemporal elasticity of substitution and the substitution elasticity of labor, respectively. Intrinsic persistence is imposed on this demand equation by the degree of the households' external habit formation denoted by $0 \leq \chi \leq 1$. Equation (2.3) is known as the hybrid NKPC where the output gap (y_t) is the driving force of inflation due to monopolistic competition and the Calvo price-setting scheme. The parameter β denotes the discount factor, where $\nu > 0$ serves as the time preference rate, i.e. $0 < \beta < 1$ holds. The price indexation parameter α displays the degree of *intrinsic persistence* in the inflation rate dynamics. The slope of the NKPC, which measures the degree of *inherited persistence* with respect to y_t , is given by the composite parameter $\kappa \geq 0$. The latter consists on the discount parameter, the degree of price indexation and the Calvo (1983) parameter of price setting, where the probability of resetting the price of a representative firm on a given frequency is denoted by $0 \leq \theta \leq 1$. According to the interest rate rule of Taylor (1993) type (cf. equation (2.4)), the monetary authority reacts directly to movements in the output gap ($\phi_y \geq 0$) and the inflation rate ($\phi_\pi \geq 0$). We assume that the exogenous driving forces in the model variables are given by a technology shock v_t^y , a cost-push shock v_t^π and a nominal interest shock v_t^i , respectively (cf. equation (2.5)). These shocks follow an autocorrelated process under consideration of an exogenous persistence parameter $0 \leq \rho^z < 1$ with $z = \{y, \pi, i\}$. Accordingly, the dynamics of the system are set in motion by one-time impulses denoted by $\varepsilon_{z,t}$ in time $t = 0$. Note that we consider a *deterministic* impulse only, i.e. the corresponding standard deviations of the shocks are assumed to be zero.²

In the *second step* we adjust the frequency-dependent variables and parameters of the NKM in quarterly magnitudes (described by the equations (2.2) to (2.9)) by h in order to transform the system in an h -economy. Note that now $0 < h < 1$ holds. It is not necessary to spell out the details here that lead from the microfoundations to the structural NKPC and dynamic IS equation in an h -economy.³ However, for completeness these microfoundations can be found in the Appendix A1. It turns out that, let's say, direct ad-hoc adjustments of the variables and parameters by h coincide with those obtained through microfoundation.

First, we discuss the representation of the NKPC in an h -economy, which is given by:

$$\Delta p_t = p_t - p_{t-h} = \frac{\beta(h)}{1 + \alpha\beta(h)} E_t \Delta p_{t+h} + \frac{\alpha}{1 + \alpha\beta(h)} \Delta p_{t-h} + \kappa(h) y_t + h v_t^\pi \quad (2.10)$$

with

$$\beta(h) = \frac{1}{1 + h\nu} \quad (2.11)$$

$$\kappa(h) = \frac{[1 - \theta(h)][1 - \theta(h)\beta(h)]}{\theta(h)[1 + \alpha\beta(h)]} (\sigma + \eta) = \frac{h^2(1 - \theta)(1 + \nu - \theta)}{(1 - h(1 - \theta))(1 + h\nu + \alpha)} (\sigma + \eta) \quad (2.12)$$

and $\Delta p_{t+h} = p_{t+h} - p_t$ as well as $\Delta p_{t-h} = p_{t-h} - p_{t-2h}$.⁴ The second part of equation (2.12) can be derived under consideration of the explicit expressions for $\theta(h)$ and $\beta(h)$. The values

²In direct comparison to the work of Franke and Sacht (2014), we allow also for a (serially correlated) technology and cost-push in addition to a nominal interest rate shock. Furthermore, backward-looking behavior in the inflation rate dynamics is incorporated by the assumption of price indexation (Christiano et. al (2005) instead of rule-of-thumb price-setting (Galí et. al (2001)). The reason for this is that we claim that price indexation is the dominant procedure in the modeling of a hybrid NKMs in the literature.

³The Calvo setting is helpful in this respect, whereas markup pricing together with Taylor's staggered wage contracts would be more difficult to treat (see Christiano (1985)).

⁴Franke and Sacht (2014) show that the solution path of Δp_t (and y_t ; see below) turns into a specific

of two of the structural parameters, which enter the NKPC, are dependent on the frequency of decision-making. First, the time preference rate ν becomes $h\nu$ in an h -economy since the household is less discounting future changes in the utility over a subperiod with length $0 < h < 1$. From this it follows that the discount factor turns into $\beta = \beta(h) = 1/(1+h\nu)$. Second, the price stickiness θ also has to be adjusted. As in a period of length h the probability for resetting the price of the firm will be $h(1 - \theta)$. We have $\theta(h) = 1 - h(1 - \theta)$ if the pure symbol θ is retained for the constituent stickiness parameter from the quarterly setting. The stickiness remains the same in the sense that on average a firm is allowed to reset the price every $1/[1 - \theta(h)]$ periods of length h , which, independently of h , means every

$$h/[1 - \theta(h)] = h/[1 - 1 + h(1 - \theta)] = 1/(1 - \theta)$$

quarters. Under consideration of $\beta(h)$ and $\theta(h)$, the slope of the NKPC turns into $\kappa(h)$. As we will discuss later, it can be shown that the slope becomes flatter on a higher frequency, i.e. as the period length decreases. Again, since the output gap exhibits no time dimension, $\kappa(h)$ is multiplied by h , therefore consider the term h^2 in equation (2.12). The parameters σ and η are not frequency-dependent since elasticities also exhibit no time-dimension. Note furthermore that according to the concept of price indexation, a fraction of firms show their willingness or ability to re-optimize their price in a *point in time*. Hence the parameter α , which denotes this fraction, is not frequency-dependent, either.

Making use of the previous expression for the high-frequency NKPC and the fact that the (natural) nominal interest rate is expressed by high-frequency magnitudes by multiplying with h , the Taylor rule for the h -economy reads

$$hi_t = h\bar{i} + \phi_\pi \Delta p_t + h\phi_y y_t + hv_t^i. \quad (2.13)$$

It is worth mentioning that the monetary policy parameter with respect to the inflation rate (ϕ_π) remains unaffected by a change in the period length, while this does not hold for the policy reaction to the output gap denoted by ϕ_y . The reason is that the inflation rate is defined as a growth and the output gap as an adjustment rate. For the latter note that the output gap is defined as the difference in the actual level of output to the actual natural level of output (i.e. the level of output in the absence of nominal rigidities) and therefore has no time-dimension. Instead, ϕ_y is multiplied by h . This procedure can be found in an overwhelming amount of numerical studies, where the parameter ϕ_y is calibrated based on, e.g. an annual nominal interest rate. In this case ϕ_y is divided by 4 in order to express the monetary authorities reaction to changes in the output gap in quarterly magnitudes, while ϕ_π remains the same (an example is given in Galí (2008, p. 56)).

The output gap as a ratio of two flow magnitudes needs *not* to be transformed since y_t is defined as a contemporaneous adjustment rate:

$$y_t = \frac{1}{1 + \chi} E_t y_{t+h} + \frac{\chi}{1 + \chi} y_{t-h} - \delta_1(h)(i_t - E_t \pi_{t+h} - \bar{i}) + \delta_2(h)v_t^y \quad (2.14)$$

with

$$\delta_1(h) = h \frac{1 - \chi}{\sigma(1 + \chi)} \quad (2.15)$$

$$\delta_2(h) = h \frac{(1 - \chi)(1 + \eta)(\rho(h)^y - 1)}{(1 + \chi)(\sigma + \eta)}. \quad (2.16)$$

differential equation in transition to a continuous-time specification of the model. More precisely, it can be shown that, under our numerical parameter scenario given here, the IRFs of the model variables converge towards finite values if the period length h tends to zero. Therefore, we refer to the ‘Observations’ (or better: ‘Lemmata’) 1 to 3 (together with the associated Appendix) presented in Franke and Sacht (2014).

The only frequency dependent components of the dynamic IS equation are the variables i_t , $E_t\pi_{t+h}$ and \bar{i} , which are all multiplied by h and the persistence parameter ρ^y , which has to be reformulated into $\rho(h)^y = 1 - h(1 - \rho^y)$ (see the derivation below). $\rho(h)^y$ is part of the composite parameter $\delta_2(h)$ due to the existence of the technology shock v_t^y in the (hybrid) dynamic IS equation. Equivalent to the price indexation parameter α , the habit formation parameter χ stands for a fraction. In more detail, in the quarterly model current consumption c_t^k of household k and past aggregate consumption C_{t-1} (scaled to the household's consumption level) enter the utility function as the difference $c_t^k - \chi C_{t-1}$. This expression requires χ to be dimensionless, as in the h -economy it simply becomes $hc_t^k - \chi hC_{t-h}$, if the consumption variables are quarterized, too. Note that in a DSGE model, the goods market is cleared in every period, i.e. $y_t = c_t$ holds. Again, further details regarding the microfoundation of the high-frequency (hybrid) dynamic IS equation can be found in the Appendix A1.

Finally, we are going to transform the AR(1) shock processes (2.5). Therefore, we express the law of motion as a *contraction rate*

$$v_t^z - v_{t-1}^z = (\rho^z - 1)v_{t-1}^z + \varepsilon_t^z. \quad (2.17)$$

Accordingly, over the subperiod h we get

$$v_t^z - v_{t-h}^z = h(\rho^z - 1)v_{t-h}^z + \varepsilon_t^z \quad (2.18)$$

which, after simple re-arrangement, leads to

$$v_t^z = \rho(h)^z v_{t-h}^z + \varepsilon_t^z \quad (2.19)$$

where

$$\rho(h)^z = 1 - h(1 - \rho^z) \quad (2.20)$$

and $z = \{y, \pi, i\}$ holds (see also Franke and Sacht (2014) as well as Sacht and Wohltmann (2013)). Note that the term $(\rho^z - 1)$ can be interpreted as the *contraction persistence*. This kind of persistence refers to the nature of the AR(1) process itself since $0 \leq \rho^z < 1$ holds, i.e. the shock dies out over a finite amount of periods. By multiplying this contraction rate with h , the shock is spread over the corresponding amount of *subperiods*. In this case the contraction rate is given by equation (2.20) and, hence, now becomes frequency-dependent. A more elaborate discussion on the effects of a change in the shock process with respect to h is given in the next section. It goes without saying, that the deterministic impulse ε^z becomes part of the agents' information *after* the shock occurs. Due to the assumption of rational expectations, the shock hits the economy surprisingly on impact, while the dynamics of the variables are perfectly predictably afterwards.

In the *third and final step* we quarterize all frequency-dependent variables in order to ensure the comparability across different h -economies. Note that here we apply the *systematic/skip sampling aggregation* scheme in order to compare different h -economies. This means that we take into consideration the value of the inflation rate in the h -economy and its corresponding counterpart in the quarterly economy on the given quarter. For a direct comparison it is necessary to divide the associated variable(s) by h . This concept is taken directly from the econometrics literature. Silvestrini and Veredas (2008, p. 459) state that the systematic/skip sampling aggregation scheme has to be applied on *stock* variables like the inflation and the nominal interest rate. The reason is that decisions based on stocks depend on the recent

realized values of these variables.⁵ Based on that, denoting the log price prevailing over the same time interval by p_t , the quarterized inflation rate is given by

$$\pi_t^{(h)} = \frac{\Delta p_t^{(h)}}{h} \quad (2.21)$$

with $\Delta p_t^{(h)} = p_t - p_{t-h}$. The superscript (h) indicates the difference in the variable of the baseline ($h = 1$) relative to the h -economy ($0 < h < 1$) model. Note that in the former case $\pi_t^{(1)} = \Delta p_t^{(1)}/1 = p_t - p_{t-1}$ holds. For clarification, in the following, we omit this superscript (except in cases where a direct comparison is necessary). Therefore the NKPC (2.10) and the Taylor rule (2.13) for $0 < h < 1$ are divided by h itself, while the dynamic IS equation remains unaffected. The *quarterized* h -economy version of the NKM is finally given by

$$y_t = \frac{1}{1+\chi} E_t y_{t+h} + \frac{\chi}{1+\chi} y_{t-h} - \delta_1(h)(i_t - E_t \pi_{t+h} - \bar{i}) + \delta_2(h) v_t^y \quad (2.22)$$

$$\pi_t = \frac{\beta(h)}{1+\alpha\beta(h)} E_t \pi_{t+h} + \frac{\alpha}{1+\alpha\beta(h)} \pi_{t-h} + \tilde{\kappa}(h) y_t + v_t^\pi \quad (2.23)$$

$$i_t = \phi_\pi \pi_t + \phi_y y_t + v_t^i. \quad (2.24)$$

$$v_t^z = \rho(h)^z v_{t-h}^z + \varepsilon_t^z \quad (2.25)$$

with (2.11), (2.15) and (2.16) as expressions for the composite parameters $\beta(h)$, $\delta_1(h)$ and $\delta_2(h)$, respectively. According to equation (2.12), $\kappa(h)$ is now redefined as

$$\tilde{\kappa}(h) = \frac{\kappa(h)}{h} = \frac{h(1-\theta)(1+\nu-\theta)}{(1-h(1-\theta))(1+h\nu+\alpha)}(\sigma+\eta). \quad (2.26)$$

Since we assume deterministic shocks only throughout Part One of this thesis we omit the expectation operator E_t . The dynamic system above is our point of departure for our analytical and numerical analysis undertaken in the following sections.

We close this section with a brief discussion on the stability of an h -economy, i.e. as $0 < h < 1$ holds. Therefore, we consider the purely forward-looking case ($\chi = \alpha = 0$) only. It is well-known (for $h = 1$) that within this model environment, the dynamic system is determinate and saddlepoint stable if the following condition is true:

$$(1 - \beta(h))\phi_y + \tilde{\kappa}(h)(\phi_\pi - 1) > 0. \quad (2.27)$$

Note that this expression is the same as shown in the seminal paper by Bullard and Mitra (2002, p. 1115) except for the frequency-dependent parameters β and κ , which have to be substituted by $\beta(h) = 1/(1+\rho h)$ and $\tilde{\kappa}(h) = \kappa(h)/h$. It goes without saying that in the case $h = 1$, this stability condition is fulfilled if the Taylor principle holds, which requires $\phi_\pi > 1$ to be true since all (composite) parameters are assumed to be equal or greater than zero. It is obvious that in the purely forward-looking h -economy model variant, the Taylor principle

⁵A large body of conflicting literature on this kind of time aggregation exists. For a survey see, again, Silvestrini and Veredas (2008). Hassler (2011) is being named as a representative of the literature, who points out that due to the existence of competing methods of aggregation, i.e. temporal aggregation (cumulation of flows), systematic/skip sampling (specific value at a point in time) or simply averaging, it is unclear which of these different schemes to choose. Drost and Nijman (1993) show that temporal aggregation and systematic/skip sampling might produce different outcomes. The reason is that information will be lost through aggregation - depending on what kind of aggregation method is applied. In Sacht and Wohltmann (2013) *temporal aggregation* is applied on the inflation rate, which is based on a specific economic interpretation. However, by direct comparison of the qualitative and quantitative observations in both studies, it turns out that there exist just minor quantitative differences in the results.

also serves as a sufficient condition for the existence of a unique solution, which holds for any value of $0 \leq h < 1$. For a more elaborate discussion on determinacy — especially concerning the hybrid specification of the high-frequency model — we refer to Flaschel et. al (2008) and Franke and Sacht (2014).⁶

2.3 The Impacts of Frequency-Dependent Persistence

To explain the results presented here (and those discussed in Chapter 3), it is important to analyze the underlying driving forces which cause the adjustments in the model under the variation of the period length h . To begin with, the structure of the shock and, in particular, the *exogenous* persistence process must be investigated. From equation (2.20) it can be seen that, when compared to its equivalent in the baseline model given by (2.5), the shock process under this specification for the high-frequency model differs from the one in the quarterly model due to the existence of h . In particular, $v_t^z = \rho^z v_{t-1}^z + \varepsilon_t^z$ is reformulated into $v_t^z = \rho(h)^z v_{t-h}^z + \varepsilon_t^z$ with $\rho(h)^z = 1 - h(1 - \rho^z)$ where z denotes the three different shocks. As a consequence, the history of the shock process is changed if the baseline model is reformulated into an h-economy framework. To see this, consider the expectation value of AR(1) process v_t^z given by the solution with respect to the initial shock in $t = 0$ (cf. equation (2.20); see Appendix A2 for a proof):

$$E_0 v_t^z = [\rho(h)^z]^t \varepsilon_0^z, \quad (2.28)$$

where E_0 denotes the expectation operator conditional on the information given in the initial period $t = 0$. As we turn to an investigation on a higher frequency relative to the baseline period length, the dynamics in the shock process are spread over all *subperiods*.⁷ This observation is mapped into an increase of the weight on the previous realization of the shock as h (v_{t-h}^z) declines. From this it follows that *less* of the impact of the initial impulse (ε_0^z) has vanished relative to the baseline period length in the transition from one subperiod to another. More precisely, the difference in $v_t^z - v_{t-h}^z$ becomes *smaller* as h decreases since $\rho(h)^z > \rho^z$ holds for $0 < h < 1$ and all values of ρ^z . Due to the fact that the contraction persistence $\rho(h)^z$ is decomposed in a regular $(1 - \rho^z)$ and a frequency dependent (h) term, it can be denoted as the *frequency-dependent ‘contraction’ persistence* (FCP). According to equation (2.28) this kind of updating-process of the system’s exogenous driving force, applied in every period, must lead to higher persistence in an h-economy environment due to the increase in the ‘life span’ of the shock itself.

By investigating the dynamics in case of various shocks, *inherited* persistence must also be taken into account. This kind of persistence displays the cross-relationship between the movements in the output gap and the inflation rate. It is measured by the slope of the NKPC, i.e. the frequency-dependent composite parameter $\tilde{\kappa}(h)$. Note that this parameter is crucial for the implementation of a(n optimal) monetary policy strategy induced by the central bank. The latter has direct control over the nominal interest rate, which therefore serves as its primary policy instrument. However, the inflation rate can only be influenced

⁶Note that, given a hybrid specification of the NKM, a check on determinacy is not straightforward since an intensive analysis of the (in)stable Eigenvalues is strictly required in this case.

⁷Obviously, the shock process v_t^z displays the exogenous impact on the system in every subperiod up to t , e.g. for every month, week or day. In Sacht and Wohltmann (2013), $t = sh$ is assumed which leads to $s = 0, 1, \dots, t/h$. Here, v_t^z displays the value of the shock in the h-economy relative to every benchmark period (which is assumed also to be a quarter in their study). We consider this specification explicitly as we compute the value of the loss (relations) in the next Chapter.

indirectly by changing the output gap via consumption smoothing (see the next sections for a detailed analysis of the corresponding transmission channel(s)). Hence, the *effectiveness* of monetary policy relies entirely on $\tilde{\kappa}(h)$: the impact of the change in the output gap depends on the slope of the NKPC. A high value of the latter might lead to a stronger stabilisation effect on the inflation rate as a shock occurs. For clarification we decompose $\tilde{\kappa}(h)$ into

$$\tilde{\kappa}(h) = \tilde{\lambda}(h)(\sigma + \eta) \quad (2.29)$$

with

$$\tilde{\lambda}(h) = \frac{h(1-\theta)(1+\nu-\theta)}{(1-h(1-\theta))(1+h\nu+\alpha)}, \quad (2.30)$$

since the elasticities σ and η do not depend on h and account just for scale effects. Based on equation (2.30) we are able to discuss the change in the slope with respect to an increase in the degree of price stickiness and indexation as well as the period length. Applying the total differential on $\tilde{\lambda}(h)$ this leads to

$$d\tilde{\lambda}(h) = \frac{\partial\tilde{\lambda}(h)}{\partial\alpha}d\alpha + \frac{\partial\tilde{\lambda}(h)}{\partial\theta}d\theta + \frac{\partial\tilde{\lambda}(h)}{\partial h}dh.$$

Since all parameters in $\tilde{\lambda}(h)$ are bounded between 0 and 1, it can be easily checked that the following results regarding the sign of the partial derivatives hold, respectively:⁸

$$\frac{\partial\tilde{\lambda}(h)}{\partial\alpha} = \frac{h(\theta-1)(1-\theta+\nu)}{[1+h(\theta-1)](1+\alpha+h\nu)^2} < 0 \quad (2.31)$$

$$\frac{\partial\tilde{\lambda}(h)}{\partial\theta} = \frac{h^2(\theta-1) - h[2(1-\theta) + \nu]}{[1+h(\theta-1)]^2(1+\alpha+h\nu)} < 0 \quad (2.32)$$

$$\frac{\partial\tilde{\lambda}(h)}{\partial h} = -\frac{(\theta-1)(1-\theta+\nu)[1+\alpha+h^2\nu(1-\theta)]}{[1+h(\theta-1)]^2(1+\alpha+h\nu)^2} > 0. \quad (2.33)$$

Note that the sign of these derivations remains unaltered in quarterly magnitudes, i.e. for $h = 1$ and $dh = 0$. According to the above expressions, the change in the slope is now decomposed into three terms, which measure the impact of the change in α , θ and h . Due to the consideration of h in $\tilde{\lambda}(h)$, equation (2.33) indicates a *frequency-dependent 'inherited' persistence* (FIP) effect.

With respect to the expression (2.31), an increase in the fraction of firms, which set their actual price level equal to the previous one, strengthens the importance of backward-looking behaviour in the price adjustment process. This means that changes in the real marginal product (approximated by the output gap) become less considered the higher the non-frequency dependent *intrinsic* persistence will be. In other words, the more firms are indexing their prices to past realisations of the same, the more the impact of economic activity on the price-setting scheme is dampened.

Equivalently, according to the expression (2.32), the higher the probability of *not* setting the prices the lower the impact of the change in the output gap induced by the central bank. From an economic point of view, a high degree of price stickiness makes it difficult for the policy maker to influence inflation rate dynamics. The reason is that the price-setting scheme becomes increasingly degenerated from the movement in the real marginal cost the

⁸Note that for the Calvo parameter in quarterly magnitudes $0 \leq \theta \leq 1$ holds. Hence, this results in $(\theta-1) \leq 0$ and $[1+h(\theta-1)] > 0$ for all values of h over its admissible range.

higher the likelihood for sticky prices will be. As a result the volatility in inflation increases with a higher degree of price stickiness.

An equivalent observation holds according to the expression (2.33): the higher the frequency of decision making, i.e. the *lower* the period length h , the flatter the slope of the NKPC will be. This result can be explained by the direct FIP effect on inflation relative to α and θ . Economically speaking the central bank must change its instrument variable more often to incorporate the higher frequency of reallocations and transactions. Once the monetary authority decides to influence economic activity they become bounded. This means that the central bank has to consider the circumstance that this intervention affects different fractions (about the same size) of agents over a corresponding amount of points in time (which depends directly on the period length). Hence, only a small fraction of firms is able to reset their prices on a single market day. As a result, *ceteris paribus*, the response of the inflation rate to a change in the output gap is lower on a higher frequency.

Given the expressions (2.31) to (2.33) it can be seen that the policy maker faces a (frequency-dependent) trade-off along several dimensions. For example, a decrease in the price stickiness can be offset by a decrease in the period length. This means that even under a high degree of flexible prices, the stabilisation of the inflation rate might become hindered if the transactions are spread over more market days relative to the quarterly case. This effect is weakened as the degree of price indexation decreases. The reason is that the change in the real marginal costs (output gap) is more pronounced the more forward-looking the NKPC will be. In this case, the decrease in h can be offset or even over-compensated by the compound effect induced by the decrease in α and θ . More precisely, in this specific case the central bank faces a situation where its policy strategy is more effective the more the dynamics in inflation depend on economic activity – even on a higher frequency. The question arises if even in the polar case of a purely forward-looking NKM with almost perfect price flexibility, monetary policy can be effective on a nearly continuous time scale, i.e. as $\alpha = 0$, $\theta \approx 0$, $h \approx 0$ hold.⁹ We are going to investigate numerically related questions in the next section. Independently of the FIP effect, the volatility in the economic variables is always increasing as h decreases due to the FCP effect, i.e. the increase in the persistence of the shock process.

For a deeper understanding of both kinds of frequency-dependent persistence effects, we discuss this issue in greater detail. From an economic point of view both effects result from the change in economic activity depending on the period length, i.e. it reflects the higher frequencies in transactions and adjustments. In particular, households and firms are re-optimizing their plans automatically after a shock occurs in order to meet the basic assumption of general equilibria on the goods and labor market in every period. In a baseline NKM all reallocation processes are made by all agents within a quarter on a specific market day. However, as already stated, in an h -economy framework these reallocations are spread over the entire quarter, i.e. a h fraction (about the same size) of agents are re-optimizing on a specific market day within a month ($h = 1/3$), week ($h = 1/12$) or day ($h = 1/90$) respectively.¹⁰

Figure 2.1 describes this issue in detail by considering the following example. Let us assume that the representative firm is able to adjust its nominal price level only every three quarters.

⁹Note that such an scenario is superfluous if $\theta = h = 0$ holds due to model dichotomy. For $\theta = 0$ this means that monetary policy is of no use (in the standard NKM) as prices are *fully* flexible since the nominal variables adjust one to one with the price level as a shock occurs. For the analysis of h -economy model in continuous time ($h = 0$) see Posch et al. (2011) as well as Franke and Sacht (2014).

¹⁰It is assumed that the reallocation process takes place in a *point* of time. In this respect, it is irrelevant if we refer in particular to a begin- or end-of-the-period concept.

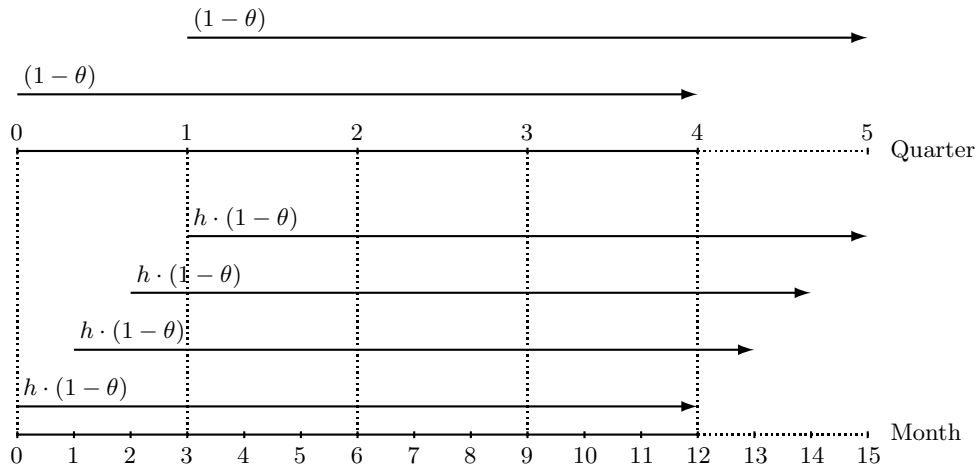


Figure 2.1: Frequency of price adjustments in the quarterly NKM versus an h -economy ($h = 1/3$).

Note: This graph shows an example for the relationship between the amount of market days and price adjustments. The arrows at the *top* denote the frequency of price adjustments in a *quarterly* economy and the arrows at the *bottom* denote the (selected) frequency of price adjustments in a *monthly* economy.

Hence, if we consider only a quarterly model with $\theta = 2/3$ and $h = 1$, the firms are able to adjust their prices in January, April, July or October respectively (so only four market days exist at all), with probability $h(1 - \theta) = (1 - 2/3) = 1/3$. In a monthly economy the probability to adjust the price changes to $h(1 - \theta) = 1/3(1 - 2/3) = 1/9$, which means that the probability is lower in every month relative to a quarter as long as the shock process does not vanish. In the case of a weakly economy ($h = 1/12$) the probability declines further and so on.

This example shows how the FCP and FIP effects have an impact on the transition from a low to a high-frequency economy. For example, under consideration of the price stickiness some but not all firms are able to react to additional news which arrive during the period (e.g. at the beginning of the next month). Hence the corresponding proportion of firms which is able to adjust on each single market day (month) must obviously be smaller than in the baseline case (quarter). From that it follows, that the duration of price adjustments increases with a decrease in the period-length h . The same argument also holds for transactions on the goods market. The reason is that the volatility in the economy increases due to the optimization behavior of firms and households (expressed by the change in their optimization plans). Hence the corresponding time spans lead to a higher shock persistence, i.e. the change in the variables becomes smoother over time. In addition, later on, we might observe stronger impact effects at time $t = 0$ as the period length decreases. A general intuition for that is if the future is one month or one day ahead it has a greater impact on today's choices than if it is one quarter away (cf. Anagnostopoulos and Giannitsarou (2010a, p. 12)).

It can be stated that the consideration of the period length sheds a different light on the discussion of monetary policy in (baseline) NKMs. In fact, a decrease in h might dampen the influence of the central bank to stabilize the economy. Our discussion in this section also fill the gap in the work by Franke and Sacht (2014) which lacks on an economic rationale

for explaining the results reported in their paper. In order to examine the quantitative and qualitative observations with respect to a variation in the period length and both FCP and FIP effects, we are going to analyze IRFs in the next section. Furthermore, the FCP and FIP effects also play an important role in explaining the observations made with respect to *optimal* monetary policy in Chapter 3.

2.4 Numerical Analysis of the Baseline NKM

In the following we are going to analyse the impact of all three shocks on the dynamics of the model for various kinds of h-economies. Therefore, we compare the corresponding IRFs with those of the baseline NKM for $h = 1$. Within this investigation we allow for two different specifications in which the underlying structure of the (high-frequency) model consists either on backward- and forward-looking elements ($0 \leq \chi \leq 1, 0 \leq \alpha \leq 1$) or forward-looking elements only in which we omit habit formation in consumption and price indexation ($\chi = \alpha = 0$). In the first case we refer to the *hybrid* and in the latter case to the *purely forward-looking* NKM. While it is well-known that the hybrid NKM accounts for persistence in the economic indicators, we show that inertia is indeed a significant characteristic of the IRFs in a high-frequency environment of the purely forward-looking NKM – even in the case of a non-autocorrelated shock ($\rho^z = 0$) due to the FCP effect. The state-space representation of the dynamic system (2.22) to (2.25) can be expressed by

$$0 = A X_t^{TR} + B X_{t+h}^{TR} + C X_{t-h}^{TR} + D V_t \quad (2.34)$$

$$V_t = N V_{t-h} + \Xi_t \quad (2.35)$$

with $X_t^{TR} = (y_t^{TR}, \pi_t^{TR})'$, $V_t = (v_t^i, v_t^y, v_t^\pi)'$ and $\Xi_t = (\varepsilon_t^i, \varepsilon_t^y, \varepsilon_t^\pi)'$. The system matrices A, B, C, D and N comprise all corresponding frequency-dependent (deep) parameters of the model.¹¹ The solution to (2.34) together with (2.35) is given by

$$X_t^{TR} = \Omega X_{t-h}^{TR} + \Phi V_t \quad (2.36)$$

where $\Omega \in \mathbb{R}^{3 \times 3}$ and $\Phi \in \mathbb{R}^{2 \times 3}$ are the solution matrices. In case of a purely forward-looking model (where $C = \mathbf{0}$ holds, which results in $\Omega = \mathbf{0}$) we obtain

$$X_t^{TR} = \Gamma V_t. \quad (2.37)$$

Hence, the dynamics in the output gap y_t^{TR} and the inflation rate π_t^{TR} for $0 < h < 1$ are described by the reduced solutions (2.36) and (2.37) for the hybrid and purely forward-looking case, respectively. A detailed description of the numerical derivation of the solution matrices Ω , Φ and Γ can be found in the Appendix A3. In particular, the solution matrices are computed numerically by applying the *method of undetermined coefficients* (also known as the “guess-and-verify method”; McCallum (1983)) and the *brute force iteration procedure* (Binder and Pesaran (1995, p. 155, fn. 26)). As an advantage over the analytical solution based on a the Schur- or Jordan decomposition method (for the hybrid NKM), we claim that the numerical solution exhibits a better manageability since the corresponding computer codes are easy to implement.¹² However, it is confirmed that the numerical and analytical

¹¹The superscript *TR* indicates that here we study the dynamics under consideration of an ad-hoc nominal interest rule of Taylor type. We make this reference in order to distinguish this non-optimal monetary policy regime from the optimal ones discussed in the next Chapter.

¹²The MATLAB codes, which are used to produce the numerical results in this and the following Chapter can be downloaded from the authors' webpage at <http://www.makro-vwl.uni-kiel.de/de/team/dipl.-vw.-stephen-sacht>.

solution procedures (presented in Sacht and Wohltmann (2013)) lead to the same results for Ω , Φ and Γ .¹³ All of the upcoming figures presented here also show the dynamics in the nominal interest rate, the real interest rate and the price level. Therefore, we plug the reduced-form solutions of y_t^{TR} and π_t^{TR} in the Taylor rule (2.24). The real interest rate is then given by the difference $i_t^{TR} - \pi_{t-h}^{TR}$ and the price level is simply computed by $p_t^{TR} = \sum_{k=0}^t \pi_k^{TR}$.

ν	Households' time preference rate	0.010
σ	Inverse intertemporal elasticity of substitution in consumption	1.000
η	Intertemporal elasticity of substitution of labour	1.000
θ	Calvo degree of price stickiness	0.667
α	Degree of Price Indexation	0.500
χ	Habit-formation parameter in consumption	0.500
\bar{i}	Natural Interest Rate	0.000
ϕ_π	Weight on inflation in the Taylor rule	1.500
ϕ_y	Weight on the output gap in the Taylor rule	0.125
ρ^z	Persistence in the shock process	0.500
ε^z	Impulse associated with the shock process	1.000

Table 2.1: Numerical parameter scenario.

Note: In the purely forward-looking NKM the parameters χ and α are set to 0. In case of a non-autocorrelated shock ρ^z is set to 0. The parameters of the shock process ρ^z and ε^z are identical across all shocks $z = \{y, \pi, i\}$.

The numerical setting in our simulations of the model relies on parameter values adopted from Franke and Sacht (2014). They are collected in Table 2.1, where the frequency-dependent coefficients are based on the quarterly time unit. A value of household's discount rate ν of about 1%, log utility ($\sigma = 1$), and a unitary Frisch elasticity of labour supply ($\eta = 1$) are standard. The policy coefficients ϕ_π and ϕ_y are the classical values which can be found in Taylor (1993). A standard price rigidity of $\theta = 0.667$ implies an average price duration of three quarters. These parameters are borrowed from Galí (2008, p.52). The values for α and χ are in line with those in literature and, in particular, χ is close to the one taken from Smets and Wouters (2003, p.1143). We simply assume that $\bar{i} = 0$ holds. Finally, the initial shock ε^z is calibrated on a moderate value of 1. Note that all different shocks (indicated by the superscript z) exhibit the same size of persistence and initial disturbance. Furthermore, it goes without saying that in the purely forward-looking NKM the parameters χ and α are set to zero. In the following we put emphasis on the analysis of one-off (impulse) shocks only, i.e. $\rho^z = 0$ holds. However, we briefly compare the results to the ones obtained in case of an autocorrelated shock $0 < \rho^z < 1$. Hereby, the value of $\rho^z = 0.5$ is associated with

¹³As a drawback of the numerical solution approach presented here, this procedure might demand a specific amount of main memory and time for execution. While this problem does not appear when computing all IRFs presented in this thesis, for the evaluation of the (so-called) welfare functions in Chapter 3 the associated analytical solutions being more appropriate to use in that case. For a detail discussion we refer to the next Chapter.

moderately persistent shocks. For a clear arrangement, all corresponding Figures can be found in Appendix A4. The analysis of all kind of shocks considered here have been heavily discussed for a quarterly economy ($h = 1$). We just name Galí (2008, his Chapter 3) and Walsh (2010, his Chapter 8) as being representative studies.

2.4.1 Nominal Interest Rate Shock

We start with analysing the impact of a nominal interest rate shock under variation of the period length. The corresponding IRFs for the purely forward-looking model are shown in Figure 2.2.¹⁴ On quarterly magnitudes ($h = 1$) an increase in the nominal interest rate by the central bank leads to an increase in the real interest rate due to the Taylor principle ($\phi_\pi > 1$). As a result the output gap and the inflation rate decline on impact, while a permanent decrease in the price level can be observed. The transmission channel can be explained as follows. In general, the household decides either to purchase consumption goods or bonds. The latter being held for one period and therefore can be indentured as a(n indirect) saving option. Since the household receives an interest payment when selling the bonds in the next period, an increase in the corresponding real interest rate increases the attractiveness for holding bonds. Hence, the demand for goods must decline at the same proportion as the amount of bonds increases. This effect is well-known as the intertemporal consumption smoothing effect, which can be explained by the Euler consumption equation (see equation (2.43) given in the Appendix A1). Under consideration of the Calvo (1983) price setting scheme, the firms which are able to set the price, decrease the latter as demand drops. The remaining firms face no price adjustment probability and, therefore, reduce the supply of goods. Hence, the goods market becomes cleared again after the shock occurs.

As we reduce the period length to a monthly ($h = 1/3$) and even a weekly ($h = 1/12$) magnitude, several observations are worth mentioning. First, although the shock is a non-autocorrelated one, all economic indicators exhibit a moderate degree of persistence. This is caused by the FCP effect, which leads to a smooth monotonic movement in the variables compared to a rapid jump from the initial period to the next in the case $h = 1$. Note that inflation inertia amplifies the pressure on the price level: the shorter the period length, the stronger the (negative) effect on the price level will be. Second, with respect to the impact effects, we observe a slightly stronger reaction in the real interest rate and the output gap, while the decrease in the inflation rate is less pronounced in an h-economy framework. However, the quantitative magnitude is quite small, i.e. the values of the inflation rate on impact are hard to distinguish in transition from a low to a high frequency. As we interpret a nominal interest rate shock as being a possible monetary policy strategy of the central bank, a nearly non-existing impact on the inflation rate seems to be appropriate for the monetary authority. It is meant by that the inflation rate becomes less destabilized on a higher frequency *on impact*. This observation is grounded on the FIP effect: the change in the output gap is transmitted less into inflation dynamics since the slope of the NKPC becomes flatter as h decreases. Nevertheless, the permanent drop in the price level is much stronger on a higher frequency — this can be seen as a severe problem of the central bank,

¹⁴We have to indicate, that the time scale is adjusted for all values of h in order to allow for a graphical comparison of the IRFs for different frequencies. The interested reader might study the corresponding computer codes, used to reproduce the following Figures, for more information. Furthermore, it must be stated that possible intersections of different IRFs are hardly to be considered for an analysis. The reason is that we have the more observations, the lower h will be. In the polar case of $h = 1$ only quarterly values can be used for interpretation. Therefore, we do not draw a line in order to connect the quarterly values but use a square as a marker instead.

i.e. as the prevention of permanent strong changes in the price level being one of its policy objectives.

In Figure 2.3 the IRFs for a non-autocorrelated shock within the hybrid NKM are displayed. It can be seen that under the standard calibration of the parameters applied here, the dynamics of the output gap and the inflation rate in the h-economy models become hump-shaped. In particular, troughs can be observed. In the quarterly model we observe a monotonic movement of these variables back to the steady state. These results mimic those reported in Franke and Sacht (2014). The opposite holds for the nominal and real interest rate, where we observe an undershooting in both variables in the case $h = 1$ – and just slightly in a monthly but significantly not in a weekly economy. The real interest rate responses stronger on impact to a decrease in h , which leads to stronger decline in the output gap in period $t = 0$. However, the impact effects on the output gap are quantitatively less pronounced for all values of h relative to the forward-looking case. Again, the impact effects on the inflation rate are hard to distinguish across different frequencies. This can be once more explained by the FIP effect, where less of the change in the output gap is transmitted into the inflation rate on a higher frequency. Interestingly, the values of the nominal and real interest rate in the monthly economy lie below the ones given in the weekly economy, roughly before time $t = 1$, where the opposite holds after the first quarter. An exception are the dynamics in the output gap and the inflation rate, where the IRFs for the case $h = 1/12$ are significantly below the ones for $h = 1/3$. This could probably be explained by the moderate degree of intrinsic persistence. As we had shown in the previous section, an increase in α reduces the effectiveness of monetary policy intervention, where this effect is amplified by the FIP one, i.e. as the period length decreases. Most likely, at a specific value of h , the FIP effect dominates the intrinsic persistence one and causes the, let's say, asymmetric movements as we go from a monthly to a weekly economy. Finally, the quantitative effects on the development in the price level are stronger for all values of h compared to the forward-looking case. Hence, the failure of the central bank for price level stabilisation is more distinctive within a hybrid specification of the NKM.

Figures 2.4 and 2.5 represent the IRFs for an *autocorrelated* shock in the purely forward-looking and hybrid NKM, respectively. As we can see, the quantitative effects are not surprisingly more pronounced if a moderate persistence in the shock is observed. This holds especially for the price level (therefore consider the IRFs for $h = 1/12$) in both cases. In the forward-looking case, a convergence/overlapping in the IRFs can be identified as h decreases. This holds also in the hybrid NKM, where the now *amplified* FCP effects (note that $\rho(h)^i$ increases due to an increase in ρ^i) might offset the dominance of the FIP over the intrinsic persistence effect as described above for the non-autocorrelated case. Furthermore, we observe over- and undershooting behaviour in the IRFs across different values of h for the nominal and real interest rate. The result that no distinctive differences in the impact effects of the inflation rate occur for different values of h , prevails in the case $\rho^i > 0$. Both observations hold for both specifications of the model.

Hence, the ineffectiveness of monetary policy on inflation stabilisation on impact is independent of the FCP effect. However, the latter amplifies the quantitative effect on the price level dramatically as discussed above. To sum up, in comparison to the non-autocorrelated shock we observe quantitatively stronger destabilisation effects in the output gap and the inflation rate, while the results regarding the qualitative effects (due to the existence of troughs) remain. With respect to the latter observation it can be said that the impact of the intrinsic persistence together with the FCP effect strengthens the humped-shaped movements in the

h-economies.

2.4.2 Technology Shock

Figure 2.6 depicts the adjustments in the model variables in case of a non-autocorrelated technology shock in the forward-looking NKM. As usual, first, we describe the transmission channel based on the case $h = 1$. An increase in ε^y leads to an increase in the efficiency of production. This affects potential output positively, while actual output remains unaffected, where the latter reacts one-to-one to changes in private consumption due to the assumption of a general equilibrium. As a result the output gap declines on impact. On the demand side, households face a decrease in the gross natural real interest rate. This means that the interest payment on bonds drops in the long run, i.e. the desired amount of bonds held in the steady state declines (to see this consider, again, the Euler consumption equation (2.43) given in the Appendix A1). Hence, the long-run demand for goods increases and coincides with the higher level of steady state (potential) output. Since the output gap drops and firms which are not able to set the price, lower the supply of good, while the remaining firms lower its price. The central bank responds to the decline in the output gap by lowering the nominal and, hence, the real interest rate in order to boost actual output based on the consumption smoothing effect.

On a higher frequency we observe persistence in all variables due to the FCP effect. Furthermore, less pronounced effects in all variables can be observed in this case. Especially with respect to the real interest rate, this variable rather increases than decreases (for $h = 1/12$) on impact. The weaker impact effect on the output gap relative to the one observed in the case of the nominal interest rate shock, can be explained by the (inverse) elasticities of consumption σ and labour η . In order to see this, consider the relation in front of the expression for the shock process v_t^y in the dynamic IS curve (see equation (2.16)) given by

$$h[\rho(h)^y - 1] \frac{1 + \eta}{\sigma + \eta} \quad (2.38)$$

if no habit formation is assumed ($\chi = 0$). Under consideration of our parameter scenario, where $\sigma = \eta = 1$ ($\rho^y = 0$) holds, the values of previous equation being $|1|$, $|2/9|$ and $|11/144|$ in a quarterly, monthly and weekly economy, respectively. Hence, ceteris paribus, the shorter the period length, the weaker the impact of the shock on the output gap will be. The quantitative impact effects, therefore, depend on the corresponding elasticities and the *reinforced* FCP effect, which is displayed by the existence in $\rho(h)^y$ in the expression (2.38) and the shock process v_t^y itself. This reinforced FCP effect dominates the FIP one: the associated impact effect on the inflation rate is also weak on a higher relative to a lower frequency like for the output gap. In other words, although the effectiveness of monetary policy is weakened as h increases, the corresponding relative strong boost in technology feeds into inflation dynamics. The dominance of the (reinforced) FCP over the FIP effect also leads to a less pronounced drop in the steady state price level on a higher relative to a lower frequency. Due to the persistence in the inflation rate based on the FCP effect, the central bank is able to maintain a higher level in the long-run relative to the case $h = 1$.

As we turn to an analysis of the same shock within a hybrid specification of the NKM (see Figure 2.7), overall weaker impact effects relative to the purely forward-looking case can be observed. This can be explained by the distinctive troughs which display the interaction between intrinsic persistence and the (reinforced) FCP effect: due to backward-looking behaviour this induces rapid changes in economic activity. Hence, the future (non-)linear

movements of the inflation rate are transmitted into the real interest rate and, therefore, dampen the decrease in the output gap. Note that the real interest rate in monthly magnitudes remains roughly unchanged on impact. This effect is more pronounced on a higher frequency because of $\rho(h)^y$ being considered in the expression for the composite parameter given by (2.38) and the shock process v_t^y . Due to the higher persistence in the inflation rate in the case $h = 1$ relative to both h-economies, the price level converges to a higher steady state value in quarterly magnitudes. This is just the opposite result as we observed in the purely forward-looking model. This is most likely stemming from the dominance of the FIP over the (reinforced) FCP effect since the FIP one is now amplified by the increase in α . It follows that the central bank faces a greater destabilisation of the price level on higher frequency in case of backward-looking behaviour of agents being considered.

Figures 2.8 and 2.9 present the IRFs for an *autocorrelated* shock in the purely forward-looking and hybrid NKM, respectively. While obviously the persistence in all variables increases, the qualitative results remain almost the same in comparison to the case $\rho^y = 0$; note that the real interest rate in the hybrid case is now also positive on impact. Two exceptions can be identified with respect to the steady state values of the price level. In the purely forward-looking case, all IRFs converge to the same price level in the long-run. Within the hybrid specification, the adjustments in the price level are of almost the same magnitude in early periods, while there is a divergence in the IRFs for later periods. It seems to be that the FCP and FIP effect are offsetting each other in the former case, while in the latter the FIP effect probably dominates the FCP one. However, a clear interpretation can be hardly obtained: while additional intrinsic persistence strengthens the FIP effect, the FCP effect is also boosted by the fact that $\rho^y > 0$ holds. As a result, the movements of the price level and, hence, a successful stabilisation policy of the monetary policy on a low relative to a high frequency depends on the interaction of α and $\rho(h)^y$ in this case.

2.4.3 Cost-Push Shock

Figure 2.10 depicts the adjustments in the model variables in case of a non-autocorrelated technology shock in the forward-looking NKM. For all values of h , a cost-push shock leads to an increase in the price level induced by firms, which are able to set the price. As a result the inflation rate increases on impact. Since the central bank can influence the inflation rate only *indirectly* via the output gap it faces a *trade-off*. In order to dampen the boost in the inflation rate, monetary authority must rise the nominal and – due to the Taylor principle – real interest rate. However, this leads to a decrease in the output gap since now it is less attractive to consume more goods instead of purchasing bonds. In other words, the households smooth their consumption due to an increase in the interest payments on bonds. The trade-off is now described by the fact that the output gap and the inflation rate move into opposite directions. Note that in case of a cost-push shock in order to target a lower level of the inflation rate after the shock occurs, the central bank must allow for a negative output gap (on impact).

In transition to an h-economy environment, we observe stronger impact effects in all variables. The persistence also increases due to the FCP effect. The quantitative change in the impact effects implies an amplification in the policy trade-off. Since monetary policy becomes less effective due to the FIP effect, the central bank must allow for a more aggressive response of its policy instrument to the shock. More precisely, the real interest rate increases on higher magnitude the shorter the period length will be. It follows that a much stronger drop in the output gap, which is needed to affect the inflation rate. In combination with

the FCP, the FIP effect leads to a more destabilizing scenario in an h-economy relative to a quarterly one. This can also be seen by considering the massive increase in the price level for $h = 1/12$.

As we turn to the hybrid specification of the NKM, which is depicted in Figure 2.11, we observe again that troughs in the dynamics of the variables occur. Besides these qualitative changes, the quantitative difference compared to the case $\rho(h)^y$ are distinctive. The reason is that due to the (moderate) degree of intrinsic persistence more rapid changes in the transactions on a higher frequency take place. Since the existence of backward-looking behaviour in price setting amplifies the FIP effect, monetary policy becomes now less effective in stabilizing the inflation rate as h decreases. Under consideration of an autocorrelated shock, the reported qualitative and quantitative effects are strengthened – again, distinctive troughs emerge in the hybrid case. This can be seen by a brief inspection of the corresponding IRFs for an autocorrelated shock in the purely forward-looking and hybrid NKM, respectively, which are depicted in the Figures 2.12 and 2.13.

2.5 Conclusion

In this Chapter we study the differences in the purely forward-looking and hybrid specification of the baseline NKM based on various shocks and under different frequencies of decision making. Therefore, we show how the structural representation of the baseline NKM (which is given in quarterly magnitudes by convention) must be adjusted by the parameter which denotes the period length given by h . In order to explain our numerical results we identify two frequency-dependent persistence effects. The first one consists on the additional degree of persistence in the shock process, which comes into play as the dynamics in the shock are spread over all subperiods as the period length declines. We call this one the frequency-dependent contraction persistence (FCP) effect. It displays the impact of the increase in the amount of market days on the dynamics of the model. As a result the time of convergence to the steady state increases on a higher frequency since the frequency of transactions and adjustments increases. The second one has an impact on the pass-through of changes in the output gap into inflation dynamics via the slope of the NKPC. As a result the central bank must intervene more often to smaller fractions of price setters on a higher frequency. We call this one the frequency-dependent inherited persistence (FIP) effect. This effect can be amplified or reduced by the non-frequency-dependent intrinsic persistence effect displayed by the degree of price stickiness as well as the effect stemming from the change in the price stickiness. As the period length decreases, the amount of transactions and reallocation processes are spread over all subperiods, which is equivalent to the FCP effect with respect to the price setting scheme. Hence, the driving force of the inflation rate, the output gap, explains less of the dynamics of the former on a higher frequency.

Within our numerical study, we observe an overall increase in the persistence in all variables – even in the case of a non-autocorrelated shock – due to the FCP effect. Across all shocks, the interaction of both persistence effects causes differences in the impact effects of the variables and more distinct hump-shaped adjustments in the hybrid NKM. This holds especially if an autocorrelated shock process is assumed. Furthermore, strong changes in the long-run steady state of the price level occur. The latter case indicates a severe problem as price stability is the main target of the central bank. This problem is much more obvious in case of a cost-push shock, where the monetary authority faces a trade-off between inflation rate and output gap stabilisation. This trade-off is strengthened on a higher frequency since the

strong output gap reaction (on impact) does not feed into the NKPC due to the FIP effect, while the inflation rate increases dramatically. We are going to investigate this scenario in much more detail as we consider optimal monetary policy strategies in the next Chapter.

The results of our numerical exercises resuscitate the question, “which interval best represents agents’ decision-making process?” (Aadland, 2001, p. 291). One answer is more practically oriented: use calibration or estimation methods to find out if variations of the period length can improve the matching of certain empirical moments or the value of an objective function in general. However, research in this direction seems to be rare.¹⁵

Future theoretical and empirical investigations might also consider an asymmetric differentiation of the decision intervals of households and firms. Our numerical results show that the more information on prices and output is available, the increase in the amount of transactions and reallocation processes dampens the possibility of the central bank to stabilize the economic indicators. This can be seen as being equivalent to the results by Winkler and Wohltmann (2012), who show that an anticipated shock in an optimal monetary policy scenario causes a higher welfare loss (relative to a non-anticipated one) due of the adjustments of the economic indicators before the shock actually occurs. Due to this as the central bank faces rapid changes in the output gap and the inflation rate, the question arises, if this result prevails as we consider a heterogeneous approach regarding the period length. It means that e.g. the central bank might react on lower frequency compared to the other agents in the economy. We leave such kind of analysis to further research.

According to Franke and Sacht (2014), a theoretical answer returns to Foley’s principle and requires robustness of the period on which one decides to settle down.¹⁶ Accordingly, a period might be called robust if it is an upper-bound on the length of the decision intervals with, in our case, essentially similar IRFs. From this point of view, the main conclusion is that, in the present modelling framework, the conventional quarter cannot be claimed to be a robust period length. The more general message is that DSGE modelling might be more sensitive to the possible “pitfalls of timing misspecification” (Christiano (1985, p. 397)). In particular, it would be an important question how seriously the structure of optimal monetary policy rules might be affected by changes in the length of the period. As we stated above, we seek to answer this question in the following Chapter.

2.6 Appendix A

2.6.1 A1: Microfoundations of the Baseline NKM in the Case $0 < h \leq 1$

The numerical results presented in this Chapter are derived under consideration of the analytic solutions by Woodford (2003, his Chapter 6) and Galí (2008, his Chapters 3 and

¹⁵Christiano (1985) and Aadland (2001) may be recalled, again, as the only two references regarding this question that we know. More modestly, in Chapter 4 of this thesis, we estimate a high-frequency New-Keynesian Phillips curve on daily inflation data for Argentina. In particular, the results for the Calvo price stickiness parameter are quite in line with microeconomic evidence. The matching of empirical moments (not in an h -economy framework) will be discussed in Part Two of this thesis.

¹⁶Foley emphasises that “[n]o substantive prediction or explanation in a well-defined macroeconomic period model should depend on the real time length of the period.” (1975, p. 310). While Franke and Sacht (2014) disprove this statement to be true based on an elaborate analytical and numerical analysis of a nominal interest rate shock, we show in this Chapter that this obviously also holds in the cases of a technology and a cost-push shock.

4), along with the specifications given in Smets and Wouters (2003) and Christiano et. al (2005), who all provide explicit microfoundations of the (hybrid) version of the standard NKM in *quarterly* magnitudes. Here, those results are adjusted towards the transition of the model's structural equations into their *h*-economy counterparts. The definition of all variables and parameters are (if not done here) given in the main text.

Households: We consider a standard utility function of the representative household subject to the underlying frequency given by¹⁷

$$U_t = E_t \sum_{k=0}^{\infty} \beta(h)^{hk} \left[\frac{1}{1-\sigma} (C_{t+hk} - \chi C_{t+hk-h})^{1-\sigma} - \frac{1}{1+\eta} N_{t+hk}^{1+\eta} \right] \quad (2.39)$$

where C_{t+hk} (C_{t+hk-h}) denotes the current (previous) level of (aggregate) consumption, where the difference $C_{t+hk} - \chi C_{t+hk-h}$ displays habit formation in consumption of the household. N_{t+hk} denotes employment in terms of hours worked. The Euler equation is the result of the first order condition with respect to the amount of bonds B_t subjected to the budget constraint

$$C_{t+hk} = -\frac{B_{t+hk}}{P_{t+hk}} + \frac{W_{t+hk}}{P_{t+hk}} N_{t+hk} + (1 + hi_{t+hk-h}) \frac{B_{t+hk-h}}{P_{t+hk}} + \Pi_{t+hk}^r \quad (2.40)$$

where W_{t+hk} and Π_{t+hk}^r stand for the nominal wage and firms real profits, respectively. The first order condition is then given by

$$\begin{aligned} \frac{\partial U_t}{\partial C_{t+hk}} &= \frac{1}{1-\sigma} (C_t - \chi C_{t-h})^{-\sigma} \left(-\frac{1}{P_t} \right) + \beta(h)^h \frac{1}{1-\sigma} (E_t C_{t+h} - \chi C_t)^{-\sigma} \frac{1 + hi_t}{E_t P_{t+h}} = 0 \\ &\Rightarrow \frac{1}{1-\sigma} (C_t - \chi C_{t-h})^{-\sigma} \frac{1}{P_t} = \beta(h)^h \frac{1}{1-\sigma} (E_t C_{t+h} - \chi C_t)^{-\sigma} \frac{1 + hi_t}{E_t P_{t+h}} \\ &\Rightarrow (C_t - \chi C_{t-h})^{-\sigma} = \beta(h)^h \frac{1}{1-\sigma} (E_t C_{t+h} - \chi C_t)^{-\sigma} (1 + hi_t) E_t \left(\frac{P_t}{P_{t+h}} \right) \end{aligned} \quad (2.41)$$

for $k = 0$ and $k = 1$. Given the Fisher (1930) equation

$$(1 + hi_t) E_t \left(\frac{P_t}{P_{t+h}} \right) = 1 + hr_t = R_t, \quad (2.42)$$

equation (2.41) can be rewritten as

$$(C_t - \chi C_{t-h})^{-\sigma} = \beta(h)^h \frac{1}{1-\sigma} (E_t C_{t+h} - \chi C_t)^{-\sigma} R_t \quad (2.43)$$

where r_t denotes the real interest rate and R_t the gross real interest rate. Log-linearization of the previous equation leads to

$$-\sigma \log(C_t - \chi C_{t-h}) = h \log \beta(h) - \log \left(\frac{1}{1-\sigma} \right) + \sigma \log(E_t C_{t+h} - \chi C_t) + \log R_t \quad (2.44)$$

where $h \log \beta(h)$ and $\log \left(\frac{1}{1-\sigma} \right)$ have to be crossed out since h is constant. By applying the log-linearization technique (cf. Ascari and Ropele (2003), pp. 7) this leads to

$$-\sigma \frac{\bar{c}}{(1-\chi)\bar{c}} \hat{c}_t + \sigma \frac{\chi \bar{c}}{(1-\chi)\bar{c}} \hat{c}_{t-h} = -\sigma \frac{\bar{c}}{(1-\chi)\bar{c}} E_t \hat{c}_{t+h} + \sigma \frac{\chi \bar{c}}{(1-\chi)\bar{c}} \hat{c}_t + \frac{\bar{R}}{\bar{R}} \hat{r}_t \quad (2.45)$$

¹⁷Note that non log-linearized variables like e.g. the aggregate price level P_t are denoted in capital letters. The logarithmic deviation of a variable from its steady state value, e.g. $\hat{c}_t = \log \left(\frac{c_t}{\bar{c}} \right)$, is denoted by lower case letters. Later on, we simply consider $\hat{c}_t = c_t$ only.

where \bar{c} (\bar{R}) denotes the steady state value in consumption (the gross real interest rate). Hence, the actual level of private consumption in relation to its steady state value (given in percent) is then expressed through

$$\begin{aligned} -\sigma\hat{c}_t + \sigma\chi\hat{c}_{t-h} &= -\sigma E_t\hat{c}_{t+h} + \sigma\chi\hat{c}_t + (1-\chi)\hat{r}_t \\ \Rightarrow (1+\chi)\hat{c}_t &= \chi\hat{c}_{t-h} + E_t\hat{c}_{t+h} - \frac{1-\chi}{\sigma}\hat{r}_t \\ \Rightarrow \hat{c}_t &= \frac{1}{1+\chi}E_t\hat{c}_{t+h} + \frac{\chi}{1+\chi}\hat{c}_{t-h} - \frac{1-\chi}{\sigma(1+\chi)}\hat{r}_t. \end{aligned} \quad (2.46)$$

The rate of change \hat{r}_t is expressed through $h(i_t - E_t\pi_{t+h} - \bar{i})$. To see this, consider (cf. equation (2.42))

$$\begin{aligned} \hat{r}_t &= \log \frac{R_t}{\bar{R}} \\ \Rightarrow \hat{r}_t &= \log \left(\frac{(1+hi_t)E_t\left(\frac{P_t}{P_{t+h}}\right)}{(1+h\bar{i})} \right) \\ \Rightarrow \hat{r}_t &= \log(1+hi_t) + \log E_t \left(\frac{P_t}{P_{t+h}} \right) - \log(1+h\bar{i}) \\ \Rightarrow \hat{r}_t &\approx hi_t - (E_t \log P_{t+h} - \log P_t) - h\bar{i}. \end{aligned} \quad (2.47)$$

Under the assumption of rational expectations $E_t P_t = P_t$ holds. We set $\log E_t P_{t+h} - \log P_t = hE_t\pi_{t+h}$ (therefore note that $\pi_t = (p_t - p_{t-h})/h$ holds). In the long-run the nominal interest rate equals the (non time-varying) natural rate of interest, i.e. $h\bar{i}$ holds. Plugging (2.47) into the log-linearized Euler equation (2.46) under consideration of the goods market clearing condition and adding the natural level of output (which equals the steady state level of consumption) on both sides leads to

$$y_t = \frac{1}{1+\chi}E_t y_{t+h} + \frac{\chi}{1+\chi}y_{t-h} - \frac{h(1-\chi)}{\sigma(1+\chi)}(i_t - E_t\pi_{t+h} - \tilde{r}_t) \quad (2.48)$$

where the natural rate of the *gross real* interest rate is given by (cf. Galí (2008), p. 49)

$$\tilde{r}_t = \bar{i} + \sigma \left(\frac{1+\eta}{\sigma+\eta} \right) E_t(v_{t+h}^y - v_t^y) \quad (2.49)$$

where v_t^y denotes a shock to technology, which measures the impact of an innovation on the efficiency of the production process (cf. equation (2.18)). The corresponding law of motion follows an AR(1) process with

$$v_t^y = \rho(h)v_{t-h}^y + \varepsilon_t^y \leftrightarrow v_{t+h}^y = \rho(h)v_t^y + \varepsilon_{t+h}^y \quad (2.50)$$

and $\rho(h)^y = 1 - h(1 - \rho^y)$. As we consider an one-time (deterministic) impulse only, $\varepsilon_{t+h}^y = 0$ holds. Plugging (2.50) into (2.49) leads to

$$\tilde{r}_t = \bar{i} + \sigma \left(\frac{1+\eta}{\sigma+\eta} \right) (\rho(h)^y - 1)v_t^y. \quad (2.51)$$

The hybrid dynamic IS equation for the h -economy is finally given by

$$y_t = \frac{1}{1+\chi}E_t y_{t+h} + \frac{\chi}{1+\chi}y_{t-h} - \delta_1(h)(i_t - E_t\pi_{t+h} - \bar{i}) + \delta_2(h)v_t^y \quad (2.52)$$

with

$$\begin{aligned}\delta_1(h) &= h \frac{1 - \chi}{\sigma(1 + \chi)} \\ \delta_2(h) &= h \frac{(1 - \chi)(1 + \eta)(\rho(h)^y - 1)}{(1 + \chi)(\sigma + \eta)}.\end{aligned}$$

Note that (2.52) equals its representation in the baseline (quarterly) economy if $h = 1$ holds:

$$y_t = \frac{1}{1 + \chi} E_t y_{t+1} + \frac{\chi}{1 + \chi} y_{t-1} - \delta_1(i_t - E_t \pi_{t+1} - \bar{i}) + \delta_2 v_t^y \quad (2.53)$$

with

$$\begin{aligned}\delta_1 &= \frac{1 - \chi}{\sigma(1 + \chi)} \\ \delta_2 &= \frac{(1 - \chi)(1 + \eta)(\rho^y - 1)}{(1 + \chi)(\sigma + \eta)}.\end{aligned}$$

Furthermore under the assumption of a Walrasian labor market

$$-\frac{\frac{\partial U_t}{\partial N_t}}{\frac{\partial U_t}{\partial C_t}} = \frac{W_t}{P_t} \quad (2.54)$$

holds. Under consideration of household's utility function (2.39) it follows (in log-linearized terms):

$$w_t - p_t = \sigma c_t + \eta n_t. \quad (2.55)$$

Firms: In the following we will derive the purely forward-looking NKPC. We show that when applying the transformation procedure for the h-economy framework (in particular the steps (1) to (3) described in section 2.2), the adjustments of the frequency-dependent variables and parameter will lead directly to the final high-frequency representation of the NKPC. Hence, we will leave out the more time-consuming derivation of the hybrid NKPC (see e.g. Christiano et al. (2005)) and spell out the change in the frequency-dependent components for $0 < h \leq 1$ based on the solution for the purely forward-looking case only. According to Walsh (2010, Chapter 5) the following loss function of firms has to be minimized:¹⁸

$$V_t = \sum_{k=0}^{\infty} \beta(h)^{hk} E_t (z_{t+hk} - p_{t+hk}^*)^2. \quad (2.56)$$

It is the aim of a representative firm to minimize the expected price distortion (deviation of the future log-price level z_{t+hk} from the optimal log-price level p_{t+hk}^* , i.e. the price firms would set in period $t + hk$ if there were no price rigidity) given above by changing its own price z_{t+hk} . Under consideration of price stickiness, z_{t+hk} remains unchanged over time with a probability $\theta(h)$. In this case (2.56) is reformulated into

$$V_t = \sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} E_t (z_t - p_{t+hk}^*)^2. \quad (2.57)$$

¹⁸The time preference rate for a period of length h is $h\nu$. Hence the transformation of β leads to $\beta(h) = 1/(1 + h\nu)$. Furthermore recall that $\theta(h) = 1 - h(1 - \theta)$ holds.

Hence,

$$\begin{aligned} \frac{dV_t}{dz_t} &= 2 \sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} E_t(z_t - p_{t+hk}^*) = 0 \\ \Rightarrow \sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} E_t(z_t) &= \sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} E_t(p_{t+hk}^*). \end{aligned} \quad (2.58)$$

As we taking into account that $E_t(z_t) = z_t$ holds and that the corresponding term is given by the following expression of a geometric sum

$$\sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} = \frac{1}{1 - \theta(h)\beta(h)} \quad (2.59)$$

we get

$$z_t = [1 - \theta(h)\beta(h)] \sum_{k=0}^{\infty} [\theta(h)\beta(h)]^{hk} E_t(p_{t+hk}^*). \quad (2.60)$$

Equation (2.60) is the forward-solution of the dynamic equation for the optimal reset price z_t given by

$$z_t = [\theta(h)\beta(h)] E_t(z_{t+h}) + [1 - \theta(h)\beta(h)](\mu + mc_t) \quad (2.61)$$

where $p_t^* = \mu + mc_t$ denotes the optimal price set by the firm under monopolistic competition as the sum of the nominal marginal costs mc_t and the desired mark-up μ . The aggregate price level p_t is the weighted sum of the desired (lagged) price level under consideration of the probability of $h(1 - \theta)$ $[\theta(h)]$ that firms are [not] able to choose their reset price z_t :

$$p_t = \theta(h)p_{t-h} + h(1 - \theta)z_t. \quad (2.62)$$

Solving for z_t and $E_t(z_{t+h})$ respectively leads to

$$z_t = \frac{1}{h(1 - \theta)}(p_t - \theta(h)p_{t-h}) \quad \text{and} \quad E_t(z_{t+h}) = \frac{1}{h(1 - \theta)}(E_t p_{t+h} - \theta(h)p_t). \quad (2.63)$$

Substituting this into (2.61) gives

$$\frac{1}{h(1 - \theta)}(p_t - \theta(h)p_{t-h}) = \frac{[\theta(h)\beta(h)]}{h(1 - \theta)}(E_t p_{t+h} - \theta(h)p_t) + [1 - \theta(h)\beta(h)](\mu + mc_t). \quad (2.64)$$

By adding $(p_{t-h} - p_{t-h})$ on the left-hand side and $(p_t - p_t)$ on the right-hand side of the

previous equation this leads to

$$\begin{aligned}
\frac{1}{h(1-\theta)}(p_t - p_{t-h} - \theta(h)p_{t-h} + p_{t-h}) &= \frac{[\theta(h)\beta(h)]}{h(1-\theta)}(E_t p_{t+h} - p_t - \theta(h)p_t + p_t) \\
&\quad + [1 - \theta(h)\beta(h)](\mu + mc_t) \\
\Rightarrow \frac{1}{h(1-\theta)}(p_t - p_{t-h} + (1 - \theta(h))p_{t-h}) &= \frac{[\theta(h)\beta(h)]}{h(1-\theta)}(E_t p_{t+h} - p_t + (1 - \theta(h))p_t) \\
&\quad + [1 - \theta(h)\beta(h)](\mu + mc_t) \\
\Rightarrow \frac{1}{h(1-\theta)}(\Delta p_t + (1 - \theta(h))p_{t-h}) &= \frac{[\theta(h)\beta(h)]}{h(1-\theta)}(\Delta p_{t+h} + (1 - \theta(h))p_t) \\
&\quad + [1 - \theta(h)\beta(h)](\mu + mc_t) \\
\Rightarrow \frac{1}{h(1-\theta)}\Delta p_t + \frac{(1 - \theta(h))}{h(1-\theta)}p_{t-h} &= \frac{[\theta(h)\beta(h)]}{h(1-\theta)}\Delta p_{t+h} + \frac{[\theta(h)\beta(h)](1 - \theta(h))}{h(1-\theta)}p_t \\
&\quad + [1 - \theta(h)\beta(h)](\mu + mc_t) \\
\Rightarrow \frac{1}{(1 - \theta(h))}\Delta p_t + p_{t-h} &= \frac{[\theta(h)\beta(h)]}{(1 - \theta(h))}\Delta p_{t+h} + [\theta(h)\beta(h)]p_t \\
&\quad + \frac{h(1-\theta)[1 - \theta(h)\beta(h)]}{(1 - \theta(h))}(\mu + mc_t) \tag{2.65}
\end{aligned}$$

where $\Delta p_t = p_t - p_{t-h}$ ($\Delta p_{t+h} = E_t p_{t+h} - p_t$) denotes the contemporaneous (one-period ahead) inflation rate in the h -economy. Subtracting p_t from both sides leads to

$$\begin{aligned}
\frac{1}{(1 - \theta(h))}\Delta p_t + p_{t-h} - p_t &= \frac{[\theta(h)\beta(h)]}{(1 - \theta(h))}\Delta p_{t+h} + [\theta(h)\beta(h)]p_t - p_t \\
&\quad + \frac{h(1-\theta)[1 - \theta(h)\beta(h)]}{(1 - \theta(h))}(\mu + mc_t) \\
\Rightarrow \left(\frac{1}{(1 - \theta(h))} - 1\right)\Delta p_t &= \frac{[\theta(h)\beta(h)]}{(1 - \theta(h))}\Delta p_{t+h} + ([\theta(h)\beta(h)] - 1)p_t \\
&\quad + \frac{h(1-\theta)[1 - \theta(h)\beta(h)]}{(1 - \theta(h))}(\mu + mc_t) \\
\Rightarrow \frac{\theta(h)}{(1 - \theta(h))}\Delta p_t &= \frac{[\theta(h)\beta(h)]}{(1 - \theta(h))}\Delta p_{t+h} - [1 - \theta(h)\beta(h)]p_t \\
&\quad + \frac{h(1-\theta)[1 - \theta(h)\beta(h)]}{(1 - \theta(h))}(\mu + mc_t). \tag{2.66}
\end{aligned}$$

The expression of the inflation rate in the h -economy is then given by

$$\begin{aligned}
\Delta p_t &= \frac{[\theta(h)\beta(h)]}{\theta(h)}\Delta p_{t+h} - \frac{(1 - \theta(h))[1 - \theta(h)\beta(h)]}{\theta(h)}p_t \\
&\quad + \frac{h(1-\theta)[1 - \theta(h)\beta(h)]}{\theta(h)}(\mu + mc_t) \\
\Rightarrow \Delta p_t &= \beta(h)\Delta p_{t+h} + \frac{h(1-\theta)[1 - \theta(h)\beta(h)]}{\theta(h)}\left(\mu + mc_t - \frac{(1 - \theta(h))}{h(1-\theta)}p_t\right) \\
\Rightarrow \Delta p_t &= \beta(h)\Delta p_{t+h} + \frac{[1 - \theta(h)][1 - \theta(h)\beta(h)]}{\theta(h)}(\mu + mc_t - p_t). \tag{2.67}
\end{aligned}$$

Note that $1 - \theta(h) = 1 - [1 - h(1 - \theta)] = h(1 - \theta)$ holds. To derive the NKPC in its h -economy representation it is necessary to introduce the output gap into the inflation equation above,

what in fact does not change the structure of the NKPC concerning the period length because the corresponding elasticities σ and η are independent of h . To see this we claim that the representative firm uses labor N_t and technology A_t as input factors such that its production function (assuming constant return of scales) is simply given by

$$\hat{X}_t = A_t N_t \quad (2.68)$$

where \hat{X}_t denotes the actual level of output. Equation (2.68) in log-linearized terms equals to

$$y_t = a_t + n_t. \quad (2.69)$$

Real marginal costs can be expressed through

$$mc_t - p_t = mc_t^r = (w_t - p_t) - mpn_t \quad (2.70)$$

where mpn_t denotes the marginal product of labor as the result of deviating (2.68) with respect to N_t and linearizing it:

$$mpn_t = \frac{\partial Y_t}{\partial N_t} = a_t. \quad (2.71)$$

After applying some algebra, using (2.55), (2.69) and (2.71), real marginal costs are given by

$$mc_t^r = (\sigma + \eta)x_t - (1 + \eta)a_t. \quad (2.72)$$

Using the following expression for real marginal cost in the steady state

$$\bar{m}c^r = -\mu = (\sigma + \eta)\bar{x}_t - (1 + \eta)a_t \quad (2.73)$$

leads to the equation for the (time-varying) natural level of output:¹⁹

$$\bar{x}_t = \frac{1 + \eta}{\sigma + \eta}a_t - \frac{\mu}{\sigma + \eta}. \quad (2.74)$$

Finally the deviation of (2.72) from (2.73) gives

$$\hat{m}c_t^r = mc_t^r - \bar{m}c^r = (\sigma + \eta)y_t. \quad (2.75)$$

where $y_t = x_t - \bar{x}_t$ denotes the output gap. Note that $mc_t^r = \hat{m}c_t^r + \bar{m}c^r = \hat{m}c_t^r - \mu$ holds. Hence (2.67) can be reformulated into

$$\Delta p_t = \beta(h)\Delta p_{t+h} + \frac{[1 - \theta(h)][1 - \theta(h)\beta(h)]}{\theta(h)}(\sigma + \eta)y_t \quad (2.76)$$

or more precisely after dividing equation (2.76) by h , the *quarterized* NKPC is finally given by

$$\pi_t^{(h)} = \beta(h)E_t\pi_{t+h}^{(h)} + \frac{[1 - \theta(h)][1 - \theta(h)\beta(h)]}{h\theta(h)}(\sigma + \eta)y_t. \quad (2.77)$$

It can be mentioned that in the benchmark case ($h = 1$) the previous equation will collapse into the more familiar expression

$$\pi_t^{(1)} = \beta E_t\pi_{t+1}^{(1)} + \frac{(1 - \theta)(1 - \theta\beta)}{\theta}(\sigma + \eta)y_t. \quad (2.78)$$

¹⁹Note that a_t can be interpreted as an autocorrelated technology shock and therefore equals v_t^y known from the derivation of the (high-frequency) hybrid dynamic IS equation above.

