

ESSAYS ON  
LABOR MARKET RESPONSES TO  
THE GREAT RECESSION  
AND ON THE  
EFFECTS OF MONETARY  
UNIFICATION ON MACROECONOMIC  
STABILITY

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# CHAPTER 1

## Outline

This dissertation consists of two parts. Part I deals with labor market responses to the Great Recession (Chapters 2 and 3). Part II deals with the effects of monetary unification on macroeconomic stability (Chapters 4 and 5).

### **Part I**

Starting in 2008/2009, the world economy experienced the most severe recession since the Great Depression 80 years before. As a result of this so-called Great Recession, unemployment rates rose sharply around the world and—as of this writing—are still elevated way above their pre-crisis levels in a number of countries. However, the response of unemployment rates and other labor market indicators to the Great Recession differed markedly across countries, especially in Europe. Part I of this dissertation contributes to understanding the reasons for this heterogeneity of labor market responses. Chapter 2 focuses on three labor market institutions: Firing costs, short-time work, and the wage formation process. Chapter 3 focuses on one of the most surprising labor market responses to the Great Recession, namely the exceptional performance of the German labor market, which became known as the “German labor market miracle.” The methodological focus of Part I is predominantly empirical.

### **Chapter 2**

This chapter analyzes the role of three labor market institutions in explaining the heterogeneous labor market responses across Europe to the Great Recession, namely the level of firing costs, the existence of short-time work, and the wage formation process. It combines two different perspectives, a structural dynamic model perspective and an empirical cross-country perspective. First, a dynamic stochastic general equilibrium (DSGE) model with microfounded labor market

frictions is used to simulate the effects of the three labor market institutions during a recession. Second, the predictions of the model are compared to the actual outcomes from selected EU countries.

The main findings are the following: First, in line with the theoretical model, countries with higher firing costs experienced a smaller reduction of employment in response to the Great Recession. In this sense, firing costs can act as a shock absorber. However, the theoretical model suggests that firing costs also delay the subsequent recovery, i.e., it takes more time for employment to return to its pre-crisis level. Second, in line with the theoretical model, countries with important short-time working schemes experienced a larger reduction in output, but a smaller reduction of employment. And third, countries with a smaller increase in unit labor costs (i.e., wage moderation) prior to the Great Recession experienced a smaller reduction of employment during the Great Recession. Such a wage-moderation scenario was especially relevant for the German labor market. This is discussed in more detail in Chapter 3.

This chapter is based on a joint paper with Jens Boysen-Hogrefe, Wolfgang Lechthaler, and Christian Merkl published under the title “The role of labor market institutions in the Great Recession” in *Applied Economics Quarterly* 56(61), *Supplement*, pp. 65-88, June 2010. My contribution to this paper consisted of the entire empirical analysis, from collecting the data to writing the corresponding text passages.

### **Chapter 3**

This chapter lays out the various reasons for the exceptional performance of the German labor market during and after the Great Recession. The reference point of the analysis is provided by an empirical labor market model of both total hours worked and employment. Dynamic simulations of the crisis period are conducted to assess how surprising the reaction of the labor market was.

The main findings are the following: The most important precondition for the minor reaction of employment in response to the Great Recession was the pronounced wage moderation observed in the years before, which constitutes a distinct difference to all other recessions in Germany. Beyond that, the flexibility of adjusting working time, which had increased considerably in the ten years before, facilitated labor hoarding. In contrast, short-time work plays a minor role in explaining the difference from previous recessions, since this in-

strument has long been available to firms in Germany and its use has not been extraordinary compared with earlier recessions.

This chapter is based on a joint paper with Jens Boysen-Hogrefe published under the title “The German labour market miracle” in *National Institute Economic Review* 214, pp. R38-R50, October 2010. I authored most of this paper, including the survey on short-time work in Germany, the estimation of the empirical labor market model, and the exposition of the wage-moderation and internal-flexibility arguments. My co-author’s contribution consisted mainly of the two subsections labeled “International perspective”, respectively.

## Part II

The Great Recession not only affected labor markets, it also revealed deeper economic problems in several countries within the European Monetary Union. Questions were raised whether these countries would be better off leaving the monetary union, thereby restoring their monetary independence and regaining the possibility of devaluing their currencies. The loss of monetary independence with its negative consequences for macroeconomic stability has long been considered one of the main costs associated with monetary unification. Part II of this dissertation revisits this traditional argument of the Optimum Currency Area (OCA) theory. Chapter 4 focuses on a benefit of monetary unification that has the potential to overcompensate the loss of monetary independence, but which has not been acknowledged by OCA theory so far. Chapter 5 focuses on the important role of monetary policy for whether countries benefit from monetary unification. The methodological focus of both chapters is exclusively theoretical. The results are derived from New Keynesian DSGE models. These models have two advantages in the OCA context. First, a welfare loss function derived from the utility of households provides a well-defined welfare measure to assess the costs and benefits of monetary unification. Second, this welfare measure depends directly on the degree of macroeconomic stability, which in turn depends on the international monetary regime. Thus, the main cost associated with monetary unification—the loss of monetary independence—can be evaluated explicitly.

## Chapter 4

This chapter uses a standard two-country New Keynesian DSGE model to analyze the conditions under which countries benefit from forming a monetary union. In contrast to the bulk of the literature, monetary policy is not assumed to be conducted optimally, but instead follows Taylor-type interest rate rules.

The main findings are the following: When prices are relatively sticky, countries gain in welfare by forming a monetary union. The gain in welfare comes from higher inflation stability, which outweighs the costs of higher output-gap and terms-of-trade-gap instability. Two effects are responsible for this higher inflation stability. First, the stabilizing property of a flexible nominal exchange rate declines as prices become stickier. Second, fixing the nominal exchange rate entails the inherent benefit of stabilizing inflation expectations. This inherent benefit of a monetary union has not been acknowledged by OCA theory so far.

This chapter is based on my single-authored paper “Monetary union and macroeconomic stabilization” published as Kiel Working Paper 1881, Kiel Institute for the World Economy, November 2013.

## Chapter 5

This chapter elaborates on Chapter 4, but it uses a more general version of the two-country New Keynesian DSGE model. This model allows, among other things, for trade imbalances, deviations from purchasing power parity (i.e., real exchange rate fluctuations), and more variation in the conduct of monetary policy.

The main findings are the following: Monetary policy is one of the most important OCA criteria. That is, whether countries benefit from forming a monetary union depends critically on the way monetary policy is conducted. When countries are unable or unwilling to implement monetary policy optimally, for whatever reason, they are likely to benefit from forming a monetary union. This is mainly because monetary policy determines whether and to what extent a flexible nominal exchange rate fosters or hampers macroeconomic stabilization, even if monetary policy does not target the nominal exchange rate explicitly.

This chapter is based on my single-authored paper “Monetary policy as an optimum currency area criterion” published as Kiel Working Paper 1969, Kiel Institute for the World Economy, November 2014.

## **Part I.**

# LABOR MARKET RESPONSES TO THE GREAT RECESSION

# The Role of Labor Market Institutions in the Great Recession

## Abstract

The recent Great Recession had very heterogeneous effects on the labor market outcomes in industrialized countries. We analyze the role of three labor market institutions in this context, namely the level of firing costs, the existence of short-time work and the wage formation process. This paper combines two different perspectives, a structural dynamic model perspective and an empirical cross-country perspective. Using the Lechthaler, Merkl, and Snower (2010) model, we first simulate the effects of the three labor market institutions during a recession. Using the panel of the EU-15 countries without Luxembourg, we then test the predictions of the model. Indeed, we find evidence that the three labor market institutions can partially explain the different labor market reactions across countries during the Great Recession. However, further empirical research is needed, as more data can be expected to become available, especially with respect to the use of short-time work in different countries.

Keywords: Employment; firing costs; short-time work; unit labor costs.

JEL classification: E24, E32, J64.

This paper was joint work with Jens Boysen-Hogrefe, Wolfgang Lechthaler, and Christian Merkl and published under the title “The role of labor market institutions in the Great Recession” in *Applied Economics Quarterly*, volume 56, issue 61, Supplement, pp. 65-88, June 2010, Duncker & Humblot, Berlin.

# The German Labor Market Miracle

## Abstract

This paper lays out the various reasons for the exceptional performance of the German labor market during and after the Great Recession of 2008/2009. The reference point of our analysis is provided by an empirical model of both total hours worked and employment. We conduct dynamic simulations of the crisis period to assess how surprising the reaction of the labor market really was. We argue that the most important precondition for the minor reaction of employment during this crisis was the pronounced wage moderation observed in the years before, which constitutes a distinct difference to all other recessions in Germany. Beyond that, the flexibility of adjusting working time, which has increased considerably during the past ten years, facilitated a tendency to labor hoarding. In contrast, short-time work plays a minor role in explaining the difference from previous recessions, since this instrument has always been available to firms in Germany and its use has not been extraordinary compared with earlier recessions.

Keywords: Labor market; Great Recession; employment; wage moderation; short-time work.

JEL classification: E24, E37, J23, J30.

This paper was joint work with Jens Boysen-Hogrefe and published under the title “The German labour market miracle” in *National Institute Economic Review*, volume 214, pp. R38-R50, October 2010, SAGE Publications, London.

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## **Part II.**

# THE EFFECTS OF MONETARY UNIFICATION ON MACROECONOMIC STABILITY



# Monetary Union and Macroeconomic Stabilization

## Abstract

It is conventionally held that countries are worse off by forming a monetary union when it comes to macroeconomic stabilization. However, this conventional view relies on assuming that monetary policy is conducted optimally. Relaxing the assumption of optimal monetary policy not only uncovers that countries do benefit from forming a monetary union under fairly general conditions. More importantly, it also reveals that a monetary union entails the inherent benefit of stabilizing private-sector expectations about future inflation. As a result, inflation rates are more stable in a monetary union.

Keywords: Monetary union; macroeconomic stabilization; welfare analysis; history dependence; inflation expectations.

JEL classification: F33, F41, E52.

This paper was published under the title “Monetary union and macroeconomic stabilization” as Kiel Working Paper 1881, Kiel Institute for the World Economy, Kiel.

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For an earlier version, see “When do countries benefit from forming a monetary union?” Beiträge zur Jahrestagung des Vereins für Socialpolitik 2013: Wettbewerbspolitik und Regulierung in einer globalen Wirtschaftsordnung - Session: Monetary Policy II, No. B01-V2.

<http://hdl.handle.net/10419/79787>.

## 4.1. Introduction

When do countries benefit from forming a monetary union? This question is at least as old as the Optimum Currency Area (OCA) literature initiated by Mundell (1961). One of the key insights of this literature is that for asymmetric countries to benefit from forming a monetary union, prices and wages have to be flexible and production factors have to be mobile. More recently, the New Keynesian literature, by using dynamic stochastic general equilibrium (DSGE) models, has come to the consensus that, from the perspective of macroeconomic stabilization, forming a monetary union makes countries generally worse off in terms of welfare.<sup>1</sup> This is because countries relinquish one of their most important policy instruments for macroeconomic stabilization, namely the short-term nominal interest rate controlled by their national central bank.

