

Three Essays on the Role of Intermediaries

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Needless to say, I am responsible for the remaining errors.

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Note on co-authorship:

Chapter 3 is based on a paper jointly written with Horst Raff and Nicolas Schmitt. My contribution to this chapter was to derive a more general result by introducing a non-lumpy trade case and a model with continuous uncertainty.

1 General Introduction

An intermediary is a third party that offers intermediation services between sellers and buyers or that purchases from suppliers and then resells to buyers.

Generally speaking, intermediaries can play many roles in a transaction whether it is an international transaction or simply a domestic one. Spulber (1996, p.136) summarizes four actions of intermediaries: "setting prices and clearing markets; providing liquidity and immediacy; coordinating buyers and sellers; and guaranteeing quality and monitoring performance." This dissertation aims at further enhancing our understanding of how intermediaries can in different scenarios affect supplier's behavior and what determines their effects on the economy and social welfare.

In principle, for any transaction involving intermediaries, there would be negotiations between suppliers and intermediaries. The outcome of the negotiations are usually influenced by the buyer power of intermediaries. Conversely when making optimal decisions, the suppliers must be concerned about the buyer power of intermediaries. Consequently, it is important to identify how the buyer power of intermediaries affects supplier behavior. In particular, chapter 2 deals with the effects on supplier incentives to invest in innovation.

Depending on different model settings, a variety of theoretical findings exists in the recent theoretical literature. The common view is that a large intermediary is able to obtain advantageous terms (e.g., price discounts) from their suppliers by exercising buyer power. However, there is a wide variety of explanations for the effects of buyer power on the supplier's innovation. Actually when endogenizing the buyer power of the intermediary, the previous theoretical models just consider a situation, in which two buyers merge into one group. This simplifies the analysis, however it neglects the

competition between strong merged buyers and small independent buyers. Therefore, chapter 2 proceeds by first examining the competition between a big merged buyer and small independent buyers, rather than focusing solely on a big merged buyer. The relative status of a large buyer is given more attention in chapter 2. Eventually, chapter 2 shows that there is an inverted-U-shaped relationship between the buyer power of intermediaries and supplier innovation.

In chapter 2, the main function of intermediaries is “setting prices and clearing markets”. In the following chapters 3 and 4, this dissertation highlights another function of intermediaries, i.e. to “...hold inventories to provide liquidity or availability of goods and services (see Spulber,1996, p.135).”

In chapter 3, we^{1.1} focus on the intermediary in the context of international trade. An intermediary could be located in the seller market (an export intermediary), in the buyer market (an import intermediary), or involve an intermediary in both markets. Specifically, we are interested in the role of export and import intermediaries in allocating inventories across countries and over time. We identify circumstances under which it would pay a producer to use either an export or an import intermediary. Moreover, we ask what effects the presence of such an intermediary has on the volume and on the volatility of trade, on the size of trade inventories, and on social welfare.

We show that it pays a manufacturer to sell through a wholesaler rather than export directly provided there is enough demand volatility. We also find that an export wholesaler has indeed an edge over an import wholesaler when demands across different markets are negatively correlated, while an import wholesaler has an edge if this correlation is positive. In addition, an important outcome of our model is the finding that wholesalers essentially smooth trade and decrease the need to maintain large inventory. As shown by Alessandria, Kaboski and Midrigan (2010a, p.2305), international trade is often lumpy^{1.2}. Therefore, we also pay attention to the trade lumpiness in chapter 3. As a trade becomes lumpy; that is, international shipments become both larger when they

^{1.1}This chapter is based on a paper that is joint work with Horst Raff and Nicolas Schmitt.

^{1.2}Using monthly data on the universe of US exports for goods in narrowly defined categories (ten-digit Harmonized System code), Alessandria, Kaboski and Midrigan show that “annual trade is highly concentrated in a few months. The bulk of trade (85 percent) is accounted for by only three months of the year; the top month of the year accounts for 50 percent of that year’s trade on average. No trade is recorded in half of the months. The infrequency and high concentration of these trade flows in a few months of the year reflect the role of economies of scale in international trade.”

occur and possibly less regular. We show that it decreases the manufacturer's incentives to channel trade through a wholesaler, whether it is an import or an export wholesaler. However, when a manufacturer still chooses to channel trade through a wholesaler, lumpy trade makes the use of an import wholesaler much more desirable.

Finally in chapter 3, we introduce a more general model to identify sufficient conditions under which intermediation may raise or lower the volume of trade and social welfare. These social welfare results show that a manufacturer's private incentive regarding the organization of international sales can be aligned with society's interests or not.

In chapter 4, this dissertation will further focus on the issue of the volatility of trade, which has been mentioned but not deeply investigated in chapter 3. Jay Forrester first introduced the definition of bullwhip effect in 1961. He characterized it as an amplification of demand variability from a downstream site to an upstream site. Since then, the bullwhip effect has been observed and recognized in a managerial context, as well as in various companies and industries. For a long time, the bullwhip effect has been an operational management concept. There exists a massive amount of articles on this topic. However, in most of the operational management literature, demand is characterized by an exogenous process. An intermediary does not need to make an optimal pricing decision. By assuming an exogenous demand process, it might be easy to measure the volatility of demand in each echelon of the supply chain. It would, however, make readers lose sight of how the intermediaries' behavior in a supply chain drives the dynamics of trade flows and then causes the bullwhip effect. On the other hand, some industries do not exhibit a bullwhip effect. By using industry-level U.S. data, Cachon, Randall et al. (2007, p.457) find "wholesale industries exhibit a bullwhip effect, but retail industries generally do not exhibit the effect, nor do most manufacturing industries." To our knowledge, there is no theoretical model in economics or operation management that attempts to give an explanation for why some industries exhibit a bullwhip effect while others do not. It is then reasonable for us to investigate the underlying rationale.

Therefore, chapter 4 develops a new theoretical model of the bullwhip effect, where the intermediary makes optimal pricing and inventory management decisions. By introducing a trade collapse case, which is ignored in previous literature, our model shows that there will be a reverse bullwhip effect if the persistence of demand shocks is small, and a bullwhip effect will be present when a large persistence of the demand shock exists. In

addition, it is shown that the degree of the bullwhip effect increases as the persistence of demand shocks goes up. Therefore, this chapter provides a new explanation for why some industries exhibit the bullwhip effect while others do not.

2 Buyer Power of Intermediaries and Supplier Incentives

Abstract: In this chapter, we develop a model to analyze the effects of buyer power on supplier incentives. It is shown that there is an inverted-U-shaped relationship between the buyer power of intermediaries and supplier innovation. We introduce the Nash Bargaining Solution to characterize the process of negotiation between buyers and supplier. Rather than focusing solely on a big merged buyer, we proceed by first examining the competition between a big merged buyer and small independent buyers. We also indicate how the supplier's innovation influences the buyer's outside option as the structure of the downstream market becomes more concentrated. Whether or not one merger is powerful enough to weaken a supplier's innovation depends on the relative size of the big merged buyer. Thus, when antitrust authorities investigate a merger case, they should evaluate how much the merger would affect market structure.

2.1 Introduction

In today's market, many industries are facing increasing downstream concentration. An apparent example is the emergence of large retail chains, particularly in the fast-moving consumer goods industry.^{2.1} Over the last several years, such concentration has drawn the attention of economists and antitrust authorities, who are concerned about the effect of the retailers' increasing buyer power over their suppliers, and its effects on the suppliers' incentives to innovate.

There are various ways to study the sources and effects of buyer power. In general, a supplier negotiates with downstream buyers, and the Nash Bargaining Solution is used to characterize the negotiation process. The bargaining position or buyer power is thus determined by the outside-option payoffs. Economists usually model a process of downstream market concentration to endogenize the bargaining position, and then show how the supplier makes the optimal decision for innovation. Depending on different model settings, a variety of theoretical findings exists in recent literature. The common view is that a large buyer is able to obtain advantageous terms (e.g., price discounts) from their suppliers by exercising buyer power. However, there is a wide variety of explanations for the source of buyer power, and the most controversial issue is the effects of buyer power on the supplier's investment in innovation.

Mazarotto (2004) finds that retailers may get price discounts in negotiations with the manufacturer if they merge locally, i.e., they merge with competing retailers. He also finds that the supplier's response, when facing a merged buyer, is to reduce its effort to bring down the unit production cost. DeGraba (2003), Chae and Heidhues (2004) find that big buyers may receive discounts if the buyers and or suppliers are risk averse. Also, the dynamic welfare effects of big buyers will be negative because the supplier's incentives to reduce his unit costs are weakened. Inderst and Wey (2003)'s model shows the link between the supplier's production technology choice and the presence of downstream buyer power. This model demonstrates that a big buyer will obtain a discount if total industry profit is strictly concave in the number of active stores. Surprisingly, it shows the dynamic welfare effects are positive, the presence of large buyers may strengthen

^{2.1}The concentration ratio of the five largest retailer (C5) in member countries of the EU is on average 50% (IGD European Grocery Retailing, 2005). The UK's top four grocery retailers account for 65% of total retail sales (the Competition Commission, 2008, p.29). In the US, C8 was 17.5% in 2007, instead of 15.3% in 2002. See the US census Bureau, Retail Trade <http://www.census.gov/econ/concentration.html>

the supplier's incentives to switch to a technology with relatively high fixed costs but low unit costs (less convex). Inderst and Wey (2011) consider cross-border mergers with assumptions of constant unit production costs and multiple downstream markets. They find a big buyer's threat of integrating backwards to be more credible than the threat of a small buyer. Hence, big buyers are more likely to receive discounts. Again, the welfare effects are found to be positive, the supplier's incentives to reduce its unit costs will increase as the buyers grow larger. Inderst and Shaffer (2007) find cross-border mergers to be profitable for retailers, because they can commit to stocking the same product at all locations and therefore make the suppliers compete harder for the contracts. However, since the merged buyers find it profitable to reduce the number of products supplied, the welfare effect is negative. Furthermore, the suppliers might find it optimal to choose inefficient product characteristics when they face big buyers.

Interestingly, various case studies and empirical data studies contradict each other. A US Federal Trade Commission (FTC 2000, p. 57) report states that when facing increasingly powerful buyers, suppliers respond by under-investing in innovation or production. In a European Commission (EC 1999, p. 4) report, buyer power may force manufacturers "to reduce investments in new products or product improvements, advertising and brand building." Nevertheless, Iacovone et al. (2010), in a study of Wal-Mart in Mexico, show that producers tend to invest more when facing a powerful buyer such as Wal-Mart. By using a unique dataset containing 1,129 observations of German firms from manufacturing and service sectors, Köhler and Rammer (2012, p.16) find that "buyer power will have a more positive effect on suppliers' innovation incentives if price competition in the downstream market is high."

We wonder, are the theories mentioned above missing something? When we consider the buyer power issue, we should not only focus on the large buyer and supplier, but also on the "small" independent buyers. The relative status of a large buyer should be given more attention. Unfortunately, articles considering the role of this relative status are rare.^{2.2}

This chapter presents a theoretical model to show the non-monotonic effects of buyer

^{2.2}Although Inderst and Wey (2011) notice it, they just consider isolated markets. In each market, there are two competing firms. This simplified assumption would make the effect of a merger so limited, it cannot characterize the case of an extremely powerful buyer. Moreover, this assumption ignores the change of the market structure as the concentration increases.

power on a supplier's innovation activities. There is one upstream supplier who can decide whether or not to invest in innovation to reduce marginal costs. In the downstream market, there are several competing buyers (or retailers) who negotiate with the supplier about the price of goods that will finally be sold to consumers.

The supplier's innovation has the following effects. At first, given a certain degree of market concentration, lower marginal costs will increase industry profits. This is naturally good for both sides, buyers and supplier. Furthermore, a decrease in marginal costs will make the buyers' outside option less valuable. In fact, if the negotiations with one buyer break down, then the supplier can sell the lower-marginal-cost goods to his rivals, and the competition the buyer faces will be much fiercer.^{2,3} However, as the market concentration increases, the changes of the innovation effect on both the supplier and buyers' sides are not easy to identify. The innovation effect on total market profit increases in the degree of market concentration, but the acceleration rate will be zero when it becomes a monopoly market. This is the positive effect for the supplier. The innovation effect on the large buyer's outside option value is ambiguous as the market becomes more concentrated. It depends on the relative status of independent buyers and large buyer. In cases where the buyer is not so strong (large), the number of other independent retailers is relatively large in this downstream market. This implies that the buyer will face a more competitive market if the supplier sells the lower-marginal-cost goods to the other unmerged independent buyers. When this not-so-strong buyer merges with another independent retailer, the downstream market structure will not change dramatically, but the not-so-strong buyer's market share will increase substantially^{2,4}. The change of innovation effect on the large buyer's outside option value is mainly presented via the change of the not-so-strong buyer's market share. Then the effect of cost reduction on a buyer's outside option value will increase as the not-so-strong buyer becomes larger. Nevertheless, when this buyer is strong (large), the result will be the opposite. In this case, the downstream market structure experiences a dramatic change. The number of other independent retailers becomes relatively small, thus the sales of the lower-marginal-cost goods through these independent retailers do not have a great impact on the strong buyer. This

^{2,3}Inderst and Wey (2011) have a similar discussion; a buyer that chooses to switch to another supplier will be at an increased disadvantage *vis-a-vis* competing firms the more the supplier has invested to reduce marginal costs.

^{2,4}For instance, the number of stores the not-so-strong retailer owns increases from 1 to 2, his market share increases roughly 100%.

implies the innovation effect becomes weaker as the market becomes more concentrated. This chapter serves to provide antitrust authorities a diversity of perspectives on merger cases. As such, antitrust authorities should consider the relative size of a merger, and verify whether one merger is too powerful to stimulate supplier's innovation.

The rest of the chapter is organized as follows. Section 2.2 presents the model and Section 2.3 analyzes the equilibria of the model. Section 2.4 examines the effects of buyer power on supplier incentives and gives the core results of this chapter. Section 2.5 provides a linear inverse demand function case. Section 2.6 concludes. All details about proofs are provided in the Appendix.

2.2 The Model

Consider an industry consisting only one upstream supplier. At the beginning, the supplier can produce with an old technology at a marginal cost level \bar{c} , and there is no fixed production cost. The supplier decides to invest in innovation in order to reduce the marginal cost from \bar{c} to c , which costs $I = I(\Delta c)$, $\Delta c = \bar{c} - c$, $I' > 0$, $I'' > 0$. After innovation, the supplier can produce with the new technology at a marginal cost level c and sell the products to the downstream market. We assume this new technology will be protected by a patent, no one else can use this new technology.

In the downstream market, there are n identical stores. If m stores merge into one group, we call it "m-retailer". The other $n - m$ stores are independent, and each of them can be considered as an independent retailer. Therefore there are $n - m + 1$ players in the downstream market, denoted by retailer i , where

$$i = \underbrace{1, 2, \dots, n - m}_{n - m \text{ independent retailers}}, \underbrace{n - m + 1}_{m\text{-retailer}}.$$

We label the m-retailer as retailer $n - m + 1$ and the independent retailers as retailer $1, 2, \dots, n - m$. Each independent retailer has only one store, while the m-retailer has m stores. For computational purposes, it is appropriate to consider m as a continuous variable, although it only has an economic meaning when it is a natural number. We assume here that $m \in [1, n]$. Obviously, as m increases, the downstream market becomes more concentrated; its structure changes as the m-retailer grows larger.

The supplier negotiates simultaneously with the $n - m + 1$ downstream retailers. They

negotiate over two-part tariffs $t_i = F_i + v_i q_i$, where v_i is the wholesale price and F_i is the fixed fee paid by retailer i . If the negotiations with the m-retailer break down, the m-retailer can invest, by paying a fixed cost f , in using the old technology for its own production; the marginal cost for the m-retailer in this case is equal to \bar{c} . We also assume for simplicity that an independent retailer is relatively small, it cannot afford the fixed cost f to produce itself. If the negotiations with an independent retailer break down, there is no outside option for the independent retailer. This implies that only the m-retailer has an outside option during the process of negotiation. For convenience, here we consider a situation, in which when a retailer i does not sign a contract with the supplier, the other retailers still sign the contract at the anticipated equilibrium level. This means we assume non-contingent contracts among supplier and retailers. In this case, each retailer i will pay the "all-buyers-active" equilibrium^{2.5} fixed fees, even if the supplier's negotiation with another retailer breaks down.

Cournot competition among these $n - m + 1$ retailers occurs in the final market, i.e. these n stores sell products to final consumers by the inverse demand function:

$$\begin{aligned}
 P &= P(Q) = P\left(\sum_i Q_i\right) \\
 &= P\left(\sum_{i=1}^{n-m} Q_i + Q_{n-m+1}\right) \\
 &= P\left(\sum_{i=1}^{n-m} Q_i + \sum_{j=1}^m Q_{n-m+1,j}\right)
 \end{aligned} \tag{2.1}$$

where $Q_{n-m+1,j}$ is quantity of product sold by each store in the m-retailer.

In order to investigate this model further, we need to make the following standard assumptions:

A1: $P'(Q) < 0$.

A2: $P'(Q) + QP''(Q) < 0$

This means that each downstream firm's reaction curve slopes downward. Equivalently, an increase in its rivals' output lowers firm i 's marginal revenue.

^{2.5} "all-buyers-active" equilibrium means that each retailer signs a contract with the supplier.

$$\mathbf{A3:} \quad 0 \leq 2P''(Q) + QP'''(Q) \leq \frac{Q[P''(Q)]^2}{P'(Q)} + P''(Q)$$

Here $0 \leq 2P''(Q) + QP'''(Q)$ means the marginal effect mentioned in A2 weakly decreases in Q , and $2P''(Q) + QP'''(Q) \leq \frac{Q[P''(Q)]^2}{P'(Q)} + P''(Q)$ has no economic meaning, it just makes the calculation more tractable. Mathematically, it means the convexity (or the elasticity of the slope) of the inverse demand function weakly decreases in Q . We will give an example to show that these assumptions are reasonable.

A4: A unique Cournot equilibrium exists.

A5: All firms have positive market shares at the Cournot equilibrium.

The overall strategic interaction between supplier and retailers can be represented by the following sequential game: in stage 1, the supplier decide how much to invest in innovation in order to reduce the marginal cost. In stage 2, the supplier negotiates simultaneously with the $n - m + 1$ downstream retailers over two-part tariffs. In stage 3, each retailer decides how much to purchase from the supplier, and then resells to the final market, where the Cournot competition occurs.

2.3 Equilibrium

In this section, we characterize the equilibria of the sequential game and derive the supplier's optimal decision of Δc^* . Solving the game backwards, we begin with the Cournot competition in the final market at stage 3, and then move to the process of negotiation at stage 2. We then turn to the supplier's maximization problem at stage 1.

2.3.1 Stage 3: Cournot competition

Here we analyze the game backward, starting at stage 3. Given the wholesale price v_{n-m+1} , the maximization problem for the m -retailer is

$$\max_{Q_{n-m+1,j}} \sum_{j=1}^m Q_{n-m+1,j} [P(Q) - v_{n-m+1}]$$

Since the n downstream stores are identical, we can use symmetry with the maximization problem and write the first order condition as:

$$P(Q) - v_{n-m+1} + mQ_m P'(Q) = 0 \tag{2.2}$$

Q_m is the optimal quantity for each store of the m-retailer.

Given the wholesale price v_i for independent retailer $i = 1, 2, \dots, n - m$, the maximization problem is

$$\max_{Q_i} Q_i [P(Q) - v_i]$$

The first order condition is

$$Q_i P'(Q) + P(Q) - v_i = 0 \quad (2.3)$$

By using the fact that $\sum_{i=1}^{n-m} Q_i + mQ_m = Q$ and equations (2.2) and (2.3), we have

$$(n - m + 1) P(Q) + Q P'(Q) - v_{n-m+1} - \sum_{i=1}^{n-m} v_i = 0 \quad (2.4)$$

Therefore, the aggregate market quantity in Cournot equilibrium is determined by equation (2.4).

2.3.2 Stage 2: Bargaining Solution

In this subsection, we use the Nash Bargaining Solution method to characterize the process of negotiation. In order to avoid double marginalization, we have assumed that the players negotiate over two-part tariffs. In equilibrium, the players then set the wholesale prices equal to marginal cost, i.e. $v_i = c$. The bargain problems reduces to split the realized surplus through the fixed transfer F_i .

We write the Nash Bargaining Solution to the bargaining problem of the m-retailer and the supplier as

$$F_{n-m+1} = \arg \max_{F_{n-m+1}} (\pi_{n-m+1} - F_{n-m+1} - \pi_{n-m+1}^{out}) \left(F_{n-m+1} + \sum_{i=1}^{n-m} F_i - \sum_{i=1}^{n-m} F_i^d \right) \quad (2.5)$$

F_{n-m+1} is the fixed transfer that the supplier can receive from the m-retailer. π_{n-m+1} is the total operating profit of the m-retailer's stores. π_{n-m+1}^{out} is the outside option value for the m-retailer. If the negotiations with the m-retailer break down, the m-retailer can

invest, by paying a fixed cost f , in using the old technology for its own production; the marginal cost for m-retailer in this case is equal to \bar{c} . However the other $n - m$ independent retailers can continue buying products from the supplier at wholesale price c .

$\sum_{i=1}^{n-m} F_i$ is the total transfer that the supplier can receive from the $n - m$ independent retailers. $\sum_{i=1}^{n-m} F_i^d$ is the disagreement point of the independent retailers. Since we have assumed non-contingent contracts and the "all-buyers-active" equilibrium, we know that $\sum_{i=1}^{n-m} F_i = \sum_{i=1}^{n-m} F_i^d$. Solving this bargaining problem above, we have

$$F_{n-m+1} = \frac{\pi_{n-m+1} - \pi_{n-m+1}^{out}}{2} \quad (2.6)$$

We have assumed that there is no outside option for an independent retailer. Similarly we have

$$F_i = \frac{\pi_i}{2}, \forall i = 1, 2, \dots, n - m$$

In order to know the value of π_{n-m+1}^{out} , we need to calculate the so-called "outside" equilibrium. In fact, this conceptual equilibrium plays a very important role in the process of bargaining. The supplier's innovation influences the bargaining positions of both sides through π_{n-m+1}^{out} . In this conceptual equilibrium, we denote the optimal output for each store of the m-retailer by Q_m^{out} , and the aggregate market quantity by Q^{out} . From equation (2.2) and (2.4), Q_m^{out}, Q^{out} are determined by:

$$P(Q^{out}) - \bar{c} + mQ_m^{out} P'(Q^{out}) = 0 \quad (2.7)$$

$$(n - m + 1) P(Q^{out}) + Q^{out} P'(Q^{out}) - \bar{c} - (n - m)(\bar{c} - \Delta c) = 0 \quad (2.8)$$

then the outside option value for the m-retailer is

$$\pi_{n-m+1}^{out} = mQ_m^{out} [(P(Q^{out}) - \bar{c})] - f$$

2.3.3 Stage 1: Supplier's Problem

The total revenue which the supplier receives from $n - m + 1$ downstream retailers is

$$\frac{\pi_{n-m+1} - \pi_{n-m+1}^{out}}{2} + \frac{1}{2} \sum_{i=1}^{n-m} \pi_i = \frac{1}{2} \left[\left(\sum_i \pi_i \right) - \pi_{n-m+1}^{out} \right]$$

the supplier's maximization problem can thus be written as:

$$\max_{\Delta c} \frac{1}{2} \left[\left(\sum_i \pi_i \right) - \pi_{n-m+1}^{out} \right] - I(\Delta c) \quad (2.9)$$

Solving this maximization problem, we can obtain this optimal solution Δc^* . We then finish the analysis of this sequential game.

2.4 The Effects of Buyer Power on Supplier Incentives

The main contribution of this chapter is to show that the optimal solution Δc^* is not monotonic as m increases. In order to do this, it is necessary to prove that the first order derivative of $\left(\sum_i \pi_i \right) - \pi_{n-m+1}^{out}$ with respect to Δc is a non-monotonic function of m .

In fact, $\sum_i \pi_i$ is the total profit in the downstream market, we denote it by π . Then we have the following proposition.

Proposition 2.1 *Under Assumptions A1-A5, we have*

$$\frac{\partial \pi}{\partial \Delta c} = Q \left[\frac{2P'(Q) + QP''(Q)}{(n-m+2)P'(Q) + QP''(Q)} \right] > 0$$

and

$$\frac{\partial \pi_{n-m+1}^{out}}{\partial \Delta c} = - \frac{(n-m)mQ_m^{out} [mQ_m^{out} P''(Q^{out}) + 2P'(Q^{out})]}{(n-m+2)P'(Q^{out}) + Q^{out} P''(Q^{out})} < 0$$

Proof. see Appendix. ■

Proposition 2.1 means that innovation would make the total profit in this industry larger. On the other hand, innovation will weaken the outside option value for the m-retailer, which follows our intuition. This can be easily explained: when the negotiation

with the m-retailer breaks down, the wholesale price for the independent retailers is lower than the m-retailer's marginal cost. Proposition 2.1 directly shows that

$$\frac{\partial}{\partial \Delta c} \left[\left(\sum_i \pi_i \right) - \pi_{n-m+1}^{out} \right] > 0$$

This implies that innovation will increase the total revenue which the supplier receives. Consequently from (2.9), we can understand the trade-off between total innovation benefits and their cost.

From now on, we write $E(Q) = -\frac{QP''(Q)}{P'(Q)}$ for the elasticity of the slope of the inverse demand curve. Mathematically, $E(Q)$ is the convexity of the inverse demand function. From A2, $E(Q) < 1$.

Lemma 2.1 *Under Assumptions A1-A3, we have $E'(Q) \leq 0$.*

Proof.

$$E'(Q) = \frac{-P'(Q)[P''(Q) + QP'''(Q)] + Q[P''(Q)]^2}{[P'(Q)]^2}$$

From A1-A3, we can have the following properties,

$$\begin{aligned} \Rightarrow 2P''(Q) + QP'''(Q) &\leq \frac{Q[P''(Q)]^2}{P'(Q)} + P''(Q) \\ \Rightarrow -P'(Q)[P''(Q) + QP'''(Q)] + Q[P''(Q)]^2 &\leq 0 \\ \Rightarrow E'(Q) &\leq 0. \end{aligned}$$

■

If we write $s_m = \frac{mQ_m^{out}}{Q^{out}}$ for the m-retailer's market share in the conceptual "outside" equilibrium, we get

$$\frac{\partial \pi}{\partial \Delta c} = Q \left[\frac{2 - E(Q)}{(n - m + 2) - E(Q)} \right] \doteq \phi \quad (2.10)$$

$$-\frac{\partial \pi_{n-m+1}^{out}}{\partial \Delta c} = \frac{(n - m) s_m Q^{out} [2 - s_m E(Q^{out})]}{[(n - m + 2) - E(Q^{out})]} \doteq \theta \quad (2.11)$$

Then

$$\frac{\partial (\pi - \pi_{n-m+1}^{out})}{\partial \Delta c} = \phi + \theta \doteq \Omega \quad (2.12)$$

From Proposition 2.1 , we know $\phi > 0, \theta > 0$, for $\forall m \leq n$.

Lemma 2.2 *Under Assumptions A1-A5, if $n > 3$, then we have $\frac{\partial \phi}{\partial m} > 0$ for any m , $\lim_{m \rightarrow 1} \frac{\partial \theta}{\partial m} > 0$; and $\lim_{m \rightarrow n} \frac{\partial \phi}{\partial m} = 0, \lim_{m \rightarrow n} \frac{\partial \theta}{\partial m} = -Q^{out} < 0$. Thus the following claim holds:*

$$\lim_{m \rightarrow 1} \frac{\partial \Omega}{\partial m} > 0, \lim_{m \rightarrow n} \frac{\partial \Omega}{\partial m} < 0, \text{ if } n > 3 \quad (2.13)$$

Proof. see Appendix. ■

One key result from Lemma 2.2 is that the innovation effect on the m-retailer's outside option, θ , is a non-monotonic function of m . In order to see the reasoning behind it^{2,6}, we write

$$\begin{aligned} \frac{\partial \log \theta}{\partial m} &= \frac{\theta'}{\theta} = \frac{\partial}{\partial m} \log s_m + \frac{\partial}{\partial m} \log (n - m) \\ &+ \frac{\partial}{\partial m} \log Q^{out} + \frac{\partial}{\partial m} \log [2 - s_m E(Q^{out})] \\ &- \frac{\partial}{\partial m} \log [(n - m + 2) - E(Q^{out})] \end{aligned}$$

In the case that the m-retailer is not as powerful (m is close to 1 and n is large enough), the rate of change in the m-retailer's market share s_m will be an absolutely large positive value, as the number of merged stores increases from m to $m + 1$. However, the structure of the downstream market does not change much, since the m-retailer is relatively weak in this market. This implies that the terms $\log (n - m)$, $\log Q^{out}$ and $\log [(n - m + 2) - E(Q^{out})]$, which are determined by the structure of the market, do not experience a large rate of change. For the term $2 - s_m E(Q^{out})$, we know that $E(Q^{out}) < 1$ and the value of s_m would be small, then $2 - s_m E(Q^{out})$ does not have a large rate of change either. The positive rate of change in s_m would dominate the sum of all the other terms'. That means the value of $\frac{\theta'}{\theta} > 0$, therefore the innovation effect on m-retailer's outside option increases in m , if m-retailer is not powerful. However, in the case that the m-retailer is very powerful in the downstream market (m is close to n), the negative rate

^{2,6}In this chapter, we just wish to find some sufficient conditions to illustrate this analysis. Therefore we do not consider the case in which $E'(Q) > 0$. It will take plenty of non-economic trivial calculations.

of change^{2.7} in $n - m$ would dominate the sum of all the other terms', so that the value of $\frac{\theta'}{\theta}$ is negative. Hence, the innovation effect on the m-retailer's outside option decreases in m , if m-retailer is extremely powerful.