It can be stated, that in case of the NKPC (and, of course, for the dynamic IS equation) it is not necessary to spell out the details here, that lead from the microfoundation of this equation in an h-economy. Instead — and as already mentioned — a high-frequency representation of the NKPC can be achieved by adjusting the frequency-dependent components directly with respect to h . We make use of this fact as we turn to the derivation the *hybrid* NKPC (e.g. Christiano et al. (2005)) for $0 < h \leq 1$ (where an exogenous shock is simply added under consideration of $h v_t^\pi$):

$$\Delta p_t = \frac{\beta(h)}{1 + \alpha\beta(h)} \Delta p_{t+h} + \frac{\alpha}{1 + \alpha\beta(h)} \Delta p_{t-h} + \kappa(h) y_t + h v_t^\pi \quad (2.79)$$

with

$$\beta(h) = \frac{1}{1 + h\nu} \quad (2.80)$$

$$\kappa(h) = \frac{[1 - \theta(h)][1 - \theta(h)\beta(h)]}{\theta(h)[1 + \alpha\beta(h)]} (\sigma + \eta) = \frac{h^2(1 - \theta)(1 + \nu - \theta)}{(1 - h(1 - \theta))(1 + h\nu + \alpha)} (\sigma + \eta) \quad (2.81)$$

and $\Delta p_{t-h} = p_{t-h} - p_{t-2h}$. Note that now the price indexation parameter α has to be taken into account. Since α denotes the *fraction* of firms which set its actual price level to the previous one, this parameter is *not* frequency-dependent. Furthermore, the *quarterized* hybrid NKPC is given by

$$\pi_t^{(h)} = \frac{\beta(h)}{1 + \alpha\beta(h)} E_t \pi_{t+h}^{(h)} + \frac{\alpha}{1 + \alpha\beta(h)} \pi_{t-h}^{(h)} + \tilde{\kappa}(h) y_t + v_t^\pi \quad (2.82)$$

where

$$\tilde{\kappa}(h) = \frac{\kappa(h)}{h} = \frac{h(1 - \theta)(1 + \nu - \theta)}{(1 - h(1 - \theta))(1 + h\nu + \alpha)} (\sigma + \eta)$$

holds. For the baseline quarterly model ($h = 1$) we get

$$\pi_t^{(1)} = \frac{\beta}{1 + \alpha\beta} E_t \pi_{t+h}^{(1)} + \frac{\alpha}{1 + \alpha\beta} \pi_{t-h}^{(1)} + \kappa y_t + v_t^\pi \quad (2.83)$$

with

$$\beta = \frac{1}{1 + \nu} \quad (2.84)$$

$$\kappa = \frac{(1 - \theta)(1 - \theta\beta)}{\theta(1 + \beta\alpha)} (\sigma + \eta). \quad (2.85)$$

2.6.2 A2: Solution of the Shock Process in the Case $0 < h \leq 1$

According to the equations (2.17) to (2.19) given in section 2.2, the AR(1) shock process

$$v_t^z = \rho(h)^z v_{t-h}^z + \varepsilon_t^z \quad (2.86)$$

with

$$\rho(h)^z = 1 - h(1 - \rho^z)$$

and $z = \{i, y, \pi\}$, emerges when the updating of v_t^z in the AR(1) process is done every $0 < h \leq 1$ time units. The solution of (2.86) is then given by

$$E_0 v_t^z = [\rho(h)^z]^t \varepsilon_0^z. \quad (2.87)$$

Note that we assume a deterministic impulse in all of the shock processes z such that

$$\varepsilon_t^z = \begin{cases} \varepsilon_0^z & \text{for } t = 0 \\ 0 & \text{for } t > 0 \end{cases} \quad (2.88)$$

holds. According to Sacht and Wohltmann (2013), equation (2.87) can be derived as follows. For a clear arrangement, in the following we omit the superscript z . Based on (2.86) we get

$$\begin{aligned} v_{t-h} &= \rho(h)v_{t-2h} + \varepsilon_{t-h} \\ v_{t-2h} &= \rho(h)v_{t-3h} + \varepsilon_{t-2h} \\ &\vdots \\ v_{t-sh} &= \rho(h)v_{t-(s+1)h} + \varepsilon_{t-sh} \end{aligned}$$

with $s \in \{0, 1, 2, \dots, t\}$, where t denotes the subperiod with respect to h . The *backward-solution* is given recursively by

$$\begin{aligned} v_t &= \rho(h)[\rho(h)v_{t-2h} + \varepsilon_{t-h}] + \varepsilon_t \\ \Rightarrow v_t &= \rho(h)^2 v_{t-2h} + \rho(h)\varepsilon_{t-h} + \varepsilon_t \\ \Rightarrow v_t &= \rho(h)^2 [\rho(h)v_{t-3h} + \varepsilon_{t-2h}] + \rho(h)\varepsilon_{t-h} + \varepsilon_t \\ \Rightarrow v_t &= \rho(h)^3 v_{t-3h} + \rho(h)^2 \varepsilon_{t-2h} + \rho(h)\varepsilon_{t-h} + \varepsilon_t \\ &\vdots \\ \Rightarrow v_t &= \rho(h)^s v_{t-sh} + \rho(h)^{s-1} \varepsilon_{t-(s-1)h} + \rho(h)^{s-2} \varepsilon_{t-(s-2)h} + \dots \\ &\quad + \rho(h)^{s-(s-1)} \varepsilon_{t-(s-(s-1))h} + \rho(h)^{s-s} \varepsilon_{t-(s-s)h} \\ \Rightarrow v_t &= \rho(h)^s [\rho(h)v_{t-(s+1)h} + \varepsilon_{t-sh}] + \rho(h)^{s-1} \varepsilon_{t-(s-1)h} + \rho(h)^{s-2} \varepsilon_{t-(s-2)h} + \dots \\ &\quad + \rho(h)\varepsilon_{t-h} + \varepsilon_t \\ \Rightarrow v_t &= \varepsilon_t + \rho(h)\varepsilon_{t-h} + \rho(h)^2 \varepsilon_{t-2h} + \rho(h)^3 \varepsilon_{t-3h} + \dots + \rho(h)^s \varepsilon_{t-sh} + \dots \\ &\quad + \rho(h)^{t-1} \varepsilon_{t-(t-1)h} + \rho(h)^t \varepsilon_0 \\ \Rightarrow v_t &= \sum_{s=0}^t \rho(h)^s \varepsilon_{t-sh}. \end{aligned}$$

Finally, by taking expectations on the previous expression we get

$$E_0 v_t = \sum_{s=0}^t \rho(h)^s E_0 \varepsilon_{t-sh} = \rho(h)^t \varepsilon_0 \quad (2.89)$$

since $E_0 \varepsilon_s = 0$ holds for $s > 0$. Hereby, E_0 denotes the expectation operator conditional on the information given in the initial period $t = 0$.

2.6.3 A3: Solution of the Baseline NKM in the Case $0 < h \leq 1$

In general, the state space representation of the NKM for $0 < h \leq 1$ is given by

$$0 = A X_t^{TR} + B X_{t+h}^{TR} + C X_{t-h}^{TR} + D V_t \quad (2.90)$$

$$V_t = N V_{t-h} + \Xi_t \quad (2.91)$$

with

$$\begin{aligned}
A &= \begin{pmatrix} 1 + \delta_1(h)\phi_y & \delta_1(h)\phi_\pi \\ -\tilde{\kappa}(h) & 1 \end{pmatrix} \\
B &= \begin{pmatrix} -\frac{1}{1+\chi} & -\delta_1(h) \\ 0 & -\frac{\beta(h)}{1+\beta(h)\alpha} \end{pmatrix} \\
C &= \begin{pmatrix} -\frac{\chi}{1+\chi} & 0 \\ 0 & -\frac{\alpha}{1+\beta(h)\alpha} \end{pmatrix} \\
D &= \begin{pmatrix} \delta_1(h) & -\delta_2(h)(\rho(h)^y - 1) & 0 \\ 0 & 0 & -1 \end{pmatrix} \\
N &= \begin{pmatrix} \rho(h)^i & 0 & 0 \\ 0 & \rho(h)^y & 0 \\ 0 & 0 & \rho(h)^\pi \end{pmatrix}
\end{aligned}$$

and $X_t^{TR} = (y_t^{TR}, \pi_t^{TR})'$, $V_t = (v_t^i, v_t^y, v_t^\pi)'$ and $\Xi_t = (\varepsilon_t^i, \varepsilon_t^y, \varepsilon_t^\pi)'$. The superscript TR indicates that we consider a (high-frequency) forward-looking/hybrid NKM, where an (non-optimal) ad-hoc Taylor rule is assumed.

Hybrid NKM: Since the state space representation (2.90) exhibits forward-and backward-looking elements, as stated in the text we apply the method of undetermined coefficients in combination with the brute force iteration procedure in order to solve the model. First, we guess that the law of motion which describes the analytical ‘solution’ is given by

$$X_t^{TR} = \Omega X_{t-h}^{TR} + \Phi V_t \quad (2.92)$$

where $\Omega \in \mathbb{R}^{3 \times 3}$ and $\Phi \in \mathbb{R}^{2 \times 3}$ are the corresponding solution matrices. The former is a stable matrix as long as its determinant is not equal to zero, which ensures the invertibility of Ω . In particular, stability of Ω requires the sufficient condition $\phi_\pi > 1$, i.e. the ‘Taylor principle’, to be fulfilled (cf. the discussion with respect to equation (2.27)). We substitute (2.92) into (2.90), which leads to

$$A(\Omega X_{t-h}^{TR} + \Phi V_t) + B(\Omega X_t^{TR} + \Phi V_{t+h}) + C X_{t-h}^{TR} + D V_t = 0.$$

This is equivalent to

$$A(\Omega X_{t-h}^{TR} + \Phi V_t) + B[\Omega(\Omega X_{t-h}^{TR} + \Phi V_t) + \Phi(NV_t + \Xi_{t+h})] + C X_{t-h}^{TR} + D V_t = 0.$$

The reduced-form solution can be rewritten as

$$(A\Omega + B\Omega^2 + C)X_{t-h}^{TR} + (A\Phi + B\Omega\Phi + B\Phi N + D)V_t = 0. \quad (2.93)$$

Since Ξ_t consists on deterministic one-time impulse shocks, i.e. $\Xi_{t+h} = 0$ holds. Thus the solution matrix Ω can be (uniquely) determined by

$$B\Omega^2 + A\Omega + C = 0. \quad (2.94)$$

The previous expression turns out to be a quadratic matrix equation. Hence, an analytical solution for Ω can be hardly obtained since the solution to equation (7.40) reads

$$\Omega = -(B\Omega + A)^{-1}C. \quad (2.95)$$

Instead we are going to solve this quadratic matrix equation numerically. Therefore, we apply the brute force iteration procedure. Hence an equivalent recursive relation of (2.95) is given by

$$\Omega_n = -(B\Omega_{n-1} + A)^{-1}C \quad (2.96)$$

with an arbitrary number of iteration steps N , i.e. $n = \{1, 2, \dots, N\}$. We define as the initial value $\Omega_0 = \varsigma I$ with I being the identity matrix and $0 \leq \varsigma \leq 1$, where we set $\varsigma = 0.8$. The iteration process (2.96) proceeds until $\|\Omega_n - \Omega_{n-1}\| < \varrho$ holds, where ϱ is an arbitrarily small number (we set $\varrho = 0.1^6$). Given the solution for Ω , for the computation of Φ applying the same recursive numerical solution method is required. This can be seen by the ‘solution’ for Φ :

$$\Phi = -(A + B\Omega)^{-1}(D + B\Phi N). \quad (2.97)$$

In order to check if both matrices serve as necessary and sufficient conditions for describing the model dynamics, Ω and Φ are plugged into the reduced-form solution (2.92), which then has to be equal to zero.²⁰ This is indeed true for all possible combinations of the parameters, including the numerical parameter set assumed throughout this Chapter (cf. Table 2.1).

Purely forward-looking NKM: For completeness, in the following we report the computation of the solution matrix in the purely forward-looking case, i.e. as $\chi = \alpha = 0$ holds. According to the equations (2.90) with $C = \mathbf{0}$ and (2.92) with $\Omega = \mathbf{0}$, respectively, we guess that the reduced-form solution is given by

$$X_t^{TR} = \Gamma V_t \quad (2.98)$$

where $\Gamma \in \mathbb{R}^{2 \times 3}$ is the corresponding solution matrix in the purely forward-looking case. Inserting (2.98) into (2.90) results in

$$\begin{aligned} A\Gamma V_t + B\Gamma V_{t+h} + DV_t &= 0 \\ \Rightarrow A\Gamma V_t + B\Gamma[NV_t + \Xi_{t+h}] + DV_t &= 0 \\ \Rightarrow (A\Gamma + B\Gamma N + D)V_t &= 0. \end{aligned}$$

Equivalent to Φ in the previous case, it is not possible to isolate Γ . Therefore, here we also apply the brute force iteration method. The solution for Γ can then be computed recursively by

$$\Gamma_n = -A^{-1}(D + B\Gamma_{n-1}N). \quad (2.99)$$

It is obvious that equation (2.97) collapsed into equation (2.99) as $\Omega = \mathbf{0}$ holds (and being substituted by Γ).²¹ According to equation (2.98), under consideration of the solution for the shock process for $0 < h \leq 1$ (cf. equation (2.89)), the reduced-form solution for the output gap and the inflation rate in the purely forward-looking case are then given by

$$X_t^{TR} = \Gamma V_t = \Gamma E_0 \begin{pmatrix} v_t^i \\ v_t^y \\ v_t^\pi \end{pmatrix} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \end{pmatrix} \begin{pmatrix} [\rho(h)^i]^t \varepsilon_0^i \\ [\rho(h)^y]^t \varepsilon_0^y \\ [\rho(h)^\pi]^t \varepsilon_0^\pi \end{pmatrix}. \quad (2.100)$$

²⁰Furthermore, note that the solution by applying the brute force iteration procedure equals the one under the Schur decomposition method. This is confirmed when comparing the outcome for (2.95) and (2.97) to the ones, which are computed by using the numerical software programme MATLAB DYNARE (which makes use of the Schur decomposition).

²¹In particular, we search for the fix points of Ω_n and Γ_n according to (2.96) and (2.99), respectively, such that $\Omega_N = f(\Omega_N) = -(B\Omega_N + A)^{-1}C$ and $\Gamma_N = f(\Gamma_N) = -A^{-1}(D + B\Gamma_N N)$ hold.

2.6.4 A4: Impulse Response Functions (TR) in the Case $0 < h \leq 1$

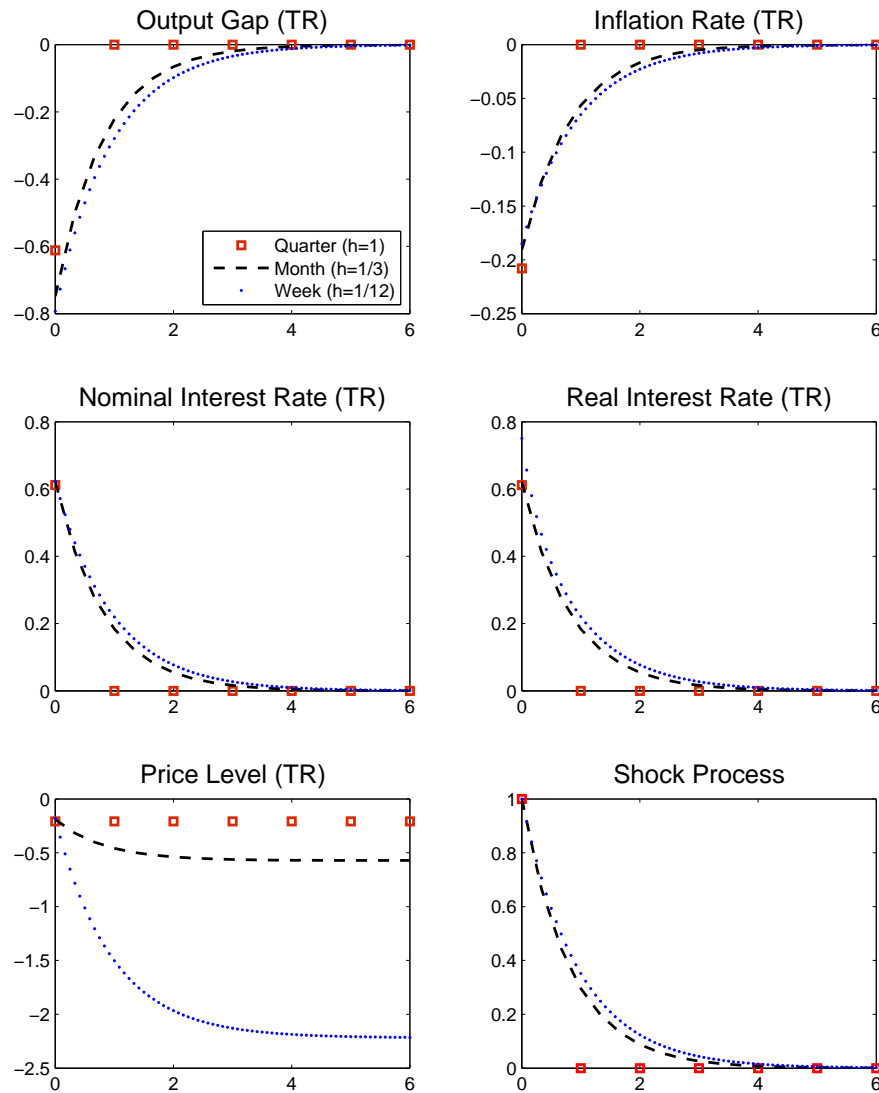


Figure 2.2: IRFs in case of a *non*-autocorrelated shock to the *nominal interest rate* in the *forward-looking* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

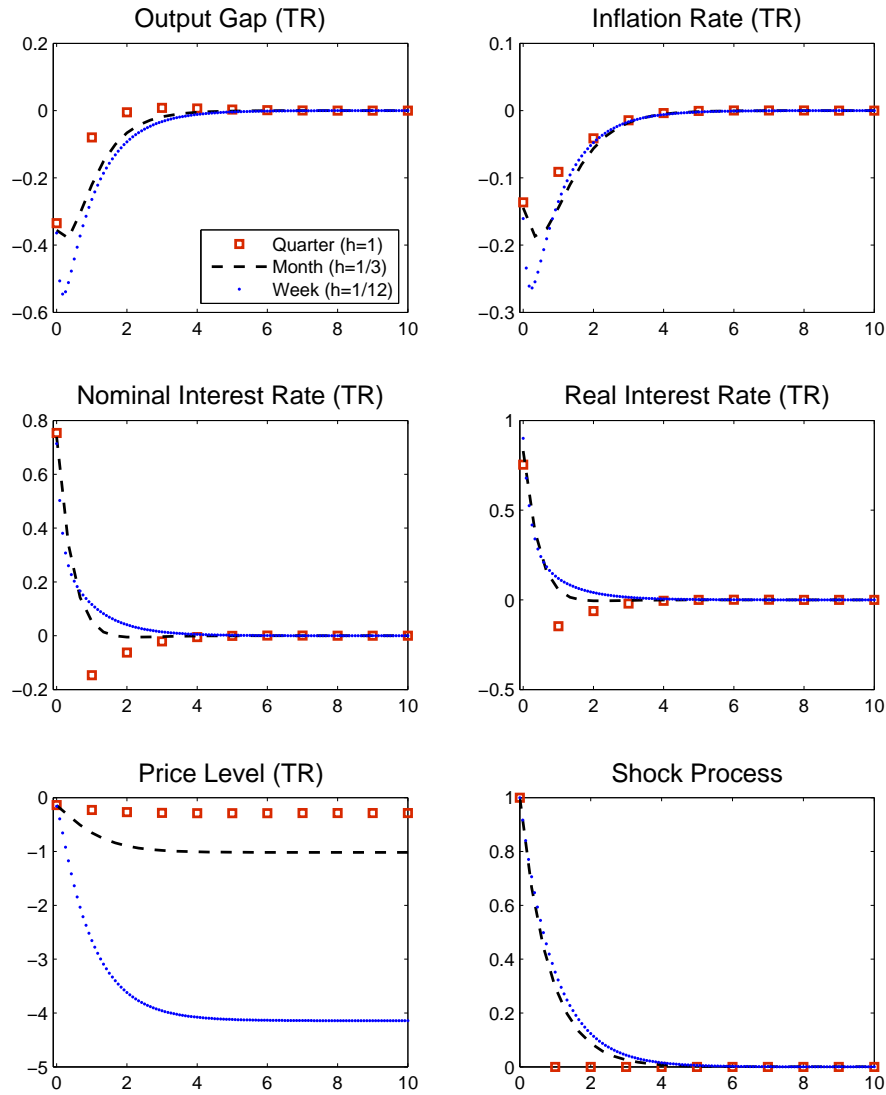


Figure 2.3: IRFs in case of a *non-autocorrelated* shock to the *nominal interest rate* in the *hybrid NKM (TR)*.

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

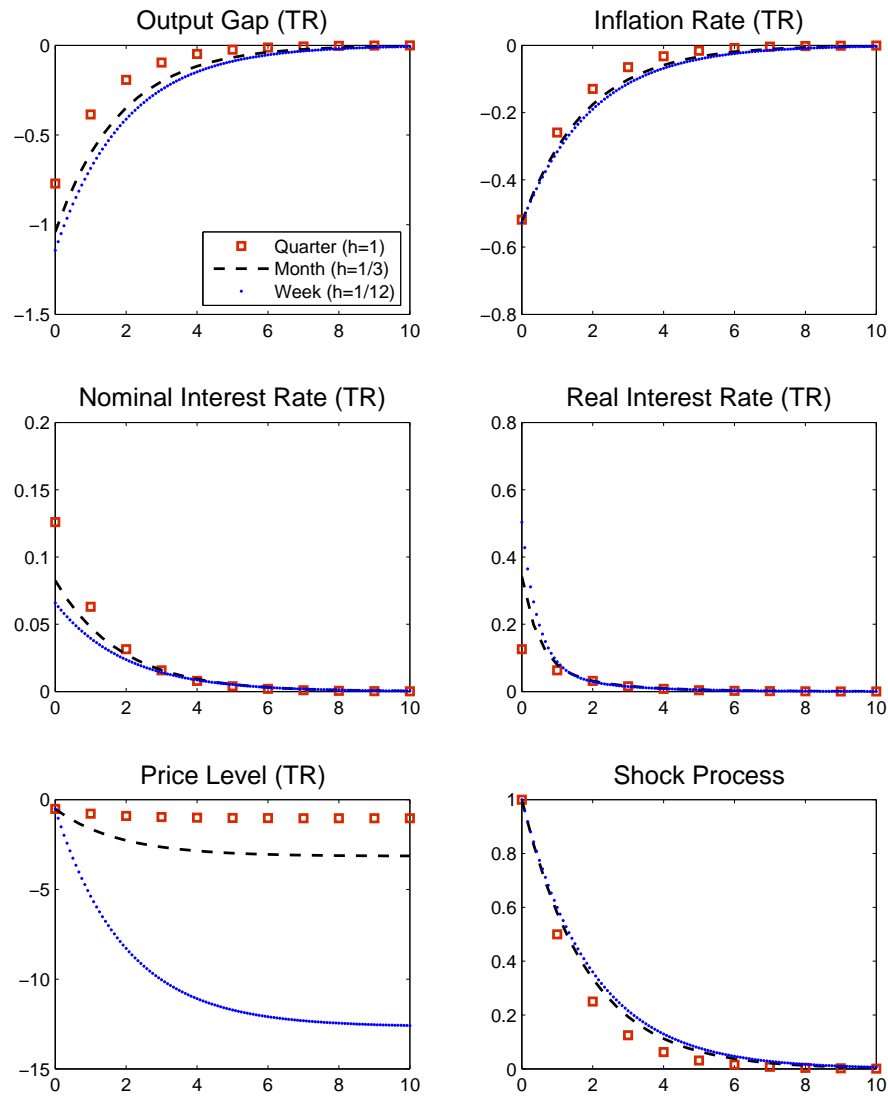


Figure 2.4: IRFs in case of an *autocorrelated* shock to the *nominal interest rate* in the *forward-looking* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(ages) are displayed on the horizontal and vertical axis, respectively.

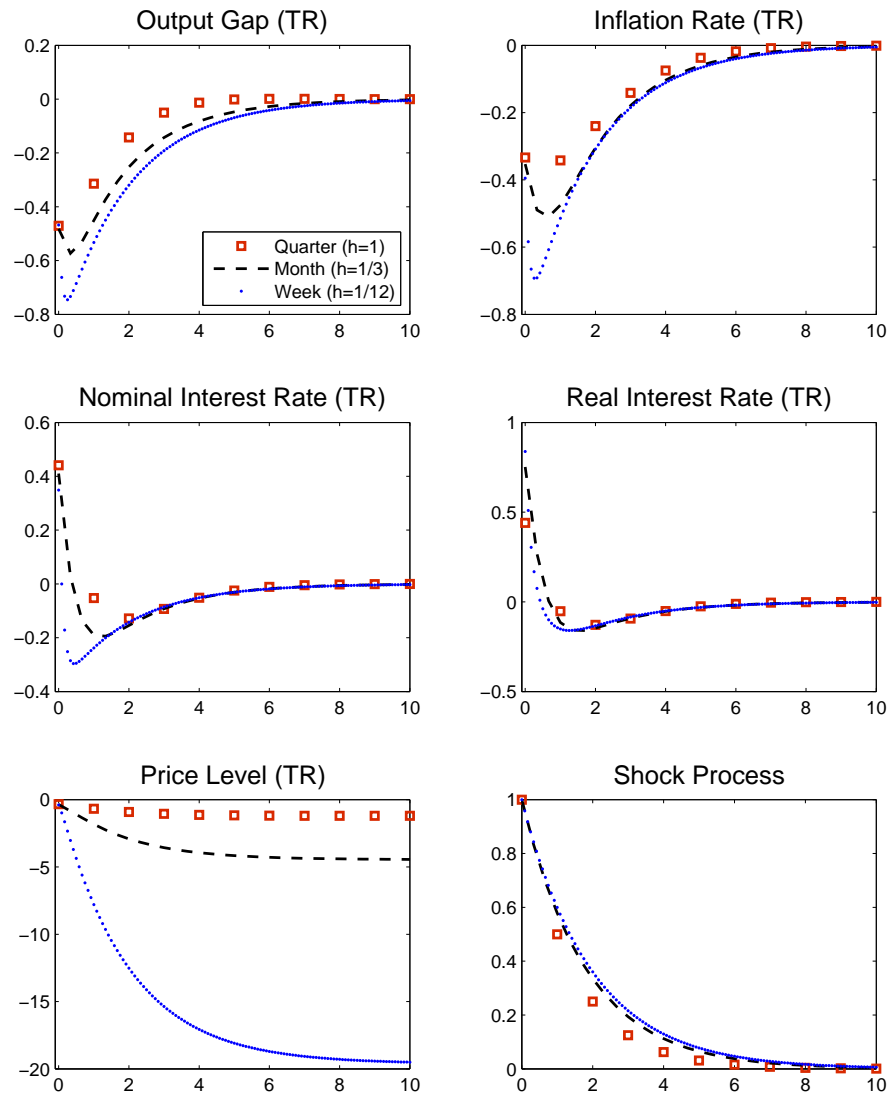


Figure 2.5: IRFs in case of an *autocorrelated* shock to the *nominal interest rate* in the *hybrid NKM (TR)*.

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed lines* depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

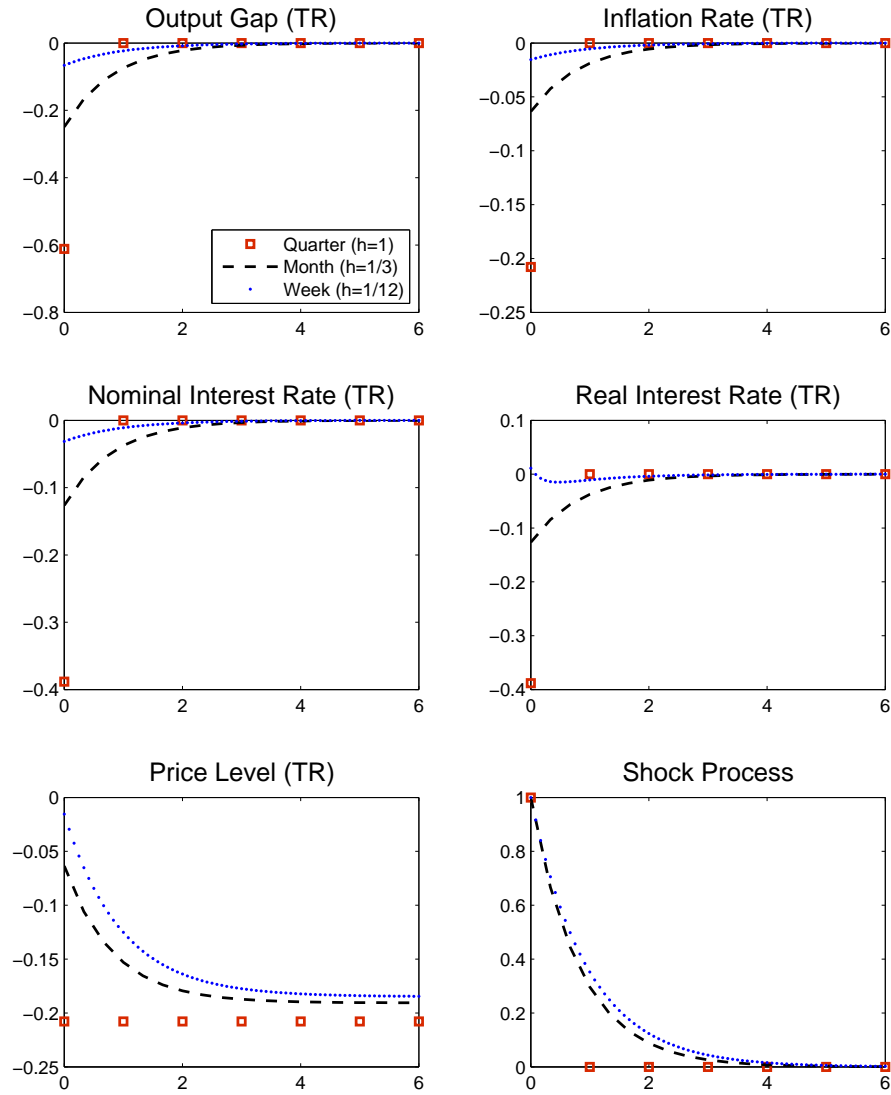


Figure 2.6: IRFs in case of a *non-autocorrelated technology shock* in the *forward-looking NKM (TR)*.

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* line depicts the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent (age points) are displayed on the horizontal and vertical axis, respectively.

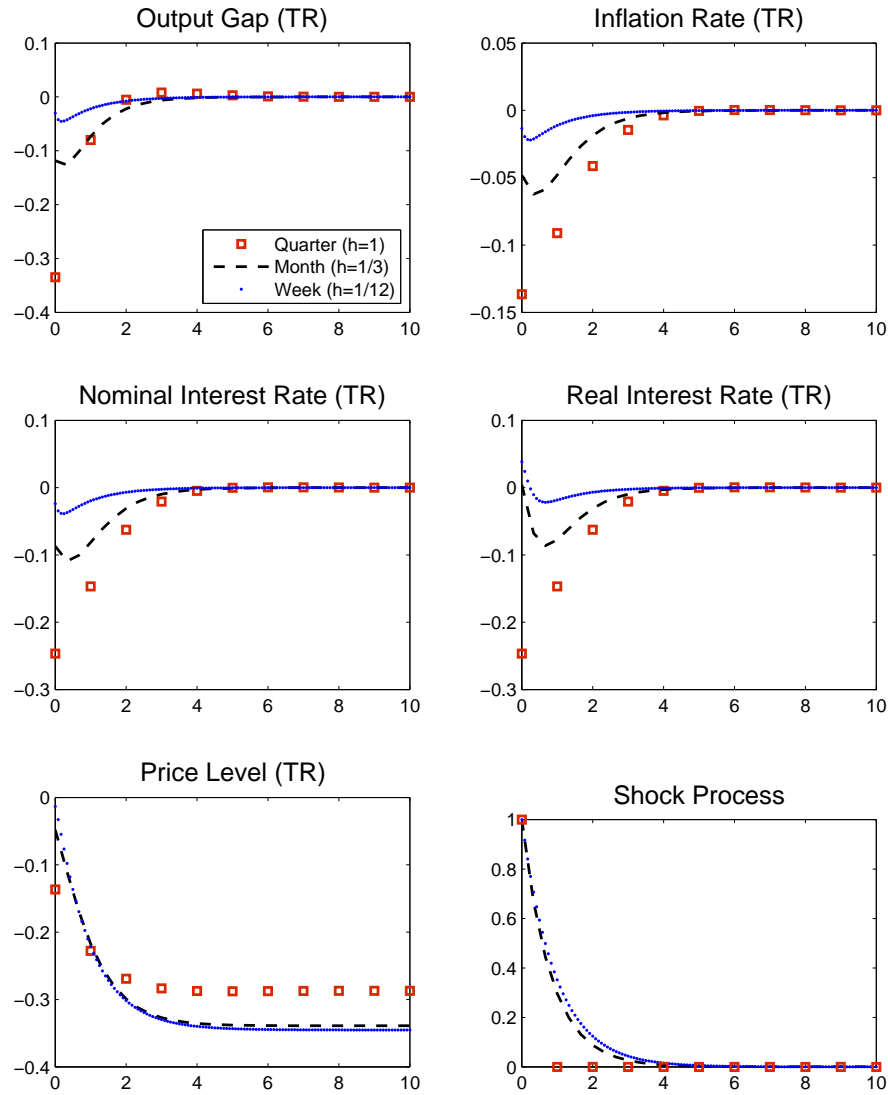


Figure 2.7: IRFs in case of a *non-autocorrelated technology* shock in the *hybrid NKM* (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent (age points) are displayed on the horizontal and vertical axis, respectively.

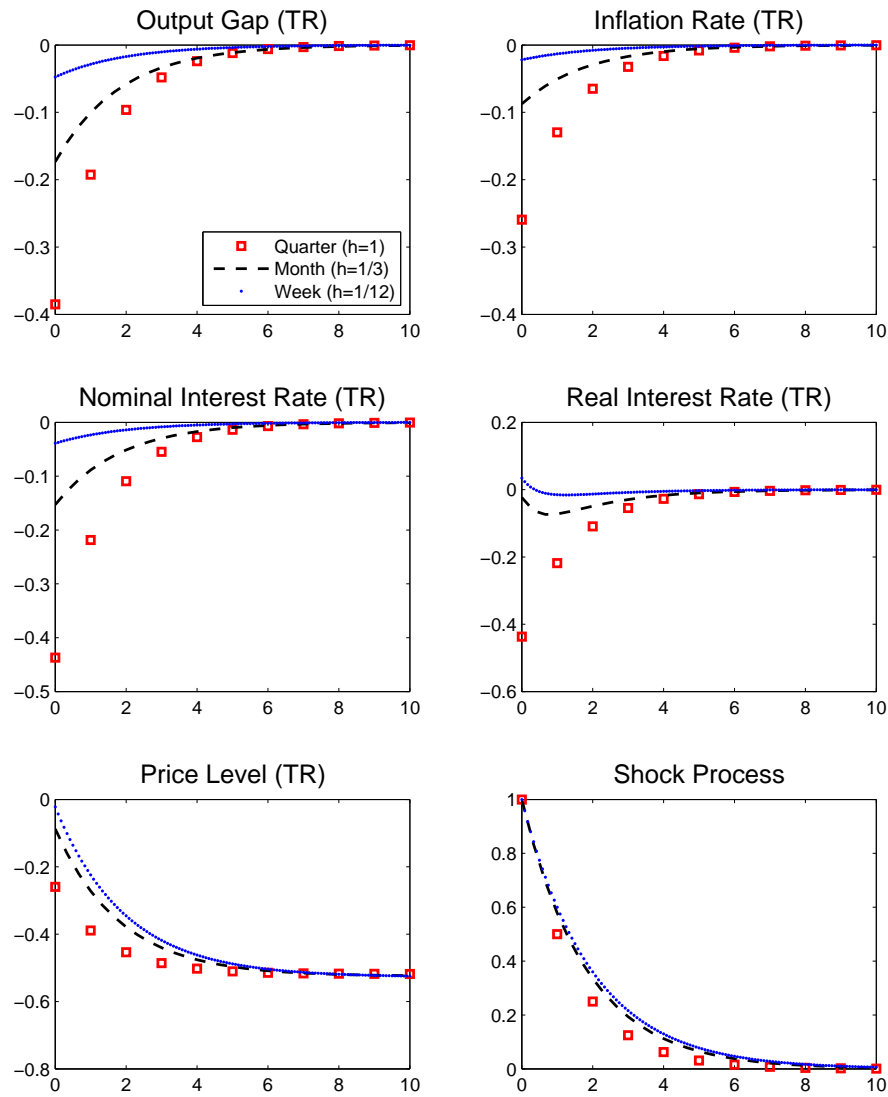


Figure 2.8: IRFs in case of an *autocorrelated technology* shock in the *forward-looking* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

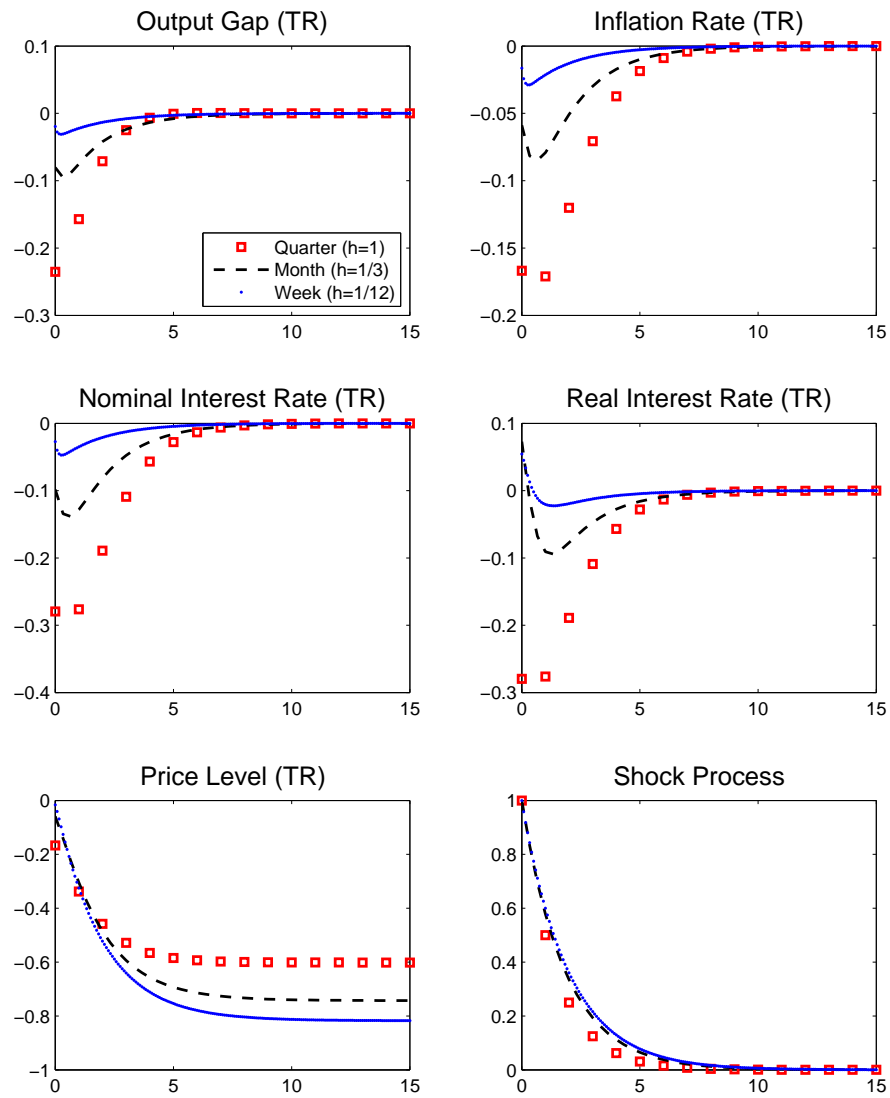


Figure 2.9: IRFs in case of an *autocorrelated technology* shock in the *hybrid* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

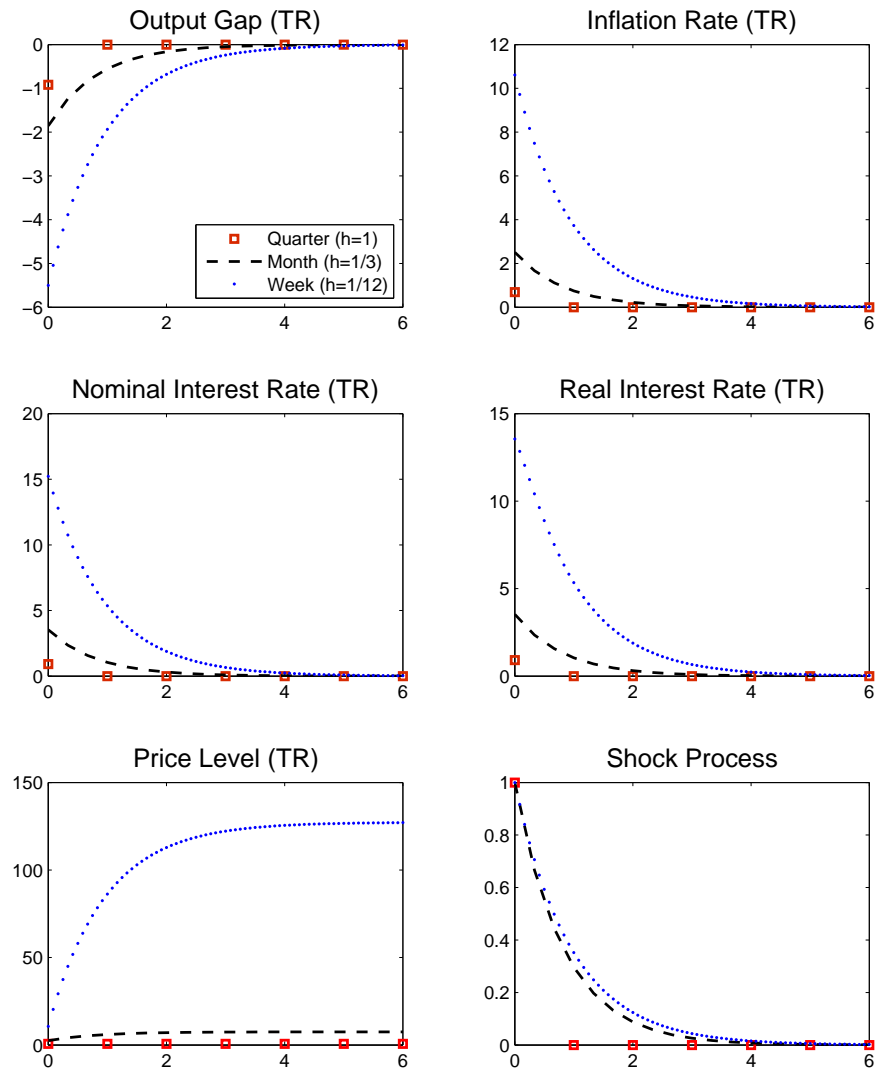


Figure 2.10: IRFs in case of a *non*-autocorrelated *cost-push* shock in the *forward-looking* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent (age points) are displayed on the horizontal and vertical axis, respectively.

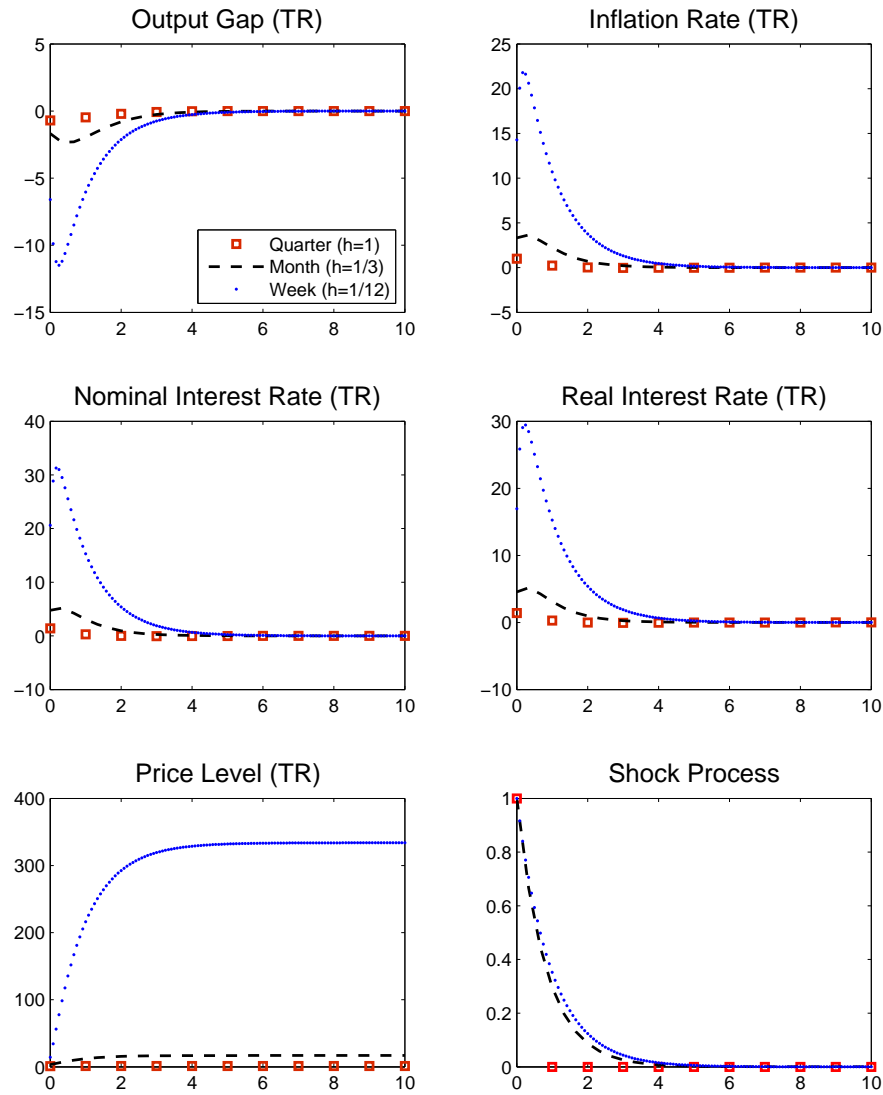


Figure 2.11: IRFs in case of a *non-autocorrelated cost-push* shock in the *hybrid* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

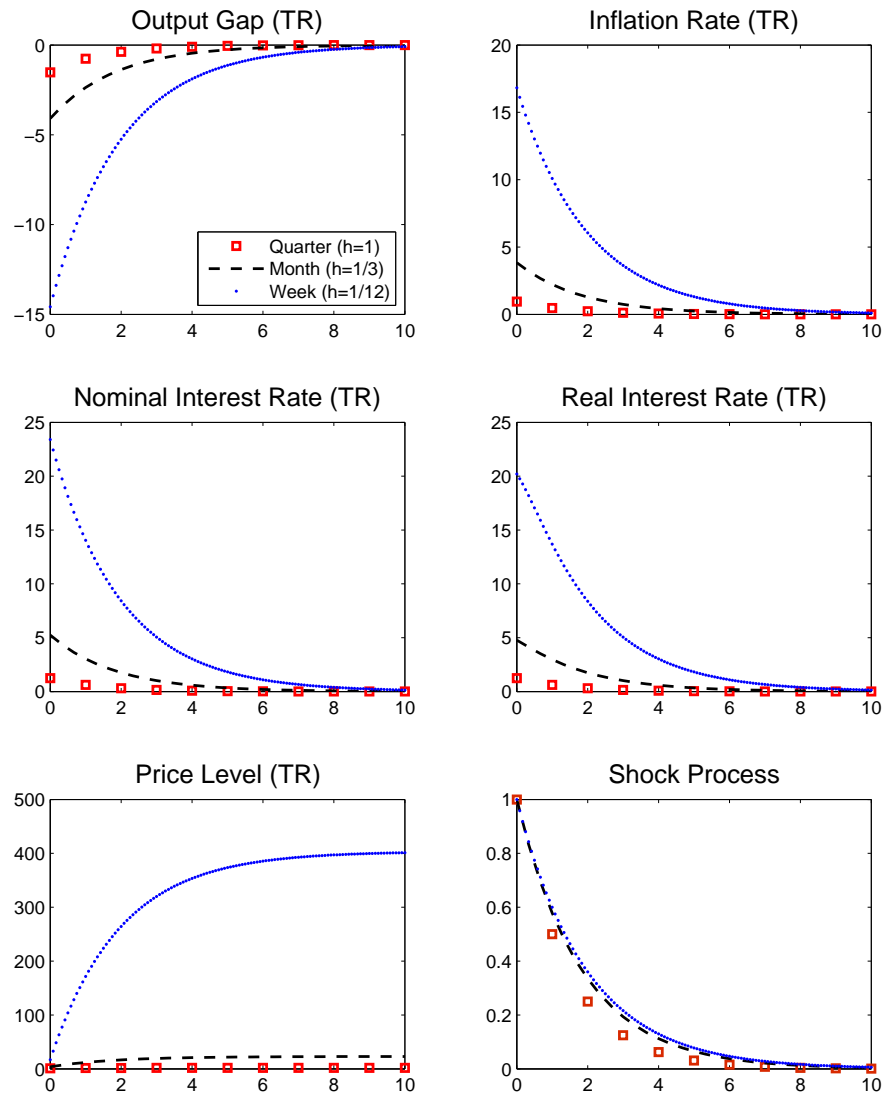


Figure 2.12: IRFs in case of an *autocorrelated cost-push* shock in the *forward-looking* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

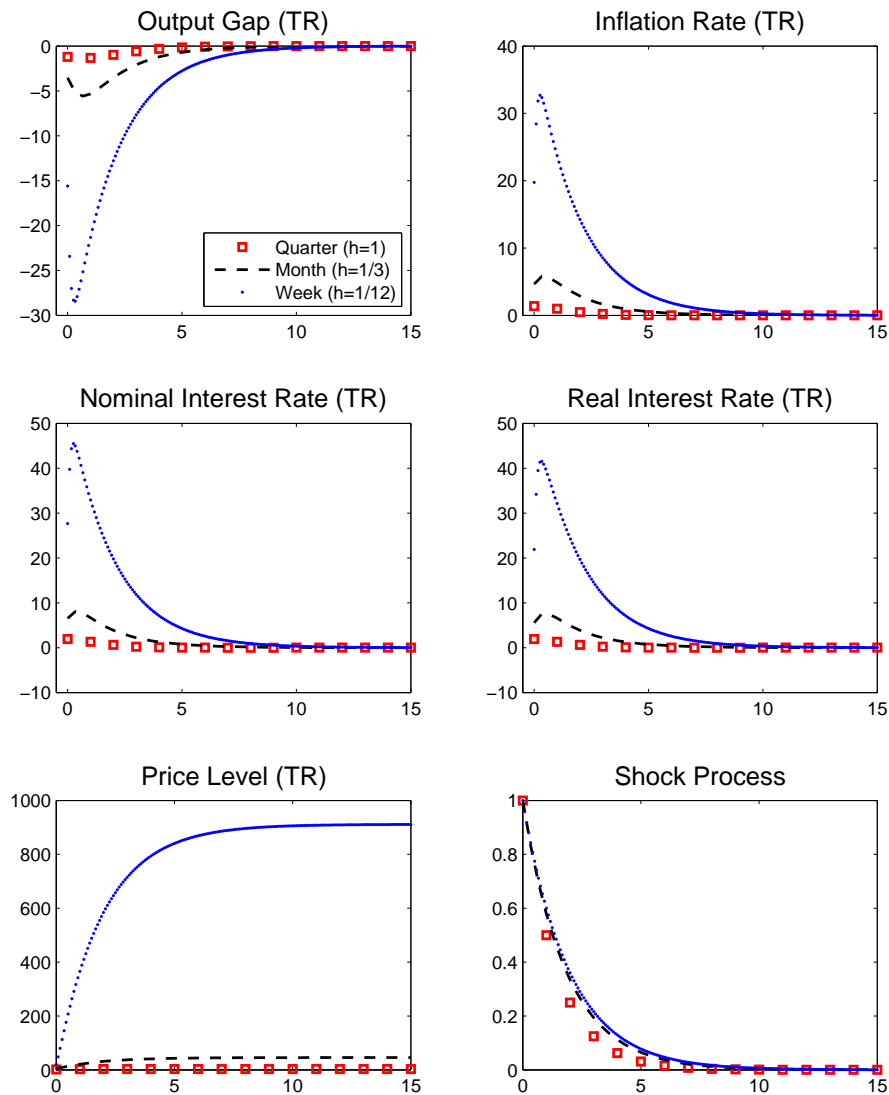


Figure 2.13: IRFs in case of an *autocorrelated cost-push* shock in the *hybrid* NKM (TR).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent (age points) are displayed on the horizontal and vertical axis, respectively.

3 Optimal Monetary Policy Responses and Welfare Analysis within the High-Frequency New-Keynesian Framework

3.1 Introduction

In this Chapter we investigate the *welfare effects* of monetary policy operations within an h-economy. Hereby we focus on the purely forward-looking baseline NKM and the impact of a *cost-push shock* on economic dynamics. The reasons for taking this experimental arrangement under consideration are manifold.

A cost-push shock has gained relative importance since it stands for the source of stagflation, i.e. a simultaneous increase and decrease in the inflation rate and the output gap, respectively. As we discussed in the previous Chapter, in order to dampen the increase in inflation, the monetary authority faces a policy trade-off: it must reduce the output gap as inflations' inherited driving force by a subsequent increase in the nominal (real) interest rate after the shock occurs. This upward movement in the policy instrument is the only reliable option the central bank will have within the baseline three-equations NKM. This observation is confirmed by an overwhelming number of studies on *optimal* monetary policy (OMP) in the literature (see Woodford (2003), Winkler and Wohltmann (2008) and Walsh (2010) among others). The term 'optimal' refers to the aim of the central bank to minimize the value of an intertemporal criterion function. This function in its basic representation consists of the discounted variability of the inflation rate and the output gap. Since the central bank seeks to minimize these kinds of variability, the welfare function is simply called a *loss* function. In the following we refer to the welfare and loss function as being synonyms. Furthermore, it is assumed that there exists a prevalence of inflation rate over output gap stabilisation by the central bank, expressed by a higher weight in the loss function on inflation rate movements. Obviously, the policy trade-off calls for a more elaborate discussion on the design of optimal monetary policy relative to demand (technology) or interest rate shocks. The latter both can be easily offset on a quarterly magnitude by the central bank, since here the dynamics in inflation and the output gap are rectified. Nevertheless, as we had seen (based on our analysis of IRFs in the previous Chapter) there exist quantitative and qualitative differences across higher frequencies – at least due to the FCP effect – in all shocks. However, the *ineffectiveness* of monetary policy is much more apparent in the case of a cost-push shock.

From an empirical point of view, supply or cost-push shocks account to a large extent for the variability in economic dynamics, which holds at least for the two oil price shocks within the Great Inflation period in the 1970's (Hamilton and Herrera (2004)). Smets and Wouters (2007, pp. 598) identify supply shocks being significant driving forces in output growth and inflation dynamics over the time horizon from 1966:1 to 2004:4 in the US. In addition, Smets

and Wouters (2003, p. 1162) show that cost-push shocks account mainly for the variation in inflation in the Euro Area between 1980:2 and 1999:4.¹

Throughout this Chapter we investigate how the OMP responses of the inflation rate and the output gap to a cost-push shock are going to change, as we increase the frequency in decision making. Therefore, again, IRFs are considered and an economic rationale based on different kinds of frequency-dependent persistency is given. In particular, we discuss the economic dynamics under two different OMP regimes. Essentially, in the regime *Discretion* the central bank must take the expectations of the households as given, while in the (pre-) *Commitment* regime expectations can be influenced by the monetary authority. It is well known that the outcome with respect to the value of the loss function in quarterly magnitudes, is higher under *Discretion* compared to *Commitment*. While this can be explained by the ability of the central bank to influence the expectation channel of monetary policy, it is an open question if this result prevails on a higher frequency. This is of serious concern, as we observe a more likely discretionary monetary policy conducted by the European Central Bank (ECB) in the aftermath of the financial and sovereign debt crisis in the Euro Area. Finally, we apply a sensitivity analysis, where we evaluate the values of the loss function across all policy regimes under variation of the Calvo parameter of price stickiness and the autocorrelation parameter in the shock process. Such a theoretical investigation is non-existent in the literature so far – except for the work by Sacht and Wohltmann (2013), where their contribution can be seen as closely related to ours.

There exist a large number of empirical studies, which deal with the impact of monetary policy on real variables under consideration of high-frequency data. For example, Faust et al. (2004) estimate the effect of monetary policy announcements on the expected future values of nominal interest rate and the price level. This is done via the matching of IRFs. In their work they use high-frequency data on the prices of Fed Funds futures contracts in a vector autoregression model. The authors report evidence for a small impact of monetary policy shocks on output. Rosa (2013) applies ordinary least squares under consideration of high-frequency data on energy prices. He shows that asset prices respond on a high-frequency due to sudden monetary policy announcements. Finally, Assenmacher-Wesche and Gerlach (2008) estimate a two-pillar NKPC of the European central bank (ECB), where they account for the ECBs simultaneous analysis of monetary and economic indicators. In their study they investigate the impact of high-frequency (e.g. exchange rates, import and fresh food prices and value-added taxes) as well as low-frequency (money and real output growth and the rate of velocity) components on the inflation process in the Euro Area. The authors check on co-integration of the inflation rate relative to these components under consideration of spectral regression techniques in order to account for the different time horizons. They find that low-frequency movements of money and real output growth as well as high-frequency fluctuations of the output gap account for the variation in the inflation rate over the period from 1970 to 2004.

In order to keep our analysis straightforward, the purely forward-looking specifications of both the baseline NKM and the h -economy model (known from the previous Chapter) are considered only. As we would turn to a hybrid specification of these model variants,

¹The definition of a cost-push or supply shock is ambiguous and depends on the structural representation of the NKPC. For example, Smets and Wouters (2007) investigate a NKM with sticky prices and wages, where supply shocks are represented as price and wage mark-up shocks. Galí et al. (2012) consider non-investment specific technology and price mark-up shocks, while wage mark-up shocks are sorted into labor shocks only. While this kind of differentiation is reasonable when a shock decomposition analysis is applied, however, for our theoretical discussion here a shock expressed by an AR(1) process is sufficient.

backward-looking elements have to be considered explicitly. In this case the computation of the policy functions (as an outcome of dynamic programming) under Discretion becomes non-trivial. According to the seminal work of Woodford (2003), he shows that under Discretion a policy function, being optimal in a certain period, becomes non-optimal in subsequent periods as new information arrives. Hence, policy makers must be aware of the change in the optimal behaviour of private agents because the decisions of the latter are affected by previous states. Two technical approaches from the literature deal with this problem. The first one makes use of the Bellman equation within optimal control theory. Here the policy maker applies a single optimization in the initial period and sticks to the resulting policy function in all periods after. This procedure meets Bellman's 'principle of optimality'.² Problem-specific solution algorithms are provided by e.g. Söderlind (1999) among others. Note that here the decisions of policy makers and private agents are undertaken simultaneously.

In contradiction, as we allow the policy maker to make its decisions first, we end up in a Stackelberg-Nash equilibria, where the current policy maker is the Stackelberg leader and the private agents as well as the future policy maker are the Stackelberg follower. For this case, solution algorithms are provided by e.g. Dennis (2007) among others. Similar to Söderlind (1999), Dennis (2007) considers the structural representation of the model directly instead of its associated state-space representation. As far as we know, an intensive discussion on the outcome and the computational properties (in terms of convergence and computational speed) of both classes of algorithms is missing in the literature so far. Rather than undertake an exercise, which consists on the comparison of different solution techniques for discretionary monetary policy, we account for clearness of our approach. The purely forward-looking specification is sufficient in this respect since the solution is less time-consuming. As in Chapter 2, across all policy regimes, the policy functions are computed via the method of undetermined coefficients in combination with the brute force iteration procedure. In general, this proceeding can be seen as a natural starting point for the analysis of OMP in DSGE models - independently, of course, of the underlying timing convention.³

The remainder of this Chapter is structured as follows. In the next section we discuss the design of OMP within a baseline purely forward-looking NKM in its h -economy variant. Furthermore, along with the presentation of the analytical results, we give an economic explanation for the upcoming results based on the change in the frequency-dependent persistence parameters. Section 3.3 provides the numerical analysis of both OMP regimes in an h -economy environment based on IRFs. In section 3.4 we apply a numerical welfare analysis, where we investigate the value of the welfare loss (relations) together with a sensitivity analysis with respect to the degree of price stickiness and the autocorrelation parameter in the shock process. Section 3.5 concludes. Appendix B contains the reduced-form solutions to the corresponding minimization problems of both OMP regimes and all Figures.

²Bellman (1957, p. 83) states that "[a]n optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision." The work on dynamic programming with respect to discretionary policy was pioneered by Oudiz and Sachs (1985), who study a finite instead of an infinite horizon problem.

³It must be emphasised that the above statement does not apply to the hybrid (baseline) NKM under Commitment due to an explicit consideration of the expectation channel in this OMP regime. This means that future states of the variables in the model can be controlled by the central bank based on previous realizations. For consistency and clearness of our study, where we compare the numerical outcomes given by the OMP regimes Discretion *and* Commitment, we leave this kind of investigation to further research.

3.2 Optimal Monetary Policy Responses in an h -Economy

3.2.1 The Design of Optimal Monetary Policy

For our analysis, we refer to purely forward-looking NKM ($\chi = \alpha = 0$) in its (quarterized) high-frequency specification ($0 < h \leq 1$) known from the previous Chapter without changes in the definition of the parameters (cf. section 2.2):

$$y_t^j = y_{t+h}^j - \delta_1(h)(i_t^j - \pi_{t+h}^j - \bar{i}) \quad (3.1)$$

$$\pi_t^j = \beta(h)\pi_{t+h}^j + \tilde{\kappa}(h)y_t^j + v_t^\pi \quad (3.2)$$

$$i_t^j = \phi_\pi \pi_t^j + \phi_y y_t^j. \quad (3.3)$$

$$v_t^\pi = \rho(h)^\pi v_{t-h}^\pi + \varepsilon_t^\pi \quad (3.4)$$

where $j = \{D, C\}$ indicates the dynamics of the inflation rate and the output gap associated with the corresponding OMP regimes *D*iscretion and *C*ommitment. In order to account for a(n autocorrelated) cost-push shock only, the autocorrelated nominal interest rate (v_t^i) and the technology (v_t^y) shocks are now discarded from the Taylor rule and the dynamic IS equation, respectively. We omit the expectation operator in front of the forward-looking terms since we only consider a *deterministic* shock. As already discussed, as the shock occurs, this leads to a boost in (future) inflation on impact, which forces the central bank to increase the nominal interest rate according to the Taylor rule. As a result the output gap declines on impact.⁴ This leads to a decrease in the inflation rate and so forth. However, as the inflation rate and the output gap move in opposite directions, the central bank faces a policy trade-off if stagflation occurs, which can be interpreted as an output-inflation trade-off of monetary policy (cf. section 2.4.3).

The welfare effects of OMP are in general measured by the following intertemporal function:

$$L_t^j = E_t \sum_{k=0}^{\infty} \beta(h)^{hk} \{h\bar{\alpha}_1(\pi_{t+hk}^j - \bar{\pi}^j)^2 + \bar{\alpha}_2(y_{t+hk}^j - \bar{y}^j)^2\} \quad (3.5)$$

where $\bar{\alpha}_1$ and $\bar{\alpha}_2$ denote the weights on inflation rate and output gap stabilisation, respectively. It must be emphasized that we do not allow for nominal interest rate stabilisation as done in various papers on OMP. According to equation (3.5) the aim of the central bank is to minimize the quadratic deviations of inflation and the output gap from their fixed steady state values $\bar{\pi}$ and \bar{y} , respectively. Therefore, equation (3.5) can be denoted as a loss function. For simplicity we assume for both OMP regimes that $\bar{\pi}^j = \bar{y}^j = 0$ holds.⁵ Again, for a direct comparison of two h -economies, the stock variable π_{t+hk}^j is uniformly expressed in *quarterized* magnitudes. Note that

$$\pi_{t+hk}^j = (p_{t+hk}^j - p_{t+hk-h}^j)/h \quad (3.6)$$

⁴Note that ceteris paribus the cost-push shock triggers an increase in the real interest rate denoted by $r_t^j = i_t^j - E_t[\pi_{t+h}^j]$. According to the Euler equation as theoretical basement of the dynamic IS curve, future consumption becomes more attractive, which would lead to a decrease in y_t^j . Hence $|di_t^j| > |dE_t[\pi_{t+h}^j]|$ holds if dynamic stability is ensured, i.e. as the Taylor principle (i.e. $\phi_\pi > 1$) holds.

⁵We depart from the strand of the literature on OMP, where a second-order approximation of household's utility function is considered as a loss function (cf. Woodford (2003)). We also do not consider the so-called Ramsey approach (Kahn et al. (2003)). Instead, we follow an overwhelming number of studies, where an ad-hoc formulation of the loss function of type (3.5) is assumed. Essentially, equation (3.5) serves as a good compromise since its structure mimics a (microfounded) second-order approximation of household's utility function (cf. Walsh (2010) and Galí (2008)).

is then considered with respect to the loss function (3.5), i.e. the latter must be augmented by h in order to ensure mathematical consistency. Therefore, based on the following definition

$$(p_{t+hk}^j - p_{t+hk-h}^j)^2 = (\Delta p_{t+hk}^j)^2 \quad (3.7)$$

it follows for $\bar{\pi}^j = 0$ that

$$h(\pi_{t+hk}^j)^2 = h \left(\frac{(\Delta p_{t+hk}^j)}{h} \right)^2 = h \frac{(\Delta p_{t+hk}^j)^2}{h^2} = \frac{(\Delta p_{t+hk}^j)^2}{h}$$

holds. Hence, according to the systematic/skip sampling aggregation scheme, the *squared deviations* of the inflation rate (from zero) are expressed in quarterized magnitudes.⁶ Since the output gap is a dimensionless adjustment rate, the weight $\bar{\alpha}_2$ is not augmented by h . According to optimal control theory, the loss function has to be minimized subjected to the equations for y_t^j and π_t^j . This approach differs across the monetary policy regimes to be considered (see also the next section as well as the Appendices B1 and B2).

An important assumption must be made with respect to the solution of the shock process:

$$E_0 v_t^\pi = [\rho(h)^\pi]^{t/h} \varepsilon_0^\pi, \quad (3.8)$$

again, E_0 denotes the expectation operator conditional on the information given in the initial period. Note here that the parameter $\rho(h)^\pi$, which measures the FCP effect, is taken to the power of t/h instead of period t only. Hence, the period has to be multiplied by the frequency of decision making, i.e. $t \cdot f$ holds. This means that all points in time which correspond to a *quarter* on a higher frequency are *selected*. While this step is not necessary for the computation of the IRFs (where in the previous Chapter we just consider period t only), it is required for the analysis of welfare effects on different frequencies. The reason is that in order to compare the values of the loss functions along the dimensions of the period length and the OMP regimes, the quarterly realizations of the variables have to be considered only. In particular, since the quarterly NKM with $h = 1$ serves as our baseline model, for a direct comparison of the corresponding loss functions for $0 < h \leq 1$, the shock process must be adjusted accordingly. Therefore, it must be emphasized that we take the entire stream of realizations of the variables over an infinite amount of periods, namely quarters, into account. Accordingly, the loss function is described by the weighted sum of these realizations. Without an adjustment of the shock process, this will lead to biased results as the period length is varied.

3.2.2 Analytical Results

In this section, the key equations for the OMP regimes Discretion and Commitment are revealed. Further explorations can be found in more detail in the Appendices B1 and B2. One of our *main results* indicates that the loss *increases* as the period length *decreases*. This general observation implies that welfare is *always* higher (based on the value of the

⁶It is easy to see that $(\Delta p_{t+hk}^j)^2$ should not be divided by h^2 , what probably would be a natural way to think of. Therefore, note that in this case we get $f^2 \cdot (\Delta p_{t+hk}^j)^2$, where $f = 1/h$ defines the frequency of decision making. In e.g. a monthly economy ($f = 3$) this leads to $f^2 \cdot (\Delta p_{t+hk}^j)^2 = 9 \cdot (\Delta p_{t+hk}^j)^2$ which is simply implausible since a quarter consists on 3 and not 9 months. In other words, in order to compare the values of the stock variables in an h-economy to its counterpart given in quarterly magnitudes, this requires the correct amount of subperiods to be considered on a higher frequency for aggregation. This holds regardless as we consider the level or squared deviation in this stock variable.

loss function) on a quarterly magnitude ($h = 1$) relative to the case where subperiods ($h < 1$) are considered. This can be explained by the FCP effect, where the increase in the persistence of the shock process leads to a stronger destabilisation effect on the economy – which holds independently for the OMP regimes Discretion and Commitment. In the following, we shed a light on the question how this increase in destabilisation can be explained by other (frequency-dependent) influences besides the FCP effect.

We start by discussing the case of *Discretion*. The minimization problem associated with this OMP regime is a static one. The reason is, that households' expectations on future movements of the output gap and the inflation rate are pre-existing for the central bank due to a lack of its credibility. Given the expectations, the monetary authority is forced to re-optimize every single period. Technically speaking, the loss function (3.5) is minimized with respect to the dynamic IS and NKPC, where all forward-looking terms are treated as being fixed expressions. As a result of the optimization, the relationship between the inflation rate and the output gap in case of a cost-push shock is described by the so-called *targeting rule*:

$$y_t^D = -\xi(h)\pi_t^D \quad (3.9)$$

with

$$\xi(h) = \frac{h\tilde{\kappa}(h)\bar{\alpha}_1}{\bar{\alpha}_2} = \frac{\kappa(h)\bar{\alpha}_1}{\bar{\alpha}_2}$$

and $\tilde{\kappa}(h) = \kappa(h)/h$. A targeting rule describes the optimal adjustment path of variables, which depend directly on each other. According to equation (3.9), an increase in the inflation rate requires a decrease in the output gap in order to dampen inflation after the cost-push shock occurs. Since inflation rate and output gap stabilisation being the target of the central bank, this type of rule is different to the so-called instrument rule. The standard Taylor rule or an Optimal Simple Rule serve as examples of the latter.⁷ The (frequency-dependent) composite parameter $\xi(h)$ is known as the *stabilisation bias*. It measures the effectiveness of OMP, where a high value of $\xi(h)$ indicates a decrease in the stabilisation bias and a stronger reduction in the output gap, which leads to less pronounced increase in the inflation rate and vice versa. Hence, OMP becomes increasingly effective the higher $\xi(h)$ will be.

Under (pre-) *Commitment*, the central bank's reputation is confirmed, i.e. the households believe in the monetary policy announcement regarding the stabilisation of the inflation rate and the output gap. Hence, the monetary authority is able to control for the expectation channel of monetary policy, where private sector expectations on future movements in y_t^C and π_t^C can be influenced over time. It follows that the minimization problem becomes indeed intertemporal, allowing the central bank to manipulate households' expectations. The corresponding targeting rule reads

$$y_t^C = -\xi(h)\pi_t^C + \psi(h)y_{t-h}^C \quad (3.10)$$

with

$$\psi(h) = \beta(h)^{(1-h)}.$$

The previous equation accounts for *history dependence*, i.e. the optimal reaction of y_t^C to changes in π_t^C does not depend only on contemporaneous but also on past movements of the

⁷See also Svensson (1999) as well as Svensson and Woodford (2005) for a discussion of targeting versus instrument rules. In the case of an Optimal Simple Rule, the optimal values for the monetary policy parameters ϕ_π and ϕ_y stem from the minimization of the loss function under consideration of the corresponding solution paths for the inflation rate and the output gap given the Taylor rule (cf. Sacht and Wohltmann (2013)).

output gap. The reason is given by the *timeless perspective* policy (see Woodford (2003, p. 473)), which indicates that, by committing herself to a long-run optimal rule (linked to the reputation of the agents), the central bank is able to react smoothly to shocks. According to equation (3.10) the monetary authority is able to stabilize the inflation rate by allowing for a smaller decline in the output gap compared to the OMP regime Discretion. In total, the deviations in both variables are smaller under the OMP regime Commitment relative to Discretion since the central bank commits itself to lower future values of the output gap and, hence, the inflation rate (below their corresponding targets given by $\bar{\pi}^j = \bar{y}^j = 0$).

Equivalently to our discussion in Chapter 2, setting $h = 1$ leads to the well-known expression for the reduced-form solutions and targeting rules in the Discretion and Commitment cases reported by an overwhelming amount of studies (see Woodford (2003), Galí (2008, his Chapters 3 and 4) and Walsh (2010, his Chapters 7 and 8) among others). From the same literature it is known, that due to the structure of the OMP regimes Discretion and Commitment, the former generates a higher welfare loss compared to the latter ($L_{(t)}^D > L_{(t)}^C$). In section 3.4 we will analytically (and numerically) show that this result prevails as we allow for a decrease in the period length.

It is worth mentioning that the weight $\psi(h)$ indicates a *frequency-dependent history (dependence) persistence* (FHP) effect. First of all, in his seminal paper, Leitemo (2008) shows that in a purely forward-looking model, the maximum degree of history dependence is required in order to achieve the lowest welfare loss in the OMP regime Commitment. While this means that $\psi(h) = 1$ holds, this condition is valid only in the baseline case of $h = 1$. In transition to an h-economy, we observe that the FHP effect is *dampened* as the period length declines.⁸ As a result, the stabilisation effect on π_t^C is reduced as h decreases, i.e. on a higher frequency relative to a quarter, the inertia in the targeting rule (3.10) is less considered. However, the quantitative effects are quite small. This can be seen by the upper panel in Figure 3.1, which depicts the change in the degree of history dependence with respect to h . Here a non-linear development in the weight can be observed, where the minimum is reached at a value of $h = 0.5$. It follows from this that the dampening effect on the history dependence and, therefore, the reduced stabilisation effect is not dramatic as h decreases. Most importantly, a degree of history dependence below unity induces a long-run deviation in the price level from its initial steady state value. Hence, in an h-economy the central bank fails to stabilize the price level in OMP regime Commitment, which does not hold in the case $h = 1$ (therefore consider also the IRFs presented in the next section).

On the contrary, the FIP effect has a distinct negative impact on the effectiveness of OMP. The latter is weakened in transition from a lower to a higher frequency. In particular, as the period length h decreases – ceteris paribus – this leads to a decline in the slope of the NKPC measured by $\kappa(h)$. This can be explained by the increasing amount of transactions and reallocations the central bank has to react to more frequently in an h-economy (cf. section 2.3). As a result the stabilisation bias is amplified, i.e. $\xi(h)$ declines for given values of $\bar{\alpha}_1$ and $\bar{\alpha}_2$. Compared to the FHP effect, the influence of the FIP effect on the stabilisation of the inflation rate is remarkable. It can be seen from the solid line (where $\bar{\alpha}_1 = 1$ and $\bar{\alpha}_2 = 0.05$ hold) in the lower panel of Figure 3.1 that the decline in $\xi(h)$ is quite strong as the period length decreases.