However, this consensus is based on the assumption that the central bank conducts monetary policy optimally. While constituting a very useful theoretical benchmark from a normative perspective, the assumption of optimal monetary policy entails at least two important disadvantages in the OCA context. On the one hand, it is widely acknowledged that optimal monetary policy typically involves severe practical limitations, in particular very demanding information requirements. For example, the central bank needs to be able to observe the households' welfare function or the flexible-price equilibrium of the economy, i.e., the equilibrium that would prevail under completely flexible prices.<sup>2</sup> On the other hand, the assumption of optimal monetary policy precludes the possibility to assess if and how the welfare performance of a monetary union depends on the way monetary policy is conducted, since deviations from optimality are ruled out by assumption. Thus, it seems at least debatable whether optimal monetary policy is the best modeling choice when one wants to know the conditions under which countries benefit from forming a monetary union.<sup>3</sup>

In light of these two disadvantages, I take a different approach in this paper by assuming that monetary policy follows Taylor-type interest rate rules, according to which it responds only to observable variables, such as inflation

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<sup>1</sup>See surveys by Corsetti (2008), Dellas and Tavlas (2009), and Beetsma and Giuliodori (2010).

<sup>2</sup>For more practical shortcomings of optimal monetary policy, see Gali (2008, Ch. 4.3.2).

<sup>3</sup>In my view, this doubt is supported by Adao, Correia, and Teles (2009), who conclude that "every currency area is an optimal currency area," after having shown that the exchange rate regime is *irrelevant* for stabilization policy if optimal monetary policy is complemented by optimal fiscal policy.

or output. By being able to vary the coefficients that determine the response of monetary policy to changes in the respective variables, these interest rate rules are general enough to allow for a great flexibility in specifying the behavior of monetary policy. For example, how aggressive is monetary policy in its response to inflation? Does it respond to output? If so, how strongly? Or does monetary policy smooth interest rates and to what extent? In this sense, the way monetary policy is conducted can be viewed just as any other country characteristic, such as its size or the degree of price stickiness in its economy. It is then possible to assess if and how the welfare performance of a monetary union depends on the behavior of monetary policy—something that is not possible under the assumption of optimal monetary policy. As it turns out, the behavior of monetary policy is absolutely critical for the welfare performance of a monetary union.

Given this different approach, the main finding of this paper is as follows: In the standard two-country New Keynesian DSGE model, in which monetary policy follows interest rate rules, countries may gain in welfare by forming a monetary union. The gain in welfare comes from a higher stability of inflation rates, which outweighs the costs of higher output-gap and terms-of-trade-gap instability. Whether countries gain in welfare depends strongly on the degree of price stickiness. When prices are relatively sticky, countries are better off forming a monetary union; when prices are relatively flexible, countries are better off maintaining a flexible exchange rate.

Two effects are responsible for this higher stability of inflation rates. First, the benefit of maintaining a flexible exchange rate diminishes as prices become stickier, since the nominal exchange rate inherits the stickiness of goods prices. As a result, an increasing degree of price stickiness reduces the effectiveness of the nominal exchange rate as a stabilization mechanism. Second, forming a monetary union entails an inherent benefit.<sup>4</sup> Since the nominal exchange rate is fixed, the terms of trade and, therefore, the inflation rates display an inertial or history-dependent behavior. This history dependence has the advantage of affecting the inflation expectations of price setters in such a way as to lower the responsiveness of inflation to changing economic conditions. The higher the degree of price stickiness is, the stronger this effect is. As a result, inflation rates are more stable in a monetary union.

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<sup>4</sup>“Inherent” refers to the fact that the benefit is not modeled explicitly, like a reduction in transaction costs, but emerges from within the model.

This second effect corresponds to an effect that is well-known from the analysis of optimal monetary policy in a closed economy. There, optimal monetary policy *under discretion* is inferior to optimal monetary policy *under commitment* because the former does not influence the inflation expectations of price setters in a favorable way. It suffers from the so-called stabilization bias. In contrast, optimal monetary policy under commitment induces history dependence into the economy and, therefore, exploits the fact that price setters are forward-looking. The intuition in this paper is completely analogous. Forming a monetary union may be superior to maintaining a flexible exchange rate because fixing the exchange rate induces history dependence.

This benefit, which manifests itself in a higher stability of inflation rates and which is related to the *stabilization bias*, obtains in addition to the benefit of eliminating a potential *inflation bias*, which is stressed by Alesina and Barro (2002) and Cooley and Quadrini (2003).<sup>5</sup> Whereas the latter benefit has been acknowledged in the literature (e.g., Dellas and Tavlas, 2009; Beetsma and Giuliodori, 2010), the former still seems to be unknown.

It is important to realize that both effects described above are endogenous to the model. The first effect is due to the presence of the uncovered interest parity condition on the one hand and monetary policy following Taylor-type interest rate rules on the other hand.<sup>6</sup> The second effect is due to the fact that price setters are forward-looking in the presence of nominal price rigidities. All these features belong to the core of new open economy macroeconomics (NOEM) models and, therefore, are present also in many medium-to-large-scale models that are built around this core.<sup>7</sup>

Clearly, the finding that countries benefit from forming a monetary union when prices are relatively sticky but not when prices are relatively flexible stands in contrast to the predictions of the traditional OCA theory. Probably the most important reason for this discrepancy is the fact that expectations are treated as endogenous in New Keynesian models, unlike in the theoretical framework of the traditional OCA literature, in which expectations are treated as exogenous. Since the inherent benefit of monetary unions works through expectations, this

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<sup>5</sup>Giavazzi and Pagano (1988)'s "advantage of tying one's hands" follows the same logic, although they refer to the former European Monetary System (1979–1999).

<sup>6</sup>Notably, the uncovered interest parity condition need not hold exactly for this effect to exist. It suffices for the interest rates and the nominal exchange rate to be linked.

<sup>7</sup>The introduction of nominal price rigidities in the spirit of Calvo (1983) into NOEM models goes back to Kollmann (2001), Gali and Monacelli (2005), and Clarida, Gali, and Gertler (2002).

channel is naturally missing in models without such an expectational feedback mechanism.<sup>8</sup>

Another important finding of this paper is that whether forming a monetary union is beneficial or not depends heavily on the way monetary policy is conducted. When monetary policy responds to inflation aggressively or when it implements a high degree of interest rate smoothing, then maintaining a flexible exchange rate is superior. Generally, it is the quality of monetary policy that is crucial for the welfare ranking between the monetary union and the flexible exchange rate regime. Since monetary policy is more potent under the flexible exchange rate regime, it is also more harmful when not conducted properly. This is because the effects of monetary policy are reinforced by the nominal exchange rate. In this sense, a monetary union provides a protective mechanism against monetary policy mistakes.

This paper is related along several dimensions to the New Keynesian literature that analyzes the conditions under which countries benefit from forming a monetary union. In this literature, only a few studies have considered an environment without optimal monetary policy: Dellas and Tavlas (2005), Dellas (2006), and Ferreira-Lopes (2010). The models used in these studies, as well as their findings are, on the one hand, quite diverse. On the other hand, none of these studies addresses the inherent benefit of monetary unions, the role of the degree of price stickiness, nor the closely related issue of the inherited stickiness of the exchange rate, all of which are crucial for the welfare ranking between the monetary union and the flexible exchange rate regime.

Several studies have introduced explicit benefits of monetary unions to create a counterpart to the cost of giving up national monetary policy as a stabilization device. Such explicit benefits of monetary unions include the elimination of shocks to the uncovered interest parity condition (Kollmann, 2004), the gain in potential output (Ca'Zorzi, De Santis, and Zampolli, 2005), the gain in central bank credibility (Clerc, Dellas, and Loisel, 2011), and the possibility of higher consumption risk sharing across countries (Ching and Devereux, 2003). In contrast, no explicit benefits are introduced into the model employed in this paper. The benefit of stabilizing inflation expectations is inherent to monetary unions as a result of a fixed nominal exchange rate.

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<sup>8</sup>See King (1993) for a critical assessment of the Old Keynesian, IS-LM models with respect to their treatment of expectations.

This paper is also related to Monacelli (2004).<sup>9</sup> Among other things, he finds that in a small open economy a fixed exchange rate regime induces inertia into the economy. On the one hand, I show that this benefit carries over to a two-country environment and is inherent to monetary union regimes as well.<sup>10</sup> On the other hand, I show that it does not hinge upon the stationarity of the price level, as stressed by Monacelli (2004). Stationarity of the price level is a special feature of the small open economy environment and does not carry over to a two-country setting employed here. Also, Monacelli (2004) does not address the role of the degree of price stickiness nor the related issue of the inherited stickiness of the exchange rate.

The rest of this paper is organized as follows. Section 4.2 outlines briefly the structure of the model. Section 4.3 provides important analytical results in the case of symmetric countries and presents the welfare results graphically. Section 4.4 presents the results in the case of asymmetric countries. Section 4.5 relates the results to the traditional OCA theory. Section 4.6 concludes.

## 4.2. Model

The model I use is a completely standard two-country New Keynesian DSGE model. Thus, I keep the description very brief. It features two international monetary regimes:

1. A monetary union (MU) regime: Both countries share the same currency. A common monetary policy governs the common nominal interest rate.
2. A flexible exchange rate (FX) regime: Each country maintains its national currency and conducts its own, independent monetary policy. Nominal interest rates are country-specific. The nominal exchange rate between the two currencies is flexible.

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<sup>9</sup>For a small open economy, a fixed exchange rate regime may dominate a flexible exchange rate regime with optimal monetary policy under discretion. A flexible exchange rate regime with optimal monetary policy under commitment, however, always dominates the other two regimes. Comparing the same three regimes, Soffritti and Zanetti (2008) come to a different conclusion, namely that a fixed exchange rate regime fares worse than the two flexible exchange rate regimes. One possible explanation for the different finding could be the different weight attached to the output-gap variance relative to the weight attached to the inflation variance in the welfare loss function, which is ad hoc in both studies. Another explanation could be the different assumption about whether the rest of the world is also subject to shocks or not.

<sup>10</sup>In general, fixed exchange rate regimes and monetary union regimes do not coincide. This depends on how the fixed exchange rate regime is implemented.