The other results $\frac{\partial \phi}{\partial m} > 0$ and $\lim_{m \rightarrow n} \frac{\partial \phi}{\partial m} = 0$ in Lemma 2.2 are also very interesting. They are related to the issue of "market concentration and innovation". We find that the innovation effect on total market profit increases in the degree of market concentration, but the acceleration rate will be zero when it becomes a monopoly market.

In sum, Lemma 2.2 shows that the total innovation effect on supplier's profit is a non-monotonic function of m . With the assumption $I'' > 0$, the optimal solution $\Delta \bar{c}^*$ is not monotonic as m increases. Thus, we immediately have the next proposition.

Proposition 2.2 *There is a non-monotonic relationship between buyer power of intermediaries and supplier innovation. More specifically, (i) a larger buyer will have a positive effect on the supplier's innovation incentive if the downstream market is less concentrated, (ii) while a larger buyer will negatively affect a supplier's innovation incentive if the downstream market is extremely concentrated.*

Proposition 2.2 provides antitrust authorities multiple perspectives on merger cases. Antitrust authorities should consider the relative size of merger, and verify whether one merger is too powerful to stimulate a supplier's innovation. Meanwhile, the first part of our results is consistent with the empirical findings in Christian Köhler and Christian Rammer (2012). In other words, when the antitrust authorities evaluate a proposed merger, they should investigate whether it dramatically changes the structure of market.

2.5 Example: Linear Inverse Demand Function

In this section, we will use a linear inverse demand function to investigate this issue. Although Proposition 2.2 shows a non-monotonic effect of buyer power on supplier's incentive, we still have no global insights about what happens when m increases from 1 to n . One mathematical reason is that we need further assumptions of fourth order derivatives of the inverse demand function. Fortunately, if we focus on those inverse

^{2.7}for instance, a change from 2 to 1, or 1 to 0.

demand functions which are usually used in economics, the buyer power effect on the supplier can be explicitly demonstrated.

Here we assume $P = 1 - Q$, $I(\Delta c) = \frac{\eta}{2} (\bar{c} - c)^2$, $\eta > \frac{1}{4}$. From (2.4), we have

$$Q = \frac{(1 + n - m) - v_{n-m+1} - \sum_{i=1}^{n-m} v_i}{2 + n - m}$$

Then from (2.2) and (2.3)

$$Q_m = \frac{(1 + n - m)(1 - v_{n-m+1}) - \sum_{i=1}^{n-m} v_i}{m(2 + n - m)}$$

$$Q_i = \frac{2(1 - v_i) - (1 - v_{n-m+1})}{2 + n - m}, i = 1, 2, \dots, n - m$$

Notice that in equilibrium $v_i = v_{n-m+1} = \bar{c} - \Delta c$, but in the conceptual "outside" equilibrium $v_i = \bar{c} - \Delta c$, for $i = 1, 2, \dots, n - m$; while $v_{n-m+1} = \bar{c}$.

The total profit of the m-retailer is

$$\begin{aligned} \pi_{n-m+1} &= mQ_m(1 - Q - v_{n-m+1}) = (mQ_m)^2 \\ &= \left[\frac{(1 + n - m)(1 - v_{n-m+1}) - \sum_{i=1}^{n-m} v_i}{2 + n - m} \right]^2 \Bigg|_{v_i=v_{n-m+1}=\bar{c}-\Delta c} \\ &= \left(\frac{1 - \bar{c} + \Delta c}{2 + n - m} \right)^2 \end{aligned}$$

and the outside option yields profit

$$\begin{aligned}
\pi_{n-m+1}^{out} &= \left[\frac{(1+n-m)(1-v_{n-m+1}) - \sum_{i=1}^{n-m} v_i}{2+n-m} \right]^2 - f \\
&= \left[\frac{(1+n-m)(1-\bar{c}) - (n-m)(1-\bar{c}+\Delta c)}{2+n-m} \right]^2 - f \\
&= \left[\frac{(1-\bar{c}) - (n-m)\Delta c}{2+n-m} \right]^2 - f
\end{aligned}$$

where $v_i = \bar{c} - \Delta c$, for $i = 1, 2, \dots, n-m$; and $v_{n-m+1} = \bar{c}$.

The profit for each independent store $i = 1, 2, \dots, n-m$ is

$$\begin{aligned}
\pi_i &= Q_i(1-Q-v_i) = (Q_i)^2 \\
&= \left[\frac{2(1-v_i) - (1-v_{n-m+1})}{2+n-m} \right]^2 \Big|_{v_i=v_{n-m+1}=\bar{c}-\Delta c} \\
&= \left(\frac{1-\bar{c}+\Delta c}{2+n-m} \right)^2
\end{aligned}$$

Easily we can have that the total profit $\pi = (1+n-m) \left(\frac{1-\bar{c}+\Delta c}{2+n-m} \right)^2$, then Proposition 2.1 can be demonstrated in the following two inequalities:

$$\frac{\partial \pi}{\partial \Delta c} = \frac{2(1+n-m)}{(2+n-m)^2} (1-\bar{c}+\Delta c) > 0$$

and

$$\frac{\partial \pi_{n-m+1}^{out}}{\partial \Delta c} = -\frac{2(n-m)}{(2+n-m)^2} [(1-\bar{c}) - (n-m)\Delta c] < 0$$

The supplier's problem can be written as

$$\max_{\Delta c} \frac{\pi_{n-m+1} - \pi_{n-m+1}^{out}}{2} + \frac{(n-m)\pi_i}{2} - I(\Delta c)$$

where $I(\Delta c) = \frac{\eta}{2} (\Delta c)^2$. The first order condition is

$$\begin{aligned}
& (n - m + 1) \left(\frac{1 - \bar{c} + \Delta c}{2 + n - m} \right) \left(\frac{1}{2 + n - m} \right) \\
& - \left[\frac{(1 - \bar{c}) - (n - m) \Delta c}{2 + n - m} \right] \left(-\frac{n - m}{2 + n - m} \right) - \eta \Delta c = 0 \\
\Rightarrow \Delta c^* &= \frac{2(n - m) + 1}{(\eta + 1)(n - m)^2 + (4\eta - 1)(n - m + 1)} (1 - \bar{c}) \doteq \Phi(m) (1 - \bar{c})
\end{aligned}$$

Then we just need to observe the function $\Phi(m)$ to understand the properties of optimal solution Δc^* .

$$\Phi'(m) = \frac{2(\eta + 1) [(n - m)^2 + (n - m)] - (4\eta - 1)}{[(\eta + 1)(n - m)^2 + (4\eta - 1)(n - m + 1)]^2}$$

Therefore, $\lim_{m \rightarrow 1} \Phi'(m) > 0$, $\lim_{m \rightarrow n} \Phi'(m) < 0$. It implies that Proposition 2.2 holds in this linear demand function case. The expression of $\Phi'(m)$ also shows that $\Phi'(m) > 0$, if $m \leq n - 1$; and $\Phi'(m) < 0$, if $m = n$. Therefore, in the case of a linear inverse demand function, the optimal investment in cost reduction will fall after rise, as the degree of market concentration increases.

To illustrate our results graphically, we set the key parameter values in the calibration as follows: $n = 3, \eta = 6$.

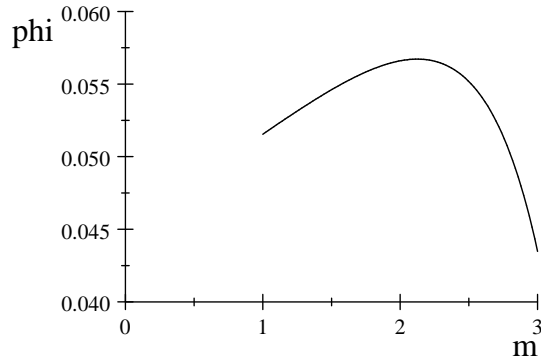


Figure 2.1: A non-monotonic effect of buyer power

2.6 Concluding Remarks

This chapter studies the impact of buyer power on a supplier's incentive to innovate. Rather than focusing only on big merged buyers, we start by examining competition among big a merged buyer and small independent buyers. If a merger cannot change the market structure dramatically, it will have a positive effect on a supplier's innovation incentive; while a merger will negatively affect supplier's innovation if it changes the market structure fundamentally. The buyer's outside option plays a critical role through our analysis. Unlike the other existing papers which simply assume that two firms or stores merge into one, our model explores a gradual transformation in the market structure. Therefore, antitrust authorities can have a diversity of perspectives on horizontal merger control.

In this chapter, we focus on innovation for the purpose of reducing marginal cost. There may be other types of innovation, for instance, quality improvement or the development of new products. We do not claim that our results can be applied in those two cases. How buyer power affects these two incentives is still open for consideration. Nevertheless, it is important to study the relative status of independent buyers and the large buyer.

Finally, there is only one supplier in our model. Is our result still valid in the case of the multiple suppliers? Furthermore, if we use a cooperative game setting instead of Nash Bargaining to revisit this issue, can we get some more fruitful results? All of these questions are worthy of further research.

3 Inventories and International Trade Volatility: The Role of Export and Import Wholesalers

Abstract:^{3.1} The chapter develops a simple theoretical model to examine the role of export and import wholesalers in allocating inventories across countries and over time. Selling internationally through export- and import-wholesalers is shown to be beneficial when demand volatility is high as wholesalers smooth trade and lower inventory levels with respect to direct manufacturing exports. When trade becomes lumpy, we show that the incentives to use intermediaries in general and export-wholesalers in particular are lessened. In addition, depending on product characteristics and trade lumpiness, the use of wholesalers within a distribution supply chain may raise or lower the volume of international trade and social welfare in the destination market.

^{3.1}This chapter is based on a paper that is joint work with Horst Raff and Nicolas Schmitt.

3.1 Introduction

Intermediaries have played an active role in international trade throughout history. Even with today's technologies that facilitate international contacts between buyers and sellers, intermediaries continue to take care of a significant share of transactions across national boundaries in all countries and for most products.^{3.2}

In principle, for any international transaction involving an intermediary, this intermediary could be located in the seller market (an export intermediary), in the buyer market (an import intermediary), or involve an intermediary in both markets. The task of this chapter is to identify circumstances under which it would pay a producer to use either an export or an import intermediary. Moreover, we ask what effects the presence of such an intermediary has on the volume and on the volatility of trade, on the size of trade inventories, and on social welfare.

In this chapter, we define an intermediary as a wholesaler who has title to a product and who buys and sells it on his own behalf earning a trade margin in the process. We ignore ownership issues, and thus a wholesaler can be part of a vertically integrated firm—such as a manufacturer supplying its own wholesale services, or a retailer who has integrated backward into wholesaling—, or be an independent merchant dealing at arm's length with upstream and downstream firms (see Kleinert and Toubal, 2013 on this issue).

The literature has established that, in general, wholesalers can play many different roles in a transaction. They range from allocating goods, providing price and product information, certifying product quality to managing inventory (Spulber, 1999). Depending on the circumstances, each of these roles can be important for any transaction whether it is an international or a domestic transaction. In this chapter, we seek advantages for wholesalers relative to direct exchange that are directly connected to two specific characteristics of international markets. First, buyers (whether wholesalers or other downstream customers) place orders and the producer delivers the goods before the state of the demand is known. Hence, we are interested in international markets characterized by (i) a significant lag between production and consumption, and (ii) uncertainty regarding the state of the demand. Second, international trade also often involves a delivery lag

^{3.2}In Canada for instance, the average share of wholesale trade (NAICS code 41) was 35.2% of the value of imports and 12.7% of the value of exports over the period 1998-2001. This share was 68.4% for imports of Apparel, and 49.4% for exports of farm products (without grain) (Hays, 2005).

(international transportation typically takes time) and is often lumpy.^{3.3} As shown by Alessandria, Kaboski and Midrigan (2010a), these two features matter a great deal for international trade. Moreover, each of them leads to the existence of inventory. Obviously someone must hold inventories (at least for storable products) when there are lags between production and consumption. In this chapter, the candidates for holding inventories are import wholesalers, export wholesalers, or else, when manufacturers directly exports their products, downstream firms in the destination market, such as retailers.

Two short examples may be useful to illustrate these cases. At the end of the 1990s, the US toy market represented nearly half of the world toy market, and it was dominated by two large firms (Mattel and Hasbro) whose production had largely moved to Asia because of low production costs. Moving production far from consumers was not without loss of flexibility in managing supply as production quantities had to be specified months before the holiday season and there were ‘long transit times, custom delays, quota restrictions, and communication barriers [making] managing the supply of product flowing from Asia a challenge’ (Johnson, 2001, p. 118). Moreover, the demand for toys is characterized by a heavy concentration of sales in November and December (45% of US annual sales during that time; Johnson, 2001), a very uncertain success rate as very few toys are typically successful during the Christmas season, and a heavy emphasis on new toys as the most successful ones are rarely so over multiple years. Clearly, these features call less for an emphasis on inventory management across sales periods than for an allocation of inventories across markets to make sure that supply meets demand in each of them. This is precisely what the main toy manufacturers put in place: large wholesaling facilities able to divert products ‘to Europe or the US depending on inventory needs’ (Johnson, p. 120). This example is best associated with the type of export-wholesaling activity that we have in mind in this chapter.

On the import-wholesaling side, consider the case of O’Neill Inc., a US manufacturer

^{3.3}The World Bank (see www.doingbusiness.org/data/exploretopics/trading-across-borders) finds that the combination of container ocean shipping and administrative border requirements leads to an average delivery lag of 10-11 days for exports from and imports to high income OECD countries and a lag of 31-37 days for exports from and imports to South Asian or Sub-Sahara African countries. See also Hummels and Schaur (2013). Arguments have been made that the recent economic crisis has led financially-strapped ocean carriers to cut fuel costs by reducing vessel speeds from 25 to 22 knots, which adds up to three days to the 11-12 days voyage between Asia and North America, for instance (Bonney and Leach, 2010). Crista, et al. (2012) show that ocean shipping is used for 50.2% of world international trade (values), ranging from 28.3% of North American exports to 89.8% of exports from Oceania.

of apparel and accessories for water sports. It also manufactures its products in Asia and like for toys, the lead time for production is long (3 months) also forcing this manufacturer to produce well before the selling season. But it allows for two types of orders from US retailers (Cachon, 2004): one placed well before the selling season (with pre-book discount and delivery guarantee) in which case the retailers manage inventories and bear the risks associated with demand uncertainty; a case of ‘direct exports’ in our terminology. The other type of order can be made on short notice and is honored provided inventory is available in its distribution centre in San Diego. In this case, it is the wholesaling unit which acts as an importer, manages inventories and bears the risk.

This chapter is thus concerned with the role of wholesalers in managing inventory and, specifically, in allocating inventory across markets and across time. We want to identify an advantage for wholesalers in managing inventory relative to downstream firms, say retailers or downstream producers who in principle could also manage inventory.^{3.4} Following the market microstructure literature (Spulber, 1999), our theory of wholesaling is built on the assumption that wholesalers possess market power.^{3.5} In fact, we show that market power creates a role for wholesalers in managing inventories even if they do not possess a superior technology relative to downstream firms.

Specifically, the benefits to producers of using a wholesaler to manage inventories come from the intermediary’s ability to adjust prices once the demand has been revealed (price delegation) and from the fact that the intermediary may have better incentives than downstream firms to allocate goods across markets and across time. In turn whether a manufacturer sells directly to foreign buyers or delegates this task to an import or to an export wholesaler influences the size of these inventories and the volume as well as the volatility of international trade.

We develop our results in a model in which a manufacturer may choose to export directly to competitive downstream firms operating in different countries and across several time periods, knowing that these firms may not sell the entire order they place in

^{3.4}Inventories could, of course, also be held by producers. However, for goods that need to be produced before demand is known, it is more difficult to justify why a producer should rely exclusively on its inventory once the demand is realized. Thus we make a distinction between production and sales but this does not mean we are imposing any particular restriction on the ownership of these two activities.

^{3.5}Spulber (1999, p. xvii) argues that any market microstructure theory requires intermediaries to have at least some market power so that they can set prices and balance supply and demand across time. Providing "immediacy services", that is, standing ready to buy and sell goods at different points in time is seen as a key role of wholesalers, comparable to that of market makers in financial markets.

a given period and thus may keep the unsold portion of their order as inventory for the following period. Alternatively a manufacturer may want to sell to an export wholesaler who then dispatches the goods he has ordered to different destination markets by selling to downstream firms, or else to an import wholesaler located in each destination market who then manages inventories himself rather than letting the downstream firms do it.

Our analysis proceeds in three steps. In a first step, we show that in a simple model with linear demand and additive demand uncertainty it pays a manufacturer to sell through a wholesaler rather than export directly provided there is enough demand volatility. This is not an entirely surprising result insofar as, unlike the manufacturer, a wholesaler is able to adjust wholesale prices and shipments to downstream firms after having learned about the state of the demand even if he takes possession of his order before demand is known. Although using a wholesaler is costly, the wholesaler's informational advantage over the manufacturer is more evident the more volatile the demand is. But this is not enough in itself. The advantage of an import wholesaler over downstream firms is also to behave in a way which is well aligned with the manufacturer's incentives regarding how, in each period, it determines what volume of goods should be sold and thus dispatched to the downstream firms. This is because an import wholesaler faces a downward sloping demand and not, like competitive downstream firms, a perfectly elastic demand.

The advantage of an export wholesaler is similar except that an export wholesaler is particularly well positioned to dispatch products to different destination markets in every period and thus to take advantage of possible differences in demands across destination markets. We find that an export wholesaler has indeed an edge over an import wholesaler when demands across different markets are negatively correlated, while an import wholesaler has an edge if this correlation is positive. In other words it pays to use an export wholesaler when there is an opportunity for spatial arbitrage.

Critically, although in our simple version of the model, the total volume of trade is for all intent and purpose the same whether wholesalers are involved or not, the main impact of wholesalers is to decrease both trade volatility and inventory size with respect to the case where the manufacturer sells directly to foreign downstream firms. Hence, an important outcome of our model is the finding that *wholesalers essentially smooth trade and decrease the need to maintain large inventories.*

In a second step we introduce an international trade lag that comes on top of the

production lag. In other words, suppose that by itself international trade takes time. Not surprisingly this forces anyone receiving import shipments, whether the downstream firms or import wholesalers, to hold larger inventories simply because trade shipments need to cover more than one sales period. As a result trade becomes lumpy; that is, international shipments become both larger when they occur and possibly less regular. More importantly for our purpose, lumpy trade has implications for the manufacturer's choice of foreign trade partners. We show that it decreases the manufacturer's incentives to channel trade through a wholesaler, whether it is an import or an export wholesaler. However, when a manufacturer still chooses to channel trade through a wholesaler, lumpy trade makes the use of an import wholesaler much more desirable.

Both results come from the same source: shipments needed to cover multiple sales periods necessarily reduce a wholesaler's relative advantage to exploit information about demands, whether this wholesaler imports or exports. However this is particularly problematic for an export wholesaler because, unlike an import wholesaler, lumpy trade forces an export wholesaler to let the downstream firms play an active role in the management of inventories. This is not desirable from the manufacturer's point of view. Thus, with lumpy trade, it is the import wholesaler's ability to retain control of the management of inventories that makes it particularly valuable to the manufacturer.

Clearly, our simple model with linear demands and additive demand uncertainty produces interesting results not only about the general role of wholesalers in international trade, but especially about the different roles of import and export wholesalers. But it does not produce very interesting welfare results. This is because, regardless of the channel of international trade, the total expected volume of trade remains essentially the same.

In a third step we therefore generalize the model in order to evaluate the effect of import wholesalers on the volume of trade and social welfare. There are two ways to address these issues: one is to keep the same type of demand uncertainty (additive) and to consider more general demands. The other is to consider multiplicative demand uncertainty. In the first case, we derive sufficient conditions regarding demand under which an import wholesaler reduces expected social welfare in the destination countries when trade is non-lumpy. Essentially under these conditions total trade is lower when import wholesalers are involved than when they are not. Interestingly, when trade is lumpy the same conditions produce the opposite result: the expected trade volume and

social welfare rise with import wholesalers relative to direct trade.

In the second case, multiplicative demand uncertainty can be viewed as uncertainty about consumers' willingness to pay. Thus there is uncertainty about consumers tastes and about the distribution of tastes across markets. In this case, we show that linear demand is a sufficient condition for social welfare to *decrease* with the presence of an import wholesaler even if trade is lumpy.

These social welfare results show that, depending on the type of product that is traded (which depends on the type of demand and on the source of the uncertainty), a manufacturer's private incentive regarding the organization of international sales can be aligned with society's interests or not. In our model, this reflects an important trade-off associated with intermediation between improving the allocation of goods and inventories across markets or time and generating inefficiencies through market power.

It should be clear that the type of wholesalers we are dealing with in this chapter are merchants, not agents or brokers, as they own the product they trade and manage inventories. Obviously intermediaries have other important roles to play in international markets other than managing inventory. Search and matching are obvious ones (see Antras and Costinot, 2011; Blum, Claro and Horstmann, 2012). However, search and matching do not require the presence of merchants and thus of intermediaries owning the products and managing inventories. Indeed being an agent or a broker would suffice. It is why search and matching are not part of our model.

There is now evidence that inventories, whoever holds them, are larger for products that are traded internationally and that they strongly influence the dynamics of international trade flows. Alessandria et al. (2010a), for instance, report detailed observations analyzed by Hall and Rust (2000) regarding a US wholesaler buying and selling 2200 products in the steel industry.^{3.6} They report that, on average, 'international orders tend to be about 50 percent larger and occur nearly half as frequently as domestic orders' (p. 2310). This has a direct impact on the level of inventories which are larger for foreign products than for domestic ones. These observations are very much consistent with the presence of lags and lumpy shipments associated with international trade and with the role of wholesalers in providing 'immediacy services'.

^{3.6}There were about 5000 wholesalers in the US in this industry in 1998-99 holding 7-8 million tons of steel as inventory and shipping 29 million tons out of the 127 million tons of steel consumed in the US making these firms the largest customer group of the steel mills (Hall and Rust, 2000, p175).

Intermediaries' inventory decisions are critically important for the understanding of the dynamics of international trade flows. Altomonte et al. (2012) argue that inventory decisions within international supply chains that include a manufacturer, a wholesaler and foreign retailers, are key to understand the trade dynamics during the great trade collapse of 2008-09 and subsequent recovery.^{3.7} They claim in particular that rapid trade collapses and recoveries can, on average, be associated with a supply chain belonging to a multinational and its affiliates. This is because having a common flow of information about the state of final demand allows for lower levels of inventory at each link along the supply chain with respect to those where firms deal at arm's length. In this last case, the so-called 'bullwhip effect' (Forrester, 1961) is more likely to occur leading demand shocks at the consumer level to translate into greater demand variability and larger inventories as one moves upstream within the supply chain.

One interpretation of our results is that the extent of a supply chain, that is how many links it has between production and consumption, does influence trade flows and their timing because it will influence who manages inventories and this in itself has an impact on the dynamics of international trade flows irrespective of the ownership structure of a supply chain.

To our knowledge, there is no paper in international trade attempting to distinguish export and import wholesalers. There is, however, a recent and rapidly growing literature on intermediation in international trade.^{3.8} This literature has focused its attention on three different aspects of intermediation. First, there is a literature providing evidence on the importance of intermediation in international markets. This is the case for the US (Bernard, Jensen and Schott, 2009; Bernard, Jensen, Redding and Schott, 2010a, 2010b), for Italy (Bernard, Grazi and Tomasi, 2011), for Chile (Blum, Claro and Horstmann, 2010) and for Hong Kong (Feenstra and Hanson, 2004) among others. Second, there is a literature investigating direct versus indirect exports (Ahn, Khandelwal and Wei, 2011; Akerman, 2011; Blum, Claro and Horstmann, 2012; Crozet, Lalanne and Poncet, 2013; Felbermayr and Jung, 2011; Krautheim, 2013; Schroeder, Trabold and Trueswetter,

^{3.7}See also Alessandria et al. (2010b) on the great trade collapse and inventories.

^{3.8}There is also a recent literature placing a large emphasis on the role of inventories to explain international trade flows, such as during the great trade collapse, and showing that importers hold more inventories stocks than non-importers (Alessandria, Kaboski and Midrigan, 2014, 2011, 2010a,b; Altomonte et al., 2012). Although intermediaries play a role in this literature, they are not the primary focus.

2005 among others). These papers typically assume that intermediaries are located in the export market.^{3.9} Third, there is a literature where producers have no choice but must use intermediaries to reach consumers. The location of these intermediaries is also generally taken as given (Antras and Costinot, 2011; Raff and Schmitt, 2005, 2006, 2007, 2009, 2012, 2013).^{3.10}

The chapter is organized as follows. In Section 3.2 we present a simple model featuring linear demand and additive demand shocks. In Section 3.3 we solve for the equilibrium of this model assuming that there is a time lag between production and consumption but no time lag in trade. We study the role of import and export wholesalers and examine the connection between inventories and the volatility of international trade. We also show how intermediaries may reduce trade volatility. In Section 3.4 we examine how the equilibrium of this simple model changes when, in addition to a production lag, we have a trade lag and lumpy trade. In particular, we show how trade lumpiness affects the relative merits of import and export wholesalers and the merit of intermediation itself. In Section 3.5 we introduce a more general model to identify sufficient conditions under which intermediation may raise or lower the volume of trade and social welfare. Section 3.6 contains conclusions, and the Appendix collects the proofs of our propositions.

3.2 A Simple Model

Consider an upstream producer (hereafter, called the manufacturer) who supplies his product to a continuum of perfectly competitive downstream firms in two identical countries, denoted α and β . Downstream firms—either manufacturers purchasing an in-

^{3.9}An exception is Blum, Claro and Horstmann (2012) which looks at a matching data set between exporters in one country (Chile, Argentina) and importers in another country (Columbia, Chile).

^{3.10}There is also a long literature in management science looking at a variety of theoretical aspects linked to demand uncertainty in distribution channels, intermediation and inventories. Some aspects have similarities with our concerns, especially issues such as who should hold inventories and own products once the demand uncertainty is resolved, the strategic use of inventories, new internet-based distribution channels such as drop shipping allowing wholesalers to ship products directly to consumers at the retailer's request, shipments to replenish retailers' inventories decided and managed by an upstream firm and not by retailers, the management of risk within a supply chain, and the role of intermediation when the environment changes. See Belavina and Girotra (2012); Bialogorsky and Koenigsberg (2010), Chen and Gavirneni (2010) and references herein for recent examples. However this literature has a very different focus with respect to this chapter since it deals mainly with issues linked to management strategies. As a result it mostly deals with static models and does not offer insights on international trade issues and social welfare.

intermediate good or retailers purchasing a final good—in turn sell to consumers. The manufacturer may sell directly to the downstream firms or sell goods to wholesalers that then resell to the downstream firms. All market participants are risk neutral.

The markets we consider have three important features. First, final demand in the two countries is random. Second, orders have to be placed and goods have to be produced before demand is known. This time lag between production and consumption, in combination with demand uncertainty, gives rise to inventory. Third, in addition to the production lag there may also be a time lag involved in shipping goods between the origin and the destination countries α and β . A sufficiently long time lag makes trade lumpy providing an additional reason for keeping inventory.

We capture the dynamics of inventory holding by assuming that there are two periods, $t = 1, 2$, in which production and consumption take place. Production takes place at the beginning of each period before demand is known. Consumption occurs at the end of each period. The production lag thus happens within each period. In addition there may be a trade lag, in which case goods may only be shipped at the beginning of period 1 for consumption in periods 1 and 2; this is the case of lumpy trade. When goods can be shipped each period, we call this non-lumpy trade.

Demand in country $i = \alpha, \beta$ at time $t = 1, 2$ is given by the linear inverse demand function: $p_{it} = A - s_{it} + \varepsilon_{it}$, where s_{it} denotes final sales and p_{it} is the consumer price. The random variables $\varepsilon_{it} \in [-d, d]$ are intertemporally independent, but in each period may be correlated across the two countries. In particular, for $t = 1, 2$, let $f(\varepsilon_{\alpha t}, \varepsilon_{\beta t})$ denote the joint density function and assume that the marginal densities are uniform so that $f_{\alpha}(\varepsilon_{\alpha t}) = \int_{-d}^d f(\varepsilon_{\alpha t}, \varepsilon_{\beta t}) d\varepsilon_{\beta t} \equiv \frac{1}{2d}$ and $f_{\beta}(\varepsilon_{\beta t}) = \int_{-d}^d f(\varepsilon_{\alpha t}, \varepsilon_{\beta t}) d\varepsilon_{\alpha t} \equiv \frac{1}{2d}$. Furthermore, suppose that $f_t(\varepsilon_{\alpha t}, \varepsilon_{\beta t}) \neq f_{\alpha}(\varepsilon_{\alpha t}) f_{\beta}(\varepsilon_{\beta t})$, and let the correlation coefficient between $\varepsilon_{\alpha t}$ and $\varepsilon_{\beta t}$ be ρ for $t = 1, 2$. Markets are spatially and intertemporally segmented so that there is no consumer arbitrage across space or time. We also rule out parallel trade across countries by downstream firms.