This result is obviously strengthened by high values for the weight on output gap stabilisation

⁸In his paper, Leitemo (2008) sets the discount factor given in quarterly magnitudes equal to unity i.e. $\beta(1) = 1$ holds. While he argues that this assumption is grounded on the matter of ‘convenience’ (p. 268), it is obviously the case, that this treatment is misleading as an h-economy is considered.

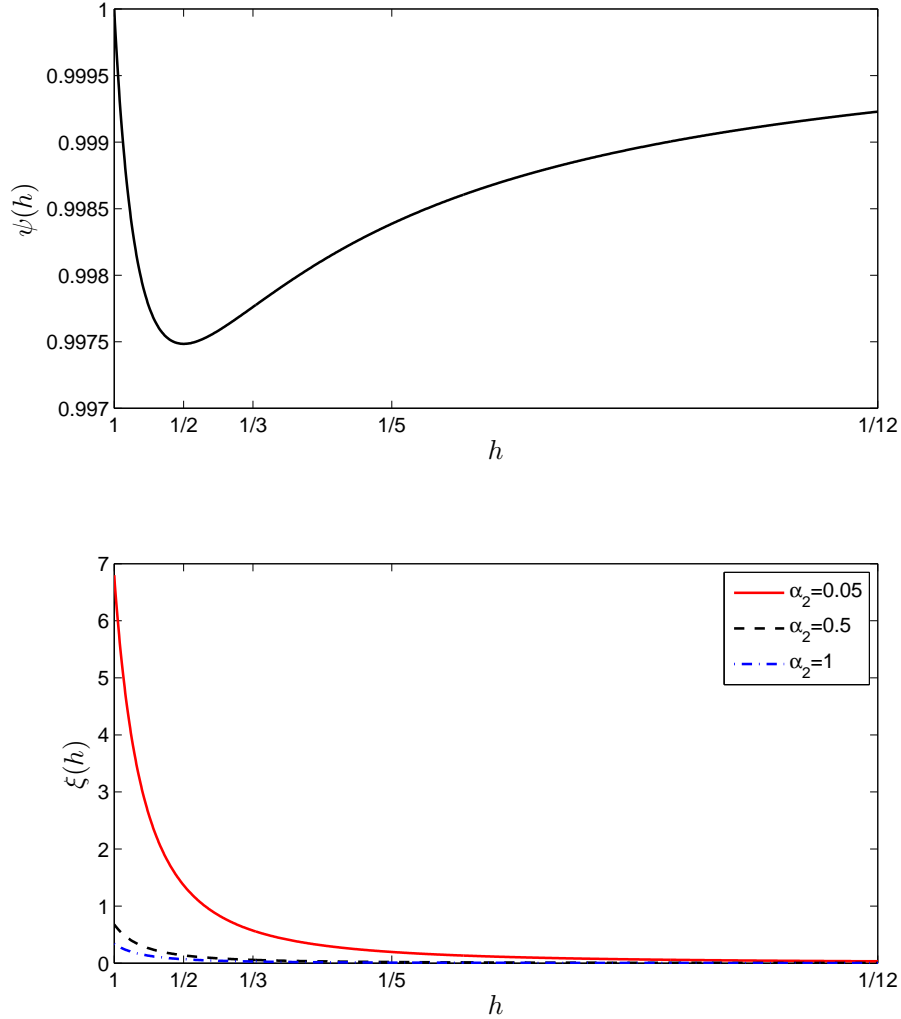


Figure 3.1: Development of the weights $\psi(h)$ and $\xi(h)$.

Note: The Figure depicts the development in the weights $\psi(h) = \beta(h)^{(1-h)}$ (upper panel) and $\xi(h)$ (lower panel) — as a component of the targeting rule under the OMP regime Commitment (cf. equation (3.10)) — with respect to h . The scale of the horizontal axis is limited on the interval $h \in [1, \dots, 1/12]$, while it is confirmed that $\psi(h) = 1$ holds for $h = 0$. The *solid* line in the lower panel depicts the development of $\xi(h)$ for $\bar{\alpha}_2 = 0.05$. The *dashed* line in the lower panel depicts the development of $\xi(h)$ for $\bar{\alpha}_2 = 0.5$. The *dashed/dotted* line in the lower panel depicts the development of $\xi(h)$ for $\bar{\alpha}_2 = 1$. The associated parameter values are taken from Table 2.1 together with $\bar{\alpha}_1 = 1$.

measured by $\bar{\alpha}_2$. In reality, it can be stated that the ECB follows a *strict* inflation target ($\bar{\alpha}_1 > 0, \bar{\alpha}_2 \approx 0$), while the latter is more *flexible* with respect to the US Federal Reserve Bank (FED; $\bar{\alpha}_1 > 0, \bar{\alpha}_2 > 0$). As we can see from equations (3.9) and (3.10), the more flexible the inflation targeting is this leads to a more destabilizing effect. The reason is, that for high values of $\bar{\alpha}_2$, the central bank is cautious about a strong negative output gap needed for stabilizing the inflation rate. Hence, the drop in the output gap is less pronounced as being necessary, which leads to a smaller dampening effect on the inflation rate. Furthermore, high values of $\bar{\alpha}_2$ amplify the impact of the FIP effect on the stabilisation bias, i.e. $\xi(h)$ becomes close to zero even on a low frequency (that is, as h is close to 1). The dashed and dashed/dotted lines in the lower panel of Figure 3.1, which represent the developments in $\xi(h)$ for $\bar{\alpha}_2 = 0.5$ and $\bar{\alpha}_2 = 1$, respectively, show this explicitly. Technically, this observation can be easily explained: as we turn to a more flexible inflation targeting environment, $\xi(h)$ becomes more compressed as h decreases relative to low values of $\bar{\alpha}_2$ given.

As we jointly consider the impacts of the FIP and FHP effects together with the variation in the degree of inflation targeting, several observations are worth mentioning. First, while the increase in the stabilisation bias (due to the FIP effect) has a stronger impact in the OMP regime Discretion compared to the Commitment one, in the latter case the FHP effect ensures the lowest welfare loss. Although, the reduction of the output gap is dampened on impact on a higher frequency, a decrease in h leads only to a small decrease in history dependence. Hence, the central bank is still able to reduce the inflation rate (and the output gap) below its target and, therefore, maintains a less destabilizing adjustment of the variable compared to the discretionary case. Second, in conjunction with the FCP effect, the almost unchanged degree in the history dependence leads to more pronounced humped-shaped adjustments in all variables under Commitment.

Finally, regarding the OMP regime Commitment only, the FHP effect *offsets* (or even dominates) the FIP effects in the case of strict inflation targeting, while the opposite holds under a flexible inflation targeting scenario. Therefore, note that a high value of $\xi(h)$ leads to a stronger reduction in the output gap relative to when this value is low. However, this strong reduction feeds back into the targeting rule (3.10) in the next period via the past value of the output gap, which is multiplied by $\psi(h)$. While this feedback effect is more pronounced for $h = 1$, we are able to see from Figure 3.1 that for $\bar{\alpha}_2 = 0.05$ the smooth decrease in $\xi(h)$ leads to a quantitatively strong consideration of the non-linear movement in $\psi(h)$ in the targeting rule (3.10). This observation helps to explain the corresponding non-linear development we obtain within our welfare analysis, where we study the difference in L^D compared to L^C for the case $\bar{\alpha}_2 = 0.05$. As we turn to higher values of $\bar{\alpha}_2$, this non-linear movement in $\psi(h)$ is less considered since $\xi(h)$ decreases rapidly for even high values of h . Under flexible inflation targeting, the weak reduction in the output gap feeds less back into the targeting rule and, therefore, dampens the FHP effect.

3.3 Numerical Results: Impulse Response Functions

In the following we shed a light on the dynamics of the model variables as a result of the output-inflation trade-off linked to the cost-push shock. Furthermore, we discuss the implications for OMP based on our investigations undertaken in the previous section. First, we study the IRFs for all model variables based on the reduced-form solutions for y_t^j and π_t^j . Based on these solutions, the dynamics of the nominal interest rate is simply computed by solving the dynamic IS equation (3.1) for i_t^j . Therefore, the dynamics of the nominal

interest rate mimics the monetary policy reaction given by the associated targeting rules (3.9) and (3.10). The real interest rate is then given by the difference ($r_t^j = i_t^j - \pi_{t-h}^j$) and the price level is simply computed by $p_t^j = \sum_{k=0}^t \pi_k^j$. We adopt the calibration of the parameters from the previous Chapter, where the corresponding values are given in Table 2.1. In addition we follow the ECB strategy when analyzing the IRFs by assuming strict inflation targeting, where $\bar{\alpha}_1 = 1$ and $\bar{\alpha}_2 = 0.05$ hold. However, as we turn to the investigation of the welfare effects of both OMP regimes in the next section, we apply a sensitivity analysis regarding different values of $\bar{\alpha}_2$. For a clear arrangement, the corresponding Figures for a non-autocorrelated shock process ($\rho^\pi = 0$) and the autocorrelated case ($\rho^\pi = 0.5$) can be found in Appendix B3.

The IRFs for the OMP regime *Discretion* in case of a *non*-autocorrelated shock process are depicted in Figure 3.2. In a quarterly economy ($h = 1$), we observe a large drop in y_t^D , which leads to a weak increase in π_t^D on impact according to the targeting rule (3.9). Since the latter exhibits no history dependence, all variables converge back to their long-run steady states in next quarter. As we turn to a monthly ($h = 1/3$) and weekly ($h = 1/12$) frequency, the impact effects on y_t^D become weaker the more h is reduced. This can be explained by the FIP effect, where the stabilisation bias increases on a higher frequency. This leads to a more destabilizing impact effect regarding π_t^D . It can be seen that the quantitative differences are not negligible – even in the case of a transition from monthly to weekly magnitudes. In the same vein, the movements in i_t^D and r_t^D become ambiguous: both variables increase stronger on impact in the monthly economy compared to $h = 1$, while in a weekly economy even a strong drop on impact (in conjunction with hump-shaped movements afterwards) can be observed. The latter observation coincides with the quite weak impact effect on y_t^D . This can be interpreted as an indication for the central bank loosing control over stabilizing the economy on a higher frequency. The increase in p_t^D emerges for all values of h , while the development is more pronounced quantitatively on a higher frequency. The dramatic upward-movement in p_t^D is grounded on the increase in the persistence of π_t^D due to the FCP effect.

In case of an *autocorrelated* shock (see Figure 3.3) the quantitative effects are more pronounced. Interestingly, y_t^D is further reduced on impact for $h = 1/3$ compared to $h = 1$. Accordingly, i_t^D and r_t^D exhibit a stronger increase on impact in a monthly compared to a quarterly economy. It can be guessed, that the increase in ρ^π amplifies the FCP effect, which leads to a dominance of this (now) *amplified* effect over the FIP one on relative high frequencies.

Figure 3.4 shows the dynamics in the OMP regime *Commitment* in case of a *non*-autocorrelated shock process. In the case $h = 1$, less strong reactions on impact for y_t^C and π_t^C are observed compared to Discretion. This can be explained by the history dependence in the targeting rule (3.10), which causes (more) persistence in the variables. The inertia in y_t^C mimics the commitment of the central bank to dampen future values of the inflation rate below its target. Furthermore, the p_t^C converges back to its steady state value due to history dependence. Again, on a higher frequency the reduction in y_t^C is less pronounced, while inertia is amplified (in all variables due to the FCP effect). Based on the FIP effect, the central bank fails to stabilize the inflation rate on impact also under Commitment. However, since the degree of history dependence is quantitatively less affected as h decreases, the corresponding FHP effect explains the increase in the persistence in y_t^C and π_t^C on higher frequencies. Especially, in the case $h = 1/12$ we see that π_t^C is pushed below its target for several periods. As we will see in the next section, the low impact effects together with the

increase in inertia lead to a *lower* value of the loss function under Commitment relative to Discretion – even within an h -economy environment.

The hump-shaped behaviour of π_t^C leads also to the convergence of p_t^C back to its initial steady state value in the case $h = 1$. It is worth mentioning, that this result *does not* hold on a higher frequency. Again, due to the FHP effect, the decrease in the degree of history dependence for $0 < h < 1$ hinders the one-to-one response to y_{t-h}^C , i.e. as $\psi(h) < 1$ holds (cf. targeting rule (3.10)). Finally, like under Discretion, the movements in i_t^C and r_t^C change qualitatively in transition from a lower to a higher frequency.

In case of an *autocorrelated shock*, Figure 3.5 shows that the amplified FCP effect itself amplifies the FHP one: inertia increases rapidly, while the corresponding hump-shaped movements in all variables are more pronounced. It goes without saying that the quantitative effects are strengthened, while p_t^C (still) does not converge back to its initial steady state.⁹

We also make a comparison of the IRFs for a daily economy ($h = 1/90$). Here we also compute the dynamics based on the forward-looking version of the baseline NKM with an ad-hoc Taylor rule assumed (labeled by *TR*) known from the previous Chapter. According to the Figures 3.6 ($\rho^\pi = 0$) and 3.7 ($\rho^\pi = 0.5$), not surprisingly, more pronounced effects are quantitatively observed for most of the variables (especially for the output gap) in the TR case compared to both OMP regimes. Since no minimization of the loss function is targeted by the central bank under consideration of an ad-hoc Taylor rule, it is well known that the corresponding loss is higher than in both OMP regimes – this result holds independently for any length of the period. Despite this observation, in general, across all specifications the quantitative effects on a daily frequency are quite strong due to the amplified FCP effect.

3.4 Numerical Results: Welfare Analysis

In this section we study the differences in the value of the loss functions given the reduced-form solutions (targeting rules) for the inflation rate and the output gap under Discretion and Commitment, respectively. All upcoming numerical results are obtained under consideration of the analytical solutions (for the loss functions) given in Sacht and Wohltmann (2013), which had been modified for the systematic/skip sampling instead of the temporal aggregation scheme. The reason for this is that our numerical solution approach presented here demand a large amount of main memory and time for execution. This can be explained by the computation of the loss function, where a discounted sum of all weighted squared deviations in the inflation rate and the output gap must be considered. However, the analytical solutions are grounded on the expression for the loss functions based on the relationship of all associated parameters of the model. In this case no streams of values over an infinite time horizon must be added up. However, it is confirmed that the numerical and analytical (based on Sacht and Wohltmann (2013)) solution procedures lead to the same results.¹⁰ All corresponding Figures can be found in the Appendix B4.

⁹In Figure 3.4 (3.5) we limit the horizontal axis to 15 (20) quarters with respect to p_t^C , in order to account for the less humped-shaped movements of the variable in the cases $h = 1$ and $h = 1/3$. However, it is confirmed that the price level does not converge back to zero as $0 < h < 1$ holds, i.e. in an h -economy.

¹⁰As already stated, this problem does not appear when computing all IRFs presented in this thesis. The related reduced-form solutions are given by the equations (3.21) and (3.22) for Discretion as well as (3.40) and (3.41) for Commitment – all given in the Appendices B1 and B2, respectively. The corresponding MATLAB codes can be downloaded from the authors' webpage at <http://www.makro-vwl.uni-kiel.de/de/team/dipl.-vw.-stephen-sacht>.

Non-Autocorrelated Cost-Push Shock ($\rho^\pi = 0$):					
$\bar{\alpha}_2 = 0.05$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	0.0555	0.6138	1.2747	4.8546	16.2144
$(L^D - L^C)/L^C$	22.53 %	85.47 %	96.68 %	57.81 %	18.94 %
$\bar{\alpha}_2 = 0.5$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	0.1888	0.5614	0.7973	1.8668	4.6486
$(L^D - L^C)/L^C$	30.28 %	39.04 %	33.65 %	16.15 %	4.78 %
$\bar{\alpha}_2 = 1$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	0.1805	0.4491	0.6090	1.3163	2.9710
$(L^D - L^C)/L^C$	25.21 %	27.98 %	23.46 %	10.86 %	3.00 %

Table 3.1: Selected values of the loss relations for $\rho^\pi = 0$.

Note: L^D and L^C denote the loss for the optimal policy regimes Discretion (D) and Commitment (C), respectively. The difference $L^D - L^C$ denotes the *absolute* loss relation. The difference $(L^D - L^C)/L^C$ denotes the *relative* loss relation.

We consider the *absolute* and *relative* loss relations of Discretion compared to Commitment. The former is given by the difference $L^D - L^C$, while for the latter the percentaged change in the difference with respect to the loss under Commitment is computed, i.e. $(L^D - L^C)/L^C$ holds. We study the developments in these differences with respect to the variation in the period length for the cases of a (non-)autocorrelated shock. Figure 3.8 contains the related diagrams. According to both upper panels, the *absolute* loss relations are positive and increase monotonically as h decreases. The first observation confirms that the loss under Discretion is always higher compared to the one under Commitment – regardless of the length of the period. The second observation emphasizes one of our main results, that the more information and transactions (reallocations) is available/are observed on a higher frequency, the higher the welfare loss will be. This can be again explained by the FCP and FIP effects, where the former leads to an overall increase in the persistence of the shock, while the latter leads to an increase of the stabilisation bias. Due to the FHP effect this increase in the stabilisation bias is dampened under Commitment, where the weight on the past value of the output gap is only marginally affected by the decrease in h . Hence, the stabilisation of the economic indicators is much more successful under Commitment, which results in a positive value in the absolute loss relation. As we increase the weight on output stabilisation ($\bar{\alpha}_2$), the loss relation curve becomes flatter, i.e. the absolute difference becomes less pronounced. The reason is that this change in the inflation targeting regime leads to a stronger consideration of the fluctuations in the output gap by the central bank. Since the

FIP effect becomes amplified by this (the stabilisation bias increases much further), both OMP regimes are affected. For the case $\rho^\pi = 0.5$ we see stronger quantitative effects due to the (now) amplified FCP effect.¹¹

Autocorrelated Cost-Push Shock ($\rho^\pi = 0.5$):					
$\bar{\alpha}_2 = 0.05$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	0.1300	2.6918	7.3873	40.0296	158.5244
$(L^D - L^C)/L^C$	30.58 %	152.00 %	203.60 %	133.24 %	40.90 %
$\bar{\alpha}_2 = 0.5$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	1.2859	4.7321	7.1605	18.6095	49.6553
$(L^D - L^C)/L^C$	74.18 %	89.58 %	76.33 %	34.89 %	10.00 %
$\bar{\alpha}_2 = 1$					
h	1	1/2	1/3	1/12	1/90
$L^D - L^C$	1.5672	4.1744	5.8594	13.5675	32.1513
$(L^D - L^C)/L^C$	68.66 %	64.56 %	52.72 %	23.20 %	6.25 %

Table 3.2: Selected values of the loss relations for $\rho^\pi = 0.5$.

Note: L^D and L^C denote the loss for the optimal policy regimes Discretion (D) and Commitment (C), respectively. The difference $L^D - L^C$ denotes the *absolute* loss relation. The difference $(L^D - L^C)/L^C$ denotes the *relative* loss relation.

Both middle panels in Figure 3.8 depict the relative loss relations with respect to ρ^π . We observe a non-linear development of the *relative* loss relations, which is much more pronounced in the case $\bar{\alpha}_2 = 0.05$. As we already explained in section 3.2.2, the FHP effect offsets and even dominates the FIP effect under strict inflation targeting over a specific interval of h . As the period length decreases, the non-linear movement in the degree of history dependence becomes more considered the lower $\bar{\alpha}_2$ will be for high to moderate values of h . Due to the low value of the stabilisation bias, the strong reduction in the output gap on impact feeds into the targeting rule (3.10) for Commitment via the past value of this variable. The additional degree of inertia due to the FHP effect leads to a stronger stabilisation effect under Commitment.

At a specific value of h , a maximum in the relative loss relation Discretion/Commitment is reached. We denote this value by \hat{h} . Beyond this point, the FIP effect dominates the FHP one: the stabilisation bias increases more rapidly the more h approaches to zero. Hence, the impact effect on the output gap becomes more and more negligible in this case (cf. Figure

¹¹We limit the scale of the upper panels by $h \in [0.2, 1]$ to ensure a clear arrangement. We do so because for values of h close to 0, the loss relation curve, which represents the case $\bar{\alpha}_2 = 0.05$, increases rapidly compared to the other ones. However, in the Tables 3.1 and 3.2 we report also the values for the case $h = 1/90 = 0.01$.

3.4). Note that under Discretion this observation does not hold since history dependence is not considered in the corresponding targeting rule (3.9). As we turn to a more flexible inflation targeting scenario, the FIP effect has a stronger impact on the relative loss relations. While the FHP effect is less considered (i.e. the non-linear movement in the relative loss relation almost vanishes), the differences between Discretion and Commitment decrease since the stabilisation bias increases rapidly.

If an *autocorrelated* shock is assumed, the FCP effect amplifies the FHP effect, which can be seen especially in the case $\bar{\alpha}_2 = 0.05$. Here, the relative loss relation peaks at a value of 203.60 % ($h = 1/3$). This value is taken from Table 3.2, where we together with Table 3.1 report the values of both loss relations for selected values of h . From these Tables, we can see that the absolute differences in the loss are low for the case $h = 1$, while it is increasing strongly as h decreases. However, this development is dampened as $\bar{\alpha}_2$ increases but it is still remarkable for values of h close to 0 (especially in the case of an autocorrelated shock, cf. Table 3.2). The values of the relative loss relations indicate a loss under Discretion being less than 100 % (higher) relative to Commitment – except for the strict inflation targeting scenario as an autocorrelated shock occurs. In this case the OMP regime Commitment is strongly superior to the Discretion one, which holds mainly on a monthly frequency.

As we can see from both middle panels, the value of \tilde{h} (especially in the case $\bar{\alpha}_2 = 0.05$) slightly moves to the left as ρ^π increases. The lower panel of Figure 3.8 shows this in more detail. In general, quite high values of ρ^π induce quite low values of h needed to maintain an (relative) advantage of the OMP regime Commitment over Discretion, i.e. \tilde{h} decreases as ρ^π increases. Furthermore, the higher the value of $\bar{\alpha}_2$, the higher \tilde{h} will be. Finally, at a maximum value of $\rho^\pi = 0.99$ we observe almost a convergence in \tilde{h} across different values of $\bar{\alpha}_2$.¹² Based on these observations it can be stated that the (amplified) FCP effect has an impact on the interaction of the FIP and the FHP effect.

We close this section by investigating the impact of a variation in the price stickiness measured by the frequency-dependent Calvo parameter denoted by $\theta(h)$. The latter probability consists on the corresponding value of this parameter in quarterly magnitudes given by θ . So far, we discuss the FIP effect as being described by the change in the slope of the NKPC. However, besides the degree of price indexation (which is set to zero in this Chapter), the degree of price stickiness plays also an important role via the FIP effect. The higher the probability of the firm to *not* adjust their prices the lower the dampening effect of reducing the output gap by the central bank will be. From an economic point of view, a high degree of price stickiness makes it difficult for the policy maker to influence inflation rate dynamics. The reason is that the price setting scheme becomes increasingly degenerated from the movement in the real marginal cost (output gap) the higher the likelihood for sticky prices will be. As a result the welfare loss increases with a higher degree of price stickiness, i.e. as θ increases.

Applying a sensitivity analysis with respect to θ is meaningful since the empirical evidence regarding this parameter is ambiguous. While $\theta \approx 2/3$ is a generally accepted benchmark that is supported by several empirical studies (Galí et al. (2001, p. 1255), Christiano et al. (2005, p. 18) or Álvarez et al. (2006, p. 578)), one can also find lower and higher degrees of stickiness in the literature. Fabiani et al. (2007, p. 41) analyze data from surveys of 11000 firms which were conducted by the national banks of 9 European countries. The median

¹²Note that we do not study the case $\rho^\pi = 1$ since we consider a *temporary* shock only. Therefore, we discard the possibility of a random walk in the shock process since in this case the dynamic system becomes obviously hard to evaluate empirically (which is not done in this thesis).

number of price changes per year in these different countries is equal to one, which implies $\theta \approx 3/4$. By estimating a DSGE model with sticky prices and wages for the Euro Area with Bayesian techniques, Smets and Wouters (2003, p. 1144) obtain a much longer average duration of price contracts of two and a half years, or $\theta \approx 9/10$. In contrast, from the data of the US Bureau of Labor Statistics, Bils and Klenow (2004, p. 953) derive evidence for firms in the US changing their price every two quarters or even less, according to which θ can become as low as $1/2$ and less. Therefore, we consider for our simulation a range of θ between 0.5 and 0.9.¹³

In the following we repeat our computation exercise with respect to the relative loss relations under variation of θ .¹⁴ The results are depicted in Figure 3.9. The results from the two dimensional investigations are resembled: the FHP effect dominates the FIP effect with respect to h on a lower to a moderate frequency, i.e. the corresponding loss relation increases towards \tilde{h} , which represents a peak on a low value of h . This holds only for a low value of $\bar{\alpha}_2$. As the latter increases the differences are high for almost all values of h – but only for low values of θ . As the degree of price stickiness increases, the stabilisation bias increases. In this case the advantage of the OMP regime Commitment over Discretion vanishes as monetary policy becomes more *ineffective*. This holds independently for the variation in $\bar{\alpha}_2$.

More pronounced effects can be observed if an *autocorrelated* shock occurs. Again, the values of the relative loss relation decreases in transition from more a flexible to a more sticky price scenario. If prices are close to being fixed, the OMP regime Commitment is nearly as being effective as the Discretion one. Therefore the FIP effect with respect to h becomes amplified as θ increases. Therefore, the central bank can be advised, first, to ensure a strict inflation targeting scenario and, second, to react more frequently to the fluctuations in the economic indicators than on a quarterly base. However, the effectiveness of such kinds of strategies can be still hindered if prices become more rigid. Furthermore, keep in mind that the absolute loss is *always higher* on a higher relative to a lower frequency due to the FCP effect.

Finally, we study the change in the value of \tilde{h} with respect to θ , $\bar{\alpha}_2$ and ρ^π . Figure 3.10 applies. We observe a convergence in \tilde{h} as ρ^π approaches to the value of 0.99. A decrease in the degree of price stickiness requires a low value of h in order to maintain a maximum in the relative loss relations. In the other way around, the more prices are rigid the more a lower frequency is required to observe a maximum. The reason is that in this case the FHP effect dominates the FIP one *only* for high values of h since the increase in θ leads to stronger increase in the stabilisation bias due to the (now) reinforced FIP effect. This observations put an emphasis on the role of the degree of price stickiness – together with the FCP effect (see above) – for the interaction of the FIP and the FHP effect.

3.5 Conclusion

In this Chapter we study the OMP responses to a cost-push shock within an h-economy environment. Our analytical and numerical results reveal the impact of frequency-dependent

¹³In the next Chapter we report a value of θ close to or even equal to zero in the case of Argentina, which stems from the estimation of a high-frequency (i.e. daily) NKPC. We omit this polar case in our sensitivity analysis here, where we consider observations for the industrialized countries, the United States and the Euro Area only.

¹⁴Similar results can be found in Franke and Sacht (2014). In particular, in their study an equivalent three dimensional numerical investigation (given in their Appendix) in the case of a nominal interest rate shock in the (baseline) NKM with an ad-hoc Taylor rule (regime *TR*) is applied.

persistence on the effectiveness of OMP in transition from a lower to a higher frequency. Therefore, the OMP regimes Discretion and Commitment are applied under consideration of a purely forward-looking NKM. We show that the central bank fails to stabilize the inflation rate and the output gap on a higher frequency – even if its response is stemming from welfare maximization. Our main observation indicates that the welfare loss is increasing rapidly as the period length declines. This can be explained by the increase in the information available, which causes rapid changes in the inflation rate and the output gap. This must lead to destabilisation in the economy, which is analogous to the impact of an anticipated shock in quarterly magnitudes as analyzed by Winkler and Wohltmann (2012).

Given that the welfare loss increases as the period length decreases, our results call for an optimal response of the central bank in terms of a timeless perspective. Here, the central bank makes use of the expectation channel of monetary policy in order to generate smooth dynamics of the economic indicators over time. In comparison, a discretionary optimal monetary policy response, where expectations of agents are given for the central bank, leads to a higher welfare loss. The overall increase in the welfare loss is triggered by the frequency-dependent contraction persistence (FCP) effect, where the persistence of the shock process is amplified by a decrease in the period length.

In particular, the magnitudes of the reduction of the output gap needed in order to dampen the increase of the inflation rate are small on a higher frequency. The reason is that the stabilisation bias is increasing in this case due to the frequency-dependent inherited persistence (FIP) effect. While in this case the pass-through of changes in the output gap into inflation dynamics is hindered, this outcome is more severe in the OMP regime Discretion relative to Commitment. In the latter case, the frequency-dependent history (dependence) persistence (FHP), which describes the decrease in the history dependence with respect to the period length, is less affected by the frequency of decision making. Hence, the welfare loss under Commitment is lower relative to the one under Discretion, which holds independently of the period length. The difference is strengthened in transition from a strict to a flexible inflation targeting. This is confirmed by various robustness checks, where we also find evidence for an increase in the stabilisation bias due to an increase in the price stickiness. In addition, as an interesting result we observe that the price level does not converge back to its initial steady state level under Commitment due to the FCP effect, which does not hold on quarterly magnitudes. According to this, the recent discretionary policy conducted by the ECB in the aftermath of the financial and sovereign debt crisis in the Euro Area can be seen as being harmful with respect to welfare maximization – regardless of low- or high-frequent movements in the economic indicators being considered in general.

Sacht and Wohltmann (2013) show that similar results can be obtained when an Optimal Simple Rule is applied. Here, while the specification of an ad-hoc Taylor rule is assumed, the optimal values of the corresponding weights on the inflation rate and the output gap are given by the outcome of the minimization of the loss function. In their paper, the authors report qualitative equivalent results with respect to the absolute and relative loss relations, i.e. that the welfare loss under Commitment is lower compared to the case of an Optimal Simple Rule. Only small quantitative differences are observed in general for the remaining loss relations, which are stemming from the temporal aggregation scheme applied in their paper.

In particular, one seeks to identify an amount of Optimal Simple Rules, which represent a good approximation of the (in reality unobservable) targeting rule under Commitment. It means that as an outcome, this specific rule becomes as close as possible in describing the

dynamics, which are obtained under the OMP regime Commitment (cf. Wohltmann and Winkler (2009) as well as Winkler and Wohltmann (2012)). It is meant by that a specific instrument rule must be imposed under consideration of the optimal weights in the loss function. An example is given by a Taylor rule with interest smoothing, where the central bank responds partly also to the previous value of the nominal interest rate. This has not been done so far when studying different frequencies of decision making. In addition, the volatility of the nominal interest rate can also be taken into account as part of the central bank's welfare function. We leave this to further research.

We like also to emphasise, that the analysis of the OMP responses within the *hybrid* NKM under Commitment would be fruitful. In this case the investigation does not require the corresponding solution methods for the computation of the policy functions like in the case of Discretion – see our discussion with respect to the approaches provided by Söderlind (1999) and Dennis (2007) given in the Introduction of this Chapter. Note that, given a hybrid specification of the NKM under Commitment, the degree of price indexation now plays a significant role in the determination of the movements of the variables on a higher frequency. According to our analysis of the cost-push shock based on an ad-hoc Taylor rule (cf. section 2.4.3), we guess that the existence of intrinsic persistence will amplify the FIP effect, which leads to a decrease in the effectiveness of OMP along with stronger hump-shaped adjustments of the variables being observed. However, those thoughts call for a more elaborate investigation in further research.

In addition, the question arises if the central bank has to be forced to intervene on a lower frequency when facing high welfare losses on a higher one. This implies that it could be welfare increasing to adjust the monetary policy instrument (via the targeting rule) on e.g. a monthly or quarterly basis as information for the remaining economic indicators are available on a, let's say, daily basis. In this case the central bank monitors the adjustments on a higher frequency and react to them on a lower one in order to avoid strong destabilisation effects in case of a cost push shock. This treatment calls for an heterogeneous specification of the period length across agents in the economy. We leave such kind of investigation also to further research. We would like to emphasise that the availability of daily information on movements in the economic indicators is not an unrealistic assumption to make. This holds especially for the (Argentine) inflation rate as we discuss in the next Chapter.

3.6 Appendix B

3.6.1 B1: Solution of the OMP Regime Discretion in the Case $0 < h \leq 1$

For a clear arrangement, the expectation operator E_t on the future values of the variables is omitted throughout this Appendix. The corresponding Lagrangian under Discretion is given by

$$L_t^D = h\bar{\alpha}_1(\pi_t^D)^2 + \bar{\alpha}_2(y_t^D)^2 + \tilde{\theta}_t(y_t^D + \delta_1(h)(i_t^D - \bar{i}) - f_t) + \tilde{\rho}_t(\pi_t^D - \tilde{\kappa}(h)y_t^D - g_t) + F_t \quad (3.11)$$

where $\tilde{\theta}_t$ and $\tilde{\rho}_t$ denote the Lagrangian multipliers (which are predetermined under Discretion), where their initial values in $t = 0$ are equal to 0. In the OMP regime Discretion, the central bank must take all expectations regarding the future values of the economic

indicators as given:

$$\begin{aligned} F_t &= E_t \sum_{k=1}^{\infty} \beta(h)^{hk} \{h\bar{\alpha}_1(\pi_{t+hk}^D)^2 + \bar{\alpha}_2(y_{t+hk}^D)^2\} \\ f_t &= y_{t+h}^D + \delta_1(h)\pi_{t+h}^D \\ g_t &= \beta(h)\pi_{t+h}^D + v_t^\pi. \end{aligned} \quad (3.12)$$

The first-order conditions read

$$\frac{\partial L_t^D}{\partial \pi_t^D} = 2h\bar{\alpha}_1\pi_t^D + \tilde{\rho}_t = 0 \quad (3.13)$$

$$\frac{\partial L_t^D}{\partial y_t^D} = 2\bar{\alpha}_2y_t^D + \tilde{\theta}_t - \tilde{\rho}_t\tilde{\kappa}(h) = 0 \quad (3.14)$$

$$\frac{\partial L_t^D}{\partial \tilde{\theta}_t} = y_t^D + \delta_1(h)(i_t^D - \bar{i}) - f_t = 0 \quad (3.15)$$

$$\frac{\partial L_t^D}{\partial \tilde{\rho}_t} = \pi_t^D - \tilde{\kappa}(h)y_t^D - g_t = 0. \quad (3.16)$$

In addition, $\frac{\partial L_t^D}{\partial i_t^D} = \tilde{\theta}_t = 0$ holds, since interest rate stabilisation is not the aim of the central bank (cf. Walsh (2010, p. 358)). Solving equation (3.14) for $\tilde{\rho}_t$ and plug it into (3.13) results in the targeting rule (3.9) under Discretion:

$$\pi_t^D = -\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1}y_t^D \Leftrightarrow y_t^D = -\frac{h\tilde{\kappa}(h)\bar{\alpha}_1}{\bar{\alpha}_2}\pi_t^D \left(= -\frac{\kappa(h)\bar{\alpha}_1}{\bar{\alpha}_2}\pi_t^D \right). \quad (3.17)$$

Note that $\tilde{\kappa}(h) = \kappa(h)/h$ holds. Inserting the first part of the targeting rule into the NKPC (cf. equation (3.16)) leads to

$$\begin{aligned} -\left(\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1}\right)y_t^D + \left(\beta(h)\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1}\right)y_{t+h}^D - \tilde{\kappa}(h)y_t^D - v_t^\pi &= 0 \\ \Rightarrow \left(\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1} + \tilde{\kappa}(h)\right)y_t^D - \left(\beta(h)\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1}\right)y_{t+h}^D + v_t^\pi &= 0 \\ \Rightarrow d_1y_t^D + d_2y_{t+h}^D + v_t^\pi &= 0 \end{aligned} \quad (3.18)$$

with

$$\begin{aligned} d_0 &= -\frac{\bar{\alpha}_2}{h\tilde{\kappa}(h)\bar{\alpha}_1} \\ d_1 &= \tilde{\kappa}(h) - d_0 \\ d_2 &= \beta(h)d_0. \end{aligned}$$

Based on the method of undetermined coefficients (McCallum (1983)), we guess that the dynamics in y_t^D are described by the following law of motion:

$$y_t^D = d_3v_t^\pi. \quad (3.19)$$

By plugging equation (3.19) into equation (3.18) we get the solution for d_3 :

$$\begin{aligned} d_1d_3v_t^\pi + d_2d_3v_{t+h}^\pi + v_t^\pi &= 0 \\ \Rightarrow d_2d_3(\rho(h)^\pi v_t^\pi + \varepsilon_{t+h}^\pi) + (d_1d_3 + 1)v_t^\pi &= 0 \\ \Rightarrow [(d_2\rho(h)^\pi + d_1)d_3 + 1]v_t^\pi &= 0 \\ \Rightarrow -\frac{1}{d_1 + d_2\rho(h)^\pi} &= d_3. \end{aligned} \quad (3.20)$$

Note that in case of an (deterministic) impulse shock $\varepsilon_{t+h}^\pi = 0$ holds. By plugging equation (3.19) together with equation (3.20) into the targeting rule (3.17) we obtain

$$x_t^D = \Gamma_D v_t^\pi = \begin{pmatrix} 1 \\ d_0 \end{pmatrix} d_3 v_t^\pi$$

with $x_t^D = (y_t^D, \pi_t^D)'$. The reduced-form solution for the output gap and the inflation rate under Discretion are then explicitly given by

$$y_t^D = -\frac{\kappa(h)\bar{\alpha}_1}{[1 - \beta(h)\rho(h)^\pi]\bar{\alpha}_2 + \{[\kappa(h)^2]/h\}\bar{\alpha}_1} [\rho(h)^\pi]^{t/h} \varepsilon_0^\pi \quad (3.21)$$

$$\pi_t^D = \frac{\bar{\alpha}_2}{[1 - \beta(h)\rho(h)^\pi]\bar{\alpha}_2 + \{[\kappa(h)^2]/h\}\bar{\alpha}_1} [\rho(h)^\pi]^{t/h} \varepsilon_0^\pi \quad (3.22)$$

where $\tilde{\kappa}(h) = \kappa(h)/h$ has been already applied. Therefore consider also the forward solution of the shock process for $0 < h \leq 1$ given by¹⁵

$$E_0 v_t^\pi = [\rho(h)^\pi]^{t/h} \varepsilon_0^\pi. \quad (3.23)$$

3.6.2 B2: Solution of the OMP Regime Commitment in the Case $0 < h \leq 1$

The Lagrangian under Commitment is given by

$$\begin{aligned} L_t^C = E_t \sum_{k=0}^{\infty} & \beta(h)^{hk} [h\bar{\alpha}_1(\pi_{t+hk}^C)^2 + \bar{\alpha}_2(y_{t+hk}^C)^2 \\ & + \bar{\theta}_{t+h+hk}(y_{t+hk}^C - y_{t+h+hk}^C + \delta_1(h)(i_{t+hk}^C - \pi_{t+h+hk}^C - \bar{i})) \\ & + \bar{\rho}_{t+h+hk}(\pi_{t+hk}^C - \beta(h)\pi_{t+h+hk}^C - \tilde{\kappa}(h)y_{t+hk}^C + v_{t+hk}^\pi)] \end{aligned} \quad (3.24)$$

where $\bar{\theta}_{t+h+hk}$ and $\bar{\rho}_{t+h+hk}$, again, denote the corresponding Lagrangian multipliers (which are, again, predetermined since π_t^C and y_t^C are forward-looking), where their initial values are equal to 0. The first-order conditions read

$$\begin{aligned} \frac{\partial L_t^C}{\partial \pi_{t+hk}^C} &= \beta(h)^{hk} (2h\bar{\alpha}_1\pi_{t+hk}^C) - \beta(h)^{hk-h}\bar{\theta}_{t+hk}\delta_1(h) \\ &+ \bar{\rho}_{t+h+hk} - \beta(h)^{hk-h+1}\bar{\rho}_{t+hk} = 0 \end{aligned} \quad (3.25)$$

$$\frac{\partial L_t^C}{\partial y_{t+hk}^C} = \beta(h)^{hk} (2\bar{\alpha}_2 y_{t+hk}^C + \bar{\theta}_{t+h+hk} - \tilde{\kappa}(h)\bar{\rho}_{t+h+hk}) - \beta(h)^{hk-h}\bar{\theta}_{t+hk} = 0 \quad (3.26)$$

$$\frac{\partial L_t^C}{\partial i_{t+hk}^C} = \bar{\theta}_{t+h+hk} = 0 \quad \forall h = 0, \dots, 1; k = 0, \dots, \infty \quad (3.27)$$

$$\frac{\partial L_t^C}{\partial \bar{\theta}_{t+h+hk}} = y_{t+hk}^C - y_{t+h+hk}^C + \delta_1(h)(i_{t+hk}^C - \pi_{t+h+hk}^C - \bar{i}) = 0 \quad (3.28)$$

$$\frac{\partial L_t^C}{\partial \bar{\rho}_{t+h+hk}} = \pi_{t+hk}^C - \beta(h)\pi_{t+h+hk}^C - \tilde{\kappa}(h)y_{t+hk}^C - v_{t+hk}^\pi = 0. \quad (3.29)$$

By dividing (3.25) and (3.26) by $\beta(h)^{hk}$ and taking into account that $k = 0$ holds, it follows

$$0 = 2h\bar{\alpha}_1\pi_t^C + \bar{\rho}_{t+h} - \bar{\rho}_t\beta(h)^{1-h} \quad (3.30)$$

$$\bar{\rho}_{t+h} = \frac{2\bar{\alpha}_2}{\tilde{\kappa}(h)} y_t^C \Leftrightarrow \bar{\rho}_t = \frac{2\bar{\alpha}_2}{\tilde{\kappa}(h)} y_{t-h}^C. \quad (3.31)$$

¹⁵Therefore, consider the derivation of the shock process under $0 < h \leq 1$ which is provided in Appendix A2.

Plugging (3.31) into (3.30) results in the targeting rule under Commitment:

$$\pi^C = -\frac{\bar{\alpha}_2}{h\bar{\alpha}_1\tilde{\kappa}(h)} \left(y_t^C - \beta(h)^{(1-h)} y_{t-h}^C \right) \Leftrightarrow y_t^C = -\frac{\bar{\alpha}_1\kappa(h)}{\bar{\alpha}_2} \pi_t^C + \beta(h)^{(1-h)} y_{t-h}^C. \quad (3.32)$$

Note, again, that $\tilde{\kappa}(h) = \kappa(h)/h$ holds. The expression in front of the brackets is equal to d_0 given under Discretion. Hence, the stabilisation bias being the same in both OMP regimes. Inserting the first part of the previous equation into the NKPC (cf. equation (3.29) for $k = 0$) leads to

$$\begin{aligned} & -\frac{\bar{\alpha}_2}{h\bar{\alpha}_1\tilde{\kappa}(h)} \left(y_t^C - \beta(h)^{(1-h)} y_{t-h}^C \right) - \tilde{\kappa}(h) y_t^C \\ & + \frac{\beta(h)\bar{\alpha}_2}{h\bar{\alpha}_1\tilde{\kappa}(h)} \left(y_{t+h}^C - \beta(h)^{(1-h)} y_t^C \right) = v_t^\pi \\ \Rightarrow & y_t^C \left(-\frac{\bar{\alpha}_2}{h\bar{\alpha}_1\tilde{\kappa}(h)} - \tilde{\kappa}(h) - \beta(h)^{(1-h)} \frac{\beta(h)\bar{\alpha}_2}{h\bar{\alpha}_1\tilde{\kappa}(h)} \right) \\ & + y_{t-h}^C \left(\frac{\bar{\alpha}_2}{h\bar{\alpha}_1\tilde{\kappa}(h)} \beta(h)^{(1-h)} \right) + y_{t+h}^C \left(\frac{\beta(h)\bar{\alpha}_2}{h\bar{\alpha}_1\tilde{\kappa}(h)} \right) = v_t^\pi \\ \Rightarrow & y_t^C \left(1 + \frac{h\bar{\alpha}_1\tilde{\kappa}(h)^2}{\bar{\alpha}_2} + \beta(h)^{(2-h)} \right) - \beta(h)^{(1-h)} y_{t-h}^C \\ & - \beta(h) y_{t+h}^C + \frac{h\bar{\alpha}_1\tilde{\kappa}(h)}{\bar{\alpha}_2} v_t^\pi = 0 \\ \Rightarrow & c_1 y_t^C + c_2 y_{t-h}^C + c_3 y_{t+h}^C + c_0 v_t^\pi = 0 \end{aligned} \quad (3.33)$$

with

$$\begin{aligned} c_0 &= \frac{h\bar{\alpha}_1\tilde{\kappa}(h)}{\bar{\alpha}_2} \\ c_1 &= 1 + \frac{h\bar{\alpha}_1\tilde{\kappa}(h)^2}{\bar{\alpha}_2} + \beta(h)^{(2-h)} \\ c_2 &= -\beta(h)^{(1-h)} \\ c_3 &= -\beta(h). \end{aligned}$$

Based on the method of undetermined coefficients, we guess that the dynamics in y_t^C are described by the following law of motion:

$$y_t^C = \tilde{c}_1 y_{t-h}^C + \tilde{c}_2 v_t^\pi. \quad (3.34)$$

By plugging equation (3.34) into equation (3.33) we are able to compute the solution for \tilde{c}_1 and \tilde{c}_2 :

$$\begin{aligned} & c_1(\tilde{c}_1 y_{t-h}^C + \tilde{c}_2 v_t^\pi) + c_2 y_{t-h}^C + c_3[\tilde{c}_1 y_t^C + \tilde{c}_2(\rho(h)^\pi v_t^\pi + \varepsilon_{t+h}^\pi)] + c_0 v_t^\pi = 0 \\ \Rightarrow & c_1(\tilde{c}_1 y_{t-h}^C + \tilde{c}_2 v_t^\pi) + c_2 y_{t-h}^C + c_3[\tilde{c}_1(\tilde{c}_1 y_{t-h}^C + \tilde{c}_2 v_t^\pi) + \tilde{c}_2 \rho(h)^\pi v_t^\pi] + c_0 v_t^\pi = 0 \\ \Rightarrow & (c_1 \tilde{c}_1 + c_2 + c_3 \tilde{c}_1^2) y_{t-h}^C + [c_1 \tilde{c}_2 + c_3(\tilde{c}_1 \tilde{c}_2 + \tilde{c}_2 \rho(h)^\pi) + c_0] v_t^\pi = 0. \end{aligned} \quad (3.35)$$

Note, again, that in case of a (deterministic) impulse shock $\varepsilon_{t+h}^\pi = 0$ holds. The first term in equation (3.35) displays a characteristic polynomial:

$$c_3 \tilde{c}_1^2 + c_1 \tilde{c}_1 + c_2 = 0 \Leftrightarrow \tilde{c}_1^2 + \frac{c_1}{c_3} \tilde{c}_1 + \frac{c_2}{c_3} = 0 \quad (3.36)$$

with

$$\frac{c1}{c3} = -\frac{1 + [h\bar{\alpha}_1\tilde{\kappa}(h)^2]\bar{\alpha}_2^{-1} + \beta(h)^{(2-h)}}{\beta(h)}$$

$$\frac{c2}{c3} = \frac{1}{\beta(h)^h}.$$

Based on the numerical parameter scenario given, we choose the specific solution of (3.36) which ensures dynamic stability, i.e. $0 < \tilde{c}_1 < 1$ holds.¹⁶ Given the solution of \tilde{c}_1 , we are able to compute the one of \tilde{c}_2 , which follows from the second term in equation (3.35):

$$c_1\tilde{c}_2 + c_3[\tilde{c}_1\tilde{c}_2 + \tilde{c}_2\rho(h)^\pi] + c_0 \leftrightarrow \tilde{c}_2 = -\frac{c_0}{c_1 + [\tilde{c}_1 + \rho(h)^\pi]c_3}. \quad (3.37)$$

The dynamics in the inflation rate under Commitment are now described by (cf. targeting rule (3.10))

$$\pi_t^C = -\frac{1}{c_0} \left[(\tilde{c}_1 - \beta(h)^{(1-h)})y_{t-h}^C + \tilde{c}_2v_t^\pi \right]. \quad (3.38)$$

The laws of motion in x_t^C can be summarized by

$$x_t^C = \Gamma_{C,1}x_{t-h}^C + \Gamma_{C,2}v_t^\pi \quad (3.39)$$

where

$$\Gamma_{C,1} = \begin{pmatrix} \tilde{c}_1 & 0 \\ -\frac{\tilde{c}_1 - \beta(h)^{(1-h)}}{c_0} & 0 \end{pmatrix}$$

$$\Gamma_{C,2} = \begin{pmatrix} \tilde{c}_2 \\ -\frac{\tilde{c}_2}{c_0} \end{pmatrix}$$

with $x_t^C = (y_t^C, \pi_t^C)'$ and $x_{t-h}^C = (y_{t-h}^C, \pi_{t-h}^C)'$ hold. More explicitly, the reduced-form solution for the output gap can be written as (cf. equation (3.34))

$$y_t^C = \frac{\tilde{c}_1^{(t+1)/h} - [\rho(h)^\pi]^{(t+1)/h}}{\tilde{c}_1 - \rho(h)^\pi} \tilde{c}_2 \varepsilon_0^\pi. \quad (3.40)$$

Plugging this law of motion into equation (3.38), after some re-arrangement we get the reduced-form solution for the inflation rate:

$$\pi_t^C = -\frac{\tilde{c} \tilde{c}_1^{t/h} - (\tilde{c} - \tilde{c}_2)[\rho(h)^\pi]^{t/h}}{c_0} \varepsilon_0^\pi \quad (3.41)$$

with

$$\tilde{c} = \frac{(\tilde{c}_1 - \beta(h)^{(1-h)})\tilde{c}_2}{\tilde{c}_1 - \rho(h)^\pi}.$$

The previous expression make, again, use of the forward solution of the shock process for $0 < h \leq 1$ given by

$$E_0 v_t^\pi = [\rho(h)^\pi]^{t/h} \varepsilon_0^\pi. \quad (3.42)$$

¹⁶Obviously, this observation serves as a necessary but not sufficient condition since it depends on the underlying parameter scenario. In our case we obtain numerically that $\tilde{c}_{1,1}$ is the *unstable* root of (3.36), while $\tilde{c}_{1,2}$ it the *stable* one, e.g. $0 < \tilde{c}_{1,2} < 1$ holds. This is true for all values of h over its admissible range. It is also confirmed that when plugging both solutions into the characteristic polynomial above, this results in 0 in both cases. However, in general, analytical solutions for $\tilde{c}_{1,1}$ and $\tilde{c}_{1,2}$ are hard to find since the non-trivial expressions $\frac{c1}{c3}$ and $\frac{c2}{c3}$ are considered to be part of the associated quadratic formula. Therefore, we rely on the numerical computation of the roots but give the strong advise to check on the (stable) solutions for any given parameter scenario. For a clear arrangement in the following we define $\tilde{c}_1 := \tilde{c}_{1,2}$.

3.6.3 B3: Impulse Response Functions (D,C) in the Case $0 < h \leq 1$

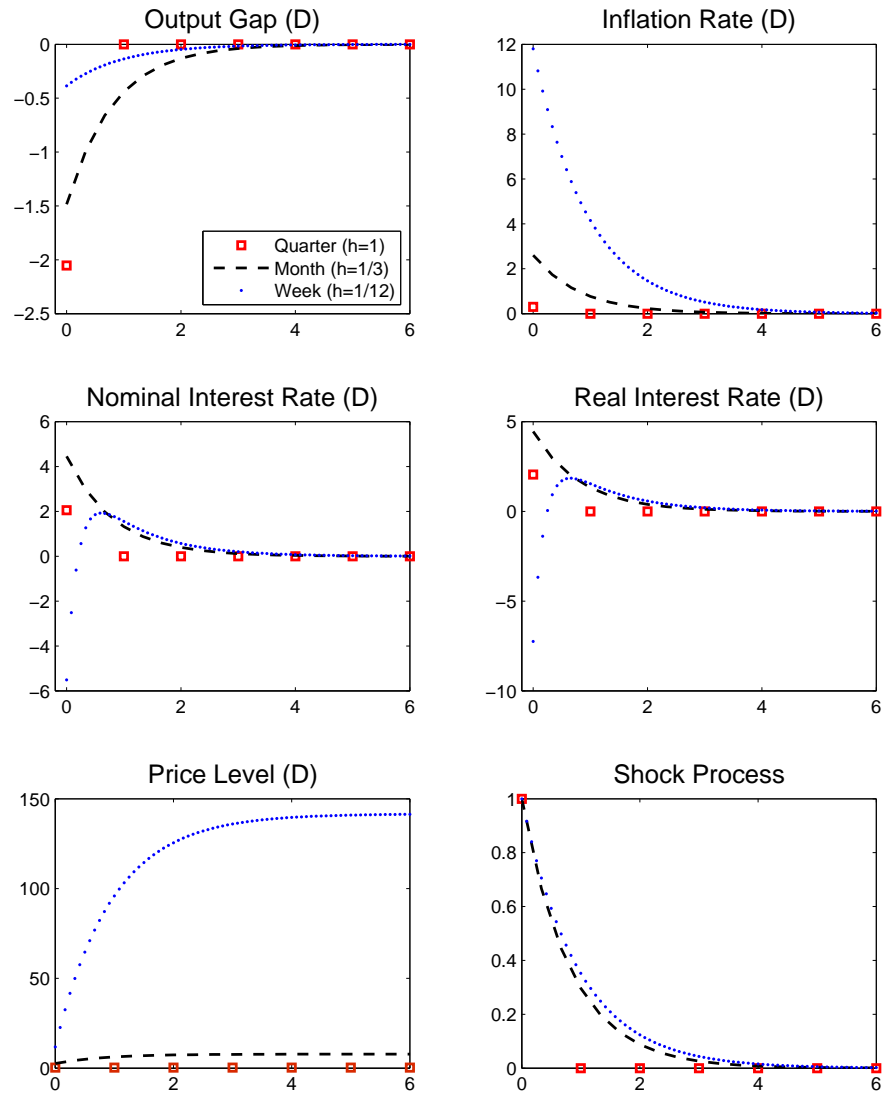


Figure 3.2: IRFs in case of a *non-autocorrelated* cost-push shock in the NKM under the OMP regime Discretion (D).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

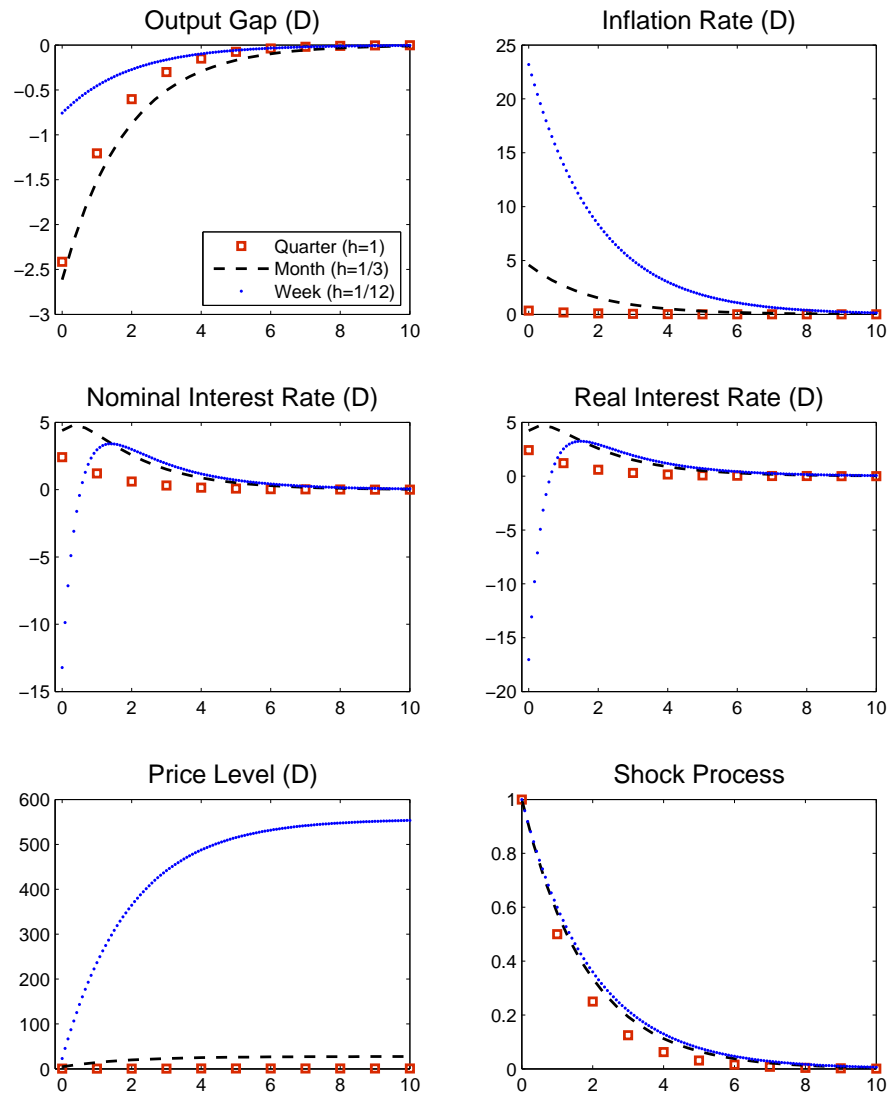


Figure 3.3: IRFs in case of an *autocorrelated* cost-push shock in the NKM under the OMP regime Discretion (D).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

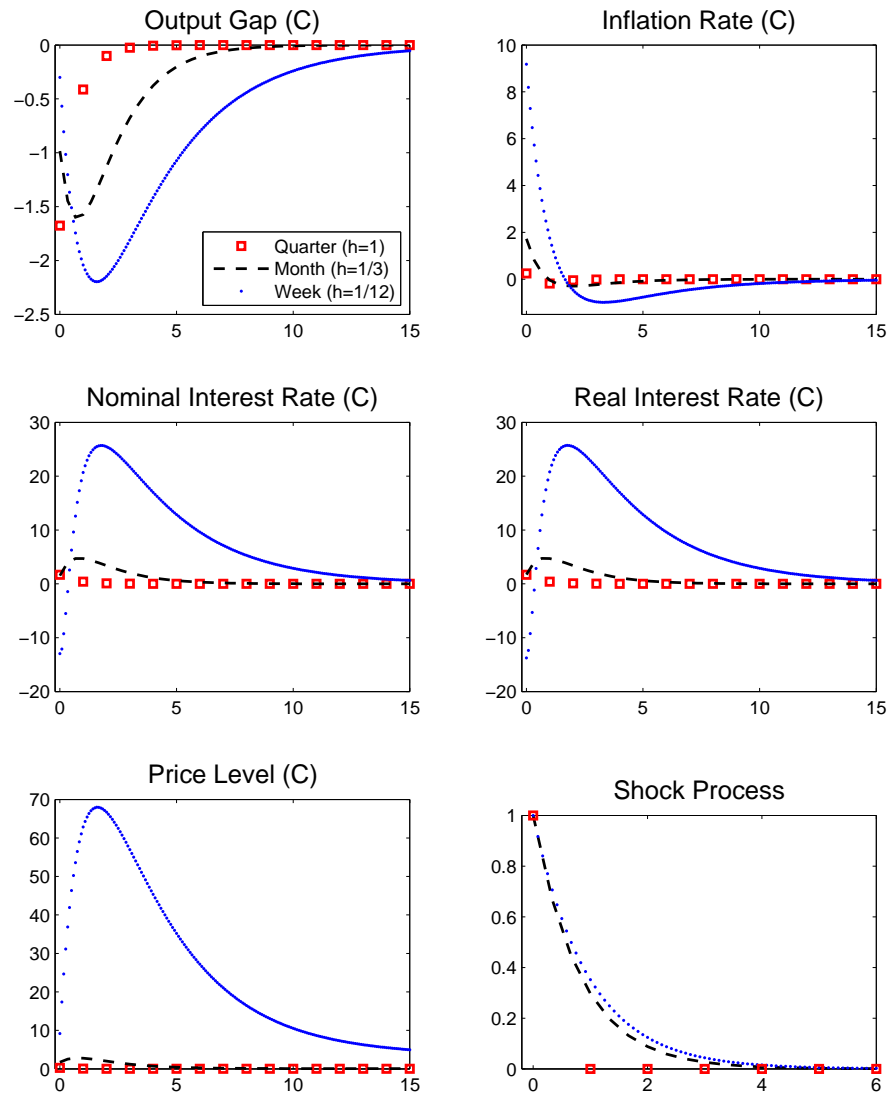


Figure 3.4: IRFs in case of a *non*-autocorrelated cost-push shock in the NKM under the OMP regime Commitment (C).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

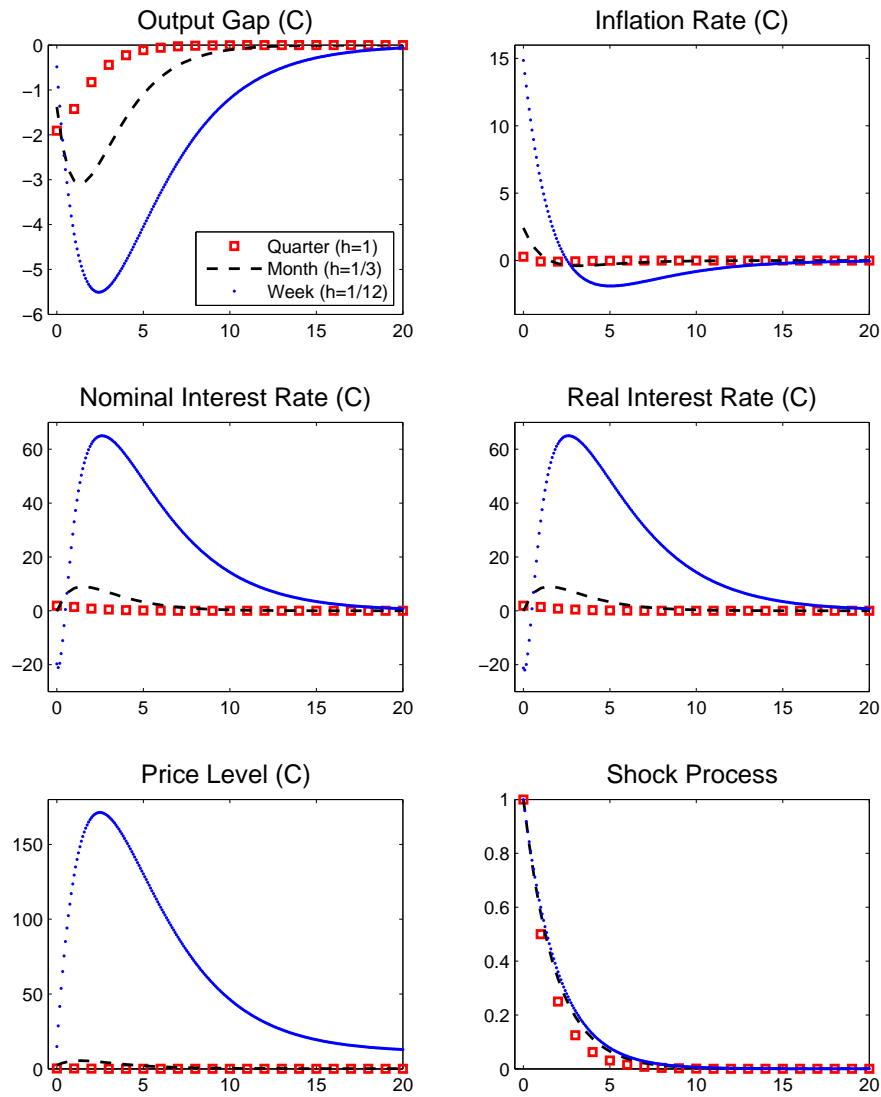


Figure 3.5: IRFs in case of an *autocorrelated* cost-push shock in the NKM under the OMP regime Commitment (C).

Note: The *quarterly* ($h = 1$) and *weekly* ($h = 1/12$) realizations are marked with *squares* and *dots*, respectively, while the *dashed* lines depict the IRFs in *monthly* ($h = 1/3$) magnitudes. The time in quarters and the change in percent(age points) are displayed on the horizontal and vertical axis, respectively.

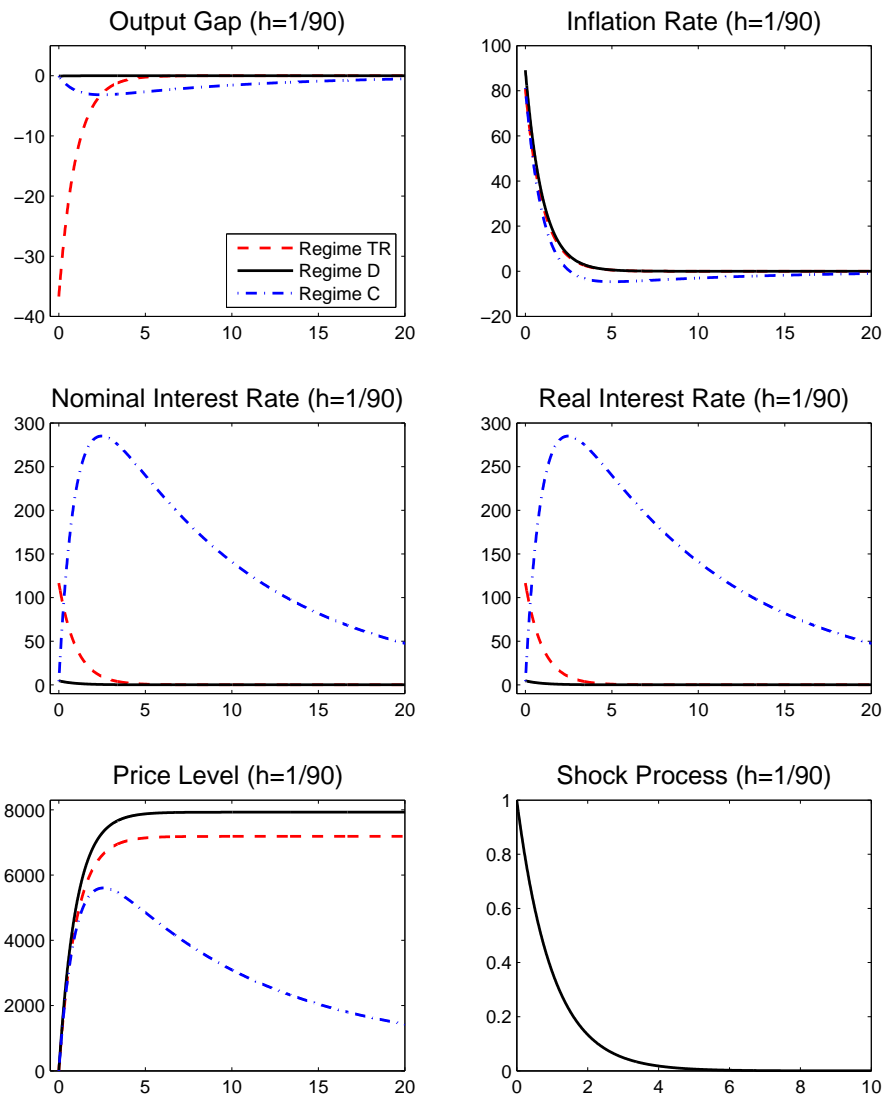


Figure 3.6: *Daily* ($h = 1/90$) IRFs in case of a *non*-autocorrelated cost-push shock in the baseline NKM (TR) and under the OMP regimes Discretion (D) and Commitment (C).

Note: The *dashed* line represents the daily IRFs under the regime TR. The *solid* line represents the daily IRFs under the regime Discretion (D). The *dashed/dotted* line represents the daily IRFs under the regime Commitment (C). The time in quarters and the change in percent (age points) are displayed on the horizontal and vertical axis, respectively.

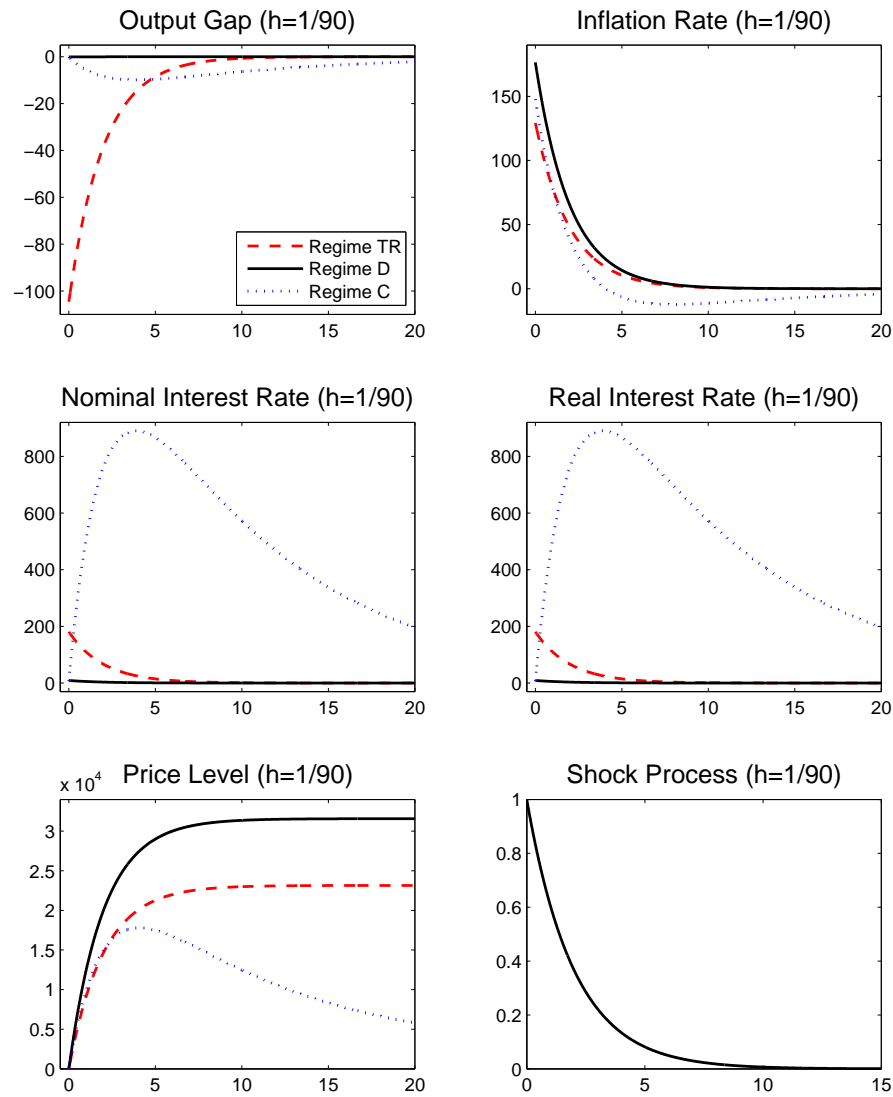


Figure 3.7: *Daily* ($h = 1/90$) IRFs in case of an *autocorrelated* cost-push shock in the baseline NKM (TR) and under the OMP regimes Discretion (D) and Commitment (C).

Note: The *dashed* line represents the daily IRFs under the regime TR. The *solid* line represents the daily IRFs under the regime Discretion (D). The *dashed/dotted* line represents the daily IRFs under the regime Commitment (C). The time in quarters and the change in percent (age points) are displayed on the horizontal and vertical axis, respectively.

3.6.4 B4: Development in the Loss Relations D/C

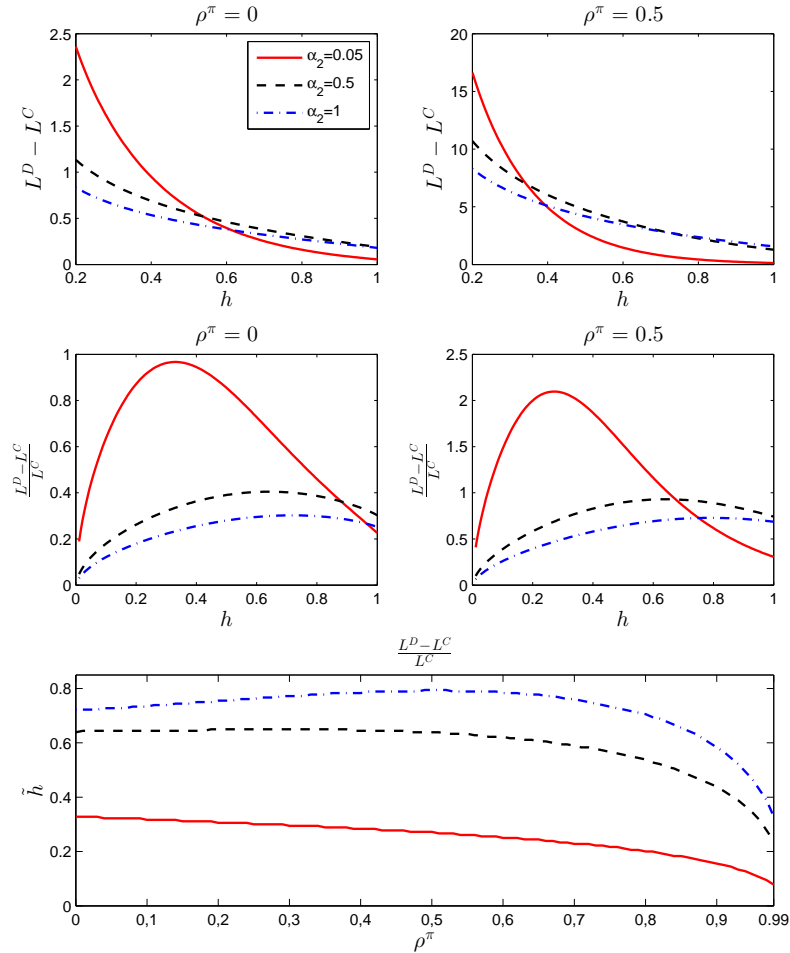


Figure 3.8: Development and maximum of the loss relation(s)
Discretion(D)/Commitment(C).

Note: The *upper* and *middle* panels depict the development of the absolute and relative *loss relations* Discretion (D)/Commitment (C) with respect to h and under variation of $\bar{\alpha}_2$ in the case of a non-autocorrelated ($\rho^\pi = 0$; left panels) and an autocorrelated ($\rho^\pi = 0.5$; right panels) cost-push shock. L^D and L^C denote the loss for the OMP regimes Discretion and Commitment, respectively. The *lower* panel depicts the value of h associated with the maximum in the relative loss relation Discretion/Commitment (denoted as \tilde{h}) under variation of $\bar{\alpha}_2$ and ρ^π . The *solid* line represents the loss relations/changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.05$. The *dashed* line represents the loss relations/changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.5$. The *dashed/dotted* line represents the loss relations/changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 1$.