The model is described in detail in Benigno (2004) and in Benigno and Benigno (2008) and it includes a microfounded, quadratic welfare measure. Under both regimes, the model economy features two countries with trade in consumption goods (as opposed to trade in intermediate goods). Consumption preferences are of the Cobb-Douglas type and are, in addition, identical across countries, i.e., there is no home bias in consumption. These preferences imply that risk sharing is perfect in the sense that consumption is equal across countries at all times. Purchasing power parity holds, i.e., the real exchange rate is constant. While these assumptions are clearly restrictive, they greatly simplify the analysis, and relaxing them to allow for a home bias in consumption (i.e., deviations from purchasing power parity and a variable real exchange rate) does not alter the findings significantly. The only factor of production is labor, which is immobile between countries. The only rigidity is the nominal price rigidity in the spirit of Calvo (1983).

Under the FX regime, prices are set in the currency of the producer's country ("producer currency pricing"), i.e., the producer does not discriminate the price between countries. The nominal exchange rate converts the price into foreign currency, i.e., the law of one price holds and exchange rate pass-through is complete. Given the same consumption preferences as under the MU regime, purchasing power parity holds as well.

In both regimes, monetary policy is conducted via Taylor-type interest rate rules. Importantly, I assume that monetary policy is not able to observe the flexible-price equilibrium of the economy, in particular the flexible-price interest rate and flexible-price output. Thus, monetary policy reacts to inflation and to output (deviation from the steady state), not to the output gap (deviation from flexible-price output). The only shocks considered are country-specific productivity shocks. However, the findings are robust with respect to other shocks, such as government-spending shocks or cost-push shocks.<sup>11</sup>

### 4.2.1. Model equations

The equations of the complete log-linearized model are displayed below (for the full derivation, see Appendices B.1 and B.2). Deviations of the logarithm of a variable  $X_t$  from its steady state are denoted by  $\tilde{X}_t$  under flexible prices and

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<sup>11</sup>In fact, under cost-push shocks the case for a monetary union becomes even stronger.

by  $\hat{X}_t$  under sticky prices.<sup>12</sup> Variables and parameters are defined in Tables 4.1 and 4.2, respectively.

**Table 4.1.: Variables**

$C_t$	Consumption (identical across countries)
$Y_t^i$	Output of country $i = H, F$
$Y_t^W$	World output (weighted average of country-specific output)
$\pi_t^i$	Producer price inflation in country $i = H, F$
$\pi_t^W$	World inflation (weighted average of country-specific inflation)
$\pi_t^R$	Inflation differential between the two countries $\pi_t^F - \pi_t^H$
$R_t^i$	Nominal interest rate in country $i = H, F$
$R_t$	Nominal interest rate in monetary union
$T_t$	Terms of trade
$S_t$	Nominal exchange rate
$\bar{Y}_t^i$	Productivity shock in country $i = H, F$
$\nu_t^i$	White noise process in country $i = H, F$

**Table 4.2.: Parameters**

$\rho$	Inverse of intertemporal elasticity of substitution in consumption
$n$	Country size measured by population
$\beta$	Discount factor
$\eta$	Inverse of elasticity of producing the differentiated good
$\sigma$	Elasticity of substitution between differentiated goods
$\alpha^i$	Probability of not being able to reset the price in country $i = H, F$
$\phi_\pi$	Inflation coefficient in interest rate rule
$\phi_Y$	Output coefficient in interest rate rule
$\phi_R$	Interest rate smoothing coefficient in interest rate rule
$\rho_i$	Persistence of productivity shock in country $i = H, F$
$k_C^i$	$k_C^i = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i} \frac{\rho+\eta}{1+\sigma\eta}$
$k_T^i$	$k_T^i = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i} \frac{1+\eta}{1+\sigma\eta}$

<sup>12</sup>Notation is adopted from Benigno (2004).



**Flexible-price equilibrium under both regimes**

Under completely flexible prices, the model equations are identical for both the FX and MU regime and are given by

$$\tilde{C}_t = \frac{\eta}{\rho + \eta} \bar{Y}_t^W \quad (4.2.1)$$

$$\tilde{T}_t = -\frac{\eta}{1 + \eta} \bar{Y}_t^R \quad (4.2.2)$$

$$\tilde{Y}_t^W = \frac{\eta}{\rho + \eta} \bar{Y}_t^W \quad (4.2.3)$$

$$\bar{Y}_t^W = n\bar{Y}_t^H + (1 - n)\bar{Y}_t^F \quad (4.2.4)$$

$$\bar{Y}_t^R = \bar{Y}_t^F - \bar{Y}_t^H \quad (4.2.5)$$

$$\bar{Y}_t^i = \rho_i \bar{Y}_{t-1}^i + \nu_t^i, \quad (4.2.6)$$

for  $i = H, F$ .

**Sticky-price equilibrium under the MU regime**

Under sticky prices, the model equations for the MU regime are given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} (\hat{R}_t - E_t \pi_{t+1}^W) \quad (4.2.7)$$

$$\hat{Y}_t^H = (1 - n)\hat{T}_t + \hat{C}_t \quad (4.2.8)$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t \quad (4.2.9)$$

$$\pi_t^H = (1 - n)k_T^H (\hat{T}_t - \tilde{T}_t) + k_C^H (\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^H \quad (4.2.10)$$

$$\pi_t^F = -nk_T^F (\hat{T}_t - \tilde{T}_t) + k_C^F (\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^F \quad (4.2.11)$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H \quad (4.2.12)$$

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R) (\phi_\pi \pi_t^W + \phi_Y \hat{Y}_t^W) \quad (4.2.13)$$

$$\pi_t^W = n\pi_t^H + (1 - n)\pi_t^F \quad (4.2.14)$$

$$\hat{Y}_t^W = n\hat{Y}_t^H + (1 - n)\hat{Y}_t^F. \quad (4.2.15)$$

### Sticky-price equilibrium under the FX regime

The model equations for the FX regime are given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( n(\hat{R}_t^H - E_t \pi_{t+1}^H) + (1-n)(\hat{R}_t^F - E_t \pi_{t+1}^F) \right) \quad (4.2.16)$$

$$\hat{Y}_t^H = (1-n)\hat{T}_t + \hat{C}_t \quad (4.2.17)$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t \quad (4.2.18)$$

$$\pi_t^H = (1-n)k_T^H(\hat{T}_t - \tilde{T}_t) + k_C^H(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^H \quad (4.2.19)$$

$$\pi_t^F = -nk_T^F(\hat{T}_t - \tilde{T}_t) + k_C^F(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^F \quad (4.2.20)$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta \hat{S}_t \quad (4.2.21)$$

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t^H - \hat{R}_t^F \quad (4.2.22)$$

$$\hat{R}_t^H = \phi_R \hat{R}_{t-1}^H + (1-\phi_R)(\phi_\pi \pi_t^H + \phi_Y \hat{Y}_t^H) \quad (4.2.23)$$

$$\hat{R}_t^F = \phi_R \hat{R}_{t-1}^F + (1-\phi_R)(\phi_\pi \pi_t^F + \phi_Y \hat{Y}_t^F). \quad (4.2.24)$$

#### 4.2.2. Model description

Consumption is equal across countries at all times and is described by only one Euler equation, equation (4.2.7) under the MU regime and equation (4.2.16) under the FX regime. The only difference between the two Euler equations is that under the MU regime the nominal interest rate is common to both countries. The structure of aggregate demand is the same under both regimes and given by equations (4.2.8), (4.2.9), (4.2.17), and (4.2.18). Also, the country-specific New Keynesian Phillips curves are the same under both regimes and are given by (4.2.10), (4.2.11), (4.2.19), and (4.2.20). In contrast to a closed-economy framework, not only the consumption gap but also the terms-of-trade gap (difference between sticky-price and flexible-price terms of trade) matters for producer price inflation.<sup>13</sup> The terms-of-trade identity is given by (4.2.12) under the MU regime and by (4.2.21) under the FX regime, the difference between the two being the presence of the nominal exchange rate in the latter. Equation (4.2.22) is the uncovered interest parity condition. The expected change in the nominal exchange rate corresponds to the interest rate differential across countries. Finally, monetary policy is conducted via Taylor-type interest

<sup>13</sup>Note that the consumption gap is equal to the world output gap:  $\hat{C}_t - \tilde{C}_t = \hat{Y}_t^W - \tilde{Y}_t^W$ . Accordingly, the New Keynesian Phillips curves can be expressed in terms of the world output gap as well.

rate rules, given by equation (4.2.13) under the MU regime and by equations (4.2.23) and (4.2.24) under the FX regime.

Under flexible prices, prices are set as a markup over marginal costs, monetary policy is neutral, and consumption, output, and the terms of trade are driven by productivity shocks only, given by equations (4.2.1), (4.2.2), and (4.2.3). Since money is neutral under flexible prices, the international monetary regime does not affect real variables, which therefore behave identically under both monetary regimes.

### 4.2.3. Welfare loss function

The welfare analysis follows the logic of the familiar linear-quadratic approach, according to which the log-linear model equations are used to evaluate a quadratic welfare loss measure (Woodford, 2003). The world welfare loss function is given by the discounted value of a weighted average across countries of the average utility flow of agents using a second-order Taylor series expansion.<sup>14</sup> Throughout the paper, it is assumed that the distortion induced by monopolistic competition is completely offset by an appropriate subsidy (see Appendix B.3 for the full derivation). Thus,

$$\begin{aligned}
 W_t = & -\frac{1}{2} \left( (\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta)n(1 - n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\
 & + \sigma(1 + \sigma\eta)n \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H\beta)} \text{var} \pi_t^H \\
 & \left. + \sigma(1 + \sigma\eta)(1 - n) \frac{\alpha^F}{(1 - \alpha^F)(1 - \alpha^F\beta)} \text{var} \pi_t^F \right) \\
 & + t.i.p. + O(\|\xi\|^3). \tag{4.2.25}
 \end{aligned}$$

As in the closed economy, the welfare loss depends on the inflation rate and the consumption gap.<sup>15</sup> In the open economy, the welfare loss depends additionally on the terms-of-trade gap. Intuitively, when the terms of trade deviate from the terms of trade that would prevail under flexible prices, the resulting al-

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<sup>14</sup>Computing country-specific welfare would complicate the calculations significantly because more accurate approximations of the non-linear model equations would be necessary (Benigno and Woodford, 2005). This is beyond the scope of this paper.

<sup>15</sup>In the basic closed-economy framework, consumption usually equals output. Note also that the welfare loss function (4.2.25) can be expressed alternatively in terms of the world output gap or the country-specific output gaps (see equation B.129). The specification in terms of the consumption gap was chosen for analytical convenience.

location of production across countries is inefficient due to the presence of price stickiness. The weights in front of each component of the welfare loss function are functions of the deep parameters of the model. The term *t.i.p.* contains all the terms that are independent of monetary policy and the international monetary regime in place. The term  $O(\|\xi\|^3)$  contains third and higher order terms, which can be neglected as long as the model equations are log-linear, i.e., first-order approximations to the non-linear equilibrium conditions.