Our assumptions about the production and distribution technologies are as simple as possible. The manufacturer incurs a constant unit cost of production c , and wholesaling involves a per-unit cost c_w . The marginal production cost of downstream firms is normalized to zero, as is the trade cost and the cost of holding inventory. There is no discounting.

In order to simplify the analysis even further, we make the additional assumption

that the demand shock is not too big:

$$d \leq \min \left\{ \frac{2(c + c_w)}{3}, \frac{A - c}{4}, \frac{A - (c + c_w)}{2} \right\}. \quad (\text{A3.1})$$

This assumption rules out situations in which demand is (i) so high that there are stock-outs in equilibrium, (ii) so low that when competitive firms unload goods at the end of period 2 the consumer price drops to zero, and (iii) so low in period 1 that trade in period 2 collapses. We discuss later what happens when we drop some of these assumptions.

We let the manufacturer set a two-part tariff, consisting of a producer price and a fixed payment or transfer. The notation we use is P_{it} for the producer price and T_{it} for the transfer, with $i = \alpha, \beta$ and $t = 1, 2$. The two-part tariff rules out any double marginalization if the manufacturer sells through wholesalers. Wholesalers charge a wholesale price w_{it} ; this is without loss of generality, as they would not be able to demand a positive fixed payment from competitive downstream firms that earn zero profits in equilibrium.

Next we describe the timing of events. We begin with the case in which the manufacturer deals directly with the downstream firms (and hence does not use a wholesaler). If trade is non-lumpy, then at the beginning of period 1 the manufacturer announces a the two-part tariff (P_{i1}, T_{i1}) . Downstream firms in country i order and take possession of q_{i1} units of goods before demand in period 1 is known; then period-one demand is revealed and the downstream firms sell $s_{i1} \leq q_{i1}$ in period 1, holding unsold units as inventory for period 2. In period 2, the manufacturer sets (P_{i2}, T_{i2}) , and downstream firms order quantity q_{i2} , again before period-two demand is known. Demand in period 2 is then revealed and downstream firms sell $s_{i2} \leq q_{i2} + (q_{i1} - s_{i1})$. If trade is lumpy, then downstream firms may not reorder goods in period 2, but instead order a quantity Q_i in period 1 and sell in period 2 whatever inventory is left over from period 1 so that $s_{i2} \leq Q_i - s_{i1}$.

Now consider the potential role of wholesalers in improving the allocation of goods across countries and across periods. To distinguish very precisely between export and import wholesalers we give them distinct roles: an export wholesaler allocates goods across export destinations; an import wholesaler allocates goods across time in a given destination market. An export wholesaler thus is a firm that takes possession of goods in the country of origin and allocates them across the two countries but not across

periods. Hence, an export wholesaler does not hold any inventory. By contrast an import wholesaler is a firm that takes possession of goods in the country of destination. There it may allocate goods across periods by holding inventory. But it does not allocate goods across countries.^{3.11} Both types of wholesalers, just like downstream firms, take possession of goods before the state of demand in a period is revealed. But unlike downstream firms an export wholesaler is able to allocate goods across countries once demand has become known. An import wholesaler, like downstream firms, may keep inventory and thus allocate goods across periods once demand has been revealed.

Recall that the manufacturer sets a two-part tariff and it thus able to extract wholesalers' entire profit. The manufacturer has therefore no incentive to use more than one export wholesaler and, in the case of import wholesalers, more than one in each destination country.

The timing in the case of an export wholesaler and non-lumpy trade is as follows. In period 1 after observing the two-part tariff (P_1, T_1) the export wholesaler orders and takes possession of quantity q_1 . After demand has been revealed, he sets wholesale prices $w_{\alpha 1}$ and $w_{\beta 1}$ to allocate q_1 across countries α and β . Downstream firms then sell output to consumers, where it has to be the case that $s_{\alpha 1} + s_{\beta 1} \leq q_1$. In period 2, the export wholesaler reorders quantity q_2 at the producer tariff (P_2, T_2) and resells it to downstream firms at wholesale prices $w_{\alpha 2}$ and $w_{\beta 2}$ so that $s_{\alpha 2} + s_{\beta 2} \leq q_2$. In the case of lumpy trade, the export wholesaler, having ordered Q_1 in period 1 can ship goods only once after observing demand in period 1. That is, he ships Q_{i1} to country i at wholesale price w_i , where $Q_{\alpha 1} + Q_{\beta 1} \leq Q_1$. Downstream firms in country $i = \alpha, \beta$ then allocate inventory across periods such that their sales satisfy $s_{i1} + s_{i2} \leq Q_{i1}$.

An import wholesaler takes over from the downstream firms the inventory holding across periods; the downstream firms sell in each period whatever they buy from the wholesaler in that period. Thus, at the beginning of period 1 before demand is known the import wholesaler in country i orders and takes possession of quantity q_{i1} paying the producer tariff (P_{i1}, T_{i1}) ; then demand in period 1 is revealed and the wholesaler sets wholesale price w_{i1} , and downstream firms realize sales of $s_{i1} \leq q_{i1}$. The import wholesaler

^{3.11}In other words, it is the ability of allocating goods across time that primarily characterizes our import wholesaler. It does not mean that an import wholesaler could not spatially allocate goods too, but that this role is secondary and thus ignored here. The same is true for our export wholesaler: the spatial allocation of goods is seen as the main role and the allocation across time is secondary, and thus ignored in our analysis.

may reorder goods in period 2 at the producer tariff (P_{i2}, T_{i2}) . Denote this quantity by q_{i2} . Finally demand in period 2 is revealed and the wholesaler sets wholesale price w_{i2} so that the downstream firms sell $s_{i2} \leq q_{i2} + (q_{i1} - s_{i1})$. In the case of lumpy trade, the import wholesaler in country i orders a quantity Q_i in period 1 before demand in that period is known. After period-1 demand is revealed, the import wholesaler determines how much of this quantity to sell to downstream firms in period 1 at wholesale price w_{i1} and how much inventory to keep for period 2. The downstream firms' sales have to satisfy $s_{i1} + s_{i2} \leq Q_i$.

In all the cases analyzed below, we assume that the total trade flow over both periods is strictly equal to total sales. In other words, all the units shipped to a country are consumed by the end of period 2.

In the next section we derive the equilibrium of this simple model for the case in which there is only a production lag but no trade lag so that trade is non-lumpy. In Section 3.4 we consider the case of lumpy trade. Note that our simple model with linear demand and an additive demand shock has a feature that makes it particularly suitable to identify circumstances under which manufacturers will export directly, use an export wholesaler, or use import wholesalers: trade lumpiness has no and intermediation only a trivial effect on the volume of trade and welfare. Such effects do, however, exist in a more general model, to which we turn in Section 3.5.

3.3 Equilibrium with Non-Lumpy Trade

3.3.1 Direct Exports

Competitive firms are price-takers and in equilibrium will order goods until their expected profit is zero. In both countries, the manufacturer therefore charges a zero fixed fee $T_t = 0$ in each period $t = 1, 2$ (we omit the subscript i for now). Consider then first what happens in period 2 in one of the countries. After observing demand the competitive firms will sell all of the products on hand, and hence $s_2 = q_2 + q_1 - s_1$. The competitive downstream firms order goods before the demand shock is realized; given a producer price P_2 , their expected profit is zero when the quantity ordered, q_2 , satisfies:

$$E_2 (A - s_2 + \varepsilon_2) s_2 - P_2 q_2 = 0.$$

Given this demand from the downstream firms, the expected manufacturer profit in period 2 is:

$$\begin{aligned} E_2(\pi_2) &= P_2 q_2 - c q_2 \\ &= E_2(A - s_2 + \varepsilon_2) s_2 - c(s_1 + s_2 - q_1). \end{aligned}$$

The profit maximizing expected sales volume is hence $s_2 = \frac{A-c}{2}$, and the expected consumer price is $E_2(p_2) = A - s_2 = \frac{A+c}{2}$.

In period 1, after ε_1 has been revealed, competitive firms have to decide how much to sell and how much inventory to keep for period 2. Being price takers they will sell as long as the price in period 1 exceeds the expected price in period 2, and otherwise hold goods in inventory for period 2. In equilibrium, the price in period 1 thus satisfies $p_1 = E_2(p_2) = \frac{A+c}{2}$.^{3.12} The first-period price thus satisfies

$$A - s_1 + \varepsilon_1 = \frac{A+c}{2},$$

and therefore $s_1 = \frac{A-c}{2} + \varepsilon_1$.

Since the leftover can be stored for next selling period at no cost, it is optimal for the competitive firms to avoid any possibility of stock-outs by ordering $q_1 = \frac{A-c}{2} + d$ in the first period and $q_2 = s_2 - (q_1 - s_1)$ in period 2. With $s_2 = \frac{A-c}{2}$, the order in period 2 is $q_2 = \frac{A-c}{2} + \varepsilon_1 - d$ and the total order over both periods is $q_1 + q_2 = A - c + \varepsilon_1$. These orders of course also correspond to international trade volumes.

In period 1 the competitive downstream firms expect to earn a total revenue equal to $\int_{-d}^d \frac{A+c}{2} (A - c + \varepsilon_1) \frac{1}{2d} d\varepsilon_1$. They order an overall quantity such that their expected profit is zero:

$$\int_{-d}^d \frac{A+c}{2} (A - c + \varepsilon_1) \frac{1}{2d} d\varepsilon_1 - P_1 q_1 - P_2 q_2 = 0.$$

^{3.12}Note that, if all of the other competitive firms behave in this way, the best response is also to follow this strategy for any given individual competitive firm.

In period 1 the expected manufacturer profit can hence be written as:

$$\begin{aligned}
& P_1 q_1 + P_2 q_2 - c(q_1 + q_2) \\
&= \int_{-d}^d \left(\frac{A+c}{2} - c \right) (A - c + \varepsilon_1) \frac{1}{2d} d\varepsilon_1 \\
&= \frac{(A-c)^2}{2}
\end{aligned}$$

Summing this profit over both destination markets we obtain a total expected manufacturer profit from direct trade of

$$E(\pi_{direct}^{nl}) = (A - c)^2. \quad (3.1)$$

The total expected volume of exports to the two countries is equal to $2(A - c)$ but what is interesting is less the total expected volume of trade than the trade volatility implied by the model. In other words, we would like to examine how the actual volume of trade reacts to demand shocks, and thus how volatility in final demand translates into volatility in trade.

The key to understanding this connection is to consider how much inventory downstream firms hold in period 1 after the shock ε_1 has been realized and the quantity s_1 has been sold. Denoting this inventory level by $I_1^{direct} \equiv q_1 - s_1$, we have

$$I_1^{direct} = d - \varepsilon_1. \quad (3.2)$$

Exports in period 2 are equal to $q_2^{direct} = s_2 - I_1^{direct}$ so that

$$q_2^{direct} = \frac{A-c}{2} - (d - \varepsilon_1). \quad (3.3)$$

Thus a demand shock ε_1 in period 1 leads to a shock to inventory, and the latter directly turns into a shock to the volume of trade in the subsequent period.^{3.13} From (3.3) we can

^{3.13}It is easy to see that this argument does not depend on having only a two-period model. Even in a setting with more periods downstream firms have an incentive to hold inventory so as to equalize prices across time. Thus a demand shock in one period will affect trade in the subsequent period in the same way.

compute the variance of trade which is equal to

$$\sigma_{q_2}^{direct} = \frac{d^2}{3}. \quad (3.4)$$

It is important to keep in mind that the equilibrium level of inventory is determined by downstream firms' incentive to hold inventory so as to equalize the observed first-period price to the expected second-period price. As we shall see below, wholesalers face different economic incentives to hold inventory than downstream firms and the volatility of trade will hence be different in the case of import or export wholesalers.

3.3.2 Import Wholesalers

Suppose that trade is intermediated by import wholesalers, one in each destination country. We consider one of the countries and drop the country subscript again. Consider the behavior of the competitive downstream firms in period 2. After demand has been revealed and, given a wholesale price w_2 , the competitive firms earn zero profit when they order from the wholesaler and then sell an amount s_2 satisfying $(A - s_2 + \varepsilon_2) s_2 - w_2 s_2 = 0$. Therefore the expected revenue of the import wholesaler in period 2 is

$$E_2 [(A - s_2 + \varepsilon_2) s_2] = (A - s_2) s_2, \quad (3.5)$$

and the expected marginal revenue is

$$E(MR_2) = A - 2s_2. \quad (3.6)$$

To determine the wholesaler's optimal sales, assume for the moment that the manufacturer charges the same producer price in both periods: $P_1 = P_2 = P$; we argue below that this is indeed optimal for the manufacturer. Expected second-period sales are then profit-maximizing for the import wholesaler when his expected marginal revenue is equal to his marginal cost, which is given by the sum of producer price and the marginal wholesale cost: $P + c_w$. The optimal level of sales hence is $s_2 = (A - (P + c_w)) / 2$.

In period 1, after ε_1 has been revealed, the import wholesaler sets a wholesale price w_1 and competitive firms purchase and sell quantity s_1 , such that $(A - s_1 + \varepsilon_1) s_1 - w_1 s_1 = 0$.

The wholesaler's revenue hence is equal to

$$R_1 = (A - s_1 + \varepsilon_1) s_1, \quad (3.7)$$

and the corresponding marginal revenue is

$$MR_1 = A - 2s_1 + \varepsilon_1. \quad (3.8)$$

Equating this marginal revenue with $P + c_w$ yields as optimal period 1 sales $s_1 = (A - (P + c_w) + \varepsilon_1) / 2$. In order to rule out the possibility of a "stock-out", the import wholesaler will order $q_1 = (A - (P + c_w) + d) / 2$ in period 1 and top this up with an expected order of $q_2 = s_2 - (q_1 - s_1)$ in period 2.

Finally consider the manufacturer's problem of choosing P and T . The manufacturer can set T so as to extract the wholesaler's entire expected profit. It is then obviously optimal for the manufacture to set $P = c$ so as to avoid any double-marginalization problem. This can be formally shown by finding first the total expected profit of the wholesaler for a given producer price P and transfer T to the manufacturer:

$$\begin{aligned} & \underbrace{\int_{-d}^d \left(A - \frac{A - (P + c_w) + \varepsilon_1}{2} + \varepsilon_1 - (P + c_w) \right) \left(\frac{A - (P + c_w) + \varepsilon_1}{2} \right) \frac{1}{2d} d\varepsilon_1}_{\text{expected revenue in period t=1}} \\ & + \underbrace{\left(A - \frac{A - (P + c_w)}{2} - (P + c_w) \right) \left(\frac{A - (P + c_w)}{2} \right)}_{\text{expected revenue in period t=2}} - T \\ & = \frac{[A - (P + c_w)]^2}{2} + \frac{d^2}{12} - T; \end{aligned}$$

and then by solving

$$\begin{aligned} & \max_P \int_{-d}^{+d} (P - c) \left[A - (P + c_w) + \frac{\varepsilon_1}{2} \right] \frac{1}{2d} d\varepsilon_1 + \frac{[A - (P + c_w)]^2}{2} + \frac{d^2}{12}; \\ & \max_P (P - c) [A - (P + c_w)] + \frac{[A - (P + c_w)]^2}{2} + \frac{d^2}{12}. \end{aligned}$$

With $P = c$, the total expected volume of exports to the two countries is simply

$2[A - (c + c_w)]$. This means that, with respect to direct exports, the volume of trade is only trivially affected by having import wholesalers. This is of course a feature of our simple model, a feature that will remain true in the presence of export wholesalers or of lumpy trade. However this does not imply that intermediation has only a trivial impact on the manufacturer's expected profit. Indeed the manufacturer's expected profit when trading with the help of an import wholesaler in each of the two destination countries is given by:

$$E(\pi_{im}^{nl}) = [A - (c + c_w)]^2 + \frac{d^2}{6}. \quad (3.9)$$

Using an import wholesaler to intermediate trade has the obvious drawback that the wholesaler has to be compensated in equilibrium for the marginal wholesale cost. This resource cost of intermediation, c_w per unit, has to be weighed against the benefit of intermediation. As compared to direct exports, this benefit consists of the import wholesaler's ability to adjust optimally the quantity sold and hence the markup in period 1 after demand has been revealed. The gross benefit of using a wholesaler, $d^2/6$, will exceed the total resource cost of intermediation if the variance of sales and hence d is sufficiently large.

The inventory held by an import wholesaler in period 1 is given by $I_1^{im} \equiv q_1 - s_1$ and thus by

$$I_1^{im} = \frac{1}{2}(d - \varepsilon_1), \quad (3.10)$$

which is only half as big as the inventory held by downstream firms in the case of direct exports. This is the outcome of two effects. One is that the first-period order is smaller when it is done by an import wholesaler than under direct exports. This is of course due to the additional resource cost associated with the import wholesaler but this is still the case even in the absence of this resource cost. The second effect is that the demand uncertainty has half the marginal impact on sales when they are intermediated by an import wholesaler than in the case of direct exports. In other words, in the best case scenario, the entire order is sold in the first period and these sales are less than they are under direct exports, while in the worst case scenario, first period sales are greater than under direct exports (at least when c_w is low enough). This necessarily results in a lower inventory at the end of the first period. These effects are chiefly due to the fact that an import wholesaler equalizes marginal revenues across periods, not prices.

Obviously, this has an impact on the second-period sales and on trade volatility. With an import wholesaler, the trade volume in period 2 is equal to $q_2^{im} = s_2 - I_1^{im}$, or

$$q_2^{im} = \frac{A - (c + c_w)}{2} - \frac{1}{2}(d - \varepsilon_1). \quad (3.11)$$

Thus the variance of trade when it is intermediated by an import wholesaler is

$$\sigma_{q_2}^{im} = \frac{d^2}{12} = \frac{\sigma_{q_2}^{direct}}{4}, \quad (3.12)$$

and thus only a quarter as high as the variance in case of direct exports.

The reason for the lower volatility of trade in the case of import wholesalers is of course the same as for having a lower level of inventory: an import wholesaler equalizes marginal revenues across periods. Hence, since overall trade is the same under direct exports and under import intermediation (at least when $c_w = 0$), an import wholesaler is able to smooth trade, and to bring smaller variations in sales and inventories than competitive downstream firms.

3.3.3 Export Wholesaler

Since we have assumed that an export wholesaler allocates goods spatially but not intertemporally, he will allocate all the products in his possession after the demand shocks have been resolved in each period. There is thus no inventory and analyzing this scenario is equivalent to considering twice a one-period case. So we omit the time subscript t for the moment.

Consider an export wholesaler who has taken delivery of a quantity of goods, q , and has to set wholesale prices so as to allocate this quantity among the two countries such that $s_\alpha + s_\beta = q$. Facing a wholesale price w_i competitive downstream firms in country $i = \alpha, \beta$ will order and sell output until profit is equal to zero: $(A - s_i + \varepsilon_i) s_i - w_i s_i = 0$. The wholesaler hence earns a total revenue in the two countries of

$$R = (A - s_\alpha + \varepsilon_\alpha) s_\alpha + (A - s_\beta + \varepsilon_\beta) s_\beta. \quad (3.13)$$

To maximize this revenue, the wholesale prices should be set so as to equalize marginal

revenues across the two countries:

$$A - 2s_\alpha + \varepsilon_\alpha = A - 2s_\beta + \varepsilon_\beta.$$

Together with the condition $s_\alpha + s_\beta = q$, this implies

$$s_\alpha = \frac{2q + (\varepsilon_\alpha - \varepsilon_\beta)}{4}, \quad s_\beta = \frac{2q - (\varepsilon_\alpha - \varepsilon_\beta)}{4}.$$

Using these values in (3.13) we obtain

$$R = \frac{8Aq + 4q(\varepsilon_\alpha + \varepsilon_\beta) - 4q^2 + (\varepsilon_\alpha - \varepsilon_\beta)^2}{8}.$$

Given a producer price P and transfer T , the export wholesaler's problem is to choose q to maximize

$$\int_{-d}^d \int_{-d}^d \frac{8Aq + 4q(\varepsilon_\alpha + \varepsilon_\beta) - 4q^2 + (\varepsilon_\alpha - \varepsilon_\beta)^2}{8} f(\varepsilon_\alpha, \varepsilon_\beta) d\varepsilon_\alpha d\varepsilon_\beta - q(P + c_w) - T.$$

The first-order condition for this problem yields

$$q = A - (P + c_w),$$

and the expected profit of the export wholesaler is

$$\begin{aligned} & \frac{[A - (P + c_w)]^2}{2} + \int_{-d}^d \int_{-d}^d \frac{(\varepsilon_\alpha - \varepsilon_\beta)^2}{8} f(\varepsilon_\alpha, \varepsilon_\beta) d\varepsilon_\alpha d\varepsilon_\beta - T \\ &= \frac{[A - (P + c_w)]^2}{2} + \frac{d^2}{12} (1 - \rho) - T. \end{aligned}$$

Finally the manufacturer has to set P and T , and it is straightforward to see that it is optimal for him to set $P = c$ and to extract the wholesaler's profit by charging a transfer

$$T = \frac{[A - (c + c_w)]^2}{2} + \frac{d^2}{12} (1 - \rho).$$

Summing across both periods, the total manufacturer profit when exporting through an

export wholesaler is therefore

$$E(\pi_{ex}^{nl}) = [A - (c + c_w)]^2 + \frac{d^2}{6}(1 - \rho). \quad (3.14)$$

The potential advantage of using an export wholesaler clearly consists of his ability to allocate goods across the two countries after demand has been realized. This advantage, specifically $d^2(1 - \rho)/6$, is positive as long as demands across the two countries are not perfectly correlated so that $\rho < 1$. And for $\rho < 1$ the benefit of using an export wholesaler compared to direct exports exceeds the resource cost of intermediation if the variance of demand is sufficiently large.

Since the export wholesaler does not hold inventory, the trade volatility in period 2 cannot depend on the demand shocks in period 1 and thus $\sigma_{q_2}^{ex} = 0$: international trade is completely shielded from demand shocks.

3.3.4 Intermediation and Trade Volatility with Non-Lumpy Trade

We can now summarize our findings regarding the manufacturer's choice of trade channel and of the effects of this trade channel on inventory and trade volatility. Our first result deals with inventory and trade volatility. We find:

Proposition 3.1 *Trade is more volatile and inventory associated with international trade is larger in the case of direct exports than when trade is intermediated by an import or an export wholesaler.*

Given that wholesalers reduce trade volatility, it is natural to ask whether the manufacturer will find it advantageous to use wholesalers to that effect. In other words, is it in the interest of the manufacturer to use wholesalers when trade would otherwise be more volatile. The answer is provided in the following proposition:

Proposition 3.2 *The manufacturer chooses to sell through a wholesaler when the variance of demand (and thus d) is sufficiently large.*

Which kind of wholesaler will the manufacturer choose in this case? Comparing the manufacturer's profits from trade through export versus trade through import wholesalers, we find:

Proposition 3.3 *Import wholesalers are used when demand is positively correlated across countries ($\rho > 0$). An export wholesaler is used when demand is negatively correlated across countries ($\rho < 0$).*

Other papers in trade have examined a manufacturer's choice between exporting directly or indirectly. However, the economic mechanism behind this choice is very different from that studied in the existing literature. Whereas direct and indirect exports by manufacturers are typically based on a manufacturer having a different export technology with respect to the one used by intermediaries, all the agents in our model have the same trade technology. Here these effects rest solely on the existence of lags between production and consumption and on the necessity for a manufacturer to produce and to sell its products before the demand is known. Even if channeling trade through an intermediary is costly, it may still be optimal for a manufacturer to use an intermediary when the demand volatility is sufficiently high. It is because an intermediary's incentives are more closely aligned with those of the manufacturer than the incentives of competitive downstream firms, resulting in a better spatial or intertemporal management of trade flows and inventories from the manufacturer's point of view. This results in higher profits for the manufacturer and this even if overall trade is not greater (indeed it could even be smaller) than without intermediaries. An advantage of this approach is that it makes possible to disentangle the relative role of an import and an export wholesaler, something that has, to our knowledge, not been considered either in the literature. The other advantage is its simplicity. We now turn to extensions of the model. We start by introducing lumpy trade.

3.4 Equilibrium with Lumpy Trade

We concentrate on the similarities and differences relative to the case of non-lumpy trade.^{3.14} Consider first the case of direct exports. Recall that in this scenario the manufacturer sets a producer price P_i , then competitive firms in each country $i = \alpha, \beta$ order quantity Q_i . In period 1, demand shock ε_{i1} becomes resolved, competitive firms decide how much to sell in period 1, and the leftover will be sold in period 2. The key result is that in our simple model trade lumpiness has no effect on the manufacturer's

^{3.14}Detailed derivations of the results for direct exports, trade through import wholesalers, and trade through an export wholesaler are provided in the Appendix.

profit. So for the manufacturer the total expected profit from exporting directly to two countries is

$$E(\pi_{direct}^l) = (A - c)^2. \quad (3.15)$$

The reason for this is that competitive firms face the same incentives to divide sales between the two periods, namely to sell in period 1 as long as the first period price p_1 exceeds the expected second-period price $E(p_2)$; otherwise, they will hold inventory for sale in the next period. This implies that the competitive firms' profits are zero for the same quantity ordered from the manufacturer.

When trade is intermediated by import wholesalers, the incentive to divide sales across periods is different. That is, an import wholesaler allocates output across periods until the marginal revenue in period 1 is equal to expected marginal revenue in period 2. The total expected manufacturer's profit from exporting to both countries via import wholesalers is then

$$E(\pi_{imp}^l) = [A - (c + c_w)]^2 + \frac{d^2}{12}. \quad (3.16)$$

Like in the case of non-lumpy trade, an import wholesaler will be used when the variance of demand is sufficiently big relative to the cost of wholesaling. Notice, however, that the gross benefit of import wholesalers, $d^2/12$, is now smaller than in the case of non-lumpy trade. The reason is, of course, that after observing the first-period demand shock the import wholesaler can still adjust sales and the mark-up in period 1, but now has to do this under the constraint that the marginal revenue in that period has to equal expected marginal revenue in period 2. In the case of non-lumpy trade the import wholesaler was free to choose the quantity that maximized first-period profit without having to obey this constraint, since it could not only move inventory into the second but also reorder goods before the start of period 2.

Finally consider trade intermediated by an export wholesaler. The optimal strategy of the export wholesaler after he has observed the demand shocks $\varepsilon_{\alpha 1}$ and $\varepsilon_{\beta 1}$ is to set wholesale prices so that the competitive firms order quantities Q_α and Q_β that equalize marginal revenues in the two countries. Thus the expected profit of the manufacturer is

$$E(\pi_{exp}^l) = [A - (c + c_w)]^2 + \frac{d^2}{24}(1 - \rho). \quad (3.17)$$

Like in the case of non-lumpy trade, trade through an export wholesaler is profitable

if the variance of demand is large relative to the resource cost of wholesaling, provided demands in the two countries are not perfectly correlated. The gross benefit of using an export wholesaler is $d^2(1 - \rho)/24$ and hence only a quarter as big as in the case of non-lumpy trade. There are two reasons for this: first, the wholesaler in the case of lumpy trade has only one opportunity to allocate inventory across countries, namely after observing demand in period 1, compared with two opportunities in the case of non-lumpy trade. Second, once the export wholesaler has completed this allocation, the competitive downstream firms in the two countries face the "wrong" incentives—wrong from the point of view of the manufacturer and of the wholesaler—to allocate inventory across time, namely by selling until the first-period price is equal to the second-period price. This intertemporal "misallocation" of inventory does not take place when there is non-lumpy trade.

It is immediate from the above discussion that trade lumpiness changes the relative benefit of using import compared to export wholesalers. In particular, trade lumpiness makes import wholesalers more attractive compared to export wholesalers. We may state:

Proposition 3.4 *The manufacturer strictly prefers import wholesalers to an export wholesaler unless demand shocks in the two countries are perfectly negatively correlated ($\rho = -1$).*

This is an interesting result because it indicates that lumpy trade penalizes more an export than an import wholesaler. It is the case because, unlike an import wholesaler, the export wholesaler loses control of the intertemporal allocation of products once they are shipped.

A second interesting result comes out of the analysis. We have seen that the benefit of using an import or export wholesaler is lower in the case of lumpy trade than with non-lumpy trade. That is, they are only used for higher levels of the variance of demand. The consequence of this is immediate:

Proposition 3.5 *Trade lumpiness reduces the likelihood that trade is intermediated by wholesalers.*

In other words, lumpy trade makes direct trade surprisingly more likely to occur and thus makes inventories more likely to be managed by the downstream firms themselves.