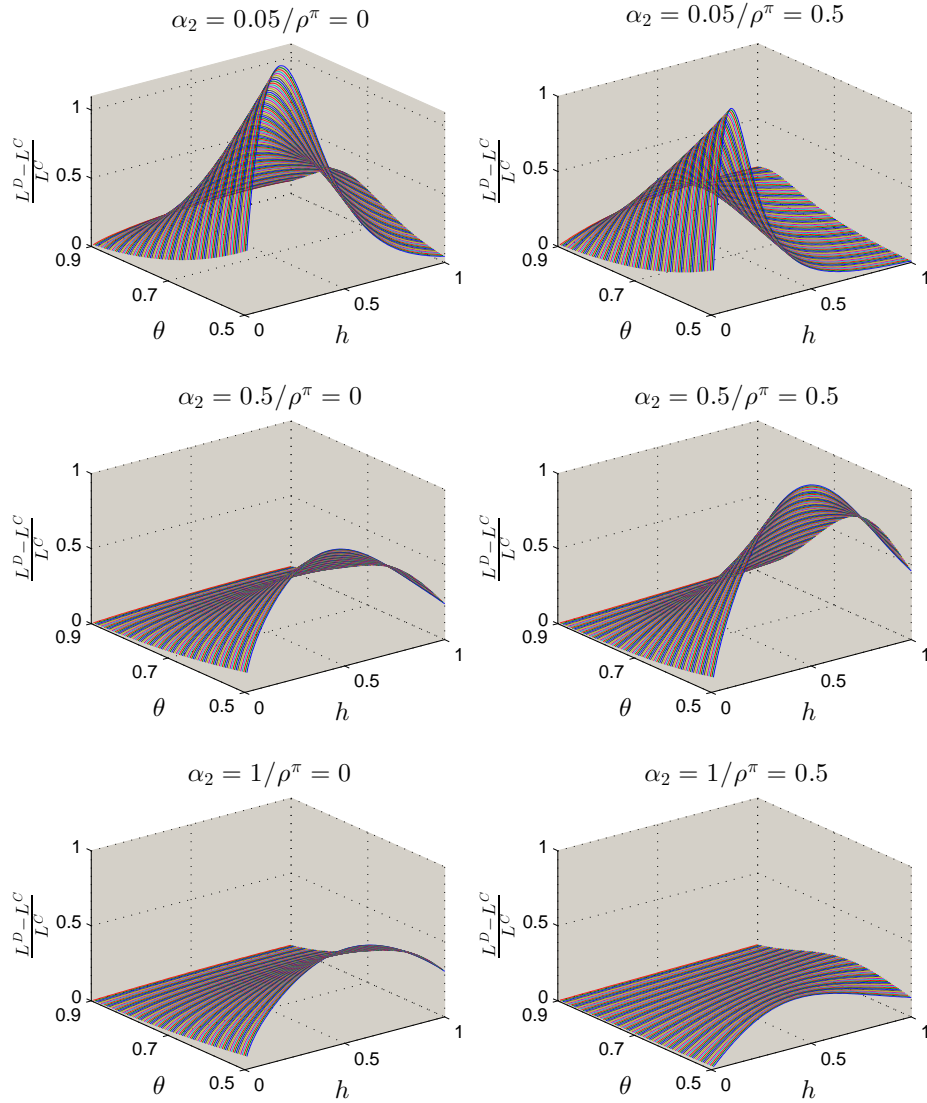


Figure 3.9: Development of the (relative) loss relation Discretion (D)/Commitment (C) in the case of an (*non*)-autocorrelated cost-push shock under variation of θ .

Note: The Figure depicts the development of the relative loss relation Discretion (D)/Commitment (C) with respect to h and under variation of $\bar{\alpha}_2$ and θ in the case of a non-autocorrelated ($\rho^\pi = 0$; left panels) and an autocorrelated ($\rho^\pi = 0.5$; right panels) cost-push shock. L^D and L^C denote the loss for the OMP regimes Discretion and Commitment, respectively. The *upper panels* represent the loss relations for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.05$. The *middle panels* represent the loss relations for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.5$. The *lower panels* represent the loss relations for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 1$.

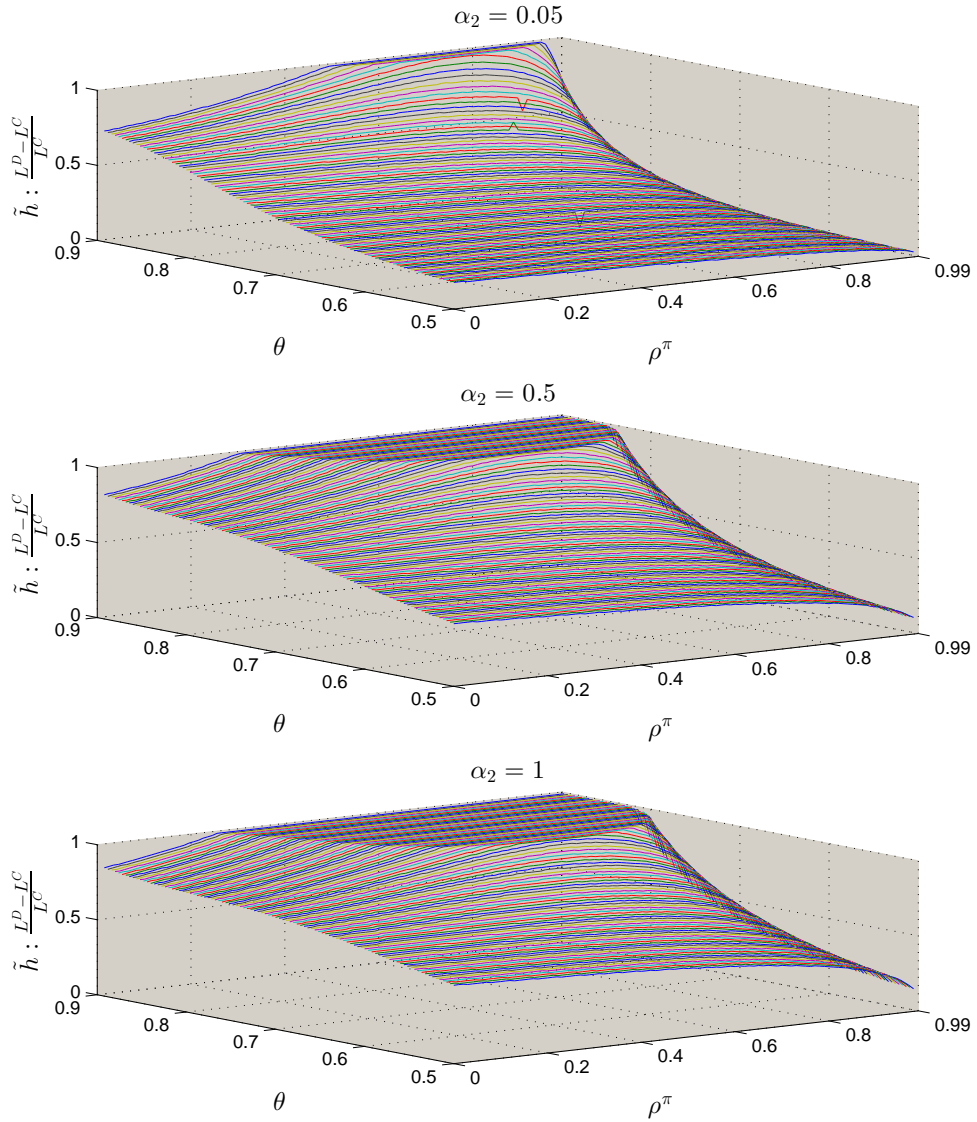


Figure 3.10: Value of h associated with the maximum in the loss relation Discretion (D)/Commitment (C) under variation of $\bar{\alpha}_2$, θ and ρ^π .

Note: The Figure depicts the value of h associated with the maximum in the loss relation (\tilde{h}) Discretion (D)/Commitment (C) under variation of $\bar{\alpha}_2$, θ and ρ^π . L^D and L^C denotes the loss for the OMP regimes Discretion and Commitment, respectively. The *upper panel* represents the changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.05$. The *middle panel* represents the changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 0.5$. The *lower panel* represents the changes in \tilde{h} for the monetary policy reaction to the output gap for $\bar{\alpha}_2 = 1$.

4 Estimation of a High-Frequency New-Keynesian Phillips Curve

This paper has been published in *Empirical Economics*, Vol. 46(2), pp. 607-628, Springer.
Co-author of this paper is Steffen Ahrens. It can be downloaded via <http://link.springer.com/article/10.1007/s00181-013-0684-7> (DOI: 10.1007/s00181-013-0684-7).

Part II: Moment-Matching Estimation of the Baseline New-Keynesian Model

5 Moment-Matching versus Bayesian Estimation: Backward-Looking Behaviour in the Baseline New-Keynesian Model

5.1 Introduction

As mentioned throughout this thesis so far, the New-Keynesian modelling of DSGE with its nominal rigidities and incomplete markets is still the ruling paradigm in contemporary macroeconomics. The fundamental three-equations versions for output, inflation and the interest rate, as a point of departure, are most valuable in shaping the theoretical discussion on monetary policy and other topics. Over the last decade these models have also been extensively subjected to econometric investigations, where system estimations (as opposed to single-equations estimations) gained in importance. First maximum likelihood and more recently Bayesian estimations crystallized as the most popular methods and by now have become so dominant that other techniques are at risk of eking out a marginal existence.

In this Chapter we introduce an alternative estimation approach, which can be sorted into minimum distance procedures and here, in particular, the Method of Moments; where we in the following also refer to as *Moment-Matching* (MM). This technique concentrates on a number of statistics, also called moments, that summarize salient features of the dynamic systems. MM seeks to identify numerical parameter values such that, as measured by a suitable loss function, these model-generated moments come as close as possible to their empirical counterparts. Besides possibly the mean values of some of the variables, the moments that are most often referred to are either IRFs or autocovariance functions (of vectors of variables), which convey similar information if the same shocks are underlying.

It may now be supposed that MM and likelihood methods do not necessarily stand in marked contrast but, with the autocovariances as moments, even amount to much the same thing. To quote Ríos-Rull et al. (2011, p. 18): “The likelihood function . . . peaks near parameter values for which the model-implied autocovariance function of the observables matches the sample autocovariance function as closely as possible in terms of a statistical metric. It does so by forcing each shock in the model to contribute particular autocovariance features, which in total have to mimic the sample autocovariances.”¹

The statement is based on the fact that the Gaussian likelihood function of a state space model has a linear decomposition in the frequency domain. The latter involves the spectral density matrices over the Fourier frequencies of the empirical and model-generated time series, respectively, which, in essence, simply repackage the autocovariances by using sine and cosine functions as weights. Thus, the parameter estimates may be said to be “ulti-

¹A similar characterization is given by Schorfheide (2008, pp. 398 and 402).

mately determined by the implicit weighting of the discrepancy between sample and DSGE model implied autocovariance functions . . . encoded in the likelihood function” (Ríos-Rull et al. (2011, p. 20); although the authors make no explicit reference to the connection just mentioned).²

In practice, however, the correspondence between MM and the Gaussian likelihood is not likely to be a perfect one. First, the theoretical connection between the time-domain and frequency-domain decomposition of the Gaussian density function is an asymptotic result, while the empirical macroeconomic time series are typically rather short. Second, the data-dependent weighting scheme in the MM loss function for the autocovariances can be somewhat different from the implicit, likelihood-implied weighting scheme.³

A third issue is that in the aforementioned correspondence the Gaussian likelihood includes all of the Fourier frequencies and accordingly relies on all of the autocovariances up to their maximal lag. By contrast, MM estimations are only based on a limited number of lags for their autocovariances.

The choice of the moments and its limitation is actually a point that critics of MM brand as arbitrary. In this respect the philosophy of MM may be taken into account. MM is deliberately a limited-information method, grounded in the insight that structural modelling can at best succeed in matching *some* of the ‘stylized facts’ of an actual economy. MM, then, requires the researcher to make up his or her mind about the dimensions along which the model should be most realistic, possibly at the cost of neglecting other aspects. It is just this explicitness and furthermore the easy interpretation of the matching criterion that, from an economic point of view, can be considered to be strong arguments in favour of MM. This begins informally with an inspection of diagrams that compare the profiles of the theoretical to the empirical autocovariances, but there are also more formal methods to assess a model’s goodness of fit. In addition, by looking at the single moments more detailed information becomes available and, beyond a statement of a model’s overall performance, one can learn something about its particular merits and demerits without much technical effort. This is sufficient justification for using MM as a method in its own right, apart from its generally higher robustness if a model is misspecified.⁴

Realizing that ideally likelihood estimations and covariance matching lead to similar results but that practically, with the short data series at hand, this relationship will be more or less distorted, in this Chapter we set out for a closer investigation into this question. To this end, we consider the quarterly baseline NKM, concentrate on a finite set of the covariances of its three key variables, and contrast Bayesian estimations on US data with the results from the MM estimations. As far as we know, such a direct comparison has not been under-

²Two papers that, after mentioning the connection, employ the frequency-domain decomposition of the Gaussian density function for their estimations are Christiano and Vigfusson (2003) and Sala (2011). Incidentally, the role of the autocovariances is more clearly seen in these papers if one compares their specification of the sample periodogram with the expressions given in, e.g., Hamilton (1994, p. 158 for the univariate case, and Section 10.4 for the multivariate case). In case the likelihood is computed with the aid of the Kalman filter, the latter also provides a general (though rather involved) algorithm for factoring the autocovariance-generating function for the observed variables (see Hamilton (1994, pp. 392ff).)

³In particular, this may be the case when owing to singularity problems MM cannot employ an asymptotically optimal weighting matrix but instead resorts to a diagonal matrix.

⁴The selection of the moments is to some degree comparable to the Bayesian likelihood estimations in the frequency domain by Sala (2011), when some frequencies are eliminated from the density function; for example, the low or/and the high frequencies. Thus, similar to Wen (1998), Sala’s objective function could be viewed as a transformation of the Method of Moments from the time domain to the frequency domain. Interestingly, Sala also discusses the autocovariances and autocorrelations of the model as they are implied by his estimations over different frequency bands.

taken before. It will be worth trying since, to anticipate our main finding, the two types of estimation do seem to focus on rather different features of the data. Noting this, we do *not* want to imply that one method is superior to the other.

Specifically, the direct matching of the covariance profiles is entirely satisfactory, which is remarkable in itself, whereas the moments implied by the Bayesian parameters show some deficiencies. Moreover, some of the parameter estimates are quite distinct, especially the ones in the NKPC that are primarily responsible for inflation persistence, namely, the degree of backward-looking behaviour versus the serial correlation in the shock process.

The remainder of this Chapter is organized as follows. The next section introduces the MM estimation procedure together with a sketch of the bootstrap re-estimations that we will make use of. In section 5.3 we explain the Bayesian approach in a greater detail. In section 5.4 we provide a critical discussion on the validity of the MM approach with respect to sufficiency and transparency. Section 5.5 describes the baseline NKM (known from the previous Chapters in) gap notation under investigation. Note that we consider the specification of the model in *quarterly* magnitudes only. The estimations themselves are presented in Sections 5.6 and 5.7, where Section 5.6 deals with the period of the so-called Great Inflation and Section 5.7 with the Great Moderation. Section 5.6 constitutes the main part of this chapter and is therefore subdivided into several subsections. After contrasting the Bayesian with the MM estimation in Section 5.6.1, the next subsection examines in greater detail the problem of disentangling the endogenous and exogenous sources of inflation persistence. In Section 5.6.3 we temporarily step outside the model and ask if a still higher (composite) coefficient on lagged inflation would outperform the previous matching. Back in the original framework, Section 5.6.4 seeks to identify the central moments for the results and Section 5.6.5 is concerned with an evaluation of the estimated parameters. The organization of Section 5.7 is similar but the presentation can be much shorter here. In section 5.7.1 we provide a discussion of both estimation procedures regarding reliability of the estimation results presented. Section 5.8 concludes. Details regarding the priors of our Bayesian estimations are relegated to the corresponding Appendix D. Since we focus on the introductory presentation of the moment-matching approach, a more technical analysis — especially with respect to a detailed description of a model comparison exercise applying later in this Chapter — has been undertaken in Franke et al. (2012). Further references to this paper will be made throughout the Chapter.

5.2 The Moment-Matching Estimation Approach

As indicated in the Introduction, the MM estimation procedure computes a number of summary statistics for a model, i.e. moments, and searches for a set of parameter values that minimize a distance between them and their empirical counterparts. The method has also been applied to NKMs. The major part of this work is concerned with the matching of the corresponding IRFs via the MM approach, where almost all of these contributions consider the responses to a monetary policy shock as the only impulse.⁵ An exception is Altig et al. (2011), who add two technology shocks to it. As an estimation, this treatment is somewhat special since a good matching of one type of IRF does not necessarily imply a similar good match of another type. In this respect our situation will be different in that we deal with an elementary NKM that has been subjected to a Bayesian estimation before. So

⁵Besides the early contribution by Rotemberg and Woodford (1997), examples from the last few years are Christiano et al. (2005), Boivin and Giannoni (2006), Henzel et al. (2009), Hülsewig et al. (2009). In contrast, Avouyi-Dovi and Matheron (2007) study the responses to a technology shock.

the model has already as many shock processes prespecified as there are observable variables, namely three: output, inflation and the interest rate (in gap form). This allows us to consider a broader range of dynamic properties, which are conveniently summarized by the second moments of the variables up to a certain maximal lag. Incidentally, these covariances contain similar information to the IRFs of the three variables of the model.

Such a choice of moments has been more customary for the (S)MM estimation of, in a wider sense, real business cycle models (the ‘S’ refers to the cases where the covariances cannot be computed analytically but must be simulated; see also our contribution in Chapter 7).⁶ Closest to our work is the MM estimation of a NKM by Matheron and Poilly (2009). Their model is, however, richer than ours and instead of the output gap as a level variable they are interested in the comovements of the output growth rate. Hence one would have to be careful with a comparison of their results and ours.⁷

It may be emphasized that we fix our moments in advance and their number will not be too small, either. This commitment is different from an explicit moment selection procedure as it was, for example, used by Karamé et al. (2008). They begin with a large set of moments, estimate their model on them, and then step by step discard the moments which the model reproduces most poorly until an over-identification test fails to reject the model any longer.

Given that we deal with quarterly US data and that the business cycles in the US take between five and ten years (roughly), the covariances underlying the estimations should not use too long a lag horizon. A maximal lag of two years appears to be a reasonable compromise. Denoting the output gap and the two gaps of inflation and the interest rate in quarter t by x_t , $\hat{\pi}_t$ and \hat{r}_t , respectively, our moments will be $\text{Cov}(p_t, q_{t-h})$ for $p, q = \{x, \hat{\pi}, \hat{r}\}$ and lag length $h = 0, 1, \dots, 8$.⁸ Hence there is a total of $n_m = 78$ moments to match: 9 covariance profiles with (1+8) lags, minus 3 moments to avoid double counting the zero lags in the cross relationships. Generally, these moments are collected in a column vector $m \in \mathbb{R}^{n_m}$.

Regarding the model, let there be n_θ parameters to be estimated, that is, a vector $\theta \in \Theta \subset \mathbb{R}^{n_\theta}$, where Θ is the admissible parameter set. Since the reduced-form solution of a log-linearized DSGE model takes the form of a vector autoregression, the expected values of the covariances of its variables need not be simulated but can be computed analytically.⁹ This saves us from the problem of sample variability. Extracting from the resulting matrix expressions the covariances that we require, we make their dependence on the particular values of θ explicit by writing $m = m(\theta)$ for them. On the other hand, let T be the length of the sample period and $m_T^{emp} \in \mathbb{R}^{n_m}$ the vector of these empirical moments.

The distance between the vectors of the model-generated and sample moments is measured by a quadratic loss function that is characterized by an $(n_m \times n_m)$ weighting matrix W . Accordingly, the model is estimated by the set of parameters $\hat{\theta}$ that minimize this distance

⁶These applications seem rather scattered, and they typically use only first- or second-order covariances; see Jonsson and Klein (1996), Hairault et al. (1997), Collard et al. (2002) and, more recently, Karamé et al. (2008), Gorodnichenko and Ng (2010), Kim and Ruge-Murcia (2011), Ruge-Murcia (2012), Ambler et al. (2012).

⁷Another difference is that they do not match the empirical second moments directly (which we do) but instead they apply the Indirect Inference (II) approach by deriving the moments from the estimation of a canonical VAR.

⁸The latter h has not to be confused with the period length as defined throughout Part One in this thesis.

⁹See, e.g., Lütkepohl (2007, pp. 26f). Franke et al. (2012) contains a self-contained description of the entire procedure in the present case.

over the admissible set Θ :

$$\hat{\theta} = \arg \min_{\theta \in \Theta} J(\theta; m_T^{emp}, W) := \arg \min_{\theta \in \Theta} [m(\theta) - m_T^{emp}]' W [m(\theta) - m_T^{emp}] \quad (5.1)$$

Solving the minimization problem is not a straightforward matter.¹⁰ Given the relatively high number of parameters that we will have to estimate, there is for functions of the present type (just as it is typically the case for likelihood functions) a great danger of multiple local extrema, possibly also located at a farther distance from one another. Our search therefore proceeds in two steps. First, in order to reduce the risk of being trapped in a wrong region of the parameter space, simulated annealing is used as a globally effective procedure.¹¹

After several repeated attempts we can thus identify a suitable region in the parameter space to concentrate on. Here a locally more effective procedure can be put into operation, for which we employ the Nelder-Mead simplex algorithm (see Press et al. (1986, pp. 289–293); it has broader scope than gradient methods to escape from small local valleys). Actually, the algorithm is repeatedly restarted upon convergence until no more noteworthy improvement in the minimization occurs. Combining the two search strategies, we can be rather confident that for all practical purposes the global minimum of (5.1) has indeed been found.

We still have to specify the weighting matrix W in (5.1). As it is usually done when the moments are highly dependent, it is set up as the diagonal matrix with the reciprocals of the sample variances of the single moments on its main diagonal. Obviously, the less precisely a moment is estimated from the data, that is, the higher is its variance, the lower is the weight attached to it in the loss function. Since these variances are the basis for the confidence intervals of the moments, it may be stated that the model-generated moments $m_i(\hat{\theta})$ obtained from the estimated parameters lie “as much as possible inside these confidence intervals” (Christiano et al. (2005, p. 17)). Nevertheless, a formulation of this kind, which with almost the same words can also be found in several other applications, should not be interpreted too narrowly. In particular, a minimum of the loss function in (5.1) need not automatically minimize the number of moments outside the confidence intervals.

It is well-known that under standard regularity conditions the parameter estimates $\hat{\theta}$ are consistent and asymptotically follow a normal distribution around the (pseudo-) true parameter vector θ^o .¹² One problem with this proposition is that the small-sample distribution of the estimates is not always well approximated by the asymptotic normal distribution (Ruge-Murcia (2012) provides evidence for this). Even more serious is our finding that the

¹⁰Note that the expression $\arg \min$ denotes the ‘argument of the minimum’. Equivalently, we get

$$J(\hat{\theta}; m_T^{emp}, W) = \min_{\theta \in \Theta} J(\theta; m_T^{emp}, W). \quad (5.2)$$

Hence, given the admissible set Θ , $\arg \min$ of J becomes the set of parameters $\hat{\theta}$, which leads to the minimum of the function J .

¹¹In detail, we apply the algorithm put forward by Corona et al. (1987) and essentially use the same ‘tuning parameters’ as Goffe et al. (1994) and Goffe (1996). The most critical of these parameters are the reduction factor r_T and the initial temperature T_o . We set $r_T = 0.75$, a conservative value “which is suitable for a function one has little experience with” (Goffe, 1996, p. 172). T_o is obtained endogenously. First the median loss M of 500 widely dispersed parameter vectors is computed. Requiring that the algorithm’s (desirable) probability of accepting an increase in the loss is about 0.50 — at temperature T_o and at the same step sizes that in the initial procedure have scaled the changes in the single parameter values — subsequently Boltzmann’s formula $\exp(-M/T_o) = 0.50$ is solved for T_o .

¹²According to Lee and Ingram (1991, pp. 200), these regularity conditions imply that the consistency of the parameter estimates is fulfilled if the simulated and empirical time series are independent of each other and stationary. Furthermore, the structural model has to be identified and the estimated weighting matrix must converge in probability to its true value (see also Hansen (1982)).

parameter vector in the central estimations will be a corner solution of (5.1), which destroys the basis for the asymptotic theory if we do not simply wish to escape the problem by exogenously fixing the critical parameters.

For this reason we make use of a (parametric) bootstrap procedure to construct confidence intervals for the estimated parameters.¹³ It is based on the null hypothesis that the estimated model is the true data generating process. Thus, we take the estimated parameters $\hat{\theta}$ and, starting from the steady state (i.e. the zero vector), run a stochastic simulation of the model over $500 + T$ periods, from which the first 500 periods are discarded to rule out any transient effects. The underlying random number sequence may be identified by an integer index b . Repeating this a great number of times B , with different random number seeds of course, $b = 1, \dots, B$ artificial time series of length T are obtained. For each of them we compute the vector of the resulting moments, denoted as m_T^b , use their variances to set up the sample-specific diagonal weighting matrix W^b , and then minimize the function $J(\theta; m_T^b, W^b)$ over the parameter space Θ . In this way a frequency distribution of re-estimated parameters

$$\{ \hat{\theta}^b : b = 1, \dots, B \} \quad (5.3)$$

is obtained. (5.3) can serve as a proxy for the probability distribution of the model's parameter estimates, from which it is easy to establish, say, 95% confidence intervals for the i -th component of the originally estimated vector $\hat{\theta}$. In finer detail, two methods can be employed for this, the standard percentile interval and Hall's (1992) percentile confidence interval. Hall's method has the advantage that it is asymptotically correct, but it may violate the admissible range of a parameter if it is estimated at or near a boundary value. Therefore we will use Hall's interval if no such violation occurs and the standard interval otherwise.¹⁴

The bootstrap re-estimation experiment also gives us some information about the plausibility of the model. We only have to consider the frequency distribution of the values of the loss function,

$$J^b = J(\hat{\theta}^b; m_T^b, W^b), \quad b = 1, \dots, B \quad (5.4)$$

and compare, let us say, the 95% quantile $J_{0.95}$ of (5.4) to the value $\hat{J} := J(\hat{\theta}; m_T^{emp}, W)$ that was obtained from the original estimation on the empirical moments in (5.1). At the conventional 5% significance level the model would have to be rejected as being inconsistent with the data if \hat{J} exceeds $J_{0.95}$, otherwise it would have passed the test. In this manner we can also readily construct a p -value of the model. It is given by the value of p that equates the $(1-p)$ -quantile of the distribution $\{J^b\}$ to \hat{J} , which says that if \hat{J} were employed as a benchmark for model rejection, then p is the error rate of falsely rejecting the null hypothesis that the model is true. Hence, in short, the higher this p -value the better the fit will be.

It goes without saying that these statements are conditional on the special choice of the moments that the model is required to match. Certainly, if more data were available or more moments were added to our list, the p -value would dwindle.

5.3 The Bayesian Approach

It is our aim in this Chapter to compare the empirical outcome (in terms of the parameter estimates) of the MM approach relative to the one when the *Bayesian estimation* technique

¹³Others might prefer to call this a 'Monte Carlo' procedure.

¹⁴The details are also spelled out in Appendix A2 in Franke et al. (2012).

is applied. Over the last decade Bayesian estimation became the most popular method for the statistical evaluation of (structural) parameters in DSGE models, replacing the pure maximum likelihood method and the GMM estimation method. According to Griffoli (2011, p. 80) Bayesian estimation is "somewhere in between calibration and maximum likelihood estimation". This can be explained by the fact that within the Bayesian framework the likelihood function is reweighed by a prior density. The latter captures the pre-sample beliefs about an a priori distribution of the parameter values, which are going to be estimated (Fernández-Villaverde (2010, p. 9)). Since it imposes restrictions on the starting values for estimation, this step can be seen as traditional calibration technique. The prior information is then used to reweigh the likelihood function in order to add information that is not contained in the sample (An and Schorfheide (2007, p. 125)). Analytically these relationships are reflected in the well-known Bayes theorem. It describes the posterior distribution of the model parameters of interest as an outcome of the corresponding prior distribution and the likelihood of the model given the data:

$$p(\theta|Y_t) = \frac{p(Y_t|\theta)p(\theta)}{p(Y_t)} \quad (5.5)$$

where $p(\theta|Y_t)$, $p(Y_t|\theta)$, $p(\theta)$, $p(Y_t)$, describe the posterior distribution conditional on the data Y_t , the probability of the data given the parameter vector θ , the prior distribution associated with θ and the marginal likelihood of the data conditional on the structural (economic) model, respectively. The latter, $p(Y_t)$, is computed via the integral of the nominator in the previous equation with respect to θ :

$$p(Y_t) = \int_{\Theta} p(\theta; Y_t) d\theta \quad (5.6)$$

where Θ denotes the whole parameter space. Since $p(Y_t)$ is always constant, it can be discarded within the identification of $p(\theta|Y_t)$. However, as we will see later the marginal likelihood is a measurement for the goodness of fit of the model to the data. Since $p(Y_t|\theta)$ is nothing else than the likelihood of the data we may write:

$$p(\theta|Y_t) \propto L(\theta|Y_t)p(\theta) \quad (5.7)$$

i.e. the posterior distribution is proportional to the product of the likelihood and the prior information. It follows that $L(\theta|Y_t)p(\theta) \equiv p(Y_t|\theta)$ holds since the functional form of $p(\theta)$ — which consists on the mean values, standard deviations and distributions of the parameters a priori — differs conditional on the information considered by the researcher. It follows from (5.7) that the likelihood function must be evaluated first in order to compute the posterior distribution of the parameters of interest. The Bayesian estimation techniques is based on the following steps (see Canova (2007, pp. 441) for a more detailed description):

- (1) *Given the (log-)linear approximation of the model, its reduced-form — which describes (in general) the relationship between observed (control) variables and predetermined (state) variables — must be computed.*

Appropriate solution methods are e.g. the analytical-based Jordan, the (Generalized) Schur decomposition method (in case of hybrid specifications, i.e. lead and lags in the model equations) or the numerical-based brute force iteration method. For an overview of a variety of solution methods to multi-variate difference equation models see Anderson (2008). The linearization of the model equations around their steady states (by applying the well-known log-linearization technique or the total differential) is required in order to analyze its state-space representation with appropriate filter techniques like the Kalman filter (An and Schorfheide (2007, p. 120)). In general, there exist different filter techniques in order to compute the likelihood in respect to the (non-)linearity of the model (see point (3)). If necessary, the control variables must be detrended in order to ensure the stationarity of the time series. The corresponding law of motions given by the reduced-form of the model are linked to the observables taken from the data via a specification of the associated measurement equations.

- (2) *Specification of the prior information $p(\theta)$ regarding the structural parameters θ , i.e. their first and second moments as well as their distribution.*

The choice of the prior information reflects beliefs about the true parameter values. Fernández-Villaverde (2010, p. 10) states that the judgement must be grounded on a plausible explanation regarding the underlying theoretical perception. In the majority of studies concerned with Bayesian estimation, prior information is taken directly from the literature based on micro evidence. Nevertheless, due to the subjective assessment which kind of prior knowledge to be used, this can lead to different outcome. See point (7) and section 6.2.2 for an intensive discussion of this issue.

- (3) *Evaluation of the joint likelihood $L(\theta|Y_t)$ of the observable endogenous variables via appropriate filter techniques like the Kalman or Particle filter. The latter is required if the model is non-linear and/or if the associated shocks are not normal distributed.*

It is well known that the evaluation of the likelihood function is not a trivial task. While the MM approach presented here just matches specific moments of limited lags imposed by the researcher, within the maximum likelihood approach all moments in the frequency domain are considered. However, both estimation procedures have the drawback that the corresponding methods used to minimize the loss function (via e.g. the Nelder-Mead simplex algorithm in the MM case) or to maximize the likelihood function may get stuck in local maxima and minima.¹⁵ This statement holds for a variety of optimization methods like when e.g. the method of simulated annealing is applied (Fernández-Villaverde (2010, p. 7)). One might argue that this problem is more severe in the Bayesian (or plain maximum likelihood) case due to the screening of the whole frequency domain as stated in the introduction. In this case no structural information given by the data like the second moments of a time series is taken into account (to achieve a higher degree of transparency; cf. section 5.4) explicitly. If a model turns out to be linear, it is appropriate to apply the Kalman filter only, in order to evaluate the likelihood function within the Bayesian approach. In case of non-linearities the so-called Particle filter must be considered, which requires a high

¹⁵Note that the Nelder-Mead simplex algorithm can be applied explicitly on non-linear optimization problems (see again Press et al. (1986, pp. 289–293) for a description of this approach).

degree of computational power. See Chapter 6 of this thesis for a less time consuming estimation of a non-linear DSGE model via MM. For a detailed description of the Kalman and Particle filter techniques we refer to DeJong and Dave (2008, their Chapter 11) and Canova (2007, pp. 214).

- (4) *Computation of the posterior mode after evaluation of the likelihood and exploration of the posterior distribution by applying Monte-Carlo Markov-Chain methods.*

Given the value of the likelihood, the proportionality (5.7) in logarithmic terms is maximized in order to calculate the posterior distribution. By maximization of $L(\theta|Y_t)p(\theta)$ the posterior mode $\hat{\theta}$ is explored. Given the latter, the approximated standard errors are computed under consideration of the inverse Hessian, i.e. the matrix of second derivatives evaluated at the posterior mode. In the next step the most common used Random-Walk Metropolis-Hastings Algorithm is applied: once we have the posterior mode we draw candidates (or proposals) θ^* from a random-walk proposal density over a finite step of iterations i within a Markov chain.¹⁶ Then we compare θ_i^* with θ_{i-1}^* (where $\hat{\theta} = \theta_0^*$ is the starting value) in order to check if the current candidate leads to an increase in the posterior distribution. If this is the case we accept θ_i^* with probability of one. Otherwise we accept the same candidate with less than one if no increase is observed.¹⁷ Hence, getting stuck in local maxima becomes a rare event since although the algorithm always leans towards the higher regions of the posterior distribution, it put also a little emphasis on the lower ones (Fernández-Villaverde (2010, p. 21)). Applying the algorithm will result in the posterior distribution based on the realizations generated through the draws of the parameters. An and Schorfheide (2007, p. 132) state that under standard regularity conditions $p(\theta|Y_t)$ will be asymptotically normal, i.e. a Gaussian approximation around the posterior mode will be constructed. In addition, test procedures developed by Geweke (1999) are provided in order to investigate if the Markov chain generated by the Metropolis-Hastings algorithm converge to the (true) posterior distribution. For all estimations in this study convergence is confirmed, although we do not report the results of these tests in connection to the Bayesian estimations applied here. This can be checked easily by looking at the properties of the posterior distribution, which indicates that the posterior means differ from the prior ones and that there exist narrow posterior standard deviations (see Appendix E2 regarding our estimation results presented in Chapter 6).

- (5) *Computation of the marginal likelihood (here denoted henceforth as Υ) for exploring the goodness of fit of the model to the data.*

As we can see from equation (5.7), the marginal likelihood $p(Y_t)$ can be neglected when computing the posterior distribution since it is constant. However, the term provides important information about how the model describes the data conditional on the estimated parameter values simply by construction. Once we have evaluated the right hand side of the proportionality (5.7) — which stands for the posterior kernel (the un-normalized posterior distribution) — in the steps (3) and (4), the marginal likelihood is given by the integral over this expression. However, this integral is not easy to compute either. Therefore, we apply the Laplace approximation procedure as it

¹⁶A detailed description can be found in An and Schorfheide (2007, p. 131), Canova (2007, p. 442) and DeJong and Dave (2008, pp. 234) among others.

¹⁷The corresponding acceptance ratio is obtained by the covariance matrix of the proposal density. This matrix must be scaled in such a way that the appropriate acceptance ratio of proposals lie between 25 % and 40 % (cf. Griffoli (2011, p. 51)). As an reliable compromise in this study we consider a acceptance ratio of 30 %.

is common in the literature. Here the posterior kernel is approximated via a functional form of a Gaussian type. Griffoli (2011, pp. 86) emphasises that one advantage of the Laplace approximation procedure is given by the fact that it demands a low degree of computational speed. Furthermore, it requires just the posterior mode $\hat{\theta}$ as an input. As we turn to an interpretation, the higher the (logarithmic) marginal likelihood, the better the fit of the model to the data (conditional on the prior information) will be.

- (6) *Calculation of the posterior probability for model comparison, i.e. the probability that the reference model is superior in describing the data compared to (an) alternative model(s).*

Given the marginal likelihoods of a reference and an alternative model, the researcher can compare how well these models describe the data. Model comparison can be done under consideration of the Bayes factor, i.e. the ratio of marginal likelihoods of the reference and (an) alternative model(s) given by Υ_i with $i = \{R = [\text{Reference}], A = [\text{Alternative}]\}$. To put it differently, this ratio contrasts the probability of the data given the null hypothesis (the reference model) to the probability of the data given the alternative hypothesis (the alternative model). Note that here we consider the case of two-model comparison only. The Bayes factor is then weighted by the prior probability of the null hypothesis, i.e. the ratio Λ_R/Λ_A . This so called prior odds of the null hypothesis reflects the researcher's belief in the *truth* of the null hypothesis relative to the alternative one or - in other words - that the reference model is judged to be superior in describing the data a priori.¹⁸ The *posterior odds* of the null hypothesis is then defined by (Goodman (1999, p. 1005)):

$$\Xi_{R,A} = \exp(\Upsilon_R - \Upsilon_A) \cdot \frac{\Lambda_R}{\Lambda_A}. \quad (5.8)$$

Due to the fact that we do not claim that one model is more likely (or true) over the other we set $\Lambda_R = \Lambda_A = 0.5$ in advance. In order to evaluate the precision of the outcome of the reference model compared to the alternative model we calculate the corresponding *posterior probability* of the null hypothesis to be true:

$$\Pi(R) = \frac{\Xi_{R,A}}{1 + \Xi_{R,A}}. \quad (5.9)$$

This expression can be interpreted as follows. The higher the value of $\Pi(R)$ is above 0.5, the more convincing the evidence in favor for the reference model will be and vice versa. Note that the probability that the null hypothesis — which means that the reference model (R) is judged to be superior in describing the data compared to the alternative one (A) — is true, is about equal size at the beginning of the experiment. Then, the calculation of $\Pi(R)$ provides a clear description of the relative goodness of fit. As we will see in section 6.2.2, even a slight difference in the marginal likelihoods across models will lead to strong changes in the reliability of the null hypothesis.

- (7) *Examination of the sensitivity of the results to the choice of the prior information.*

In terms of an identification problem (i.e. different values of structural parameters lead to the same joint distribution for the observables), An and Schorheide (2007, p.

¹⁸Obviously, this specific prior probability has not to be mistaken as the vector of prior information $p(\theta)$. As stated above, Λ_R/Λ_A reflects the probability of *the structural model* to be true, while $p(\theta)$ contains the *functional form* of the parameters (a priori) to be estimated.

127) state that "the posterior distribution is well defined as the joint prior distribution of the parameters is proper". Clearly, it is an open question what 'proper' means in this context — especially if prior information is freely chosen by the researcher. This calls for a robustness check of the results due to the change in $p(\theta)$. For example, Lombardi and Nicoletti (2012) discuss the sensitivity of posterior estimation results to the choice of different expressions of the prior knowledge. In contrast, Del Negro and Schorfheide (2008) provide an explicit method for constructing prior distributions based on the beliefs regarding macroeconomic indicators. Basically the authors emphasize the difference between micro- and macro priors, where the latter are identified based on macroeconomic time series. In Chapter 6 we shed a light on the issue how moment-matching helps to determine the prior information in a pre-step to Bayesian estimation. In particular, we make use of the point estimates from MM and specify them as prior means within subsequent Bayesian estimation. Those so-called macro-priors are obtained directly from macroeconomic time series and as we will see, this kind of prior knowledge leads to a fit of the model (evaluated by the relative measure for the goodness of fit $\Xi(R)$), which is superior relative to the standard case where micropriors are considered.

Our Bayesian estimation results presented in this (and the following) Chapter are obtained by using the software package DYNARE. The latter is widely used for estimating DSGE models via the Bayesian methodology in academics and practice. DYNARE includes several techniques embedded as codes in the MATLAB environment in order to apply all steps from (1) to (7) – except for step (6), where we have to compute the posterior probability on our own via the equations (5.8) and (5.9).¹⁹

5.4 A Critical Discussion on Sufficiency and Transparency

According to the explorations in this Chapter, in the following we discuss the underlying question which kind of estimation approach be more appropriate. Furthermore, which results are more reliable or simply, in what method you believe in (most)? Although, we would like to make no direct statement about the superiority of one estimation approach over the other, the question is of serious concern. An econometrician who believes only in the Bayesian approach states that the key quantity within this method is the likelihood function, where this statistic (i.e. a function of the random sample) is *sufficient*. This term means that no additional information from the same sample size is provided by other statistics (Fisher (1922) and Mittelhammer (1999, in his section 7.4)). In other words, the likelihood function, associated to standard Bayesian techniques we presented here, is sufficient in respect to the description of the joint distribution of the estimated parameter values.²⁰ The econometrician then will argue that if one believes that the underlying model is the *true* one, alternative approaches to estimate (system-based) models can not be superior.

¹⁹The software is freely available at <http://www.dynare.org>. Note that in DYNARE the *logarithm* of the marginal likelihoods is reported. Since according to (6) the Bayes factor describes the non-logarithmic probability ratio, we have to exponentiate Υ_i for all i , which means that $\frac{\exp \Upsilon_R}{\exp \Upsilon_A} = \exp(\Upsilon_R - \Upsilon_A)$ holds.

²⁰The word *standard* means that a likelihood function reflects a normal (Gaussian) distribution of the unknown parameter vector θ . Unlike other families of distributions (e.g. a Beta or Uniform one) the mean equals the median due to symmetry and the mass of probability is centered around this mean value. These properties lead to an appropriate statistical description of the distribution of the random variables in θ .

While the argument for likelihood functions which are based on sufficient statistic is striking, the moment-matching approach indeed can be seen as a reliable alternative approach: the choice of what kind of moments are used within estimation is linked to a specific *economic reasoning*. In particular, within MM the focus is on the moments of interest connected to the business cycle. This stands in contradiction to the Bayesian approach, where the whole frequency domain is taken into account for optimization. According to this both approaches impose *different* prior information (likelihood evaluation versus moment selection) on the data, while the MM approach is more (let's say) narrow with respect to the data considered. Furthermore, by consideration of the prior information $p(\theta)$, the evaluation of the mode of the posterior distributions within Bayesian estimation is restricted to a subregion of the parameter space, where the distribution of θ is set by the researcher. However, the estimation outcome is sensitive to the choice of prior distributions associated with the underlying economic interpretation of the model's structural parameters. It is still an open question what criteria is suited best in order to identify the most accurate prior information (see Chapter 6, where we present a potential solution to this problem). In addition, Bayesian estimation could suffer from 'uninformative' priors, i.e. that the estimated posterior becomes quite similar to the prior distribution.²¹

The strong prediction that a model is the true one stands in contradiction to the more reliable statement that the same model is misspecified - where we believe that it will be hard to disagree on the latter in general. Even though this point is more a methodological issue, applying any kind of optimization technique is not a trivial task since (as already stated) it may get stuck in a local rather than a global maximum (or minimum). The reason is twofold. First, e.g. the whole parameter space is considered within maximum likelihood based on the starting values as well as lower and upper boundaries imposed on θ . Second, the probability of being in a local maximum (or minimum) could be positive related with the degree of dimensionality of the parameter space. In this respect estimating a medium- or even a small-scale model goes along with the uncertainty regarding the identification of a global maximum.

Although the MM approach is not excluded by this shortcomings, we claim that a global maximum might be easier to find due to the feature of *transparency*: by comparing the empirical to the simulated covariance profiles (after optimization) it is simple to judge the fitting of the model to the data. In other words, if the value of the loss function $J(\theta)$ is low and the matching of the simulated to the empirical generated profiles is fairly good (or even excellent) for the moments of interest, this consists of a high probability of uniqueness of the parameter estimates. This statement is, however, not strong enough to rule out the possibility of a local minimum of the loss function. One might think about a situation where a value of the loss function $J^1(\theta)$ is slightly lower than the previous value $J^0(\theta)$, while the improvement in the matching is negligible low. Of course, this requires a sensitivity analysis regarding different starting values for θ . A possible solution to this identification problem might be found in Franke (2012a), who enlarges the investigations done in this Chapter by applying the simulating annealing search procedure. It is based on a specific stochastic algorithm which is applied for determination of the starting values and identification of multiple local extrema. The author judges the procedure to be fully credible in order "[t]o guard against a possible miss of a competitive local minimum obtained from a suitable combination" (p. 20) of different pairs of the backward-looking model components. Nevertheless, the lack of

²¹ Obviously, by assuming quite diffuse prior information, e.g. if all elements in θ are uniformly distributed with a high standard deviation, this might avoid high sensitivity in the outcome while there is no restriction imposed on the parameter space. In this case, of course, we simply end up with maximum likelihood.

a mathematical formalization (if any exists) for the identification of global extrema within the MM approach, gives rise to further research in this field.²²

5.5 The Baseline NKM in Gap Notation

It should be explicitly made clear from the beginning that our estimations are concerned with the baseline NKM (in quarterly magnitudes) known from Part One of this thesis, in which also inflation and the interest rate are already formulated in *gaps*.²³ They refer to concepts that for simplicity may be called trend rates, designated π_t^* and r_t^* , respectively. The latter may be zero, positive constants, or they may vary over time. In any case, what is showing up in the three key equations of the model are not the raw rates of inflation and interest π_t and r_t (identical to their deviations from the zero steady state values as discussed in Part One), but the inflation gap $\hat{\pi}_t := \pi_t - \pi_t^*$ and the interest rate gap $\hat{r}_t := r_t - r_t^*$. There are several ways to interpret the occurrence of these more general gaps in, especially, the NKPC, and the persuasiveness of the microfoundations presently available for them in the literature is still another issue. We nevertheless join most of the empirical applications and leave this discussion aside.

The crucial point is that possible trend variations in some interpretation or another are treated as purely exogenous or incorporated into the shock processes, so that π_t^* and r_t^* can remain in the background. Whether or to what extent endogenous or stochastic trends, in a model as well as in the data, will alter the results of the present analysis could be investigated in a second stage of research.²⁴ Even if the new results were robust, noticeably different and perhaps preferred because of their greater familiarity, our investigations would still be useful as we would have learned more about the conditions for these “better” estimations to come about.

Regarding possible sources of persistence in the endogenous variables, which we then try to disentangle in the estimations, our focus is on the NKPC. We include both lagged inflation in its deterministic core and serial correlation in the exogenous shocks. This is in contrast to the common practice that from the outset assumes either white noise shocks or purely forward-looking price setting behaviour.²⁵ On the other hand, the random shocks in the dynamic IS equation and the Taylor rule are supposed to be i.i.d. and persistence is only brought about by a lagged output gap and a lagged rate of interest, respectively. Denoting

²²This debate on the validity of the MM approach is not conclusive. For an example, in section 7.4 we provide an elaborate discussion on why the MM approach is appropriate to use when it comes to the estimation of a *non-linear* system-of-equation model in comparison to standard procedures known from the literature.

²³As for example remarked by Cogley et al. (2010, p. 43, fn. 1) when discussing inflation persistence, it is not always completely plain in the literature whether the focus is on raw inflation or the inflation gap.

²⁴Ireland (2007) and, more ambitiously, Cogley and Sbordone (2008) are two proposals of how to endogenize trend inflation as the target set by the central bank. Ireland (2007, p. 1864), however, concludes from his estimations that still “considerable uncertainty remains about the true source of movements in the Federal Reserve’s inflation target”. Laubach and Williams (2003) and Messonnier and Renne (2007) are attempts at an estimation of a time-varying natural rate of interest.

²⁵In similar models to ours, examples of excluding autocorrelated shocks in a hybrid NKPC are Lindé (2005), Cho and Moreno (2006) or Salemi (2006), while the purely forward-looking models studied by, e.g., Lubik and Schorfheide (2004), Del Negro and Schorfheide (2004), Schorfheide (2005) allow for some persistence in the shock process. These references have been chosen from the compilation in Schorfheide (2008, p. 421, Table 3).

the output gap in period t by x_t , the model thus reads,

$$x_t = \frac{1}{1+\chi} E_t x_{t+1} + \frac{\chi}{1+\chi} x_{t-1} - \tau (\hat{r}_t - E_t \hat{\pi}_{t+1}) + \varepsilon_{x,t} \quad (5.10)$$

$$\hat{\pi}_t = \frac{\beta}{1+\alpha\beta} E_t \hat{\pi}_{t+1} + \frac{\alpha}{1+\alpha\beta} \hat{\pi}_{t-1} + \kappa x_t + v_{\pi,t} \quad (5.11)$$

$$\hat{r}_t = \phi_r \hat{r}_{t-1} + (1-\phi_r) (\phi_\pi \hat{\pi}_t + \phi_x x_t) + \varepsilon_{r,t} \quad (5.12)$$

$$v_{\pi,t} = \rho_\pi v_{\pi,t-1} + \varepsilon_{\pi,t}. \quad (5.13)$$

The time unit is to be thought of as one quarter. The three (mutually uncorrelated) shocks $\varepsilon_{z,t}$ are normally distributed around zero with variances σ_z^2 ($z = \pi, x, r$). All of the parameters are nonnegative. Specifically, β is the discount factor, κ a composite parameter that depends on the degree of price stickiness and assumptions on the production technology of firms, the coefficient α represents the degree of price indexation ($0 \leq \alpha \leq 1$), and the persistence in the supply shocks is given by the autocorrelation ρ_π ($0 \leq \rho_\pi < 1$). In the IS equation, χ is the representative household's degree of habit formation ($0 \leq \chi \leq 1$) and τ a composite parameter containing its intertemporal elasticity of substitution.²⁶ In the Taylor rule, ϕ_r determines the degree of interest rate smoothing ($0 \leq \phi_r < 1$), and ϕ_x and ϕ_π are the policy coefficients that measure the central bank's reactions to contemporaneous output and inflation.

The model's representation here is quite similar to the (quarterly) one given in the Chapters 2 and 3. The differences can be motivated by empirical reasoning. First, we account for time-varying steady state values in all variables by considering their corresponding gap specifications. Second, the Taylor rule exhibits inertia, which is a stylized fact observed in the data – note that we consider an impulse shock in equation (5.12) instead of an autocorrelated shock process. Finally, we reduce the amount of parameters to be estimated by considering the composite parameters κ and τ . We do so in order to avoid non-linear relationships in the parameters for price stickiness, habit formation and various elasticities.

The empirical data on which the estimations of the system (5.10) to (5.13) are carried out derive from real GDP, the GDP price deflator, and the Federal Funds Rate. Their exogenous trend rates should be moderately flexible and are therefore specified by the convenient Hodrick-Prescott filter (as usual, although debatable, the smoothing parameter is $\lambda=1600$). The total sample period covers the time from 1960 to 2007.²⁷

Despite concentrating on trend deviations instead of levels, one has to be aware that there are still great changes over these years in the variances of the three variables and partly also in the entire pattern of their cross covariances. This makes it expedient to subdivide the period into two subsamples, which are commonly referred to as the periods of the Great Inflation (GI) and the Great Moderation (GM). We define the former by the interval 1960:1–1979:2

²⁶It depends on the particular kind of microfoundations whether or not α and χ also enter the determination of the composite parameters κ and τ , respectively, and whether the latter continue to be positive and well-defined in the polar cases $\alpha=1$ or $\chi=1$. In the estimations, however, κ and τ will not be subjected to any theoretical constraints in this respect. Again, explicit expressions of these parameters are presented in the Chapters 2 and 3.

²⁷The Hodrick-Prescott trend is computed over a longer period, to avoid end-of-period effects. The time series of the raw and filtered data as well as the covariances can be downloaded from http://www.bwl.uni-kiel.de/gwif/downloads_papers.php?lang=en. However, as mentioned by Chiaie (2009) among others, the estimation outcome with respect to DSGE models is sensitive to the choice of data detrending and filtering methods. In fact, the posterior point estimates might vary whether e.g. the Hodrick-Prescott or a linear trend filter technique is applied. Although, we have to admit that this issue is of serious concern, we stick to the majority of empirical studies, where the Hodrick-Prescott filter is used.

and the latter by 1982:4–2007:2; the time in between is excluded because of its idiosyncrasy (Bernanke and Mihov (1998)). To give an immediate example for the need of the subdivision, the standard deviation of the annualized inflation gap in GI is 1.41% versus 0.77% in GM; for the output gap it is 1.77% in GI versus 1.15% in GM.

5.6 The Great Inflation Period

5.6.1 On the Model's Goodness of Fit

The NKM given by the equations (5.10) to (5.13) includes 12 structural parameters. Among them, the discount factor β is not a very critical coefficient and is therefore directly calibrated at $\beta = 0.99$. So the following 11 parameters remain to be estimated: α , κ , ρ_π , σ_π in the NKPC and its shock process; χ , τ , σ_x in the dynamic IS equation; and ϕ_π , ϕ_x , ϕ_r , σ_r in the Taylor rule. The inflation and interest rate gap are annualized, which may be taken into account when considering the order of magnitude of κ , τ , ϕ_x and the two noise levels σ_π , σ_r .

We begin with a Bayesian estimation of the model, for which we choose priors that are quite in line with other applications (they are documented in the Appendix D1). Alternatively, in various combinations, we shifted some of the prior distributions closer to the parameter values obtained by MM. In all of these cases, however, the marginal likelihood was distinctly worse. Thus, our *Bayesian reference* estimation (R=BR) is the first column in Table 5.1, which records the means of the posterior parameter distributions.

The estimated parameters are not dramatically different from other Bayesian results in the literature. In particular, regarding the sources of inflation persistence, low coefficients on expected inflation in the NKPC (i.e., low values of α) and substantial autocorrelation ρ_π in the shock process are typical for them.²⁸ It is, however, interesting to note an exception to this rule. Del Negro et al. (2007, p. 132, Table 1) obtain high price indexation ($\alpha=0.76$) and low shock persistence ($\rho_\pi=0.12$), despite their setting of rather opposite priors.²⁹ This outcome exemplifies that even within the Bayesian framework, the tendency towards a purely forward-looking NKPC with persistent random shocks is possibly not an unequivocally established property, yet.

The original motivation for this Chapter was to check the role of α and ρ_π from a different perspective, by using an alternative estimation approach. The pivotal result of our MM estimation is given in the second column of Table 5.1, which we will refer to as estimation A, or model A. As a matter of fact, the most immediate observation is on α and ρ_π , for which the contrast to the Bayesian estimation could not be more striking: α is estimated at its maximum value of unity and ρ_π at its minimum value of zero.

Before we turn to a more comprehensive discussion of these parameters and the other results in the Table (especially the unfamiliar absence of monetary policy shocks, $\sigma_r=0$, in model A), let us consider the moment-matching properties of estimations BR and A. Regarding an assessment of the general goodness of fit of the models, the likelihood will be disregarded

²⁸For examples from more general models, see Smets and Wouters (2003, 2007), Adolfson et al. (2007), Benati and Surico (2009), Fève et al. (2009), Cogley et al. (2010). Apart from the determination of trend inflation, estimation BR can be directly compared to Castelnuovo's (2010) results for his so-called TI model, on which he (arguably) imposes $\alpha=0$.

²⁹The present symbols α and ρ_π correspond to their ι_p and ρ_{λ_f} (we refer to Table 6.3 in Chapter 6 for more details).

	Bayesian	Moment-Matching		
	BR	A	B	C
α	0.074 0.000 – 0.156	1.000 0.585 – 1.000	1.000	0.000
κ	0.209 0.125 – 0.290	0.052 0.020 – 0.196	0.055	0.279
ρ_π	0.570 0.452 – 0.693	0.000 0.000 – 0.487	0.234	0.716
σ_π	0.694 0.524 – 0.864	0.614 0.394 – 0.937	0.405	0.716
χ	0.767 0.689 – 0.850	1.000 0.779 – 1.000	1.000	1.000
τ	0.048 0.030 – 0.067	0.105 0.020 – 0.159	0.113	0.070
σ_x	0.552 0.465 – 0.637	0.519 0.222 – 0.781	0.511	0.336
ϕ_π	1.387 1.124 – 1.644	1.324 1.187 – 1.586	1.289	1.238
ϕ_x	0.759 0.314 – 1.193	0.000 0.000 – 0.207	0.000	0.000
ϕ_r	0.742 0.668 – 0.816	0.314 0.054 – 0.414	0.256	0.394
σ_r	0.745 0.643 – 0.841	0.000 0.000 – 0.627	0.745	0.270
J	213.5	49.1	50.6	124.0
MCI missed	36	0	0	5
p -value	—	8.6%	—	—

Table 5.1: Parameter estimates for GI.

Note: The discount factor is $\beta = 0.99$ throughout. In estimation B and C, σ_r and α are fixed at 0.745 and zero, respectively. The bold face figures emphasize certain results (model A) or the exogenous parameter setting (model B and C). The smaller typeface indicates the 95% confidence intervals. The second-last row reports the number of moments ('M') that miss the confidence intervals ('CI') of the empirical moments.

and the covariance matching will from now on be our only criterion. According to the loss statistics J in Table 5.1, the overall matching implied by BR is not only worse, which is trivial, but the difference appears to be nonnegligible. Figure 5.1 provides more specific information about it. The thin dashed lines in the diagrams are the empirical auto- and cross-covariances up to a lag of 20 quarters of the interest rate, output and inflation (since there will be no more risk of confusion, we will in the following omit the expression ‘gap’ when discussing these variables). The shaded areas are the 95% confidence bands around them. The bold (red) lines depict the moments obtained from the MM estimation A, while the dotted (blue) lines are the moments to which the Bayesian estimation BR gives rise. Recall that underlying MM are only the first eight lags.

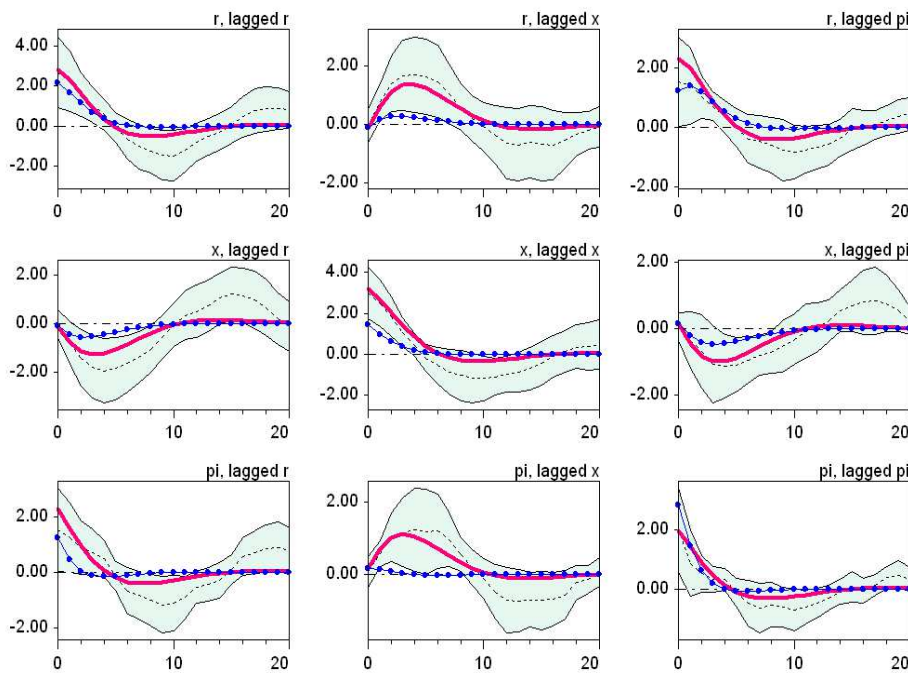


Figure 5.1: Estimated versus empirical covariance profiles (GI).

Note: The bold (red) lines result from the MM estimation A of Table 5.1, the solid (blue) lines with dots from the Bayesian reference estimation BR. The shaded areas are the 95% confidence bands around the empirical moments (the thin dotted lines).

Inspecting the performance of the MM estimation with the naked eye, the match it achieves looks very close over the first few lags and still fairly close over the higher lags until the maximal lag of 8 quarters. It is actually *remarkable* that all of the moments are contained within the confidence intervals of the empirical moments. From lag 10 on, the covariances of the model remain close to zero, while there is more structure in the empirical statistics—although their confidence bands still include the zero line. In sum, the covariance profiles brought about by estimation A can at least qualitatively be judged to be rather satisfactory.

Accepting the approach, the match could in fact provide a real challenge for other models of similar complexity.³⁰

In more accurate terms it can be said that an econometric test is not able to discard model A as outright incompatible with the data. Focussing on the present choice of moments, the p -value introduced in Section 5.2, i.e. the error rate of falsely rejecting the null that the model is true, is computed to be higher than five percent; the last row in Table 5.1 reports a p -value of 8.6%. The distribution $J^b = J(\hat{\theta}^b; m_T^b, W^b)$ of equation (5.4) from which this value derives is shown in the top-left panel in Figure 5.5 further below, together with the 95% quantile $J_{0.95} = 58.8$ and a bar indicating the originally estimated loss of $\hat{J} = 49.1$ – given the good reproduction of the empirical covariances, even a higher p -value might have been expected.

Returning to the Bayesian estimation BR, do its parameters generate similar covariances to those of estimation A? An immediate answer based on the comparison of the values of the loss function in Table 5.1 and a visual inspection of Figure 5.1 would probably be in the negative. While the auto-covariances of inflation and the interest rate, and to a lesser extent also the output gap, are quite close, several of the cross-covariances of the three variables have different patterns of persistence. In particular, referring to the discussion in the Introduction, we infer from Figure 5.1 that the implicit weighting scheme of the likelihood function in BR takes almost no account of the covariances $\text{Cov}(\hat{r}_t, x_{t-h})$ and $\text{Cov}(x_t, \hat{\pi}_{t\pm h})$; in contrast to the MM estimation, all of these statistics show very little deviations from zero.

The covariance profiles of BR in Figure 5.1 derive from a particular set of parameters. Since a Bayesian estimation delivers a large set of acceptable parameter vectors, it may be argued that not a majority, but still a sizeable fraction of them gives rise to more satisfactory moments. To scrutinize this idea we make a number of *posterior predictive checks*.³¹ The graphical approach of bivariate scatter plots proposed by An and Schorfheide (2007, pp. 144f) will do, for which we consider the following four pairs of covariances: the variances of x_t and $\hat{\pi}_t$; the variance of \hat{r}_t and $\text{Cov}(\hat{r}_t, \hat{r}_{t-4})$; $\text{Cov}(\hat{r}_t, x_{t-4})$ and $\text{Cov}(x_t, \hat{r}_{t-4})$ as a succinct representation of the lead-lag relationships between output and interest rates; and $\text{Cov}(\hat{\pi}_t, x_{t-4})$ and $\text{Cov}(x_t, \hat{\pi}_{t-4})$ as a representation of the output-inflation nexus. Accordingly, we take the parameter vectors that were selected from the Metropolis-Hastings algorithm to constitute the posterior distributions, compute the eight moments just mentioned for each of the vectors, and plot them pairwise as indicated. This is done in Figure 5.2. For a direct assessment of their absolute fit, the intersection of the two solid lines in a panel signifies the observed sample moments, and the two dotted lines the corresponding moments generated by model A.

The equal scale of the four panels makes it clear that the predictions for the three variances are much more dispersed than the covariances with the four-quarter lags, the distributions of which are actually fairly narrow. The distribution of the two auto-covariances of the interest rate in the top-right panel is largely satisfactory, but the moments in the other three panels stay consistently away from both the empirical moments and the ones of model A. In connection with the profiles in Figure 5.1 it is thus, in particular, seen that the

³⁰Perhaps the heterodox, non-linear and purely ‘backward-looking’ 3-equations models by De Grauwe (2011) or Franke (2012b). With respect to the former, this model will indeed be estimated via (S)MM in Chapter 6. Besides, the feature that all moments of model A are contained in the empirical confidence intervals may not be overrated since the latter are based on a short data series, while the moments computed for the model are the asymptotic covariances.

³¹They are “a valuable tool in applied Bayesian analysis though they have not been used much in the context of DSGE models” (An and Schorfheide (2007, p. 143)).

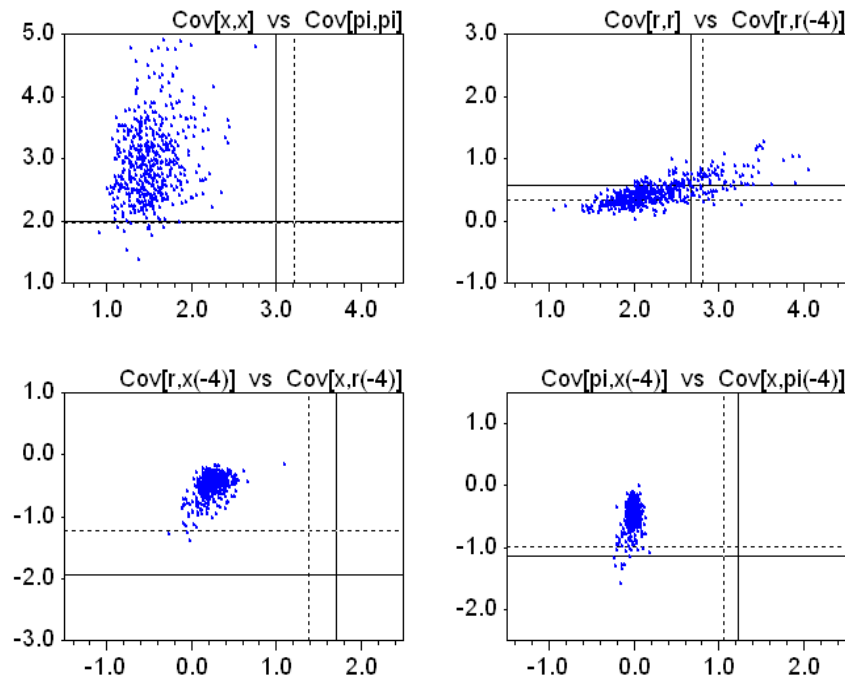


Figure 5.2: Posterior predictive check for estimation BR (GI).

Note: Moments implied by draws from the posterior distribution for BR. Solid (dotted) lines signify the observed sample moments (the moments generated by model A).

Bayesian parameters do not properly reproduce the lead-lag relationships in the output-inflation nexus. The deficiency is nonnegligible since after all, this nexus is at the heart of monetary policy.

It could nevertheless be objected that Figure 5.2 gives an overly dramatic impression since only a small subset of the moments happen to lie outside the empirical 95% confidence intervals. In a final step we therefore come back to a direct comparison of the covariance profiles of the (point) estimations BR and A and ask if their differences can also be classified as significant according to some rigorous econometric criterion. For this purpose a test procedure proposed by Hnatkovska et al. (2012) can be employed, which incidentally remains valid for misspecified models. With a little trick, the covariances implied by BR can be forced into their moment-matching framework. It is then found that the null hypothesis that models BR and A have equal moments can be safely rejected.³² On this basis we reach the firm conclusion that, in the present case of GI, a Bayesian likelihood estimation and an MM estimation *do not* essentially amount to the same thing. They rather give rise to

³²It may be added that a result of unequal moments does not necessarily imply that the superior estimation is also significantly better (from an MM point of view, of course). In fact, we were not able to confirm this for model A; a second test in this direction does not seem to have sufficient power (cf. Franke et al. (2012, their section 4.3)).

significantly different covariance structures.

5.6.2 Price Indexation versus Shock Persistence

The MM estimation makes a definite statement about the relative importance of price indexation and the shock autocorrelation as the two main sources of persistence in the NKPC. As noted above, the outcome of $\alpha = 1$ and $\rho_\pi = 0$ is the exact opposite of the message from the Bayesian estimation BR and many other results in the literature, where no significant evidence for backward-looking behaviour in the NKPC is found.³³ The reliability of the special MM corner solution should therefore be further scrutinized.

Because of their common role to generate persistence in the NKPC, the two parameters α and ρ_π are the first candidates the variations of which might give rise to multiple local minima. To check this ambiguity the following complementary estimations are conducted: both α and ρ_π are treated as exogenous parameters, a grid of pairs (α, ρ_π) on $\tilde{R} := [0.70, 1.00] \times [0.00, 0.70]$ is considered, and the nine remaining parameters are re-estimated for each of these points. What is obtained in this way is a perfectly smooth surface of the resulting losses J above the rectangle \tilde{R} , there is no local valley in its interior, and the corner $(\alpha, \rho_\pi) = (1.00, 0.00)$ constitutes a clear minimum of J over \tilde{R} . Indeed, as we allow α being greater than one (against general convention), J will decrease further (cf. section 5.6.3).

More specifically, it can be stated that for fixed values of α the variations in ρ_π have only a minor impact on the minimized losses (at least as long as $0.00 \leq \rho_\pi \leq 0.50$, otherwise the match worsens anyway). For fixed values of ρ_π , on the other hand, a decrease in α always causes a deterioration in the goodness of fit, which is also more serious than any (positive or negative) effects originating with ρ_π . Price indexation α is thus a crucial parameter for the moment-matching, and higher persistence in the shocks is not nearly capable of making up for the negative effects of lower values of α .³⁴

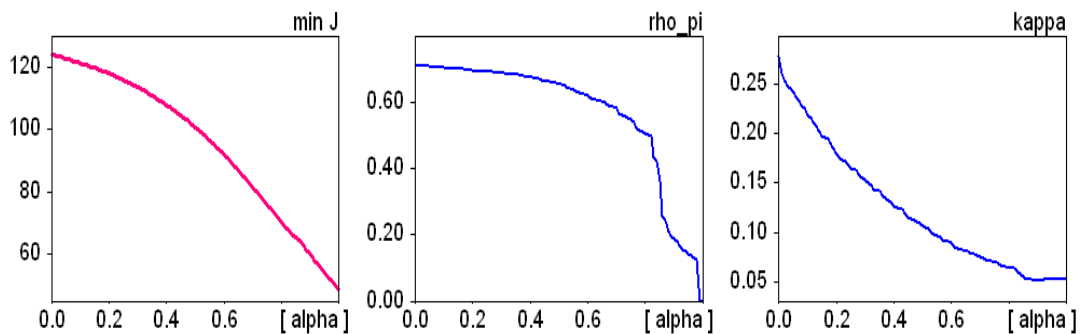


Figure 5.3: MM estimation of the model under exogenous variations of α (GI).

After emphasizing the role of indexation as a parameter of primary concern in the NKPC, it is now interesting to see the changes in the estimation results when only α is exogenously varied and J is minimized across the remaining ten parameters, including ρ_π . Figure 5.3 presents the most important reactions. First of all, as α decreases, the loss function in

³³See, in particular, Ireland (2007, p. 1864) and Cogley and Sbordone (2008, p. 2113) for similar price setting specifications.

³⁴Franke et al. (2012) illustrate these features in their Figure 2.

the left panel is steadily rising over the entire admissible range from unity down to zero. This underlines what has just been said about the dominance of the effects from α over the effects from ρ_π , not only partially but over the full domain of α . Besides, the deterioration of $J=49.1$ at $\alpha=1$ to $J=124.0$ at zero indexation appears to be nonnegligible; cf. estimation C in Table 5.1. As worked out in Franke et al. (2012), the differences in the covariance profiles of model A and C are indeed significant, even though (similar to the situation in fn. 32) the match of the latter cannot be established to be significantly worse, too.

The next effect of interest are the implied changes in the autocorrelation ρ_π of the supply shocks. As expected, lower indexation gives more scope for higher shock persistence, and again this holds over the entire range of α ; see the middle panel in Figure 5.3. It is, however, remarkable that between $\alpha = 0.99$ and $\alpha = 0.98$ an almost discontinuous change in the optimal value of ρ_π occurs, when ρ_π jumps from 0.000 to 0.126. The reason for this is that the functions $\rho_\pi \mapsto J(\rho_\pi)$ for fixed values of α are all very flat in that region, which implies that already small changes in their shape brought about by small changes in α can shift the minimum of these functions considerably.

Our reasoning concerning the NKPC has so far left aside the output gap as a source of inherited persistence. The third panel in Figure 5.3 for the optimal values of the parameter κ reveals a stronger influence of this variable as additional compensation for a reduced persistence from price indexation (which nevertheless cannot avoid the rising loss).

The results illustrated in these three panels can be viewed as an extension of Fuhrer's (2006) analysis of the factors contributing to inflation persistence. For this, he concentrates on the autocorrelations of the inflation rate as they are brought about by a hybrid NKPC and a simple AR(1) process for the driving variable.³⁵ From his GMM and also maximum likelihood estimations, Fuhrer likewise concludes that little is inherited from the persistence of (the shock and) the driving variable—and if so, this deteriorates the performance of the model. Hence, “the predominant source of inflation persistence in the NKPC is the lagged inflation term” (Fuhrer, 2006, p. 79). Actually, his coefficient on lagged inflation is typically even higher than 0.5025, which is the maximal value that we can possibly get in equation (5.11) when $\alpha=1$. This is a numerical issue that we turn to in the next subsection.

5.6.3 Admitting Stronger Backward-Looking Behaviour

Having identified the momentous role of full indexation in the price adjustments of the non-optimizing firms, we may take one step further. In fact, the unchecked fall of the function $\alpha \mapsto \min J$ towards the end-point $\alpha = 1$ in the left panel of Figure 5.3 suggests that still higher values of α would lead to an additional improvement in the matching of the moments. This idea could be pursued in another framework that allows for wider intervals of the two coefficients on expected and lagged inflation in the NKPC. In the simplest case, a parameter $\mu \in [0, 1]$ may be introduced and the coefficients on $E_t \hat{\pi}_{t+1}$ and $\hat{\pi}_{t-1}$ directly specified as $(\beta - \mu)$ and μ , respectively, without caring much about the exact microfoundations.³⁶

The range of the composite coefficients on the two inflation rates could also be extended if, to economize on notation, we temporarily leave the economic interpretation of the parameter α aside and allow it to exceed unity. Formally, the equation for the NKPC (5.11) need

³⁵Fuhrer assumes white-noise i.i.d. shocks and makes a remark that the serial correlation that might be added here will plausibly be relatively low (Fuhrer (2006, p. 70)).

³⁶This is the version that, without discussing further details of its theoretical background, Fuhrer (2006, p. 53) presents as the “canonical hybrid New Keynesian Phillips curve”. Similarly so Cho and Moreno (2006).

not be altered then. Carrying out the estimation once more without the upper bound does indeed drive α further up to a value distinctly above unity (see model D in Table 5.2). As a result, the composite coefficient on lagged inflation now clearly dominates its counterpart on expected inflation, that is, $\alpha=2.760$ increases it to 0.74.

Model	α	κ	σ_π	χ	τ	σ_x	ϕ_π	ϕ_x	ϕ_r	J
A	1.000	0.052	0.614	1.000	0.105	0.519	1.324	0.000	0.314	49.1
D	2.760	0.131	0.416	1.000	0.183	0.430	1.105	0.000	0.163	23.4
E	1.000	0.046	0.568	1.282	0.110	0.409	1.540	0.140	0.429	42.7
F	3.242	0.123	0.313	1.440	0.192	0.366	1.521	0.000	0.081	9.1

Table 5.2: Estimations when the constraints on α and χ are dropped (GI).

Note: In all four cases, $\rho_\pi = 0$ and $\sigma_r = 0$ results. Values of α and χ exceeding one are admitted for notational convenience (they are not meant to have a meaningful economic interpretation). In model F, the implied coefficients on lagged output and lagged inflation in (5.10) and (5.11) are 0.59 and 0.77, respectively. Bold face figures emphasize the kind of ‘excessive’ backward-looking behaviour in the estimations.