In the special case when prices are equally rigid in both countries ( $\alpha^H = \alpha^F = \alpha$ ), the welfare loss function simplifies to

$$\begin{aligned} W_t = & -\frac{1}{2} \left( (\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta)n(1 - n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\ & \left. + \sigma(1 + \sigma\eta) \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)} \left[ \text{var} \pi_t^W + n(1 - n) \text{var} \pi_t^R \right] \right) \\ & + \textit{t.i.p.} + O(\|\xi\|^3). \end{aligned} \quad (4.2.26)$$

#### 4.2.4. Calibration

The values for the baseline calibration are taken from Benigno (2004), except for the interest rate rule coefficients (Table 4.3). A value of 0.99 for the discount factor  $\beta$  implies a steady-state real interest rate of around 4.1 percent annually. A value of 7.66 for the elasticity of substitution between differentiated goods  $\sigma$  implies a steady-state markup of prices over marginal costs of 15 percent. A value of 0.75 for the probability of not being able to reset the price  $\alpha^i$  implies an average duration of price contracts of 4 quarters. Following Rotemberg and Woodford (1998) and Benigno (2004), the inverse of the elasticity of producing the differentiated good  $\eta$  is calculated as

$$\eta = \epsilon_{wy} - \rho + \frac{1 - \gamma}{\gamma}, \quad (4.2.27)$$

where  $\epsilon_{wy}$  denotes the elasticity of the average real wage with respect to production and  $\gamma$  denotes the labor income share.

Under the baseline calibration, monetary policy responds to inflation ( $\phi_\pi = 1.5$ ), but it does not react to output ( $\phi_Y = 0$ ) and does not engage in interest rate smoothing ( $\phi_R = 0$ ). I assume throughout the paper that all interest rate rule coefficients are identical across countries and regimes.

**Table 4.3.:** *Baseline calibration*

$\rho$	1/6	Inv. of intertemporal elasticity of substitution in consumption
$n$	0.5	Country size measured by population
$\beta$	0.99	Discount factor
$\eta$	0.67	Inv. of elasticity of producing the differentiated good
$\epsilon_{wy}$	0.5	Production elasticity of average real wage
$\gamma$	0.75	Labor income share
$\sigma$	7.66	Elasticity of substitution between differentiated goods
$\alpha^i$	0.75	Probability of not being able to reset the price
$\phi_\pi$	1.5	Inflation coefficient in interest rate rule
$\phi_Y$	0	Output coefficient in interest rate rule
$\phi_R$	0	Interest rate smoothing coefficient in interest rate rule
$\rho_i$	0.9	Persistence of productivity shock
$\text{var } \nu_t^i$	1	Variance of white noise process
$\text{corr}(\nu_t^H, \nu_t^F)$	0	Correlation between country-specific white noise processes

I consider a broad range of values for the parameters of the model to check for the validity of the results (Table 4.4). In particular, the interest rate rules will also feature a reaction to output and interest rate smoothing.

**Table 4.4.:** *Parameter range*

$\rho$	[0.1,1.1]	Inv. of intertemporal elasticity of substitution in consumption
$n$	[0.05,0.95]	Country size measured by population
$\beta$	[0.9,1.0]	Discount factor
$\eta$	[0.2,3.0]	Inv. of elasticity of producing the differentiated good
$\epsilon_{wy}$	[0.2,1.2]	Production elasticity of average real wage
$\gamma$	[0.5,0.9]	Labor income share
$\sigma$	[5,25]	Elasticity of substitution between differentiated goods
$\alpha^i$	[0.05,0.95]	Probability of not being able to reset the price
$\phi_\pi$	[1.1,3.5]	Inflation coefficient in interest rate rule
$\phi_Y$	[0,3]	Output coefficient in interest rate rule
$\phi_R$	[0,0.95]	Interest rate smoothing coefficient in interest rate rule

### 4.3. Results under symmetry

First, I conduct the analysis under the assumption that the two countries are symmetric (except for country size  $n$ ). In particular, the degree of price stickiness is equal across countries. The world welfare loss function under symmetry,

equation (4.2.26), is repeated here for convenience:

$$\begin{aligned}
 W_t = & -\frac{1}{2} \left( (\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta)n(1 - n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\
 & \left. + \sigma(1 + \sigma\eta) \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)} \left[ \text{var} \pi_t^W + n(1 - n) \text{var} \pi_t^R \right] \right) \\
 & + t.i.p. + O(\|\xi\|^3). \tag{4.3.1}
 \end{aligned}$$

It contains four components: the variance of the consumption gap ( $\hat{C}_t - \tilde{C}_t$ ), the variance of the terms-of-trade gap ( $\hat{T}_t - \tilde{T}_t$ ), the variance of the world inflation rate ( $\pi_t^W$ ), and the variance of the inflation differential ( $\pi_t^R$ ).

### 4.3.1. Analytical results

The analytical results in this subsection are crucial to understanding the main finding of the paper. I derive the recursive law of motion (RLOM) of the model equations for each monetary regime using the method of undetermined coefficients to obtain the analytical expressions for the variances contained in the welfare loss function.<sup>16</sup> The derivations are based on the assumption that the degree of price stickiness and the persistence of productivity shocks are identical across countries ( $\alpha^H = \alpha^F$  and  $\rho_H = \rho_F$ ) and that monetary policy does not engage in interest rate smoothing ( $\phi_R = 0$ ).

Fortunately, it is not necessary to derive the RLOM for the variables consumption and world inflation, since they both behave identically across monetary regimes. To see this for the MU regime, eliminate the nominal interest rate  $\hat{R}_t$  in the Euler equation (4.2.7) by inserting the interest rate rule (4.2.13) and the equations for aggregate demand (4.2.8) and (4.2.9):

$$\rho E_t \hat{C}_{t+1} = (\rho + \phi_Y) \hat{C}_t + \phi_\pi \pi_t^W - E_t \pi_{t+1}^W. \tag{4.3.2}$$

The same equation is obtained completely analogously for the FX regime.

For world inflation, inserting the New Keynesian Phillips curves, which are identical across regimes, into the definition of world inflation  $\pi_t^W = n\pi_t^H + (1 - n)\pi_t^F$ , where  $\alpha^H = \alpha^F = \alpha$  due to symmetry and therefore  $k_T^H = k_T^F = k_T$  and

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<sup>16</sup>Benigno and Benigno (2008) use a similar approach in describing the behavior of the nominal exchange rate under different interest rate rules.

$k_C^H = k_C^F = k_C$ , yields

$$\pi_t^W = k_C(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^W. \quad (4.3.3)$$

The reason why world inflation is the same under both the MU and the FX regimes is that the terms of trade vanish from the equation when the degree of price stickiness is equal across countries. The fact that both consumption and world inflation behave identically across monetary regimes implies that the variance of consumption and the variance of world inflation are identical as well. As a result, they do not produce differences in welfare across the two regimes.

For the remaining two variables, the terms of trade and the inflation differential, the reduced system of equations under the MU regime is given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \quad (4.3.4)$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R. \quad (4.3.5)$$

The reduced system of equations under the FX regime is given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \quad (4.3.6)$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R + \Delta \hat{S}_t \quad (4.3.7)$$

$$E_t \Delta \hat{S}_{t+1} = -\phi_\pi \pi_t^R + \phi_Y \hat{T}_t. \quad (4.3.8)$$

Equations (4.3.4) and (4.3.6) are obtained by subtracting the New Keynesian Phillips curve of country  $H$  from that of country  $F$ . Equations (4.3.5) and (4.3.7) are the terms-of-trade identities. Equation (4.3.8) is obtained by inserting the interest rate rules (4.2.23) and (4.2.24) and the equations for aggregate demand (4.2.17) and (4.2.18) into the uncovered interest parity condition (4.2.22).

The RLOM under the MU regime is, then, given by (see Appendix B.4 for the entire derivation)

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \quad (4.3.9)$$

$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t, \quad (4.3.10)$$

with coefficients

$$b_1 = \frac{1 + k_T + \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta}$$

$$b_2 = \frac{1 + k_T - \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta}$$

$$c_1 = c_2 = c = \frac{k_T}{1 + k_T + \beta(1 - \rho_H - b_1)}.$$

The RLOM under the FX regime is given by

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \quad (4.3.11)$$

$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t \quad (4.3.12)$$

$$\Delta \hat{S}_t = b_3 \hat{T}_{t-1} + c_3 \tilde{T}_t, \quad (4.3.13)$$

with coefficients

$$b_1 = 0$$

$$b_2 = 0$$

$$b_3 = -1$$

$$c_1 = \frac{(\phi_\pi - \rho_H)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}$$

$$c_2 = \frac{(1 - \rho_H + \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}$$

$$c_3 = \frac{(\phi_\pi - 1 - \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}.$$



Consequently, the variances of the terms-of-trade gap and the variances of the inflation differential under each regime are given by

$$\text{var}_{MU}(\hat{T}_t - \tilde{T}_t) = \left[ \frac{(1 + \rho_H b_1)c^2}{(1 - b_1^2)(1 - \rho_H b_1)} - \frac{2c}{1 - \rho_H b_1} + 1 \right] \text{var} \tilde{T}_t \quad (4.3.14)$$

$$\text{var}_{FX}(\hat{T}_t - \tilde{T}_t) = (c_1 - 1)^2 \text{var} \tilde{T}_t \quad (4.3.15)$$

$$\text{var}_{MU} \pi_t^R = \frac{2c^2(1 - \rho_H)}{(1 + b_1)(1 - \rho_H b_1)} \text{var} \tilde{T}_t \quad (4.3.16)$$

$$\text{var}_{FX} \pi_t^R = c_2^2 \text{var} \tilde{T}_t \quad (4.3.17)$$

$$\text{var} \tilde{T}_t = \frac{1}{1 - \rho_H^2} \left( \frac{\eta}{1 + \eta} \right)^2 \left[ \text{var} \nu_t^H + \text{var} \nu_t^F - 2 \text{cov}(\nu_t^H, \nu_t^F) \right]. \quad (4.3.18)$$

Two important differences exist between the MU and FX regime. First, in contrast to the MU regime, there is no persistence in the terms of trade nor in the inflation differential under the FX regime ( $b_1 = b_2 = 0$ ). Hence, once the shock has vanished, both variables return immediately to their steady state. This is due to the nominal exchange rate. Intuitively, the coefficient  $b_3 = -1$  implies that, if the terms of trade were, for example, one percent below the steady state in the previous period, the nominal exchange rate would increase by one percent in the current period, so that the terms of trade are at steady state. Naturally, this mechanism is absent under the MU regime, since the nominal exchange rate is fixed. Thus, both the terms of trade and the inflation differential are inertial or history-dependent in the sense that they depend on the realization of the terms of trade in the previous period. While the inertia of the terms of trade in the context of a monetary union has been recognized before (e.g., Benigno, 2004; Pappa, 2004), it was regarded solely as an additional distortion in the economy. However, as will be shown below, the inertia of the terms of trade will also prove to be beneficial, namely from the perspective of stabilizing inflation expectations.