The reason is clear: a wholesaler, whether on the import or on the export side, is more constrained with lumpy than with non-lumpy trade and is thus not as useful to a manufacturer to manage inventories within a supply chain.

3.5 Welfare Effects of Intermediation

Our simple model is useful for exploring the role of wholesalers in managing inventory and the effect of different inventory management arrangements (inventory held by downstream firms versus import wholesalers) on trade volatility. However, the model also has a special feature, namely that it yields only trivial results regarding the effect of wholesalers on the volume of trade and on social welfare in the destination countries. In particular, intermediation only affects the trade volume and social welfare in the destination markets to the extent that it involves a resource cost.

In this section we show that for more general demand functions or demand shocks intermediation does affect the volume of trade and social welfare even if we ignore any resource cost of wholesaling and thus assume that $c_w = 0$. We concentrate on the comparison between direct exports and indirect trade through import wholesalers only and we look at two separate cases: one where we keep the additive uncertainty but we relax the assumption of linear demand, and the other where we replace additive uncertainty by multiplicative uncertainty.

Regarding the first case, consider a general demand function in country $i = \alpha, \beta$ at time $t = 1, 2$: $p_{it} = p_{it}(s_{it}) + \varepsilon_{it}$, with $p'_{it} < 0$. We make the following assumption, ignoring country and time subscripts:

$$\begin{aligned} p''(s) \geq 0; p'(s) + sp''(s) \leq 0; p'''(s) \leq 0; MR(s) = sp'(s) + p(s) > 0; & \quad (\text{A3.2}) \\ MR'(s) = 2p'(s) + sp''(s) < 0; MR''(s) = 3p''(s) + sp'''(s) \geq 0; & \\ p''(s) + sp'''(s) - s(p''(s))^2/p'(s) \leq 0 & \end{aligned}$$

This assumption is satisfied, for instance, for linear demand functions. Obviously, the linear demand case is a specific and extreme case of (A3.2) since it requires some of these assumptions to be strictly equal to zero.

We can then show the following result:

Proposition 3.6 *Suppose $c_w = 0$, the demand uncertainty is additive, and the demand satisfies Assumption (A3.2). Then an import wholesaler reduces the expected volume of trade and expected social welfare in the destination country relative to direct exports if trade is non-lumpy, and it increases the expected volume of trade and expected social welfare in the destination country if trade is lumpy.*

Proof: see Appendix.

The result that the trade volume and welfare effects of import wholesalers are sensitive to the degree of trade lumpiness reflects the fact that import wholesalers play two different roles. First, they allow the manufacturer to better exercise market power in the destination countries. Specifically by delegating pricing to a wholesaler with better information about the realization of demand permits more exact pricing to extract surplus from consumers. Second, import wholesalers improve the allocation of goods across time and thus reduce the amount of inventory required to avoid stockouts. The relative importance of these two roles depends on the lumpiness of trade. In particular, if trade is non-lumpy, then import wholesalers are able to exploit demand information each period to optimally exercise market power; the allocation of goods across time is not so important, as goods can be reordered in period two. The opposite is true for lumpy trade: the role in improving the allocation of goods across time now takes center stage, because goods cannot be reordered in period two; naturally there is less scope to adjust prices to new demand information.

To drive home the point even more clearly that there is a trade-off between a better exercise of market power and improved intertemporal allocation of goods, consider the case of multiplicative demand uncertainty and hence a demand function: $p_{it} = \varepsilon_{it}p_{it}(s_{it})$ with $p'_{it} < 0$. This type of uncertainty is best interpreted as uncertainty about consumers' willingness to pay. Thus there is uncertainty about consumers tastes and about the distribution of tastes across markets. In this case, we would expect the price setting role of import wholesalers to be especially important, as wholesalers are able to adjust prices after observing consumers' willingness to pay. Delegating pricing to import wholesalers thus allows the manufacturer to optimally price discriminate across destination markets, possibly to the detriment of consumers.

The following proposition reveals two things: it shows that the result that an import wholesaler tends to raise the expected volume of trade carries over to the case of multi-

plicative demand uncertainty. And it confirms the intuition that import wholesalers may reduce social welfare even in the case of lumpy trade; a sufficient conditions for this is linear demand:

Proposition 3.7 *Suppose that $c_w = 0$, the demand uncertainty is multiplicative, and trade is lumpy. If demand satisfies Assumption (A3.2), then an import wholesaler weakly increases the expected volume of trade relative to direct exports. If demand is linear, the expected volume of trade is the same in the case of an import wholesaler and in the case of direct exports, but an import wholesaler decreases expected social welfare in the destination country.*

Proof: see Appendix.

3.6 Concluding Remarks

This chapter shows that adding one more agent within a distribution chain in between a manufacturer and its customers is often an optimal strategy to follow for manufacturers in an environment where orders must be placed before demand is known. This is the case even if adding intermediation is costly and may even decrease the overall volume of sales. This is an important and especially relevant result in an international trade context as lags between production and consumption, whether because of the nature of production, the location of that production or the transportation technology, make them far more prevalent than in a domestic market environment. The benefit brought by intermediation is shown to be especially important when demand volatility is high, as an intermediary is able to smooth trade and to lower the need to maintain inventories within a distribution chain with respect to one where such an intermediary is not present. It is then not surprising that export and import wholesaling continues to be an important economic activity around the world. In fact a corollary of our results is that during a dramatic shock such as the trade collapse and rebound of 2008-09, a trade collapse and the subsequent rebound should be expected to be weaker within a distribution chain that involves export or import wholesalers than without them.

But we go further. We also show that import and export wholesalers do not play the same role as far as a manufacturer is concerned. While import wholesalers are particularly useful whenever intertemporal arbitrage can be exploited, export wholesalers are

useful when international spatial arbitrage is likely to be present. Interestingly, lumpy trade, often associated with international transportation lags and relevant in our model insofar as it affects the ability of having enough products on hand to meet demand during a sales period, is shown to decrease a manufacturer's incentives to channel sales through an intermediary, whether it is on the import or the export side, but it does more so with respect to an export wholesaler. That is, lumpy trade leads to less intermediation in general, and shifts the focus toward import-type intermediation. If this may not be viewed as particularly surprising, it nevertheless has interesting implications. In particular, if one accepts the fact that 'globalization' has reduced trade lumpiness, for instance, through the more intensive use of international air transportation, our results suggest that globalization has created more room, not less, for intermediaries to be active in the international market place. Moreover, these intermediaries should become more mixed in terms of export and import wholesaling activities. It is not that a given intermediary necessarily takes on both more export and import activities, or even that in a given industry, there are more of both types of intermediaries, but simply that by increasing the benefit associated with their use, there is a more widespread use of both types of intermediaries. The argument that intermediaries are more useful than ever and this even if transport costs and barriers to trade have come down is not new. In fact Belavina and Girotra (2012) argue that intermediaries help adapting to a volatile environment even for firms that are much larger than the intermediaries they typically use.

While testing directly some of the above results and hypotheses is beyond the scope of this chapter, it is interesting to note that our theoretical results are consistent with the empirical results about drop-shipping provided by Randall, Netessine and Rudi (2006). As mentioned earlier, drop-shipping is an arrangement whereby a retailer forwards buyers' orders to a wholesaler who then ships the product from its own inventory. This internet-based arrangement makes possible for a retailer to avoid holding any inventory.^{3.15} The authors then compare this arrangement with one where the retailer is a traditional one that holds its own inventories. This is a similar structure to ours in so far as a traditional retailer holding and managing inventories corresponds to our case of direct exports, while the drop-shipping arrangement corresponds to the case where a wholesaler manages inventories. The authors do find empirical evidence that retailers who manage their own

^{3.15}See PRWeb (2012) for an example of drop-shipping in an international context.

inventories (direct exports in our case) face lower demand uncertainty than the retailers who do not (but wholesalers do). This is very much consistent with our result that using wholesalers as managers of inventories is optimal when there is high demand uncertainty. They also find that the greater the number of retailers, the greater the use of drop-shipping. Although our retailers are perfectly competitive and thus we have no particular result on that dimension, it is interesting to note that the fundamental reason why wholesalers might be needed is because retailers, as price takers, do not have the same incentives as a manufacturer or a wholesaler. In that sense this empirical finding is also consistent with our theoretical results. Finally, while most drop shipping wholesalers supplying the U.S. market are still located in the United States, improved logistics has led to the emergence of large drop shipping wholesalers in China competing successfully in the U.S. market. This is consistent with the notion that reduced lumpiness of international trade tends to give a larger role to export as opposed to import wholesalers. Obviously a lot more needs to be done on the empirical side, especially in the context of international markets, as intermediation is here to stay and, as we have shown, their presence affects trade dynamics.

The final point is about the volume of trade and social welfare. While most of our results are derived in a simple environment that essentially keeps constant the volume of trade and social welfare regardless of the presence of intermediaries, we are still able to relax a few assumptions to show that, in the case of import-wholesalers, the volume of trade and social welfare can either go up or down depending on product and international trade characteristics. In particular we show that there is a fundamental trade-off between an efficient intertemporal allocation of products and the exploitation of market power associated with import wholesalers and that this trade off is influenced by both the product characteristics (reflected by the shape of the demand and the type of uncertainty associated with this demand) and the degree of international trade lumpiness. Everything else being equal, a demand uncertainty reflecting uncertainty associated with consumers' willingness to pay in a market, surely a feature more associated with differentiated than with homogeneous products, tends to push down both the volume of trade and welfare in the presence of an import-wholesaler as compared to direct exports simply because it allows the wholesaler to exploit better market power through time than manufacturers are able to do through direct sales. However, lumpy trade, by itself, tends to limit the ability to exploit market power as compared to non-lumpy trade simply because an import

wholesaler is forced to deal with an uncertain intertemporal allocation of products out of a single shipment.

This first attempt to uncover the role of import and export wholesalers has revealed a rich set of results. But it is only a first step and more work is required both at the theoretical and at the empirical level. Indeed the widespread use of intermediaries suggests that they have an impact at the aggregate level whether it is through the dynamics of trade, the volume of trade or social welfare.

4 An Economic Model of the Bullwhip Effect

Abstract: This chapter develops a new theoretical model of the bullwhip effect, where firms engage in profit-maximization pricing and inventory management. By introducing a trade collapse case, which is ignored in previous literature, our model shows that a bullwhip effect will be present when demand shocks exhibit strong persistence, and there will be a reverse bullwhip effect if the persistence of demand shocks is small. In addition, it is shown that the degree of the bullwhip effect increases as the persistence of demand shocks goes up. Therefore, this chapter provides a new explanation for why some industries exhibit a bullwhip effect while others do not.

4.1 Introduction

Jay Forrester first introduced the definition of the bullwhip effect in 1961. He characterized it as an amplification of demand variability from a downstream site to an upstream site. Since then, the bullwhip effect has been observed in various companies and industries and widely discussed in the management literature. Hammond (1994) observes an amplification of demand variability in the case of Barilla's pasta. Procter & Gamble found that "the diaper orders issued by the distributors have a degree of variability that cannot be explained by consumer demand fluctuations alone. (See Lee et al. 1997, p.546)" At the industry level, semiconductor and DRAM industries are more volatile than the PC industry (Terwiesch et al. 2005 and Lee et al. 1997). Several recent empirical studies based on macroeconomic data also show an aggregate bullwhip effect.^{4.1} The "great trade collapse" after the Lehman Brother Bankruptcy in global supply chains has been considered as one of the most remarkable features. One stylized fact is that, compared with final goods, intermediates experienced a larger drop followed by a faster rebound in trade.^{4.2} The volatility of trade after the Lehman shock is higher for upstream industries (Zavacka, 2012).

Lee et al. (1997) outline four causes of the bullwhip effect: demand signal processing, a rationing game, order batching, and price variations. Specially, Zavacka (2012) and Altomonte, Di Mauro, Ottaviano, et al. (2012) point out that the demand signal processing becomes particularly relevant in context of the 2008-09 trade collapse. When facing a positive demand shock, the intermediary does not only sell more in the current period to satisfy the observed demand shock, but it also updates the next period demand forecast and increases replenishment further in anticipation of higher future demand.

If the story could stop here, it might sound good: we have found and empirically tested the bullwhip effect, and we know how to interpret it. Unfortunately, some industries do not exhibit a bullwhip effect. By using industry-level U.S. data, Cachon, Randall et al. (2007, p.457) find "wholesale industries exhibit a bullwhip effect, but retail industries generally do not exhibit the effect, nor do most manufacturing industries." Even Zavacka

^{4.1}See Zavacka, 2012; Altomonte, Di Mauro, Ottaviano, et al. 2012.

^{4.2}In the United States for instance, from July 2008 to February 2009, US real imports and exports each fell by about 24 %, while manufacturing production fell 12 %. The rebound was equally impressive, with import and exports expanding about 20% between May 2009 and May 2010, while manufacturing production rebounded only by 10 % (Alessandrial et al. 2011).

(2012, p.26) also finds "the volatility of a downstream production stage might exceed the volatility of its suppliers if the upstream producers operate in networks of chains with uncorrelated demands. The analysis is limited by a somewhat simplistic inventory rule." As she argues, she uses an alternative theory to interpret this reverse bullwhip effect which seems limited and inconsistent with her explanation for the bullwhip effect.

To our knowledge, there is no theoretical model in economics or operations management that attempts to give an explanation for why some industries exhibit a bullwhip effect while others do not. It is then reasonable for us to investigate the underlying rationale. Particularly, we want to comprehend the exact relationship between a (reverse or) bullwhip effect and trade collapse. Furthermore, in most the operations management literature, demand uncertainty is characterized by an exogenous process, for example $D_t = d + \rho D_{t-1} + u_t$. An intermediary does not need to make an optimal pricing decision. By assuming an exogenous demand process, it might be easy to calculate the variance of demand in each echelon of the supply chain. It would, however, make readers lose sight of economic factors driving the intermediaries' behavior. Therefore, not only do we find a tractable way of comparing variances, we also develop a new theoretical model of the bullwhip effect in which the intermediary makes the optimal pricing and inventory management decisions.^{4.3}

This chapter presents a two-period model to show the existence of a bullwhip effect, and explains why and where a reverse bullwhip effect may also occur. In period 1, an intermediary initially takes possession of some inventory. After first period demand uncertainty has been resolved, an intermediary sells the product to a downstream final market and decides whether and how much to purchase from an upstream producer for the next period. In period 2, the intermediary will sell all of inventory he holds, i.e. the leftover at the end of period 1 plus the replenishment. If first period demand turns out to be very high, there will be a stockout case: the initial inventory will be sold out, and the only optimal decision left for the intermediary is how much to reorder for period 2. If first period demand is at an intermediate level, the intermediary will set the profit-maximization wholesale price and decide how much to reorder for period 2.

^{4.3}Kahn (1987) provided a general model, in which the producer needs to make decisions on both of these two issues. However, because he neglected the non-production case, (analogous to the trade collapse case in our context), his conclusion has many limitations. See Reagan (1982) for the rigorous mathematical proof of this issue.

If demand in the first period is very low, the intermediary will not reorder and simply allocate the inventory he possesses across two periods. We call this the trade collapse case, and its occurrence is the most critical point in this chapter. In this situation, the replenishment is constantly equal to zero, whereas the sales in period 1 increase with the demand (i.e. a higher demand uncertainty leads to more sales in period 1). That means that in the trade collapse case, the replenishment is less sensitive to the demand uncertainty than sales; In the other two cases, the replenishment is more sensitive to the demand uncertainty than sales, as long as the persistence of demand shocks is larger than zero. This sensitivity analysis can help us to detect the existence of a (reverse) bullwhip effect^{4.4}, and understand the underlying economics. The model further shows us that the volatility of orders is increasing, while the volatility of sales is decreasing in the persistence of the demand shock. The larger the persistence of the demand shock, the stronger is the influence of the first period shock, and the optimal inventory level for the next period will be more sensitive to the demand shock. However in the trade collapse case, the intermediary will allocate products across two periods more equally when facing a larger persistence of the demand shock.

The rest of the chapter is organized as follows. Section 4.2 introduces the model, and then provides the equilibrium of the model. Section 4.3 gives an analytical approach to the bullwhip effect. Section 4.4 concludes. All details of the proofs are provided in the Appendix.

4.2 A Simple Model

4.2.1 Model Setup

In this section, we develop a simple model in which an intermediary purchases goods from an upstream producer and then sells them to a downstream final market.

We are only interested in "additive" and "multiplicative" demand uncertainty, this means the inverse demand function $P(s, \varepsilon)$ is in the form of $p(s) + \varepsilon$ or $\varepsilon p(s)$, in which $p'_s < 0$, $sp'_s + p > 0$, $p'_s + sp''_s < 0$. The revenue function is $R(s, \varepsilon) = P(s, \varepsilon) \cdot s$. Since the uncertainty is additive or multiplicative, it is reasonable to assume $E[R(s, \varepsilon)] = R[s, E(\varepsilon)]$. The demand uncertainty may be serially correlated, which means $\varepsilon_1 = v_1$,

^{4.4}One mathematical lemma will be used, see details in section 4.3.

$\varepsilon_2 = \rho(\varepsilon_1 - \bar{v}) + v_2$, where $E(v_t) = \bar{v}$, and v_t is i.i.d.. The cumulative distribution is $F(\cdot)$, the support of this distribution is $[v_l, v_h]$. Here we assume the persistence of demand shock $\rho \in [0, 1]$.

In period 1, the intermediary initially takes possession of I_1 goods. After demand uncertainty ε_1 has been resolved, the intermediary sells s_1 products to the downstream final market and decides whether and how much to purchase from the upstream producer for the next period, denoted by q_1 . The contract between the intermediary and the producer follows the two-part tariff form, $T = q_1 h + f$, where h is the producer price, and f is the fixed transfer from intermediary to producer. The marginal cost for the producer is c .

We then distinguish between three cases in period 1. If the intermediary does not order from the producer, we call it the "trade collapse" case. In this case, the intermediary observes that ε_1 is low, and the inventory I_1 is relatively large. It is optimal for the intermediary not to order from the producer, but instead to use the initial inventory to cover two selling periods. If the intermediary orders from the producer, but he does not sell all of I_1 in the first period, we call it the "normal" case. In this case, ε_1 reaches an intermediate level, and the intermediary cannot use I_1 to cover two periods. He has to order from the producer for the next period. However, ε_1 is not so high that the intermediary cannot satisfy the first period demand by selling all of inventory I_1 . The last possibility is that ε_1 is so high that the intermediary sells all of inventory I_1 in period 1, and continues to order from the original producer, we call this the "stockout" case.

In period 2, the demand shock ε_2 is realized, and the intermediary gets the amount of q_1 goods, then the inventory held by the intermediary is $I_2 = I_1 - s_1 + q_1$. No goods will be left at the end of period 2; the intermediary will sell all of the products he possesses.

The overall strategic interaction can be represented by the following sequential game: At the beginning of period 1, the intermediary takes possession of I_1 goods, then the demand shock ε_1 is realized, and the producer provides the two-part tariff contract T to the intermediary. In the next stage, the intermediary makes the optimal decisions on s_1 and q_1 , then one of the three cases classified above occurs. In period 2, the intermediary sells all of the inventory he holds.

4.2.2 Equilibrium

In this subsection, we characterize the equilibria of the sequential game and derive how the intermediary makes optimal pricing and inventory management decisions. Solving the game backwards, we begin with the optimal inventory level for the intermediary in period 2 given the ε_2 and the producer price h , and then move to period 1. We then investigate the behavior of the intermediary in these three cases: trade collapse, normal and stockout; and then calculate the two-part tariff contract T in equilibrium. Eventually, we show which of the three cases classified above occurs, given I_1 and the realized ε_1 .

In period 2, given inventory level I_2 held by the intermediary and the realized demand shock $\varepsilon_2 = \rho(\varepsilon_1 - \bar{v}) + v_2$, the revenue of the intermediary is $R(I_2, \varepsilon_2)$. Therefore, the expected revenue for the intermediary at the end of period 1 is $E[R(I_2, \varepsilon_2)] = R[I_2, \rho\varepsilon_1 + (1 - \rho)\bar{v}]$.

Then we move to period 1. If the intermediary orders from a producer (in the normal and stockout case) in period 1, the marginal expected revenue in period 2 should be equal to h , i.e.

$$\frac{\partial}{\partial I_2} R[I_2, \rho\varepsilon_1 + (1 - \rho)\bar{v}] = h \quad (4.1)$$

where I_2 is the available inventory for the intermediary in period 2. Then we obtain the so called optimal inventory level $I_2^o(\varepsilon_1, h, \rho)$, which satisfies equation (4.1). Furthermore, in the normal case, the revenue for the intermediary in period 1 is $R(s_1, \varepsilon_1)$, and the marginal revenue should be equal to h as well^{4.5}, that is

$$\frac{\partial}{\partial s_1} R(s_1, \varepsilon_1) = h \quad (4.2)$$

We call $s_1^o(\varepsilon_1, h)$ the optimal sales for period 1, which satisfies equation (4.2).

We now start to calculate the producer price h in equilibrium. In the stockout case, actual sales are $s_1^s = I_1$, and replenishment is $q_1^s = I_2^o(\varepsilon_1, h, \rho)$. The sum of revenue in period 1 and the expected revenue in period 2 is

$$(\Sigma R^s) = R(I_1, \varepsilon_1) + R[I_2^o(\varepsilon_1, h, \rho), \rho\varepsilon_1 + (1 - \rho)\bar{v}]$$

^{4.5}These two first order conditions follow from the optimality criterion of dynamic programming.

The producer's problem is

$$\max_h \underbrace{q_1^s (h - c)}_{\text{revenue minus cost}} + \underbrace{(\Sigma R^s) - q_1^s h}_{\text{fixed transfer from wholesaler}}$$

The first order condition is

$$\frac{\partial I_2^o}{\partial h} (-c) + \frac{\partial (\Sigma R^s)}{\partial I_2^o} \frac{\partial I_2^o}{\partial h} = 0$$

From (4.1) and the fact that $R(I_1, \varepsilon_1)$ does not depend on h in the stockout case, we have $\frac{\partial (\Sigma R^s)}{\partial I_2^o} = h$. The first order condition can then be simplified as $\frac{\partial I_2^o}{\partial h} (h - c) = 0$, thus $h = c$.

In the normal case, actual sales are $s_1^n = s_1^o(\varepsilon_1, h)$, and replenishment is $q_1^n = I_2^o(\varepsilon_1, h, \rho) - [I_1 - s_1^o(\varepsilon_1, h)]$, in which $I_1 - s_1^o(\varepsilon_1, h)$ is the leftover in period $t = 1$. The sum of revenue in period 1 and the expected revenue in period 2 is

$$(\Sigma R^n) = R[s_1^o(\varepsilon_1, h), \varepsilon_1] + R[I_2^o(\varepsilon_1, h, \rho), \rho\varepsilon_1 + (1 - \rho)\bar{v}]$$

In this case, the producer's problem is

$$\max_h q_1^n (h - c) + (\Sigma R^n) - q_1^n h$$

The first order condition is

$$\begin{aligned} & \frac{\partial q_1^n}{\partial h} (-c) + \frac{\partial R(s_1^o, \varepsilon_1)}{\partial s_1^o} \frac{\partial s_1^o}{\partial h} + \frac{\partial R(I_2^o, \rho\varepsilon_1 + (1 - \rho)\bar{v})}{\partial I_2^o} \frac{\partial I_2^o}{\partial h} \\ & = (-c) \left(\frac{\partial I_2^o}{\partial h} + \frac{\partial s_1^o}{\partial h} \right) + \frac{\partial R(s_1^o, \varepsilon_1)}{\partial s_1^o} \frac{\partial s_1^o}{\partial h} + \frac{\partial R(I_2^o, \rho\varepsilon_1 + (1 - \rho)\bar{v})}{\partial I_2^o} \frac{\partial I_2^o}{\partial h} = 0 \end{aligned}$$

Combining (4.1) and (4.2), we have $\frac{\partial R(s_1^o, \varepsilon_1)}{\partial s_1^o} = \frac{\partial R(I_2^o, \rho\varepsilon_1 + (1 - \rho)\bar{v})}{\partial I_2^o} = h$, the first order condition can then be simplified as $(h - c) \left(\frac{\partial I_2^o}{\partial h} + \frac{\partial s_1^o}{\partial h} \right) = 0$, thus $h = c$.

Therefore, in equilibrium $h = c$. Plug $h = c$ into (4.1) and (4.2) in equilibrium, we have two implicit functions of I_2^o and s_1^o , from which we can obtain the optimal inventory level $I_2^o(\varepsilon_1, c, \rho)$ and the optimal sales $s_1^o(\varepsilon_1, c)$.

In the trade collapse case, the story is slightly different. Given ε_1 , the intermediary allocates the inventory I_1 between two periods by equalizing "marginal revenue", which means

$$\frac{\partial}{\partial s_1} R(s_1^c, \varepsilon_1) = \frac{\partial}{\partial s_2} R[s_2^c, \rho\varepsilon_1 + (1 - \rho)\bar{v}], \quad (4.3)$$

$$\text{where } s_1^c + s_2^c = I_1 \quad (4.4)$$

Let the solution to equations (4.3) and (4.4) be defined by $s_1^c = s_1^c(I_1, \varepsilon_1, \rho)$. In this case, the replenishment $q_1^c = 0$, the sum of revenue in period $t = 1$ and the expected revenue in period $t = 2$ is

$$(\Sigma R^c) = R(s_1^c, \varepsilon_1) + R[I_1 - s_1^c, \rho\varepsilon_1 + (1 - \rho)\bar{v}]$$

Here we finish analysis of the three possible cases classified above. Which of the three cases occurs depends on the quantity of inventory the intermediary possesses I_1 and the realized demand shocks ε_1 . If $I_1 > I_2^o(\varepsilon_1, c, \rho) + s_1^o(\varepsilon_1, c)$, we obtain the trade collapse case; if $I_1 < s_1^o(\varepsilon_1, c)$, there will be a stockout case. The cut-off values of ε_1 for these two cases are $\varepsilon_1^c(I_1, c, \rho)$ and $\varepsilon_1^s(I_1, c)$, and are respectively determined by

$$I_1 = I_2^o(\varepsilon_1^c, c, \rho) + s_1^o(\varepsilon_1^c, c) \quad (4.5)$$

$$I_1 = s_1^o(\varepsilon_1^s, c) \quad (4.6)$$

We have shown each player's (producer and intermediary) optimal behavior in equilibrium. In the next section, we will look more closely at the (reverse) bullwhip effect. In order to do that, we need to know the volatility of sales and replenishment. However, since we make very general assumptions regarding $p(s)$ and $F(\cdot)$ in this chapter, it's impossible to calculate the variances of sales and replenishment directly. Although used effectively in other research, the brutal force solution is not recommended; it involves a tedious calculation, and makes readers lose sight of economic factors driving the intermediaries' behavior. Therefore we provide a new method of comparing these two variances in the next section. This will not only remove difficulties in calculating the variances of functions and make the comparison of them tractable, but also show the mechanism of

the bullwhip effect.

4.3 The Analytics of the Bullwhip Effect and the Reverse Bullwhip Effect

4.3.1 Existence of the Bullwhip Effect and the Reverse Bullwhip Effect

In this section, we will analyze the so called "bullwhip effect". There are two ways of measuring the (reverse) bullwhip effect: amplification ratio = $\frac{Var(q_1)}{Var(s_1)}$ and amplification difference = $Var(q_1) - Var(s_1)$. Since the aim of this chapter is to investigate the existence of (reverse) bullwhip effect, we do not compute this ratio or difference, we just need to know which one of $Var(q_1)$ and $Var(s_1)$ is larger. We say that there is a bullwhip effect if $Var(q_1) > Var(s_1)$. In the case of $Var(q_1) < Var(s_1)$, we say that there is a reverse bullwhip effect. We first analyze the scenario in which $v_l < \varepsilon_1^c(I_1, c, \rho) < \varepsilon_1^s(I_1, c) < v_h$. This implies that the three cases introduced in the last section are possible in period 1. The other scenarios, in which the trade collapse or stockout cases will not exist, will be discussed at the end of this section. We can write replenishment q_1 and sales s_1 in period 1 as two piecewise-defined functions of ε_1 ,

$$q_1(\varepsilon_1) = \begin{cases} 0 & \text{if } v_l < \varepsilon_1 \leq \varepsilon_1^c(I_1, c, \rho) \\ I_2^o(\varepsilon_1, c, \rho) - [I_1 - s_1^o(\varepsilon_1, c)] & \text{if } \varepsilon_1^c(I_1, c, \rho) \leq \varepsilon_1 \leq \varepsilon_1^s(I_1, c) \\ I_2^o(\varepsilon_1, c, \rho) & \text{if } \varepsilon_1^s(I_1, c) \leq \varepsilon_1 < v_h \end{cases} \quad (4.7)$$

and

$$s_1(\varepsilon_1) = \begin{cases} s_1^c(I_1, \varepsilon_1, \rho) & \text{if } v_l < \varepsilon_1 \leq \varepsilon_1^c(I_1, c, \rho) \\ s_1^o(\varepsilon_1, c) & \text{if } \varepsilon_1^c(I_1, c, \rho) \leq \varepsilon_1 \leq \varepsilon_1^s(I_1, c) \\ I_1 & \text{if } \varepsilon_1^s(I_1, c) \leq \varepsilon_1 < v_h \end{cases} \quad (4.8)$$

The following lemma is a useful tool throughout our analysis. (For a detailed proof, see J. Gurland, 1976)

Lemma 4.1 *Let X be a random variable, and let n, m be continuous functions on \mathbb{R} . If n is monotonically increasing and m monotonically decreasing, then*

$$E[n(X)m(X)] \leq E[n(X)]E[m(X)]$$

If n , m are both monotonically increasing or decreasing, then

$$E [n(X) m(X)] \geq E [n(X)] E [m(X)]$$

Moreover, in both cases, if both functions are strictly monotone, the inequality is strict.