While any serial correlation in the supply shocks continues to be absent, $\rho_\pi = 0$ again, it is perhaps somewhat surprising to note that the influence of the inherited persistence in the NKPC increases, too, rather than decreases, i.e. the estimate of the slope coefficient κ more than doubles from 0.052 to 0.131. Simultaneously, the role of the shocks themselves is diminished (σ_π is reduced). As far as the matching is concerned, the improvement to which the higher values of α can give rise vis-à-vis model A is more than only marginal as the loss J declines by more than 50 percent.

Since it proved helpful to step outside the original interpretation of the NKPC, the same should next be tried with the habit parameter χ in the dynamic IS equation, which so far was consistently estimated at its upper bound $\chi=1$. Reintroducing the upper bound $\alpha=1$ in the NKPC, model E in Table 5.2 shows that also in this way a better fit can be obtained, although with $J=42.7$ much less so than with model D. The improvement is brought about by $\chi = 1.282$, by which the coefficient on lagged output in the dynamic IS equation rises from 0.50 to 0.56.

Lastly, it is only natural to drop the upper bounds for both parameters α and χ simultaneously, which constitutes our model F. The inertia thus made possible do not tend to replace each other but α as well as χ experience a further moderate increase. Interestingly, almost no more persistence is now required on the part of the interest rate ($\phi_r = 0.081$), and the noise levels σ_π and σ_x of the exogenous shocks can further subside. Hence the deterministic core of the model gains in importance.

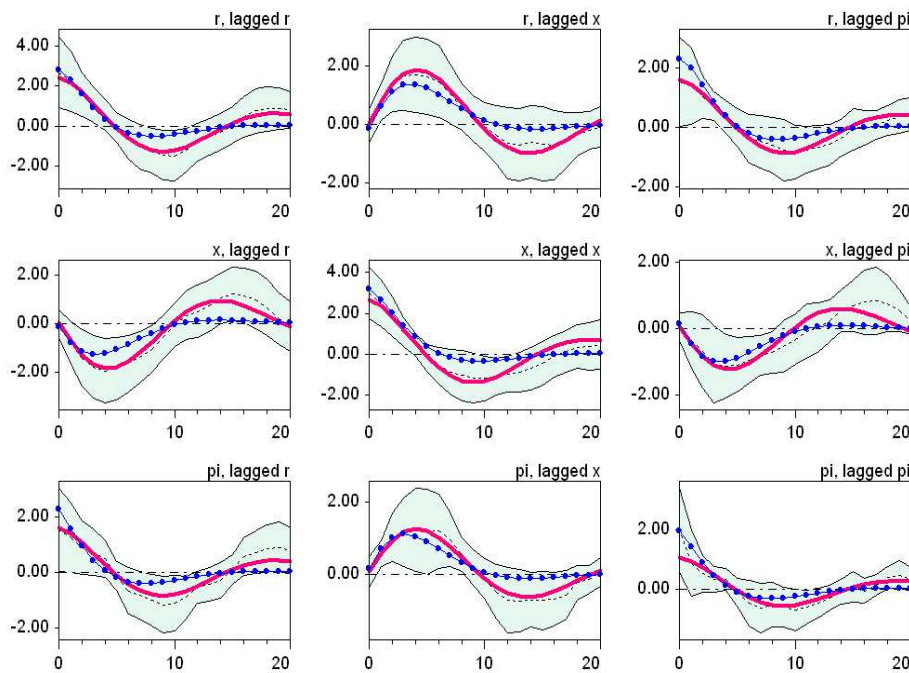


Figure 5.4: Covariance profiles of model F from Table 5.2 (bold lines).

Note: The solid (blue) lines with dots represent MM estimation A.

Most remarkable of all, however, is the final improvement in the performance of the hybrid NKM in gap notation that is thus achieved. Not only that the two persistence effects from higher values of α and χ do not cancel out, they rather reinforce each other. That is, if starting from model A each effect were hypothetically maintained irrespective of the rest, the value of J would fall to $49.1 - (49.1 - 23.4) - (49.1 - 42.7) = 17.0$. Instead, estimation F reduces the value of the loss function further down to 9.1. With respect to model A this is as strong an improvement as 81%.

The fit of model F could therefore be summarized as, we dare say, *excellent*. The diagrams of the covariances in Figure 5.4 illustrate this to the naked eye. If there still is something to be desired it is a higher variance of the inflation rate in the lower-right panel. We would also like to emphasize that, compared to model A, model F captures much more of the variations of the covariances at the higher lags beyond the two-years horizon, although these moments play no role in the loss function.

Despite the large difference in the minimized loss, the superiority of estimation F to estimation A over the 8-lag horizon in Figure 5.4 appears perhaps less substantial. The formal econometric model comparison test nevertheless establishes that their covariance profiles are significantly different. Moreover, it makes sure that also the match of the unconstrained model is significantly better than that of our reference estimation A, which previously seemed so satisfactory.³⁷ In sum, we find strong evidence for the beneficial role of backward-looking

³⁷The statistical details are provided in Franke et al. (2012). However, this superiority does not necessarily

behaviour in the NKPC and dynamic IS equation – if a moment-matching perspective is adopted. Our investigation even calls for a reconsideration of the microfoundations that would permit the resulting coefficients on lagged inflation and lagged output to become larger than one-half.

5.6.4 Which are the Central Moments for the Results?

After temporarily transgressing the interpretational framework for the indexation and habit persistence parameters, we return to the main estimation A with its estimates $\alpha = 1$ and $\chi = 1$. In order to gain more insight into the reasons why, by contrast, the Bayesian estimation yields a value of α close to zero, we take up the observation in Section 5.6.1 that the likelihood in the latter implies low weights of the cross relationships $\text{Cov}(\hat{r}_t, x_{t-h})$ and $\text{Cov}(x_t, \hat{\pi}_{t\pm h})$, which are therefore only poorly reproduced. So far, however, it is not clear why they should also have a bearing on the forward-looking nature of the NKPC. To address this issue we more specifically ask which of the second moments that have been employed for MM are favourable for getting, in particular, a low price indexation α , and which are the most important covariances counteracting this tendency.

	Covariances underlying		
	$(\hat{\pi}_t, \hat{\pi}_{t-h})$ $0 \leq h \leq 8$	+	+
		$(\hat{r}_t, \hat{r}_t), (x_t, x_t)$	$(\hat{r}_t, x_t), (x_t, \hat{\pi}_t)$
α :	0.00	0.00	1.00
κ :	0.09	0.28	0.04
ρ_π :	0.00	0.00	0.00
$J^{(78)}$:	17742	628	90

Table 5.3: Selected estimations with a small set of moments.

Note: The latter h denotes the *lag* length.

Let us start with the auto-covariances of inflation, $\text{Cov}(\hat{\pi}_t, \hat{\pi}_{t-h})$, as the only (nine) moments on which we now re-estimate the model. With so many structural parameters available, this profile can be very well matched, but the rest of the original 78 moments is badly missed. As the first column of Table 5.3 shows, the loss associated with all of the 78 moments, which for greater clarity may here be designated $J = J^{(78)}$, would make this estimation completely unacceptable. What the rudimentary estimation, however, indicates is that the good match of the inflation profile is, in particular, accomplished by $\alpha = 0$; although it will also be noted that for this alone no serial correlation in the supply shocks is needed ($\rho_\pi = 0$), and only a limited inherited persistence from the output gap (as measured by the slope coefficient κ).

imply a better p -value from the re-estimations of model F. We actually obtain $p = 7.9\%$ for it, which is lower than the 8.6% of model A in Table 5.1.

In the first instance, the extremely poor match is due to the fact that the other covariances are not properly anchored. As a minimal device to correct the failure, we take a second step and add the variances of the interest rate and the output gap to the previous auto-covariances. From the second column in Table 5.3 we learn that the resulting match is still unsatisfactory but much better than before. Moreover, the zero estimates of α and ρ_π are maintained, and the slope $\kappa=0.28$ has shifted into the range of the Bayesian estimation BR.

For a further improvement, some cross relationships should be taken into account. In light of the abovementioned observation, we choose the covariances that the Bayesian approach largely neglects, yet without any lags. Accordingly, $\text{Cov}(\hat{r}_t, x_t)$ and $\text{Cov}(x_t, \hat{\pi}_t)$ are added to the moments from step 2. As it turns out, we can view these two covariances as the crucial moments that move the MM estimation towards model A. First, α jumps to unity, ρ_π remains at zero, and κ is of the same order of magnitude as in model A. Second, there is another great progress in the goodness of fit; the loss $J=J^{(78)}=90$ is here already lower as it results for estimation BR as well as the MM estimation C that has $\alpha=0$ imposed (cf. Table 5.1).

To sum up this little experiment, the moments that are mainly responsible for bringing about the NKPC parameters similar to those from the Bayesian estimation are the auto-covariances of inflation and the variances of the other two variables of the model. However, as soon as they are combined with only the unlagged cross-covariances $\text{Cov}(\hat{r}_t, x_t)$ and $\text{Cov}(x_t, \hat{\pi}_t)$, the foundation stone for obtaining the backward-looking behaviour is laid.

5.6.5 Evaluation of the Estimated Parameters

As interesting as the MM estimations may be so far, the complete absence of stochastic noise in the monetary policy rule found in model A, $\sigma_r = 0$, is possibly somewhat irritating. Although, different from the likelihood approach, we need not be concerned about a stochastic singularity in this way, we should ask how heavily the results are dependent on this feature. To this end we impose a nonnegligible noise level $\sigma_r = 0.745$ on the Taylor rule—most naturally the same that the Bayesian estimation delivered—and re-estimate the remaining parameters by MM. The outcome is shown as model B in Table 5.1.

Three points deserve to be emphasized. First, the deterioration in the match from model A to B is rather minor; actually, the increase in the loss from 49.1 to 50.6 would not be easy to recognize in a diagram like Figure 5.1. Second, price indexation remains at its maximal level $\alpha = 1$ (just as the habit persistence parameter $\chi = 1$ in the dynamic IS equation). Interestingly, a moderate shock persistence ρ_π proves now to be helpful in addition. Third, the other parameters are fairly robust, too. The policy coefficient ϕ_x on the output gap is especially remarkable. In estimation A as well as B it is at its lower bound zero, which is in striking contrast to the Bayesian BR estimate $\phi_x = 0.759$ and the straightforward conventional wisdom that over the Great Inflation period the central bank paid (perhaps unduly) strong attention to the variations of economic activity. The fact that the interest rate in model A and B does not need to inherit persistence from output is all the more surprising as, compared to BR, the inflation coefficient ϕ_π is the same order of magnitude and own-persistence ϕ_r in the Taylor rule is even weaker.

Presently, we take the unfamiliar (as well as familiar) results as a matter the robustness of which needs to be checked with alternative versions of the Taylor rule. For example, interest rate smoothing might take another form than the lagged interest rate or another dating of output and inflation in the rule might be investigated. Also, the central bank may not

react to the rather noisy quarter-on-quarter inflation rates but to the four-quarter rates.³⁸ Incidentally, this smoothing could leave more scope for the quarterly noise of monetary policy shocks, i.e. for $\sigma_\tau > 0$. On the basis of some preliminary explorations we expect that the price indexation in the NKPC will not be much affected by these respecifications but, of course, this needs to be tested, too.

The observation that variations of σ_τ have only a limited effect on the loss function, or at least on the minimized loss, brings us to the general question of the accuracy of the estimated parameters. As indicated in Section 5.2, re-estimations on model-generated moments are used to construct 95% confidence intervals for them.³⁹ A sample size of $B = 1000$ is sufficient for the bootstrap procedure. In this way we arrive at the intervals given in column A of Table 5.1.

It will be noted that most of the confidence intervals of the MM estimation are wider than those from its Bayesian counterpart BR. This especially holds true for the intensity σ_π , σ_x , σ_τ of the random shocks. It may even be suspected that a relatively weak identification of the stochastic noise levels is a more general problem for the moment-matching methodology. Some hints from complementary likelihood estimations to delimit these levels might then perhaps be helpful. In particular, the high number of parameters estimated might lead to wide confidence intervals. It seems to be reasonable at first glance, to fix the monetary policy parameters in the Taylor rule in order to check if the confidence intervals will become narrow in the case of a reduction in the parameter space.

There are two exceptions for which MM yields a narrower confidence interval than BR. This is the case for the policy coefficients ϕ_π and ϕ_x . Regarding the responsiveness in the Taylor rule to inflation, both the Bayesian and MM estimation have ϕ_π bounded away from unity, so that indeterminacy is no issue. As far as economic activity is concerned, the no-response result in model A is now more firmly established and it is distinctly different from the central bank's active engagement according to BR.

Figure 5.5 shows the frequency distributions for the parameters that have originally been estimated at their lower or upper bound.⁴⁰ It is seen here that the occurrence of values of ρ_π and σ_τ close to zero is certainly nonnegligible, but higher values are by no means an exception, either. On the other hand, it is absolutely typical that α and χ are estimated close to unity, and ϕ_x close to zero. The width of the confidence interval $[0.585, 1]$ in Table 5.1 for the price indexation α appears thus somewhat exaggerated; given the shape of its distributions, we would not hesitate very much to think of α as being practically equal to unity. On the whole, apart from the problem with the stochastic noise, this and the other parameter estimates appear to be reasonably precise.

Clearly, the re-estimated parameter values are not all independent of each other. Table 5.4 records the pairs of parameters with the highest correlation coefficients (in modulus; the upper figures in a cell refer to the GI period). Not surprisingly in the light of the discussion on the sources of persistence in the NKPC, there is a *negative relationship* between the

³⁸Preliminary results from Bayesian estimation for the *four-quarter rates* specification turn out to be ambiguous. Based on the marginal likelihood, for the GI period it holds that the fit of this model variant is superior to the *quarter-on-quarter* one. For the GM period just the opposite holds. However, further investigations are needed. It must be stated that σ_τ turns out to be significantly greater than zero as we combine the MM approach with the Bayesian one. Further details are given in section 6.2.2.

³⁹If a parameter has originally been estimated at an end-point of its admissible interval, we consider the 95% of the re-estimates from that boundary upward or downward, respectively.

⁴⁰The density functions are estimated by means of the Epanechnikov kernel (see Davidson and MacKinnon (2004, pp. 678–683) for the computational details).

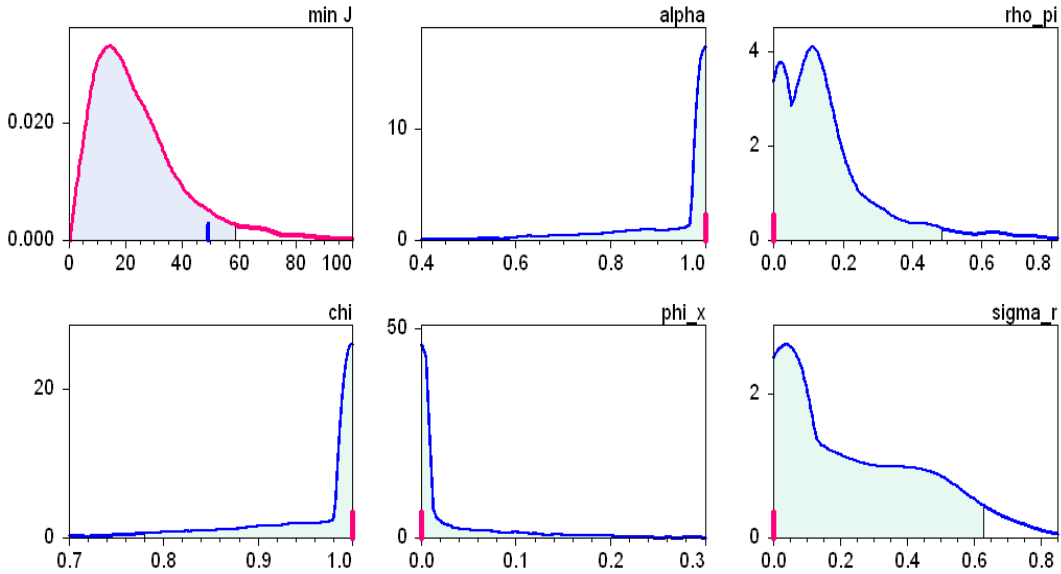


Figure 5.5: Selected frequency distributions of the bootstrap re-estimations (5.3) and (5.4) of model A (GI).

Note: The bold bars at the bottom indicate the estimates on the empirical moments, the shaded areas show a 95% probability mass of the distributions.

indexation α and the serial correlation ρ_π in the shock process. This means that a given degree of inertia in the inflation dynamics can be sustained for a combination of a low value of α and a high value of ρ_π – and vice versa. The fact that the connection is not tighter is in accord with the flatter part of the function $\rho_\pi = \rho_\pi(\alpha)$ in the middle panel in Figure 5.3. It is also apparent that higher values of ρ_π should induce lower levels σ_π of the white noise innovations.

Less obvious is the tendency that the two parameters measuring the exogenous persistence (ρ_π) and the inherited persistence (κ) move in the same direction. Furthermore, from the monotonic relationship between α and κ in the right-hand panel in Figure 5.3 above one would have also expected a certain correlation between these two parameters; actually, however, the two are virtually independent.

Only a few other relationships are worth mentioning. One of them includes the parameters of the dynamic IS equation and says that stronger demand shocks have to be mitigated by less persistence as it is brought about by the habit parameter χ . The relationships in the Taylor rule between the coefficients ϕ_π and ϕ_x themselves as well as between them and the degree ϕ_r of sluggishness are all positive, which is perhaps not immediately apparent. While these are all relationships between parameters within the same part of the model, there are two (but no more) relationships across different equations: both of them between the NKPC and the monetary policy shocks, where it is remarkable that stronger shocks go along with lower values of κ and ρ_π ; on the other hand, α is not affected by these shocks in either direction.

	ρ_π	σ_π	σ_x	σ_r	ϕ_x	ϕ_r
α	-0.36					
	-0.62					
κ	0.46			-0.41		
	0.22			-0.11		
ρ_π		-0.41		-0.39		
		-0.39		0.01		
χ			-0.44			
			-0.16			
τ			0.14			
			0.45			
ϕ_π					0.43	0.43
					-0.26	0.20
ϕ_x						0.63
						0.59

Table 5.4: Main correlations between the re-estimated parameters of model A in GI (upper figure in each cell) and GM (lower figure).

5.7 The Great Moderation Period

In this section we consider the GM period, where in other respects we can proceed along the same lines as above. Accordingly, our main interest is a comparison of a showcase MM estimation A with a Bayesian reference estimation BR. They are shown in the first two columns of Table 5.5. As a first point we would like to stress that, regarding the NKPC, a similar result to the GI sample prevails. The bold face figures in the table accentuate that estimation A, contrary to BR, needs no persistence from the supply shocks ($\rho_\pi = 0$); again it is a high degree of price indexation α that takes care of this, although it is here not maximal.

Another feature of estimation A is a corroboration of the intuition that inflation during the GM period was less exposed to exogenous shocks than during GI; cf. the estimate of the noise level σ_π , which is reduced by two-thirds vis-à-vis the GI estimation A in Table 5.1 (the Bayesian estimations yield a much lower reduction). Also the driving variables in the NKPC and the dynamic IS equation have a somewhat weaker influence than in GI, evidenced by lower estimates of κ and τ and narrower confidence intervals. On the other hand, the Taylor rule exhibits stronger persistence ϕ_r . In addition, the interest rate now responds to the output gap, even quite strongly ($\phi_x \approx 1$ versus $\phi_x = 0$ in GI), while the estimated coefficient on the inflation gap ϕ_π has a similar order of magnitude to GI. These statements have, however, to be qualified since—in striking contrast to the Bayesian reference estimation in Table 5.5 and to model A in GI as well—both of these parameter estimates have extremely wide confidence intervals. In our moment-matching estimation approach and with the present specification of the Taylor rule, we have therefore no firm basis to compare the stance of monetary policy

	Bayesian	Moment-Matching		
	BR	A	B	C
α	0.033 0.000 – 0.071	0.816 0.475 – 1.000	0.000	0.863
κ	0.163 0.103 – 0.221	0.030 0.000 – 0.046	0.139	0.020
ρ_π	0.389 0.274 – 0.510	0.000 0.000 – 0.453	0.712	0.000
σ_π	0.517 0.420 – 0.611	0.200 0.140 – 0.373	0.176	0.163
χ	0.825 0.759 – 0.891	1.000 0.669 – 1.000	1.000	∞
τ	0.017 0.009 – 0.025	0.047 0.000 – 0.085	0.045	0.275
σ_x	0.346 0.296 – 0.399	0.532 0.295 – 0.702	0.515	0.555
ϕ_π	1.181 1.001 – 1.383	1.626 0.295 – 3.746	2.412	1.418
ϕ_x	1.014 0.602 – 1.419	1.031 0.176 – 2.129	0.664	1.296
ϕ_r	0.814 0.762 – 0.867	0.776 0.673 – 0.958	0.753	0.760
σ_r	0.449 0.395 – 0.502	0.472 0.296 – 0.942	0.527	0.348
J	170.1	54.1	68.4	39.6
MCI missed	15	3	4	2
p -value	—	5.4%	—	—

Table 5.5: Parameter estimates for GM.

Note: In estimation B, α is fixed at 0.00, in estimation C the constraint on χ is dropped. The smaller typeface indicates the confidence intervals, the bold face figures serve for emphasis. The second-last row reports the number of moments ('M') that miss the confidence intervals ('CI') of the empirical moments.

in GI and GM. Incidentally, the width of the confidence intervals for ϕ_π and ϕ_x is not so much different from the intervals that Cho and Moreno (2006, pp. 1467ff, Tables 2, 4, 5) obtain from their maximum likelihood bootstrap re-estimations of a similar three-equations model, on which they impose $\rho_\pi = 0$ (their sample period is 1980:4–2000:1).⁴¹

With respect to the correlation coefficients between the re-estimated parameters a few things are different from the GI sample (cf. Table 5.4 in the previous section). Most conspicuously, the relationship between ϕ_π and ϕ_x in the Taylor rule turns from positive to negative. There is moreover no longer a connection between the intensity of the monetary policy shocks and the NKPC, i.e. between σ_r and ρ_π . Note that σ_r is now clearly bounded away from zero. The relationships between κ and ρ_π , κ and σ_r , and ϕ_π and ϕ_r become weaker in GM, whereas the negative relationship between α and ρ_π becomes stronger, possibly because the upper bound on α has largely ceased to be effective. Lastly, within the dynamic IS equation the effect from the noise level σ_x has shifted from χ to τ , i.e. from endogenous persistence to persistence inherited from the real interest rate.

Let us then turn to the goodness of fit of estimation A in GM. With a minimized value $J = 54.1$ of the loss function versus $J = 49.1$ in Table 5.1, it appears slightly worse than estimation A in GI. The distribution of the minimized losses from the bootstrap re-estimations shows a minor shift to the left, yielding a 95% quantile $J_{0.95} = 55.0$ versus 58.8 in GI. As a consequence, the moment-specific p -value for GM results like $p = 5.4\%$, as compared to 8.6% for GI.⁴²

Considering the matching of the single moments, there are now three moments that miss the empirical confidence intervals, *versus* none in GI. Figure 5.6 shows that responsible for this is the steep initial decline of the auto-covariance profile of the inflation gap, which means that in GM there is noticeably less persistence in $\hat{\pi}_t$ than in GI (the first-order serial correlation is 0.85 in GI and 0.50 in GM). As it turns out, the estimation is not too well prepared for that, so that one may be even tempted to say that in its entirety estimation A tends to exhibit too much, rather than too little, inflation persistence. Specifically, it seeks to find a compromise by first strongly underestimating the level of the variance of $\hat{\pi}_t$, the corresponding t -statistic being -3.17 , and then moderately overestimating $\text{Cov}(\hat{\pi}_t, \hat{\pi}_{t-1})$ and $\text{Cov}(\hat{\pi}_t, \hat{\pi}_{t-2})$ with a t -statistic around 2.30 in both cases; see the bold (red) line in Figure 5.6 (unfortunately, the miss of $\text{Cov}(\hat{\pi}_t, \hat{\pi}_{t-1})$ is hard to recognize).⁴³

⁴¹Several of the low estimates of ϕ_π for model A might imply indeterminacy with one stable root too many in the Blanchard-Kahn condition. This poses no problem for us since the solution matrix Ω in the quadratic matrix equation for the deterministic part of the model was computed by employing the brute force iteration procedure. In the case of multiple solutions it selects one of the solution matrices automatically and, as we have checked by a number of examples, the most appropriate one. This means that Ω changes continuously when, say, *ceteris paribus* variations of ϕ_π lead the system from determinacy to indeterminacy. For an overview of the brute-force iteration method with respect to the NKM presented here, we refer to the Appendices A3 and F1 (in case of a non-linear version of this model). By the way, we found a high robustness of the convergence with respect to variations in the initial conditions, which is in contrast to the sufficient, somewhat special conditions for local convergence given by Bai et al. (2005, pp. 116f).

⁴²Cho and Moreno (2006) evaluate their three-equations model by bootstrapping and re-estimating the model and a low-order unconstrained VAR, from which subsequently a likelihood ratio test statistic can be computed. The resulting p -value is zero for their base model with its i.i.d. random shocks, but interestingly, with $p = 3.9\%$ (Table 6 on p. 1474, panels A and B) this statistic is not too different from ours if they admit auto- as well as cross-correlations in the three shock variables (which on the other hand are features that our estimates can completely dispense with).

⁴³With different weighting schemes in the loss function and skillful non-linear transformations of the critical components, we also tried to force all of the model-generated moments into the empirical confidence intervals. The best we could achieve in this way (at the price of an increase in the loss to $J = 72.8$) is a miss of just one confidence interval, the one for $\text{Cov}(\hat{\pi}_t, \hat{\pi}_{t-4})$ with a t -statistic of -3.63 (see Franke et al. (2012, Table 5)).

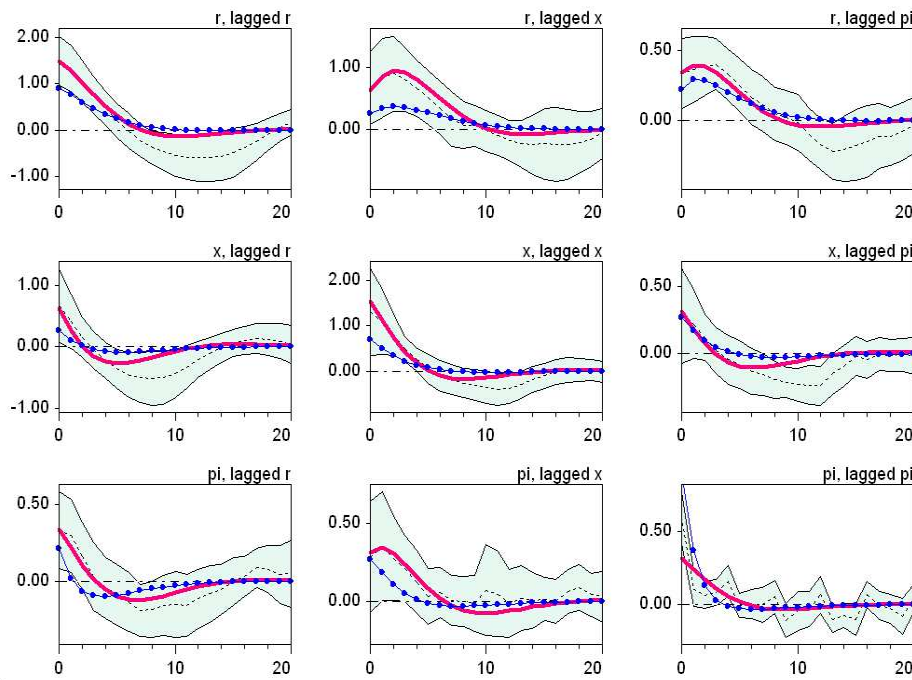


Figure 5.6: Estimated versus empirical covariance profiles (GM).

Note: The bold (red) line results from the MM estimation A of Table 5.5, the solid (blue) line with dots from the Bayesian reference estimation BR. The shaded area is the 95% confidence band around the empirical moments.

Regarding the auto-covariance pattern of inflation, the Bayesian reference estimation BR proves to be somewhat superior to model A; see the dotted (blue) line in Figure 5.6. For the other types of moments, BR displays a similar inferiority to that in GI. It seems, however, to be less pronounced. Informally, the impression is supported by the considerably lower loss differential of $(170.1 - 54.1) = 116.0$ in GM versus $(213.5 - 49.1) = 164.4$ in GI. In fact, the econometric test that was already used above assures us that in GM (unlike GI) the covariance profiles of estimations BR and A cannot be significantly told apart. It is nonetheless remarkable that the thus indistinguishable moments are generated by two parameter sets representing (almost) purely forward-looking behaviour in the NKPC on the one hand, and largely backward-looking behaviour on the other hand.

Besides, taking up the idea of bivariate scatter plots for a posterior check of selected moments as they are induced by the Bayesian estimation BR, similar characteristics to those of Figure 5.2 are obtained—only on a smaller scale and the clusters of points are closer to the empirical moments (this is documented in Figure 5.8 given in the Appendix D2).

After discussing the main estimation A, we can turn to studying the effects of varying degrees of price indexation α . Figure 5.7 is the counterpart of Figure 5.3 for GI in Section 5.6.2. Now, for GM, the function $\alpha \mapsto \min J$ has an interior minimum, although the left-hand panel in Figure 5.7 indicates that the performance of the model for $\alpha=1$ is not much worse. Also to

the left of the estimated (i.e. minimizing) α , the deterioration of J is not very dramatic. The details of this MM estimation with its imposition of the purely forward-looking behaviour are given in column B in Table 5.5. As the loss of estimation B is lower than that of BR and the moments generated by the latter are not significantly different from those of estimation A, there is no significant difference between the moments of estimations A and B, either.⁴⁴ This notwithstanding, $\alpha = 0$ is far outside estimation A's confidence interval for α . On the other hand, although $\alpha = 0$ is contained in the confidence interval of BR, there are several other parameters of model B that do not fall into the Bayesian confidence intervals.

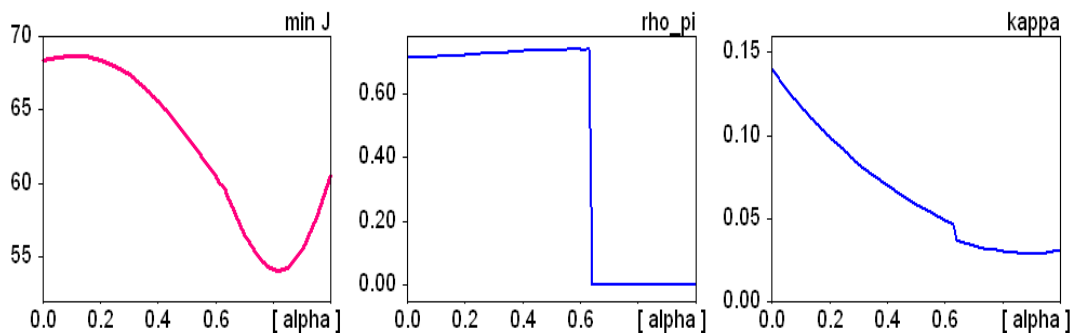


Figure 5.7: MM estimation of the model under exogenous variations of α (GM).

Regarding the estimates of ρ_π and κ that are associated with the exogenous variations in α , Figure 5.7 shares with Figure 5.3 the feature that these parameters are low if α is high and vice versa. Only the pattern of the reactions of the shock persistence ρ_π in GM is somewhat different from GI. Instead of the monotonic increase of ρ_π as α decreases, there are now practically just two different states: the estimated ρ_π is zero for $0.64 \leq \alpha \leq 1$, and it marginally falls (rather than increases) from 0.739 to 0.712 as α decreases from 0.63 down to zero. The virtually discontinuous jump of ρ_π between 0.63 and 0.64 is furthermore so strong that it makes itself also felt in the estimates of κ (and other parameters).

At the end of the section, we again step outside the interpretational framework for the two parameters α and χ and admit values exceeding unity for them. Estimation C in the last column of Table 5.5 shows that also in GM an improved match is possible in this way, though it is certainly less pronounced than in GI. The price indexation α makes no use of the relaxation of its constraint; even if the minimum search procedure for the loss function initializes α considerably above unity, the parameter soon returns into a region of roughly 0.80 or 0.90 (before the other parameters settle down on the final values of the estimation). By contrast, the habit persistence χ in the IS equation strongly tends away from unity. This goes along with a strong increase in the coefficient τ , stronger than the increase of τ from estimation A to F in Table 5.2 for GI.

Re-initializing the search algorithm again and again, the coefficient χ can practically be said to head towards infinity, which only means that the IS equation places the entire weight on the lagged output gap. As far as we know, a complete absence of forward-looking behaviour in the dynamic IS equation has not yet been obtained in the estimation of NKMs of similar complexity.

⁴⁴Strictly speaking, this requires an extra run of the econometric testing procedure, which formally confirmed the statement.

5.8 Conclusion

Being concerned with the estimation of contemporary macroeconomic DSGE models, the main purpose of this Chapter was a challenge of the dominant position of the Bayesian approach. Our alternative was the Method of Moments, also known as Moment-Matching (MM). In the present application it seeks to match the model-generated second moments of the economic variables to their empirical counterparts. Besides the relatively low computational cost, a main advantage of the method is its transparency. MM allows the researcher to concentrate on what he or she considers to be the most important stylized facts of the economy, but it also requires him or her to make them perfectly explicit. While in the end the choice of moments is a matter of judgement, it is a useful and informative decision to make since a model, at whatever level of complexity, cannot possibly reproduce all of the empirical regularities that we observe. In addition, the MM approach provides an intuitive notion of the goodness of fit of a model, which may be checked by visual inspection of the covariance structure or more formally by an econometric assessment of the minimized value of a loss function.

A novel feature of the contribution in this Chapter is that it directly contrasts MM with the Bayesian estimation (BE) results. Based on our results reported in this Chapter, it can be occasionally argued that high values of the likelihood function imply a low discrepancy between sample and model-implied second moments. Within the framework of a baseline three-equations model of the New-Keynesian macroeconomic consensus, our estimations for the Great Inflation (GI) sample period disproved this claim. Franke et al. (2012) develop a formal econometric test and show that the covariance profiles generated by the Bayesian and the MM estimation are significantly different - which can be seen in this Chapter already by the naked eye. For the Great Moderation (GM), on the other hand, the two profiles could not be significantly told apart; at least the test did not have sufficient power for that.

While it is trivial that the different covariance structures of the two estimation methods in GI are obtained from different parameter sets, similar differences in the parameter estimates were also found for GM. Special emphasis in this respect was placed on a comparison of the degree of backward-looking behaviour in the hybrid NKPC. A typical result of many (though not all) Bayesian estimations, to which our investigation was no exception, is that lagged inflation tends to play only a minor role in the NKPC. Moreover, Milani emphasizes in his survey that “[...] ad-hoc indexation rules have been disproved many times” (2012, p. 11). Inflation persistence is here brought about by serial correlation in the shock process, besides the inherited persistence from the output gap. Our MM estimations may add new insights into this discussion. In fact, they found strong evidence to exactly the contrary. With $\alpha \approx 0.80$ the degree of price indexation is high in the GM period, and it is estimated at its maximal value of $\alpha = 1.00$ in the GI sample. Moreover, in both cases the supply side shocks are white noise and inherited persistence is weak.

We subsequently took one step further and showed that if, hypothetically, the parameter α were permitted to exceed unity, then in GI it would be higher than 3. This means that the composite coefficient on lagged inflation in the NKPC would be larger than 0.75. The habit persistence parameter χ in the dynamic IS equation, by the way, would also be higher than one if this upper bound were dropped. In GM, the dynamic IS equation then even turned out to be purely backward-looking. The much stronger role for the backward-looking elements in the model, already in the presence of the constraints $\alpha \leq 1$ and $\chi \leq 1$, is all the more important since the matching of the empirical moments proves to be fairly good.

The general qualitative impression was supported by (moment-specific) p -values above the 5% significance level (even though we might have hoped for more).

With respect to our results obtained in Chapter 2 of this thesis, the question of whether the model exhibits a hybrid NKPC or not is of serious concern when conducting (optimal) monetary policy. While our results based on MM are in favour for a hybrid specification, it must be said that a value of the degree of price indexation close to its upper boundary point leads to a distinct reduction of the effectiveness of policy measures. This holds especially in the case where we admit a degree of price indexation being greater than one. Since the pass-through of output gap movements into the inflation rate dynamics is damped as the degree of price indexation increases, this effect can be amplified by the frequency-dependent inherited persistence (FIP) effect. On the opposite, our Bayesian results (and the ones taken from the majority of contributions in the literature) show a different picture, where an almost purely forward-looking NKPC (i.e. $\alpha \approx 0$) must be assumed. Obviously, in this case the interplay between the degree of price indexation and the FIP effect plays only a minor (or even no) role.

Some problems showed up in the monetary policy rule, where in GI there was no need for any direct shocks at all, while in GM the two coefficients ϕ_x and ϕ_π on output and inflation had unpleasantly wide confidence intervals. Here we argued that before putting all of the blame on the estimation method, alternative specifications of the Taylor rule should be investigated. More generally, the usually negative valuation of imprecise parameter estimates might also be reversed. They could rather be indicative of some scope for the model to take account of *additional and qualitatively different moments* without too seriously affecting the match of the original moments. In the first instance we are thinking of the different type of ‘raggedness’ (i.e. peaks and troughs) in the quarterly series of inflation and the output gap, a feature that has played hardly any role in the literature so far – the only contribution in this respect, we know, is the work by Franke (2012a).

Apart from these problems, future research may turn also to the following two issues. First, reconsider the microfoundations for lagged inflation and output in the NKPC and dynamic IS equation, which still are arguably ad-hoc – whether they could possibly allow for coefficients on these variables that are larger than one-half.⁴⁵ Second, apply the MM approach to models with a richer theoretical structure, which would also extend more than ‘linearly’ the number of additional (cross-) covariances that may be included in the estimations. The obvious question would then be whether or not the present results will survive.

Finally, we would like to emphasise that, although the MM approach might be criticised by not relying on sufficient statistics, it serves as an alternative estimation technique – especially if it is applied in conjuncture with Bayesian estimation and in order to estimate (highly) non-linear DSGE models. This will be shown in the following two Chapters.

⁴⁵For the ad-hoc nature of the common microfoundations of a hybrid NKPC, see Rudd and Whelan (2005, pp. 20f), which is the longer version of Rudd and Whelan (2007, p. 163, fn 7). An interesting new concept to make the NKPC more flexible is the hazard function studied by Sheedy (2010), although it comes at the cost of a more complicated structure of lagged and also expected inflation.

5.9 Appendix D

5.9.1 D1: Prior Densities of the Bayesian Reference Estimations

The prior densities for GI are essentially taken over from Castelnuovo (2010), which are quite in line with the recent literature. One exception is that we mistrust his relatively high estimate of the policy parameter ϕ_π in the GI period, the posterior mode of which—guided by his prior normal distribution around 1.70—amounts to more than 1.80. Following the results by Lubik and Schorfheide (2007) and Benati and Surico (2009), we prefer a lower prior mean and decide on $\phi_\pi \sim N(1.3, 0.2)$ for this distribution.

α	κ	ρ_π	σ_π
$\beta(0.3, 0.2)$	$\Gamma(0.4, 0.1)$	$\beta(0.6, 0.1)$	$I\Gamma(1.0, 8.0)$
χ	τ	—	σ_x
$\beta(0.5, 0.1)$	$\Gamma(0.037, 0.0125)$	—	$I\Gamma(0.25, 2.0)$
ϕ_π	ϕ_x	ϕ_r	σ_r
$N(1.3, 0.2)$	$\Gamma(1.2, 0.8)$	$\beta(0.5, 0.28)$	$I\Gamma(1.0, 8.0)$

Table 5.6: Prior densities of the BR estimations in Tables 5.1 and 5.5.

Note: β , N , Γ and $I\Gamma$ denotes a Beta, Normal (Gaussian), Gamma and Inverse Gamma distribution, respectively.

Regarding the prior for the price indexation parameter α we cannot draw on Castelnuovo (2010) since, basically (apart from some other specification details), he alternatively fixes α either at zero or one. As his results, like the ones by Ireland (2007) and Cogley and Sbordone (2008) mentioned in the text, favour the purely forward-looking NKPC with $\alpha=0$, we choose a prior mean less than 0.50 but still with some scope for α to move to higher values in the estimation process. So we assume $\alpha \sim \beta(0.3, 0.2)$. Nevertheless, as seen from Table 5.1, the estimation shows a strong tendency for α to lean against zero again. To be self-contained, these and the other priors are listed in Table A5.6; first row in the cells.⁴⁶ We checked that the posterior densities to which they give rise are in fact well-behaved. This concerns their relationship to the prior densities as well as the convergence checks by Brooks and Gelman (1998), which are summarized in the uni- and multivariate diagnostics provided by Dynare and are provided upon request.

⁴⁶Note that our rates of interest and inflation are annualized, while Castelnuovo's (2010) are not.

5.9.2 D2: Posterior Predictive Check for GM

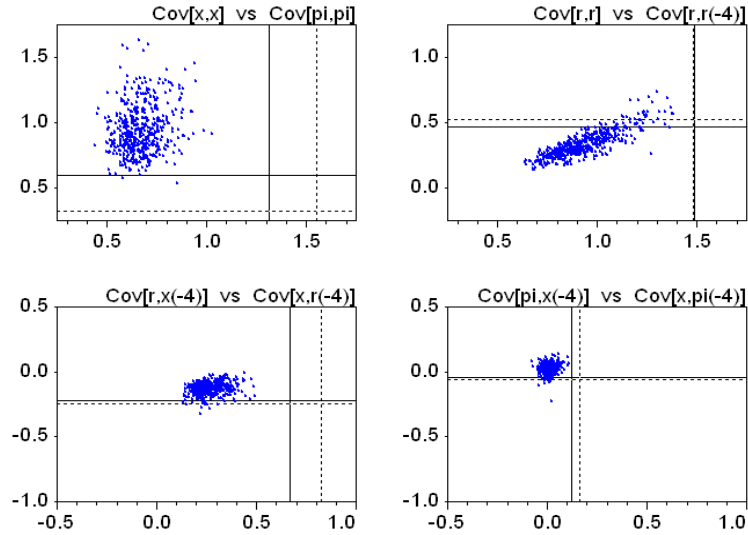


Figure 5.8: Posterior predictive check for estimation BR (GM).

Note: Moments implied by draws from the posterior distribution for BR in Table 5.5. Solid (dotted) lines signify the observed sample moments (the moments generated by model A).

6 Identification of Prior Information via Moment-Matching

6.1 Introduction

As we stated in the previous Chapter, the estimation outcome within the Bayesian approach is sensitive to the choice of prior knowledge. In the majority of studies each parameter of interest is assumed to be independently distributed a priori. The associated sufficient statistics of these so-called *micropriors* are taken from microeconomic evidence. For example an overwhelming amount of empirical studies report a value of the quarterly discount factor β being equal to 0.99 mostly based on descriptive statistics and without consideration of other (deep) parameters. Lombardi and Nicoletti (2012, pp. 294; *LN* henceforth) claim that for some parameters this kind of information is ambiguous due to different estimation outcomes across several studies (e.g. for the Calvo parameter of price setting) while for others it is simply rare (e.g. for the first and second moments of shock processes). Furthermore, they mention that the property of micropriors to be independent put a restriction on the dynamics of the model variables a priori after a shock occurs. In order to avoid these shortcomings the evaluation of macroeconomic variables based on econometric time series analysis, e.g. the moments of observable variables, is considered.

Taking this information into account will lead to the formulation of so-called *macropriors*, which can be then used as prior information within the Bayesian approach. LN extract macropriors from IRFs and state that this kind of priors help to clarify how the observables react to a specific shock a priori without explicit consideration of a data-based pre-sample. Similarly, Del Negro and Schorfheide (2008; *DNS* henceforth) formulate macropriors based on the second moments of the data. Furthermore *DNS* (p. 1193) emphasize that the problem of choosing micropriors is given by “the mechanical use of identical prior distribution for alternative model specifications”.

The question arises to what extent the choice of micro- or macropriors has an influence on the estimation outcome. In particular, how sensitive will the estimation results be to a change in the functional form of the micropriors? With respect to our investigations, which had been undertaken in the previous Chapter: is the MM approach able to identify sufficient macropriors, i.e. prior information which leads to higher values of the (logarithmic) marginal likelihood within Bayesian estimation as in the case of micropriors? In order to answer these questions, in this Chapter we apply a sensitivity analysis regarding different kinds of prior information. Then we compare the corresponding outcome (in terms of the goodness of fit) to the one when a ‘standard’ prior information, which is in line with the recent literature and documented in Castelnovo (2010), is taken into account. The underlying model is directly taken from the previous Chapter. It consists on the dynamic system given by the equations (5.10) to (5.13) without any change in the notation of the associated variables and parameters.

We account mainly for two things. First, we investigate the impact of the transformation of

those model parameters which are bounded to the unit interval as discussed in the paper by *LN*. The idea is to allow for a more diffuse prior distribution of the parameters of interest through transformation (e.g. assuming a Gamma instead of a Beta distribution). Hence, we compare the estimation outcome under three alternative sets of micropriors, namely the baseline estimation (BR) taken from the previous Chapter and two specifications, which differ with respect to the transformation of the price indexation parameter. Second, we shed a light on the investigation of a combination of both MM and Bayesian (maximum likelihood) techniques. Since our parameter estimates obtained via MM account for the second moments of the underlying time-series they can be identified as macropriors for Bayesian estimation. In this respect we define a two-stage estimation procedure: we take the point estimates from the MM estimates for the GI and GM periods, respectively, and consider them as prior mean values of the parameters within Bayesian estimation. For a robustness check, we vary the priors with respect to the standard deviation of the persistence parameters α , χ , and ρ_π in order to allow for a more diffuse prior information.

In both experiments we seek to answer the question which kind of prior information is most reliable in order to ensure the best fit of the structural model to the data measured by the (logarithmic) marginal likelihood Υ . We also report the posterior probability $\Pi(BR)$, where the reference model is given by the Bayesian estimation of the model using the standard prior information (cf. section 5.3).

The remainder of this Chapter is organized as follows. In the following section we present the transformation rules for the parameters, which are either bounded below by zero or on the unit interval. In addition, we introduce the two-stage estimation procedure, where the prior mean values for Bayesian estimation are given by the estimates obtained from MM. Finally, we compare all estimation outcomes relative to the ones from Bayesian estimation, where standard prior information from the literature is considered. In section 6.3 we contrast our estimation outcomes for the endo- and exogenous persistence parameters (by applying the two-stage estimation procedure as described above) to the results taken from the literature. Section 6.4 concludes. Appendix E contains the Tables regarding the prior information for the GI and GM periods, respectively. Furthermore, the graphical representations of the posterior densities for all parameters to be estimated across all specifications are presented here.

6.2 Micro- versus Macropriors

6.2.1 Micropriors: The Impact of Transformation

In the following we consider the experimental set-up from the previous Chapter with respect to the underlying NKM and the empirical data. Furthermore, we make use of the same notation of the variables (expressed in gaps) and parameters known from Chapter 5. We follow *LN* (p. 299) and transform the parameters which are either bounded below by zero or on the unit interval. This is especially of interest in such cases presented here, where we find the price indexation parameter $\alpha \approx 0$ based on standard, i.e. non-transformed priors (BR) and $\alpha = \chi = 1, \rho_\pi = \phi_x = \sigma_r = 0$ based on MM in the GI period (cf. estimation *A* provided in Table 5.1).

For the GM period we find $\alpha > 0, \tau \approx 0$ via BR and $\rho_\pi = 0, \kappa \approx 0$ and even $\chi = \infty$ via MM (cf. estimation *C* provided in Table 5.5). It goes without saying, that in both cases we

choose the MM estimation outcome associated to the lowest value of J . By applying the transformation rules presented below within the Bayesian estimation, we are able to avoid narrow restrictions on those parameters of interest due to a change in their functional form. In this respect transformation means that e.g. a Gamma instead of a Beta distribution is assumed, where the former covers the range from a lower boundary point to unity, while the latter is bounded on the unit interval. In other words, we allow for differences in the micropriors in order to search for the posterior mean values over a broader region of the parameter space relative to the standard prior information we had considered so far. In the end we perform a model comparison exercise.

First, we consider the following relationship:

$$\tilde{k} = f(k) = \frac{1}{1-k} \text{ with } k = \{\chi, \rho_\pi, \phi_r\} \quad (6.1)$$

where $\tilde{k} = \{\tilde{\chi}, \tilde{\rho}_\pi, \tilde{\phi}_r\}$ represents the transformed counterparts of χ, ρ_π and ϕ_r . The corresponding functional form of the priors is given by $\tilde{k} \sim \Gamma(2, 0.99, 1, \infty)$.¹ Given the posterior mean of \tilde{k} , the value of the original parameters can be easily computed:

$$k = f^{-1}(\tilde{k}) = \frac{\tilde{k} - 1}{\tilde{k}}. \quad (6.2)$$

Note that, although all three original parameter values are bounded on the unit interval, their transformed counterparts are not. To see this consider both polar cases where the posterior mean values of \tilde{k} are either equal to one or an arbitrary high number (even close to infinity), which is possible under the assumption of the Gamma distribution. According to the transformation rule (6.2), k becomes then zero or one, respectively.

In contrast τ and κ are bounded below by zero only. The relationship between the original parameters $z = \{\tau, \kappa\}$ and their transformed counterparts $\tilde{z} = \{\tilde{\tau}, \tilde{\kappa}\}$ reads

$$\tilde{z} = f(z) = \frac{1}{z} \quad (6.3)$$

where $\tilde{z} \sim \Gamma(2, 0.99, 0, \infty)$ holds. Accordingly, the corresponding transformation rule is given by

$$z = f^{-1}(\tilde{z}) = \frac{1}{\tilde{z}}. \quad (6.4)$$

Suppose that the posterior mean values of \tilde{z} are zero, infinity, one or less than one. After transformation z becomes infinity, zero, one or greater than one, respectively, according to (6.4).

Regarding the parameter of price indexation α , we distinguish between two specifications based on the transformation rules (6.2) and (6.4):

$$\text{T1: } \alpha = \frac{\tilde{\alpha} - 1}{\tilde{\alpha}}, \quad \tilde{\alpha} \sim \Gamma(2, 0.99, 1, \infty) \quad (6.5)$$

$$\text{T2: } \alpha = \frac{1}{\tilde{\alpha}}, \quad \tilde{\alpha} \sim \Gamma(2, 0.99, 0, \infty) \quad (6.6)$$

Allowing for two different transformations of α is motivated by our findings in the previous Chapter. In the first case T1 we stick to the economic rationale for the price indexation

¹According to this expression the priors $\tilde{\chi}, \tilde{\rho}_\pi$ and $\tilde{\phi}_r$ follow a Gamma distribution with a mean value of 2, a standard deviation of 0.99 and are bounded between unity and infinity.

parameter bounded on the unit interval. In the second case T2 we account for our observation from section 5.6.3 where we report evidence for a stronger backward-looking behavior in the NKPC, i.e. $\alpha > 1$. Finally, we are going to investigate, what kind of specification leads to a better fit of the model to the data relative to the other one. Therefore, specification T1 makes use of the transformed micropriors given by (6.1), (6.3) and (6.5) while for specification T2 we consider (6.1), (6.3) and (6.6). Accordingly, we report the posterior point estimates and their corresponding standard deviations, which are computed by applying the transformation rules (6.2) and (6.4). Note that we set all prior mean values equal to 2 which (regardless of the type of transformation) leads to a moderate value of the parameters equal to 0.5 a priori. Across both specifications the functional forms of $\phi_\pi, \phi_x, \sigma_\pi, \sigma_x$ and σ_r remain unchanged as motivated by *LN* (p. 300).

The estimation outcomes for T1 and T2 are reported in the third and fourth column of the Tables 6.1 (GI) and 6.2 (GM). For each period we compare our results to the one in the second column where for the BR case the prior information given by Castelnuovo (2010) is used. We find strong evidence for the superiority of BR for the GI period relative to T1 and T2. Even though it can be seen that the marginal likelihood Υ is higher in the BR case, in addition, the posterior probability $\Pi(BR)$ is also strictly in favor of the use of the standard prior information: a value of unity indicates that the probability that the null hypothesis is true — i.e. the reference model BR is superior in describing the data relative to the alternative models T1 and T2 — is given by hundred percent. Furthermore, the value of the loss function J of BR is (slightly) below the ones of T1 and T2. Moreover, the number of moments that missed the confidence intervals of the empirical moments (MCI) is significantly higher for T1 and T2. Note that here we calculate J according to equation (5.1) from Chapter 5 under consideration of the vector of second moments $m(\hat{\theta})$ given the point estimates $\hat{\theta}$. As we turn to a closer look of the point estimates it can be seen that most of them remain almost unchanged across specifications — remarkable differences can be observed regarding α (significantly greater than zero in T1), τ (nearly three times larger in T1 and T2 compared to BR) and σ_π (larger in T1 and T2 compared to BR).

The results above are confirmed for the GM period. Based on the values for $\Pi(BR)$, the Bayesian estimation of the model using the standard prior information still outperforms the ones using transformed parameters, i.e. the null hypothesis that the reference model BR is the true model can not be rejected. Like for the GI period there are no remarkable differences in the point estimates across specifications with only a few exceptions (see above) — in particular for ρ_π (significantly lower in T1 and T2 compared to BR), while τ remains nearly unchanged. To sum up, in all cases the evidence is in favor of this kind of micropriors which are used in the reference model BR. In other words, the marginal likelihood Υ does not increase when a transformation (i.e. a change in the functional form) of the parameters as in T1 and T2 is applied. This holds independently for the assumption regarding the economic rationale for the price indexation parameter, i.e. if it is assumed to be true (T1: $0 \leq \alpha \leq 1$) or not (T2: $\alpha > 1$).²

²It is indeed the case as we consider T2 as the reference model, the posterior probability ($\Upsilon(T2)$ versus $\Upsilon(T1)$) is given by $\Pi(T2) = 0.887$ for the GI and even $\Pi(T2) = 0.974$ for the GM period. Hence, allowing for a (non-plausible) value of α greater than 1 leads to a better goodness of fit. However, these observations are pure hypotheticalal since — as we discussed above — both estimations (T1 and T2) are inferior relative to estimation BR.

GI	Bayesian Prior Information			MM Prior Information			
	BR	T1	T2	V0	V1	V2	V3
α	0.074	0.300	0.137	0.790	0.783	0.610	0.580
	0.000-0.156	0.194-0.700	0.000-0.307	0.625-0.984	0.615-0.983	0.375-0.863	0.324-0.849
κ	0.209	0.202	0.222	0.044	0.044	0.037	0.038
	0.125-0.290	0.141-0.353	0.156-0.395	0.010-0.075	0.010-0.076	0.005-0.066	0.005-0.068
ρ_π	0.570	0.410	0.543	0.052	0.064	0.103	0.135
	0.452-0.693	0.012-0.555	0.230-0.665	0.000-0.111	0.000-0.137	0.000-0.225	0.000-0.286
σ_π	0.694	0.867	0.801	0.771	0.767	0.760	0.753
	0.524-0.864	0.659-1.073	0.592-1.013	0.674-0.871	0.670-0.868	0.653-0.863	0.646-0.866
χ	0.767	0.852	0.853	0.955	0.956	0.948	0.949
	0.689-0.850	0.751-0.894	0.743-0.894	0.908-0.999	0.909-0.999	0.895-1.000	0.896-0.999
τ	0.048	0.135	0.136	0.085	0.085	0.082	0.082
	0.030-0.067	0.105-0.185	0.107-0.189	0.053-0.118	0.052-0.118	0.050-0.115	0.049-0.114
σ_x	0.552	0.617	0.625	0.481	0.481	0.482	0.483
	0.465-0.637	0.504-0.721	0.511-0.735	0.420-0.541	0.418-0.538	0.420-0.538	0.421-0.540
ϕ_π	1.387	1.430	1.433	1.265	1.263	1.262	1.260
	1.124-1.644	1.142-1.707	1.143-1.704	1.147-1.385	1.145-1.384	1.142-1.381	1.141-1.384
ϕ_x	0.759	0.639	0.649	0.211	0.209	0.213	0.211
	0.314-1.193	0.301-0.993	0.285-0.993	0.039-0.371	0.037-0.366	0.043-0.373	0.034-0.365
ϕ_r	0.742	0.663	0.668	0.586	0.584	0.587	0.586
	0.668-0.816	0.546-0.730	0.556-0.735	0.511-0.665	0.507-0.661	0.511-0.665	0.511-0.668
σ_r	0.745	0.786	0.781	0.788	0.787	0.786	0.785
	0.643-0.841	0.675-0.900	0.672-0.892	0.678-0.899	0.676-.894	0.675-0.898	0.675-0.895
Υ	-341.129	-349.755	-347.690	-337.356	-337.944	-337.205	-337.633
$\Pi(BR)$	0.500	1.000	1.000	0.023	0.039	0.020	0.030
J	213.512	237.934	222.343	99.280	100.291	131.282	134.565
MCI missed	23	32	26	3	4	9	11

Table 6.1: Bayesian estimation: sensitivity analysis regarding the prior information (GI).

Note: The results for T1 and T2 are computed by applying the formulas (6.2) and (6.4) given the transformed posterior point estimates and their corresponding standard deviations. Υ denotes the logarithmic marginal likelihood evaluated via the Laplace approximation (see step (5) given in section 5.3 for an explanation). The posterior probability of the null hypothesis (i.e. the reference model BR is the true model) is given by $\Pi(BR) = \frac{\Xi_{BR,i}}{1+\Xi_{BR,i}}$ with $\Xi_{BR,i} = \exp(\Upsilon_{BR} - \Upsilon_i) \cdot \frac{\Lambda_{BR}}{\Lambda_i}$ and $i = \{T1, T2, V0, V1, V2, V3\}$. By construction $\Xi_{BR, BR} = \exp(\Upsilon_{BR} - \Upsilon_{BR}) \cdot \frac{\Lambda_{BR}}{\Lambda_{BR}} = 1$ holds, which leads to $\Pi(BR) = \frac{\Xi_{BR, BR}}{1+\Xi_{BR, BR}} = 0.5$. The definitions of all of these expressions are explained in step (6) given in section 5.3.

6.2.2 Macropriors: An Investigation via MoMBay

In our second experiment we describe a two-stage estimation approach. At the first stage, the parameters of interest are estimated via MM. Then the resulting point estimates are adopted as prior mean values within the Bayesian estimation. Since these point estimates are the result of the matching of the second moments of the structural model to its empirical counterparts, they can be identified as macropriors. The prior mean values (obtained via MM) can be seen as appropriate starting values which might help to reduce the uncertainty in the researcher's choice of the prior information. We call this procedure a *Moment-Matching* based Bayesian estimation approach or simply *MoMBay* for short.

In order to judge the reliability of MoMBay, we compare the outcome of this approach to the one when using micropriors reported in BR based on Castelnuovo (2010) evaluated via the (logarithmic) marginal likelihood Υ and the posterior probability $\Pi(BR)$. Obviously, we have to determine the functional form of the priors first. The prior mean values are equal to the results of the parameter estimates from the MM estimation A for the GI and the GM period (cf. Tables 5.1 and 5.5 of Chapter 5), respectively. Regarding the prior distributions of the parameters we adopt the one from BR. Due to the fact that as an outcome of the A estimations some parameter values are close to or even hit their boundary points (i.e. $\alpha, \kappa, \rho_\pi, \chi, \phi_x$ and σ_r in the GI case as well as κ, ρ_π, χ and τ in the GM case), within Bayesian estimation the prior mean values are set to 0.1 and 0.9, respectively. For the sake of uniformity all corresponding prior values of the standard deviations are set to 0.08.³ We call this specification V0. Furthermore we consider three additional specifications where we account for more diffuse prior information regarding the extrinsic (ρ_π) and intrinsic (α and χ) persistence parameters. Within specification V1 we increase the prior mean and the standard deviation of ρ_π to 0.2 and 0.15 respectively. Within specification V2, $\rho_\pi \sim \beta(0.2, 0.15)$ together with the intrinsic persistence parameters $\alpha, \chi \sim \beta(0.8, 0.15)$ hold. Finally, in specification V3, $\rho_\pi \sim \beta(0.3, 0.2)$ together with $\alpha, \chi \sim \beta(0.8, 0.15)$ is applied. The functional forms of all other parameters remain the same as specified in V0. An overview of all prior densities are given in the Tables 6.4 (GI) and 6.5 (GM) in Appendix E1.

This procedure is motivated by the observation that given the BR estimation results, the dynamics in inflation is explained heavily by extrinsic rather than intrinsic persistence, i.e. $\alpha \approx 0$ and $\rho_\pi \gg 0$ hold (while χ is high). In this respect we try to investigate if this result still holds, when MoMBay is applied or if there is evidence just for the opposite, as reported in the MM case given the marginal likelihood. In other words, we seek to answer the question if, as an outcome of MoMBay, the posterior mean values of the three persistence parameters tend to depart from their prior ones (taken from MM; see above) to the left (α and χ) and right (ρ_π) of the posterior distribution. In such a case the results from Bayesian estimation are confirmed under consideration of macro- rather than micropriors - if, of course, the value of the posterior probability will be in favor of the estimations based on MoMBay.

The results for V0 till V3 are reported in the fifth to eighth columns of Table 6.1 (GI) and 6.2 (GM). It turns out that the posterior probability $\Pi(BR)$ is in favor for all specifications where macropriors are considered. Independently of the underlying sample period the following observation holds: Given the values of $\Pi(BR)$ the probability that BR is the true model varies between 2 to 3.9 percent in the GI case and be even 0 percent in the GM case. This

³This procedure ensures that the inverse Hessian matrix of the second derivatives (evaluated at the posterior mode) will be positive-definite which is a requirement for applying the Random-Walk Metropolis-Hastings algorithm (cf. section 5.3).

GM	Bayesian Prior Information			MM Prior Information			
	BR	T1	T2	V0	V1	V2	V3
α	0.033	0.224	0.057	0.502	0.501	0.285	0.279
	0.000-0.071	0.160-0.366	0.000-0.120	0.364-0.639	0.362-0.641	0.144-0.424	0.137-0.419
κ	0.163	0.155	0.178	0.026	0.026	0.030	0.030
	0.103-0.221	0.119-0.224	0.134-0.267	0.004-0.046	0.004-0.046	0.007-0.052	0.006-0.053
ρ_π	0.389	0.103	0.218	0.030	0.035	0.059	0.066
	0.274-0.510	0.000-0.201	0.000-0.340	0.000-0.064	0.000-0.078	0.001-0.123	0.000-0.143
σ_π	0.517	0.635	0.644	0.528	0.526	0.567	0.563
	0.420-0.611	0.535-0.730	0.512-0.768	0.460-0.593	0.460-0.594	0.486-0.646	0.482-0.644
χ	0.825	0.880	0.881	0.961	0.960	0.943	0.945
	0.759-0.891	0.804-0.913	0.808-0.913	0.917-1.000	0.917-1.000	0.874-0.999	0.879-1.000
τ	0.017	0.081	0.080	0.015	0.015	0.012	0.012
	0.009-0.025	0.066-0.103	0.065-0.100	0.004-0.026	0.003-0.026	0.000-0.021	0.000-0.021
σ_x	0.346	0.437	0.435	0.347	0.347	0.347	0.347
	0.296-0.399	0.366-0.504	0.363-0.504	0.307-0.386	0.309-0.386	0.308-0.386	0.307-0.386
ϕ_π	1.181	1.258	1.243	1.595	1.596	1.596	1.594
	1.001-1.383	0.965-1.510	0.962-1.505	1.465-1.727	1.467-1.729	1.465-1.727	1.459-1.725
ϕ_x	1.014	0.944	0.917	1.038	1.040	1.035	1.037
	0.602-1.419	0.587-1.230	0.558-1.254	0.909-1.166	0.910-1.167	0.912-1.168	0.910-1.166
ϕ_r	0.814	0.731	0.728	0.861	0.861	0.861	0.861
	0.762-0.867	0.659-0.778	0.656-0.774	0.821-0.900	0.822-0.900	0.822-0.898	0.825-0.902
σ_r	0.449	0.537	0.538	0.491	0.491	0.489	0.491
	0.395-0.502	0.466-0.604	0.467-0.601	0.436-0.544	0.438-0.546	0.437-0.543	0.438-0.554
Υ	-299.756	-327.730	-324.108	-287.655	-288.385	-283.164	-283.719
$\Pi(BR)$	0.500	1.000	1.000	0.000	0.000	0.000	0.000
J	170.098	246.827	243.162	154.373	154.873	152.265	153.968
MCI missed	15	25	27	14	13	16	16

Table 6.2: Bayesian estimation: sensitivity analysis regarding the prior information (GM).

Note: The results for T1 and T2 are computed by applying the formulas (6.2) and (6.4) given the transformed posterior point estimates and their corresponding standard deviations. Υ denotes the logarithmic marginal likelihood evaluated via the Laplace approximation (see step (5) given in section 5.3 for an explanation). The posterior probability of the null hypothesis (i.e. the reference model BR is the true model) is given by $\Pi(BR) = \frac{\Xi_{BR,i}}{1+\Xi_{BR,i}}$ with $\Xi_{BR,i} = \exp(\Upsilon_{BR} - \Upsilon_i) \cdot \frac{\Lambda_{BR}}{\Lambda_i}$ and $i = \{T1, T2, V0, V1, V2, V3\}$. By construction $\Xi_{BR, BR} = \exp(\Upsilon_{BR} - \Upsilon_{BR}) \cdot \frac{\Lambda_{BR}}{\Lambda_{BR}} = 1$ holds, which leads to $\Pi(BR) = \frac{\Xi_{BR, BR}}{1+\Xi_{BR, BR}} = 0.5$. The definitions of all of these expressions are explained in step (6) given in section 5.3.

leads to the conclusion that the specifications V0 to V3 are able to describe the data well. Moreover, applying macro- rather than micropriors within Bayesian estimation results in a slight (GI) and distinct (GM) improvement in the goodness of fit in terms of Υ .⁴

Four observations are also worth mentioning. First, although the values of the marginal likelihoods Υ do not differ strongly across V0 to V3 and especially in contrast to BR (when looking at GI; see the corresponding entries in Table 6.1), the effect on $\Pi(BR)$ is indeed remarkable. Hence, even a ‘small’ difference in the marginal likelihood across models (e.g. for GI: $|\Upsilon(BR)| - |\Upsilon(V2)| = |3.924|$) leads to strong changes in the reliability of the null hypothesis measured by $\Pi(BR)$. Second, the goodness of fit of V0 to V3 is also confirmed when looking at the corresponding indices known from the MM approach: higher values of Υ correspond with lower values for J and MCI . This means that the matching of the simulated to the empirical moments is — at least in the GI case — very good when macropriors are considered (given J together with MCI). Third, some of the point estimates of the parameters evaluated via MoMBay differ significantly from the ones of the BR model (see below). However, the point estimates remain unchanged the more we choose relative diffuse priors for the persistence parameters, i.e. as we go from V0 to V3. Exceptions are given by the values for α and ρ_π : The price indexation parameter is estimated to be roughly 30 (78) percent *bigger* in V0 and V1 compared to V2 and V3 in the Great Inflation (Moderation) period. Regarding the shock persistence this previous observation is mirror inverted, since the parameter is estimated to be roughly 51 to 53 percent *lower* in V0 and V1 compared to V2 and V3 in the GI and GM period, respectively. Finally, these observations support the analysis undertaken by *LN* who claim that prior knowledge is sensitive to the use of macropriors, i.e. the moments of observable variables. In the following we will discuss the results for the GI and GM period separately in greater detail.

According to Table 6.1 specification V0 provides a higher value for Υ (-337.356) relative to BR (-341.129) in the GI case. Although the marginal likelihood for V2 (-337.205) is slightly higher than for V0, we claim that V0 is the most appropriate candidate within MoMBay since the values of J and MCI are the lowest. As we turn to a comparison of the point estimates between BR and V0, differences can be identified. The results for α (0.790) and ρ_π (0.052) reflect our observations from the previous Chapter. In this respect using point estimates evaluated via MM as prior information within Bayesian estimation, this leads to distinct evidence for intrinsic rather than extrinsic persistence in the baseline NKM. Hence, concerning the source of persistence, the outcome of the plain MM approach (estimation A given in Table 5.1) is confirmed through MoMBay. More precisely, the Bayesian approach largely confirms the MM prior information, which is approved by the values of Υ and $\Pi(BR)$. Furthermore, the monetary policy parameter ϕ_x as well as the standard deviation of the interest rate shock σ_r are estimated to be significantly greater than zero (as reported in Chapter 5 for GI based on the corresponding estimation A). While the difference in V0 compared to the reference MM estimation A for ϕ_x is moderate (0.211), the value of σ_r is quite high (0.788). Not surprisingly, these results depend heavily on the fact that the macropriors are non-diffuse, i.e. small standard deviations are applied for all parameters within specification V0. Nevertheless, recall that all measurements for the goodness of fit presented here prove that the reliability of V0 over BR in describing the data is unambiguous.

⁴Of course, the expressions ‘slight’ and ‘distinct’ seem to be a little bit broad in this context. The computation of the posterior probability given a V -specification, i.e. V0 to V3, as being the reference model could help to get things straight. For a clear arrangement we do not apply such kind of analysis, which can be motivated by our observations discussed in section 6.3.

The outcome for the GM case is depicted in Table 6.2. Here we claim that specification V2 is superior over BR in terms of the log marginal likelihood (-283.164 vs. -299.756) and the values for J (152.265 vs. 170.098). However, it must be emphasized that the matching seems not to be a good one since the values for J are quite high compared to the reference estimation A ($J = 54.1$; Table 5.5). This can also be seen by looking at the values for MCI : the number of moments missed the confidence interval of the empirical moments is higher for V2 (16) than for BR (15). The lowest values is actually given for the specification V1 (13). A reason for this can probably be found in the steep initial decline in the auto-covariance profile in the inflation gap as discussed in section 5.7 (cf. Figure 5.6). Concerning the discussion of the point estimates one observation is worth mentioning. As we compare the estimates of α in V2 (0.285) to the reference estimation A (0.816) and BR (0.033), we come to the conclusion that, first, allowing for more diffuse macropriors (as in V2), α tends towards a lower value than 0.5 and second, there is a low degree of (intrinsic, extrinsic *and* inherited) persistence in the inflation gap, even though the standard deviation in the shock σ_π is quite large.⁵ While the other parameter estimates remain nearly the same for V2 in contrast to the reference estimation A (and BR), the low estimated value of α sheds a new light on the issue of backward-looking behavior in the NKPC. In particular, persistence plays a *minor* role in describing the dynamics in the inflation gap as indicated by evidence from Bayesian (MoMBay) as well as MM. This observation is even more surprising as $\Pi(BR)$ is strictly in favor of specification V2. Nevertheless, it is an open question if this result still holds if e.g. the level of inflation instead of the gap is considered. We leave this to further research.

6.3 Comparison with other Empirical Studies

Next, we seek to compare our results to the most prominent studies from the literature. In *DNS* three different groups of model parameters are considered, which describe the steady-state relationships, exogenous processes and (roughly speaking) the degree of persistence. Prior information on these three groups is either given by micro- or macropriors. In the former case standard priors from the literature are used. In contrast, macropriors (primarily for the parameter of the exogenous processes) are constructed based on a quasi-likelihood function where the second-moments of an auxiliary vector autoregression (VAR) model are taken into account.⁶ A similar procedure is also adopted in *LN*. In addition, the authors introduce an estimation approach where the macropriors are constructed through the minimization of IRFs — which is indeed similar to the matching of second-moments presented here. Following *DNS* (p. 1196), the authors then divide the vector of structural parameters into two blocks of parameters, which represent the steady state relationships among

⁵Note that the prior mean value of α is set from 0.816 to 0.8 across the alternative specifications (see the first row in Table 6.5 given in Appendix E1). As it can be seen by the graphical comparison of the corresponding prior to the posterior distribution in Figure 6.13 which is provided in Appendix E2, the posterior mean value of α indeed tends to the left (relative to the prior one) when applying the Bayesian approach.

⁶A quasi-likelihood function shows similar properties to the standard log likelihood function except for the specification of the parameters' distribution. Instead a quasi-likelihood function consists on a relationship between the variance and the mean of the observations only (see Wedderburn (1974) for a detailed description of this approach). A quasi-likelihood function is most often used in conjunction with an auxiliary model for the purpose of estimation. The corresponding procedure is known as *Indirect Inference* (II). While this approach allows endogenizing the choice of moments (rather than chosen arbitrarily by the researcher within MM), it becomes an open question which kind of auxiliary model will be a good approximation to a DSGE model (*LN* (p. 298, fn. 9)). An overview of the II approach is provided by Carrasco and Florens (2002). A definition and a more elaborate discussion on the comparison of II to MM (when estimating a non-linear model) is given in section 7.4.

observable variables and the law of motion of the exogenous processes, respectively. These two blocks can be either related or non-related to each other. In both studies the regular algorithm within Bayesian estimation (cf. section 5.3) is modified in order to account for the macropriors as discussed above.

We emphasize that MoMBay is located in-between both methods presented in DNS and LN. Obviously, like in *DNS* the second-moments of observables are considered while as in *LN* IRFs are matched. In comparison to *DNS*, we apply our estimation procedure directly (i.e. without use of an auxiliary model) under consideration of the MM approach, which is close to the GMM estimation approach (instead of a specific likelihood function) like in *LN*. Both studies report evidence for differences in the posterior estimates with respect to the underlying prior information. Although, these studies are closely related to our experimental setting presented here, the interpretation of the results must be done with some caution. First, *DNS* investigate a small-scale NKM with sticky prices *and* wages. Furthermore, in their paper they report the outcome of three so-called Low Rigidities, High Rigidities and Agnostic scenarios with respect to micro- and macropriors. Across these scenarios the prior information for the Calvo parameter of price and wage stickiness becomes more diffuse. Note that in our study we consider the baseline NKM with sticky prices only while the slope of the NKPC is characterised by the composite parameter κ . Hence, we do not control *directly* for the Calvo parameter of price stickiness within our estimation. LN also work with the baseline NKM but assume a purely forward-looking dynamic IS curve while there exists only a non-autocorrelated (one-off) shock in the hybrid NKPC. However, besides price indexation, persistence is brought into inflation by considering an autocorrelated preference and technology shock. Needless to say, that in both studies the levels (given as demeaned values) rather than the (time-dependent) gaps of the variables are taken into account. Table 6.3 summarizes the differences in the estimation procedures.

Although there exist structural differences in the setting of our experiments compared to the ones in *DNS* and *LN*, the results help to motivate a fruitful discussion on micro- vs. macropriors. For the sake of clarity (and in order to ensure the comparability with our results), we focus on the outcome for the price indexation parameter (here: α , *LN*: ω , *DNS*: ι_p) and the persistence parameter in the cost-push/mark-up shock (here: ρ_π , *LN*: none, *DNS*: ρ_λ) only. The estimated values for DNS are given in Table 6 of their paper, where they discuss the results using micropriors (denoted as \wp_s) and macropriors (denoted as \wp_{QL}) expressed by (Low Rigidities/Agnostic/High Rigidities). In LN their Table B2 (micropriors) and Table B3 together with Table B4 (macropriors) given in their Appendix B are primarily of interest.