Second, in contrast to the MU regime, monetary policy is able to influence the terms-of-trade gap and the inflation differential under the FX regime, i.e., monetary policy is more potent under the FX regime. Technically, the variance of the terms-of-trade gap and of the inflation differential depend on the interest rate rule coefficients  $\phi_\pi$  and  $\phi_Y$ . Moreover, if monetary policy is extremely aggressive towards inflation under the FX regime ( $\phi_\pi \rightarrow \infty$ ), the variance of the terms-of-trade gap and of the inflation differential converge towards zero (since

$c_1 \rightarrow 1$  and  $c_2 \rightarrow 0$ ).<sup>17</sup> Thus, the efficient equilibrium can be approximated arbitrarily well, reducing the welfare loss to zero. In contrast, the variance of the terms-of-trade gap and of the inflation differential under the MU regime cannot be zero, and therefore the efficient equilibrium is not feasible. Thus, only the FX regime features the so-called divine coincidence.<sup>18</sup>

### 4.3.2. Price stickiness

The analytical expressions for the variances can be used to derive the condition under which world welfare is larger in one or the other monetary regime. Unfortunately, the resulting condition is a complex inequality that provides hardly any intuition. In the following, I thus compute the welfare losses numerically and display the results graphically. The deep parameters are calibrated according to the baseline calibration (Table 4.3), except for the parameters of interest, which take on a broad range of values (Table 4.4).

Whether the world welfare loss is higher in one than in the other monetary regime depends crucially on the Calvo parameter  $\alpha$ , i.e., the degree of price stickiness in both economies (Figure 4.1). In both regimes, the world welfare loss is increasing in the degree of price stickiness.<sup>19</sup> When the degree of price stickiness is rather low, the world welfare loss is higher under the MU regime than under the FX regime. The countries are better off with their own currency and their own independent monetary policy. However, beyond a certain threshold ( $\alpha \approx 0.5$ ), where the degree of price stickiness is rather high, the world welfare loss is higher under the FX regime than under the MU regime. The countries are better off forming a monetary union with one currency and one common monetary authority. Quantitatively, the difference in welfare between the two monetary regimes can be substantial. Under the baseline calibration, the welfare loss under the MU regime is roughly 40 percent lower than under the FX regime (0.8/1.3).

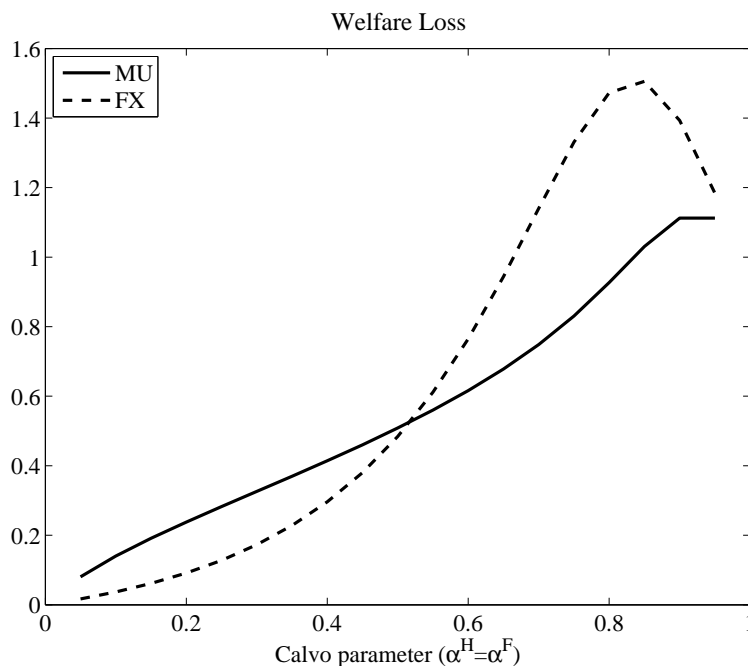
As described above, two components of the world welfare loss function (4.3.1) behave identically across monetary regimes and, therefore, cannot create welfare differences across regimes: the consumption gap and the world inflation rate

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<sup>17</sup>Benigno and Benigno (2008) reach the same result for interest rate rules containing the output gap (deviation from flexible-price output).

<sup>18</sup>See Corsetti, Dedola, and Leduc (2011) for details on the divine coincidence in open economies.

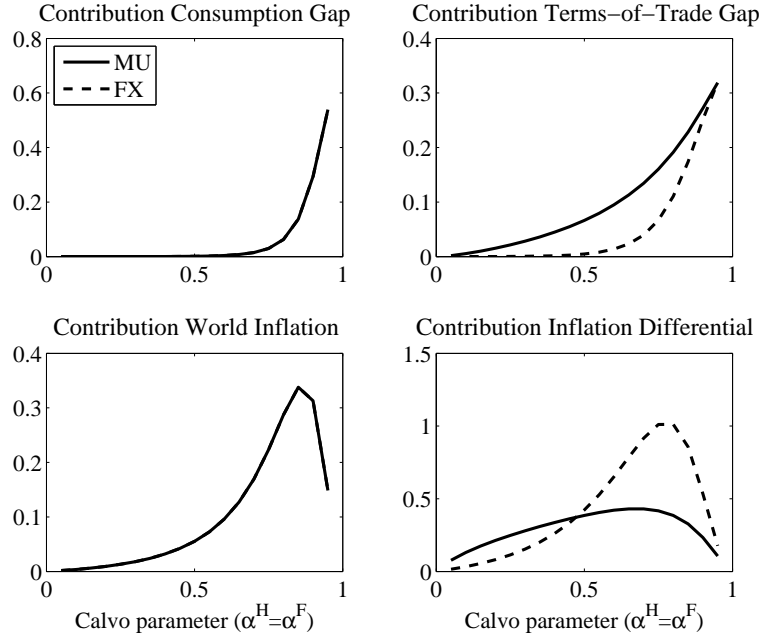
<sup>19</sup>This feature is common to the closed-economy setup of the basic New Keynesian model, as in Galí (2008).

**Figure 4.1.:** *World welfare loss as a function of the degree of price stickiness ( $\alpha^H = \alpha^F$ )*

(Figure 4.2, upper and lower left panel). However, this does not hold for the terms-of-trade gap and the inflation differential (Figure 4.2, upper and lower right panel). The contribution of the terms-of-trade gap to the world welfare loss is higher under the MU regime than under the FX regime regardless of the degree of price stickiness.<sup>20</sup> This indicates that the MU regime entails costs. However, the contribution of the terms-of-trade gap is much smaller than the contribution of the inflation differential. This is due to the fact that agents attach by far the highest weight to inflation, which is traditionally the case in microfounded welfare measures derived from New Keynesian models. Therefore, the inflation differential is the key to understanding the above finding that the MU regime yields higher world welfare when prices are relatively sticky. In fact, the pattern in the lower right panel of Figure 4.2 closely resembles the pattern in Figure 4.1, with a similar threshold value of  $\alpha \approx 0.5$ . This indicates that the MU regime entails benefits.

The contribution of a component to the world welfare loss is the product of the variance of that component and its weight. The weight and variance of the inflation differential show opposite patterns with respect to price stickiness.

<sup>20</sup> This holds for the contribution of the country-specific output gaps as well. The corresponding graphs are available upon request.

**Figure 4.2.:** Contributions to world welfare loss in Figure 4.1


Whereas the variance decreases with a rising degree of price stickiness (Figure 4.3), the weight increases (Figure 4.4). Thus, although the variance decreases with the degree of price stickiness, which per se enhances the agents' welfare, the agents attach a higher weight to inflation as prices become stickier.<sup>21</sup>

Since the weight of the inflation differential is identical across the two monetary regimes (this holds for all components), it is the variance that causes the difference in welfare. Whereas the variance of the inflation differential is higher under the MU regime for lower degrees of price stickiness, it is higher under the FX regime for higher degrees of price stickiness. That is, inflation rates are more stable under the MU regime when prices are relatively sticky.

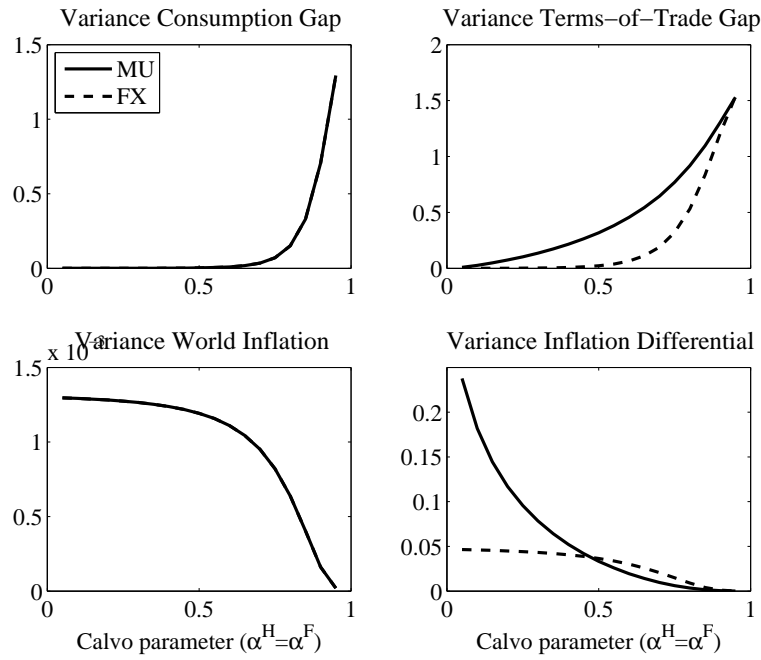
The inflation differential in period  $t$  can be expressed as the sum of current and discounted expected future terms-of-trade gaps. Solving equation (4.3.4) forward, which holds under both regimes, gives

$$\pi_t^R = -k_T E_t \sum_{k=0}^{\infty} \beta^k (\hat{T}_{t+k} - \tilde{T}_{t+k}). \quad (4.3.19)$$

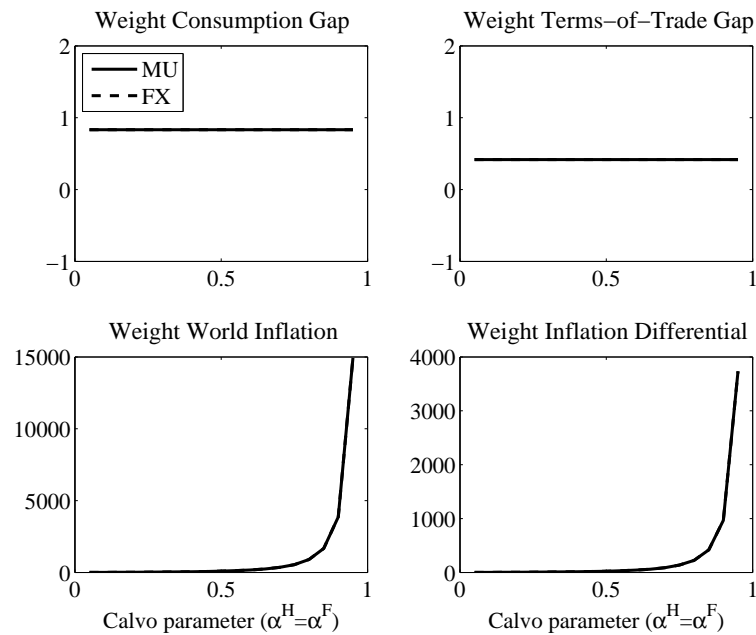
Accordingly, current and expected future terms-of-trade gaps are the only determinant for the current inflation differential. As shown next, two endogenous

<sup>21</sup>The agents attach a higher weight to inflation as prices become stickier because the degree of inefficient price dispersion between differentiated goods is increasing in the degree of price stickiness for a given level of aggregate inflation (Woodford, 2003).