With Lemma 4.1 we can reduce our problem to just analyzing the sales and replenishment sensitivity to the first period demand shock. Doing so provides two benefits. First, it is mathematically easier than calculating the expectation term in the formula directly. Second, and more important, by analyzing the sensitivity to demand shock, we can offer a better picture of the underlying mechanism and a deeper understanding of the role of the intermediary in the bullwhip effect. This can be shown in the following proposition and its proof.

Proposition 4.1 *If $\rho = 0$, there is a reverse bullwhip effect; if $\rho = 1$, there is a bullwhip effect.*

Proof. Case 1, $\rho = 0$: For $\varepsilon_1 \in [v_l, \varepsilon_1^c]$, we obtain the trade collapse case, and $q_1(\varepsilon_1) = 0$. From equation (4.3), s_1 is determined by

$$\begin{aligned} \frac{\partial}{\partial s_1} R(s_1^c, \varepsilon_1) &= \frac{\partial}{\partial s_2} R(s_2^c, \bar{v}) \\ s_1^c + s_2^c &= I_1 \end{aligned}$$

By using the implicit function theorem, we have $\frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} > 0$ in this interval and $\frac{\partial q_1(\varepsilon_1)}{\partial \varepsilon_1} = 0$.

For $\varepsilon_1 \in [\varepsilon_1^c, \varepsilon_1^s]$, we have the normal case. $s_1 = s_1^o(\varepsilon_1, c)$ is determined by

$$\frac{\partial}{\partial s_1} R(s_1, \varepsilon_1) = c$$

Using the implicit function theorem again, we have $\frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} > 0$ in this interval. The replenishment is $q_1(\varepsilon_1) = I_2^o(\varepsilon_1, c, \rho) - [I_1 - s_1^o(\varepsilon_1, c)]$. From equation (4.1), $I_2^o(\varepsilon_1, c, \rho)$ is determined by

$$\frac{\partial}{\partial I_2} R(I_2, \bar{v}) = c$$

Obviously, we see that $I_2^o(\varepsilon_1, c, \rho)$ does not depend on the value of ε_1 if $\rho = 0$, i.e.

$\frac{\partial I_2^o(\varepsilon_1, c, \rho)}{\partial \varepsilon_1} = 0$ in the interval. Therefore $\frac{\partial q_1(\varepsilon_1)}{\partial \varepsilon_1} = 0 - \left[0 - \frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1}\right] = \frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} > 0$ in this interval.

For $\varepsilon_1 \in [\varepsilon_1^s, v_h]$, we obtain the stockout case, $s_1(\varepsilon_1) = I_1$ is a constant, and $q_1(\varepsilon_1) = I_2^o(\varepsilon_1, c, \rho)$. As before we have $\frac{\partial I_2^o(\varepsilon_1, c, \rho)}{\partial \varepsilon_1} = 0$, if $\rho = 0$. Therefore $\frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} = \frac{\partial q_1(\varepsilon_1)}{\partial \varepsilon_1} = 0$ in this interval.

In sum, $\forall \varepsilon_1, \frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} \geq \frac{\partial q_1(\varepsilon_1)}{\partial \varepsilon_1} \geq 0$, and in some non-trivial areas, these inequalities are strict.

Here we define a function $g(\lambda, \varepsilon_1) = \lambda q_1(\varepsilon_1) + (1 - \lambda) s_1(\varepsilon_1)$, $\lambda \in (0, 1)$. Easily we can check that $E\left(\frac{\partial [g(\lambda, \varepsilon_1)]^2}{\partial \lambda}\right) = \frac{\partial E[g(\lambda, \varepsilon_1)^2]}{\partial \lambda}$; and $\frac{\partial}{\partial \lambda} E[g(\lambda, \varepsilon_1)] = E\left[\frac{\partial}{\partial \lambda} g(\lambda, \varepsilon_1)\right]$, then

$$\begin{aligned} \frac{\partial}{\partial \lambda} Var[g(\lambda, \varepsilon_1)] &= \frac{\partial}{\partial \lambda} \{E[g(\lambda, \varepsilon_1)^2] - (E[g(\lambda, \varepsilon_1)])^2\} \\ &= E\left[\frac{\partial}{\partial \lambda} [g(\lambda, \varepsilon_1)]^2\right] - 2E[g(\lambda, \varepsilon_1)] \frac{\partial}{\partial \lambda} E[g(\lambda, \varepsilon_1)] \\ &= E\left[2g(\lambda, \varepsilon_1) \frac{\partial}{\partial \lambda} g(\lambda, \varepsilon_1)\right] - 2E[g(\lambda, \varepsilon_1)] E\left[\frac{\partial}{\partial \lambda} g(\lambda, \varepsilon_1)\right] \end{aligned}$$

Furthermore,

$$\begin{aligned} \frac{\partial}{\partial \varepsilon_1} g(\lambda, \varepsilon_1) &= \lambda \frac{\partial}{\partial \varepsilon_1} [q_1(\varepsilon_1)] + (1 - \lambda) \frac{\partial}{\partial \varepsilon_1} [s_1(\varepsilon_1)] \geq 0 \\ \frac{\partial}{\partial \varepsilon_1} \left[\frac{\partial}{\partial \lambda} g(\lambda, \varepsilon_1)\right] &= \frac{\partial q_1(\varepsilon_1)}{\partial \varepsilon_1} - \frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} \leq 0 \end{aligned}$$

and in some non-trivial areas, these two inequalities are strict.

From Lemma 4.1, we know that

$$E\left[g(\lambda, \varepsilon_1) \frac{\partial}{\partial \lambda} g(\lambda, \varepsilon_1)\right] - E[g(\lambda, \varepsilon_1)] E\left[\frac{\partial}{\partial \lambda} g(\lambda, \varepsilon_1)\right] < 0$$

Thus we prove that $\frac{\partial}{\partial \lambda} Var[g(\lambda, \varepsilon_1)] < 0$, which implies

$$Var[g(1, \varepsilon_1)] < Var[g(0, \varepsilon_1)]$$

Thus $Var[q_1(\varepsilon_1)] < Var[s_1(\varepsilon_1)]$, there is a reverse bullwhip effect.

Case 2, $\rho = 1$: For $\varepsilon_1 \in [v_l, \varepsilon_1^c]$, $q_1(\varepsilon_1) = 0$, and $s_1(\varepsilon_1)$ is determined by

$$\begin{aligned}\frac{\partial}{\partial s_1} R(s_1^c, \varepsilon_1) &= \frac{\partial}{\partial s_2} R(s_2^c, \varepsilon_1) \\ s_1^c + s_2^c &= I_1\end{aligned}$$

Clearly $s_1^c = \frac{1}{2}I_1$. This means given I_1 , the first period sale does not depend on the realized value of ε_1 . Therefore in this interval,

$$\frac{\partial q_1(\varepsilon_1)}{\partial \varepsilon_1} = \frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} = 0$$

For $\varepsilon_1 \in [\varepsilon_1^c, \varepsilon_1^s]$, $s_1(\varepsilon_1) = s_1^o(\varepsilon_1, c)$ is determined by $\frac{\partial}{\partial s_1} R(s_1, \varepsilon_1) = c$, and $\frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} > 0$. The replenishment is $q_1 = I_2^o(\varepsilon_1, c, \rho) - [I_1 - s_1^o(\varepsilon_1, c)]$, where $I_2^o(\varepsilon_1, c, \rho)$ is determined by

$$\frac{\partial}{\partial I_2} R(I_2, \varepsilon_1) = c$$

Therefore $I_2^o(\varepsilon_1, h, \rho) = s_1^o(\varepsilon_1, h)$, and $\frac{\partial q_1(\varepsilon_1)}{\partial \varepsilon_1} = 2\frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} > 0$ in this interval.

For $\varepsilon_1 \in [\varepsilon_1^s, v_h]$, $s_1(\varepsilon_1) = I_1$ is a constant, thus $\frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} = 0$; and $q_1(\varepsilon_1) = I_2^o(\varepsilon_1, c, \rho)$, where $I_2^o(\varepsilon_1, c, \rho)$ is determined by

$$\frac{\partial}{\partial I_2} R(I_2, \varepsilon_1) = c$$

Thus $\frac{\partial q_1(\varepsilon_1)}{\partial \varepsilon_1} = \frac{\partial I_2^o(\varepsilon_1)}{\partial \varepsilon_1} > 0 = \frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1}$ in this interval.

Similarly, we have $\forall \varepsilon_1$, $\frac{\partial q_1(\varepsilon_1)}{\partial \varepsilon_1} \geq \frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} \geq 0$, and in some non-trivial areas, these inequalities are strict. Then we know that, in the case of $\rho = 1$,

$$\begin{aligned}\frac{\partial}{\partial \varepsilon_1} g(\lambda, \varepsilon_1) &= \lambda \frac{\partial}{\partial \varepsilon_1} [q_1(\varepsilon_1)] + (1 - \lambda) \frac{\partial}{\partial \varepsilon_1} [s_1(\varepsilon_1)] \geq 0 \\ \frac{\partial}{\partial \varepsilon_1} \left[\frac{\partial}{\partial \lambda} g(\lambda, \varepsilon_1) \right] &= \frac{\partial q_1(\varepsilon_1)}{\partial \varepsilon_1} - \frac{\partial s_1(\varepsilon_1)}{\partial \varepsilon_1} \geq 0\end{aligned}$$

and in some nontrivial areas, the inequalities above are strict.

Thus we get $\frac{\partial}{\partial \lambda} Var[g(\lambda, \varepsilon_1)] > 0$, which implies

$$Var[g(1, \varepsilon_1)] > Var[g(0, \varepsilon_1)]$$

That is $Var [q_1 (\varepsilon_1)] > Var [s_1 (\varepsilon_1)]$. Hence there is a bullwhip effect. ■

Proposition 4.1 tells us that there will be a reverse bullwhip effect if the persistence of demand shocks is zero and a bullwhip effect will occur when the demand shocks is persistent. In the case of $\rho = 0$, replenishment $q_1 (\varepsilon_1)$ will be less sensitive to the demand shock ε_1 than sales $s_1 (\varepsilon_1)$. The key point is that when the intermediary allocates inventory across two periods in the trade collapse case, replenishment $q_1 (\varepsilon_1)$ is constantly equal to zero. In the normal and stockout case, the two curves of $q_1 (\varepsilon_1)$ and sales $s_1 (\varepsilon_1)$ are parallel if $\rho = 0$, because the optimal inventory stock for next period does not depend on the current demand shock ε_1 , and the intermediary will order one more unit from the producer for the next period when facing sales of one more unit in the current period^{4,6}, i.e. there is a one-to-one relationship. Hence overall the volatility of orders is smaller than the volatility of sales. A reverse bullwhip effect exists.

On the other hand, if $\rho = 1$, this means the demand uncertainty shock ε_1 will last for two periods. In the trade collapse case, the intermediary will equalize each period sales given inventory I_1 , thus sales are equal to $\frac{1}{2}I_1$ in each period, and the two curves of $q_1 (\varepsilon_1)$ and $s_1 (\varepsilon_1)$ are parallel. However, in the normal and stockout case, the slope of $q_1 (\varepsilon_1)$ is steeper than $s_1 (\varepsilon_1)$: as ε_1 increases, the intermediary does not only satisfy the observed current demand but also increases replenishment further in anticipation of higher future demand. This implies that replenishment $q_1 (\varepsilon_1)$ will be more sensitive to the demand shock ε_1 than sales $s_1 (\varepsilon_1)$, and the volatility of orders is larger than the volatility of sales, i.e. there is a bullwhip effect.

We can illustrate these results by using a linear inverse demand function with additive demand uncertainty. The inverse demand function is $p = 1 - s + \varepsilon$, and $\bar{v} = 0$, the support of the distribution is $[-d, d]$. After the same calculation mentioned in section 4.2, we have

$$q_1 (\varepsilon_1) = \begin{cases} 0 & \text{if } \varepsilon_1 < \varepsilon_1^c \\ \frac{2-2c+(\rho+1)\varepsilon_1}{2} - I_1 & \text{if } \varepsilon_1^c < \varepsilon_1 < \varepsilon_1^s \\ \frac{1-c+\rho\varepsilon_1}{2} & \text{if } \varepsilon_1^s < \varepsilon_1 \end{cases}$$

^{4,6}That is why we can use a two-period model to investigate this issue. When we consider this problem with infinite periods dynamic programming model, the mechanism would be same, but only if the value function of the recursive maximization problem exists.

and

$$s_1(\varepsilon_1) = \begin{cases} \frac{2I_1 + (1-\rho)\varepsilon_1}{4} & \text{if } \varepsilon_1 < \varepsilon_1^c \\ \frac{1-c+\varepsilon_1}{2} & \text{if } \varepsilon_1^c < \varepsilon_1 < \varepsilon_1^s \\ I_1 & \text{if } \varepsilon_1^s < \varepsilon_1 \end{cases}$$

in which $\varepsilon_1^c = \left(\frac{2}{\rho+1}\right)(I_1 + c - 1)$, $\varepsilon_1^s = 2I_1 + c - 1$. Now let $I_1 = 0.6$ and $c = 0.1$. In the following figures^{4.7}, the solid lines are the curves of $s_1(\varepsilon_1)$; the dashed lines are the curves of $q_1(\varepsilon_1)$. The x -axis represents the value of the resolved ε_1 ; the y -axis represents the volume of sales or replenishment.

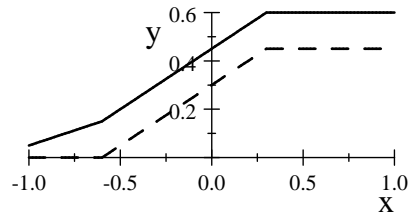


Figure 4.1: A reverse bullwhip effect: $\rho = 0$

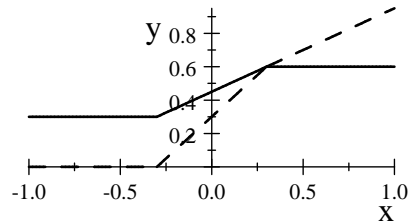


Figure 4.2: A bullwhip effect: $\rho = 1$

If $\rho = 0$, the solid line is steeper than the dashed line in the trade collapse case, and in the other two cases, these two lines are paralleled. If $\rho = 1$, these two lines are paralleled in the trade collapse case, and in the other two cases, the dashed line is steeper than the solid line.

^{4.7}Including the figure used in the next subsection.

4.3.2 Prevalence of the Bullwhip Effect and the Reverse Bullwhip Effect

We have shown that in two extreme conditions, namely $\rho = 0$ and $\rho = 1$, a reverse bullwhip effect, respectively a bullwhip effect, will occur. It is natural to try to find a cutoff value ρ^* , such that there is a reverse bullwhip effect if $\rho < \rho^*$, and a bullwhip effect is exhibited if $\rho > \rho^*$. Before we start this investigation, however, we first introduce the next proposition to help us understand how the persistence of the demand shock affects the (reverse) bullwhip effect.

Write the marginal revenue function $m(s_t) = p(s_t) + s_t p'(s_t)$, which satisfies $m'(s_t) < 0$ and $m''(s_t) > 0$. We make the additional assumption that the marginal revenue function is log-concave, that is

$$[m'(s_t)]^2 - m(s_t) m''(s_t) > 0 \quad (\text{A4.1})$$

This assumption is satisfied for linear and other not-so-convex demand functions. We also assume that if the uncertainty is additive, $\bar{v} = 0$; and if the uncertainty is multiplicative^{4,8}, $\bar{v} = 1$, $v_t \in [0.5, 2]$.

Proposition 4.2 *Under A4.1, the volatility of orders is increasing, while the volatility of sales is decreasing in the persistence of the demand shock. That is*

$$\frac{\partial \text{Var}[q_1(\varepsilon_1)]}{\partial \rho} > 0, \text{ and } \frac{\partial \text{Var}[s_1(\varepsilon_1)]}{\partial \rho} < 0.$$

Consequently, the value of $\frac{\text{Var}[q_1(\varepsilon_1)]}{\text{Var}[s_1(\varepsilon_1)]}$ or $\text{Var}[q_1(\varepsilon_1)] - \text{Var}[s_1(\varepsilon_1)]$ is increasing in ρ .

Proof. Step 1: First, we want to show that

$$\begin{aligned} \frac{\partial \text{Var}[q_1(\varepsilon_1)]}{\partial \rho} &= \frac{\partial}{\partial \rho} [E(q_1^2(\varepsilon_1)) - [E(q_1(\varepsilon_1))]^2] \\ &= \left[E\left(2q_1(\varepsilon_1) \frac{\partial}{\partial \rho} q_1(\varepsilon_1)\right) - 2E(q_1(\varepsilon_1)) E\left(\frac{\partial}{\partial \rho} q_1(\varepsilon_1)\right) \right] \end{aligned}$$

^{4,8}In this chapter, we are only interested in some sufficient conditions to show the following proposition and the intuition behind it. If the uncertainty is multiplicative, we do not consider the case of $v_t \notin [0.5, 2]$.

In fact^{4.9},

$$\begin{aligned} \frac{\partial}{\partial \rho} E(q_1^2(\varepsilon_1)) &= \int_{\varepsilon_1^c(\rho)}^{\varepsilon_1^s(\rho)} \frac{\partial}{\partial \rho} [q_1^n(\varepsilon_1)]^2 dF(v) + [q_1^n(\varepsilon_1^s(\rho))]^2 \frac{\partial \varepsilon_1^s(\rho)}{\partial \rho} \\ &\quad - [q_1^n(\varepsilon_1^c(\rho))]^2 \frac{\partial \varepsilon_1^c(\rho)}{\partial \rho} \\ &\quad + \int_{\varepsilon_1^s(\rho)}^{v_h} \frac{\partial}{\partial \rho} [q_1^s(\varepsilon_1)]^2 dF(v) - [q_1^s(\varepsilon_1^s(\rho))]^2 \frac{\partial \varepsilon_1^s(\rho)}{\partial \rho} \end{aligned}$$

The function $q_1^2(\varepsilon_1)$ is continuous at the points $\varepsilon_1^c(\rho)$ and $\varepsilon_1^s(\rho)$. This implies $[q_1^n(\varepsilon_1^c(\rho))]^2 = 0$ and $[q_1^n(\varepsilon_1^s(\rho))]^2 = [q_1^s(\varepsilon_1^s(\rho))]^2$, then we have

$$\begin{aligned} \frac{\partial}{\partial \rho} E(q_1^2(\varepsilon_1)) &= \int_{\varepsilon_1^c(\rho)}^{\varepsilon_1^s(\rho)} \frac{\partial}{\partial \rho} [q_1^n(\varepsilon_1)]^2 dF(v) + \int_{\varepsilon_1^s(\rho)}^{v_h} \frac{\partial}{\partial \rho} [q_1^s(\varepsilon_1)]^2 dF(v) \\ &= \int_{\varepsilon_1^c(\rho)}^{\varepsilon_1^s(\rho)} 2q_1^n(\varepsilon_1) \frac{\partial}{\partial \rho} q_1^n(\varepsilon_1) dF(v) + \int_{\varepsilon_1^s(\rho)}^{v_h} 2q_1^s(\varepsilon_1) \frac{\partial}{\partial \rho} q_1^s(\varepsilon_1) dF(v) \\ &= E \left[2q_1(\varepsilon_1) \frac{\partial}{\partial \rho} q_1(\varepsilon_1) \right] \end{aligned}$$

Similarly, we have

$$\begin{aligned} \frac{\partial}{\partial \rho} [E(q_1(\varepsilon_1))]^2 &= 2E(q_1(\varepsilon_1)) \frac{\partial}{\partial \rho} E(q_1(\varepsilon_1)) \\ &= 2E(q_1(\varepsilon_1)) E \left(\frac{\partial}{\partial \rho} q_1(\varepsilon_1) \right) \end{aligned}$$

Step 2: From (4.7) and (4.8), we have $\frac{\partial}{\partial \varepsilon_1} q_1(\varepsilon_1) \geq 0$. In the trade collapse case, $\frac{\partial^2}{\partial \rho \partial \varepsilon_1} q_1(\varepsilon_1) = 0$; in the stock-out case and the normal case, ρ has no effect on the first

^{4.9}From the Leibniz rule: $\frac{\partial}{\partial x} \left[\int_{g(x)}^{f(x)} v(x, \varepsilon) dF(\varepsilon) \right] = \int_{g(x)}^{f(x)} \frac{\partial v(x, \varepsilon)}{\partial x} dF(\varepsilon) + v[x, f(x)] \frac{\partial f(x)}{\partial x} - v[x, g(x)] \frac{\partial g(x)}{\partial x}$

period sales $s_1(\varepsilon_1)$, so that

$$\frac{\partial^2}{\partial \rho \partial \varepsilon_1} q_1(\varepsilon_1) = \frac{\partial^2}{\partial \rho \partial \varepsilon_1} I_2^o(\varepsilon_1, c, \rho) > 0$$

(For a detailed proof that $\frac{\partial^2}{\partial \rho \partial \varepsilon_1} I_2^o(\varepsilon_1, c, \rho) > 0$, see the Appendix). Hence we prove that both $q_1(\varepsilon_1)$ and $\frac{\partial}{\partial \rho} q_1(\varepsilon_1)$ are increasing in ε_1 . Using Lemma 4.1 again, we have $E\left(q_1(\varepsilon_1) \frac{\partial}{\partial \rho} q_1(\varepsilon_1)\right) > E(q_1(\varepsilon_1)) E\left(\frac{\partial}{\partial \rho} q_1(\varepsilon_1)\right)$, then we prove that $\frac{\partial \text{Var}[q_1(\varepsilon_1)]}{\partial \rho} > 0$.

With the same method, we can also prove $\frac{\partial \text{Var}[s_1(\varepsilon_1)]}{\partial \rho} < 0$. Note again that the first period sales $s_1(\varepsilon_1)$ in the normal and the stock-out cases do not depend on the value of ρ . Here we just need to prove that $\frac{\partial^2}{\partial \rho \partial \varepsilon_1} s_1^c(I_1, \varepsilon_1, \rho) < 0$ (see the Appendix for a proof).

■

The key point in this proof is the different reaction of $\frac{\partial I_2^o(\varepsilon_1, c, \rho)}{\partial \varepsilon_1}$ and $\frac{\partial s_1^c(I_1, \varepsilon_1, \rho)}{\partial \varepsilon_1}$ as ρ increases. $\frac{\partial I_2^o(\varepsilon_1, c, \rho)}{\partial \varepsilon_1}$ increases in response to a greater persistence of the demand shock. As ρ increases, the first period shock has a stronger influence, and the optimal inventory level for the next period I_2^o will be more sensitive to ε_1 . However in the trade collapse case, the intermediary will allocate products across two periods more equally when facing a larger ρ , this is because the larger ρ is, the smaller is the difference of ε_1 and $\rho\varepsilon_1 + (1 - \rho)\bar{v}$.

From Proposition 4.2, we immediately know that the value of $\frac{\text{Var}[q_1(\varepsilon_1)]}{\text{Var}[s_1(\varepsilon_1)]}$ or $\text{Var}[q_1(\varepsilon_1)] - \text{Var}[s_1(\varepsilon_1)]$ is increasing in ρ . Combined with Proposition 4.1, we have the following corollary:

Corollary 4.1 *There is a cutoff value ρ^* , such that there is a reverse bullwhip effect if $\rho < \rho^*$, and a bullwhip effect is exhibited if $\rho > \rho^*$.*

The above analysis has three implications. First, it explains theoretically why some industries exhibit a bullwhip effect while others do not. Second, it predicts that one empirical test for the bullwhip effect may have different results when using seasonally adjusted data instead of monthly data, consistent with findings from Cachon, Randall et al. (2007). When the bullwhip effect is evaluated with seasonally adjusted data, the prevalence of the bullwhip effect is larger. Third, IT advances and other improvements in supply chain management may fundamentally change the nature of the economy, including the persistence of the demand shock; Proposition 4.2 can predict how the bullwhip

effect will change in the future. This possibility is worth further research.^{4.10}

4.3.3 Discussion

Notice that the discussion above depends on a necessary prerequisite that there is a possibility of trade collapse case, meaning $v_l < \varepsilon_1^c(I_1, c, \rho) < \varepsilon_1^s(I_1, c) < v_h$. Otherwise as long as $\rho > 0$, replenishment will be always more sensitive to the demand shock than sales, or geometrically, the slope of $q_1(\varepsilon_1)$ would be always steeper than $s_1(\varepsilon_1)$, thus there will always be a bullwhip effect. However, if the intermediary faces the possibility of a trade collapse, there may be no order after ε_1 has been resolved, and replenishment is constantly equal to zero. The variance of replenishment may be smaller than the variance of sales. Therefore we can conclude that one sufficient condition^{4.11} for the bullwhip effect is that there is no possibility of having a trade collapse case with $\rho > 0$.

The range of $[v_l, v_h]$, the support of the demand shock distribution function, plays a crucial role. At first, the support $[v_l, v_h]$ determines whether there may be a trade collapse case or not; if a trade collapse case is impossible, then there will be a bullwhip effect as long as $\rho > 0$; if a trade collapse case is possible, then only a large persistence of demand shock will cause a bullwhip effect.

For example, take the linear inverse demand function with additive demand uncertainty, if the range of $[-d, d]$ is so small that $\varepsilon_1^c < -d$, there is no possibility of a trade collapse case, and we find the slope of $q_1(\varepsilon_1)$ is steeper than $s_1(\varepsilon_1)$ as long as $\rho > 0$. Then a bullwhip effect exists. Otherwise, the trade collapse case is possible, in which the slope of $s_1(\varepsilon_1)$ is larger than zero as long as $\rho < 1$, and a reverse bullwhip effect may exist.

How do we explain the range of $[v_l, v_h]$? In the case of a uniform distribution, a larger variance of demand shock implies a wider range of support. Therefore we can say that the more volatile the demand, the larger is the possibility of a trade collapse case, and the more likely the reverse bullwhip effect is to exist. In the case of a normal distribution, the support of the distribution is $(-\infty, \infty)$. As the variance of demand shock increases, the shape of the distribution function is less concentrated, and the probability

^{4.10}A similar idea has been shown in Wen (2005), which provides an alternative explanation for the reduction of volatility in the U.S. economy since 1984 (McConnell and Perez-Quiros 2000).

^{4.11}Alternately, we can say that reverse bullwhip effect's necessary condition is possibility of a trade collapse case.

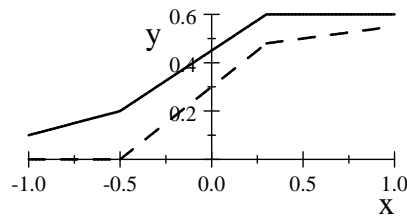


Figure 4.3: The range of the support determines a (reverse) bullwhip effect

of a trade collapse case will be larger, as well as the likelihood of the reverse bullwhip effect. Generally speaking, the larger is the volatility of the demand, the greater is the likelihood that the reverse bullwhip effect will occur. The other research referenced above^{4.12} misses the point that a trade collapse may cause reverse bullwhip effect. My model predicts that during a serious crisis, the likelihood of a reverse bullwhip effect will be larger than usual. One may wonder why the great trade collapse after 2008 is so interesting, and why the bullwhip effect is ignored before 2008 in the trade literature? The fact is that when big shocks happened in 2008, the phenomenon that intermediate goods are more volatile could not be explained by existing economic models, such as the business cycle, fixed cost capacity constraint, etc.. Then some economic models turn to explain this phenomenon by the operations management method. However, the concept of the bullwhip effect has been intensively discussed in the management literature, it's not a new way of telling this story.

4.4 Concluding Remarks

The bullwhip effect has received a great deal of attention from economists. Yet theoretical models are rarely available in the literature, and many empirical findings cannot be properly explained by theory. This chapter provides a simple two-period dynamic model of the bullwhip effect and applies the model to explain why some industries do not exhibit the bullwhip effect.

The two-period model developed in this chapter is applied to investigate the underlying economics of the bullwhip effect. In this model, the intermediary should make the optimal decisions based on pricing and inventory management. It is shown that the bull-

^{4.12} Alessandria, 2010a, 2011; Veronika Zavacka 2012; Ginamarcro Ottaviano 2012

whip effect will occur if the persistence of demand shocks is large. This result coincides with Kahn (1987) and Lee et al. (1997). However, this model is the first to introduce the "trade collapse" case which is ignored in the literature. In the trade collapse case there will be a reverse bullwhip effect if the persistence of demand shocks is small. In addition, this model shows that the volatility ratio, or bullwhip effect, increases as the persistence of demand shocks goes up. This result has implications for both theoretical and empirical research, and also predicts how the bullwhip effect will change in the future.