DNS report significant differences in the estimation results (in terms of posterior mean values and the goodness of fit) with respect to both types of priors. While across all scenarios in the case of micropriors the values of ι_p are low (0.19/0.19/0.20), using macropriors the observation becomes ambiguous (0.47/0.14/0.08). Note here, that the confidence bands are quite wide and include zero except for the Low Rigidities scenario. Hence, at least for the latter it can be stated that there exists evidence for (a moderate degree of) price indexation if macropriors are considered. One explanation for the low value of ι_p in the case of micropriors is given by the associated high values of ρ_λ where (0.86/0.56/0.41) holds - a result which is confirmed by our study (cf. both BR estimates for the GI and GM period). Interestingly, the corresponding vector in the macroprior case is given by (0.89/0.88/0.68) where the first entry indicates that a high value for ρ_λ coincides with a high value of ι_p . Hence, there exists both intrinsic and extrinsic persistence in inflation where the latter one is more distinctive.

<i>Characteristics</i>	<i>DNS</i>	<i>LN</i>	<i>MoMBay</i>
Moments	(Auto)Covariances	IRFs	(Auto)Covariances
Matching Procedure	Likelihood Maximization, Auxiliary VAR	Minimum Distance	Minimum Distance
Structural Model	Hybrid NKM (sticky wages and prices)	Partly forward-looking NKM (sticky prices only)	Hybrid NKM (sticky prices only)
Sample Period(s)	1982:4–2005:4	1960:1–2001:4	1960:1–1979:2 1982:4–2007:2
Data	Level (demeaned)	Level (demeaned)	Gap (time-dependent)
Price Indexation	ι_p	ω	α
↔ Micro:	0.19 / 0.19 / 0.20	0.77	0.074 / 0.033
↔ Macro:	0.47 / 0.14 / 0.08	0.76	0.790 / 0.285
Persistence Parameter	ρ_λ	—	ρ_π
↔ Micro:	0.86 / 0.56 / 0.41	—	0.570 / 0.389
↔ Macro:	0.89 / 0.88 / 0.68	—	0.052 / 0.059

Table 6.3: Construction of macropriors.

Note: The results using micro- and macropriors are given in the corresponding rows ‘Micro’ and ‘Macro’, respectively. The estimated values for DNS are given in Table 6 of their paper where they discuss the results using micropriors (denoted as φ_s) and macropriors (denoted as φ_{QL}) expressed by so-called (Low Rigidities/Agnostic/High Rigidities) scenarios. For LN the Table B2 (micropriors) and Table B3 together with Table B4 (macropriors) are considered. With respect to the results from MoMBay, the first entry in the row ‘Micro’ denotes the value for GI and the second for GM based on the Bayesian estimation *BR*. The first entry in the row ‘Macro’ denotes the value for GI and the second for GM based on the estimations *V0* and *V2*, respectively.

An explanation for this phenomenon is not easy to reveal because in their paper DNS focus on a discussion of price and wage rigidity only. Nevertheless, the authors show that the lowest values in both types of rigidities as well as a quite low level of habit formation (which is denoted by h in their paper) can be found in the Low Rigidities scenario. Since a low degree of price rigidity leads to a high value of the slope of the NKPC, *inherited* persistence plays a major role in explaining the dynamics in inflation.⁷ It follows from that, although inherited persistence is important, the persistence in the output gap is not high enough in order to explain the volatility in inflation. In other words, if there is empirical evidence for persistence in inflation while the habit formation parameter is estimated to be low, intrinsic

⁷Recall from DNS (p. 1195, equation (9)) that under price indexation, the slope of the NKPC is given by $\frac{(1-\zeta_p\beta)(1-\zeta_p)}{\zeta_p(1+\iota_p\beta)}$ where – for a given value of the discount parameter β and price indexation ι_p – this expression increases as price rigidity ζ_p decreases. See also our discussion on the frequency-dependent inherited persistence (FIP) effect in section 2.3.

as well as extrinsic inflation persistence must be significantly higher than zero – in order to account for inertia in the corresponding times series. Hence, both values of ι_p and ρ_λ have to be unequal to zero if h is low. Again, these observations must be interpreted with some caution: DNS (p. 1207) notice that probably the time series they use be not informative enough to analyze different scenarios since some posterior estimates mimic their prior values. Our study does not face this kind of problem — with only two exceptions. In particular, for the policy parameters ϕ_π and ϕ_x the corresponding prior distributions do not differ from its posterior counterparts. This holds only in the GM case and across the specifications V0 to V3 (cf. Figures 6.11 to 6.14 given in Appendix E2). Here it seems that the posterior distribution is driven by the data. Since the policy parameters are not primarily of interest in our study we judged this observation not to be severe. However, further investigations on the influence of non-informative times series within Bayesian estimation are always a necessary task to be undertaken in general. In particular, micropriors might be imposed on ϕ_π and ϕ_x in order to check if the problem remains and if the goodness of fit changes.

LN show that no differences in the estimations for the parameter (ω) of interest can be found. Both in the micro- and macroprior (first and second block) case, price indexation is around a high value of 0.77. Although the cost push shock is characterized by an iid-shock, we already stated above that persistence might come into play through the autoregression parameter in the preference and technology shocks, respectively, which have a (in)direct influence on the inflation dynamics (cf. the Appendix A in LN). The corresponding values are estimated around 0.83 to 0.85 (ρ_a) and 0.89 (ρ_g) across all specifications regarding the prior information. The only changes in the parameter estimates can be found in the values for price rigidity, the elasticity of labor and in the standard deviation of the technology and mark-up shock. The authors argue that this problem might be explained by the misspecification of the model described by a high cross-correlation of the shocks. Under consideration of artificial data they show that this indeed holds with exception of preference shocks. According to our Tables 6.1 and 6.2, we see that in our experiment there exist no differences in the standard deviations of the shock at all. However, we do not discard the possibility of misspecification completely since the phenomena might help to explain the changes in the composite parameter κ where the parameter of price rigidity is incorporated. In this respect we call for an explicit consideration of the slope of the NKPC in greater detail — although the number of parameters to be estimated will increase in that case, simply by adding the parameters for various elasticities (cf. the expression(s) for the NKPC in Part One of this thesis).

We sum up that the results presented in DNS and LN mimic those from the literature, where the degree of price indexation is high in the absence of persistence in the cost push (mark-up) shock. If both sources of persistence are considered, the autocorrelation parameter is estimated to be dominant, i.e. being greater than the parameter of price indexation - where the latter can also be insignificant. Moreover, both studies indicate that there exist changes in the parameter estimates if macro- instead of micropriors are applied as prior information. Our experiments (partly) confirm these observations, i.e. we provide evidence for the domination of the persistence parameter if micropriors are considered. The opposite holds under consideration of macropriors, which leads to a better goodness of fit within Bayesian estimation compared to the BR case. Therefore, we claim that the MoMBay approach can be identified as a point of departure for further research.

6.4 Conclusion

The estimation outcome within the Bayesian approach is sensitive to the choice of prior knowledge. In the majority of studies each parameter of interest is assumed to be independently distributed a priori. The associated sufficient statistics of these so-called micropriors are taken from microeconomic evidence. In contrast, prior information based on econometric time series analysis leads to the formulation of so-called macropriors. We show that while micropriors are often used in the literature, applying MM-based macropriors leads to a better fit of the structural model to the data. Therefore, the advantages of Bayesian (sufficiency) and MM estimation (transparency) can be *unified* within a two-stage estimation procedure called MoMBay, where the macroprior information for Bayesian estimation is evaluated via MM.

The MoMBay approach can be seen as an alternative approach for the estimation and evaluation of macroeconomic models in conjuncture with the Bayesian approach. The reason for this is that MoMBay reduces the parameter space from which appropriate starting values (or better: prior mean values) for the optimization within the Bayesian approach are taken from. We show that this indeed leads to a change in the estimates of some parameters. Similar observations are also made by Del Negro and Schorfheide (2008) as well as Lombardi and Nicoletti (2012) who also found differences in the posterior values under consideration of macropriors. Problems are stemming from the existence of less or even non-informative time series and misspecification of the model. A deeper investigation on these topics might be fruitful in further research.

6.5 Appendix E

6.5.1 E1: Prior Densities of the MoM Bay Estimations

GI				
	V0	V1	V2	V3
α	$\beta(0.900, 0.080)$	$\beta(0.900, 0.080)$	$\beta(\mathbf{0.800}, \mathbf{0.150})$	$\beta(\mathbf{0.800}, \mathbf{0.150})$
κ	$\Gamma(0.100, 0.080)$	$\Gamma(0.100, 0.080)$	$\Gamma(0.100, 0.080)$	$\Gamma(0.100, 0.080)$
ρ_π	$\beta(0.100, 0.080)$	$\beta(\mathbf{0.200}, \mathbf{0.150})$	$\beta(\mathbf{0.200}, \mathbf{0.150})$	$\beta(\mathbf{0.300}, \mathbf{0.200})$
σ_π	$I\Gamma(0.614, 0.080)$	$I\Gamma(0.614, 0.080)$	$I\Gamma(0.614, 0.080)$	$I\Gamma(0.614, 0.080)$
χ	$\beta(0.900, 0.080)$	$\beta(0.900, 0.080)$	$\beta(\mathbf{0.800}, \mathbf{0.150})$	$\beta(\mathbf{0.800}, \mathbf{0.150})$
τ	$\Gamma(0.105, 0.080)$	$\Gamma(0.105, 0.080)$	$\Gamma(0.105, 0.080)$	$\Gamma(0.105, 0.080)$
σ_x	$I\Gamma(0.519, 0.080)$	$I\Gamma(0.519, 0.080)$	$I\Gamma(0.519, 0.080)$	$I\Gamma(0.519, 0.080)$
ϕ_π	$N(1.324, 0.08)$	$N(1.324, 0.08)$	$N(1.324, 0.08)$	$N(1.324, 0.08)$
ϕ_x	$\Gamma(0.100, 0.080)$	$\Gamma(0.100, 0.080)$	$\Gamma(0.100, 0.080)$	$\Gamma(0.100, 0.080)$
ϕ_r	$\beta(0.314, 0.080)$	$\beta(0.314, 0.080)$	$\beta(0.314, 0.080)$	$\beta(0.314, 0.080)$
σ_r	$I\Gamma(0.100, 0.080)$	$I\Gamma(0.100, 0.080)$	$I\Gamma(0.100, 0.080)$	$I\Gamma(0.100, 0.080)$

Table 6.4: Prior densities of the MoM Bay estimations (GI).

Note: For specification V0 we adopt the point estimates of the MM estimation A (cf. Tables 5.1 and 5.5) as prior mean values within Bayesian estimation. If MM parameter estimates hit the boundary points, the corresponding prior mean values are set to 0.1 or 0.9, respectively. The same prior distributions as in the BR case are considered (cf. Table 5.6). For the sake of uniformity all corresponding prior values of the standard deviations are set to 0.08. All changes across the specifications V1 to V3 relative to V0 are given in bold type.

GM				
	V0	V1	V2	V3
α	$\beta(0.816, 0.080)$	$\beta(0.816, 0.080)$	$\beta(\mathbf{0.800}, \mathbf{0.150})$	$\beta(\mathbf{0.800}, \mathbf{0.150})$
κ	$\Gamma(0.100, 0.080)$	$\Gamma(0.100, 0.080)$	$\Gamma(0.100, 0.080)$	$\Gamma(0.100, 0.080)$
ρ_π	$\beta(0.100, 0.080)$	$\beta(\mathbf{0.200}, \mathbf{0.150})$	$\beta(\mathbf{0.200}, \mathbf{0.150})$	$\beta(\mathbf{0.300}, \mathbf{0.300})$
σ_π	$I\Gamma(0.200, 0.080)$	$I\Gamma(0.200, 0.080)$	$I\Gamma(0.200, 0.080)$	$I\Gamma(0.200, 0.080)$
χ	$\beta(0.900, 0.080)$	$\beta(0.900, 0.080)$	$\beta(\mathbf{0.800}, \mathbf{0.150})$	$\beta(0.800, 0.150)$
τ	$\Gamma(0.047, 0.080)$	$\Gamma(0.047, 0.080)$	$\Gamma(0.047, 0.080)$	$\Gamma(0.047, 0.080)$
σ_x	$I\Gamma(0.532, 0.080)$	$I\Gamma(0.532, 0.080)$	$I\Gamma(0.532, 0.080)$	$I\Gamma(0.532, 0.080)$
ϕ_π	$N(1.626, 0.08)$	$N(1.626, 0.08)$	$N(1.626, 0.08)$	$N(1.626, 0.08)$
ϕ_x	$\Gamma(1.031, 0.080)$	$\Gamma(1.031, 0.080)$	$\Gamma(1.031, 0.080)$	$\Gamma(1.031, 0.080)$
ϕ_r	$\beta(0.776, 0.080)$	$\beta(0.776, 0.080)$	$\beta(0.776, 0.080)$	$\beta(0.776, 0.080)$
σ_r	$I\Gamma(0.472, 0.080)$	$I\Gamma(0.472, 0.080)$	$I\Gamma(0.472, 0.080)$	$I\Gamma(0.472, 0.080)$

Table 6.5: Prior densities of the MoM Bay estimations (GM).

Note: See Table 6.4.

6.5.2 E2: Graphical Results – Prior versus Posterior Densities

Figures 6.1 to 6.14 show the graphical results from optimization for the GI and GM period discussed in the sections 6.2.1 and 6.2.2. In these standard graphical outputs provided by DYNARE, the dashed/dotted green line represents the posterior mode (calculated from the posterior kernel) *associated* with the posterior distribution (black line; as a result of the Random-Walk Metropolis-Hastings procedure). The red/grey line stands for the prior distribution (collected in the vector $p(\theta)$). It can be stated that all posterior distributions are close to being normal and that the deviations of the posterior mean values from the modes are negligible low i.e. the dashed/dotted green line is close to the maximum of the posterior distribution. Hence the goodness of the optimization is confirmed (cf. section 5.3): convergence checks by Brooks and Gelman (1998) which are summarized in the uni- and multivariate diagnostics (which are, again, provided upon request) are considered. In the majority of the cases the posterior distributions are not driven by the prior distributions i.e. the shape of the distributions differ and they are not close to each other. Hence, the information from the data is sufficient to ensure that the estimation of the posterior distribution is independent from the priors. The only few exceptions are given by the monetary policy parameters ϕ_π and ϕ_x in the GM case and across the specification V0 (Figure 6.11) till V3 (Figure 6.14). Here the sample is not informative enough in order to identify a discrepancy between the prior and posterior distributions.

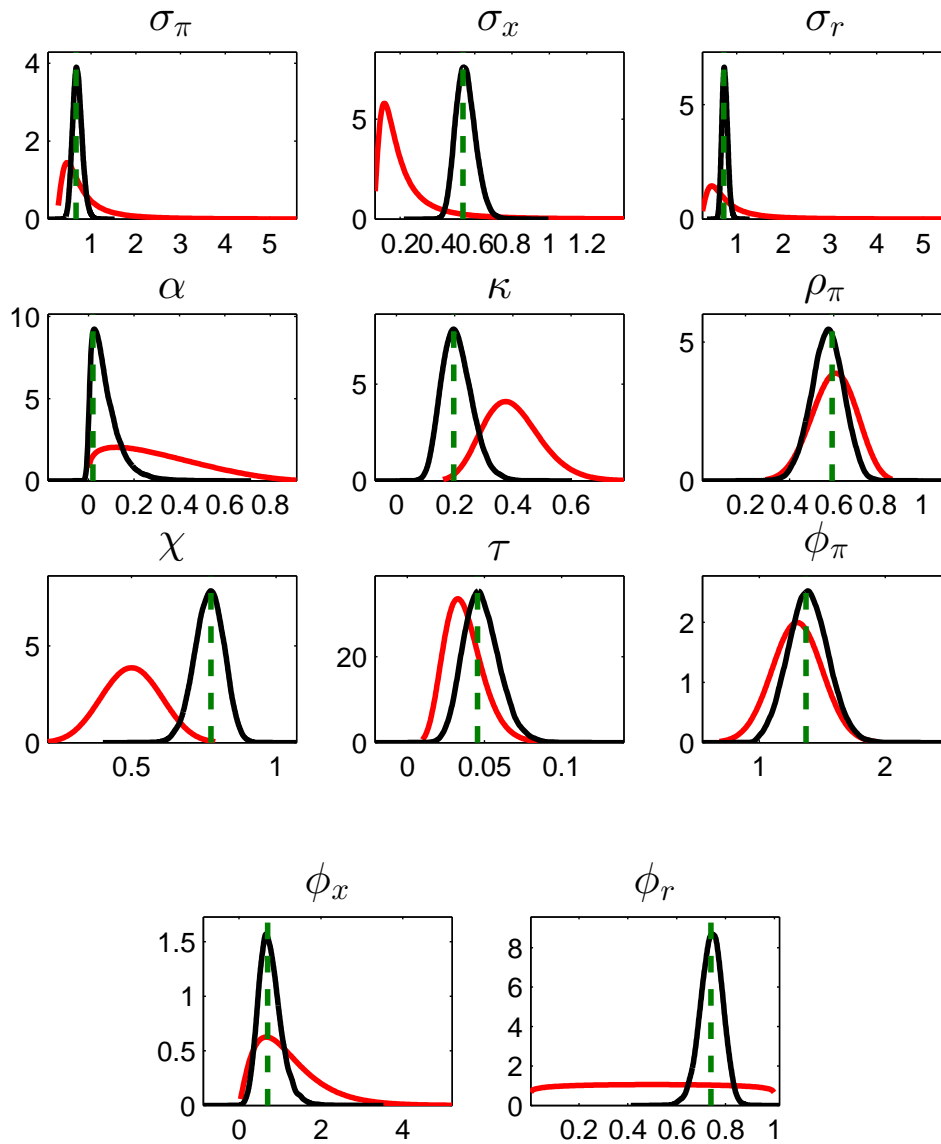


Figure 6.1: Posterior density estimates for BR (GI).

Note: The dashed/dotted green line represents the posterior mode (calculated from the posterior kernel) associated with the posterior distribution (black line; generated by the Random-Walk Metropolis-Hastings procedure). The red/grey line represents the prior distribution (see text for more information regarding the vector of prior information $p(\theta)$). This note equally applies to all of the following Figures.

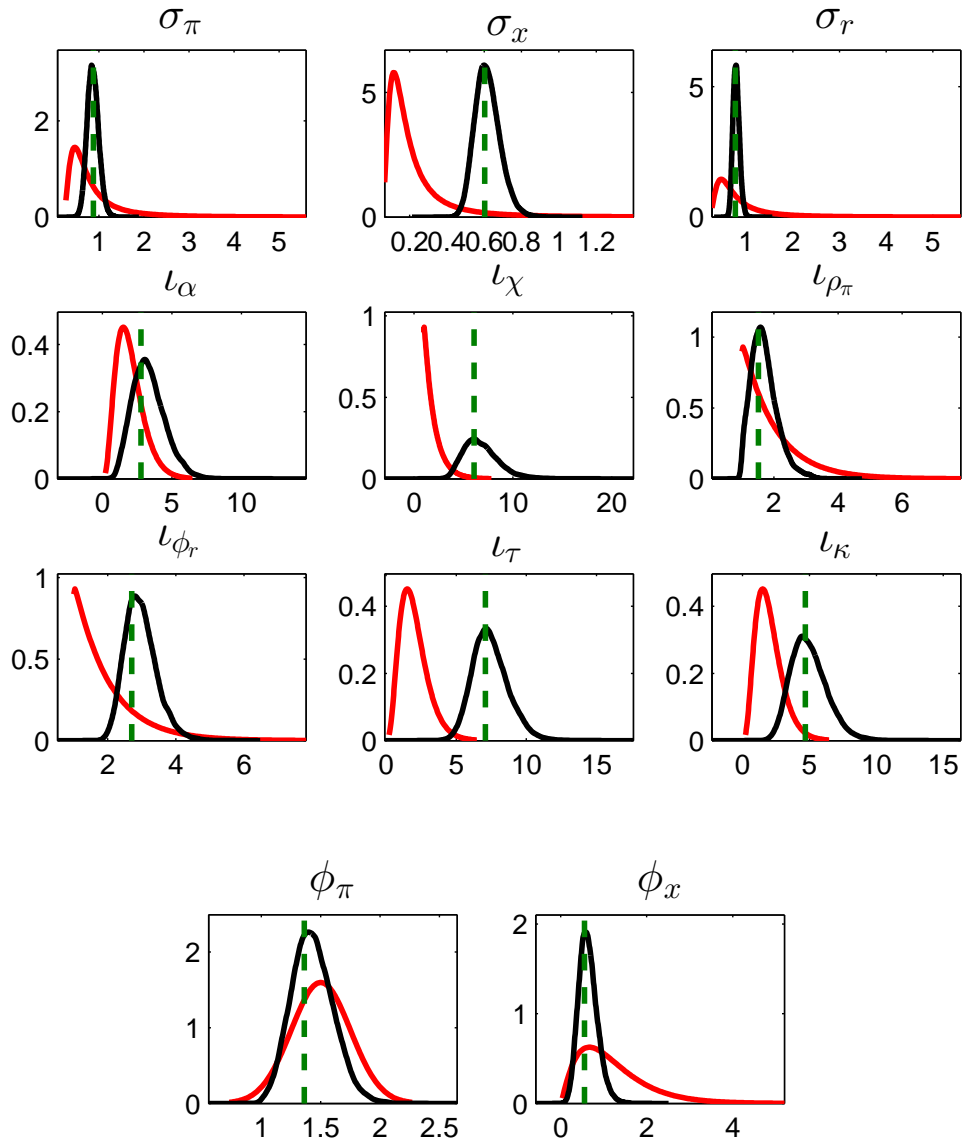


Figure 6.2: Posterior density estimates for T1 (GI).

Note: See Figure 6.1. The parameter ι_{ii} with $ii \in \{\alpha, \chi, \rho_\pi, \phi_r, \tau, \kappa\}$ denotes the transformed parameters according to the corresponding rules given in section 6.2.1.

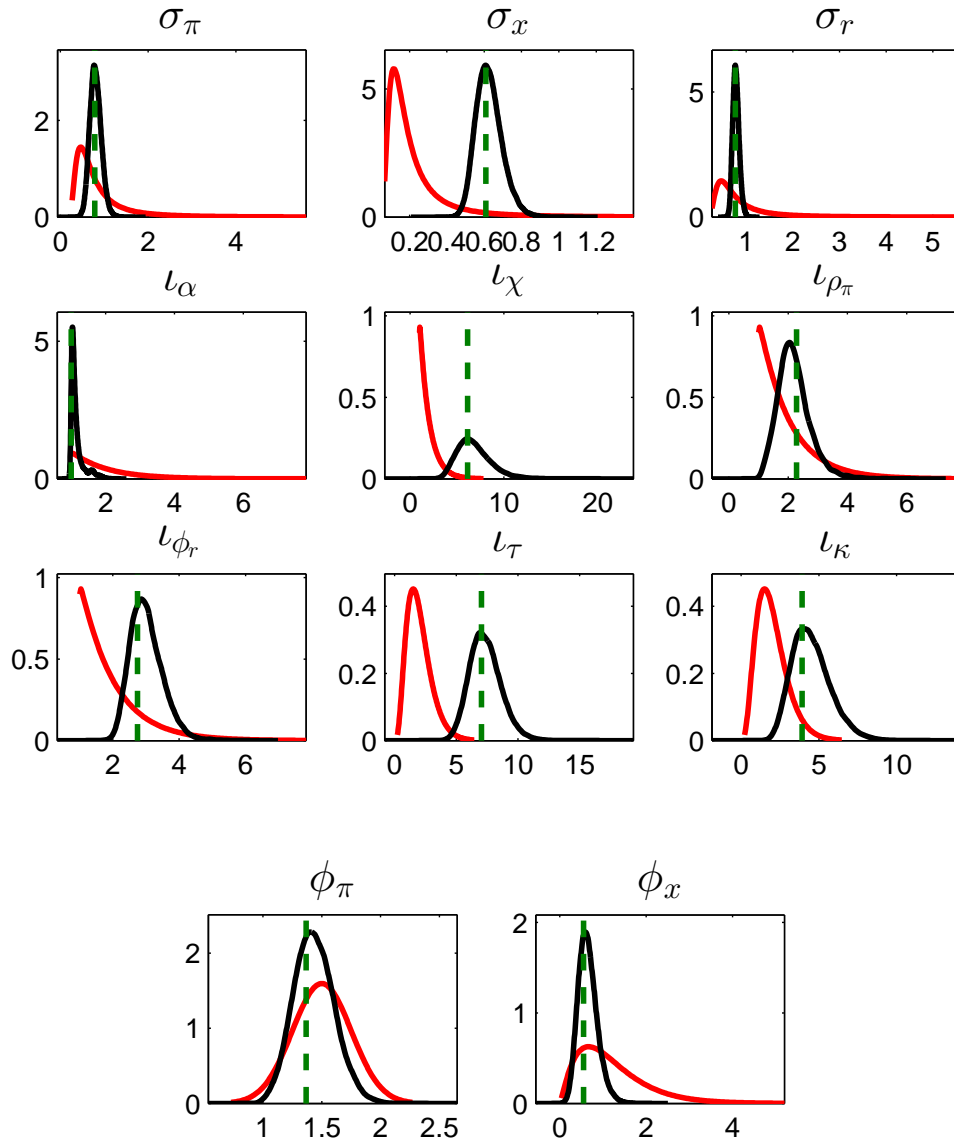


Figure 6.3: Posterior density estimates for T2 (GI).

Note: See Figure 6.1. The parameter ι_{ii} with $ii \in \{\alpha, \chi, \rho_\pi, \phi_r, \tau, \kappa\}$ denotes the transformed parameters according to the corresponding rules given in section 6.2.1.

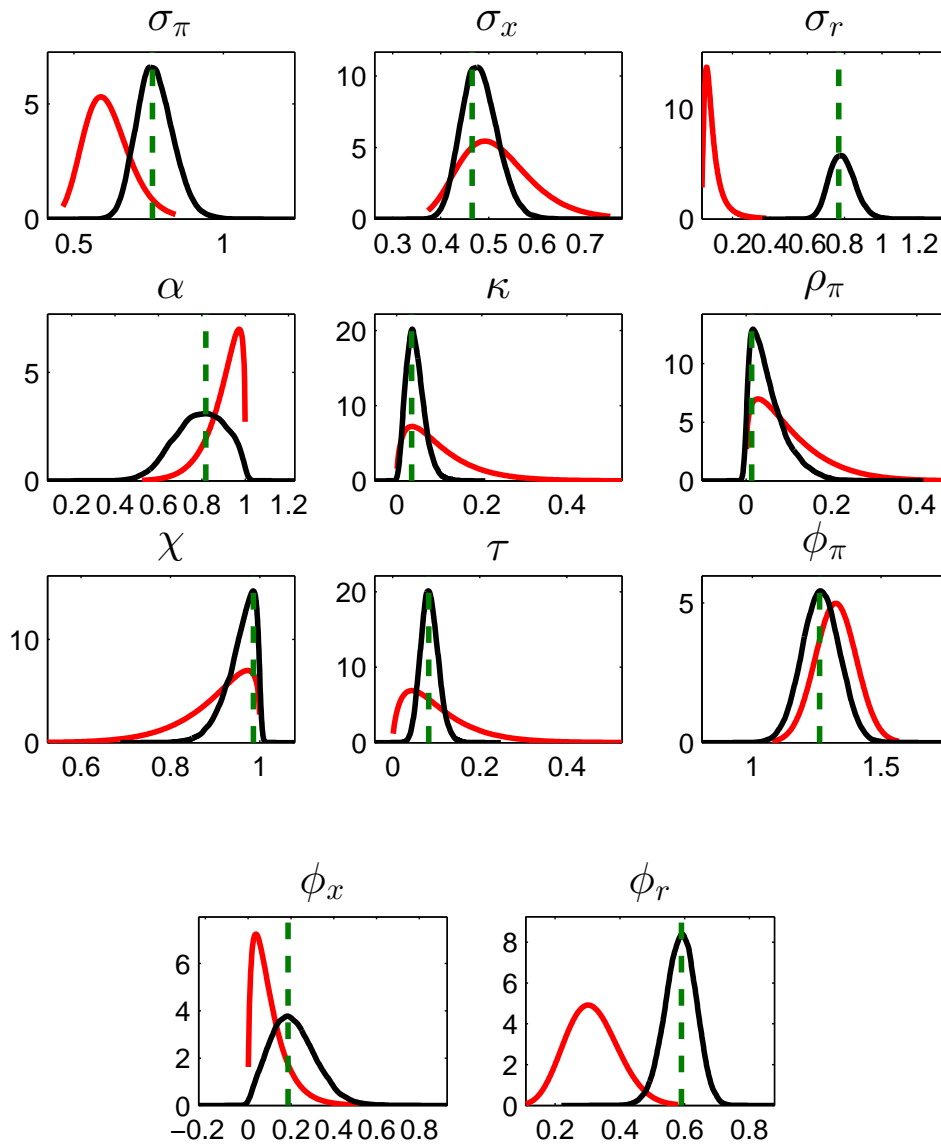


Figure 6.4: Posterior density estimates for V0 (GI).

Note: See Figure 6.1.

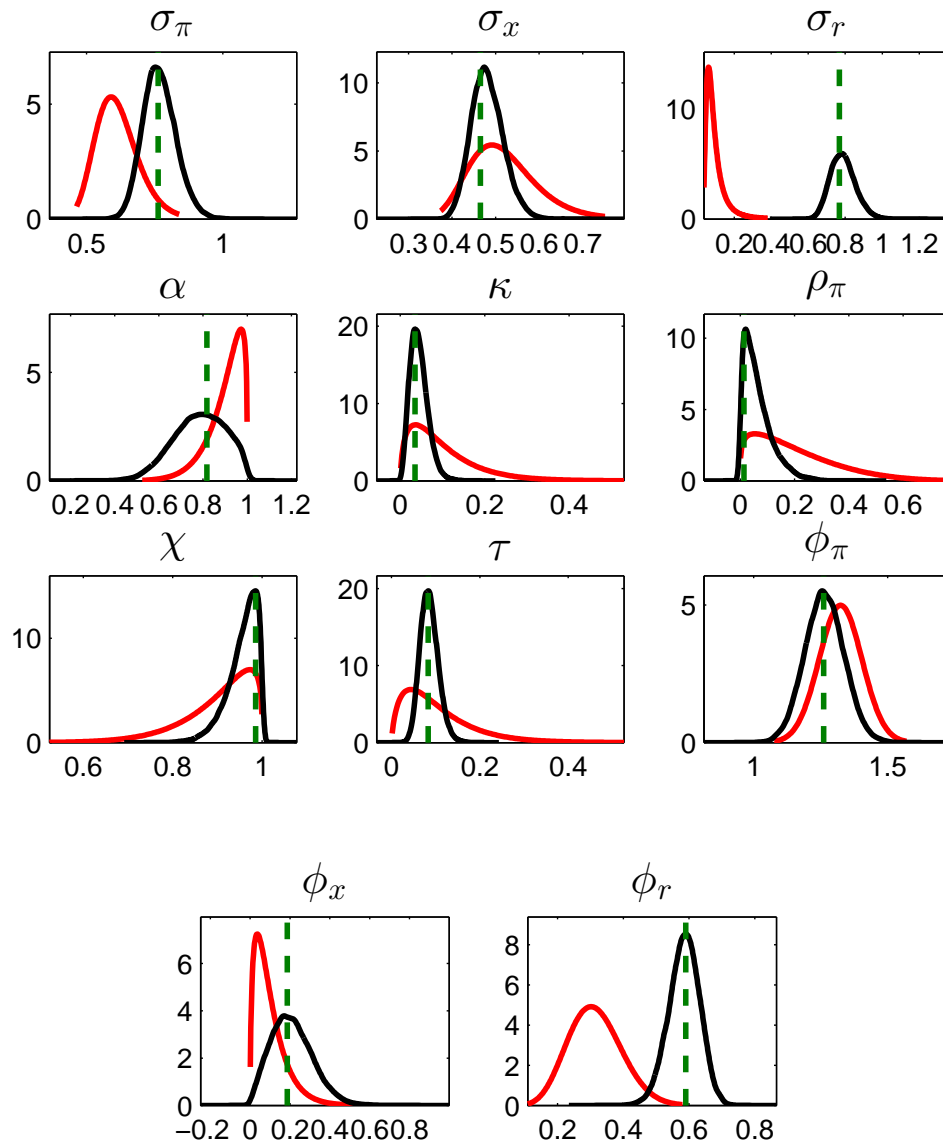


Figure 6.5: Posterior density estimates for V1 (GI).

Note: See Figure 6.1.

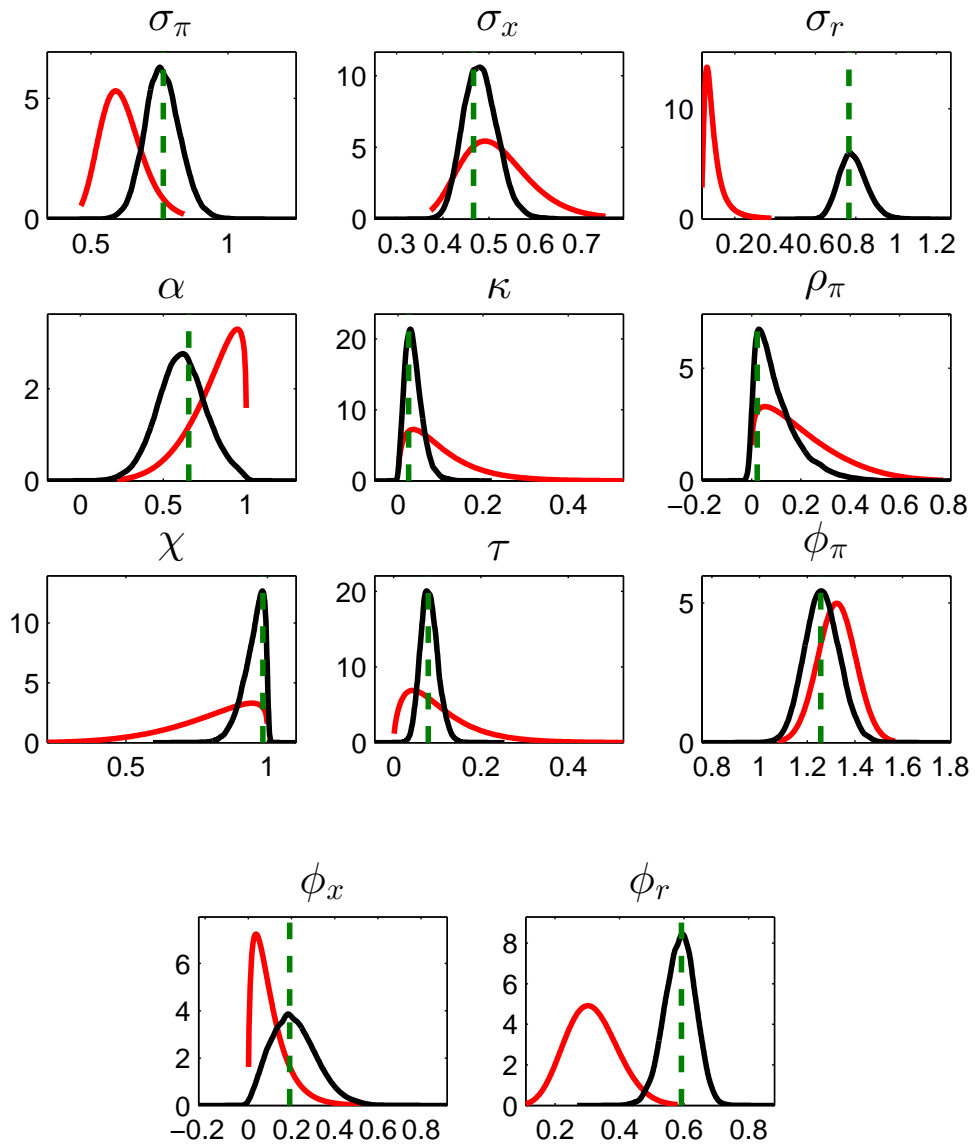


Figure 6.6: Posterior density estimates for V2 (GI).

Note: See Figure 6.1.

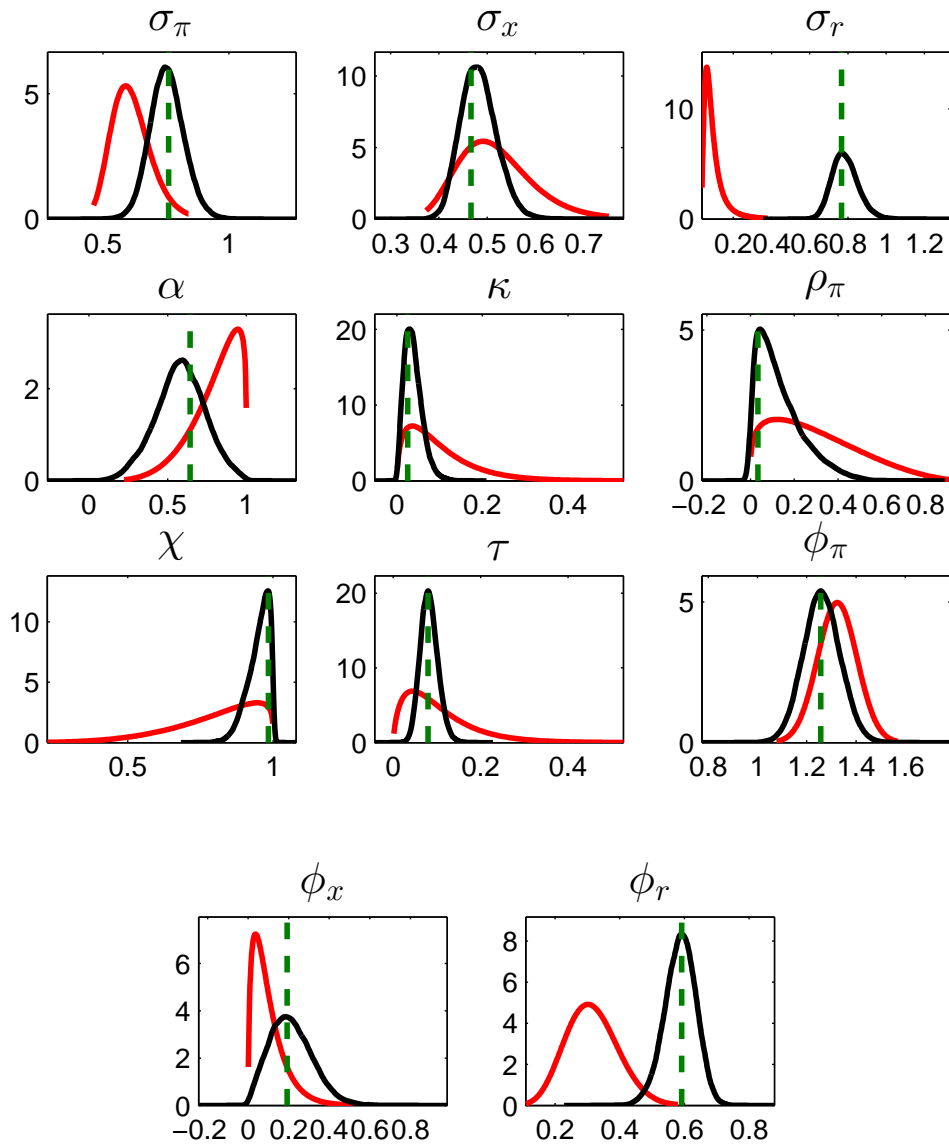


Figure 6.7: Posterior density estimates for V3 (GI).

Note: See Figure 6.1.

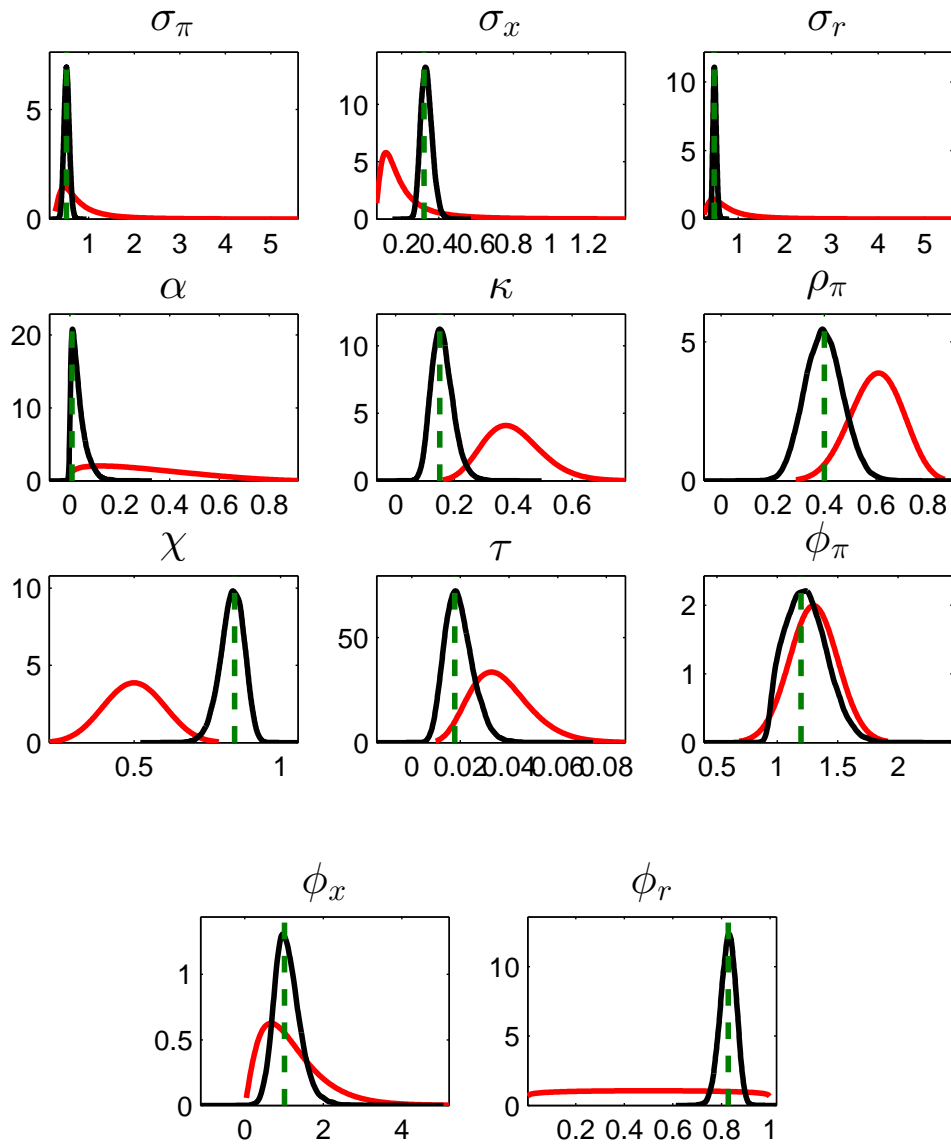


Figure 6.8: Posterior density estimates for BR (GM).

Note: See Figure 6.1.

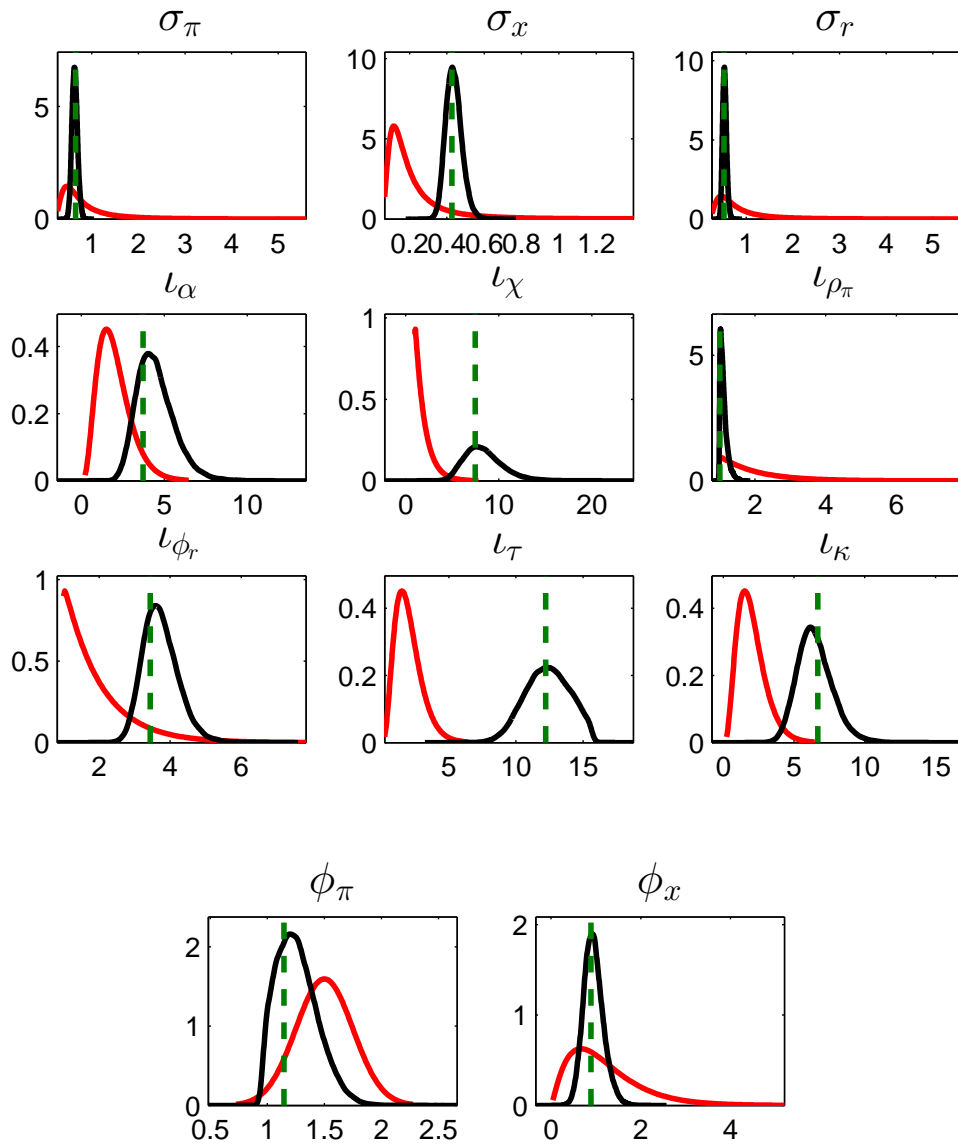


Figure 6.9: Posterior density estimates for T1 (GM).

Note: See Figure 6.1. The parameter ι_{ii} with $ii \in \{\alpha, \chi, \rho_\pi, \phi_r, \tau, \kappa\}$ denotes the transformed parameters according to the corresponding rules given in section 6.2.1.

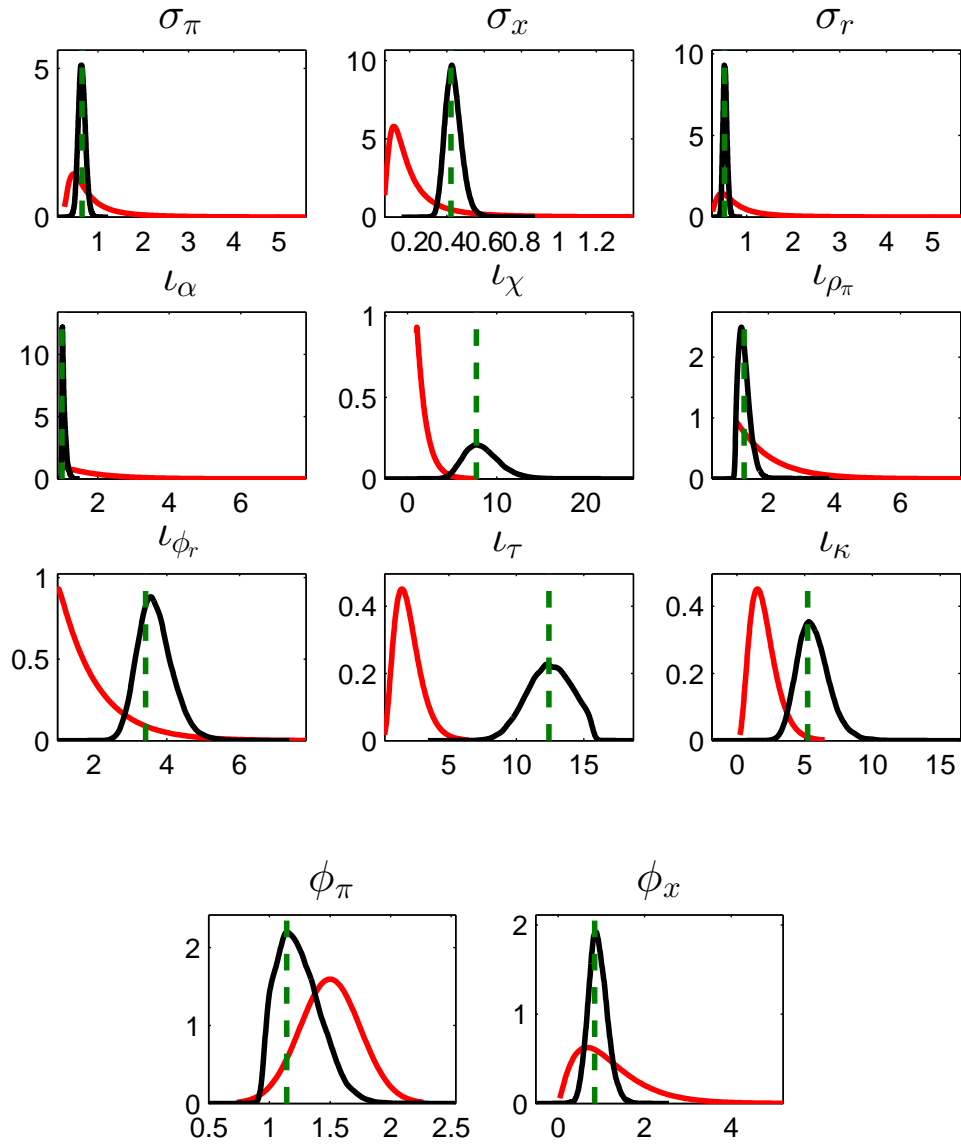


Figure 6.10: Posterior density estimates for T2 (GM).

Note: See Figure 6.1. The parameter ι_{ii} with $ii \in \{\alpha, \chi, \rho_\pi, \phi_r, \tau, \kappa\}$ denotes the transformed parameters according to the corresponding rules given in section 6.2.1.

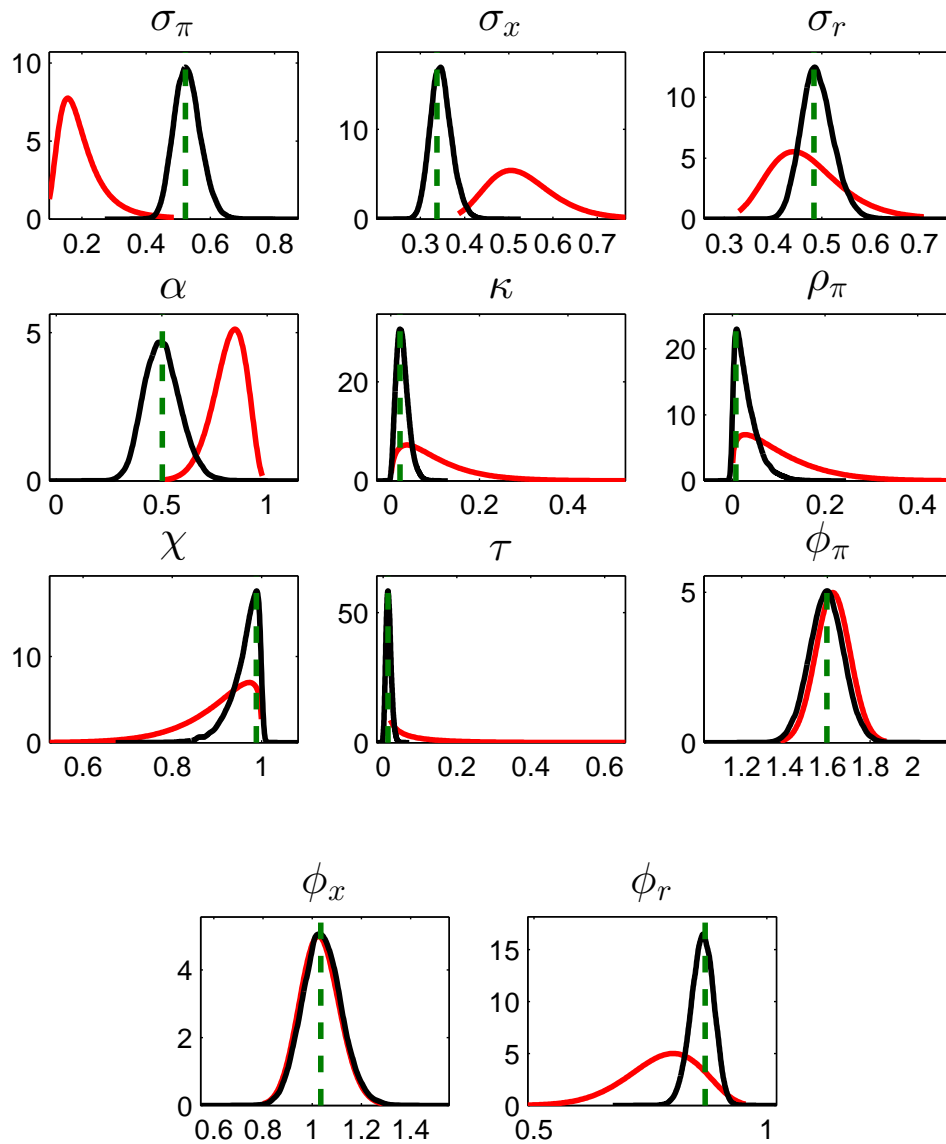


Figure 6.11: Posterior density estimates for V0 (GM).

Note: See Figure 6.1.

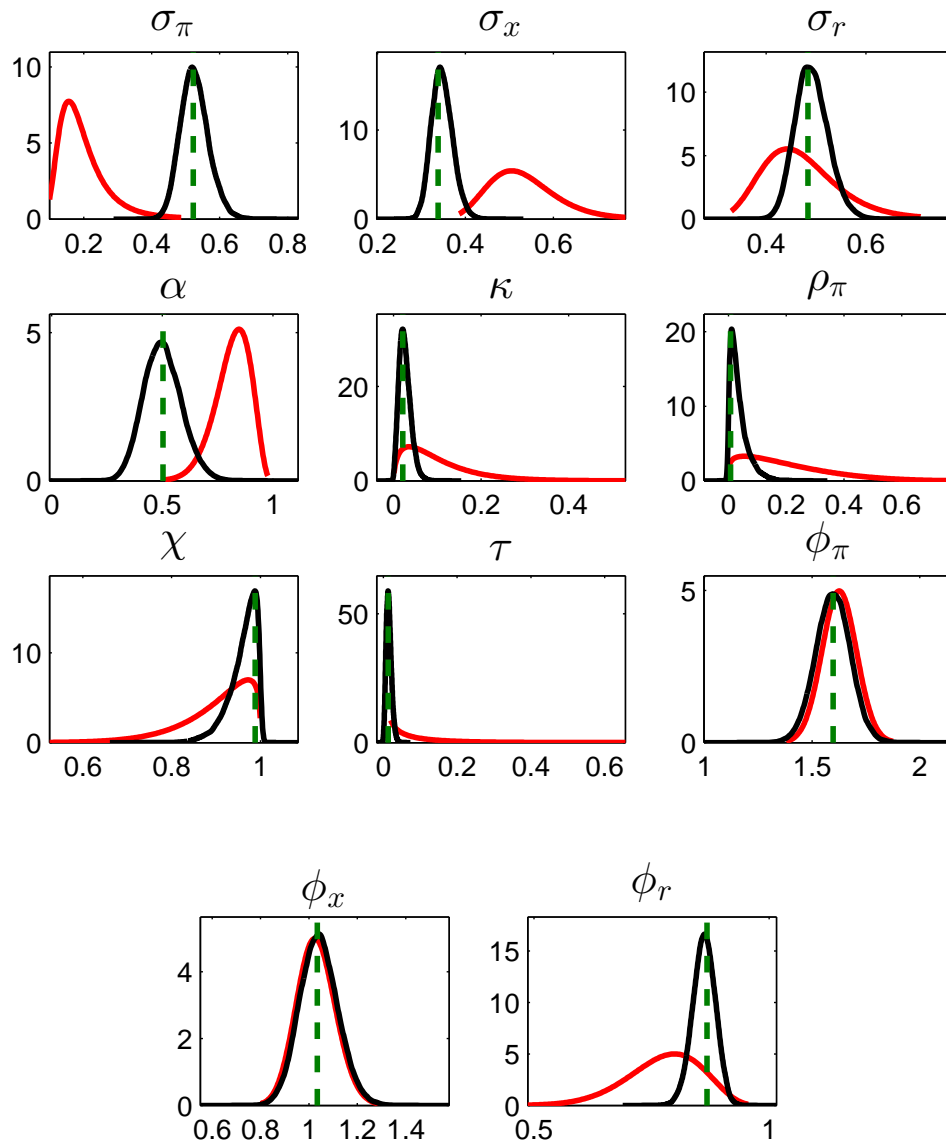


Figure 6.12: Posterior density estimates for V1 (GM).

Note: See Figure 6.1.

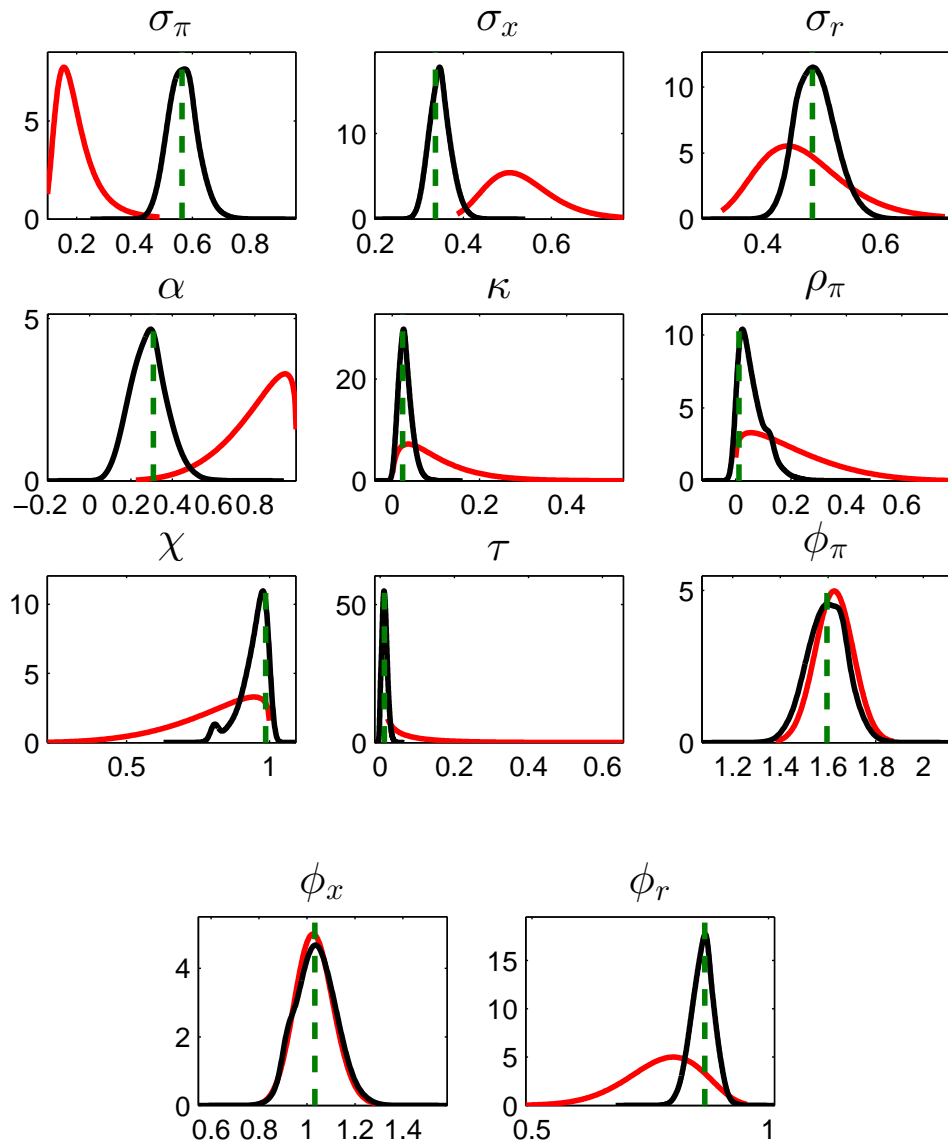


Figure 6.13: Posterior density estimates for V2 (GM).

Note: See Figure 6.1.

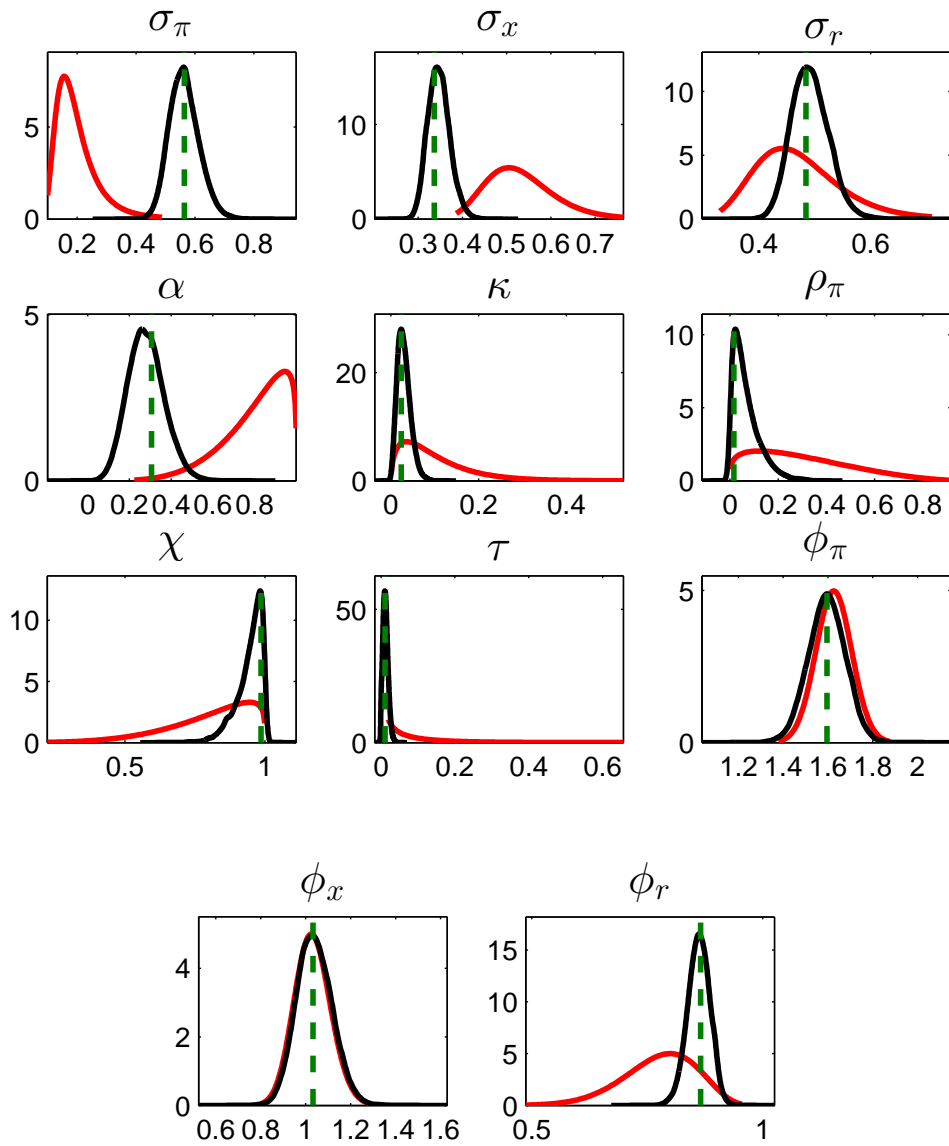


Figure 6.14: Posterior density estimates for V3 (GM).

Note: See Figure 6.1.

7 The Rational Expectation Hypothesis versus Animal Spirits within the New-Keynesian Framework: Empirical Evidence from Moment-Matching

7.1 Introduction

Rational expectations are a flexible and natural way of modelling market behaviour in DSGE models. Since the DSGE approach disposes a convenient analytical tractability under the assumption of rational expectations, this modelling framework serves as an efficient toolbox for analyzing monetary and fiscal policy measures. As Selten (2001 p. 2) states, however, "modern mainstream economic theory is largely based on an unrealistic picture of human decision theory". Indeed, the abundant evidence from experimental studies supports information processing with limited cognitive ability of agents rather than perfect information (see Hommes (2011) among others).

For the most part of the behavioral research, we can treat the realization of economic decisions as being a complex and interactive process between different types of agents. Keynes (1936) already attributed significant irrationality to human nature and discussed the impacts of waves of optimism and pessimism - so called *animal spirits* - on economic outcome. According to Akerlof and Shiller (2009), the emotional states are reflected in economic behaviour - see also Franke (2012b) for his extensive discussion about market behaviour and alternative ways to describe expectation formation processes in macroeconomic models.

The point of view taken here is that a behavioral model can provide a conceptual framework for cognitive ability as well as a substantial degree of inertia in DSGE models. According to De Grauwe (2011), if agents are known to be either optimists or pessimists, then their ability (or better: limitation) to form their expectations affects economic activities, i.e. movements in employment, the output gap and the inflation rate (gap), more appropriately than in standard rational expectation models. Indeed, it is shown in the expectation formation process under bounded rationality by De Grauwe (2011) that we can delineate dynamics of animal spirits by applying discrete choice theory on group behavior. Then the behavior of optimists and pessimists is considered to be a by-product of the switching rule based on agents' expectations. This procedure was first introduced by Brock and Hommes (1997) in their seminal paper on adaptively rational equilibrium models (see also e.g. Westerhoff (2008) as well as Lengnick and Wohltmann (2013) among others).

In his paper, De Grauwe (2011) replaces the forward-looking elements in the baseline three-equations NKM – which stem from the assumption of rational expectations – by a regime-

switching mechanism based on animal spirits. While inertia in the dynamics of the output gap and the inflation rate (gap) is observed empirically, it is well-known that the forward-looking NKM under rational expectations is not able to reproduce the corresponding IRFs without exogenous persistence induced by autocorrelated shock processes (cf. Chari et al. (2000) and Christiano et al. (2005)). De Grauwe (2011) shows theoretically that under consideration of a bounded rational expectation formation process, this specification in fact can account for intrinsic persistence in the output gap and the inflation rate – even if only non-autocorrelated exogenous shocks are considered. To the best of our knowledge, however, an empirical evaluation of a bounded rationality DSGE model of *this* type is missing in the literature so far.

In this Chapter we compare the empirical outcome of a bounded rationality DSGE model to the same model with rational expectations. In particular, similarities and dissimilarities between two polar cases of expectation formation processes will be examined: while the underlying model structure is identical to a standard hybrid three-equations NKM, the two models differ in terms of the expectation formation process, namely, rational expectations and endogenously-formed expectations using the behavioral specification by De Grauwe (2011). From this, we study his behavioral economic framework and provide an empirical investigation of bounded rationality on economic dynamics in the Euro Area from 1975Q1 to 2009Q4. Although the modification of the NKM by De Grauwe (2011) serves as an appropriate starting point for this kind of investigation, the modelling approach on behavioral heterogeneity relies on *non-linear* dynamics from discrete choice theory. We show that the *Simulated* Method of Moments (SMM) approach can be successfully used in order to deal with non-linearities in the structure of the model.

Main findings can be summarized as follows. First, empirical results from rational expectations and bounded rationality show that the model-generated auto- and cross-covariances of the output gap, the inflation gap and the nominal interest gap are able to mimic real data well. Second, the agents had expected moderate deviations of the future output gap from its steady state value over the whole time interval. Third, the main results suggest strong evidence for a static expectation formation process, i.e. the previous realization of the inflation gap is considered. This kind of expectation formation process has been discussed extensively by experimental economists (see Roos and Schmidt (2012) for an overview). Furthermore, the results indicate that the parameter estimates for the price indexation in both model specifications are close to unity. This is in line with our results given in the Chapters 5 and 6 for the rational expectations model specification. However, the result stands in contrast to other (empirical) studies in New-Keynesian literature, which discuss the evidence for a purely forward-looking specification. Finally, we offer reliable point estimates that can be used for calibration exercises in more realistic-grounded future work, e.g. studying monetary and fiscal policy analysis in a DSGE model without the assumption of rational expectations.

Indeed, a plethora of studies have been done on alternative forms of information processing mechanisms in macroeconomics; see e.g. the literature on learning (Evans and Honkapohja (2001)), rational inattention (Sims (2003)), sticky information (Mankiw and Reis (2002)) or bounded rationality in general (Sargent (1994) and Kahneman (2003)). Camerer (1998) also offers an informative overview of the discussion on this topic in economics. In addition, there exists several studies which deal with the estimation of bounded rationality (mostly partial equilibrium) models. Our approach can be seen as closely related to the work of Brock and Hommes (1997), Milani (2007), Cornea et al. (2013) and Boswijk et al. (2007).

Brock and Hommes (1997) introduce the exact switching mechanism, which is applied by

De Grauwe within the New-Keynesian framework. Brock and Hommes (1997) show that disequilibria made occur in a cobweb type demand-supply model where agents form rational and naive expectations – as being considered in De Grauwe (2011), and, of course, this Chapter. Milani (2007) estimates a linear hybrid NKM similar to the one in this Chapter with Bayesian techniques. As a feature of this model, the expectation formation process is described by constant-gain learning. According to the results of model comparison exercises, he shows that the model with constant-gain learning outperforms its rational expectation counterpart, where in the former case the learning process can generate a substantial degree of inertia in the model dynamics. However, the bounded rationality model studied in this Chapter is based on a non-linear switching rule from discrete choice theory. It is well known that Bayesian techniques can not be easily used to evaluate the non-linearities in these kind of models.

In another study, Cornea et al. (2013) estimate the NKPC for the US economy with an endogenous variation in the fractions of forward-looking fundamentalists and backward-looking native price-setters. The authors show that due to their theoretical modifications, their specification of the NKPC fits the data well. They conclude that the results are mainly driven by a regime switching mechanism, which is similar to the one presented in this Chapter. It must be emphasized that the non-linear least squares approach used in Cornea et al. (2013) is based on the assumption of Gaussian shocks, while the efficiency of this procedure is limited to the case where the number of the estimated parameters are small. The results in their paper are not affected by this shortcoming, since they consider a partial equilibrium model (for inflation rate dynamics). Thus the amount of parameters to be estimated is indeed manageable. In the same vein, Boswijk et al. (2007) estimate a dynamic single-equation asset pricing model with two types of agents. From this, behavioral heterogeneity can be described by a high degree of switching between fundamentalists and chartists. However, the major difference in our approach is that inference for group behavior is based on a system-of-equations estimation approach. Therefore, the current study aims to show that the SMM approach can be applied in order to improve the empirical shortcomings on non-linear behavioral dynamics within a highly parameterized model. To the best of our knowledge, such kind of investigation with respect to a non-linear DSGE model with *regime switching* has not been undertaken in the literature so far.

The remainder of the Chapter is structured as follows. In section 7.2 we present the baseline NKM known from the previous Chapters and discuss its two model specifications, that is, one with rational expectations and the other under consideration of animal spirits developed by De Grauwe (2011). The estimation methodology is presented in section 7.3. A discussion on the comparison between the (S)MM approach and standard estimation techniques known from the literature is provided in section 7.4. In Section 7.5 we estimate the two specifications of the model by the (S)MM approach and give an economic interpretation of the empirical results. Finally, section 7.6 concludes. The Appendix F contains the solution of the model specification under rational expectations and bounded rationality, respectively, as well as the presentation of the Delta Method.

7.2 The Model: Rational Expectations versus Bounded Rationality

The baseline three-equations NKM reads as follows:

$$y_t = \frac{1}{1+\chi} \tilde{E}_t^j y_{t+1} + \frac{\chi}{1+\chi} y_{t-1} - \tau(\hat{r}_t - \tilde{E}_t^j \hat{\pi}_{t+1}) + \varepsilon_{y,t} \quad (7.1)$$

$$\hat{\pi}_t = \frac{\nu}{1+\alpha\nu} \tilde{E}_t^j \hat{\pi}_{t+1} + \frac{\alpha}{1+\alpha\nu} \hat{\pi}_{t-1} + \kappa y_t + \varepsilon_{\hat{\pi},t} \quad (7.2)$$

$$\hat{r}_t = \phi_{\hat{r}} \hat{r}_{t-1} + (1 - \phi_{\hat{r}})(\phi_{\hat{\pi}} \hat{\pi}_t + \phi_y y_t) + \varepsilon_{\hat{r},t} \quad (7.3)$$

where the superscript $j = \{\text{RE, BoR}\}$ refers to the rational expectation (RE) and the bounded rationality (BoR) model, respectively. The corresponding expectations operator is \tilde{E}_t^j , which has to be specified for both models. It goes without saying that all variables are given in quarterly magnitudes.¹

In equation (7.1), the hybrid dynamic IS curve results from intertemporal optimization of consumption and saving, which leads to consumption smoothing. The parameter $\tau \geq 0$ denotes the inverse intertemporal elasticity of substitution in consumption. Equation (7.2) represents the hybrid NKPC where the output gap (y_t) is the driving force of inflation due to monopolistic competition and the Calvo price-setting scheme. The slope of the Phillips Curve is given by the parameter $\kappa \geq 0$. ν measures the discount factor ($0 < \nu < 1$). According to the Taylor rule with interest rate smoothing (equation (7.3)), the nominal interest gap is a predetermined variable, while the monetary authority reacts directly to contemporaneous movements in the output ($\phi_y \geq 0$) and inflation ($\phi_{\hat{\pi}} \geq 0$) gap. We account for intrinsic persistence in the stylized version of the well-known Smets and Wouters (2003, 2005 and 2007) model due to the assumption of backward-looking behavior indicated by the parameters for habit formation χ , price indexation α and interest rate smoothing $\phi_{\hat{r}}$, respectively ($0 \leq \chi \leq 1$, $0 \leq \alpha \leq 1$, $0 \leq \phi_{\hat{r}} \leq 1$). We assume that the exogenous driving forces in the model variables follow idiosyncratic shocks $\varepsilon_{z,t}$, which are drawn from normal distributions around mean zero and variance σ_z^2 with variables $z = \{y, \hat{\pi}, \hat{r}\}$.