**Figure 4.3.:** *Variances underlying the contributions in Figure 4.2*



**Figure 4.4.:** *Weights underlying the contributions in Figure 4.2*



effects that both influence the terms-of-trade gap explain why inflation rates are more stable under the MU regime when prices are relatively sticky. First, the stabilizing property of a flexible nominal exchange rate declines as prices become stickier (“inherited stickiness of the nominal exchange rate”). Second,

fixing the exchange rate entails the benefit of affecting inflation expectations in a favorable way by inducing history dependence into the economy (“inherent benefit of monetary unions”).

### Inherited stickiness of the nominal exchange rate

Whether the nominal exchange rate stabilizes or destabilizes the terms-of-trade gap, thereby facilitating an efficient allocation across countries, depends on the way monetary policy is conducted. Recall the recursive law of motion for the nominal exchange rate, (4.3.13):

$$\Delta \hat{S}_t = -\hat{T}_{t-1} + c_3 \tilde{T}_t,$$

with

$$c_3 = \frac{(\phi_\pi - 1 - \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}.$$

Under the baseline calibration, where monetary policy reacts to inflation, but not to output ( $\phi_Y = 0$ ), the coefficient  $c_3$  is unambiguously positive (since  $\phi_\pi > 1$ ) and smaller than one. Accordingly, in response to a shock that leads to an increase in the flexible-price terms of trade, the nominal exchange rate will increase as well, pushing up the sticky-price terms of trade closer to the flexible-price terms of trade. Thus, the nominal exchange rate helps to stabilize the terms-of-trade gap.

However, the stabilizing effect of the nominal exchange rate weakens as prices become stickier. The size of the response of the nominal exchange rate to a productivity shock decreases with the degree of price stickiness. Analytically, as the degree of price stickiness  $\alpha$  increases,  $k_T$  decreases and  $c_3$  decreases. In the limit, when prices are fixed ( $\alpha \rightarrow 1$ ), the nominal exchange rate is fixed as well ( $k_T \rightarrow 0, c_3 \rightarrow 0$ ).

The reason for this is that the expected change in the nominal exchange rate depends on the interest rate differential across countries according to the uncovered interest parity condition, (4.2.22):

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t^H - \hat{R}_t^F.$$

Interest rates, in turn, are set by monetary policy in response to inflation according to the interest rate rules. Therefore, an increase in price stickiness, which

reduces inflation variability, reduces interest rate variability and, ultimately, reduces the variability of the nominal exchange rate.

Thus, the nominal exchange rate inherits the stickiness of goods prices. This, in turn, hampers the stabilization of the terms-of-trade gap. Therefore, the stabilizing property of the nominal exchange rate of facilitating an efficient allocation across countries declines with the degree of price stickiness. Notably, for this effect to be effective, the uncovered interest parity condition need not hold exactly. It suffices for the interest rates and the nominal exchange rate to be linked.

### **Inherent benefit of monetary unions**

The fact that the benefit of a flexible nominal exchange rate declines with the degree of price stickiness cannot alone explain the finding that the MU regime is welfare-improving over the FX regime. For even under relatively sticky prices, the nominal exchange rate stabilizes the terms-of-trade gap at least to some extent compared to a situation with a completely fixed nominal exchange rate, as under the MU regime. This is also the reason why the variance of the terms-of-trade gap is lower under the FX regime regardless of the degree of price stickiness (Figure 4.3, upper right panel). Therefore, the MU regime must also provide a benefit.

The MU regime differs from the FX regime in one important respect, as the analytical results from Section 4.3.1 have shown. In contrast to the FX regime, the economy under the MU regime is intrinsically inertial. So, even in the presence of a one-off shock, the inflation differential and the terms-of-trade gap are persistent. As shown next, this inertia will result in a higher stability of inflation rates.

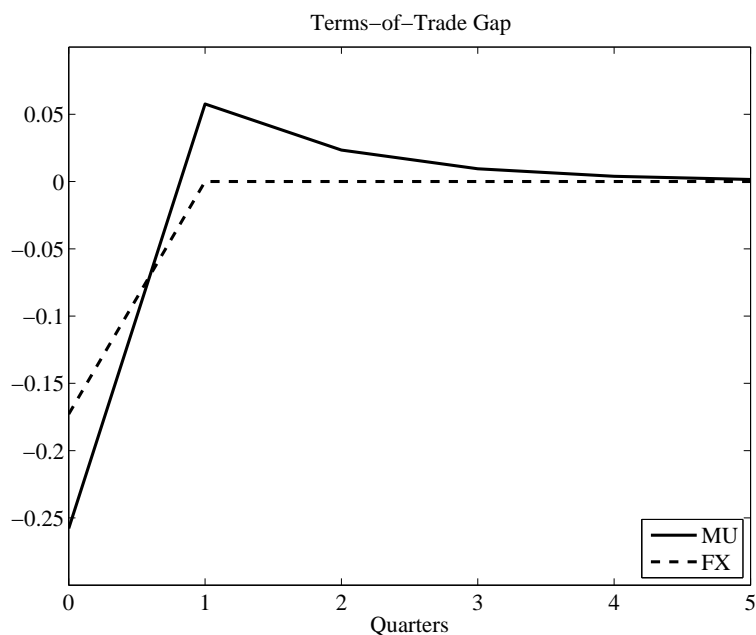
The qualitative difference between the two monetary regimes can be seen clearly by looking at the impulse response of the terms-of-trade gap to a positive one-off productivity shock in country  $H$  (Figure 4.5).<sup>22</sup> On impact, the terms-of-trade gap decreases under both regimes because the sticky-price terms of trade do not increase as much as the flexible-price terms of trade due to the stickiness of prices. However, in the following period, when the shock has vanished, the terms-of-trade gap has returned to the steady state under the FX regime, but

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<sup>22</sup>The degree of price stickiness was set low ( $\alpha = 0.2$ ) to ensure that the differences in the impulse responses are clearly visible. The differences are much smaller for higher degrees of price stickiness, but they are qualitatively the same.

not under the MU regime. Under the FX regime, it is the nominal exchange rate that brings the terms-of-trade gap automatically back to the steady state in the absence of shocks.<sup>23</sup> Under the MU regime, this mechanism is absent, since the nominal exchange rate is fixed. As a result, the terms-of-trade gap is intrinsically inertial or history-dependent.

**Figure 4.5.:** *Impulse response of the terms-of-trade gap to a positive one-off productivity shock in country H ( $\rho_H = 0$ ), with  $\alpha = 0.2$*



Importantly, the history dependence of the terms-of-trade gap manifests itself in an overshooting pattern. The terms-of-trade gap overshoots because the sticky-price terms of trade are still elevated above the steady state after the shock has vanished, whereas the flexible-price terms of trade are back at the steady state. In subsequent periods, the terms-of-trade gap converges back to the steady state.

The qualitative difference in the dynamics between the two monetary regimes prevails in situations in which the productivity shock itself is persistent (Figure 4.6, left panel).<sup>24</sup> Whereas the terms-of-trade gap converges monotonically back to the steady state under the FX regime, it overshoots the steady state under the MU regime.

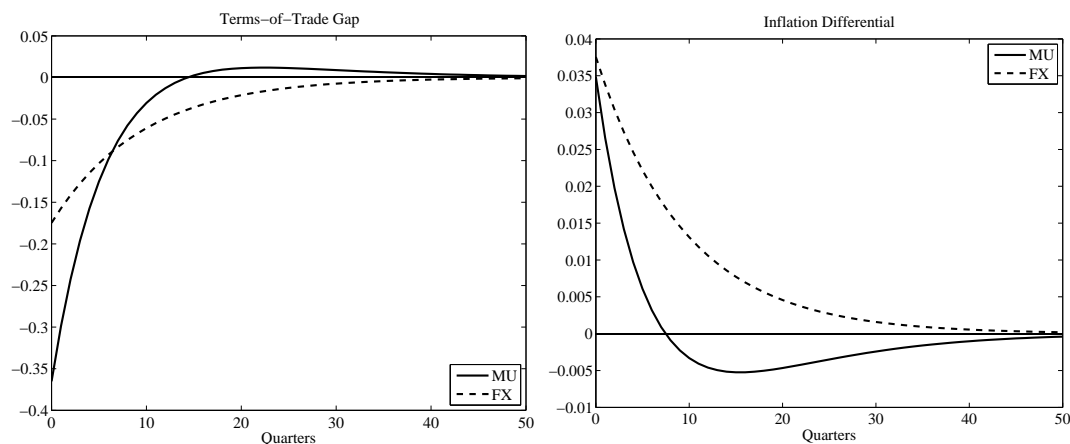
<sup>23</sup>Interestingly, this mechanism is independent of the interest rate rule coefficients  $\phi_\pi$  and  $\phi_Y$  (recall the RLOM coefficient  $b_3 = -1$ ).

<sup>24</sup>The calibration underlying the impulse responses in Figure 4.6 is now identical to the calibration underlying the welfare results in Figure 4.1 through 4.4.



Since the inflation differential is determined by the terms-of-trade gap and its expected future path (recall equation 4.3.19), it exhibits the same qualitative difference. Accordingly, under the FX regime the inflation differential increases on impact and converges monotonically back to the steady state (Figure 4.6, right panel). In contrast, under the MU regime, price setters adjust their prices less in the initial period despite the stronger initial change in the terms-of-trade gap because they anticipate the future overshooting of the terms-of-trade gap. In subsequent periods, inflation approaches the steady state faster than under the FX regime and eventually overshoots the steady state as well.<sup>25</sup> As a result, the variance of the inflation differential, i.e., the sum of squared deviations of the inflation differential from zero, is lower under the MU regime than under the FX regime.