Finally, the mathematical method used in this chapter provides a tractable way of comparing volatilities. It allows the reader to better understand the mechanism of the bullwhip effect.

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Appendix

A Appendix to Chapter 2

A.1 Proof of Proposition 2.1

$$\pi = \left(\sum_i \pi_i \right) = Q [P(Q) - (\bar{c} - \Delta c)]$$

$$\frac{\partial \pi}{\partial \Delta c} = \frac{\partial Q}{\partial \Delta c} [P(Q) - (\bar{c} - \Delta c)] + Q \left[P'(Q) \frac{\partial Q}{\partial \Delta c} + 1 \right] \quad (\text{A.1})$$

Note that in equilibrium $v_i = \bar{c} - \Delta c, \forall i$. However, in the conceptual outside option equilibrium $v_i = \bar{c} - \Delta c, i = 1, 2, \dots, n - m$ and $v_{n-m+1} = \bar{c}$.

From (2.4), Q is determined by

$$(n - m + 1) P(Q) + Q P'(Q) - (n - m + 1) (\bar{c} - \Delta c) = 0 \quad (\text{A.2})$$

Differentiating (A.2) with respect to Δc ,

$$\begin{aligned} (n - m + 2) P'(Q) \frac{\partial Q}{\partial \Delta c} + Q P''(Q) \frac{\partial Q}{\partial \Delta c} + (n - m + 1) &= 0 \\ \Rightarrow \frac{\partial Q}{\partial \Delta c} &= - \frac{n - m + 1}{(n - m + 2) P'(Q) + Q P''(Q)} \end{aligned} \quad (\text{A.3})$$

Plug (A.3) into (A.1), we have:

$$\begin{aligned} \frac{\partial \pi}{\partial \Delta c} &= - \frac{(n - m + 1) [P(Q) - (\bar{c} - \Delta c)]}{(n - m + 2) P'(Q) + Q P''(Q)} \\ &+ Q \left[\frac{-(n - m + 1) P'(Q) + (n - m + 2) P'(Q) + Q P''(Q)}{(n - m + 2) P'(Q) + Q P''(Q)} \right] \end{aligned}$$

From (2.2) and (2.3), for $i = 1, 2, \dots, n - m$, we have $Q_i = m Q_m$ and $P(Q) - (\bar{c} - \Delta c) = -Q_i P'(Q)$, in which $Q = (n - m + 1) Q_i$, therefore

$$(n - m + 1) ([P(Q) - (\bar{c} - \Delta c)]) = -Q P'(Q)$$

and

$$\begin{aligned}\frac{\partial \pi}{\partial \Delta c} &= \frac{QP'(Q)}{(n-m+2)P'(Q) + QP''(Q)} \\ &+ Q \left[\frac{P'(Q) + QP''(Q)}{(n-m+2)P'(Q) + QP''(Q)} \right] \\ &= Q \left[\frac{2P'(Q) + QP''(Q)}{(n-m+2)P'(Q) + QP''(Q)} \right]\end{aligned}$$

Now we move to π_{n-m+1}^{out} . In this case, $v_i = \bar{c} - \Delta c, i = 1, 2, \dots, n-m$ and $v_{n-m+1} = \bar{c}$. That is, in the conceptual equilibrium, the m-retailer uses the old technology \bar{c} , but the other independent stores buy the products from the supplier at the wholesale price c .

$$\pi_{n-m+1}^{out} = mQ_m^{out} [(P(Q^{out}) - \bar{c})] - f$$

Differentiating (2.8) w.r.t. Δc ,

$$\frac{\partial Q^{out}}{\partial \Delta c} = - \frac{n-m}{(n-m+2)P'(Q^{out}) + Q^{out}P''(Q^{out})}$$

Differentiating (2.2) w.r.t. Q^{out} , and consider the term mQ_m as an entirety.

$$\Rightarrow \frac{\partial mQ_m^{out}}{\partial Q^{out}} = - \frac{mQ_m^{out}P''(Q^{out}) + P'(Q^{out})}{P'(Q^{out})}$$

$$\begin{aligned}\frac{\partial \pi_{n-m+1}^{out}}{\partial \Delta c} &= \frac{\partial mQ_m^{out}}{\partial Q^{out}} [(P(Q^{out}) - \bar{c})] \frac{\partial Q^{out}}{\partial \Delta c} + mQ_m^{out}P'(Q^{out}) \frac{\partial Q^{out}}{\partial \Delta c} \\ &= \frac{\partial Q^{out}}{\partial \Delta c} \left\{ \frac{\partial mQ_m^{out}}{\partial Q^{out}} [(P(Q^{out}) - \bar{c})] + mQ_m^{out}P'(Q^{out}) \right\} \\ &= \frac{\partial Q^{out}}{\partial \Delta c} \left\{ \left(- \frac{mQ_m^{out}P''(Q^{out}) + P'(Q^{out})}{P'(Q^{out})} \right) [(P(Q^{out}) - \bar{c})] \right. \\ &\quad \left. + mQ_m^{out}P'(Q^{out}) \right\}\end{aligned}$$

From (2.2), $mQ_m^{out} P' (Q^{out}) = - [P (Q^{out}) - \bar{c}]$, then

$$\begin{aligned}
\frac{\partial \pi_{n-m+1}^{out}}{\partial \Delta c} &= \frac{\partial Q^{out}}{\partial \Delta c} mQ_m^{out} P' (Q^{out}) \left[\frac{mQ_m^{out} P'' (Q^{out}) + P' (Q^{out})}{P' (Q^{out})} + 1 \right] \\
&= \frac{\partial Q^{out}}{\partial \Delta c} mQ_m^{out} [mQ_m^{out} P'' (Q^{out}) + 2P' (Q^{out})] \\
&= - \frac{(n-m) mQ_m^{out} [mQ_m^{out} P'' (Q^{out}) + 2P' (Q^{out})]}{(n-m+2) P' (Q^{out}) + Q^{out} P'' (Q^{out})}
\end{aligned}$$

A.2 Proof of Lemma 2.2

Obviously $\phi > 0$, $\theta > 0$ for $\forall m \leq n$. By the expression of ϕ ,

$$\begin{aligned}
\frac{\partial \phi}{\partial m} &= \frac{\partial Q}{\partial m} \left[\frac{2 - E(Q)}{(n-m+2) - E(Q)} \right] \\
&+ Q \frac{[(n-m+2) - E(Q)] [-E'(Q)] \frac{\partial Q}{\partial m} + [2 - E(Q)] [1 + E'(Q)] \frac{\partial Q}{\partial m}}{[(n-m+2) - E(Q)]^2} \\
&= \frac{\partial Q}{\partial m} \left[\frac{2 - E(Q)}{(n-m+2) - E(Q)} \right] + Q \frac{(n-m) [-E'(Q)] \frac{\partial Q}{\partial m} + 2 - E(Q)}{[(n-m+2) - E(Q)]^2} \tag{A.4}
\end{aligned}$$

Differentiating (A.2) with respect to m ,

$$\begin{aligned}
\Rightarrow \frac{\partial Q}{\partial m} &= \frac{P(Q) - (\bar{c} - \Delta c)}{(n-m+2) P'(Q) + Q P''(Q)} = \frac{\frac{P(Q) - (\bar{c} - \Delta c)}{P'(Q)}}{n-m+2 - E(Q)} \\
&= \frac{-\left(\frac{1}{n-m+1}\right) Q}{n-m+2 - E(Q)} \tag{A.5}
\end{aligned}$$

Then

$$\begin{aligned}
\Rightarrow \frac{\partial \phi}{\partial m} &= \frac{\left(-\frac{1}{n-m+1} + 1\right) Q [2 - E(Q)]}{[(n-m+2) - E(Q)]^2} - \frac{Q (n-m) E'(Q) \frac{\partial Q}{\partial m}}{[(n-m+2) - E(Q)]^2} \\
&= \frac{\frac{n-m}{n-m+1} Q [2 - E(Q)] - Q (n-m) E'(Q) \frac{\partial Q}{\partial m}}{[(n-m+2) - E(Q)]^2} \tag{A.6}
\end{aligned}$$

in which

$$\begin{aligned}
& \frac{n-m}{n-m+1}Q[2-E(Q)] - Q(n-m)E'(Q)\frac{\partial Q}{\partial m} \\
&= \frac{(n-m)Q}{n-m+1}\left[2-E(Q) - E'(Q)\frac{\partial Q}{\partial m}(n-m+1)\right] \\
&= \frac{(n-m)Q}{n-m+1}\left[2-E(Q) + E'(Q)\frac{Q}{n-m+2-E(Q)}\right] \tag{A.7}
\end{aligned}$$

Since we have proved $E'(Q) < 0$ in Lemma 2.1, then we have the following inequalities:

$$\begin{aligned}
& [(n-m+2) - E(Q)]^2 \frac{\partial \phi}{\partial m} \\
&> \frac{(n-m)Q}{n-m+1}[2-E(Q) + E'(Q)Q] \\
&> \frac{(n-m)Q}{n-m+1}[1 + E'(Q)Q] \\
&> \frac{(n-m)Q}{n-m+1}[E(Q) + E'(Q)Q]
\end{aligned}$$

From A3, we have

$$\begin{aligned}
& \Rightarrow 2P''(Q) + QP'''(Q) \geq 0 \\
& \Leftrightarrow P''(Q) \geq -[P''(Q) + QP'''(Q)] \\
& \Leftrightarrow \frac{P''(Q)}{P'(Q)} < -\frac{P''(Q) + QP'''(Q)}{P'(Q)} \\
& \Rightarrow \frac{P''(Q)}{P'(Q)} < -\frac{P''(Q) + QP'''(Q)}{P'(Q)} + \frac{Q[P''(Q)]^2}{[P'(Q)]^2} \\
& \Leftrightarrow -\frac{E(Q)}{Q} < E'(Q) \\
& \Rightarrow E(Q) + E'(Q)Q > 0
\end{aligned}$$

Thus

$$\frac{\partial \phi}{\partial m} > \frac{\frac{(n-m)Q}{n-m+1}[E(Q) + E'(Q)Q]}{[(n-m+2) - E(Q)]^2} > 0, \forall m$$

and from equation (A.7),

$$\begin{aligned}
\lim_{m \rightarrow n} \frac{\partial \phi}{\partial m} &= \frac{\partial Q}{\partial m} \left[\frac{2 - E(Q)}{2 - E(Q)} \right] + Q \frac{2 - E(Q)}{[2 - E(Q)]^2} \\
&= \frac{\partial Q}{\partial m} + \frac{Q}{2 - E(Q)} \\
&= -\frac{Q}{2 - E(Q)} + \frac{Q}{2 - E(Q)} \\
&= 0
\end{aligned}$$

Now we move to analyze the performance of $\frac{\partial \theta}{\partial m}$. In fact

$$\frac{\partial \theta}{\partial m} = \frac{\Lambda}{[(n - m + 2) - E(Q^{out})]^2}$$

where

$$\begin{aligned}
\Lambda &= \left\{ \begin{aligned} &\left[-s_m Q^{out} [2 - s_m E(Q^{out})] + (n - m) \frac{\partial s_m Q^{out} [2 - s_m E(Q^{out})]}{\partial m} \right] \\ &\quad \cdot [(n - m + 2) - E(Q^{out})] \\ &- (n - m) s_m Q^{out} [2 - s_m E(Q^{out})] \left[-1 - E'(Q^{out}) \frac{\partial Q^{out}}{\partial m} \right] \end{aligned} \right\} \\
&= \left\{ \begin{aligned} &- [2 - E(Q^{out})] s_m Q^{out} [2 - s_m E(Q^{out})] \\ &+ [(n - m + 2) - E(Q^{out})] (n - m) \frac{\partial s_m Q^{out} [2 - s_m E(Q^{out})]}{\partial m} \\ &+ (n - m) s_m Q^{out} [2 - s_m E(Q^{out})] E'(Q^{out}) \frac{\partial Q^{out}}{\partial m} \end{aligned} \right\}
\end{aligned}$$

Here we treat $s_m Q^{out} [2 - s_m E(Q^{out})]$ as an entirety.

By the definition of s_m , easily we have $\lim_{m \rightarrow n} s_m = 1$, then

$$\begin{aligned}
\lim_{m \rightarrow n} \frac{\partial \theta}{\partial m} &= \frac{[2 - E(Q^{out})] [-Q^{out} [2 - E(Q^{out})]]}{[2 - E(Q^{out})]^2} \\
&= -Q^{out} < 0
\end{aligned} \tag{A.8}$$

It only remains to prove that $\lim_{m \rightarrow 1} \frac{\partial \theta}{\partial m} > 0$. In order to do this, we need to calculate

$\frac{\partial s_m Q^{out} [2 - s_m E(Q^{out})]}{\partial m}$. Actually,

$$\begin{aligned} & \frac{\partial s_m Q^{out} [2 - s_m E(Q^{out})]}{\partial m} \\ &= [2 - s_m E(Q^{out})] \frac{\partial m Q_m^{out}}{\partial m} - m Q_m^{out} E(Q^{out}) \frac{\partial s_m}{\partial m} \\ & \quad - m Q_m^{out} s_m E'(Q^{out}) \frac{\partial Q^{out}}{\partial m} \end{aligned}$$

Differentiating (2.7) with respect to m , and treat the term $m Q_m^{out}$ as an entirety,

$$\begin{aligned} \frac{\partial m Q_m^{out}}{\partial m} &= - \left[\frac{m Q_m^{out} P''(Q^{out}) + P'(Q^{out})}{P'(Q^{out})} \right] \frac{\partial Q^{out}}{\partial m} \\ &= - \left[\frac{\frac{m Q_m}{Q^{out}} Q^{out} P''(Q^{out}) + P'(Q^{out})}{P'(Q^{out})} \right] \frac{\partial Q^{out}}{\partial m} \\ &= [s_m E(Q^{out}) - 1] \frac{\partial Q^{out}}{\partial m} \end{aligned} \tag{A.9}$$

and

$$\begin{aligned} \frac{\partial s_m}{\partial m} &= \frac{Q^{out} \frac{\partial m Q_m^{out}}{\partial m} - m Q_m^{out} \frac{\partial Q^{out}}{\partial m}}{(Q^{out})^2} \\ &= \frac{\{Q^{out} [s_m E(Q^{out}) - 1] - s_m Q^{out}\} \frac{\partial Q^{out}}{\partial m}}{(Q^{out})^2} \\ &= \frac{\{s_m [E(Q^{out}) - 1] - 1\} \partial Q^{out}}{Q^{out} \partial m} \end{aligned}$$

Therefore

$$\begin{aligned}
& \frac{\partial s_m Q^{out} [2 - s_m E(Q^{out})]}{\partial m} \\
&= [2 - s_m E(Q^{out})] [s_m E(Q^{out}) - 1] \frac{\partial Q^{out}}{\partial m} \\
&- s_m E(Q^{out}) \{s_m [E(Q^{out}) - 1] - 1\} \frac{\partial Q^{out}}{\partial m} \\
&- (s_m)^2 Q^{out} E'(Q^{out}) \frac{\partial Q^{out}}{\partial m}
\end{aligned} \tag{A.10}$$

From equation (2.8),

$$\begin{aligned}
\frac{\partial Q^{out}}{\partial m} &= \frac{P(Q^{out}) - (\bar{c} - \Delta c)}{(n - m + 2) P'(Q^{out}) + Q^{out} P''(Q^{out})} \\
&= \frac{\frac{P(Q^{out}) - (\bar{c} - \Delta c)}{P'(Q^{out})}}{n - m + 2 - E(Q^{out})} \\
&= \frac{-Q_i^{out}}{n - m + 2 - E(Q^{out})} \\
&= \frac{-(1 - s_m) Q^{out}}{(n - m) [n - m + 2 - E(Q^{out})]}
\end{aligned}$$

in which Q_i^{out} is the quantity for independent store i in the outside option equilibrium, $i = 1, 2, \dots, n - m$. The second equality comes from the first order condition for independent store, which is analogous to (2.3). The last equality follows the fact that

$$(n - m) Q_i^{out} + s_m Q^{out} = Q^{out}$$

Therefore we have

$$[(n - m + 2) - E(Q^{out})] (n - m) \frac{\partial Q^{out}}{\partial m} = -(1 - s_m) Q^{out}$$

Then Λ can be simplified as

$$\begin{aligned} \Lambda = & - [2 - E(Q^{out})] s_m Q^{out} [2 - s_m E(Q^{out})] \\ & + [-(1 - s_m) Q^{out}] \left\{ \begin{array}{l} [2 - s_m E(Q^{out})] [s_m E(Q^{out}) - 1] \\ -s_m E(Q^{out}) [s_m [E(Q^{out}) - 1] - 1] - (s_m)^2 Q^{out} E'(Q^{out}) \end{array} \right\} \end{aligned} \quad (A.11)$$

$$\begin{aligned} & + s_m Q^{out} [2 - s_m E(Q^{out})] E'(Q^{out}) \frac{-(1 - s_m) Q^{out}}{[n - m + 2 - E(Q^{out})]} \\ \doteq & A(Q^{out}) + B(Q^{out}) \end{aligned} \quad (A.12)$$

where

$$\begin{aligned} A(Q^{out}) = & (1 - s_m) Q^{out} \left\{ \begin{array}{l} [2 - s_m E(Q^{out})] [1 - s_m E(Q^{out})] \\ -s_m E(Q^{out}) [1 + s_m - s_m E(Q^{out})] \end{array} \right\} \\ & - [2 - E(Q^{out})] s_m Q^{out} [2 - s_m E(Q^{out})] \end{aligned}$$

and

$$\begin{aligned} B(Q^{out}) = & (1 - s_m) Q^{out} (s_m)^2 Q^{out} E'(Q^{out}) \\ & - \frac{s_m Q^{out} [2 - s_m E(Q^{out})] E'(Q^{out}) (1 - s_m) Q^{out}}{[n - m + 2 - E(Q^{out})]} \end{aligned}$$

In fact

$$\begin{aligned} \frac{A(Q^{out})}{Q^{out}} = & (1 - s_m) \left\{ \begin{array}{l} [2 - s_m E(Q^{out})] [1 - s_m E(Q^{out})] \\ -s_m E(Q^{out}) [1 + s_m - s_m E(Q^{out})] \end{array} \right\} \\ & - [2 - E(Q^{out})] s_m [2 - s_m E(Q^{out})] \\ = & 2 + 5s_m^2 E(Q^{out}) + s_m^2 E(Q^{out})^2 + s_m^3 E(Q^{out}) \\ & - 6s_m - 2s_m E(Q^{out}) - 2s_m^3 E(Q^{out})^2 \end{aligned} \quad (A.13)$$

When $m \rightarrow 1$, then the m-retailer's market share $s_m < \frac{1}{n} < 1$, since $\bar{c} > \bar{c} - \Delta c$. With

the fact that $E(Q^{out}) < 1$,

$$\begin{aligned}
\lim_{m \rightarrow 1} \frac{A(Q^{out})}{Q^{out}} &> 2 + 5s_m^2 E(Q^{out}) + s_m^3 E(Q^{out})^2 + s_m^3 E(Q^{out})^2 \\
&\quad - 6s_m - 2s_m E(Q^{out}) - 2s_m^3 E(Q^{out})^2 \\
&= 2 + 5s_m^2 E(Q^{out}) - 6s_m - 2s_m E(Q^{out}) \\
&> 2 + 5s_m^2 E(Q^{out}) - \frac{6 + 2E(Q^{out})}{n} \\
&> 2 + 5s_m^2 E(Q^{out}) - \frac{8}{n} \\
&> 0 \text{ sufficiently, } n > 3.
\end{aligned}$$

This implies $\lim_{m \rightarrow 1} A(Q^{out}) > 0$ if $n > 3$. What is left is to show $\lim_{m \rightarrow 1} B(Q^{out}) > 0$.

$$\lim_{m \rightarrow 1} B(Q^{out}) = (1 - s_m) s_m Q^{out} E'(Q^{out}) \left[s_m - \frac{2 - s_m E(Q^{out})}{[n + 1 - E(Q^{out})]} \right]$$

and

$$\begin{aligned}
s_m - \frac{2 - s_m E(Q^{out})}{[n + 1 - E(Q^{out})]} &= \frac{s_m n + s_m - 2}{[n + 1 - E(Q^{out})]} \\
&< \frac{1 + s_m - 2}{[n + 1 - E(Q^{out})]} < 0
\end{aligned}$$

With condition that $E'(Q^{out}) < 0$, then $B(Q^{out}) > 0$.

Eventually, we have^{A.1}

$$\lim_{m \rightarrow 1} \frac{\partial \theta}{\partial m} > 0 \text{ sufficiently, if } n > 3$$

^{A.1}Note that in the linear demand case, $E(Q^{out}) = E'(Q^{out}) = 0$, then $\lim_{m \rightarrow 1} \frac{A(Q^{out})}{Q^{out}} = 2 - 6s_m$ and $\lim_{m \rightarrow 1} B(Q^{out}) = 0$. This means $\lim_{m \rightarrow 1} \frac{\partial \theta}{\partial m} > 0$ sufficiently, if $n \geq 3$.

B Appendix to Chapter 3

B.1 Equilibrium With Lumpy Trade

B.1.1 Direct Export

When demand shock ε_1 becomes resolved in period 1, the optimal strategy for each competitive firm is to sell in period 1 as long as the first period price p_1 exceeds the expected second-period price $E(p_2)$; otherwise, it will hold inventory for sale in the next period. In an equilibrium with competitive firms we hence have $p_1 = E(p_2)$, or

$$A - s_1 + \varepsilon_1 = E(A - s_2 + \varepsilon_2) = A - s_2.$$

Assuming that competitive firms have ordered the quantity Q so that $s_1 + s_2 = Q$, we have $s_1 = (Q + \varepsilon_1)/2$ and $s_2 = (Q - \varepsilon_1)/2$. The first-period price (and expected second-period price) is hence

$$\bar{p} = A - s_1 + \varepsilon_1 = \frac{2A - Q + \varepsilon_1}{2},$$

so that the total revenue after observing demand in period 1 is

$$R = r_1 + E_1(r_2) = \bar{p}Q = \frac{(2A - Q + \varepsilon_1)Q}{2}. \quad (\text{B.1})$$

Given consumer price \bar{p} and producer price P , competitive firms will order goods until their expected revenue equals cost so that profit is zero:

$$\int_{-d}^d \frac{(2A - Q + \varepsilon_1)Q}{2} \frac{1}{2d} d\varepsilon_1 - PQ = 0.$$

Thus the expected profit of the manufacturer is

$$\int_{-d}^d \frac{(2A - Q + \varepsilon_1)Q}{2} \frac{1}{2d} d\varepsilon_1 - cQ.$$

Solving the first-order condition,

$$\frac{1}{2d} \int_{-d}^d \frac{2A - 2Q + \varepsilon_1}{2} d\varepsilon_1 - c = 0,$$

yields as optimal output $Q = A - c$.

So for the manufacturer the total expected profit from exporting directly to two countries is

$$E(\pi_{direct}^l) = (A - c)^2.$$

B.1.2 Import Wholesaler

As we know from the case of non-lumpy trade, the wholesaler will earn an expected revenue in period 2 given by (3.5) and revenue (3.7) in period 1. Hence the total expected revenue of the wholesaler after observing ε_1 is:

$$R = (A - s_1 + \varepsilon_1) s_1 + (A - s_2) s_2, \quad (\text{B.2})$$

where $s_1 + s_2 = Q$ and Q is the initial quantity ordered by the wholesaler.

In period 1, the cost of ordering quantity Q is sunk and, given the revealed demand shock ε_1 , the import wholesaler allocates output across periods until the marginal revenues in period 1 (given by (3.8)) is equal to expected marginal revenue in period 2 (given by (3.6)). This means $A - 2s_1 + \varepsilon_1 = A - 2s_2$, and since $s_1 + s_2 = Q$, then $s_1 = (2Q + \varepsilon_1) / 4$ and $s_2 = (2Q - \varepsilon_1) / 4$. Using these values in (B.2) we obtain as total expected revenue of the import wholesaler in period 1

$$\begin{aligned} R &= \frac{(4A - 2Q + 3\varepsilon_1)(2Q + \varepsilon_1)}{4} + \frac{(4A - 2Q + \varepsilon_1)(2Q - \varepsilon_1)}{4} \\ &= \frac{4Q(2A + \varepsilon_1) + \varepsilon_1^2 - 4Q^2}{8}. \end{aligned}$$

Hence, at the beginning of period 1, the import wholesaler chooses Q to maximize

$$\int_{-d}^d \frac{4Q(2A + \varepsilon_1) + \varepsilon_1^2 - 4Q^2}{8} \frac{1}{2d} d\varepsilon_1 - Q(P + c_w) - T.$$

From the first-order condition

$$\int_{-d}^d \frac{4(2A + \varepsilon_1) - 8Q}{8} \frac{1}{2d} d\varepsilon_1 - (P + c_w) = 0,$$

we obtain the optimal order quantity $Q = A - (P + c_w)$ and a total expected profit of import wholesaler of

$$\frac{[A - (P + c_w)]^2}{2} + \frac{d^2}{24} - T.$$

As in the case of non-lumpy trade the manufacturer sets $P = c$, and extracts the wholesaler's profit through the transfer T . The total expected manufacturer profit from exporting to both countries via import wholesalers then is

$$E(\pi_{imp}^l) = [A - (c + c_w)]^2 + \frac{d^2}{12}.$$

B.1.3 Export Wholesaler

When facing wholesale price w_i , the competitive firms in country $i = \alpha, \beta$ will order goods until their expected profit is zero and thus

$$\frac{(2A - Q_i + \varepsilon_{i1})}{2} Q_i - w_i Q_i = 0$$

The total revenue that the export wholesaler can generate by selling to the competitive firms in the two countries then is

$$R = \frac{(2A - Q_\alpha + \varepsilon_{\alpha 1})}{2} Q_\alpha + \frac{(2A - Q_\beta + \varepsilon_{\beta 1})}{2} Q_\beta, \quad (\text{B.3})$$

where Q_α and Q_β are the quantities of products allocated to countries α and β , respectively, and $Q_\alpha + Q_\beta = Q$. After Q has been allocated, the competitive firms in each country sell quantity Q_i for two periods, and equalize the first period price and the expected second-period price as has been discussed in the direct export subsection.