Note here that we consider the gaps instead of the levels and therefore account explicitly for a time-varying trend in the inflation rate and the natural rate of interest. The corresponding gaps are simply given by taking the difference of the actual value for the output, the inflation rate and the nominal interest rate from their trends (i.e. time-varying steady state values) respectively, where the latter are computed by applying the Hodrick-Prescott filter with a standard value of the corresponding smoothing parameter of $\lambda = 1600$. Accordingly, the set of equations is used to describe the dynamics in the output gap y_t , the inflation gap $\hat{\pi}_t$ and the nominal interest rate gap \hat{r}_t , where \hat{x}_t with $x = \{\pi, r\}$ denotes the deviations in both variables from the time-varying trend explicitly.

The results of many studies show that assuming a constant trend, like a zero-inflation steady state, may provide misleading results. For example, Ascari and Ropele (2009) observe that the stability of the economic system can depend on the variation in trend inflation. Cogley and Sbordone (2008) also provide evidence for the explanation of inflation persistence by considering a time-varying trend in inflation. In the same vein, we abandon the assumption of a constant natural rate of interest as being empirically unrealistic. In this Chapter, we follow

¹Note that here we made some changes with respect to the notation of the variables and parameters in the hybrid baseline NKM (in quarterly magnitudes). We do so in order to avoid confusion concerning the point estimates in comparison to those reported in the Chapters 5 and 6.

the empirical approaches proposed by Cogley et al. (2010), Castelnuovo (2010) – besides ours applied in the previous Chapters – among others, who also consider gap specifications for the inflation (and the nominal interest) rate. Furthermore, inflation and money growth are likely to be non-stationary in the Euro Area data. If that is the case, the estimation methodology such as the (S)MM approach presented here (or GMM in general) will lead to biased estimates on non-stationary macro variables.² From this, the current study considers the gaps rather than the levels in order to ensure the behavior of the stationary times series.

To make the description of the expectation formation processes more explicit, first we examine two polar cases in the theoretical model framework of the baseline NKM. First, under rational expectations, the forward-looking terms are described by the expectations of the output gap and inflation gap at time $t + 1$ in the equations (7.1) and (7.2):

$$\tilde{E}_t^{RE} y_{t+1} = E_t y_{t+1} \quad (7.4)$$

$$\tilde{E}_t^{RE} \hat{\pi}_{t+1} = E_t \hat{\pi}_{t+1}, \quad (7.5)$$

where E_t denotes the expectations operator conditional on information given at time t . Second, as regards the other specification, we depart from rational expectations by considering the behavioral model of De Grauwe (2011). It is generally assumed that agents will be either *optimists* or *pessimists* (in the following indicated by the superscripts O and P , respectively) who form expectations based on their beliefs regarding movements in the future output gap:

$$E_t^O y_{t+1} = d_t \quad (7.6)$$

$$E_t^P y_{t+1} = -d_t, \quad (7.7)$$

where

$$d_t = \frac{1}{2} \cdot [\beta + \delta \lambda_{y,t}]. \quad (7.8)$$

Note here that the term d_t can be interpreted as "the divergence in beliefs among agents about the output gap" (De Grauwe (2011, p. 427)). In contrast to the RE model, both types of agents are uncertain about the future dynamics of the output gap and therefore predict a fixed value of y_{t+1} measured by $\beta \geq 0$. We can interpret the latter as the *predicted subjective mean value* of y_t . However, this kind of subjective forecast is generally biased and therefore depends on the volatility in the output gap, i.e. given by the unconditional standard deviation $\lambda_{y,t} \geq 0$. In this respect, the parameter $\delta \geq 0$ measures the *degree of divergence* in the movement of economic activity. Note that due to the symmetry in the divergence in beliefs, optimists expect that the output gap will differ positively from the steady state value (which for consistency is set to zero), while pessimists will expect a negative deviation by the same amount. The value of δ remains the same across both types of agents.

The expression for the *market forecast* regarding the output gap in the bounded rationality model is given by

$$\tilde{E}_t^{BoR} y_{t+1} = \alpha_{y,t}^O \cdot E_t^O y_{t+1} + \alpha_{y,t}^P \cdot E_t^P y_{t+1} = (\alpha_{y,t}^O - \alpha_{y,t}^P) \cdot d_t, \quad (7.9)$$

where $\alpha_y^O + \alpha_y^P = 1$ holds. Which kind of specific forecasting rule being chosen by the agents (i.e. equation (7.6) or (7.7)), is indicated by the probability of $\alpha_{y,t}^O$ and $\alpha_{y,t}^P$, respectively. In particular, α_y^O (or α_y^P) can also be interpreted as the probability being an optimist (or pessimist). In the following, we show explicitly how these probabilities are computed. Indeed,

²See also Russel and Banerjee (2008) as well as Assenmacher-Wesche and Gerlach (2008) among others for methodological issues related to non-stationary inflation in the US and Euro Area.

the selection of the forecasting rules (7.6) or (7.7) depends on the *forecast performances* of optimists and pessimists U_t^k (with $k = O, P$) given by the mean squared forecasting error, which can be simply updated in every period as (cf. Brock and Hommes (1997)):

$$U_t^k = \rho U_{t-1}^k - (1 - \rho)(E_{t-1}^k y_t - y_t)^2. \quad (7.10)$$

The parameter ρ measures the memory of agents ($0 \leq \rho \leq 1$). Here $\rho = 0$ means that agents have no memory of past observations, while $\rho = 1$ means that they have infinite memory instead. By applying the discrete choice theory under consideration of the forecast performances, agents revise their expectations in which different performance measures will be utilized for $\alpha_{y,t}^O$ and $\alpha_{y,t}^P$.³

$$\alpha_{y,t}^O = \frac{\exp(\gamma U_t^O)}{\exp(\gamma U_t^O) + \exp(\gamma U_t^P)} \quad (7.11)$$

$$\alpha_{y,t}^P = \frac{\exp(\gamma U_t^P)}{\exp(\gamma U_t^O) + \exp(\gamma U_t^P)} = 1 - \alpha_{y,t}^O, \quad (7.12)$$

where the parameter $\gamma \geq 0$ denotes the intensity of choice: if $\gamma = 0$, the self-selecting mechanism is purely stochastic ($\alpha_{y,t}^O = \alpha_{y,t}^P = 1/2$), whereas if $\gamma = \infty$, it is fully deterministic ($\alpha_{y,t}^O = 0$, $\alpha_{y,t}^P = 1$ or vice versa (see De Grauwe (2011), p. 429)). In other words, in the polar case where $\gamma = 0$ holds, agents are indifferent in being optimist or pessimist. Their expectation formation process is independent of their emotional state instead if $\gamma = \infty$ holds, i.e. as they react quite sensitively to infinitesimal changes in their forecast performances.

We explain this revision process as follows. Given the past value of the forecast performance (U_{t-1}^k), we see that the lower the difference between the expected value of the output gap (taken from the previous period, i.e. $E_{t-1}^k y_t = |d_{t-1}|$) and its realization in period t , the higher the corresponding forecast performance U_t^k will be. More precisely, if e.g. the optimists predict future movements in y_t more accurately compared to the pessimists, then this results in $U_t^O > U_t^P$. Hence, the pessimists revise their expectations by switching to the forecasting rule used by the optimists, which we can express as $E_t^O y_{t+1} = d_t$. Finally, this forecasting rule becomes dominant and the share of pessimists in the market decreases. According to the equations (7.10) to (7.12), we can rationalize equation (7.9) by using simple substitution. This results in a higher degree of volatility in the expectation formation process regarding the output gap relative to the outcome in the RE model – hereby we refer to Figure 7.1 given in section 7.5 for a clarification.

The same logic can be applied for the inflation gap expectations. Following De Grauwe (2011, pp. 436), we assume that agents will be either so called *inflation (gap) targeters (tar)* or *extrapolators (ext)*.⁴ In the former case, the central bank anchors expectations by announcing a target for the inflation gap $\hat{\pi}$. From the view point of the inflation targeters, we consider this pre-commitment strategy to be fully credible. Hence the corresponding forecasting rule becomes

$$E_t^{tar} \hat{\pi}_{t+1} = \hat{\pi}, \quad (7.13)$$

³See also Westerhoff (2008, p. 199) and Lengnick and Wohltmann (2013) among others for an application of discrete choice theory to models in finance and macroeconomics.

⁴The concept of behavioral heterogeneity has been widely used in financial market models, see e.g. Chiarella and He (2002) as well as Hommes (2006) among others.

where we assume $\bar{\pi} = 0$.⁵ The extrapolators form their expectations in a static way and will expect that the future value of the inflation gap equals simply its past value, i.e.

$$E_t^{ext} \hat{\pi}_{t+1} = \hat{\pi}_{t-1}. \quad (7.14)$$

This results in the market forecast for the inflation gap similar to the one given in equation (7.9):

$$\tilde{E}_t^{BoR} \hat{\pi}_{t+1} = \alpha_{\hat{\pi},t}^{tar} E_t^{tar} \hat{\pi}_{t+1} + \alpha_{\hat{\pi},t}^{ext} E_t^{ext} \hat{\pi}_{t+1} = \alpha_{\hat{\pi},t}^{tar} \bar{\pi} + \alpha_{\hat{\pi},t}^{ext} \hat{\pi}_{t-1}. \quad (7.15)$$

The forecast performances of inflation targeters and extrapolators are given by the mean squared forecasting error written as

$$U_t^s = \rho U_{t-1}^s - (1 - \rho)(E_{t-1}^s \hat{\pi}_t - \hat{\pi}_t)^2, \quad (7.16)$$

where $s = (tar, ext)$ holds.

Finally, we can write:

$$\alpha_{\hat{\pi},t}^{tar} = \frac{\exp(\gamma U_t^{tar})}{\exp(\gamma U_t^{tar}) + \exp(\gamma U_t^{ext})} \quad (7.17)$$

$$\alpha_{\hat{\pi},t}^{ext} = \frac{\exp(\gamma U_t^{ext})}{\exp(\gamma U_t^{tar}) + \exp(\gamma U_t^{ext})} = 1 - \alpha_{\hat{\pi},t}^{tar}. \quad (7.18)$$

Here $\alpha_{\hat{\pi},t}^{tar}$ denotes the probability to be an inflation targeter, which is the case if the forecast performance using the announced inflation gap target is superior to the extrapolation of the inflation gap expectations and vice versa. Note here that the memory (ρ) as well as the intensive of choice (γ) do not differ across the expectation formation processes in terms of the output and inflation gap. In the end, the BoR model turns out to be purely backward-looking (cf. the equations (7.10) and (7.16)) while both forward- and backward-looking elements are contained in the RE model. The solution to both systems can be computed numerically by backward-induction as well as the method of undetermined coefficients together with the brute force iteration procedure, respectively. Note that the BoR model does not have a simple closed-form solution. We refer to the Appendix F1 for more details.

Finally, one may argue that the BoR model presented is not suitable for e.g. policy analysis since it is not based completely on microfoundations. In particular, the expectation mechanisms are imposed ex post on a system of structural equations which themselves have been derived from maximizing behavior under the assumption of rational expectations. However, evidence from experimental economics can help us to motivate the assumption of the divergence in beliefs (which reflects guessing) and the existence of the extrapolators (which might be seen as pattern-based time-series forecasting) done by De Grauwe (2011) and adopted in our study. Roos and Schmidt (2012) find evidence for a backward-looking behavior in forming expectations by non-professionals in economic theory and policy. In their experimental study, they show that the projections of the future realizations in the output gap and the inflation rate are based either on historical patterns of the time series or - in the case of no available information - on simple guessing.

From a theoretical point of view, Branch and McGough (2009) introduce heterogeneous expectations into a New-Keynesian framework where the forward-looking expressions in the

⁵In this respect (based on an optimal monetary policy strategy), an inflation *gap* target of zero percent implies that the European central bank seeks to minimize the deviation of its (realized) target *rate* of inflation from the corresponding time-varying steady state value. Thus the deviation should be zero in the optimum.

dynamic IS curve and the NKPC are convex combinations of backward- and forward-looking behavior. The authors show that a micro-founded NKM under bounded rationality can be theoretically sound if specific axioms are considered within the optimizing behavior of households and firms. These axioms ensure the ability of agents to forecast future realizations of the output gap and the inflation rate at the micro level as well as the aggregated behavior at the macro level. In comparison, De Grauwe (2011) allows for a switching mechanism based on discrete choice theory. It is an open question if the latter fulfills the axioms imposed by Branch and McGough (2009) which may mitigate the (neglected) problems of misspecification. To sum up, there is no doubt that an extensive elaboration on the microfoundation of expectations formation is needed. However, this is still an open question among neuroscientists, since the evidence on information processing in the human brain is ambiguous (De Grauwe (2011, p. 428, fn. 4)).

7.3 The Simulated Method of Moments Approach

In this Chapter we seek to match the model-generated autocovariances of the output gap, the inflation gap and the nominal interest rate gap with their empirical counterparts. Statistical inference from the behavioral models to the distributional properties of empirical data is conducted through the minimization of the distance between these model-generated and empirical moments via the (S)MM approach.

In most cases, the moment conditions play an important role in accounting for distributional properties of empirical data X_t with $t = 1, \dots, T$, where T denotes the sample size. Indeed, the moment-matching does not require strong distributional assumptions, and its approximation to the data generating process will be as efficient as the maximum likelihood when appropriate moments are selected (see also Canova (2007, his Chapter 5)). In this respect, our study aims to show that a set of auto- and cross-covariances can be used to describe the regular properties of the business cycle.⁶ The sample covariance matrix at lag k is defined by:

$$m_t(k) = \frac{1}{T} \sum_{t=1}^{T-k} (X_t - \bar{X})(X_{t+k} - \bar{X})', \quad (7.19)$$

where $\bar{X} = (1/T) \sum_{t=1}^T X_t$ is the vector of the sample mean. The sample average of discrepancy between the model-generated and empirical moments is denoted as

$$g(\theta; X_t) \equiv \frac{1}{T} \sum_{t=1}^T (m_t^* - m_t), \quad (7.20)$$

where θ denotes a $l \times 1$ vector of unknown structural parameters. m_t^* and m_t are the *empirical* moment function and the *model-generated* moment function, respectively (cf. equation (7.19)). In the empirical analysis of the structural model, we use an explicit closed-form solution for the endogenous variables in the RE model to derive the moment conditions (see

⁶It is well known that a Gaussian distribution can be approximated by the first and second moments. In addition, if empirical observations do not follow a Gaussian process, the use of higher moments will be effective in matching the data generating process. In our empirical application, however, the sample size on the macro data being considered is not large enough in order to ensure accurate estimates on the higher moments. Thus we decide to focus on the second moments for our current study only, while further statistical analysis on the, let's say, optimal selection of the moments being left for future research.

Appendix F1 and Franke et al. (2012) for the intermediate steps needed). In the BoR case, due to its non-linear structure, no analytic solution for m_t can be obtained. Therefore, the SMM approach must be applied instead (see further below). However, before we turn to the description of SMM applied for estimating the non-linear BoR model, we briefly explain (again) the MM approach with respect to the estimation of the linear RE model.

Note here that our main objective of this study is to draw inference from the underlying model to the auto- and cross-covariances of the observations at a (fixed) lag k with $k = 0, \dots, n$.⁷ If a number of j variables is selected as the lag length, we can compute the corresponding p -dimensional vector of (empirical and simulated) moment conditions by

$$p = p(k, j) = (j \cdot k - 1) \cdot j. \quad (7.21)$$

From this, we avoid the double counting at the zero lags in the cross relationships by subtracting the term $j \cdot k$ by one. Concerning the confidence intervals for the auto- and cross-covariance moments, we use the Delta method. Here a linear approximation of the moment function around the point estimates under consideration of the corresponding gradient vector is applied (see Appendix F2 for more details).

Given that the duration of the business cycle lies between (roughly) one and eight years in the Euro Area (cf. Artis et. al (2003)), a reasonable compromise is a length of two years. According to equation (7.21), $p = 78$ moments are considered as appropriate choice for our study, where we (based on the underlying model structure) take into account a lag length of $n = k^{max} = 8$ and $j = 3$ variables. As we choose a lag structure in the auto- and cross-covariances of $n = k^{max} = 8$, we take 1/4 of the total length of a business cycle into account, where the latter is defined by the movements in the output gap from a trough to the next one. In the next section we will see empirically, the output gap exhibits similar patterns in the upswing movement from a trough to a peak over the whole time series. Therefore we claim that the relevant observations rely on $p = 78$ moments only, since repeating patterns in the time series do not exhibit additional information.⁸

With a focus on these moment conditions, we can estimate the model parameters by minimizing the following quadratic objective function:

$$J(\theta) = \arg \min_{\theta} g(\theta; X_t)' \widehat{W} g(\theta; X_t), \quad (7.22)$$

where more importance is attached to particular moment conditions according to the weighting matrix \widehat{W} (see Andrews (1991)). The kernel estimator has the following general form with the covariance matrix of the appropriately standardized moment conditions given by

$$\widehat{\Gamma}_T(h) = \frac{1}{T} \sum_{t=h+1}^T (m_t - \bar{m})(m_t - \bar{m})', \quad (7.23)$$

where \bar{m} once again denotes the sample mean. Concerning the lag length, we use a popular choice of $h \sim T^{1/3}$, that is, $h = 5$ for estimating the covariance matrix in the Euro Area

⁷The maximal number of lags (denoted by n) is chosen under consideration of the economic model and sample size. In a macroeconomic model, n captures the length of the business cycle.

⁸We claim that the business cycle can be seen approximately as a sinus function, where a fraction of 1/4 describes the upswing movement if a trough being the starting point. Hence, the transition from the peak to the second trough mimics the upswing with, of course, the opposite sign. Although, this empirical time series is not *perfectly* described by a sinus function, the length of the business cycle is close to such a representation (see the corresponding panels in Figure 7.1 given in the next section). Therefore, we judge our choice of $p = 78$ moments (or, equivalently, a fraction 1/4 of the length of the business cycle) as being valid.

(Newey and West (1994)):

$$\widehat{\Omega}_{NW} = \widehat{\Gamma}_T(0) + \sum_{h=1}^5 \left(\widehat{\Gamma}_T(h) + \widehat{\Gamma}'_T(h) \right). \quad (7.24)$$

The weighting matrix \widehat{W} is computed from the inverse of the estimated covariance matrix $\widehat{\Omega}_{NW}$. However, a high correlation between the moment conditions that we consider makes the estimated covariance matrix nearly singular. This holds since the moment conditions and the elements of the weighting matrix are highly correlated when the small sample size is used (Altonji and Segal (1996)). Note here that we select $p = 78$ moments to approximate the business cycle in the Euro Area. To circumvent the econometric issues, we use the diagonal matrix entries as the weighting scheme, while the off-diagonal components of the matrix $\widehat{W} = \widehat{\Omega}_{NW}^{-1}$ are ignored. Although the weighting matrix becomes non-optimal by applying the previous steps, it can be seen that this procedure is based on an economic (rather than a strict econometric) rationale. The estimated confidence bands (intervals), then, become wider since the sandwich elements in the covariance of parameter estimates cannot cancel out under consideration of this weighting scheme (see also Anatolyev and Gospodinov (2011)).

To make inferences about the model parameters, we need to analyze properties of sample distribution for the MM estimator. In particular, under certain regularity conditions, one can derive the following asymptotic distribution of the model parameters:

$$\sqrt{T}(\widehat{\theta}_T - \theta_0) \sim N(0, \Lambda), \quad (7.25)$$

where $\Lambda = [(DWD')^{-1}]D'W\Omega WD[(DWD')^{-1}]'$ holds. D is the gradient vector of moment functions evaluated around the point estimates. This can be written as:

$$\widehat{D} = \left. \frac{\partial m(\theta; X_T)}{\partial \theta} \right|_{\theta = \widehat{\theta}_T}. \quad (7.26)$$

Under RE, we can obtain the simple analytic moment conditions of the model as described above. However, for the *BoR model*, the analytic expressions for the moment conditions are not available due to its non-linear structure and, in particular, of discrete choice theory (see again Appendix F1). To circumvent this problem, we make use of SMM to estimate the behavioral parameters in the BoR model. The SMM approach is particularly suited to a situation where the model is easily simulated by replacing theoretical moments. Then the model-generated moments in equation (7.22) are replaced by their simulated counterparts:

$$m_t = \frac{1}{S \cdot T} \sum_{t=1}^{S \cdot T} \widetilde{m}_t. \quad (7.27)$$

In equation (7.27), we approximate the theoretical moments (m_t) based on the simulated data of \widetilde{m}_t . The simulation size is denoted by S .

The asymptotic normality of the simulated method of moments holds under certain regularity conditions (Duffie and Singleton (1993), Lee and Ingram (1991)):

$$\sqrt{T}(\widehat{\theta}_{SMM} - \theta_0) \sim N(0, \Lambda_{SMM}), \quad (7.28)$$

where $\Lambda_{SMM} = [(B'WB)^{-1}]B'W(1 + 1/S)\Omega WB[(B'WB)^{-1}]'$ holds, that is a covariance matrix of the SMM estimates. A gradient vector of the moment function is defined as $B \equiv E \left[\left. \frac{\partial m_t}{\partial \theta} \right|_{\theta = \widehat{\theta}_t} \right]$.

Since the covariance matrix becomes less accurate compared to the estimation where the analytic moments are used, the model estimation is now subjected to simulation errors. Especially, in the non-linear dynamics of the BoR model, a linear approximation from the Delta method is not suitable.⁹ To avoid a large confidence interval for the statistical inference, we compute the standard errors by using the following steps:

1. The BoR model is estimated using a simulation size of 10.
2. The estimation is iterated over 100 times.
3. We take 100 different estimates to compute the mean and standard error of parameter estimates.

From this, we can obtain the simulated intervals for the model parameters, especially for the behavioral parameters. Finally, we use the J test to evaluate compatibilities of the moment conditions:

$$\bar{J} \equiv T \cdot J(\hat{\theta}) \xrightarrow{d} \chi_{p-l}^2, \quad (7.29)$$

where the J -statistic is asymptotically χ^2 distributed with $(p-l)$ degrees of freedom, where l denotes the number of parameters to be estimated. In our study, we set the lag length for the covariance to two years. Hence, the number of moment conditions exceeds the model parameters, which results in overidentification.¹⁰ Note that the degrees of freedom is smaller in the hybrid RE compared to the hybrid BoR model (66 versus 68) due to the additional behavioral parameters, β and δ , to be estimated.

7.4 On the Validity of the Moment-Matching Approach

In this study, we make use of the MM approach, where the behavioral equations of a New-Keynesian framework are mapped into the second moments. By this we investigate if the underlying model structure is suited to explain the probability distribution of the empirical variables. Indeed, a main advantage of MM is its *transparency*: a researcher chooses the specific aspects of the data generating process, in particular, those which mimic the most important stylized facts of the economy - represented here by the second moments. While in the end the choice of moments is a matter of judgement, informative criteria for statistical inference are still based on a certain set of moments, since a model, at whatever level of complexity, cannot possibly reproduce all of the empirical regularities that we observe.

In addition, the MM approach provides an intuitive notion of the goodness of fit of the model, which may be checked by visual inspection of the covariance structure or more formally by an econometric assessment of the minimized value of the corresponding objective function.

⁹We can reduce the approximation error by increasing the simulation size in order to maintain $\frac{1}{S} \rightarrow 0$. For instance, the simulation error becomes 1% when the moments are simulated 100 times. However, the non-linearity often increases the magnitude of changes in these errors. From this, our study does not use the Delta method to measure the uncertainty of parameter estimates on the BoR model.

¹⁰However, if the off-diagonal components in the estimated Newey and West matrix $\hat{\Omega}_{NW}$ (given in equation (7.24)) are discarded, the distribution in the J -statistic is likely to have a larger dispersion than the χ^2 -distribution with degrees of freedom of $(p-l)$. Indeed, when the weighting matrix is non-optimal or some moment conditions are not valid, it is likely that the J -statistic is no longer χ^2 distributed. To examine the effectiveness of J -test, in a working paper related to this Chapter, Jang and Sacht (2012) check the validity of the weighting matrix with our chosen moment conditions via an extensive Monte Carlo study.

In particular, the graphical comparison between the match of the estimated and simulated autocovariances is direct. Thus the advantages of MM could be evaluated — besides how transparently the results are computed and documented — on its detailed diagnosis of merits and demerits of models, whose moments are hard to match like in non-linear DSGE models. In the following, we briefly compare the MM approach to well-known estimation techniques, which have been widely applied in cases of non-linear models. From this, we state that the MM approach can be successfully used to estimate non-linear bounded rational NKM in terms of the features given by transparency and muted manageability.

In fact, MM is closely related to the estimation technique known as *indirect inference* (II) in econometric theory. Here, the model is not taken directly to the observed variables compared to the MM approach. Instead, auxiliary models should be used to encompass the underlying economic model. Given the simulated moments based on the auxiliary model, within II these moments are matched to the empirical ones. This procedure indeed provides an alternative way to evaluate parameters in the case where e.g. the economic model is highly non-linear. Pioneer work in this field has been undertaken by Smith (1993) as well as Gallant and Tauchen (1996, 2010). Prominent results like the ones reported in Christiano et al. (2005) as well as Del Negro and Schorfheide (2008) are based on the matching of IRFs to estimate the moments of theoretical models.

In general, because of rapid advances in information and communication technologies during the last two decades, simulation based inference has become a popular tool for estimating complex economic models. In addition, this kind of statistical inference is considered to be more reliable when the model structure of latent (unobservable) variables has non-linear dynamics and the connection of the model from macroeconomic data is not clear (Smith (2008)). Especially, II and MM could provide more reliable parameter estimates from which we can avoid to compute analytically intractable likelihood function. In this case, there is a high chance that multiple peaks could exist over the entire parameter space and the optimization might stop at some inferior points of the model parameters (see also Gilli and Shumann (2010) for a description of heuristic optimization techniques).

In a statistical point of view, both approaches share a criterion function, where the distance between model-based (via simulation) and empirical moments is imposed by restrictions on a covariance matrix - however, it must be stated that this criterion function is based on a (quasi) likelihood function in the case of II (Gallant and Tauchen (2010)). Indeed, the use of such kind of function directly refers to GMM. It is well known that the latter is most likely applied on single-equation rather than system-of-equation models and is highly sensitive to the instruments, e.g. the lag-structure in the data used. Furthermore, the question arises which kind of auxiliary model to choose in order to generate significant and plausible parameter estimates. Thus the major concern is to find the appropriate structure of e.g. a VAR model, namely the number of lags considered.

As we consider the well known *Bayesian* approach, this procedure has the advantage that the posterior distributions of the parameters (within a system-of-equations framework) can be easily computed from user friendly software like e.g. MATLAB DYNARE. However, there are several major disadvantages when we apply Bayesian techniques to our empirical study, i.e. with respect to a non-linear bounded rationality model. To begin with, the Bayesian approach requires the choice of appropriate prior distributions associated with the underlying economic interpretation of the structural parameters. It is still an open question what criteria are suited best in order to identify the most accurate prior information. For instance, Lombardi and Nicoletti (2012) discuss the sensitivity of posterior estimation results to the

choice of different expressions of the prior knowledge. Del Negro and Schorfheide (2008) also provide an explicit method for constructing prior distributions based on the beliefs regarding macroeconomic indicators. However, we state that the Bayesian analysis is not a panacea for the BoR model, since prior information is not available at least for the behavioral parameters β and δ . Again, this holds since, so far, the existing knowledge by neuroscientists does not allow for pinning down a general micro-founded model on information processing (De Grauwe (2011)). In addition, the Bayesian estimation must be designed to cope with the shape of the prior distribution, which is often unspecified, i.e. ‘uninformative’ priors might be considered. As a result, the estimated posterior becomes quite similar to the prior distribution.

Furthermore, we need to apply a Bayesian full-information analysis where a particle filter technique must be applied for estimating the behavioral parameters in the BoR model. This is because these parameters are incorporated in non-linear logistic functions (as a result of applying discrete choice theory). Thus the estimation of a non-linear model can be subjected to e.g. an increase in approximation errors, especially when the filter method is applied for evaluating the likelihood function (DeJong and Dave (2007), their Chapter 11). In general, we state that the Bayesian approach serves more likely as an *intransparent* ‘black box’ if a particle filter technique is incorporated, when estimating (non-linear) models relative to the SMM approach.

Finally, the *non-linear least squares* (NLLS) estimation approach is also considered to be a potential candidate for evaluating non-linear models. Since the behavioral rules are determined endogenously within in the regime-switching process in the BoR model, however, parameter estimates from NLLS are likely to be biased and inefficient. In comparison to the MM approach, two remarks must be made. First, the MM approach is capable of the estimation of a larger set of parameters relative to NLLS. In this sense, a more highly structured model can be taken to the empirical data by MM. Second, it is well-known that statistical efficiency on parameter estimates in NLLS can be guaranteed only if the shocks are Gaussian distributed. Hence, possible model misspecification problems could be avoided using the MM approach.

7.5 Empirical Application to the Euro Area

7.5.1 Data

Euro Area data set is retrieved from the 10th update of the Area-Wide Model (see Fagan et al. (2001)) quarterly database. The data applied in this study cover the period from 1975:Q1 to 2009:Q4. The output gap and interest rate gap are computed from real GDP and nominal short-term interest rate, respectively. A standard smoothing parameter of $\lambda = 1600$ is used to estimate the trend of the observed data from the Hodrick-Prescott filter. The inflation measure is the quarterly log-difference of the Harmonized Index of Consumer Prices (HICP) instead of the GDP deflator. The inflation gap is also computed using the Hodrick-Prescott filter.¹¹ The sample for this data set is available from 1970:Q1 onwards. Five years are

¹¹In most cases, the HICP and GDP deflation are used to measure the consumer price index (CPI) and producer price index (PPI) inflation, respectively. For our empirical application, we resort to the HICP instead of the implicit GDP-deflator, since the former has been widely used for micro-level analysis. For instance, Forsells and Kenny (2004) show that inflation expectations can be approximated by micro-level data like consumer surveys (i.e. in the European Commission survey indicators). Also see Chapter 4 of this thesis for a more detailed discussion on using the HICP instead of the GDP-deflator in macroeconomic studies.

considered in a rolling window analysis to estimate the perceived volatility of the output gap $\lambda_{y,t}$. The underlying MATLAB codes are available upon readers request.

7.5.2 Basic Results

Table 7.1 presents the parameter estimates when the intensity of choice γ is varied over an admissible range, that is, $\gamma \in \{0.1, 1, 10, 100\}$ – where the value of 100 serves as an approximation to infinity. Hence, we cover the cases where the self-selecting mechanism is close to being purely stochastic ($\gamma = 0.1$) up to being almost fully deterministic ($\gamma = 100$). In addition, we also fix this parameter to unity, which is in line with De Grauwe (2011, p. 439) and account for a moderate degree in the intensity of choice together with a value of 10. We choose this treatment in order to check how sensitive the estimation results are with respect to the differences in the speed of switching between regimes. The difficulty for estimating γ can be seen more transparently when examining the non-linear structure of the discrete choice mechanism in Appendix F1. Several authors also point this out in their studies, e.g. Gaunersdorfer and Hommes (2007) as well as Goldbaum and Mizrach (2008).¹² Furthermore, in our application, the structural representation of the behavioral parameters can cause multiple peaks over the entire parameter space, i.e. the associated likelihood is located in uninteresting and/or implausible regions. By calibrating γ we can put focus on our objective of the empirical application, that is, the economic interpretation of the model parameters in the BoR model.

De Grauwe emphasizes in his original paper that the “heuristic model does not need lags in the transmission process to generate inertia” (2011, fn. 11, p. 443). Although it can be agreed on this from a theoretical point of view, we show empirically that this statement is most likely wrong. In order to verify this, we estimate also a purely forward-looking version of the BoR model where we assume $\chi = \alpha = 0$. We show that based on the value of J (see second last row of Table 7.1), the fit of the forward-looking specification is worse compared to the hybrid specification of the model. The results show that the backward-looking elements in the dynamic IS equation and the NKPC play indeed an important role in matching the empirical moments. Therefore we focus on the comparison of the hybrid RE to the hybrid BoR model only.

In the hybrid BoR case, we will choose those parameter estimates for interpretation, which are connected to the lowest value of the loss function given the corresponding value of γ . However, the values of the criterion function for the cases $\gamma = 0.1$ and $\gamma = 1$ are close to each other, i.e. $J = 51.51$ vs. $J = 53.72$. Given the corresponding p-values (0.905 vs. 0.861), both models exhibit almost an equal fit. We decide to interpret those point estimates based on the empirical evaluation in the case $\gamma = 1$. The reason is that the confidence bands are much more narrow in this case - especially for the parameters α and β . For the latter, the increase in uncertainty is not really surprising as γ approaches zero because the switching process becomes almost purely stochastic in that case. Here, as the intensity of choice is set close to its lower bound this means that expectations do not influence the realizations of the current output gap and inflation gap. Therefore the point estimate for β is hard to pin down. Since there is less uncertainty connected to the estimation results of the parameters as $\gamma = 1$ holds, we claim that the results are more reasonable for economic interpretation – besides the point estimates do not differ (except for β) either. In addition, the cases of strong switching processes ($\gamma = 10$ and $\gamma = 100$) do not provide a good approximation to the

¹²Gaunersdorfer et al. (2008) show that the intensity of choice being a crucial parameter with respect to (local) stability in this kind of bounded rationality model with regime-switching.

data generating ones. According to the model specification test, the corresponding p-value is very small and they are rejected as being a "true" model with a high probability.

As it is common in a persuasive amount of empirical studies, the discount parameter ν is calibrated to 0.99. Finally, the memory parameter ρ is set to zero, i.e. past errors are not taken into account (cf. the equations (7.10) and (7.16)), since this is the empirical result for all our estimations. From this, it can be concluded that a strict forgetfulness or cognitive limitation holds, which is a requirement for observing animal spirits (cf. De Grauwe (2011, p. 440)).¹³ By fixing those parameters in the final estimation, we can reduce problems in high-dimensional parameter space and focus on a particular parameter space which will be more interesting for empirical economists. Given these assumptions, we can separately obtain the estimates for remaining parameters from the RE and BoR model via (S)MM. Note here that the Taylor principle $\phi_{\hat{\pi}} > 1$, which is required to hold in order to ensure the stability of the system is always fulfilled. This is ensured by setting the lower bound of this monetary policy parameter equal to one.

7.5.3 Economic Interpretation of the Results

Several observations are worth mentioning. The parameter estimate for the degree of price indexation α is higher in the BoR (0.973) than the RE (0.765) model. It follows that the expressions, which are in front of the forward- and backward-looking terms in the Phillips Curve, indicate a (slightly) higher weight on future inflation $\tilde{E}_t^j \hat{\pi}_{t+1}$ (i.e. $\frac{\nu}{1+\alpha\nu} > \frac{\alpha}{1+\alpha\nu}$), where the result is more pronounced for the RE (0.563 > 0.437) compared to the BoR (0.504 > 0.496) model. From this, we see that there is strong evidence for a hybrid structure of the NKPC in both models. The empirical applications of the BoR model show that the dynamics of the inflation gap are also driven by the expectation formation process (i.e. the evaluation of the forecast performance) in the inflation gap, which is caused by agents' cognitive limitation. In other words, we find evidence for a static expectation formation process, since the estimated value for α is quite high: one group believes in a central bank inflation target of zero percent (equation (7.13)) while another group of the agents form their expectations in a purely static way (equation (7.14)), i.e. under consideration of $\hat{\pi}_{t-1}$.

The results on the estimate of price indexation has important implications for conducting optimal monetary policy. According to Leitemo (2008), the optimal targeting rule in the (pre-)commitment case has to be much more forward-looking from a high degree of backward-looking behavior in the NKPC. Our results indicate that this most likely holds for the RE model via MM, but this result is quite different when the Bayesian technique is applied, e.g. the parameter of price indexation is estimated to be zero (cf. Smets and Wouters (2003, 2007), Benati and Surico (2009), Cogley et al. (2010) among others as well as our results for the BR case as reported in the Chapters 5 and 6). In particular, the estimated high degree of price indexation will have a similar impact on the results from optimal monetary policy analysis (as reported in Leitemo (2008)) based on the BoR model, although such kind of investigation has not to be undertaken (to the best of our knowledge) in the literature so far.

Regarding the dynamic IS equation in the RE model, the output gap is influenced by the forward- and backward-looking terms at the same proportion, since the empirical estimates show that $\chi = 0.999$ holds. Any specific statement can not be made with respect to the im-

¹³For clarification, we apply the estimations for all calibrated values of γ and include ρ as a parameter to be estimated. It turns out that ρ being equal to zero in all cases.

Label	hybrid RE	$\gamma = 0.1$		$\gamma = 1$		$\gamma = 10$		$\gamma = 100$	
		pure BoR	hybrid BoR	pure BoR	hybrid BoR	pure BoR	hybrid BoR	pure BoR	hybrid BoR
α	0.765 0.481 - 1.000	0 0.294 - 0.468	0.931 0.140 - 0.242	0 0.219 - 0.404	0.973 0.139 - 0.211	0 0.106 - 0.603	0.856 0.071 - 0.284	0 0.0 - 0.575	0.432 0.117 - 0.421
κ	0.035 0.011 - 0.058	0.381 0.294 - 0.468	0.191 0.140 - 0.242	0.312 0.219 - 0.404	0.175 0.139 - 0.211	0.355 0.106 - 0.603	0.178 0.071 - 0.284	0.268 0.0 - 0.575	0.269 0.117 - 0.421
$\sigma_{\hat{\pi}}$	0.275 0.097 - 0.453	0.446 0.105 - 0.788	0.495 0.269 - 0.721	0.297 0.000 - 0.643	0.498 0.326 - 0.670	0.192 0.000 - 0.408	0.432 0.084 - 0.780	0.164 0.000 - 0.370	0.277 0.000 - 0.577
χ	0.999 0.349 - 1.000	0 0.000 - 1.000	0.547 0.000 - 1.000	0 0.000 - 1.000	0.451 0.000 - 1.000	0 0.000 - 1.000	0.542 0.000 - 1.000	0 0.000 - 0.842	0.351 0.000 - 0.842
τ	0.079 0.000 - 0.222	0.737 0.298 - 1.177	0.329 0.106 - 0.551	0.371 0.172 - 0.570	0.322 0.158 - 0.486	0.179 0.000 - 0.663	0.259 0.026 - 0.492	0.164 0.000 - 0.442	0.242 0.000 - 0.559
σ_y	0.561 0.354 - 0.768	0.423 0.018 - 0.827	0.669 0.364 - 0.974	0.808 0.654 - 0.962	0.793 0.630 - 0.957	0.760 0.418 - 1.102	0.835 0.623 - 1.048	0.635 0.204 - 1.065	0.756 0.490 - 1.021
$\phi_{\hat{\pi}}$	1.289 1.000 - 1.944	1.527 1.000 - 2.743	1.128 1.000 - 1.534	1.816 1.000 - 3.313	1.145 1.000 - 1.485	1.316 1.000 - 2.059	1.250 1.000 - 1.973	1.322 1.000 - 2.031	1.284 1.000 - 1.826
ϕ_y	0.497 0.058 - 0.936	0.621 0.257 - 0.985	0.681 0.438 - 0.924	0.713 0.252 - 1.175	0.674 0.443 - 0.904	0.580 0.066 - 1.094	0.608 0.239 - 0.978	0.755 0.000 - 1.911	0.589 0.181 - 0.997
$\phi_{\hat{r}}$	0.604 0.411 - 0.797	0.685 0.614 - 0.755	0.675 0.613 - 0.736	0.739 0.671 - 0.808	0.677 0.608 - 0.746	0.602 0.305 - 0.900	0.660 0.534 - 0.786	0.488 0.075 - 0.901	0.578 0.292 - 0.863
$\sigma_{\hat{r}}$	0.421 0.140 - 0.701	0.109 0.000 - 0.409	0.216 0.000 - 0.563	0.269 0.000 - 0.601	0.256 0.000 - 0.565	0.439 0.000 - 0.965	0.304 0.000 - 0.728	0.435 0.000 - 1.020	0.361 0.000 - 0.840
β	-	5.563 3.899 - 7.227	4.584 1.918 - 7.249	2.379 1.417 - 3.341	2.269 1.082 - 3.456	3.101 0.0 - 7.767	2.274 0.000 - 6.331	1.645 0.387 - 2.902	1.927 0.565 - 3.289
δ	-	0.709 0.000 - 2.123	0.688 0.000 - 2.789	0.379 0.000 - 1.278	0.488 0.000 - 1.653	1.291 0.000 - 4.455	0.465 0.000 - 1.736	0.658 0.000 - 2.655	0.386 0.000 - 1.403
J	56.31	80.04	51.51	100.88	53.72	100.52	71.67	241.61	122.93
p	0.843	0.151	0.905	0.006	0.861	0.006	0.295	0.000	0.000

Table 7.1: Estimation results (RE versus BoR model).

Note: We use $p = 78$ moments (two years), based on the SMM approach. 95% confidence interval is given in brackets. The degrees of freedom for χ^2 distribution amount to 68 (hybrid RE, pure[ly forward-looking] BoR) and 66 (hybrid BoR). The 5% critical values are 88.25 and 85.96, respectively. No memory is assumed in the BoR models ($\rho = 0$). The discount factor ν is calibrated to 0.99. The p-value is denoted by p .

pect of habit formation on the dynamics of the output gap in the BoR model, since χ is estimated to be significant in this case. Furthermore, the result on the interest rate smoothing parameter shows that there exists a moderate degree of persistence ($\phi_{\hat{r}} = 0.677$) in the nominal interest rate gap in the BoR model. This observation is consistent with the results obtained from Bayesian estimation reported in the literature. In our study, the results obtained by MM show that the point estimate for $\phi_{\hat{r}}$ is less pronounced.

The empirical estimates for κ and τ in the RE model provide an evidence of a small degree of inherited persistence, where the latter one measures the cross-relationships between output and inflation gap, respectively. These results indicate that both economic indicators respond less to the changes in the associated driving forces, i.e. the real interest rate gap and the output gap, respectively. However, this does not hold for the BoR model. Here the changes in the output gap have a strong impact on the movements in the inflation gap (indicated by $\kappa = 0.175$) compared to the RE case ($\kappa = 0.035$). For the output gap in the BoR model, inherited persistence plays a fundamental role in shaping the dynamics of this variable, which can be seen through the high values of inverse intertemporal elasticity of substitution ($\tau = 0.322$). For the RE model, this parameter is estimated to be insignificant. This implies that the tendency towards risk aversion is stronger in the BoR relative to the RE model. To sum up, our results show that in the BoR model, the cross-movements in the output and inflation gap account for the persistence in both variables (under consideration of perfect habit formation $\chi = 0.999$ in the RE case) rather than price indexation alone. This can be seen through the high values for κ and τ together with α within a bounded rational environment.

The variance in the output and inflation gap shocks are estimated to be larger for the BoR ($\sigma_y = 0.793$ and $\sigma_{\hat{\pi}} = 0.498$) than those of the RE ($\sigma_y = 0.561$ and $\sigma_{\hat{\pi}} = 0.275$) model, respectively. The results reveal that the volatilities of the output and inflation gap are closely related to the switching rules with respect to the consumption and price-setting behavior. For instance, the waves of optimism and pessimism act much like a persistent force in the output gap fluctuations going from peaks to troughs. Figure 7.1 illustrates that the peak of the fluctuation in the simulated output gap for the BoR case (middle-left panel) corresponds to the market optimism (lower-left panel) and vice versa. The qualitative interpretation remains almost the same for the inflation gap in the BoR case (middle- and lower-right panels, respectively) – but the dynamics of extrapolators are highly volatile reflecting the large variations in the second moments of the empirical inflation gap (upper-right panel). The goodness of fit of the models should not be directly compared by illustrating the simulated time series (middle-panels), but we see that the series qualitatively resemble their empirical counterparts (upper-panels). Finally, the nominal interest rate shocks $\sigma_{\hat{r}}$ in the RE model are estimated to be 0.421, while in the BoR case no interpretation can be made since the point estimate is not significant.

The remaining parameter estimates confirm the known results from the literature where the monetary authority may react slowly to changes in the output gap, while the opposite holds for the coefficient on the inflation gap (RE: $\phi_y = 0.497$ and $\phi_{\hat{\pi}} = 1.289$ vs. BoR: $\phi_y = 0.674$ and $\phi_{\hat{\pi}} = 1.145$). The results for $\phi_{\hat{\pi}}$ indicate that the Taylor principle clearly holds over the whole sample period when $\phi_{\hat{\pi}}$ is significantly higher than its lower bound, which is assumed to be equal to one. This observation holds for both cases. Nevertheless, the results for the BoR model indicate a stronger concern in the output gap movements relative to the RE model. It is worth mentioning that the estimation results indicate that the monetary policy coefficient on the output gap ϕ_y is equal to 0.673, which is in line with the observations of

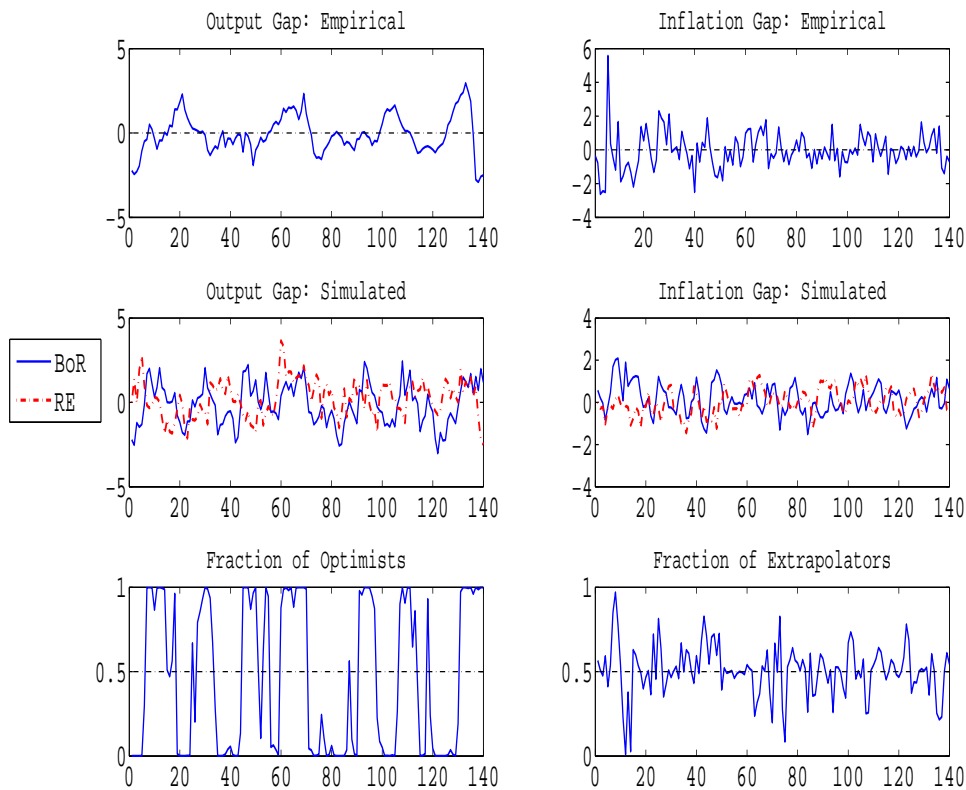


Figure 7.1: Dynamics in the output and inflation gap (RE versus BoR model).

Note: The upper and middle panels display the empirical and simulated values for the output gap (left) and the inflation gap (right), while the lower panels display the corresponding fraction of market optimists (left) and extrapolators (right). The simulated time series are computed using the point estimates for the parameters in the RE and BoR model in Table 7.1. The time in quarters is displayed on the horizontal axis.

De Grauwe (2011, pp. 443-445). His simulations show that flexible inflation targeting can reduce both output gap and inflation (gap) variability at its minimum level when ϕ_y lies in the range of 0.6 to 0.8.

The interpretation of this observation is manifold. First, consider the case of strict inflation targeting, where the central bank does not account for the volatility in the output gap.¹⁴

¹⁴Note here, that strict and flexible inflation targeting are measured by low and high values of ϕ_y in the *Taylor rule*, respectively. This definition of inflation targeting is rather uncommon in the literature, while is more connected to the corresponding weights in the *loss function* of the central bank (see also the Chapters 2 and 3 of this thesis). However, we stick to De Grauwe's (2011, pp. 443-445) interpretation here in order to build a bridge from our empirical results to his theoretical analysis.

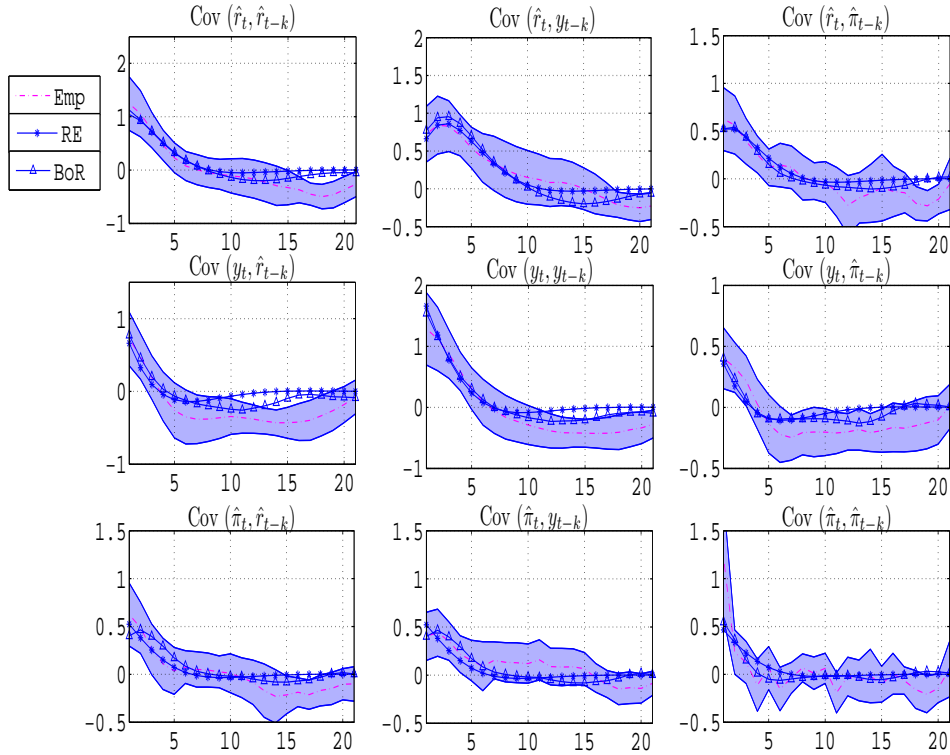


Figure 7.2: Model covariance (Cov) profiles in the Euro Area.

Note: The dashed line results from the empirical covariance estimates. The shaded area is the 95% confidence bands around the empirical moments. The triangle (BoR) and star (RE) lines indicate the model generated ones. The confidence bands are computed via the Delta method (see Appendix F2). The lag length is displayed on the horizontal axis.

As a result, the forecast performance of the optimists and pessimists are not affected since the real interest rate gap in the dynamic IS curve does not response directly to monetary policy. However, there is still an indirect effect (even highly volatile movements in y_t are not dampened by the policy makers) indicated by κ in the NKPC. Hence, due to the high degree of inherited persistence the strict inflation targeting can fail to control strong fluctuations in the output and inflation gap. Second, in the case of strong output gap stabilization (relative to the inflation gap) the central bank dampens its pre-commitment to an inflation target. The amplification effects of this kind of policy on the forecast performances of the inflation extrapolators will then result in a higher volatility of the inflation gap. In this respect, it can be concluded that our empirical findings account for neither the first nor the second extreme case, but for a moderate degree of flexible inflation targeting in the Euro Area over the observed time interval instead.

As already noted, the present study focuses on the estimation of the bounded rationality parameters. First, over the whole sample period, the optimistic agents have expected a fixed

divergence of belief indicated by $\beta = 2.269$. Roughly speaking, the optimists have been really optimistic that the future output gap will differ *positively* by slightly above one percent on average from its steady state value.¹⁵ Due to the symmetric structure of the divergence in beliefs, pessimistic agents were moderately pessimistic over the same sample period. From their point of view the future output gap was expected to be around one percent on average *below* its steady state value. The point estimate for the bounded rationality parameter δ , which measures the divergence in beliefs, is *unfortunately* insignificant. Hence, one can not make any statements about how both types of agents felt confident about their expectations. In other words, there is no clear evidence for a low or high degree of uncertainty connected to the expected future value of y_t given by β .

In line with the results for (and assumptions of) the parameters including $\rho = 0$, which indicate endogenous and inherited persistence (α, χ, κ and τ), the highly subjective expected mean value of the output gap β - in conjunction with the dynamics induced by the self-selecting mechanisms (see the corresponding fractions in the lower-panels in Figure 7.1) - explains the (high) volatility of the output gap. According to the discrete choice theory, we see that this strengthens the optimistic agents' belief about the future output gap to diverge in the data, since they can over(or under)react to the underlying shocks that occur within the Euro Area. The same observation holds for the inflation gap dynamics. The proportion of the extrapolators in the economy corresponds to the empirical inflation gap movements (cf. lower right vs. upper-right panels in Figure 7.1): the higher the fraction of extrapolators is, the more volatile the inflation gap dynamics due to static expectations, i.e. under the past realizations $\hat{\pi}_{t-1}$ will be.

The visual inspection in Figure 7.2 shows a *fairly remarkable* fit of the models to the data. The match of both models looks clearly good over the first few lags and still fairly good over the higher lags until the lag 8. Exceptions can be found in the (y_t, \hat{r}_{t-k}) , $(y_t, \hat{\pi}_{t-k})$ and $(\hat{\pi}_t, y_{t-k})$ nexus, where the simulated covariance profiles generated by both models diverge from their empirical counterparts beginning at the 3,4 and 5 lag, respectively. More precisely, the simulated covariance profiles of the RE model mimic the empirical ones for the cross-covariance $(\hat{\pi}_t, \hat{r}_{t-k})$ quite well, while the same holds for the BoR model in case of the cross-/auto-covariances $(\hat{\pi}_t, y_{t-k})$ and $(\hat{\pi}_t, \hat{\pi}_{t-k})$. In the latter case, although the sharp decline in the autocovariance of the inflation gap over the first two lags is not covered by both models, the BoR specification matches the empirical profiles *slightly better* relative to the RE one. In any case, all of the moments are *inside* the confidence intervals of the empirical moments up to lag 8. The graphical results are also confirmed by the values of the objective function J for the RE (56.31) and BoR (53.72) model in the second to last row of Table 7.1. The asymptotic χ^2 distributions for the J -test have the degrees of freedom of 68 and 66 for the RE and BoR model, respectively. Since the critical values at 5% level are 88.25 and 85.96, that is the estimated loss function values are smaller than these criteria, we do *not* reject the null hypothesis that these models can approximate the data generating process well. Moreover, the picture shows a *remarkable fit of the BoR model*, which leads to some confidence in the estimation procedure. We conclude from this observation that a bounded rationality model with cognitive limitation provides good fits for auto- and cross-covariances of the data.

Note here that the significant differences between the two models could be tested using a formal model comparison method, since the models do not have any difficulties to fit the

¹⁵Note that the expected future value of the output gap is given by $E_t^i y_{t+1} = |d_t| = \frac{1}{2}\beta$ on average with $i = \{O, P\}$.

empirical moments at the 5% significant interval (see also Jang (2012) among others). In other words, the J -test only evaluates the validity of the model along the lines of the chosen moment conditions. Therefore we cannot provide a direct comparison between the fits of the two models. More rigorous tests will be a priority in future research. Finally, our empirical results indicate that the empirical test on bounded rationality (i.e. the assumption of the divergence in beliefs) has to be treated carefully, because some parameters (including the behavioral parameter δ) within the non-linear modelling approach are *insignificant*.

7.5.4 Comparison with other Empirical Studies

Indeed, there exists a plethora of studies on the estimation of (small, medium or large) NKMs with rational expectations using Euro Area data. To the best of our knowledge, however, these studies provide some implications which differ from our contribution. While we apply a moment-based estimation under consideration of Euro Area data over a specific time interval up to the end of 2009, most of the investigations are based on GMM and the Bayesian approach, using data just to the beginning of the 21st century instead. Furthermore, we consider gap specifications of $\hat{\pi}_t$ and \hat{r}_t explicitly, while in the literature the majority of time series are not de-trended. Hence, we must indicate that a comparison of our results with those from the literature has to be done with some caution.

More generally, one of the representative studies in this field is the empirical application of Smets and Wouters (2003). Here the sample captures the period from 1980:Q2 to 1999:Q4. In their paper, they apply Bayesian estimation on a medium scale NKM for the Euro Area. Compared to the RE and BoR cases presented here, they found different values for the parameters τ and $\phi_{\hat{\pi}}$, which are estimated to be higher (0.739 and 1.684). In contrast, the estimated values for κ and ϕ_y are relatively small (0.01 and 0.10). Finally, $\phi_{\hat{r}}$ is slightly lower than in Smets and Wouters (2003, $\phi_r = 0.956$).

Moons et al. (2007) give a good overview on the results stemming from different studies using different techniques except for the Bayesian one. Most of the parameter estimates are in line with those reported in column 2 of our Table 7.1, i.e. in case of the hybrid RE model. According to Table 1 in Moons et al. (2007, p. 888), τ and κ vary in a range of (0.03, 0.08) and (0.02, 0.17), while we find $\tau = 0.079$ and $\kappa = 0.035$. The policy parameters $\phi_y = 0.561$, $\phi_{\hat{\pi}} = 1.289$ and $\phi_{\hat{r}} = 0.604$ are (slightly) below the estimates reported in Moons et al. (2007) where $\phi_y = (0.77, 0.90)$, $\phi_{\pi} = (0.87, 2.02)$ and $\phi_r = (0.77, 0.94)$ are observed. For the latter, note once again that the level and not the gap of the corresponding time series is considered. The composite parameters, which indicate backward-looking behavior in the dynamic IS curve and the NKPC, can be denoted by $\psi_1 = \frac{\chi}{1+\chi}$ and $\psi_2 = \frac{\alpha}{1+\alpha\nu}$. It can be stated that our results for the RE model, $\psi_1 = 0.5$ and $\psi_2 = 0.44$, mimic roughly those found in the literature, i.e. $\psi_1 = (0.22, 0.97)$ and $\psi_2 = (0.13, 0.54)$ hold.

It can be seen that in the case of the hybrid BoR model – presented in column 6 of Table 1 – (for $\gamma = 1$) these results differ substantially from those reported in the literature. Not surprisingly, this stems from the fact that the behavioral model of De Grauwe (2011) exhibits a different kind of expectation channel which can substitute the absence of rational expectations. Nevertheless, Moons et al. (2007) estimate a small scale open-economy NKM under consideration of a fiscal policy rule (in the spirit of the European Stability and Growth Pact) with Bayesian techniques and found that the parameter estimates are similar with our results. In particular, τ is estimated to be high (0.24) which is in line with the BoR model (0.32). The authors also find that a high value of the monetary policy coefficient concerning

the output gap is estimated to be $\phi_y = 0.75$, while our estimated value is equal to 0.67.

7.6 Conclusion

In this Chapter, we attempt to provide empirical evidence for the behavioral characteristics of the De Grauwe (2011) model. In particular, the validity of the model assumptions on the cognitive limitation (e.g. because of different individual emotional states) is empirically tested using historical Euro Area data. Our contribution relies on the application of the SMM approach, which can be used to deal with non-linear models. We focus on the estimation of the behavioral parameters in the model, which account for animal spirits in the Euro Area, i.e. we hypothesize that historical movements of macro dynamics are influenced by waves of optimism and pessimism. From this, our study aimed to examine the effects of the group behavior on the output and inflation gap. In particular, we follow the behavioral approach of De Grauwe (2011), who assumes divergence in beliefs about the future value of both variables. The corresponding decision rules for market optimists and pessimists are given by the forecast performance of two groups of agents based on discrete choice theory. To see this, we contrast a standard hybrid version of the three-equations NKM under consideration of rational expectations with a version of the same model where we assume bounded rationality in expectation formation processes using (S)MM. To the best of our knowledge, such kind of empirical study – structural estimation of a bounded rationality model under consideration of the moment conditions – has not been extensively investigated before in the literature.

Our main empirical findings show that a bounded rationality model with cognitive limitation provides a reasonable fit to auto- and cross-covariances of the Euro Area data. Therefore our empirical results of the BoR model offer some new insights into expectation formation processes for the Euro Area. In particular, we provide empirical evidence in support of De Grauwe (2011) for understanding the group's over- and under-reaction to the economy. First, over the whole time interval the agents had expected moderate deviations of the output gap from its steady state value. Second, in the absence of rational behavior we find strong evidence for a static expectation formation process regarding the inflation gap. Both observations explain a high degree of persistence in the output gap and the inflation gap. From this, we found that animal spirits strengthen the optimists' belief about the future output gap to diverge in the historical Euro Area data.

However, the estimation of the bounded rationality (i.e. under the assumption of the divergence in beliefs) model reveals that the parameters which measures the degree of habit formation in the dynamic IS curve, the standard deviation in the nominal interest rate shock and, unfortunately, the degree of the divergence in the movement of economic activity are *statistical insignificant*. In the latter case no statement can be made about the uncertainty in agents' forecast regarding the output gap. A reason for this bounded rationality parameter being insignificant could be explained by the *highly non-linear* approach for modeling expectations being considered.

This research will serve as a base for future studies. For example, the current study focused on Euro Area data in which we could use the large sample size for parameter estimation. While an estimation for e.g. the US economy would be a promising exercise to be undertaken, note that in this case the sample size must be split into two subperiods namely the Great Inflation and Moderation period, respectively. This is necessary due to the existing structural breaks in the US time series based on a change in inflation's volatility from high to low at

beginning of the 1980's. However, such an observation does not hold for the Euro Area. We state that because of the small sample size in both subperiods for the US the BoR model is hard to evaluate. A specification of both models based on the high-frequency approach, known from Part One of this thesis, might help in this respect. Nevertheless, we would like to leave such an investigation to further research.

In addition, in order to identify the effects of individual expectation formation processes on the economy, the decision rules i.e. the transition rules from one state of the economy to another can be calculated based on survey data (for example see Lux (2009)). Thus, these probabilities are then treated as exogenous and (in contrast to the De Grauwe model) are computed under consideration of the underlying time series using discrete choice theory. Finally, one can further continue the model estimation with much richer models like e.g. the medium-scale version developed by Smets and Wouters (2005, 2007). Finally, De Grauwe (2012) shows that his modification of the baseline NKM with respect to bounded rationality is able to reproduce a non-Gaussian distribution of the growth rate of output. Franke (2013) shows that there is some empirical evidence for this kind of observation. We agree with the author that it might be appropriate to consider the skewness of the distribution of this (and other) variable(s) as an additional moment to match. We leave all these issues also to future research.

7.7 Appendix F

7.7.1 F1: Solution of the Baseline NKM under RE and BoR

In general, all model specifications are described by the following system in canonical form:

$$AX_t + BX_{t-1} + CX_{t+1} + \varepsilon_t = 0, \quad (7.30)$$

where

$$X_t = \begin{pmatrix} y_t \\ \hat{\pi}_t \\ \hat{r}_t \end{pmatrix}, \quad X_{t-1} = \begin{pmatrix} y_{t-1} \\ \hat{\pi}_{t-1} \\ \hat{r}_{t-1} \end{pmatrix}, \quad X_{t+1} = \begin{pmatrix} \tilde{E}_t^j y_{t+1} \\ \tilde{E}_t^j \hat{\pi}_{t+1} \\ \tilde{E}_t^j \hat{r}_{t+1} \end{pmatrix}, \quad \varepsilon_t = \begin{pmatrix} \varepsilon_{y,t} \\ \varepsilon_{\hat{\pi},t} \\ \varepsilon_{\hat{r},t} \end{pmatrix}.$$

The corresponding system matrices are given by:

$$A = \begin{pmatrix} 1 & 0 & \tau \\ -\lambda & 1 & 0 \\ -(1 - \phi_{\hat{r}})\phi_y & -(1 - \phi_{\hat{r}})\phi_{\pi} & 1 \end{pmatrix}, \quad B = \begin{pmatrix} -\frac{\chi}{1+\chi} & 0 & 0 \\ 0 & -\frac{\alpha}{1+\alpha\nu} & 0 \\ 0 & 0 & -\phi_{\hat{r}} \end{pmatrix} \quad (7.31)$$

and

$$C = \begin{pmatrix} -\frac{1}{1+\chi} & -\tau & 0 \\ 0 & -\frac{\nu}{1+\alpha\nu} & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (7.32)$$

Recall that for the *BoR model* we assume

$$\begin{aligned}\tilde{E}_t^{BoR} y_{t+1} &= (\alpha_{y,t}^O - \alpha_{y,t}^P) d_t \\ \tilde{E}_t^{BoR} \hat{\pi}_{t+1} &= \alpha_{\hat{\pi},t}^{tar} \bar{\pi} + \alpha_{\hat{\pi},t}^{ext} \hat{\pi}_{t-1}\end{aligned}$$

with

$$d_t = \frac{1}{2}(\beta + \delta\lambda_{y,t}), \quad (7.33)$$

where we also consider the equations (7.10) to (7.18) with $\bar{\pi} = 0$. In the following, we solve for the dynamics of the system (7.30). In case of the BoR model, the ‘solution’ is given by

$$X_t = -A^{-1}[BX_{t-1} + CX_{t+1} + \varepsilon_t], \quad (7.34)$$

where the matrix A is of full rank, i.e. its determinant is not equal to zero, given the point estimates of the parameters in Table 7.1. Under consideration of the heuristics for the forecasts regarding the output and inflation gap expectations, the forward-looking term X_{t+1} is substituted by the equivalent expressions for the discrete choice mechanism given in section 7.2. It follows that the model becomes purely backward-looking and thus (7.34) can be solved by backward induction. In particular, it can be shown that the backward solution of the BoR model is based on a non-linear parameterization. To see this, the expected output gap can be rewritten as (cf. the equations (7.6) to (7.9)):

$$\tilde{E}_t^{BoR} y_{t+1} = (2\alpha_{y,t}^O - 1) \frac{1}{2}(\beta + \delta\lambda_{y,t}). \quad (7.35)$$

In addition, the probability density of optimists can be rewritten as (cf. the equations (7.11) and (7.12)):

$$\begin{aligned}\alpha_{y,t}^O &= \frac{\exp(\gamma U_t^O)}{\exp(\gamma U_t^O) + \exp(\gamma U_t^P)} = \frac{1}{1 + \exp\{\gamma U_t^P - \gamma U_t^O\}} \\ &= \frac{1}{1 + \exp\{\gamma(E_{t-1}^O y_t - y_t)^2 - \gamma(E_{t-1}^P y_t - y_t)^2\}},\end{aligned} \quad (7.36)$$

where $U_t^P = -(E_{t-1}^P y_t - y_t)^2$ and $U_t^O = -(E_{t-1}^O y_t - y_t)^2$ are applied. Note here that no memory is assumed ($\rho = 0$).¹⁶ Then we replace the term $E_{t-1}^O y_t$ by $\frac{1}{2}(\beta + \delta\lambda_{y,t-1})$. Similarly, the term $E_{t-1}^P y_t$ is replaced by $-\frac{1}{2}(\beta + \delta\lambda_{y,t-1})$. After some algebra we get

$$\tilde{E}_t^{BoR} y_{t+1} = \left[2 \left\{ \frac{1}{1 + \exp\{-2\gamma(\beta + \delta\lambda_{y,t-1})y_t\}} \right\} - 1 \right] \frac{1}{2}(\beta + \delta\lambda_{y,t}), \quad (7.37)$$

where the term inside the $[\cdot]$ brackets controls the magnitude of the changes in the divergence in beliefs, that is, $[\cdot] \in (-1, 1)$.

From this, we see that the analytical solution for y_t cannot be easily obtained due to the non-linearity in the expected future output gap of optimists. The same results hold for the derivation of the expected future output gap of pessimists. In addition, we also obtain a non-linear solution formula for the expected inflation gap (not shown here), but the non-linearity can also not be dropped in this case. Thus our study relies on the approximation of the backward solution of the BoR model based on simulations. This motivates us to apply SMM when estimating BoR models.

¹⁶We do so under consideration of our empirical results (cf. section 7.5.2). It can be stated that equation (7.36) will become more complicated in the case $\rho > 0$, while all non-linearities prevail.

In contrast, for the *RE model* we assume

$$\begin{aligned}\tilde{E}_t^{RE} y_{t+1} &= E_t y_{t+1} \\ \tilde{E}_t^{RE} \hat{\pi}_{t+1} &= E_t \hat{\pi}_{t+1},\end{aligned}$$

where E_t denotes the mathematical expectation operator. As a result, the RE model is both backward- and forward-looking. Therefore we apply the method of undetermined coefficients in order to solve the model. We guess that the law of motion, which describes the analytical solution to the model, is given by

$$X_t = \Omega X_{t-1} + \Phi \varepsilon_t, \quad (7.38)$$

where $\Omega \in \mathbb{R}^{3 \times 3}$ and $\Phi \in \mathbb{R}^{3 \times 3}$ are the solution matrices. The former is a stable matrix as long as (similar to the matrix A in the BoR case) its determinant is not equal to zero, which ensures the invertibility of Ω . Again, this is confirmed given the estimation results in Table 7.1. We substitute equation (7.38) into equation (7.30) which leads to

$$A(\Omega X_{t-1} + \Phi \varepsilon_t) + B X_{t-1} + C(\Omega X_t + \Phi E_t \varepsilon_{t+1}) + \varepsilon_t = 0.$$

This is equivalent to

$$A(\Omega X_{t-1} + \Phi \varepsilon_t) + B X_{t-1} + C(\Omega^2 X_{t-1} + \Omega \Phi \varepsilon_t + \Phi E_t \varepsilon_{t+1}) + \varepsilon_t = 0.$$

Hence, the reduced-form can be rewritten as

$$(C\Omega^2 + A\Omega + B)X_{t-1} + (A\Phi + C\Omega\Phi + I)\varepsilon_t = 0 \quad (7.39)$$

with I being the identity matrix. Note that $\varepsilon_t \sim N(0, \sigma_z^2)$ with $z = \{y, \hat{\pi}, \hat{r}\}$ and thus $E_t \varepsilon_{t+1} = 0$ holds. In order to solve equation (7.39), all the terms in brackets must be zero.¹⁷ Thus the solution matrices can be uniquely determined. We may write that as

$$C\Omega^2 + A\Omega + B = 0 \Rightarrow \Omega = -(C\Omega + A)^{-1}B. \quad (7.40)$$

In order to solve the quadratic matrix equation (7.40) numerically, we apply again the brute force iteration procedure (cf. Appendix A3 of Chapter 2). Hence an equivalent recursive relation of (7.40) is given by

$$\Omega_n = -(C\Omega_{n-1} + A)^{-1}B \quad (7.41)$$

with an arbitrary number of iteration steps N , i.e. $n = \{1, 2, \dots, N\}$. We define as the initial value $\Omega_0 = \varsigma I$ with I being the identity matrix and $0 \leq \varsigma \leq 1$, where we set $\varsigma = 0.8$. The iteration process (7.41) proceeds until $\|\Omega_n - \Omega_{n-1}\| < \varrho$ holds, where ϱ is an arbitrarily small number (we set $\varrho = 0.1^6$).¹⁸ Given the ‘solution’ of Ω , the computation of Φ is straightforward:

$$A\Phi + C\Omega_n\Phi + I = 0 \Rightarrow \Phi = -(A + C\Omega_n)^{-1}. \quad (7.42)$$

7.7.2 F2: Delta Method and Confidence Interval for Auto-/Cross-Covariances

The Delta method is a common technique for providing the first-order approximations to the variation of moments (see Chapter 5 of Davidson and MacKinnon (2004) among others). In

¹⁷Obviously the trivial solution $X_{t-1} = \Gamma_t = \varepsilon_t = 0$ is discarded.

¹⁸In particular, we search for the fix point of Ω_n according to equation (7.41), respectively, such that $\Omega_N = f(\Omega_N) = -(C\Omega_N + A)^{-1}B$ holds.

this Chapter, we compute the standard errors of the estimated auto- and cross-covariances of the data via the Delta method. The covariance is defined as:

$$\gamma_{ij}(h) = E[(X_{i,t} - \mu_i)(X_{j,t+h} - \mu_j)'], \quad t = 1, \dots, T, \quad (7.43)$$

where γ_{ij} is the auto-covariance function when $i = j$. Otherwise γ_{ij} denotes the cross-covariance between $X_{i,t}$ and $X_{j,t+h}$. h denotes the lag length and μ_i (or μ_j) is the sample mean of the variable X_i (or X_j). The covariance function in Equation (7.43) proceeds with a simple multiplication:

$$\gamma_{ij}(h) = E[X_{i,t} \cdot X'_{j,t+h}] - \mu_i \cdot E[X'_{j,t+h}] = \mu_{ij} - \mu_i \cdot \mu_j,$$

where μ_{ij} denotes $E[X_{i,t} \cdot X'_{j,t+h}]$. Now, we see that $\gamma_{ij}(h)$ is a transformed function of the population moments μ_i , μ_j and μ_{ij} . The vector μ is denoted as the collection of the moments: $\mu = [\mu_i \quad \mu_j \quad \mu_{ij}]$. The covariance function can be differentiated with respect to the vector μ , as follows:

$$D = \frac{\partial \gamma_{ij}(h)}{\partial \mu} = \begin{bmatrix} \frac{\partial \gamma_{ij}(h)}{\partial \mu_i} \\ \frac{\partial \gamma_{ij}(h)}{\partial \mu_j} \\ \frac{\partial \gamma_{ij}(h)}{\partial \mu_{ij}} \end{bmatrix} = \begin{bmatrix} -\mu_j \\ -\mu_i \\ 1 \end{bmatrix}. \quad (7.44)$$

Note that the Delta method is used to provide the asymptotic distribution for the estimate $\widehat{\gamma}_{ij}$ when matching the sample moments of the data:

$$\sqrt{T}(\gamma_{ij} - \widehat{\gamma}_{ij}) \sim N(0, D'SD), \quad (7.45)$$

where $D'SD$ is the covariance matrix of the estimated moments. As regards to some suitable lag length q , we employ a common HAC estimator introduced by Newey and West (1994) when estimating the covariance matrix of sample moments:

$$\widehat{\Sigma}_\mu = \widehat{C}(0) + \sum_{k=1}^q \left(1 - \frac{k}{q+1}\right) [\widehat{C}(k) + \widehat{C}(k)'] \quad (7.46)$$

with

$$\widehat{C}(k) = \frac{1}{T} \sum_{t=k+1}^T [f(z_t) - \widehat{\mu}][f(z_{t-k}) - \widehat{\mu}]', \quad (7.47)$$

where $f(z_t) = [X_i, X_j, X_i \cdot X_j]$ holds. The total number of lags is once again denoted by k . In particular, we follow the advice by Davidson and MacKinnon (2004, p. 364) and scale q with $T^{1/3}$. Accordingly, we set it to $q = 5$ for the Euro Area data. The optimal weight matrix is defined as $S = \widehat{\Sigma}_\mu^{-1}$, and it is used to estimate the covariance matrix of moments. As we use s_γ to denote $\sqrt{D'SD}$, then the 95% asymptotic confidence intervals for auto- and cross-covariance estimates boil down to:

$$[\gamma_{ij} - 1.96 \cdot s_\gamma, \quad \gamma_{ij} + 1.96 \cdot s_\gamma]. \quad (7.48)$$

8 Summary and Outlook

This thesis contributes to the literature in six, we think, important dimensions:

Analysis of Various Shocks within the High-Frequency Versions of the Baseline New-Keynesian Model

In *Chapter 2* we analyze a small-scale NKM with a hybrid NKPC and dynamic IS equation, with an arbitrary frequency of the agents' synchronized decision making. We study the impact of various demand and supply shocks on the dynamics of the model variables based on IRFs. We show that the IRFs of the high-frequency versions can qualitatively as well as quantitatively be fairly dissimilar from their quarterly counterparts. This can be explained by the decrease in the effectiveness of monetary policy responses to these shocks and the overall increase of inertia in the model variables. In particular, different kinds of frequency-dependent persistence effects occur, which dampen the pass-through of output gap movements into inflation rate dynamics via the slope of the NKPC as the period length decreases. Furthermore, additional persistence in the exogenous shock process on a higher frequency leads to an increase in the volatility of the economic indicators. The main conclusion is that DSGE modelling may be more sensitive to its choice of the agents' decision interval. In this respect, based on our investigations, we call for a more elaborate discussion on the timing convention in DSGE models.