**Figure 4.6.:** *Impulse responses to a positive productivity shock in country H with  $\rho_H = 0.9$  and  $\alpha = 0.75$*



To sum up: Since price setters are forward-looking, not only present, but also expected future terms-of-trade gaps matter for current inflation. Since the nominal exchange rate is fixed under the MU regime, the terms-of-trade gap overshoots in response to a shock at some point in time, which would then call for the opposite price adjustments as in the present. In anticipation of this, current price responses are smaller in magnitude than under the FX regime. As a result, inflation is more stable under the MU regime. Thus, the inherent benefit of monetary union is that it affects inflation expectations in such a way as to lower the welfare-relevant variance of inflation by inducing history dependence into the economy. The strength of this benefit increases as prices

<sup>25</sup>Throughout the paper, I use the term “overshooting” to describe both “overshooting” and “undershooting.”

become stickier, since price setters attach higher weights to future terms-of-trade gaps as the probability of being able to reset prices decreases.

The benefit of history dependence is well-known from the analysis of optimal monetary policy in a closed-economy environment. Optimal monetary policy *under discretion* is inferior from a welfare perspective to optimal policy *under commitment* because the former does not influence the inflation expectations of price setters in a favorable way. It suffers from the so-called stabilization bias.<sup>26</sup> In contrast, optimal monetary policy under commitment induces history dependence into the economy, therefore taking advantage of the fact that price setters are forward-looking. This results in a higher stability of inflation. In exactly the same sense, forming a monetary union may be superior to maintaining a flexible exchange rate under certain conditions because fixing the nominal exchange rate affects inflation expectations in a favorable way by inducing history dependence into the economy.

Notably, this benefit exists despite the fact that price levels are not stationary.<sup>27</sup> Thus, and in contrast to Monacelli (2004), the benefit does not hinge upon stationarity of price levels. This is a particular feature of the small-open-economy assumption and does not generally carry over to a two-country environment. Instead, the benefit hinges upon the overshooting pattern of the terms of trade, the anticipation of which reduces the magnitude of price changes, rendering the inflation rates more stable.

While the inertia of the terms of trade has been recognized before (e.g., Benigno, 2004; Pappa, 2004), it was regarded solely as an additional distortion in the economy, not as a benefit. The reason for this is the different assumption on monetary policy. Given that there are as many policy instruments as distortions under the FX regime but more distortions than policy instruments under the MU regime, monetary policy is more potent under the FX regime.<sup>28</sup> Under the assumption that monetary policy is conducted optimally and is equipped with the necessary information, as is the case in Pappa (2004), it is not surprising that the FX regime is superior, since monetary policy has the ability to implement the flexible-price allocation, achieving the first-best solution (divine coincidence). While the beneficial effect of the inertial terms of trade is

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<sup>26</sup>For details on the stabilization bias, see, e.g., Woodford (2003, Ch. 7), Gali (2008, Ch. 5), or Walsh (2010, Ch. 8).

<sup>27</sup>Impulse responses for price levels are available upon request.

<sup>28</sup>The fact that monetary policy is more potent under the FX regime is reminiscent of the analytical results from Section 4.3.1.

also present in these circumstances, it is dwarfed by an ideal monetary policy. By contrast, when monetary policy is not able to be conducted optimally, e.g., due to the very demanding information requirements, and instead it resorts to interest rate rules, as is the case in this paper, its abilities are more limited. In these circumstances, the beneficial effect of the inertial terms of trade may be strong enough as to render the MU regime superior.

The following section elaborates on the importance of monetary policy for the welfare performance of the MU regime.

### 4.3.3. Monetary policy

The finding that forming a monetary union is beneficial when prices are relatively sticky is very robust to the range of parameter values considered in Table 4.4.<sup>29</sup> The important exception is the parameters that govern the behavior of monetary policy, i.e., the coefficients of the interest rate rules. These will be considered next. Critically, such an analysis, in which the welfare performance of a monetary union regime is analyzed under different monetary policy designs, is not possible under optimal monetary policy—an assumption often made in the strand of the New Keynesian literature that deals with OCA issues—since deviations from optimality are ruled out by assumption.

#### Response to inflation

Whether forming a monetary union turns out to be beneficial depends crucially on the inflation coefficient  $\phi_\pi$  in the interest rate rules, i.e., on the aggressiveness of monetary policy towards inflation (Figure 4.7).<sup>30</sup> Starting out at a very low response of monetary policy to inflation ( $\phi_\pi$  above, but close to, one), the MU regime yields a lower world welfare loss for every degree of price stickiness. Increasing the aggressiveness of monetary policy a little bit results in the FX regime being superior for very low degrees of price stickiness, but inferior for higher degrees of price stickiness. As the aggressiveness of monetary policy increases further, the threshold value for  $\alpha$  increases beyond which the MU regime yields a lower world welfare loss. Eventually, beyond a certain aggressiveness of

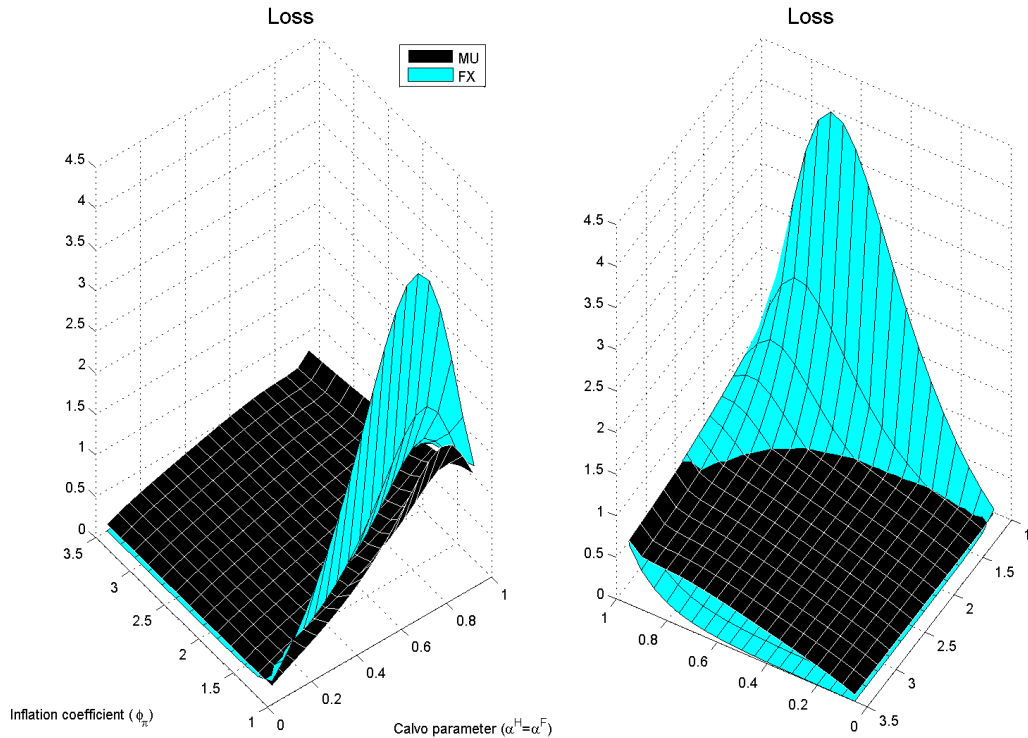
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<sup>29</sup>The corresponding graphs are available upon request.

<sup>30</sup>The graph in the right panel of Figure 4.7 is a rotation of the graph in the left panel, in order to be able to see behind the steep surface area. Cutting through the two surface areas along  $\phi_\pi = 1.5$  produces Figure 4.1.

monetary policy towards inflation ( $\phi_\pi \approx 2.5$ ), the MU regime is inferior to the FX regime regardless of the degree of price stickiness.

**Figure 4.7.:** *World welfare loss as a function of the degree of price stickiness ( $\alpha^H = \alpha^F$ ) and the inflation coefficient ( $\phi_\pi$ ), from two different angles*

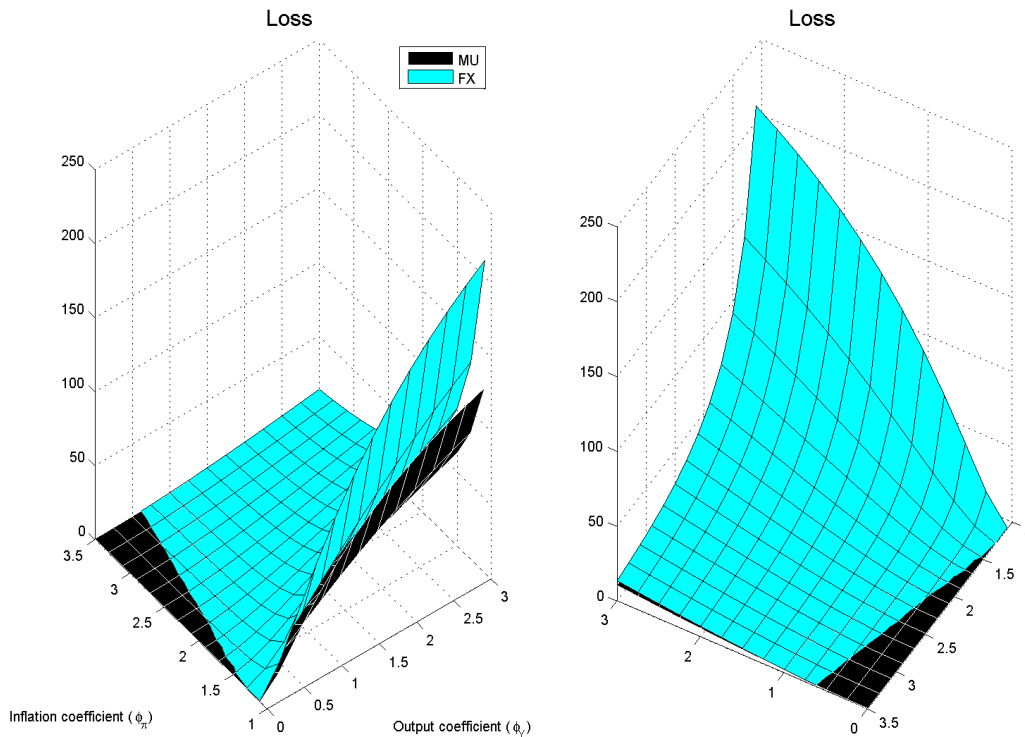


The intuition for this is as follows. Under the FX regime, when monetary policy reacts to inflation only, the nominal exchange rate stabilizes the terms-of-trade gap in response to shocks (see Section 4.3.2). The strength of this stabilizing property increases with the aggressiveness of monetary policy towards inflation, since monetary policy directly influences the nominal exchange rate via the uncovered interest parity condition. As a result, even when prices are relatively sticky, monetary policy can counteract by being more aggressive towards inflation. In the limit ( $\phi_\pi \rightarrow \infty$ ), monetary policy perfectly stabilizes all welfare-relevant variables, reducing the welfare loss to zero (divine coincidence). This is not the case under the MU regime because the common monetary policy has no influence on the terms-of-trade gap and the inflation differential when prices are equally sticky across countries (see Section 4.3.1).