Consider the optimal strategy of the export wholesaler after he has observed the demand shocks $\varepsilon_{\alpha 1}$ and $\varepsilon_{\beta 1}$. The export wholesaler sets wholesale prices so that the competitive firms order quantities Q_α and Q_β that equalize marginal revenues in the two

countries:

$$2A - 2Q_\alpha + \varepsilon_{\alpha 1} = 2A - 2Q_\beta + \varepsilon_{\beta 1}$$

Using $Q_\alpha + Q_\beta = Q$, we obtain $Q_\alpha = (2Q + (\varepsilon_{1\alpha} - \varepsilon_{1\beta})) / 4$ and $Q_\beta = (2Q - (\varepsilon_{1\alpha} - \varepsilon_{1\beta})) / 4$. Using these values in (B.3) and taking expectations, the total expected revenue of the export wholesaler

$$\int_{-d}^d \int_{-d}^d \frac{16AQ + 4Q(\varepsilon_{\alpha 1} + \varepsilon_{\beta 1}) - 4Q^2 + (\varepsilon_{\alpha 1} - \varepsilon_{\beta 1})^2}{16} f(\varepsilon_{\alpha t}, \varepsilon_{\beta t}) d\varepsilon_{\alpha 1} d\varepsilon_{\beta 1}. \quad (\text{B.4})$$

Now note that

$$\begin{aligned} & \int_{-d}^d \int_{-d}^d (\varepsilon_{\alpha 1} + \varepsilon_{\beta 1}) f(\varepsilon_{\alpha t}, \varepsilon_{\beta t}) d\varepsilon_{\alpha 1} d\varepsilon_{\beta 1} \\ &= \int_{-d}^d \int_{-d}^d \varepsilon_{\alpha 1} f(\varepsilon_{\alpha t}, \varepsilon_{\beta t}) d\varepsilon_{\alpha 1} d\varepsilon_{\beta 1} + \int_{-d}^d \int_{-d}^d \varepsilon_{\beta 1} f(\varepsilon_{\alpha t}, \varepsilon_{\beta t}) d\varepsilon_{\alpha 1} d\varepsilon_{\beta 1} \\ &= \int_{-d}^d \varepsilon_{\alpha 1} f_\alpha(\varepsilon_{\alpha t}) d\varepsilon_{\alpha 1} + \int_{-d}^d \varepsilon_{\beta 1} f_\beta(\varepsilon_{\beta t}) d\varepsilon_{\beta 1} \\ &= \int_{-d}^d \varepsilon_{\alpha 1} \frac{1}{2d} d\varepsilon_{\alpha 1} + \int_{-d}^d \varepsilon_{\beta 1} \frac{1}{2d} d\varepsilon_{\beta 1} = 0, \end{aligned}$$

and

$$\begin{aligned}
& \int_{-d}^d \int_{-d}^d (\varepsilon_{1\alpha} - \varepsilon_{1\beta})^2 f(\varepsilon_{\alpha t}, \varepsilon_{\beta t}) d\varepsilon_{\alpha 1} d\varepsilon_{\beta 1} \\
&= \int_{-d}^d \int_{-d}^d (\varepsilon_{1\alpha} - \varepsilon_{1\beta})^2 (\varepsilon_{\alpha 1}^2 + \varepsilon_{\beta 1}^2 - 2\varepsilon_{\alpha 1}\varepsilon_{\beta 1}) f(\varepsilon_{\alpha t}, \varepsilon_{\beta t}) d\varepsilon_{\alpha 1} d\varepsilon_{\beta 1} \\
&= \int_{-d}^d \varepsilon_{\alpha 1}^2 \frac{1}{2d} d\varepsilon_{\alpha 1} + \int_{-d}^d \varepsilon_{\beta 1}^2 \frac{1}{2d} d\varepsilon_{\beta 1} - 2E(\varepsilon_{\alpha 1}\varepsilon_{\beta 1}) \\
&= \frac{2d^2}{3} - 2[\text{cov}(\varepsilon_{\alpha 1}, \varepsilon_{\beta 1}) + E(\varepsilon_{\alpha 1})E(\varepsilon_{\beta 1})] \\
&= \frac{2d^2}{3} - 2\rho\sigma_{\alpha 1}\sigma_{\beta 1} \\
&= \frac{2d^2}{3}(1 - \rho).
\end{aligned}$$

Thus (B.4) reduces to

$$\frac{4AQ - Q^2}{4} + \frac{d^2}{24}(1 - \rho).$$

Setting marginal revenue equal to marginal cost ($P + c_w$) yields the wholesaler's optimal order $Q = 2(A - (P + c_w))$. We can then compute the wholesaler's expected profit, which is

$$[A - (P + c_w)]^2 + \frac{d^2}{24}(1 - \rho) - T.$$

It is again straightforward to show that the manufacturer will optimally choose $P = c$ and extract the wholesaler's profit through the transfer T . Thus the expected profit of the manufacturer is

$$E(\pi_{exp}^l) = [A - (c + c_w)]^2 + \frac{d^2}{24}(1 - \rho).$$

B.2 Proof of Proposition 3.6

B.2.1 Non-lumpy Trade

At the beginning of period 2, denote the optimal inventory level as I_2^* . From the analysis above, we know that this optimal inventory level should be the same with direct exports

and trade through an import wholesaler. I_2^o is determined by the condition $p(I_2^o) + I_2^o p'(I_2^o) = c$. In period 1 after demand has been revealed, "price equalization" across periods by downstream firms in the case of direct exports implies sales in period 1, $s_1^{direct}(\varepsilon_1)$, such that:

$$p(s_1^{direct}) + \varepsilon_1 = p(I_2^o). \quad (\text{B.5})$$

"Marginal revenue equalization" across periods by an import wholesaler implies first-period sales of $s_1^{im}(\varepsilon_1)$, such that

$$p(s_1^{im}) + s_1^{im} p'(s_1^{im}) + \varepsilon_1 = p(I_2^o) + I_2^o p'(I_2^o). \quad (\text{B.6})$$

Note that applying the implicit function theorem to (B.6) we obtain:

$$\frac{\partial s_1^{im}(\varepsilon_1)}{\partial \varepsilon_1} = -\frac{1}{2p'(s_1^{im}) + s_1^{im} p''(s_1^{im})} > 0,$$

and

$$\begin{aligned} \frac{\partial^2 s_1^{im}(\varepsilon_1)}{\partial \varepsilon_1^2} &= -\frac{-[3p''(s_1^{im}) + s_1^{im} p'''(s_1^{im})] \frac{\partial s_1^{im}(\varepsilon_1)}{\partial \varepsilon_1}}{[2p'(s_1^{im}) + s_1^{im} p''(s_1^{im})]^2} \\ &= -\frac{3p''(s_1^{im}) + s_1^{im} p'''(s_1^{im})}{[2p'(s_1^{im}) + s_1^{im} p''(s_1^{im})]^3} > 0. \end{aligned}$$

The proof now proceeds by proving several lemmas. The first two lemmas establish the result that the expected first-period price is greater in the case of an import wholesaler than in the case of direct exports.

Lemma B.1 $s_1^{im} p'(s_1^{im})$ is concave in ε_1 ; or equivalently, $-s_1^{im} p'(s_1^{im})$ is convex in ε_1 .

Proof.

$$\begin{aligned}
\frac{\partial}{\partial \varepsilon_1} [s_1^{im} p'(s_1^{im})] &= [p'(s_1^{im}) + s_1^{im} p''(s_1^{im})] \frac{\partial s_1^{im}(\varepsilon_1)}{\partial \varepsilon_1} \\
\frac{\partial^2}{\partial \varepsilon_1^2} [s_1^w p'(s_1^w)] &= [2p''(s_1^{im}) + s_1^{im} p'''(s_1^{im})] \left(\frac{\partial s_1^{im}}{\partial \varepsilon_1} \right)^2 \\
&\quad + [p'(s_1^{im}) + s_1^{im} p''(s_1^{im})] \frac{\partial^2 s_1^{im}}{\partial \varepsilon_1^2} \\
&= \frac{2p''(s_1^{im}) + s_1^{im} p'''(s_1^{im})}{[2p'(s_1^{im}) + s_1^{im} p''(s_1^{im})]^2} \\
&\quad - \frac{[3p''(s_1^{im}) + s_1^{im} p'''(s_1^{im})][p'(s_1^{im}) + s_1^{im} p''(s_1^{im})]}{[2p'(s_1^{im}) + s_1^{im} p''(s_1^{im})]^3} \\
&= \frac{\left\{ \begin{aligned} &[2p''(s_1^{im}) + s_1^{im} p'''(s_1^{im})][2p'(s_1^{im}) + s_1^{im} p''(s_1^{im})] \\ &- [3p''(s_1^{im}) + s_1^{im} p'''(s_1^{im})][p'(s_1^{im}) + s_1^{im} p''(s_1^{im})] \end{aligned} \right\}}{[2p'(s_1^{im}) + s_1^{im} p''(s_1^{im})]^3} \\
&\doteq \frac{K(s_1^{im})}{[2p'(s_1^{im}) + s_1^{im} p''(s_1^{im})]^3}
\end{aligned}$$

where

$$\begin{aligned}
K(s) &= [4p'p'' + 2s(p'')^2 + 2sp'p''' + s^2p''p'''] \\
&\quad - [3p'p'' + 3s(p'')^2 + sp'p''' + s^2p''p'''] \\
&= p'p'' - s(p'')^2 + sp'p'''.
\end{aligned}$$

By (A3.2), $0 \leq 3p''(s) + sp'''(s) \leq \frac{s[p''(s)]^2}{p'(s)} + 2p''(s)$,

$$\begin{aligned}
&\Rightarrow p'' + sp''' \leq \frac{s(p'')^2}{p'} \\
&\Leftrightarrow p'p'' - s(p'')^2 + sp'p''' \geq 0.
\end{aligned}$$

Therefore $\frac{\partial^2}{\partial \varepsilon_1^2} [s_1^{im} p'(s_1^{im})] = \frac{1}{[2p'(s_1^{im}) + s_1^{im} p''(s_1^{im})]^3} K(s_1^{im}) \leq 0$. \blacksquare

Lemma B.2 $E[p(s_1^{im})] \geq E[p(s_1^{direct})]$

Proof.

$$\begin{aligned}
& E [p (s_1^{im})] - E [p (s_1^{direct})] \\
&= E [I_2^o p' (I_2^o) - s_1^{im} p' (s_1^{im})] \\
&= I_2^o p' (I_2^o) - E [s_1^{im} p' (s_1^{im})]
\end{aligned}$$

Note that $E (\varepsilon_1) = \bar{\varepsilon} = 0$, and $s_1^{im} (\bar{\varepsilon}) = I_2^o$. Using Lemma B.1 and *Jensen's inequality*,^{B.1} we know that $E [-s_1^{im} p' (s_1^{im})] \geq -s_1^{im} (\bar{\varepsilon}) p' [s_1^{im} (\bar{\varepsilon})] = -I_2^o p' (I_2^o)$. This proves that $E [p (s_1^{im})] - E [p (s_1^{direct})] \geq 0$. ■

The next two lemmas establish that expected first-period sales are lower with an import wholesaler than with direct exports.

Lemma B.3 *If $\varepsilon_1 < 0$, then $s_1^{direct} < s_1^{im} < I_2^o$; if $\varepsilon_1 > 0$, then $s_1^{direct} > s_1^{im} > I_2^o$.*

Proof.

$$\begin{aligned}
p (s_1^{im}) - p (s_1^{direct}) &= p (I_2^o) + I_2^o p' (I_2^o) - s_1^{im} p' (s_1^{im}) - p (I_2^o) \\
&= I_2^o p' (I_2^o) - s_1^{im} p' (s_1^{im}).
\end{aligned}$$

In the case of $\varepsilon_1 < 0$: from (B.6) we know $s_1^{im} < I_2^o$. Since by (A3.2), $p' (s) + s p'' (s) < 0$, we have $I_2^o p' (I_2^o) - s_1^{im} p' (s_1^{im}) < 0$, and thus $s_1^{im} > s_1^{direct}$. Similarly for $\varepsilon_1 > 0$ we can prove $I_2^o < s_1^{im} < s_1^{direct}$. ■

Lemma B.4 $E [s_1^{im}] \leq E [s_1^{direct}]$

Proof. Mean value theorem tells us that for any ε_1 , there is a

$$\zeta (\varepsilon_1) \in [\min (s_1^{direct}, s_1^{im}), \max (s_1^{direct}, s_1^{im})]$$

such that $p' (\zeta) = \frac{p(s_1^{im}) - p(s_1^{direct})}{s_1^{im} - s_1^{direct}}$. This implies

$$\begin{aligned}
s_1^{im} - s_1^{direct} &= \frac{p (s_1^{im}) - p (s_1^{direct})}{p' (\zeta)} \\
&= \frac{I_2^o p' (I_2^o) - s_1^{im} p' (s_1^{im})}{p' (\zeta)}.
\end{aligned}$$

^{B.1}If X is a random variable and φ is a convex function, then $\varphi [E (X)] \leq E [\varphi (X)]$.

Note that

$$\frac{d}{ds} \left[\frac{1}{p'(s)} \right] = -\frac{p''(s)}{[p'(s)]^2} < 0,$$

and consider two cases: If $\varepsilon_1 < 0$, then from Lemma B.3, $s_1^{direct} < \zeta < s_1^{im} < I_2^o$ and $I_2^o p'(I_2^o) - s_1^{im} p'(s_1^{im}) < 0$. If $\varepsilon_1 > 0$, then Lemma B.3 implies $I_2^o < s_1^{im} < \zeta < s_1^{direct}$ and $I_2^o p'(I_2^o) - s_1^{im} p'(s_1^{im}) > 0$. Hence in both cases, we have

$$\begin{aligned} s_1^{im} - s_1^{direct} &= \frac{I_2^o p'(I_2^o) - s_1^{im} p'(s_1^{im})}{p'(\zeta)} \\ &\leq \frac{I_2^o p'(I_2^o) - s_1^{im} p'(s_1^{im})}{p'(I_2^o)} \\ &= I_2^o - \frac{s_1^{im} p'(s_1^{im})}{p'(I_2^o)}. \end{aligned}$$

Taking expectations yields

$$\begin{aligned} E[s_1^{im} - s_1^{direct}] &\leq E \left[I_2^o - \frac{s_1^{im} p'(s_1^{im})}{p'(I_2^o)} \right] \\ &= I_2^o - \frac{1}{p'(I_2^o)} E[s_1^{im} p'(s_1^{im})]. \end{aligned}$$

Since we know from the proof of Lemma B.2 that $E[s_1^{im} p'(s_1^{im})] \leq I_2^o p'(I_2^o)$, we can state

$$\begin{aligned} E[s_1^{im} - s_1^{direct}] &\leq I_2^o - \frac{1}{p'(I_2^o)} I_2^o p'(I_2^o) \\ &= 0. \end{aligned}$$

■

Finally note that in our two-period model, $s_2^{im} = s_2^{direct}$ and $p(s_2^{im}) = p(s_2^{direct})$. Hence Lemmas B.2 and B.4 imply directly that an import wholesaler reduces the expected volume of trade and consumer surplus relative to direct exports if trade is non-lumpy. Since the expected profits of the import wholesaler and downstream customers are zero, social welfare in the destination countries decreases when trade is intermediated by an import wholesaler.

B.2.2 Lumpy Trade

Suppose that quantity Q has been delivered to competitive downstream firms or an import wholesaler, respectively. In period 1 after demand has been revealed, "price equalization" across periods by downstream firms in the case of direct exports implies sales in period 1, $s_1^{direct}(\varepsilon_1)$, such that:

$$p(s_1^{direct}) + \varepsilon_1 = p(Q - s_1^{direct}), \quad (\text{B.7})$$

where

$$\begin{aligned} \frac{\partial s_1^{direct}}{\partial Q} &= \frac{p'(Q - s_1^{direct})}{p'(s_1^{direct}) + p'(Q - s_1^{direct})} \\ &= \end{aligned}$$

"Marginal revenue equalization" across periods by an import wholesaler implies first-period sales of $s_1^{im}(\varepsilon_1)$, such that

$$p(s_1^{im}) + s_1^{im} p'(s_1^{im}) + \varepsilon_1 = p(Q - s_1^{im}) + (Q - s_1^{im}) p'(Q - s_1^{im}), \quad (\text{B.8})$$

where

$$\begin{aligned} \frac{\partial s_1^w}{\partial Q} &= \frac{2p'(Q - s_1^{im}) + (Q - s_1^{im}) p''(Q - s_1^{im})}{2p'(s_1^{im}) + s_1^{im} p''(s_1^{im}) + 2p'(Q - s_1^{im}) + (Q - s_1^{im}) p''(Q - s_1^{im})} \\ &= \frac{1}{\frac{2p'(s_1^{im}) + s_1^{im} p''(s_1^{im})}{2p'(Q - s_1^{im}) + (Q - s_1^{im}) p''(Q - s_1^{im})} + 1} = \frac{1}{\frac{MR'(s_1^{im})}{MR'(s_2^{im})} + 1}. \end{aligned}$$

In order to compare the expected volume of trade in the case of direct exports with that in the case of an import wholesaler, we need to know the sum of marginal revenues in periods 1 and 2 for a given inventory level Q . Since marginal costs are the same in both cases, the case with the higher expected total marginal revenue yields the larger trade volume in equilibrium.

Using (B.8), the sum of marginal revenues in the case of an import wholesaler is

$$\begin{aligned}
\sum MR^{im} &= \frac{\partial}{\partial Q} [p(s_1^{im}) s_1^{im} + \varepsilon_1 s_1^{im} + (Q - s_1^{im}) p(Q - s_1^{im})] \\
&= [p(s_1^{im}) + s_1^{im} p'(s_1^{im}) + \varepsilon_1 - p(Q - s_1^{im}) - (Q - s_1^{im}) p'(Q - s_1^{im})] \frac{\partial s_1^{im}}{\partial Q} \\
&\quad + p(Q - s_1^{im}) + (Q - s_1^{im}) p'(Q - s_1^{im}) \\
&= [p(s_1^{im}) + s_1^{im} p'(s_1^{im}) + \varepsilon_1] \frac{\partial s_1^{im}}{\partial Q} \\
&\quad + [p(Q - s_1^{im}) + (Q - s_1^{im}) p'(Q - s_1^{im})] \left(1 - \frac{\partial s_1^{im}}{\partial Q}\right) \\
&= [MR(s_1^{im}) + \varepsilon_1] \frac{\partial s_1^{im}}{\partial Q} + MR(s_2^{im}) \left(1 - \frac{\partial s_1^{im}}{\partial Q}\right).
\end{aligned}$$

From (B.7) the sum of marginal revenues in the case of direct exports is

$$\begin{aligned}
\sum MR^{direct} &= [p(s_1^{direct}) + s_1^{direct} p'(s_1^{direct}) + \varepsilon_1] \frac{\partial s_1^{direct}}{\partial Q} \\
&\quad + [p(Q - s_1^{direct}) + (Q - s_1^{direct}) p'(Q - s_1^{direct})] \left(1 - \frac{\partial s_1^{direct}}{\partial Q}\right) \\
&= [MR(s_1^{direct}) + \varepsilon_1] \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{direct}) \left(1 - \frac{\partial s_1^{direct}}{\partial Q}\right).
\end{aligned}$$

We can thus write the difference of the sums of marginal revenues as

$$\begin{aligned}
\Delta &= \sum MR^{direct} - \sum MR^{im} \\
&= MR(s_1^{direct}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{direct}) \left(1 - \frac{\partial s_1^{direct}}{\partial Q}\right) \\
&\quad - MR(s_1^{im}) \frac{\partial s_1^{im}}{\partial Q} - MR(s_2^{im}) \left(1 - \frac{\partial s_1^{direct}}{\partial Q}\right) \\
&\quad + \varepsilon_1 \left(\frac{\partial s_1^{direct}}{\partial Q} - \frac{\partial s_1^{im}}{\partial Q}\right).
\end{aligned}$$

Notice that without any ambiguity we can write $\left(1 - \frac{\partial s_1^j}{\partial Q}\right) = \frac{\partial s_2^j}{\partial Q}$, $j = im, direct$.

The following lemma shows that direct exports are more sensitive to demand shocks than trade intermediated by an import wholesaler.

Lemma B.5 *Given inventory level Q and the resolved demand shock ε_1 in period 1, if $\varepsilon_1 < 0$, then $s_1^{direct} < s_1^{im}$ and $s_2^{direct} > s_2^{im}$; if $\varepsilon_1 > 0$, then $s_1^{direct} > s_1^{im}$ and $s_2^{direct} < s_2^{im}$, where $s_1^{direct} + s_2^{direct} = s_1^{im} + s_2^{im} = Q$.*

Proof. Using $s_2^{im} = Q - s_1^{im}$ and $s_2^{direct} = Q - s_1^{direct}$, we can rewrite (B.8) as

$$p(s_1^{im}) + s_1^{im} p'(s_1^{im}) + \varepsilon_1 = p(s_2^{im}) + s_2^{im} p'(s_2^{im}),$$

and

$$p(s_1^{direct}) + \varepsilon_1 = p(s_2^{direct}).$$

Then

$$p(s_1^{im}) - p(s_1^{direct}) = p(s_2^{im}) + s_2^{im} p'(s_2^{im}) - s_1^{im} p'(s_1^{im}) - p(s_2^{direct}) \doteq B.$$

If $\varepsilon_1 < 0$, then clearly $\{s_1^{direct}, s_1^{im}\} < \frac{1}{2}Q < \{s_2^{direct}, s_2^{im}\}$. Suppose $s_1^{im} < s_1^{direct}$, then $B > 0$ and $s_2^{direct} < s_2^{im}$. But note

$$\begin{aligned} B &< p(s_2^{direct}) + s_2^{direct} p'(s_2^{direct}) - s_1^{im} p'(s_1^{im}) - p(s_2^{direct}) \\ &= s_2^{direct} p'(s_2^{direct}) - s_1^{im} p'(s_1^{im}) \\ &< 0. \end{aligned}$$

The first inequality follows $MR'(s) < 0$ and $s_2^{direct} < s_2^{im}$; the second inequality follows $p' + sp'' < 0$ and $s_2^{direct} > s_1^{im}$. This means there is a contradiction, and $s_1^{direct} < s_1^{im}$.

If $\varepsilon_1 > 0$, we have $\{s_1^{direct}, s_1^{im}\} > \frac{1}{2}Q > \{s_2^{direct}, s_2^{im}\}$. Suppose $s_1^{im} > s_1^{direct}$, then $B < 0$ and $s_2^{direct} > s_2^{im}$. But

$$\begin{aligned} B &> p(s_2^{direct}) + s_2^{direct} p'(s_2^{direct}) - s_1^{im} p'(s_1^{im}) - p(s_2^{direct}) \\ &= s_2^{direct} p'(s_2^{direct}) - s_1^{im} p'(s_1^{im}) \\ &> 0. \end{aligned}$$

Hence there is a contradiction, and $s_1^{direct} > s_1^{im}$. ■

This lemma is used to prove the following result:

Lemma B.6

$$\begin{aligned} & MR(s_1^{direct}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{direct}) \frac{\partial s_2^{direct}}{\partial Q} \\ & \leq MR(s_1^{im}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{im}) \frac{\partial s_2^{direct}}{\partial Q}, \forall \varepsilon_1 \end{aligned}$$

Proof. Let $s_L^{direct} = \min\{s_1^{direct}, s_2^{direct}\}$, and $s_H^{direct} = \max\{s_1^{direct}, s_2^{direct}\}$. We may construct a function $\Gamma(x)$ with domain $x \in [0, \bar{x}]$, where $\bar{x} \doteq \frac{1}{2}Q - s_L^{direct} = s_H^{direct} - \frac{1}{2}Q$, such that

$$\begin{aligned} \Gamma(x) &= MR\left(\frac{1}{2}Q - x\right) \frac{\partial s_L^{direct}}{\partial Q} + MR\left(\frac{1}{2}Q + x\right) \frac{\partial s_H^{direct}}{\partial Q} \\ &= MR\left(\frac{1}{2}Q - x\right) \frac{p'(s_H^{direct})}{p'(s_L^{direct}) + p'(s_H^{direct})} \\ &\quad + MR\left(\frac{1}{2}Q + x\right) \frac{p'(s_L^{direct})}{p'(s_L^{direct}) + p'(s_H^{direct})}. \end{aligned}$$

We want to prove $\Gamma'(x) \leq 0$, where

$$\begin{aligned} \Gamma'(x) &= MR' \left(\frac{1}{2}Q + x \right) \frac{p'(s_L^{direct})}{p'(s_L^{direct}) + p'(s_H^{direct})} \\ &\quad - MR' \left(\frac{1}{2}Q - x \right) \frac{p'(s_H^{direct})}{p'(s_L^{direct}) + p'(s_H^{direct})}. \end{aligned}$$

Since $p'(\cdot) < 0$, this is equivalent to proving that

$$MR' \left(\frac{1}{2}Q + x \right) p'(s_L^{direct}) - MR' \left(\frac{1}{2}Q - x \right) p'(s_H^{direct}) \geq 0$$

We define $\xi(s) = -\frac{sp''(s)}{p'(s)}$ as the elasticity of the slope of the inverse demand curve. Mathematically, $\xi(s)$ is the convexity of the inverse demand function, and $\xi'(s) \leq 0$.

Then,

$$\begin{aligned}
& MR' \left(\frac{1}{2}Q + x \right) p' (s_L^{direct}) - MR' \left(\frac{1}{2}Q - x \right) p' (s_H^{direct}) \\
& \geq MR' \left(\frac{1}{2}Q + \bar{x} \right) p' (s_L^{direct}) - MR' \left(\frac{1}{2}Q - \bar{x} \right) p' (s_H^{direct}) \\
& = MR' \left(\frac{1}{2}Q + s_H^{direct} - \frac{1}{2}Q \right) p' (s_L^{direct}) \\
& \quad - MR' \left(\frac{1}{2}Q - \frac{1}{2}Q + s_L^{direct} \right) p' (s_H^{direct}) \\
& = MR' (s_H^{direct}) p' (s_L^{direct}) - MR' (s_L^{direct}) p' (s_H^{direct}) \\
& = p' (s_L^{direct}) p' (s_H^{direct}) \left[\frac{MR' (s_H^{direct})}{p' (s_H^{direct})} - \frac{MR' (s_L^{direct})}{p' (s_L^{direct})} \right] \\
& = p' (s_L^{direct}) p' (s_H^{direct}) [2 - \xi (s_H^{direct}) - 2 + \xi (s_L^{direct})] \\
& = p' (s_L^{direct}) p' (s_H^{direct}) [\xi (s_L^{direct}) - \xi (s_H^{direct})] \\
& \geq 0
\end{aligned}$$

The first inequality comes from $MR''(s) \geq 0$ and $p'(s) < 0$; the second inequality comes from $\xi'(s) \leq 0$, which holds since by Assumption (A3.2) we have $2p''(s) + sp'''(s) \leq \frac{s[p''(s)]^2}{p'(s)} + p''(s)$. This proves that $\Gamma'(x) \leq 0$.

Next define $x^{direct} \doteq |s_1^{direct} - \frac{1}{2}Q| = |s_2^{direct} - \frac{1}{2}Q|$, and $x^{im} \doteq |s_1^{im} - \frac{1}{2}Q| = |s_2^{im} - \frac{1}{2}Q|$. From Lemma B.5 we know $x^{direct} > x^{im}$.^{B.2} Since $\Gamma'(x) \leq 0$ it follows that $\Gamma(x^{direct}) \leq \Gamma(x^{im})$. This implies that $MR(s_1^{direct}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{direct}) \frac{\partial s_2^{direct}}{\partial Q} \leq MR(s_1^{im}) \frac{\partial s_1^{im}}{\partial Q} + MR(s_2^{im}) \frac{\partial s_2^{im}}{\partial Q}$. ■

^{B.2}If $\varepsilon_1 < 0$, from Lemma B.5, $s_1^{direct} < s_1^{im} < \frac{1}{2}Q < s_2^{im} < s_2^{direct}$, then $x^{direct} = \frac{1}{2}Q - s_1^{direct} = s_2^{direct} - \frac{1}{2}Q$, $x^{im} = \frac{1}{2}Q - s_1^{im} = s_2^{im} - \frac{1}{2}Q$. The case of $\varepsilon_1 > 0$ is similar.

Applying Lemma B.6, we know that

$$\begin{aligned}
\Delta &\leq MR(s_1^{im}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{im}) \left(1 - \frac{\partial s_1^{direct}}{\partial Q}\right) \\
&\quad - MR(s_1^{im}) \frac{\partial s_1^{im}}{\partial Q} - MR(s_2^{im}) \left(1 - \frac{\partial s_1^{im}}{\partial Q}\right) + \varepsilon_1 \left(\frac{\partial s_1^{direct}}{\partial Q} - \frac{\partial s_1^{im}}{\partial Q}\right) \\
&= MR(s_1^{im}) \left(\frac{\partial s_1^{direct}}{\partial Q} - \frac{\partial s_1^{im}}{\partial Q}\right) + MR(s_2^{im}) \left(\frac{\partial s_1^{im}}{\partial Q} - \frac{\partial s_1^{direct}}{\partial Q}\right) \\
&\quad + \varepsilon_1 \left(\frac{\partial s_1^{direct}}{\partial Q} - \frac{\partial s_1^{im}}{\partial Q}\right) \\
&= [MR(s_1^{im}) - MR(s_2^{im})] \left(\frac{\partial s_1^{direct}}{\partial Q} - \frac{\partial s_1^{im}}{\partial Q}\right) + \varepsilon_1 \left(\frac{\partial s_1^{direct}}{\partial Q} - \frac{\partial s_1^{im}}{\partial Q}\right) \\
&= -\varepsilon_1 \left(\frac{\partial s_1^{direct}}{\partial Q} - \frac{\partial s_1^{im}}{\partial Q}\right) + \varepsilon_1 \left(\frac{\partial s_1^{direct}}{\partial Q} - \frac{\partial s_1^{im}}{\partial Q}\right) \\
&= 0,
\end{aligned}$$

where we have used that fact that (B.8) implies $MR(s_1^{im}) - MR(s_2^{im}) = -\varepsilon_1$.

$\Delta \leq 0$ means that, given an inventory level Q , the marginal total revenue is always smaller when the manufacturer exports directly for any given ε_1 . Thus we know that at the beginning of period 1 before demand is revealed, the expected total marginal revenue is smaller in the case of direct exports than in the case where trade is intermediated by an import wholesaler. Since the manufacturer's marginal cost is the same in both cases, but expected total marginal revenue is greater in the latter case, it follows that $Q^{direct} \leq Q^{im}$.^{B.3}

Finally we want to show that if $Q^{direct} = Q^{im} = Q$, then consumer prices are higher in the case of direct exports, so that the welfare result goes through. Denote by $\Sigma P^j = p(s_1^j) + \varepsilon_1 + p(s_2^j)$, $j = direct, im$ the sum of first period price and expected second period price, given the realized ε_1 in period 1. Then

$$\Sigma P^{direct} - \Sigma P^{im} = [p(s_1^{direct}) + p(s_2^{direct})] - [p(s_1^{im}) + p(s_2^{im})].$$

Consider a function

$$\Phi(x) = p\left(\frac{1}{2}Q + x\right) + p\left(\frac{1}{2}Q - x\right).$$

^{B.3}Here we can easily check that the total revenue function is concave.

From Lemma B.5 we know that $x^{direct} > x^{im}$, where $x^{direct} = \frac{1}{2}Q - s_L^{direct} = s_H^{direct} - \frac{1}{2}Q$ and $x^{im} = \frac{1}{2}Q - s_L^{im} = s_H^{im} - \frac{1}{2}Q$. Furthermore $\Phi'(x) = p'(\frac{1}{2}Q + x) - p'(\frac{1}{2}Q - x)$, and from $p''(s) \geq 0$ we know $\Phi'(x) \geq 0$. This yields $\Phi(x^{direct}) \geq \Phi(x^{im})$, which means $\Sigma P^{direct} - \Sigma P^{im} \geq 0$.