In our view, thinking about the timing in DSGE models is important. An economist might ask how realistic e.g. the quarterly change in consumption will be rather than a more frequent decision making of households. This is even more of serious concern, as we consider intertemporal consumption smoothing, i.e. that consumption depends on the real interest rate (because of the connection to financial markets via the purchase of bonds). More important and in contraction to the analysis undertaken in this thesis, in future research one might allow for asymmetric changes in the frequency of decision making across agents. This means that all decisions and transactions could occur in a heterogeneous synchronized rhythm, e.g. on a weekly (households), a monthly or quarterly (central bank) and annual (government) frequency. The following two examples can be seen as representative work to be done in the field of high-frequency analysis. Both strengthen our observations made in Chapter 2 and call for a more detailed elaboration on the impact of the (asymmetric) change in the period length in macroeconomic models.

As a guideline, two representative papers might be mentioned. Kendrick and Amman (2011) emphasize that changes of the nominal interest rate in the US are undertaken by the Open Market Committee of the FED on a monthly frequency. In contradiction, during the first term of the Obama administration, the US Congress decided on fiscal stimulus packages annually. In their paper they call for a more frequent reaction of fiscal policy to shocks. Therefore, they introduce a fiscal policy rule equivalent to a Taylor type rule in monetary policy regimes. In their theoretical framework they show that current government expenditure should react to current and lagged output and the past value of government expenditure (Kendrick and Amman (2011, p. 8)). The transition from an annual to a quarterly frequency

regarding the fiscal policy rule can be studied under consideration of time-varying weights in a standard loss function, which includes variations of the output gap and the inflation rate (Kendrick and Amman (2011, p. 6)). Furthermore, the authors combine the fiscal policy feedback rule with the one of the monetary authorities. They emphasise that this mixed feedback rule is appropriate to stabilize the economy, although both rules are given in different frequencies.

The consideration of an analysis on a high-frequency is obviously needed when integrating financial markets into macroeconomic models. A new strand of the macroeconomic literature consists on the method of agent-based computational economics. In this kind of models the main assumption of representative agents is dropped. Instead, a group of heterogeneous agents interacts simultaneously and the market outcome is determined via the transaction of the individually acting participants in (dis)equilibrium. In their paper, Lengnick and Wohltmann (2012) model financial market dynamics in the baseline NKM under consideration of bounded rationality, i.e. non-rational expectation formation processes like in the De Grauwe (2011) model, which has been estimated in Chapter 7 of this thesis. This model allows for the endogenous development of business cycles and stock price bubbles. In particular, they consider the quarterly value of the stock price in the NKPC. However, real markets operate on a lower frequency relative to the financial market, where stocks are traded on a daily frequency. Therefore, the quarterly stock price is given by the mean of daily stock prices, calculated recursively in the agent-based model for the financial market (Lengnick and Wohltmann (2012, p. 12)). This framework gives valuable insights on the modelling of asymmetric frequencies in a DSGE model with financial frictions. However, the calculation of the mean value of a high-frequency variable (like in Lengnick and Wohltmann (2012)) is non-trivial from an econometric point of view. As we discuss in Chapter 2 the correct choice of the time aggregation scheme is ambiguous and relies on a clear definition of stocks and flows in a (macroeconomic) model. For the sake of completeness an explicit estimation of different period lengths, i.e. under an asymmetric change in the transactions over time across agents, can be done by applying the MiDas (mixed data sampling) regression. Within the MiDas approach, mixed time series in low- and high frequencies are obtained for estimation. For an introduction see Ghysels et al. (2007).

Optimal Monetary Policy Responses and Welfare Analysis within the High-Frequency Framework

In *Chapter 3* we investigate the welfare effects of monetary policy measurements within an h-economy. Hereby we focus on the purely forward-looking version of the baseline NKM from Chapter 2 and the impact of an (non-)autocorrelated cost-push shock in the NKPC on economic dynamics. OMP is characterized by the regimes Discretion and Commitment. Our results indicate that the policy maker faces a higher welfare loss on a higher relative to a lower frequency. An explanation is, again, given by the impact of the frequency-dependent persistence effects, which mimic the impact of the increase in the amount of market days on the dynamics of the model. While overall inertia in the model increases, we show that the more the pass-through of the level of the output gap is dampened on a higher frequency, this amplifies the trade-off of the central bank in case of a cost-push shock. This result is less severe in the OMP regime Commitment because of the time-invariant history dependence effect with respect to the period length. Due to the latter, the central bank is unable to stabilize the price level on its initial steady state level in the long run. Within our welfare analysis, we show that the loss (relation) is sensitive to the variation in the period length, the correlation in the shock process, the weight on the output stabilisation and the degree

of price stickiness.

It can be stated that the consideration of the period length sheds a different light on the discussion of OMP in baseline NKMs. We propose a more elaborate investigation of this issue in future research. Obviously, the more information on the adjustments in the economic indicators is available on a higher frequency, the more this leads to a stronger destabilisation in these variables and, hence, a higher welfare loss. This is analogous to the results obtained by Winkler and Wohltmann (2012), who show that in case of an anticipated shock movements in the variables already occur in the preparatory phase of the shock. This increase in the market days (similar to the existence of subperiods in an h -economy) hinders the OMP responses of the central bank. It is an open question if this result still holds on a higher frequency.

Furthermore, the volatility of the nominal interest rate can also be taken into account as part of the central bank's (welfare) loss function. While Sacht and Wohltmann (2013) study also the welfare loss in an h -economy when an Optimal Simple Rule is applied, a numerical investigation via IRFs is still missing in the literature so far. This, of course, requires an approximation of the Optimal Simple Rule to the solution of the targeting rule under Commitment (cf. Wohltmann and Winkler (2009)). Finally, a more elaborate discussion on the convergence of the dynamic system in case of a hybrid DSGE model is needed. Especially a check on the two discussed solution algorithms by Söderlind (1999) and Dennis (2007) — which must be applied in the hybrid discretionary case — under different frequencies of decision making seems to be fruitful.

Estimation of a High-Frequency New-Keynesian Phillips Curve

In *Chapter 4* we estimate a high-frequency NKPC via GMM. Allowing for higher-than-usual frequencies strongly mitigates the problems of small-sample bias and structural breaks. Applying a daily frequency allows us to obtain estimates for the Calvo parameter of nominal rigidity over a very short period — for instance for the recent financial and economic crisis — which can then be easily transformed into their low-frequency equivalences. With Argentine data from the end of 2007 to the beginning of 2011 we estimate the daily Calvo parameter and find that on average prices remain fixed for approximately two to three months which is in line with recent microeconomic evidence.

Concerning our own future research we plan to estimate the Calvo parameter of price stickiness for a number of North American and European as well as Asian Economies. Daily inflation data is at least provided for the US (via <http://www.primestats.com/>). In this respect, these investigations might guide a researcher when analysing various shocks either in a flexible or sticky price DSGE model, which depends on the estimation results.

Furthermore, for a better understanding of the implications of our results for conducting optimal monetary (and fiscal) policy one might consider a model in which firms adjust their prices fully flexible to firm-specific (idiosyncratic) shocks but sluggishly to aggregate shocks. The former captures strategic motives e.g. the (re)filling of stations where the latter stands for policy or other exogenous shocks which affect the overall economic conditions. The question arises, whether we observe price changes on a day-to-day basis due to idiosyncratic or aggregate shocks. Furthermore, which of these shocks appear more frequently? Using a model where it is assumed that aggregate shocks occur rarely while idiosyncratic shocks happen frequently, might help to understand the information content of daily and of quarterly data. Klenow and Malin (2010, p. 29) report that micro price changes in the Euro Area and the United States do not keep up with overall inflation. The authors claim that

this could be explained most probably by the dominance of idiosyncratic over aggregate shocks. In this respect the approach of Svensson and Woodford (2003) could be helpful. The authors discuss optimal monetary policy in a NKM under the assumption of a symmetric partial information distribution between agents. In particular, they account for uncertainty regarding the level of potential output and a cost-push shock which are unobservable and partially observable, respectively. Similar to our ideas on asymmetric frequencies of decision making, following Svensson and Woodford (2003) by considering a measurement equation that accounts (in our case) for data with different frequencies could shed a light on the importance of idiosyncratic or aggregate shocks.

Moment-Matching versus Bayesian Estimation: Backward-Looking Behaviour in the New-Keynesian Baseline Model

In *Chapter 5* we consider the elementary New-Keynesian three-equations model and compare its Bayesian estimation to the results from the MM approach, where we seek to match a finite set of model-generated second-moments of inflation, output and the interest rate to their empirical counterparts. We found that in the GI (though not so strongly in the GM) period, the two estimations imply a significantly different covariance structure. Regarding the parameters, special emphasis is placed on the degree of backward-looking behaviour in the NKPC. While, in line with much of the literature, it plays a minor role in the Bayesian estimations, MM reports values of the price indexation parameter close to or even at its maximal value of unity. For both the GI and GM period, these results are worth noticing since in (strong or, respectively, weak) contrast to the Bayesian parameters, the covariance matching is entirely satisfactory.

As part of our own future research agenda we tie in with preliminary research by estimating NKMs via MM. Therefore, two further projects are planned. The first project is based directly on our investigations in Chapter 5, where we undertake comparison exercises of different estimation procedures. In particular, within this project we are going to confront the empirical outcomes of maximum likelihood techniques and Indirect Inference (II) to MM, respectively. We consider the maximum likelihood technique since there is reason to believe that the emphasis must be put on the comparison of informative to non-informative prior information - rather than Bayesian versus MM estimation. Indeed, the maximum likelihood procedure does not use informative prior information and therefore can be seen as a natural starting point for a comparison to MM (even though the Bayesian approach is the most common used in the literature). Comparing II and MM seems to be reasonable from a technical point of view because the two are closely related to each other.

This analysis will shed a light on the differences and the question which approach – direct (MM) or indirect (II) moment-matching – is more appropriate in terms of tractability and in measuring the goodness of fit of the model to the data. The underlying model structure exhibits non-linearities in a small-scale model environment, where we depart from the linearization of the model equations (around their steady states) in order to prevent approximation errors linked to this procedure. Furthermore, we will step outside the baseline NKM and apply the comparison exercises on a medium-scale model, where we account for e.g. capital utilization, search-and-matching procedures when modelling frictions in employment dynamics and the zero lower bound on nominal interest rates. It is the aim of the project to strengthen the confidence in the MM approach by the scientific community; especially by emphasising its transparency and tractability. The second project is connected to our work presented in Chapter 7 (see below).

Identification of Prior Information via Moment-Matching

In *Chapter 6* we apply a sensitivity analysis regarding two types of prior information considered within Bayesian estimation. In particular, we shed a light on the impact of micro- and macropriors on the estimation outcome. First, we investigate the impact of the transformation of those model parameters which are bounded to the unit interval, in order to allow for a more diffuse prior distribution. Second, we combine both MM and Bayesian techniques in order to evaluate macropriors. In this respect we define a two-stage estimation procedure (so-called MoMBay) where we take the point estimates evaluated via MM for the GI and GM period and consider them as prior mean values of the parameters within Bayesian estimation. Finally, we compare the corresponding outcomes (in terms of the goodness of fit) of micro- and macropriors to those when the standard prior information, which is in line with the recent literature and documented in *Chapter 5*, is taken into account.

We show that while (transformed) micropriors are often used in the literature, applying macropriors evaluated via the MoMBay approach leads to a better fit of the structural model to the data. The reason for this is that the MoMBay approach reduces the parameter space from which appropriate starting values (or better: prior mean values) for the optimization within the Bayesian approach are taken from. As a general observation, however, the results from *Chapter 5* are confirmed when applying the MM-based procedure. This means there is evidence for intrinsic (degree of price indexation) rather than extrinsic (autocorrelation in the shock process) persistence — an observation which, again, stands in contradiction to the results documented in the recent literature. We claim that the MoMBay approach can be seen as an alternative procedure for the estimation and evaluation of macroeconomic models.

Similar observations are made also by Del Negro and Schorfheide (2008) as well as Lombardi and Nicoletti (2012), who found also differences in the posterior values under consideration of macropriors. Severe problems stem from the existence of less or even non-informative time series and misspecification of the model. A deeper investigation on these topics — besides a comparison exercise with level versus gap data — might be fruitful in further research.

The Rational Expectation Hypothesis versus Animal Spirits within the New-Keynesian Framework: Empirical Evidence from Moment-Matching

In *Chapter 7* we empirically examine a heterogeneous bounded rationality version of the hybrid NKM. The model is estimated via the simulated method of moments using Euro Area data from 1975Q1 to 2009Q4. It is generally assumed that agents' beliefs display waves of optimism and pessimism (so-called animal spirits) on future movements of the output and inflation gap. Our main empirical findings show that a bounded rationality model with cognitive limitation provides a reasonable fit to auto- and cross-covariances of the data. The result is mainly driven by a high degree of persistence in the output and inflation gap due to the impact of animal spirits on economic dynamics. Further, over the whole time interval the agents had expected moderate deviations of the future output gap from its steady state value. Finally, we find evidence for a static expectation formation process regarding the inflation gap.

Within our second research project (see above) we empirically investigate the (recent) dynamics of the output, the inflation and the nominal interest rate gap in the US and Euro Area based on bounded rationality NKMs. While rational expectations as a main assumption in NKMs are heavily criticized in scientific and public media today — which is mainly motivated by the financial and sovereign debt crisis started in 2008 with the downturn of

Lehman Brothers — we spotlight alternative techniques of how agents form expectations on future values of key economic indicators. Inspired by our contribution in this Chapter, it is the aim of the project to develop the NKM framework further in terms of non-rational expectation formation processes and taking the resulting structural model to the data via (S)MM. Essentially, we drop the assumption of rational expectations and consider bounded rationality in decision making instead, i.e. non-rational expectations processes.

In particular, we will take different kinds of heuristics into account, where the expectation formation lacks full information and hence herding behaviour of heterogeneous agents occurs. We will perform a horse race on a variety of specific decision rules, i.e. the bound and likelihood heuristic (Becker and Leopold-Wildburger (1996, 2000)), genetic algorithms (Hommes and Lux (2013)) and convex combinations of backward- and forward-looking behaviour (Branch and McGough (2009)). As a result the model becomes non-linear, which makes it a good candidate for estimating it via (S)MM for practical reasons, e.g. no time- and resource-consuming particle filter technique must be considered like in the Bayesian approach. In addition, we consider micro-level data like consumer surveys (Forsells and Kenny (2004)), which can be used to approximate inflation rate expectations. The (relative) performance is measured by the value of the (S)MM criterion function and a model comparison test procedure (based on p -values generated by bootstrapping methods) developed by Jang (2012). Finally, the (S)MM approach seems to be a promising candidate for estimating agent-based economic models, where Bayesian prior information for individual agents is hard to apply.

Bibliography

- AADLAND, D. (2001): High Frequency Real Business Cycles. *Journal of Monetary Economics*, Vol. 48(2), pp. 271-292.
- ABEL, A.B. (1990): Asset Prices Under Habit Formation And Catching Up With The Joneses. *The American Economic Review*, Vol. 80(2), pp. 38-42.
- ABE, N. AND TONOGLI, A. (2010): Micro And Macro Price Dynamics In Daily Data. *Journal of Monetary Economics*, Vol. 57(6), pp. 716-728.
- ADJEMIAN, S., BASTANI, H., JUILLARD, M., KARAMÉ, F., MIHOUBI, F., PERENDIA, G., PFEIFER, J., RATTO, M. AND VILLEMOT, S. (2011): *Dynare: Reference Manual, Version 4*. Dynare Working Papers No. 1, CEPREMAP.
- ADOLFSON, M., LASEEN, S., LINDÉ, J. AND VILLANI, M. (2007): Bayesian Estimation Of An Open Economy DSGE Model With Incomplete Pass-Through. *Journal of International Economics*, Vol. 72(2), pp. 481-511.
- AHRENS, S. AND SACT, S. (2014): Estimating A High-Frequency New-Keynesian Phillips Curve. *Empirical Economics*, Vol. 46(2), pp. 607-628.
- AKERLOF, G. AND SHILLER, R. (2009): *Animal Spirits. How Human Psychology Drives The Economy And Why It Matters For Global Capitalism*. Princeton University Press, New Jersey.
- ALTIG, D., CHRISTIANO, L.J., EICHENBAUM, M. AND LINDÉ, J. (2011): Firm-Specific Capital, Nominal Rigidities And The Business Cycle. *Review of Economic Dynamics*, Vol. 14(2), pp. 225-247.
- ALTONJI, J. AND SEGAL, L. (1996): Small-Sample Bias In GMM Estimation Of Covariance Structures. *Journal of Business and Economic Statistics*, Vol. 14, pp. 353-366.
- ÁLVAREZ, L.J., DHYNE, E., HOEBERICHTS, M., KWAPIL, C., BIHAN, H.L., LÜNNEMANN, P., MARTINS, F., SABBATINI, R., STAHL, H., VERMEULEN, P. AND VILMUNEN, J. (2006): Sticky Prices In The Euro Area: A Summary Of New Micro-Evidence. *Journal of the European Economic Association*, Vol. 4(2/3), pp. 575-584.
- ÁLVAREZ, L.J. (2008): What Do Micro Price Data Tell Us On The Validity Of The New Keynesian Phillips Curve?. *Economics: The Open-Access, Open-Assessment E-Journal*, Vol. 2(19), pp. 1-38.
- AMBLER, S., GUAY, A. AND PHANEUF, L. (2012): Endogenous Business Cycle Propagation And The Persistence Problem: The Role Of Labor-Market Frictions. *Journal of Economic Dynamics and Control*, Vol. 36(1), pp. 47-62.

- AN, S. AND SCHORFHEIDE, F. (2007): Bayesian Analysis Of DSGE Models. *Econometric Reviews*, Vol. 26(2-4), pp. 113-172.
- ANAGNOSTOPOULOS, A. AND GIANNITSAROU, C. (2010a): *Indeterminacy And Period Length*. Working Paper, Stony Brook and University of Cambridge (UK).
- ANAGNOSTOPOULOS, A. AND GIANNITSAROU, C. (2010b): Modeling Time And Macroeconomic Dynamics. *CEPR Discussion Paper* No. 8050, Centre for Economic Policy Research, London.
- ANATOLYEV, S. AND GOSPODINOV, N. (2011): *Methods For Estimation And Inference In Modern Econometrics*. New York: A Chapman and Hall Book.
- ANDERSON, G. S. (2008): Solving Linear Rational Expectations Models: A Horse Race. *Computational Economics*, Vol. 31(2), pp. 95-113.
- ANDREWS, D. (1991): Heteroscedasticity And Autocorrelation Consistent Covariance Matrix Estimation. *Econometrica*, Vol. 59, pp. 817-858.
- ARTIS, M.J. MARCELLINO, M. AND PROIETTI, T. (2003): *Dating The Euro Area Business Cycle*. Center for Economic Policy Research, CEPR Discussion Paper No. 3696.
- ASCARI, G. AND ROPELE, T. (2003): *Notes On Real Business Cycle*. Mimeo, University of Pavia.
- ASCARI, G. AND ROPELE, T. (2009): Trend Inflation, Taylor Principle, And Indeterminacy. *Journal of Money, Credit and Banking*, Vol. 41(8), pp. 1557-1584.
- ASSENMACHER-WESCHE, K. AND GERLACH, S. (2008): Interpreting Euro Area Inflation At High And Low Frequencies. *European Economic Review*, Vol. 52(6), pp. 964-986.
- AVOUYI-DOVI, S. AND MATHERON, J. (2007): Technology Shocks And Monetary Policy: Revisiting The Fed's Performance. *Journal of Money, Credit and Banking*, Vol. 39(2/3), pp. 471-507.
- BAI, Z.-Z., GUO, X.-X. AND YIN, J.-F. (2005): On Two Iteration Methods For The Quadratic Matrix Equations. *International Journal of Numerical Analysis and Modeling*, Vol. 2, pp. 114-122.
- BECKER, O. AND LEOPOLD-WILDBURGER, U. (1996): The Bounds And Likelihood-Procedure - A Simulation Study Concerning The Efficiency Of Visual Forecasting Techniques. *Central European Journal of Operations Research and Economics*, Vol. 4(2/3), pp. 223-229.
- BECKER, O. AND LEOPOLD-WILDBURGER, U. (2000): Erwartungsbildung Und Prognose - Ergebnisse Einer Experimentellen Studie. *Austrian Journal of Statistics*, Vol. 29(1), pp. 7-16.
- BELLMAN, R. (1957): *Dynamic Programming*. Princeton University Press, Princeton.
- BENATI, L. AND SURICO, P. (2009): VAR Analysis And The Great Moderation. *American Economic Review*, Vol. 99(4), pp. 1636-1652.

- BENHABIB, J. AND FARMER, R.E.A. (1994): Indeterminacy And Increasing Returns. *Journal of Economic Theory*, Vol. 63(1), pp. 19-41.
- BERNANKE, B. AND MIHOV, I. (1998): Measuring Monetary Policy. *Quarterly Journal of Economics*, Vol. 113(3), pp. 869-902.
- BILS, M. AND KLENOW, P.J. (2004): Some Evidence On The Importance Of Sticky Prices. *Journal of Political Economy*, Vol. 112(5), pp. 947-985.
- BINDER, M. AND PESARAN, M.H. (1995): Multivariate Rational Expectations Models And Macroeconomic Modeling: A Review And Some New Results. in: PESARAN, M.H. AND WICKENS, M. (Eds.) (1995): *Handbook Of Applied Econometrics: Macroeconomics*, Basil Blackwell, pp. 139-187.
- BOIVIN, J. AND GIANNONI, M.P. (2006): Has Monetary Policy Become More Effective?. *Review of Economics and Statistics*, Vol. 88(3), pp. 445-462.
- BOSWIJK, H.P. HOMMES, C.H. AND MANZAN, S. (2007): Behavioral Heterogeneity In Stock Prices. *Journal of Economic Dynamics and Control*, Vol. 31(6), pp. 1938-1970.
- BRANCH, W.A. AND MCGOUGH, B. (2009): A New Keynesian Model With Heterogeneous Expectations. *Journal of Economic Dynamics and Control*, Vol. 33(5), pp. 1036-1051.
- BREUSCH, T.S. (1979): Testing For Autocorrelation In Dynamic Linear Models. *Australian Economic Papers*, Vol. 17(31), pp. 334-355.
- BROCK, W.A. AND HOMMES, C.H. (1997): A Rational Route to Randomness. *Econometrica*, Vol. 65(5), pp. 1059-1095.
- BROOKS, S.P. AND GELMAN, A. (1998): General Methods For Monitoring Convergence Of Iterative Simulations. *Journal of Computational and Graphical Statistics*, Vol. 7(4), pp. 434-455.
- BULLARD, J. AND MITRA, K. (2002): Learning About Monetary Policy Rules. *Journal of Monetary Economics*, Vol. 49(6), pp. 1105-1129.
- CALVO, G.A. (1983): Staggered Prices In A Utility-Maximizing Framework. *Journal of Monetary Economics*, Vol. 12(3), pp. 383-398.
- CAMERER, C. (1998): Bounded Rationality In Individual Decision Making. *Experimental Economics*, Vol. 1, pp. 166-183.
- CANOVA, F. (2007): *Methods For Applied Macroeconomic Research*. Princeton University Press, Princeton.
- CARRASCO, M. AND FLORENS, J.-P. (2002): Simulation-Based Method Of Moments And Efficiency. *Journal of Business and Economic Statistics*, Vol. 20(4), pp. 482-492.
- CASTELNUOVO, E. (2010): Trend Inflation And Macroeconomic Volatilities In The Post WWII U.S. Economy. *North American Journal of Economics and Finance*, Vol. 21(1), pp. 19-33.

- CAVALLO, A. (2010): *Scraped Data And Sticky Prices: Frequency, Hazards, And Synchronization*. Working Paper, MIT Sloan.
- CAVALLO, A. AND RIGOBON, R. (2011): *The Distribution Of The Size Of Price Changes*. NBER Working Papers No. 16760, National Bureau of Economic Research, Inc.
- CAVALLO, A. (2012a): *Scraped Data And Sticky Prices*. Working Paper, MIT Sloan.
- CAVALLO, A. (2012b): *Online And Official Price Indexes: Measuring Argentina's Inflation*. Working Paper, MIT Sloan.
- CHARI, V.V., KEHOE, P.J. AND MCGRATTAN, E.R. (2000): Sticky Price Models Of The Business Cycle: Can The Contract Multiplier Solve The Persistence Problem?. *Econometrica*, Vol. 68(5), pp. 1151-1179.
- CHARI, V.V., KEHOE, P.J. AND MCGRATTAN, E.R. (2002): Can Sticky Price Models Generate Volatile And Persistent Real Exchange Rates?. *Review of Economic Studies*, Vol. 69(3), pp. 533-563.
- CHIAIE, S.D. (2009): *The Sensitivity Of DSGE Models' Results To Data Detrending*. Working Papers No. 157, Oesterreichische Nationalbank (Austrian Central Bank).
- CHIARELLA, C. AND HE, T. (2002): Heterogeneous Beliefs, Risk And Learning In A Simple Asset Pricing Model. *Computational Economics*, Vol. 19, pp. 95-132.
- CHO, S. AND MORENO, A. (2006): A Small-Sample Study Of The New Keynesian Macro Model. *Journal of Money, Credit and Banking*, Vol. 38(6), pp. 1461-1481.
- CHRISTIANO, L.J. (1985): A Method For Estimating The Timing Interval In A Linear Econometric Model With An Application To Taylor's Model of Staggered Contracts. *Journal of Economic Dynamics and Control*, Vol. 9(4), pp. 363-404.
- CHRISTIANO, L.J. AND VIGFUSSON, R.J. (2003): Maximum Likelihood In The Frequency Domain: The Importance Of Time-To-Plan. *Journal of Monetary Economics*, Vol. 50(4), pp. 789-815.
- CHRISTIANO, L.J., EICHENBAUM, M. AND EVANS, C.L. (2005): Nominal Rigidities And The Dynamic Effects Of A Shock To Monetary Policy. *Journal of Political Economy*, Vol. 113(1), pp. 1-45.
- CLARIDA, R., GALÍ, J. AND GERTLER, M. (2001): Optimal Monetary Policy In Open Versus Closed Economies: An Integrated Approach. *The American Economic Review*, Vol. 91(2), pp. 248-252.
- CLARIDA, R., GALÍ, J. AND GERTLER, M. (2002): A Simple Framework For International Monetary Policy Analysis. *Journal of Monetary Economics*, Vol. 49(5), pp. 879-904.
- COGLEY, T. AND SBORDONE, A. (2008): Trend Inflation, Indexation, And Inflation Persistence In The New Keynesian Phillips Curve. *American Economic Review*, Vol. 98(5), pp. 2101-2126.

- COGLEY, T., PRIMICERI, G.E. AND SARGENT, T.J. (2010): Inflation-Gap Persistence In The US. *American Economic Journal: Macroeconomics*, Vol. 2(1), pp. 43-69.
- COLLARD, F., FÈVE, P., LANGOT, F. AND PERRAUDIN, C. (2002): A Structural Model Of U.S. Aggregate Job Flows. *Journal of Applied Econometrics*, Vol. 17(3), 197-223.
- CORNEA, A. HOMMES, C.H. AND MASSARO, D. (2013): *Behavioral Heterogeneity In U.S. Inflation Dynamics*. Tinbergen Institute Discussion Papers No. 13-015/II, Tinbergen Institute.
- CORONA, A., MARCHESI, M., MARTINI, C. AND RIDELLA, S. (1987): Minimizing Multimodal Functions Of Continuous Variables With The 'Simulated Annealing' Algorithm. *ACM Transactions on Mathematical Software*, Vol. 13(3), pp. 262-280.
- D'AMATO, L., GAREGNANI, M.L. AND SOTES PALADINO, J.M. (2007): *Inflation Persistence And Changes In The Monetary Regime: The Argentine Case*. Working Paper No. 2007/23, Banco Central de la República Argentina.
- D'AMATO L. AND GAREGNANI, M.L. (2009): *Studying The Short-Run Dynamics Of Inflation: Estimating A Hybrid New-Keynesian Phillips Curve For Argentina (1993.2007)*. Working Paper No. 2009/40, Banco Central de la República Argentina.
- DAVIDSON, R. AND MACKINNON, J.G. (2004): *Econometric Theory And Methods*. Oxford University Press, Oxford.
- DE GRAUWE, P. (2011): Animal Spirits And Monetary Policy. *Economic Theory*, Vol. 47(2-3), pp. 423-457.
- DE GRAUWE, P. (2012): Booms And Busts In Economic Activity: A Behavioral Explanation. *Journal of Economic Behavior and Organization*, Vol. 83(3), pp. 484-501.
- DEJONG, D.N. AND DAVE, C. (2007): *Structural Macroeconomics*. 1st Edition, Princeton University Press, New Jersey.
- DEL NEGRO, M. AND SCHORFHEIDE, F. (2004): Priors From General Equilibrium Models For VARS. *International Economic Review*, Vol. 45(2), pp. 643-673.
- DEL NEGRO, M., SCHORFHEIDE, F., SMETS, F. AND WOUTERS, R. (2007): On The Fit Of New Keynesian Models. *Journal of Business and Economic Statistics*, Vol. 25(2), pp. 123-143.
- DEL NEGRO, M. AND SCHORFHEIDE, F. (2008): Forming Priors For DSGE Models (And How It Affects The Assessment Of Nominal Rigidities). *Journal of Monetary Economics*, Vol. 55(7), pp. 1191-1208.
- DENNIS, R. AND SÖDERSTRÖM, U. (2006): How Important Is Pre-Commitment For Monetary Policy?. *Journal of Money, Credit and Banking*, Vol. 38(4), pp. 847-872.
- DENNIS, R. (2007): Optimal Policy In Rational Expectations Models: New Solution Algorithms. *Macroeconomics Dynamics*, Vol. 11(1), pp. 31-55.

- DHYNE, E., ALVAREZ, L. J., BIHAN, H. L., VERONESE, G., DIAS, D., HOFFMANN, J., JONKER, N., LUNNEMANN, P., RUMLER, F. AND VILMUNEN, J. (2006): Price Changes In The Euro Area And The United States: Some Facts From Individual Consumer Price Data. *Journal of Economic Perspectives*, Vol. 20(2), pp. 171-192.
- DROST F.C. AND NIJMAN T.E. (1993): Temporal Aggregation Of GARCH Processes. *Econometrica*, Vol. 61(4), pp. 909-927.
- DUFFIE, D. AND SINGLETON, K. (1993): Simulated Moments Estimation Of Markov Models Of Asset Prices. *Econometrica*, Vol. 61(4), pp. 929-952.
- ELLIS, C. (2009): *Do Supermarket Prices Change From Week To Week?*. Bank of England Working Paper No. 378, Bank of England.
- ERCEG, C.J., HENDERSON, D.W. AND LEVIN, A.T. (2000): Optimal Monetary Policy With Staggered Wage And Price Contracts. *Journal of Monetary Economics*, Vol. 46(2), pp. 281-313.
- ESCUDE, G.J. (2007): *ARGEM: A DSGE Model With Banks And Monetary Policy Regimes With Two Feedback Rules, Calibrated For Argentina*. Working Paper No. 2007—21, Banco Central de la República Argentina.
- ESCUDE, G.J. (2009): *ARGEMmy: An Intermediate DSGE Calibrated/Estimated For Argentina: Two Policy Rules Are Often Better Than One*. Working Paper No. 2009—42, Banco Central de la República Argentina.
- EVANS, G. AND HONKAPOHJA, S. (2001): *Learning and Expectations in Macroeconomics*. Princeton University Press, New Jersey.
- FABIANI, S., LOUPIAS, C.S., DRUANT, M., HERNANDO, I., KWAPIL, C., LANDAU, B., MARTINS, F.M.M., MATHÄ, T., SABBATINI, R., STAHL, H. AND STOKMAN, A. (2007): Summary Of Results For The Euro Area. in: FABIANI, S., LOUPIAS, C.S., MARTINS, F.M.M. AND SABBATINI, R. (Eds.) (2007): *Pricing Decisions In The Euro Area: How Firms Set Prices And Why*. Oxford University Press, Oxford, pp. 32-68.
- FAGAN, G., HENRY, J. AND MESTRE, R. (2001): *An Area Wide Model (AWM) For The Euro Area*. ECB working paper No. 42.
- FAUST, J., SWANSON, E.T. AND WRIGHT, J.H. (2004): Identifying VARs Based On High Frequency Futures Data. *Journal of Monetary Economics*, Vol. 51(6), pp. 1107-1131.
- FERNÁNDEZ-VILLVERDE, J. AND RUBIO-RAMÍREZ, J.F. (2007): *How Structural Are Structural Parameters?*. NBER Working Papers No. 13166, National Bureau of Economic Research, Inc.
- FERNÁNDEZ-VILLVERDE, J. (2010): The Econometrics Of DSGE Models. *SERIES: Journal of the Spanish Economic Association*, Vol. 1(1), pp. 3-49.
- FÈVE, P., MATHERON, J. AND SAHUC, J.-G. (2009): Inflation Target Shocks And Monetary Policy Inertia In The Euro Area. *The Economic Journal*, Vol. 120(547), pp. 1100-1124.

- FISHER, R.A. (1922): On The Mathematical Foundations Of Theoretical Statistics. *Philosophical Transactions of the Royal Society*, Vol. 222, pp. 309-368.
- FISHER, I. (1930): *The Theory Of Interest*. Macmillan, New York.
- FLASCHEL, P., FRANKE, R. AND PROAÑO, C.R. (2008): On Equilibrium Determinacy In New Keynesian Models With Staggered Wage And Price Setting. *The B.E. Journal of Macroeconomics*, Vol. 8(1), Topics, Article 31.
- FOLEY, D.K. (1975): On Two Specifications Of Asset Equilibrium In Macroeconomic Models. *Journal of Political Economy*, Vol. 83(2), pp. 303-324.
- FORSELLS, M. AND KENNY, G. (2004): Survey Expectations, Rationality And The Dynamics Of Euro Area Inflation. *Journal of Business Cycle Measurement and Analysis*, Vol. 2004(1), pp. 13-41.
- FRANKE, R., JANG, T.-S. AND SACHT, S. (2012): *Moment Matching Versus Bayesian Estimation: Backward-Looking Behaviour In The New-Keynesian Three-Equations Model (Extended Version, Second Draft)*. Economics Working Paper No. 2012-08, Department of Economics, Christian-Albrechts-University of Kiel, October 2011/September 2012. Downloadable via: http://www.bwl.uni-kiel.de/vwlinstitute/gwif/files/papers/Franke_Jang_Sacht_Aug2012.pdf
- FRANKE, R. (2012a): *How Much Backward-Looking Is The New-Keynesian Three-Equations Model? Evidence From Moment Matching Estimations*. Working Paper, Department of Economics, Christian Albrechts University of Kiel, October 2012. Downloadable via: http://www.bwl.uni-kiel.de/vwlinstitute/gwif/files/papers/Franke_NK-MM_Oct2012.pdf
- FRANKE, R. (2012b): Microfounded Animal Spirits In The New Macroeconomic Consensus. *Studies in Nonlinear Dynamics and Econometrics*, Vol. 16(4), Article 4.
- FRANKE, R. (2013): *How Fat-Tailed Is US Output Growth?*. Working Paper, Department of Economics, Christian Albrechts University of Kiel, September 2013. Downloadable via: http://www.bwl.uni-kiel.de/vwlinstitute/gwif/files/papers/Franke_FatTailedOutput.pdf
- FRANKE, R. AND SACHT, S. (2014): Some Observations In The High-Frequency Versions Of A Standard New-Keynesian Model. *Bulletin of Economic Research*, Vol. 66(1), pp. 72-94.
- FUHRER, J.C., MOORE, G.R. AND SCHUH, S.D. (1995): Estimating The Linear-Quadratic Inventory Model Maximum Likelihood Versus Generalized Method Of Moments. *Journal of Monetary Economics*, Vol. 35(1), pp. 115-157.
- FUHRER, J. (2006): Intrinsic And Inherited Inflation Persistence. *International Journal of Central Banking*, Vol. 2(3), pp. 49-86.
- GALÍ, J. AND GERTLER, M. (1999): Inflation Dynamics: A Structural Econometric Analysis. *Journal of Monetary Economics*, Vol. 44(2), pp. 195-222.

- GALÍ, J., GERTLER, M. AND LÓPEZ-SALIDO, J.D. (2001): European Inflation Dynamics. *European Economic Review*, Vol. 45(7), pp. 1237-1270.
- GALÍ, J. AND MONACELLI, T. (2005): Monetary Policy And Exchange Rate Volatility In A Small Open Economy. *Review of Economic Studies*, Vol. 72(3), pp. 707-734.
- GALÍ, J. (2008): *Monetary Policy, Inflation, And The Business Cycle*. Princeton University Press, Princeton.
- GALÍ, G., SMETS, F. AND WOUTERS, R. (2012): *Slow Recoveries: A Structural Interpretation*. NBER Working Papers No. 18085, National Bureau of Economic Research, Inc.
- GALLANT, A.R. AND TAUCHEN, G. (1996): Which Moments To Match?. *Econometric Theory*, Vol. 12(4), pp. 657-681.
- GALLANT, A.R. AND TAUCHEN, G. (2010): Simulated Score Methods And Indirect Inference For Continuous-Time Models. in: AIT-SAHALIA, Y. AND HANSEN, L. (Eds.) (2010):: *Handbook Of Financial Econometrics: Volume 1 - Tools And Techniques*, North Holland, pp. 427-478.
- GAUNERSDORFER, A. AND HOMMES, C. (2007): A Nonlinear Structural Model For Volatility Clustering. in: TEYSSIERE, G. AND KIRMAN, A. (Eds.) (2007): *Long Memory in Economics*, Springer, pp. 265-288.
- GAUNERSDORFER, A., HOMMES, C. AND WAGENER, F.O.O. (2008): Bifurcation Routes To Volatility Clustering Under Evolutionary Learning. *Journal of Economic Behavior & Organization*, Vol. 67(1), pp. 27-47.
- GEWEKE, J. (1999): Simulation Methods For Bayesian Econometric Models: Inference, Development And Communication. *Econometric Review*, Vol. 18(1), pp. 1-73.
- GHYSELS, E., SINKO, A. AND VALKANOV, R. (2007): MIDAS Regressions: Further Results And New Directions. *Econometric Reviews*, Vol. 26(1), pp. 53-90.
- GILLI, M. AND SCHUMANN, E. (2010): Optimal Enough?. *Journal of Heuristics*, Vol. 17, pp. 373-387.
- GODFREY, L.G. (1978): Testing Against General Autoregressive And Moving Average Error Models When The Regressors Include Lagged Dependent Variables. *Econometrica*, Vol. 46(6), pp. 1293-1301.
- GOFFE, W.L., FERRIER, G.D. AND ROGERS, J. (1994): Global Optimization Of Statistical Functions With Simulated Annealing. *Journal of Econometrics*, Vol. 60(1-2), pp. 65-99.
- GOFFE, W.L. (1996): SIMANN: A Global Optimization Algorithm Using Simulated Annealing. *Studies in Nonlinear Dynamics and Econometrics*, Vol. 1(3), pp. 169-176.
- GOLDBAUM, D. AND MIZRACH, B. (2008): Estimating The Intensity Of Choice In A Dynamic Mutual Fund Allocation Decision. *Journal of Economic Dynamics and Control*, Vol. 32(11), pp. 3866-3876.

- GOODFRIEND, M. AND KING, R. (1997): The New Neoclassical Synthesis And The Role Of Monetary Policy. *NBER Macroeconomics Annual 1997*, Vol. 12, pp. 231-296, National Bureau of Economic Research, Inc.
- GOODMAN, S.N. (1999): Toward Evidence-Based Medical Statistics. 2: The Bayes Factor. *Annals of Internal Medicine*, Vol. 130(12), pp. 1005-1013.
- GORODNICHENKO, Y. AND NG, S. (2010): Estimation Of DSGE Models When The Data Are Persistent. *Journal of Monetary Economics*, Vol. 57(3), pp. 325-340.
- GOUVEA, S. (2007): *Price Rigidity In Brazil: Evidence From CPI Micro Data*. Working Papers No. 143, Central Bank of Brazil.
- GRIFFOLI, T.M. (2011): *Dynare User Guide - An Introduction To The Solution And Estimation Of DSGE Models*. Downloadable via: <http://www.dynare.org/documentation-and-support/user-guide/Dynare-UserGuide-WebBeta.pdf/view>
- HAIRAUT, J., LANGOT, F. AND PORTIER, F. (1997): Time To Implement And Aggregate Fluctuations. *Journal of Economic Dynamics and Control*, Vol. 22(1), 109-121.
- HALL P. (1992): *The Bootstrap And Edgeworth Expansion*. Springer, Berlin.
- HAMILTON, J.D. (1994): *Time Series Analysis*. Princeton University Press, Princeton.
- HAMILTON, J.D. AND HERRARA, A.M. (2004): Comment: Oil Shocks And Aggregate Macroeconomic Behavior: The Role Of Monetary Policy. *Journal of Money, Credit, and Banking*, Vol. 36(2), pp. 265-286.
- HANSEN, L.P. (1982): Large Sample Properties Of Generalized Method Of Moments Estimators. *Econometrica*, Vol. 50(4), pp. 1029-1054.
- HASSLER, U. (2011): *Temporal Aggregation Of Multiple Time Series In The Frequency Domain*. Working Paper, Goethe University Frankfurt.
- HENZEL, S., HÜLSEWIG, O., MAYER, E. AND WOLLMERSHÄUSER, T. (2009): The Price Puzzle Revisited: Can The Cost Channel Explain A Rise In Inflation After A Monetary Policy Shock?. *Journal of Macroeconomics*, Vol. 31(2), pp. 268-289.
- HINTERMAIER, T. (2005): A Sunspot Paradox. *Economics Letters*, Vol. 87(2), pp. 285-290.
- HNATKOVSKA, V., MARMER, V. AND TANG, Y. (2012): Comparison Of Misspecified Calibrated Models: The Minimum Distance Approach. *Journal of Econometrics*, Vol. 169(1), pp. 131-138.
- HODRICK, R. AND PRESCOTT, E.C. (1997): Postwar U.S. Business Cycles: An Empirical Investigation. *Journal of Money, Credit, and Banking*, Vol. 29 (1), pp. 1-16.
- HOLMBERG, K. (2006): *Derivation And Estimation Of A New Keynesian Phillips Curve In A Small Open Economy*. Working Paper No. 197, Sveriges Riksbank.

- HOMMES, H. (2006): Heterogeneous Agent Models In Economics And Finance. in: TESFATSION, L. AND JUDD, K. (Eds.) (2006): *Handbook of Computational Economics*, North Holland, Vol. 2, Chapter. 23, pp. 1109-1186.
- HOMMES, C. (2011): The Heterogeneous Expectations Hypothesis: Some Evidence From The Lab. *Journal of Economic Dynamics and Control*, Vol. 35 (1), pp. 1-24.
- HOMMES, C. AND LUX, T. (2013): Individual Expectations And Aggregate Behavior In Learning-To-Forecast Experiments. *Macroeconomic Dynamics*, Vol. 17(2), pp. 1-29.
- HÜLSEWIG, O., MAYER, E. AND WOLLMERSHÄUSER, T. (2009): Bank Behavior, Incomplete Interest Rate Pass-Through, And The Cost Channel Of Monetary Policy Transmission. *Economic Modelling*, Vol. 26(6), pp. 1310-1327.
- IRELAND, P.N. (2005): *The Monetary Transmission Mechanism*. Working Paper No. 06-1, Federal Reserve Bank of Boston.
- IRELAND, P.N. (2007): Changes In The Federal Reserve's Inflation Target: Causes And Consequences. *Journal of Money, Credit and Banking*, Vol. 39(8), pp. 1851-1882.
- JONSSON, G. AND KLEIN, P. (1996): Stochastic Fiscal Policy And The Swedish Business Cycle. *Journal of Monetary Economics*, Vol. 38(2), pp. 245-268.
- JANG, T.-S. (2012): Structural Estimation Of The New-Keynesian Model: A Formal Test Of Backward- And Forward-Looking Behavior. in: BALKE, N., CANOVA, F., MILANI, F. AND WYNNE, M.A. (Eds.) (2012): *DSGE Models In Macroeconomics: Estimation, Evaluation, And New Developments, Advances In Econometrics*, Emerald Group Publishing Limited, Vol. 28, pp. 421-467.
- JANG, T.-S. AND SACHT, S. (2012): *Identification Of Animal Spirits In A Bounded Rationality Model: An Application To The Euro Area*. Working Paper No. 1798, Kiel Institute for the World Economy.
- KAHN, A. KING, R.G. AND WOLMAN, A.L. (2003): Optimal Monetary Policy. *Review of Economic Studies*, Vol. 70(4), pp. 825-860.
- KAHNEMAN, D. (2003): Maps Of Bounded Rationality: Psychology For Behavioral Economics. *The American Economic Review*, Vol. 93(5), pp. 1449-1475.
- KARAMÉ, F., PATUREAU, L. AND SOPRASEUTH, T. (2008): Limited Participation And Exchange Rate Dynamics: Does Theory Meet The Data?. *Journal of Economic Dynamics and Control*, Vol. 32(4), pp. 1041-1087.
- KEHOE, P.J. AND MIDRIGAN, V. (2007): *Sales And The Real Effects Of Monetary Policy*. Working Paper No. 652, Federal Reserve Bank of Minneapolis.
- KENNEDY, P. (2007): *A Guide To Econometrics*. 5th Edition, Blackwell Publishing, Oxford (US).
- KENDRICK, D.A. AND AMMAN, H.M. (2011): *A Taylor Rule For Fiscal Policy*. Discussion Paper No. 11-17, Utrecht School of Economics, Tjalling C. Koopmans Research Institute.

- KEYNES, J.M. (1936): *The General Theory Of Employment, Interest And Money*. Macmillan Cambridge University Press, Cambridge(US).
- KHALAF L. AND KICHIAN, M. (2004): *Estimating New Keynesian Phillips Curves Using Exact Methods*. Working Paper No. 04-11, Bank of Canada.
- KIM, J. AND RUGE-MURCIA, F.J. (2011): Monetary Policy When Wages are Downwardly Rigid: Friedman Meets Tobin. *Journal of Economic Dynamics and Control*, Vol. 35(12), pp. 2064–2077.
- KLENOW, P.J. AND MALIN, B.A. (2010), *Microeconomic Evidence On Price-Setting*. NBER Working Paper No. 15826, National Bureau of Economic Research, Inc.
- KYDLAND, F.E. AND PRESCOTT, E.C. (1982): Time To Built And Aggregate Fluctuations. *Econometrica*, Vol. 50(6), pp. 1345-1370.
- LAUBACH, T. AND WILLIAMS, J.C. (2003): Measuring The Natural Rate Of Interest. *The Review of Economics and Statistics*, Vol. 85(4), pp. 1063-1070.
- LEE, B.S. AND INGRAM, B. (1991): Simulation Estimation Of Time-Series Models. *Journal of Econometrics*, Vol. 47(2-3), pp. 197-205.
- LEITEMO, K. AND SÖDERSTRÖM, U. (2005): Simple Monetary Policy Rules And Exchange Rate Uncertainty. *Journal of International Money and Finance*, Vol. 24(3), pp. 481-507.
- LEITEMO, K. (2008): Inflation-Targeting Rules: History-Dependent Or Forward-Looking?. *Economic Letters*, Vol. 100, pp. 267-270.
- LENGNICK, M. AND WOHLTMANN, H.-W. (2012): Agent-Based Financial Markets And New Keynesian Macroeconomics - A Synthesis. *Journal of Economic Interaction and Coordination*, Vol. 8(1), pp. 1-32.
- LINDÉ, J. (2005): Estimating New-Keynesian Phillips Curves: A Full Information Maximum Likelihood Approach. *Journal of Monetary Economics*, Vol. 52(6), pp. 1135-1149.
- LINDEMANN, L. AND SCHABERT, A. (2003): Fiscal Policy In The New Neoclassical Synthesis. *Journal of Money, Credit and Banking*, Vol. 35(6-1), pp. 911-929.
- LOMBARDI, M.J. AND NICOLETTI, G. (2012): Bayesian Prior Elicitation In DSGE Models: Macro- Vs Micropriors. *Journal of Economic Dynamics and Control*, Vol. 36(2), pp. 294-313.
- LUBIK, T. AND SCHORFHEIDE, F. (2004): Testing For Indeterminacy: An Application To U.S. Monetary Policy. *American Economic Review*, Vol. 94(1), pp. 190-217.
- LUBIK, T. AND SCHORFHEIDE, F. (2007): Do Central Banks Respond To Exchange Rate Movements? A Structural Investigation. *Journal of Monetary Economics*, Vol. 54(4), pp. 1069-1087.
- LUCAS, R.E.JR. (1973): Some International Evidence On Output-Inflation Trade-Offs. *American Economic Review*, Vol. 63, pp. 326-334.

- LUCAS, R.E.JR. (1976): Econometric Policy Evaluation: A Critique. *Carnegie-Rochester Conference Series on Public Policy*, Vol. 1(1), pp. 19-46.
- LUCAS, R.E.JR. (1982): Interest Rate And Currency Prices In A Two-Country World. *Journal of Monetary Economics*, Vol. 10(3), pp. 335-359.
- LUX, T. (2009): Rational Forecasts Or Social Opinion Dynamics? Identification Of Interaction Effects In A Business Climate Survey. *Journal of Economic Behavior and Organization*, Vol. 72(2), pp. 638-655.
- LÜTKEPOHL, H. (2007): *New Introduction To Multiple Time Series Analysis*. Springer, Berlin.
- MA, A. (2002): GMM Estimation Of The New Phillips Curve. *Economics Letters*, Vol. 76(3), pp. 411-417.
- MATHERON, J. AND POILLY, C. (2009): How Well Does A Small Structural Model With Sticky Prices And Wages Fit Postwar U.S. Data?. *Economic Modelling*, Vol. 26(1), pp. 266-284.
- MANKIW, N.G. AND REIS, R. (2002): Sticky Information Versus Sticky Prices: A Proposal To Replace The New Keynesian Phillips Curve. *The Quarterly Journal of Economics*, Vol. 117(4), pp. 1295-1328.
- MAVROIDES, S. (2007): *Testing The New Keynesian Phillips Curve Without Assuming Identification*. Working Paper No. 2006-13, Brown University.
- MCCALLUM, B.T. (1976): Rational Expectations And The Natural Rate Hypothesis: Some Consistent Estimates. *Econometrica*, Vol. 44(1), pp. 43-52.
- MCCALLUM, B.T. (1983): On Non-Uniqueness In Rational Expectations Models: An Attempt At Perspective. *Journal of Monetary Economics*, Vol. 11(2), pp. 139-168.
- MCCALLUM, B.T. (2005): *A Monetary Policy Rule For Automatic Prevention Of A Liquidity Trap*. NBER Working Papers No. 11056, National Bureau of Economic Research, Inc.
- MCCALLUM, B.T. AND NELSON, E. (2005): Targeting Versus Instrument Rules For Monetary Policy. *Review*, Federal Reserve Bank of St. Louis, September 2005, pp. 597-612.
- MEDINA, J.P., RAPPOPORT, D., AND SOTO, C. (2007): Dinámica De Ajuste De Precios: Evidencia De Datos Microeconómicos Para Chile. *Journal Economía Chilena*, Vol. 10(2), pp. 5-26.
- MESONNIER, J.-S. AND RENNE, J.-P. (2007): A Time-Varying 'Natural' Rate Of Interest For The Euro Area. *European Economic Review*, Vol. 51(7), pp. 1768-1784.
- MIHAILOV, A., RUMLER, F. AND SCHARLER, J. (2011a): The Small Open-Economy New Keynesian Phillips Curve: Empirical Evidence And Implied Inflation Dynamics. *Open Economies Review*, Vol. 22(2), pp. 317-337.

- MIHAILOV, A., RUMLER, F. AND SCHARLER, J. (2011b): Inflation Dynamics In The New EU Member States: How Relevant Are External Factors?. *Review of International Economics*, Vol. 19(1), pp. 65-76.
- MILANI, F. (2007): Expectations, Learning And Macroeconomic Persistence. *Journal of Monetary Economics*, Vol. 54, p. 2065-2082.
- MILANI, F. (2012): *The Modeling Of Expectations In Empirical DSGE Models: A Survey*. Working Paper No. 121301, Department of Economics, University of California-Irvine.
- MITTELHAMMER, R.C. (1999): *Mathematical Statistics For Economics And Business*. 3rd Edition, Springer, Berlin.
- MONACELLI, T. (2004): Into The Mussa Puzzle: Monetary Policy Regimes And The Real Exchange Rate In A Small Open Economy. *Journal of International Economics*, Vol. 62(1), pp. 191-217.
- MOONS, C., GARRETSEN, H., VAN AARLE, B. AND FORNERO, J. (2007): Monetary Policy In The New-Keynesian Model: An Application To The Euro Area. *Journal of Policy Modeling*, Vol. 29(6), pp. 879-902.
- MORANDÉ, F. AND TEJADA, M. (2008): *Price Stickiness In Emerging Economies: Empirical Evidence For Four Latin-American Countries*. Working Paper No. 286, Department of Economics, University of Chile.
- MORTENSEN, D.T. (1986): Job Search And Labor Market Analysis. in: ASHENFELTER, O. AND LAYARD, R. (Eds.) (1986): *Handbook Of Labor Economics*, North Holland, Vol. 2(2), pp. 849-919.
- NASON, M.N. AND SMITH, G.W. (2008): The New Keynesian Phillips Curve: Lessons From Single-Equation Econometric Estimation. *Economic Quarterly*, Vol. 94(4), pp. 361-395.
- NEWBY, W. AND WEST, K. (1994): Automatic Lag Selection In Covariance Matrix Estimation. *Review of Economic Studies*, Vol. 61(4), pp. 631-53.
- UDIZ, G. AND SACHS, J. (1985): International Policy Coordination In Dynamic Macroeconomic Models. in: BUITER, W. AND MARSTON, R. (Eds.) (1985): *International Economic Policy Coordination*, Cambridge University Press, Cambridge (US), pp. 275-319.
- POSCH, O. RUBIO-RAMÍREZ, J.F. AND FERNÁNDEZ-VILLAVARDE, J. (2011): *Solving The New Keynesian Model In Continuous Time*. 2011 Meeting Papers No. 829, Society for Economic Dynamics.
- PRESS, W.H., TEUKOLSKY, S., VETTERLING, W.T. AND FLANNERY, B.P. (1986): *Numerical Recipes: The Art Of Scientific Computing*. Cambridge University Press, Cambridge (UK).
- RAMOS-FRANCIA, M. AND TORRES, A. (2008): Inflation Dynamics In Mexico: A Characterization Using The New Phillips Curve. *The North American Journal of Economics and Finance*, Vol. 19(3), pp. 274-289.

- RÍOS-RULL, J-V., SCHORFHEIDE, F., FUENTES-ALBERO, C., KRYSHKO, M. AND SANTAEULÀLIA-LLOPIS, R. (2011): *Methods Versus Substance: Measuring The Effects Of Technology Shocks On Hours*. Mimeo, University of Minnesota.
- ROGERSON, R., SHIMER, R. AND WRIGHT, R. (2005): Search-Theoretic Models Of The Labor Market: A Survey. *Journal of Economic Literature*, Vol. 43(4), pp. 959-988.
- ROOS, M. W. M., SCHMIDT, U. (2012): The Importance of Time-Series Extrapolation for Macroeconomic Expectations. *German Economics Review*, Vol. 13(2), pp. 196-210.
- ROSA, C. (2013): *The High-Frequency Response Of Energy Prices To Monetary Policy: Understanding The Empirical Evidence*. Staff Report No. 598, Federal Reserve Bank of New York.
- ROTEMBERG, J.J. AND WOODFORD, M. (1997): An Optimization-Based Econometric Framework For The Evaluation Of Monetary Policy. in: BERNANKE, B. AND ROTEMBERG, J.J. (Eds.) (1997): *NBER Macroeconomics Annual*, MIT Press, Cambridge (US).
- RUDD, J. AND WHELAN, K. (2005): *Modeling Inflation Dynamics: A Critical Review Of Recent Research*. Finance and Economics Discussion Series No. 2005-66, Federal Reserve Board, Washington D.C., pp. 2005-66.
- RUDD, J. AND WHELAN, K. (2007): Modeling Inflation Dynamics: A Critical Review Of Recent Research. *Journal of Money, Credit and Banking*, Supplement to Vol. 39(1), pp. 155-170.
- RUGE-MURCIA, F. (2012): Estimating Nonlinear DSGE Models By The Simulated Method Of Moments. With An Application To Business Cycles. *Journal of Economic Dynamics and Control*, Vol. 36(6), pp. 914-938.
- RUSSELL, B. AND BANERJEE, A. (2008): The Long-Run Phillips Curve And Non-Stationary Inflation. *Journal of Macroeconomics*, Vol. 30(4), pp. 1792-1815.
- SACHT, S. AND WOHLTSMANN, H.-W. (2013): *Inflation Targeting And Welfare Analysis In High-Frequency New-Keynesian Models*. Mimeo, Department of Economics, Christian Albrechts University Kiel, November 2013.
- SACHT, S. (2014): *Analysis of Various Shocks within the High-Frequency Versions of the Baseline New-Keynesian Model*. Economics Working Paper No. 2014-02, Department of Economics, Christian Albrechts University Kiel, January 2014.
- SACHT, S. (2014): *Optimal Monetary Policy Responses and Welfare Analysis within the High-Frequency New-Keynesian Framework*. Economics Working Paper No. 2014-03, Department of Economics, Christian Albrechts University Kiel, January 2014.
- SACHT, S. (2014): *Identification of Prior Information via Moment-Matching*. Economics Working Paper No. 2014-04, Department of Economics, Christian Albrechts University Kiel, January 2014.
- SALA, L. (2011): *DSGE Models In The Frequency Domain*. Mimeo, Università Bocconi.

- SALEMI, M.K (2006): Econometric Policy Evaluation And Inverse Control. *Journal of Money, Credit and Banking*, Vol. 38(7), pp. 1737-1764.
- SARGENT, T.J. (1994): *Bounded Rationality In Macroeconomics*. Oxford University Press, New York.
- SCHMITT-GROHÉ, S. AND URIBE, M. (2004): Solving Dynamic General Equilibrium Models Using A Second-Order Approximation To The Policy Function. *Journal of Economic Dynamics and Control*, Vol. 28 (4), pp. 755-775.
- SCHMITT-GROHÉ, S. AND URIBE, M. (2007): Optimal Simple And Implementable Monetary And Fiscal Rules. *Journal of Monetary Economics*, Vol. 54(6), pp. 1702-1725.
- SCHORFHEIDE, F. (2005): Learning And Monetary Policy Shifts. *Review of Economic Dynamics*, Vol. 8(2), pp. 392-419.
- SCHORFHEIDE, F. (2008): DSGE Model-Based Estimation Of The New Keynesian Phillips Curve. *Federal Reserve Bank of Richmond Economic Quarterly*, Vol. 94(4), pp. 397-433.
- SELTEN, R. (2001): What Is Bounded Rationality?. in: GIGERENZER, G. AND SELTEN, R. (Eds.) (2001): *Bounded Rationality: The Adaptive Toolbox*, The MIT Press, Cambridge (US), pp. 13-36.
- SHEEDY, K.D. (2010): Intrinsic Inflation Persistence. *Journal of Monetary Economics*, Vol. 57(8), pp. 1049-1061.
- SILVESTRINI, A. AND VEREDAS, D. (2008): Temporal Aggregation Of Univariate And Multivariate Time Series Models: A Survey. *Journal of Economic Surveys*, Vol. 22(3), pp. 458-497.
- SIMS, C. (2003): Implications of Rational Inattention. *Journal of Monetary Economics*, Vol. 50(3), pp. 665-690.
- SMETS, F. AND WOUTERS, R. (2003): An Estimated Dynamic Stochastic General Equilibrium Model Of The Euro Area. *Journal of the European Economic Association*, Vol. 1(5), pp. 1123-1175.
- SMETS, F. AND WOUTERS, R. (2005): Comparing Shocks And Frictions In US And Euro Area Business Cycles: A Bayesian DSGE Approach. *Journal of Applied Econometrics*, Vol. 20(2), pp. 161-183.
- SMETS, F. AND WOUTERS, R. (2007): Shocks And Frictions In US Business Cycles: A Bayesian DSGE Approach. *American Economic Review*, Vol. 97(3), pp. 586-606.
- SMITH, A.A., JR.(1993): Estimating Nonlinear Time-Series Models Using Simulated Vector Autoregressions. *Journal of Applied Econometrics*, Vol. 8(1), pp. 63-84.
- SMITH, A.A., JR.(2008): Indirect Inference. *The New Palgrave Dictionary of Economics*, 2nd Edition, Macmillan, New York.

- SÖDERLIND, P. (1999): Solution And Estimation Of RE Macromodels With Optimal Policy. *European Economic Review*, Vol. 43(4-6), pp. 813-823.
- STOCK, J.H., WRIGHT, J.H. AND YOGO, M. (2002): A Survey Of Weak Instruments And Weak Identification In Generalized Method Of Moments. *Journal of Business and Economic Statistics*, Vol. 20(4), pp. 518-529.
- SVENSSON, L. (1999): Inflation Targeting As A Monetary Policy Rule. *Journal of Monetary Economics*, Vol. 43(9), pp. 607-654.
- SVENSSON, L.E.O. AND WOODFORD, M. (2003): Indicator Variables For Optimal Policy. *Journal of Monetary Economics*, Vol. 50(3), pp. 691-720.
- SVENSSON, L.E.O. AND WOODFORD, M. (2005): Implementing Optimal Monetary Policy Through Inflation-Forecast Targeting. in: BERNANKE, B. AND WOODFORD, M. (Eds.) (2005): *The Inflation Targeting Debate*, University of Chicago Press, Chicago (US).
- TAYLOR, J.B. (1980): Aggregate Dynamics And Staggered Contracts. *Journal of Political Economy*, Vol. 88(1), pp. 1-23.
- TAYLOR, J.B. (1993): Discretion Versus Policy Rules In Practice. *Carnegie-Rochester Conference Series on Public Policy*, Vol. 39(1), pp. 195-214.
- TAYLOR, J.B. (1999): Staggered Price And Wage Setting In Macroeconomics. in: TAYLOR, J.B. AND WOODFORD, M. (Eds.) (1999): *Handbook of Macroeconomics*, North Holland, Vol. 1(1), pp. 1009-1050.
- VAN ROYE, B. (forthcoming): Financial Stress And Economic Activity In Germany And The Euro Area. *Empirica*, forthcoming.
- WALSH, C. (2010): *Monetary Theory And Policy*. 3rd Edition, MIT Press, Cambridge (US).
- WEDDERBURN, R.W.M. (1974): Quasi-Likelihood Functions, Generalized Linear Models, And The Gauss-Newton Method. *Biometrika*, Vol. 61(3), pp. 439-447.
- WEN, Y. (1998): Can A Real Business Cycle Model Pass The Watson Test?. *Journal of Monetary Economics*, Vol. 42(1), pp. 185-203.
- WESTERHOFF, F. (2008): The Use of Agent-Based Financial Market Models To Test the Effectiveness of Regulatory Policies. *Journal of Economics and Statistics*, Vol. 228, pp. 195-227.
- WINKLER, R.C. AND WOHLTMANN, H.-W. (2006): *Anticipated Raw Materials Price Shocks And Monetary Policy Response - A New Keynesian Approach*. Working Paper No. 2006-19, Department of Economics, Christian Albrechts University of Kiel.
- WINKLER, R.C. AND WOHLTMANN, H.-W. (2008): *Anticipated And Unanticipated Oil Price Shocks And Optimal Monetary Policy*. Working Paper No. 2008-05, Department of Economics, Christian Albrechts University Kiel.

- WINKLER, R.C. AND WOHLTMANN, H.-W. (2012): On The (De)Stabilizing Effects of News Shocks. *Economics Letters*, Vol. 114(3), pp. 256-258.
- WOHLTMANN, H.-W. AND KRÖMER, W. (1989): On The Notion Of Time-Consistency - A Comment. *European Economic Review*, Vol. 33, pp. 1283-1288.
- WOHLTMANN, H.-W. AND WINKLER, R.C. (2009): *Rational Expectations Models With Anticipated Shocks And Optimal Policy: A General Solution Method and A New Keynesian Example*. Kiel Working Papers No. 1507, Kiel Institute for the World Economy.
- WOODFORD, M. (1999): Optimal Monetary Policy Inertia. *Manchester School*, University of Manchester, Vol. 67, pp. 1-35.
- WOODFORD, M. (2003): *Interest And Prices: Foundations Of A Theory Of Monetary Policy*. Princeton University Press, Princeton.
- WORLD BANK (2009): *World Development Indicators*. World Bank, Washington DC.
- WORLD BANK (2010): Argentina Trade Brief. *World Trade Indicators 2009/10: Country Trade Briefs*, World Bank, Washington DC.
- YAZGAN, M.E. AND YILMAZKUDAY, H. (2005): Inflation Dynamics Of Turkey: A Structural Estimation. *Studies in Nonlinear Dynamics and Econometrics*, Vol. 9(1), pp. 1-15.

Curriculum Vitae

Last Name: **Sacht**
First Name: **Stephen**
Citizenship: German
Born: June 24, 1982 in Eckernförde (Germany)
Address: Westring 275
24116 Kiel
Germany
E-Mail: sacht@economics.uni-kiel.de
stephen.sacht@ifw-kiel.de

WORK

since 10/2012 Member of the Research Area ‘Macroeconomic Policy under Market Imperfections’, Kiel Institute for the World Economy
since 04/2008 Research and Teaching Assistant at the Department of Economics, Chair of Macroeconomics (Professur für Makroökonomik), University of Kiel (Teaching on Bachelor, Master and PhD Level)
01/2007-04/2008 Student Assistant at the Chair of Macroeconomics of Prof. Dr. Hans-Werner Wohltmann, University of Kiel
10/2005-04/2008 Active Member of the Student Representatives’ Association (‘Fachschaft’) at the Department of Economics
08/2007-10/2007 Internship at Ministry of Science, Economic and Transport of Land Schleswig-Holstein, Kiel
08/2005-10/2005 Internship at Deutsche Bank AG, Kiel

EDUCATION

05/2012 Graduate of the ‘Advanced Studies Program in International Economic Policy Research’ at the Kiel Institute for the World Economy
04/2008 Diploma in Economics (‘Diplom-Volkswirt’), University of Kiel
since 10/2007 Participant in the PhD-Programme ‘Quantitative Economics’ at the University of Kiel
10/2005-04/2008 Graduate Studies in Economics with Emphasis on International Economics, Diploma Thesis on ‘Shocks and optimal monetary policy in the baseline New Keynesian model’ (Supervisor: Prof. Dr. Hans-Werner Wohltmann)
08/2006-02/2007 Studies in Economics as Erasmus Exchange Student at the University of Jönköping, Sweden
10/2003-10/2005 Undergraduate Studies in Economics at the University of Kiel

RESEARCH INTERESTS

- Bounded Rationality and Macroeconomic Dynamics
- High-Frequency Modeling of DSGE models
- Moment-Matching and Bayesian Estimation of DSGE models

PUBLICATIONS

- **Some Observations in the High-Frequency Versions of a Standard New-Keynesian Model** (with R. Franke), *Bulletin of Economic Research*, January 2014, Vol. 66(1), pp. 72-94.
- **Estimating a High-Frequency New-Keynesian Phillips Curve** (with S. Ahrens), *Empirical Economics*, March 2014, Vol. 46(2), pp. 607-628.

WORKING PAPERS

- **Analysis of Various Shocks within the High-Frequency Versions New-Keynesian Models of the Baseline New-Keynesian Model**
Economics Working Paper No. 2014-02,
Department of Economics, University of Kiel, January 2014
- **Optimal Monetary Policy Responses and Welfare Analysis within the High-Frequency New-Keynesian Framework**
Economics Working Paper No. 2014-03,
Department of Economics, University of Kiel, January 2014
- **Identification of Prior Information via Moment-Matching**
Economics Working Paper No. 2014-04,
Department of Economics, University of Kiel, January 2014
- **The Rational Expectation Hypothesis versus Animal Spirits within a New-Keynesian Framework: Empirical Evidence from Moment-Matching**
(with T.-S. Jang), Kiel Working Paper No. 1798,
Kiel Institute for the World Economy, October 2012
- **Moment Matching versus Bayesian Estimation: Backward-Looking Behaviour in the New-Keynesian Three-Equations Model**
(with R. Franke and T.-S. Jang), Economics Working Paper No. 2012-08,
Department of Economics, University of Kiel, September 2012

CONFERENCES AND WORKSHOPS

- Presentation at the Annual Meeting of the German Economic Association (Verein für Socialpolitik), University of Düsseldorf (Germany), 4.-7. September 2013 (Chairman of the self-organized Conference session on ‘Moment Matching: Applications of an Alternative Estimation Approach in Macroeconomics’)
- Presentation at the SCE 19th International Conference on Computing in Economics and Finance, Vancouver (Canada), 10.-12. July 2013
- Presentation at the Conference ‘The State of Economics after the Crisis’, Macroeconomic Policy Institute (IMK) at the Hans-Boeckler-Foundation, Research Network Macroeconomics and Macroeconomic Policies (FMM), Berlin, 25.-27. October 2012
- Presentation at the Annual Meeting of the German Economic Association (Verein für Socialpolitik), University of Göttingen (Germany), 9.-12. September 2012
- Presentation at the SCE 18th International Conference on Computing in Economics and Finance, Prague (Czech Republic), 27.-29. June 2012
- Presentation at the 17th Spring Meeting of Young Economists (SMYE), ZEW Mannheim, 26.-28. April 2012 (Chairman of the Conference session on ‘Business Cycles III’)
- Presentation at the Macroeconometric Workshop, DIW Berlin, 2.-3. December 2011
- Presentation at the Conference: Modeling High Frequency Data in Finance III, Stevens Institute of Technology (Hoboken/New Jersey/USA), 28.-31. July 2011
- Presentation at the Annual Meeting of the Swiss Society of Economics and Statistics, University of Lucerne (Switzerland), 9.-10. June 2011 (Chairman of the Conference session on ‘Macroeconomics I’)
- Presentation at the Annual Meeting of the German Economic Association (Verein für Socialpolitik), University of Kiel (Germany), 7.-10. September 2010
- Presentation at the 25th Annual Congress of the European Economic Association, University of Glasgow (UK), 23.-26. August 2010 (Chairman of the Conference session on ‘Preferences and Decision Making’)
- Presentation at the SCE 16th International Conference on Computing in Economics and Finance, City University London (UK), 15.-18. July 2010
- Presentation at the Annual Meeting of the Swiss Society of Economics and Statistics, University of Fribourg (Switzerland), 24.-25. June 2010 (Chairman of the Conference session on ‘Monetary Macroeconomics’)
- Participation at the 2. ‘Global Economic Symposium’, Plön, 9.-11. September 2009
- Participation at the Workshop ‘Dynare Summer School’, Bank of France/CEPREMAP (Paris), 22.-26. June 2009
- Participation at the EES Kick-off Workshop ‘The Labor Market and the Business Cycle’, IfW Kiel, 5.-7. March 2009
- Participation at the 3. Workshop ‘Konjunktur und Makroökonomik’, ifo Dresden, 20.-21. November 2008
- Participation at the 1. ‘Global Economic Symposium’, Plön, 4.-5. September 2008

AWARDS & GRANTS

- Graduate Student Best Paper Award 2012 of the Society of Computational Economics (for the Working Paper *The Rational Expectation Hypothesis versus Animal Spirits within a New-Keynesian Framework: Empirical Evidence from Moment-Matching*)
- Travel Grant by The U.S. National Science Foundation (for the Conference *Modeling High Frequency Data in Finance III at Stevens Institute of Technology* in 2011)
- Möller Fund Travel Grant 2010, '11 & '12 (University of Kiel)

MEMBERSHIPS

- Euro Area Business Cycle Network (EABCN)
- European Economic Association (EEA)
- German Economic Association (Verein für Socialpolitik)
- Society for Computational Economics (SCE)

FURTHER QUALIFICATIONS

- Computer Skills: MATLAB, DYNARE, Mathematica, TEX, LATEX
- Foreign Language: English (fluent)

TEACHING EXPERIENCE

- Tutorial on Introduction to Macroeconomic Theory (Grundzüge der makroökonomischen Theorie; WS 08/09–10/11)
- Tutorial on Intermediate Macroeconomics (Neue Makroökonomik; SS 09–13)
- Tutorial on Open Economy Macroeconomics (Stabilisierungspolitik; WS 09/10–13/14)
- Tutorial on Advanced Macroeconomics (within the Doctoral Programme; SS 2009)
- Seminar on Money and Credit (Geld- und Kredittheorie; WS 08/09)
- Seminar on Open Economy Macroeconomics (Stabilisierungspolitik; SS 09–13)
- Seminar on Macroeconomics (Makroökonomisches Seminar; WS 09/10–13/14)
- Seminar on Macroeconomic Dynamics and Optimal Monetary Policy (Master; WS 11/12–13/14)

OUTREACH & SERVICE

- | | |
|-----------------|--------------------------------------------------------------------------------------------------------------------|
| since 04/2008 | Student Guidance for Economics at the Faculty of Business, Economics and Social Sciences at the University of Kiel |
| 04/2008-04/2012 | Member of the Convention of the Faculty of Business, Economics and Social Sciences at the University of Kiel |

Eidesstattliche Erklärung

Ich erkläre hiermit an Eides Statt, dass ich meine Doktorarbeit *High-Frequency Analysis and Moment-Matching Estimation of the Baseline New-Keynesian Model* selbstständig und ohne fremde Hilfe angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehnenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen nach den mir angegebenen Richtlinien zitiert habe.

Kiel, 25. November 2013

(Stephen Sacht)