Combining the result that $Q^{direct} \leq Q^{im}$ with the result that if $Q^{direct} = Q^{im}$ then $\Sigma P^{direct} - \Sigma P^{im} \geq 0$ completes the proof that import wholesalers increase social welfare relative to direct exports if trade is lumpy.

B.3 Proof of Proposition 3.7

We proceed exactly as in the previous proof for the case of lumpy trade, but now consider a demand function $p_{it} = \varepsilon_{it} p_{it}(s_{it})$ with $p'_{it} < 0$ satisfying Assumption (A3.2). At the end we show that linear demand is a sufficient condition for the trade volume and welfare results.

Suppose that quantity Q has been delivered to competitive downstream firms or an import wholesaler, respectively. In period 1 after demand has been revealed, "price equalization" across periods by downstream firms in the case of direct exports implies sales in period 1, $s_1^{direct}(\varepsilon_1)$, such that:

$$\varepsilon_1 p(s_1^{direct}) = p(Q - s_1^{direct}), \quad (\text{B.9})$$

where

$$\frac{\partial s_1^{direct}}{\partial Q} = \frac{p'(Q - s_1^{direct})}{\varepsilon_1 p'(s_1^{direct}) + p'(Q - s_1^{direct})}.$$

"Marginal revenue equalization" across periods by an import wholesaler implies first-period sales of $s_1^{im}(\varepsilon_1)$, such that

$$\varepsilon_1 [p(s_1^{im}) + s_1^{im} p'(s_1^{im})] = p(Q - s_1^{im}) + (Q - s_1^{im}) p'(Q - s_1^{im}), \quad (\text{B.10})$$

where

$$\begin{aligned} \frac{\partial s_1^{im}}{\partial Q} &= \frac{2p'(Q - s_1^{im}) + (Q - s_1^{im}) p''(Q - s_1^{im})}{\varepsilon_1 [2p'(s_1^{im}) + s_1^{im} p''(s_1^{im})] + 2p'(Q - s_1^{im}) + (Q - s_1^{im}) p''(Q - s_1^{im})} \\ &= \frac{MR'(s_2^{im})}{\varepsilon_1 MR'(s_1^{im}) + MR'(s_2^{im})}. \end{aligned}$$

As before, we want to compare $\sum MR^{direct}$ with $\sum MR^{im}$. The sum of marginal revenues when trade is intermediated by an import wholesaler and ε_1 has been observed is

$$\begin{aligned}\sum MR^{im} &= \frac{\partial}{\partial Q} [\varepsilon_1 p(s_1^{im})(s_1^{im}) + p(Q - s_1^{im})(Q - s_1^{im})] \\ &= \varepsilon_1 MR(s_1^{im}) \frac{\partial s_1^{im}}{\partial Q} + MR(s_2^{im}) \left(1 - \frac{\partial s_1^{im}}{\partial Q}\right).\end{aligned}$$

The equivalent expression in the case of direct export is

$$\sum MR^{direct} = \varepsilon_1 MR(s_1^{direct}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{direct}) \left(1 - \frac{\partial s_1^{direct}}{\partial Q}\right).$$

Lemma B.7 *Given inventory level Q and the resolved demand shock ε_1 in period 1, if $\varepsilon_1 < 1$, then $s_1^{direct} < s_1^{im}$ and $s_2^{direct} > s_2^{im}$; if $\varepsilon_1 > 1$, then $s_1^{direct} > s_1^{im}$ and $s_2^{direct} < s_2^{im}$, where $s_1^{direct} + s_2^{direct} = s_1^{im} + s_2^{im} = Q$.*

Proof. Note that in this case $\bar{\varepsilon} = 1$. Using $s_2^{im} = Q - s_1^{im}$ and $s_2^{direct} = Q - s_1^{direct}$, we can rewrite (B.10) as

$$\varepsilon_1 [p(s_1^{im}) + s_1^{im} p'(s_1^{im})] = p(s_2^{im}) + s_2^{im} p'(s_2^{im})$$

and (B.9) as

$$\varepsilon_1 p(s_1^{direct}) = p(s_2^{direct}).$$

Then

$$\varepsilon_1 p(s_1^{im}) - \varepsilon_1 p(s_1^{direct}) = p(s_2^{im}) + s_2^{im} p'(s_2^{im}) - \varepsilon_1 s_1^{im} p'(s_1^{im}) - p(s_2^{direct}) \doteq C$$

If $0 < \varepsilon_1 < 1$, we have $\{s_1^{direct}, s_1^{im}\} < \frac{1}{2}Q < \{s_2^{direct}, s_2^{im}\}$. Suppose $s_1^{im} < s_1^{direct}$, then

$C > 0$ and $s_2^{direct} < s_2^{im}$. But note that

$$\begin{aligned}
C &< p(s_2^{direct}) + s_2^{direct} p'(s_2^{direct}) - \varepsilon_1 s_1^{im} p'(s_1^{im}) - p(s_2^{direct}) \\
&= s_2^{direct} p'(s_2^{direct}) - \varepsilon_1 s_1^{im} p'(s_1^{im}) \\
&< s_2^{direct} p'(s_2^{direct}) - s_1^{im} p'(s_1^{im}) \\
&< 0.
\end{aligned}$$

The first inequality follows $MR'(s) < 0$ and $s_2^{direct} < s_2^{im}$; the second inequality comes from $-s_1^{im} p'(s_1^{im}) > 0$ and $0 < \varepsilon_1 < 1$; the third inequality follows from $p' + sp'' < 0$ and $s_2^{direct} > s_1^{im}$. This means there is a contradiction, and hence $s_1^{direct} < s_1^{im}$.

Similarly, if $\varepsilon_1 > 1$, we have $\{s_1^{direct}, s_1^{im}\} > \frac{1}{2}Q > \{s_2^{direct}, s_2^{im}\}$. Suppose $s_1^{im} > s_1^{direct}$, then $C < 0$ and $s_2^{direct} > s_2^{im}$. However, in this case

$$\begin{aligned}
C &> p(s_2^{direct}) + s_2^{direct} p'(s_2^{direct}) - \varepsilon_1 s_1^{im} p'(s_1^{im}) - p(s_2^{direct}) \\
&= s_2^{direct} p'(s_2^{direct}) - \varepsilon_1 s_1^{im} p'(s_1^{im}) \\
&> s_2^{direct} p'(s_2^{direct}) - s_1^{im} p'(s_1^{im}) \\
&> 0
\end{aligned}$$

The first inequality follows $MR'(s) < 0$ and $s_2^{direct} > s_2^{im}$; the second inequality comes from $-s_1^{im} p'(s_1^{im}) > 0$ and $\varepsilon_1 > 1$; the third inequality follows $p' + sp'' < 0$ and $s_2^{direct} < s_1^{im}$. This again means there is a contradiction, and hence $s_1^{direct} > s_1^{im}$. ■

Lemma B.8

$$\begin{aligned}
&\varepsilon_1 MR(s_1^{direct}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{direct}) \frac{\partial s_2^{direct}}{\partial Q} \\
&\leq \varepsilon_1 MR(s_1^{im}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{im}) \frac{\partial s_2^{direct}}{\partial Q}, \forall \varepsilon_1
\end{aligned}$$

Proof.

$$\begin{aligned}
& \varepsilon_1 MR(s_1^{direct}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{direct}) \frac{\partial s_2^{direct}}{\partial Q} \\
&= \varepsilon_1 MR(s_1^{direct}) \frac{p'(s_2^{direct})}{\varepsilon_1 p'(s_1^{direct}) + p'(s_2^{direct})} \\
&+ MR(s_2^{direct}) \frac{\varepsilon_1 p'(s_1^{direct})}{\varepsilon_1 p'(s_1^{direct}) + p'(s_2^{direct})} \\
&= \frac{\frac{p(s_2^{direct})}{p(s_1^{direct})} [MR(s_1^{direct}) p'(s_2^{direct}) + MR(s_2^{direct}) p'(s_1^{direct})]}{\varepsilon_1 p'(s_1^d) + p'(s_2^d)} \\
&= \frac{p(s_2^{direct}) [MR(s_1^{direct}) p'(s_2^{direct}) + MR(s_2^{direct}) p'(s_1^{direct})]}{p(s_1^{direct}) [\varepsilon_1 p'(s_1^{direct}) + p'(s_2^{direct})]},
\end{aligned}$$

where we have used the fact that $\frac{p(s_2^{direct})}{p(s_1^{direct})} = \varepsilon_1$ from (B.10).

Now define a function

$$\Phi(x) = MR\left(\frac{1}{2}Q - x\right) p'(s_H^{direct}) + MR\left(\frac{1}{2}Q + x\right) p'(s_L^{direct})$$

where as before $s_L^{direct} = \min\{s_1^{direct}, s_2^{direct}\}$, and $s_H^{direct} = \max\{s_1^{direct}, s_2^{direct}\}$, and $x \leq \frac{1}{2}Q - s_L^{direct} = s_H^{direct} - \frac{1}{2}Q \doteq \bar{x}$. This function has the following properties

$$\begin{aligned}
\Phi'(x) &= MR'\left(\frac{1}{2}Q + x\right) p'(s_L^{direct}) - MR'\left(\frac{1}{2}Q - x\right) p'(s_H^{direct}) \\
\Phi''(x) &= MR''\left(\frac{1}{2}Q + x\right) p'(s_L^{direct}) + MR''\left(\frac{1}{2}Q - x\right) p'(s_H^{direct}) \leq 0.
\end{aligned}$$

It follows that

$$\begin{aligned}
\Phi'(x) &\geq \Phi'(\bar{x}) \\
&= MR'(s_H^{direct}) p'(s_L^{direct}) - MR'(s_L^{direct}) p'(s_H^{direct}) \\
&= p'(s_L^{direct}) p'(s_H^{direct}) \left[\frac{MR'(s_H^{direct})}{p'(s_H^{direct})} - \frac{MR'(s_L^{direct})}{p'(s_L^{direct})} \right] \\
&= p'(s_L^{direct}) p'(s_H^{direct}) [2 - \xi(s_H^{direct}) - 2 + \xi(s_L^{direct})] \\
&= p'(s_L^{direct}) p'(s_H^{direct}) [\xi(s_L^{direct}) - \xi(s_H^{direct})] \\
&\geq 0
\end{aligned}$$

From Lemma B.7, we know that $x^{direct} > x^{im}$, where $x^{direct} \doteq \frac{1}{2}Q - s_L^{direct} = s_H^{direct} - \frac{1}{2}Q$ and $x^{im} \doteq \frac{1}{2}Q - s_L^{im} = s_H^{im} - \frac{1}{2}Q$. Therefore

$$\begin{aligned}
\Phi(x^{direct}) &= MR(s_L^{direct}) p'(s_H^{direct}) + MR(s_H^{direct}) p'(s_L^{direct}) \\
&\geq \Phi(x^{im}) \\
&= MR(s_L^{im}) p'(s_H^{direct}) + MR(s_H^{im}) p'(s_L^{direct}).
\end{aligned}$$

Since $p(s_1^{direct}) [\varepsilon_1 p'(s_1^{direct}) + p'(s_2^{direct})] < 0$, then

$$\frac{p(s_2^{direct}) \Phi(x^{direct})}{p(s_1^{direct}) [\varepsilon_1 p'(s_1^{direct}) + p'(s_2^{direct})]} \leq \frac{p(s_2^{direct}) \Phi(x^{im})}{p(s_1^{direct}) [\varepsilon_1 p'(s_1^{direct}) + p'(s_2^{direct})]}$$

which implies the desired result that

$$\varepsilon_1 MR(s_1^{direct}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{direct}) \frac{\partial s_2^{direct}}{\partial Q} \leq \varepsilon_1 MR(s_1^{im}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{im}) \frac{\partial s_2^{direct}}{\partial Q}.$$

■

Note that $\varepsilon_1 MR(s_1^{im}) = MR(s_2^{im})$, which follows from (B.10). Combining this equality with the fact that $\frac{\partial s_1^{direct}}{\partial Q} + \frac{\partial s_2^{direct}}{\partial Q} = \frac{\partial s_1^{im}}{\partial Q} + \frac{\partial s_2^{im}}{\partial Q} = 1$ implies that

$$\varepsilon_1 MR(s_1^{im}) \frac{\partial s_1^{direct}}{\partial Q} + MR(s_2^{im}) \frac{\partial s_2^{direct}}{\partial Q} = \varepsilon_1 MR(s_1^{im}) \frac{\partial s_1^{im}}{\partial Q} + MR(s_2^{im}) \frac{\partial s_2^{im}}{\partial Q}. \quad (\text{B.11})$$

Using Lemma B.8 and (B.11) establishes that $\sum MR^{direct} < \sum MR^{im}$. Following the same arguments as in the proof of Proposition 3.6, we know that in expected terms $Q^{direct} \leq Q^{im}$.

Finally, consider again the case where $Q^{direct} = Q^{im} = Q$. Specifically consider the difference between the sums of first period prices and expected second period prices in the case of direct exports and the case where an import wholesaler is used

$$\begin{aligned} \Sigma P^{direct} - \Sigma P^{im} &= [\varepsilon_1 p(s_1^{direct}) + p(s_2^{direct})] - [\varepsilon_1 p(s_1^{im}) + p(s_2^{im})] \\ &= \left[\frac{p(s_2^{direct})}{p(s_1^{direct})} p(s_1^{direct}) + p(s_2^{direct}) \right] \\ &\quad - \left[\frac{p(s_2^{direct})}{p(s_1^{direct})} p(s_1^{im}) + p(s_2^{im}) \right] \\ &= \frac{[2p(s_2^{direct}) p(s_1^{direct}) - p(s_2^{direct}) p(s_1^{im}) - p(s_2^{im}) p(s_1^{direct})]}{p(s_1^{direct})}. \end{aligned}$$

Define a function $\Lambda(x) = p(s_H^{direct}) p(\frac{1}{2}Q - x) + p(\frac{1}{2}Q + x) p(s_L^{direct})$, where $x \leq \frac{1}{2}Q - s_L^{direct} = s_H^{direct} - \frac{1}{2}Q = \bar{x}$. We have

$$\begin{aligned} \Lambda'(x) &= p' \left(\frac{1}{2}Q + x \right) p(s_L^{direct}) - p(s_H^{direct}) p' \left(\frac{1}{2}Q - x \right) \\ \Lambda''(x) &= p'' \left(\frac{1}{2}Q + x \right) p(s_L^{direct}) + p(s_H^{direct}) p'' \left(\frac{1}{2}Q - x \right) \geq 0 \end{aligned}$$

so that

$$\begin{aligned} \Lambda'(x) &\leq \Lambda'(\bar{x}) = p'(s_H^{direct}) p(s_L^{direct}) - p(s_H^{direct}) p'(s_L^{direct}) \\ &= p(s_L^{direct}) p(s_H^{direct}) \left[\frac{p'(s_H^{direct})}{p(s_H^{direct})} - \frac{p'(s_L^{direct})}{p(s_L^{direct})} \right], \end{aligned}$$

where $\left[\frac{p'(s)}{p(s)} \right]' = \frac{p(s)p''(s) - [p'(s)]^2}{[p(s)]^2}$.

Now suppose for the moment that $p(s)p''(s) - [p'(s)]^2 \leq 0$, then $\frac{p'(s_H^{direct})}{p(s_H^{direct})} - \frac{p'(s_L^{direct})}{p(s_L^{direct})} \leq$

0, which means $\Lambda'(x) \leq 0$. In this case we get

$$\Sigma P^{direct} - \Sigma P^{im} = \frac{1}{p(s_1^{direct})} [\Lambda(x^{direct}) - \Lambda(x^{im})] \leq 0.$$

Since we assumed $Q^{direct} = Q^{im} = Q$, an import wholesaler would hence decrease social welfare relative to the case of direct exports. A sufficient condition for both $p(s)p''(s) - [p'(s)]^2 \leq 0$ and $Q^{direct} = Q^{im}$ is for demand to be linear, i.e., to have an inverse demand function of the form $p_{it} = \varepsilon_{it}(A - s_{it})$. In this case $p(s)p''(s) - [p'(s)]^2 = -1 < 0$. Moreover, $MR''(s) = \xi'(s) = 0$, which according to the proof of Lemma B.8 implies $Q^{direct} = Q^{im}$.

C Appendix to Chapter 4

C.1 Proof of $\frac{\partial^2}{\partial \rho \partial \varepsilon_1} I_2^o(\varepsilon_1^c, c, \rho) > 0$

Proof. (a) In the case of multiplicative demand uncertainty, $R_t = \varepsilon_t p(s_t) s_t$, Define $m(s_t) = p(s_t) + s_t p'(s_t)$ as the marginal revenue function without uncertainty. From equation (4.1), we have $xm(I_2^o) - c = 0$, where $x = \rho\varepsilon_1 + (1 - \rho)\bar{v}$, and $\bar{v} = 1$. Note that $\varepsilon_1 \in [0.5, 2]$ and $\rho \in [0, 1]$, then $x \in [0.5, 2]$. Assume $I_2^o = g(x)$ is determined by the implicit function $G(x, I_2^o) = xm(I_2^o) - c = 0$. Then

$$\begin{aligned}\frac{\partial I_2^o}{\partial \varepsilon_1} &= g' \rho \\ \frac{\partial^2}{\partial \rho \partial \varepsilon_1} I_2^o &= g' + \rho g''(\varepsilon_1 - \bar{v}) = g' + g'' \cdot (x - \bar{v})\end{aligned}$$

From the implicit function theorem,

$$g' = -\frac{G'_x}{G'_{I_2^o}} = -\frac{m(I_2^o)}{xm'(I_2^o)} \equiv -\frac{m}{xm'}$$

and

$$\begin{aligned}g'' &= -\frac{xm'(I_2^o)m'(I_2^o)g' - m(I_2^o)[m'(I_2^o) + xm''(I_2^o)g']}{[xm'(I_2^o)]^2} \\ &= -\frac{x(m')^2\left(-\frac{m}{xm'}\right) - m\left[m' + xm''\left(-\frac{m}{xm'}\right)\right]}{x^2(m')^2} \\ &= -\frac{-2mm' + mm''\left(\frac{m}{m'}\right)}{x^2(m')^2} \\ &= \frac{2m(m')^2 - m^2m''}{x^2(m')^3}\end{aligned}$$

Therefore

$$\begin{aligned}
\frac{\partial^2}{\partial \rho \partial \varepsilon_1} I_2^o &= g' + g'' (x - \bar{v}) \\
&= -\frac{m}{xm'} + \frac{2m(m')^2 - m^2 m''}{x^2 (m')^3} (x - 1) \\
&= \frac{xm(m')^2 - xm^2 m'' - 2m(m')^2 + m^2 m''}{x^2 (m')^3} \\
&= \frac{(x-2)m[(m')^2 - mm''] - m^2 m''}{x^2 (m')^3} > 0
\end{aligned}$$

(b) In the case of additive demand uncertainty, $R_t = [p(s_t) + \varepsilon_t] s_t$. From equation (4.1), we have $m(I_2^o) + x - c = 0$, where $x = \rho \varepsilon_1 + (1 - \rho) \bar{v}$. As proof mentioned in (a), we assume $I_2^o = g(x)$ is determined by the implicit function $G(x, I_2^o) = m(I_2^o) + x - c = 0$. Then

$$\begin{aligned}
\frac{\partial^2}{\partial \rho \partial \varepsilon_1} I_2^o &= g' + g'' \cdot (x - \bar{v}) \\
&= g' + xg''
\end{aligned}$$

and

$$\begin{aligned}
g' &= -\frac{G'_x}{G'_{I_2^o}} = -\frac{1}{m'(I_2^o)} \equiv -\frac{1}{m'} \\
g'' &= -\frac{[m''(I_2^o)g'(x)]}{[m'(I_2^o)]^2} = -\frac{-m''(-\frac{1}{m'})}{(m')^2} \\
&= -\frac{m''}{(m')^3}
\end{aligned}$$

Thus

$$\begin{aligned}
\frac{\partial^2}{\partial \rho \partial \varepsilon_1} I_2^o &= g' + x g'' = -\frac{1}{m'} - x \frac{m''}{(m')^3} \\
&= -\frac{(m')^2 + x m''}{(m')^3} = -\frac{(m')^2 + (c - m) m''}{(m')^3} \\
&= -\frac{(m')^2 - m m'' + c m''}{(m')^3} > 0
\end{aligned}$$

■

C.2 Proof of $\frac{\partial^2}{\partial \rho \partial \varepsilon_1} s_1^c < 0$

Proof. (a) In the case of multiplicative demand uncertainty, we know

$$\varepsilon_1 m(s_1^c) = [\rho \varepsilon_1 + (1 - \rho) \bar{v}] m(I_1 - s_1^c)$$

Denote $x = \frac{\rho \varepsilon_1 + (1 - \rho) \bar{v}}{\varepsilon_1}$, we then have the implicit function

$$G(x, s_1^c) = m(s_1^c) - x m(I_1 - s_1^c) = 0$$

Assume $s_1^c = g(x)$, where $g(x)$ is determined by the implicit function $G(x, s_1^c) = 0$.

$$\begin{aligned}
\frac{\partial s_1^c}{\partial \varepsilon_1} &= g' \cdot \frac{\varepsilon_1 \rho - [\rho \varepsilon_1 + (1 - \rho) \bar{v}]}{\varepsilon_1^2} \\
&= g' \cdot \frac{-(1 - \rho) \bar{v}}{\varepsilon_1^2} \\
\frac{\partial^2}{\partial \rho \partial \varepsilon_1} s_1^c &= \frac{\bar{v}}{\varepsilon_1^2} g' - \frac{(1 - \rho) \bar{v}}{\varepsilon_1^2} g'' \cdot \left(\frac{\varepsilon_1 - \bar{v}}{\varepsilon_1} \right) \\
&= \frac{\bar{v}}{\varepsilon_1^2} \left[g' - \frac{(\varepsilon_1 - \bar{v})(1 - \rho)}{\varepsilon_1} g'' \right] \\
&= \frac{\bar{v}}{\varepsilon_1^2} [g' - (1 - x) g'']
\end{aligned}$$

As before

$$\begin{aligned} g' &= -\frac{G'_x}{G'_{s_1^c}} = -\frac{-m(I_1 - s_1^c)}{m'(s_1^c) + xm'(I_1 - s_1^c)} \\ &= \frac{m(s_2^c)}{m'(s_1^c) + xm'(s_2^c)} \end{aligned}$$

and

$$\begin{aligned} g'' &= \frac{[m'(s_1^c) + xm'(s_2^c)] [-m'(s_2^c) g'] - m(s_2^c) [m''(s_1^c) g' + m'(s_2^c) - xm''(s_2^c) g']}{[m'(s_1^c) + xm'(s_2^c)]^2} \\ &= \frac{[-2m'(s_2^c) m(s_2^c)] - m(s_2^c) [m''(s_1^c) - xm''(s_2^c)] \frac{m(s_2^c)}{m'(s_1^c) + xm'(s_2^c)}}{[m'(s_1^c) + xm'(s_2^c)]^2} \\ &= \frac{[-2m'(s_2^c) m(s_2^c)] [m'(s_1^c) + xm'(s_2^c)] - m^2(s_2^c) [m''(s_1^c) - xm''(s_2^c)]}{[m'(s_1^c) + xm'(s_2^c)]^3} \end{aligned}$$

Therefore

$$\begin{aligned} &g' - (1-x)g'' \\ &= \frac{m(s_2^c)}{[m'(s_1^c) + xm'(s_2^c)]^3} \left\{ \begin{array}{l} [m'(s_1^c) + xm'(s_2^c)]^2 \\ + 2(1-x)m'(s_2^c)[m'(s_1^c) + xm'(s_2^c)] \\ + (1-x)m(s_2^c)[m''(s_1^c) - xm''(s_2^c)] \end{array} \right\} \end{aligned}$$

Note that $(1-x)m(s_2^c) = m(s_2^c) - xm(s_2^c) = m(s_2^c) - m(s_1^c)$, and $x = \frac{\rho\varepsilon_1 + (1-\rho)\bar{v}}{\varepsilon_1} = \rho + (1-\rho)\frac{1}{\varepsilon_1} \leq \rho + (1-\rho)\frac{1}{0.5} = 2 - \rho \leq 2$ then

$$\begin{aligned}
& \left\{ \begin{array}{l} [m'(s_1^c) + xm'(s_2^c)]^2 \\ +2(1-x)m'(s_2^c)[m'(s_1^c) + xm'(s_2^c)] \\ + (1-x)m(s_2^c)[m''(s_1^c) - xm''(s_2^c)] \end{array} \right\} \\
& = m'(s_1^c)^2 + x^2m'(s_2^c)^2 + 2xm'(s_1^c)m'(s_2^c) \\
& + 2(1-x)m'(s_1^c)m'(s_2^c) + 2(1-x)xm'(s_2^c)^2 \\
& + (1-x)m(s_2^c)m''(s_1^c) - (1-x)xm(s_2^c)m''(s_2^c) \\
& = m'(s_1^c)^2 + [x^2 + 2(1-x)x]m'(s_2^c)^2 + 2m'(s_1^c)m'(s_2^c) \\
& + [m(s_2^c) - m(s_1^c)]m''(s_1^c) - (1-x)xm(s_2^c)m''(s_2^c) \\
& = [m'(s_1^c)^2 - m(s_1^c)m''(s_1^c)] \\
& + x[(2-x)m'(s_2^c)^2 - (1-x)m(s_2^c)m''(s_2^c)] \\
& + 2m'(s_1^c)m'(s_2^c) + m(s_2^c)m''(s_1^c) \\
& > [m'(s_1^c)^2 - m(s_1^c)m''(s_1^c)] + xm(s_2^c)m''(s_2^c) \\
& + 2m'(s_1^c)m'(s_2^c) + m(s_2^c)m''(s_1^c) \\
& > 0
\end{aligned}$$

The first inequality comes from $m'(s_2^c)^2 > m(s_2^c)m''(s_2^c)$, and $0 < x \leq 2$; and the second inequality follows $m'(s_1^c)^2 > m(s_1^c)m''(s_1^c)$.

Therefore $g' - (1-x)g'' < 0$, as well as $\frac{\partial^2}{\partial\rho\partial\varepsilon_1}s_1^c < 0$.

(b) In the case of additive demand uncertainty, we have $G(x, s_1^c) = m(s_1^c) - m(I_1 - s_1^c) + x = 0$, where $x = \varepsilon_1 - [\rho\varepsilon_1 + (1-\rho)\bar{v}] = (1-\rho)(\varepsilon_1 - \bar{v})$. This implicit function $G(x, s_1^c) = 0$ yields $s_1^c = g(x)$, then

$$\begin{aligned}
\frac{\partial s_1^c}{\partial \varepsilon_1} &= g' \cdot (1-\rho) \\
\frac{\partial^2}{\partial \rho \partial \varepsilon_1} s_1^c &= -g' - (1-\rho)(\varepsilon_1 - \bar{v})g'' \\
&= -g' - xg''
\end{aligned}$$

and

$$\begin{aligned}
 g' &= -\frac{G'_x}{G'_{s_1^c}} = -\frac{1}{m'(s_1^c) + m'(s_2^c)} \\
 g'' &= \frac{m''(s_1^c)g' - m''(s_2^c)g'}{[m'(s_1^c) + m'(s_2^c)]^2} \\
 &= -\frac{m''(s_1^c) - m''(s_2^c)}{[m'(s_1^c) + m'(s_2^c)]^3}
 \end{aligned}$$

Then

$$\begin{aligned}
 \frac{\partial^2}{\partial \rho \partial \varepsilon_1} s_1^c &= -g' - xg'' \\
 &= \frac{1}{m'(s_1^c) + m'(s_2^c)} + \frac{x[m''(s_1^c) - m''(s_2^c)]}{[m'(s_1^c) + m'(s_2^c)]^3} \\
 &= \frac{[m'(s_1^c) + m'(s_2^c)]^2 + x[m''(s_1^c) - m''(s_2^c)]}{[m'(s_1^c) + m'(s_2^c)]^3}
 \end{aligned}$$

and

$$\begin{aligned}
 &[m'(s_1^c) + m'(s_2^c)]^2 + x[m''(s_1^c) - m''(s_2^c)] \\
 &= [m'(s_1^c) + m'(s_2^c)]^2 + [m(s_2^c) - m(s_1^c)][m''(s_1^c) - m''(s_2^c)] \\
 &= [m'(s_1^c)^2 - m(s_1^c)m''(s_1^c)] + [m'(s_2^c)^2 - m(s_2^c)m''(s_2^c)] \\
 &+ 2m'(s_1^c)m'(s_2^c) + m(s_2^c)m''(s_1^c) + m(s_1^c)m''(s_2^c) \\
 &> 0
 \end{aligned}$$

Therefore we have $\frac{\partial^2}{\partial \rho \partial \varepsilon_1} s_1^c < 0$. ■

Affirmation

I hereby affirm that I have completed my doctoral thesis entitled, "Three Essays on the Role of Intermediaries" entirely on my own and unassisted, and that I have specially marked all of the quotes I have used from other authors as well as those passages in my work that are extremely close to the thoughts presented by other authors, and listed the sources in accordance with the regulations I have been given.

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