**Response to output**

The welfare ranking between the two monetary regimes depends also on the output coefficient in the interest rate rules  $\phi_Y$ , i.e., on the aggressiveness of monetary policy towards output (Figure 4.8). For almost all the combinations of  $\phi_Y$  and  $\phi_\pi$  considered, the FX regime yields a higher world welfare loss than the MU regime, although the degree of price stickiness was deliberately chosen to favor the FX regime ( $\alpha = 0.2$ ). Increasing the degree of price stickiness would favor the MU regime further. In general, the stronger monetary policy reacts to output, the stronger it needs to react to inflation for the FX regime to remain superior. This relationship is very steep; a small increase in  $\phi_Y$  (e.g. from 0 to 0.25) requires a strong increase in  $\phi_\pi$  (from roughly 1.5 to 2.3).

**Figure 4.8.:** *World welfare loss as a function of the output coefficient ( $\phi_Y$ ) and the inflation coefficient ( $\phi_\pi$ ), with  $\alpha = 0.2$ , from two different angles*



Unlike in the case of the response to inflation, the more aggressive monetary policy reacts to output, the smaller the impact response of the nominal

exchange rate to shocks becomes (see coefficient  $c_3$  of the RLOM). When the aggressiveness of monetary policy towards output relative to inflation exceeds a certain degree ( $\phi_Y > \phi_\pi - 1$ ), the nominal exchange rate destabilizes the terms-of-trade gap in response to shocks ( $c_3 < 0$ ).

The reason for this is that from a welfare perspective, a response to output by monetary policy is detrimental. It is not the deviation of output from the steady state that is welfare-relevant, it is the deviation from the flexible-price counterpart (output gap).<sup>31</sup> For example, a positive productivity shock in country  $H$  induces an increase in output, but a decrease in the output gap, since the increase in output is lower than the increase in flexible-price output. A welfare-oriented reaction of monetary policy would require a reduction in the interest rate due to the negative output gap. Instead, monetary policy raises the interest rate due to the rise in output. As a result, the variance of the inflation differential is higher when monetary policy reacts to output ( $\phi_Y > 0$ ) than when it does not react to output ( $\phi_Y = 0$ ).<sup>32</sup>

While a reaction to output is detrimental under both regimes (in Figure 4.8, the welfare loss is increasing in the output coefficient  $\phi_Y$  under both regimes), the damage in terms of welfare is greater under the FX regime. The reason for this is that, in contrast to the MU regime, monetary policy under the FX regime affects every component of the welfare loss function (see Section 4.3.1). Thus, conducting “bad” monetary policy is more harmful under the FX regime because monetary policy is more potent in this regime. Essentially, the nominal exchange rate does not compensate for monetary policy mistakes, it reinforces them. In this sense, a monetary union provides a protective mechanism against monetary policy mistakes.

### Interest rate smoothing

The welfare ranking between the two monetary regimes depends on the degree of interest rate smoothing  $\phi_R$  as well (Figure 4.9).<sup>33</sup> When monetary policy does not engage in interest rate smoothing ( $\phi_R = 0$ ), the MU regime yields a lower welfare loss for relatively sticky prices. As the degree of interest rate smoothing increases, the threshold value for  $\alpha$  beyond which the MU regime

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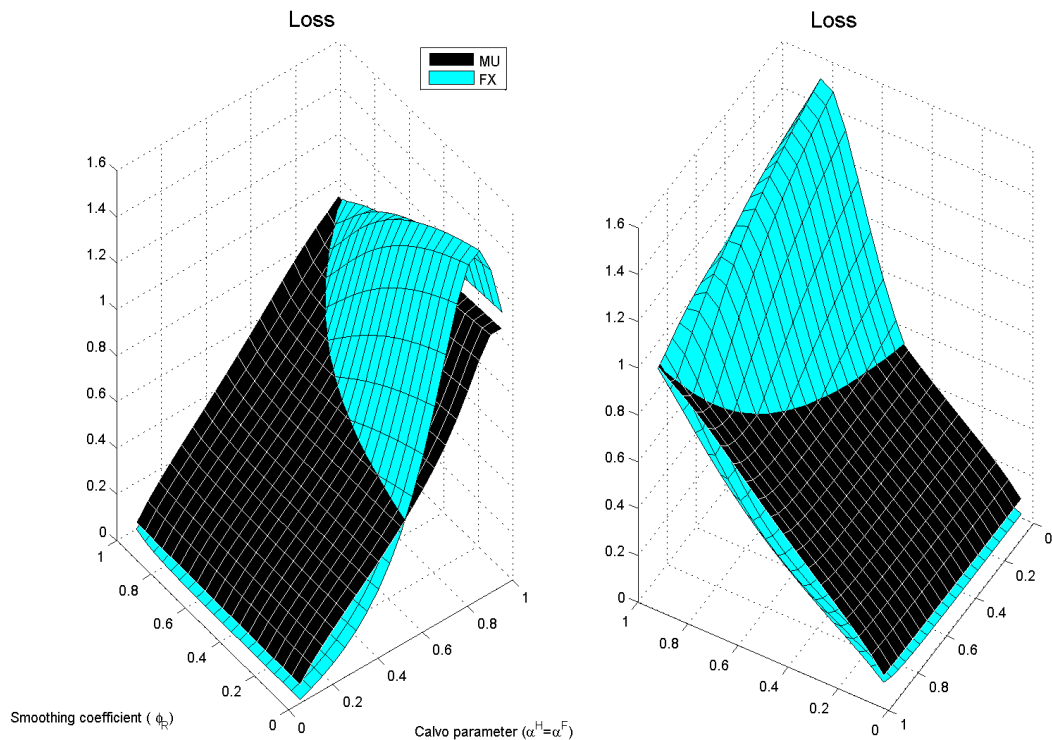
<sup>31</sup>Recall that the welfare loss function can be expressed alternatively in terms of country-specific output gaps instead of the consumption gap (see equation B.129).

<sup>32</sup>This is common to the closed-economy setup of the basic New Keynesian model, as in Galí (2008).

<sup>33</sup>Cutting through the two surface areas along  $\phi_R = 0$  produces Figure 4.1

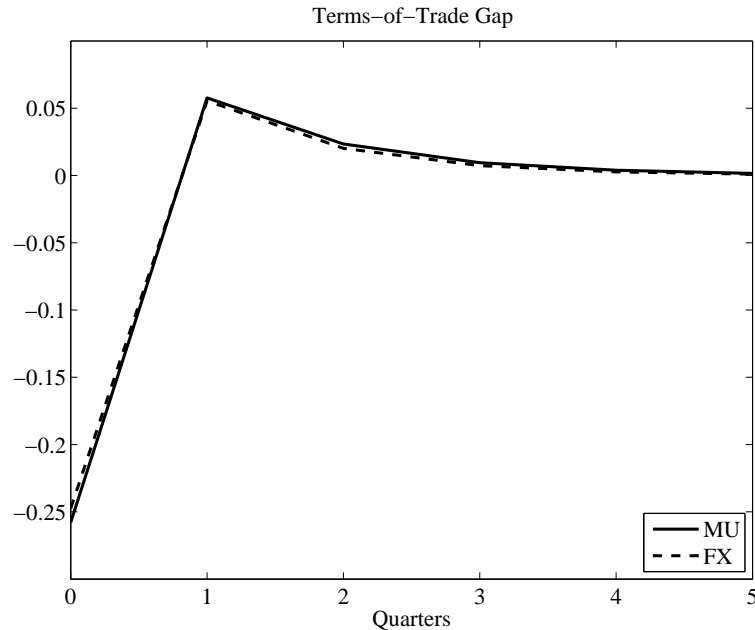
is superior increases as well. For very high degrees of interest rate smoothing, the MU regime is welfare-improving only for extremely high degrees of price stickiness. Thus, interest rate smoothing makes a beneficial monetary union less likely.

**Figure 4.9.:** *World welfare loss as a function of the degree of price stickiness ( $\alpha^H = \alpha^F$ ) and the interest rate smoothing coefficient ( $\phi_R$ ), from two different angles*



The reason for this becomes clear by looking again at the impulse response of the terms-of-trade gap to a positive one-off productivity shock in country  $H$ , but now with a relatively high degree of interest rate smoothing (Figure 4.10). The impulse response under the MU regime is identical to the situation without interest rate smoothing (Figure 4.5) because monetary policy continues to exert no influence on the terms of trade when prices are equally sticky across countries. In contrast, the impulse response under the FX regime now resembles the response under the MU regime. Although the productivity shock is one-off, the terms-of-trade gap displays inertia in the form of overshooting.

**Figure 4.10.:** *Impulse response of the terms-of-trade gap to a positive one-off productivity shock in country  $H$  ( $\rho_H = 0$ ), with  $\phi_R = 0.9$  and  $\alpha = 0.2$*



As a result, inflation expectations are affected in the same favorable way as under the MU regime, namely by inducing history dependence into the economy. Only the source of history dependence is different. Under the FX regime, monetary policy has to engage in interest rate smoothing to induce history dependence. Under the MU regime, history dependence is induced automatically by the fact that the nominal exchange rate is fixed. For the FX regime to be welfare-improving over the MU regime under relatively sticky prices, monetary policy has to implement a sufficiently high degree of interest rate smoothing. This renders the inflation differential more stable under the FX regime.

The fact that interest rate smoothing under the FX regime reduces the welfare loss by stabilizing inflation comes as no surprise. As shown by Woodford (1999), one way for monetary policy to implement the kind of history dependence that is desirable from the perspective of optimal monetary policy is to engage in interest rate smoothing by including a feedback of the current nominal interest rate to past realizations of the nominal interest rate, as is the case in the interest rate rules given by equations (4.2.23) and (4.2.24).



## 4.4. Results under asymmetry

In the following, I check whether asymmetries in country size or in the degree of price stickiness matter for the welfare ranking between the MU and FX regime.

### 4.4.1. Country size

If the two countries differ only in population size, the analytical results from Section 4.3.1 carry over. Accordingly, the RLOM under both monetary regimes, equations (4.3.9) through (4.3.13), are valid in this case. As one can see, the RLOMs are independent of the country size  $n$ . Thus, the welfare-relevant inflation differential and the terms-of-trade gap are independent of  $n$ . As a consequence, the threshold value for  $\alpha$  beyond which the MU regime yields a lower world welfare loss is completely insensitive with respect to  $n$ . Therefore, the welfare ranking between the MU and FX regime does not depend on country size.

### 4.4.2. Price stickiness

The world welfare loss function under different degrees of price stickiness across the two countries is given by equation (4.2.25) and repeated here for convenience:

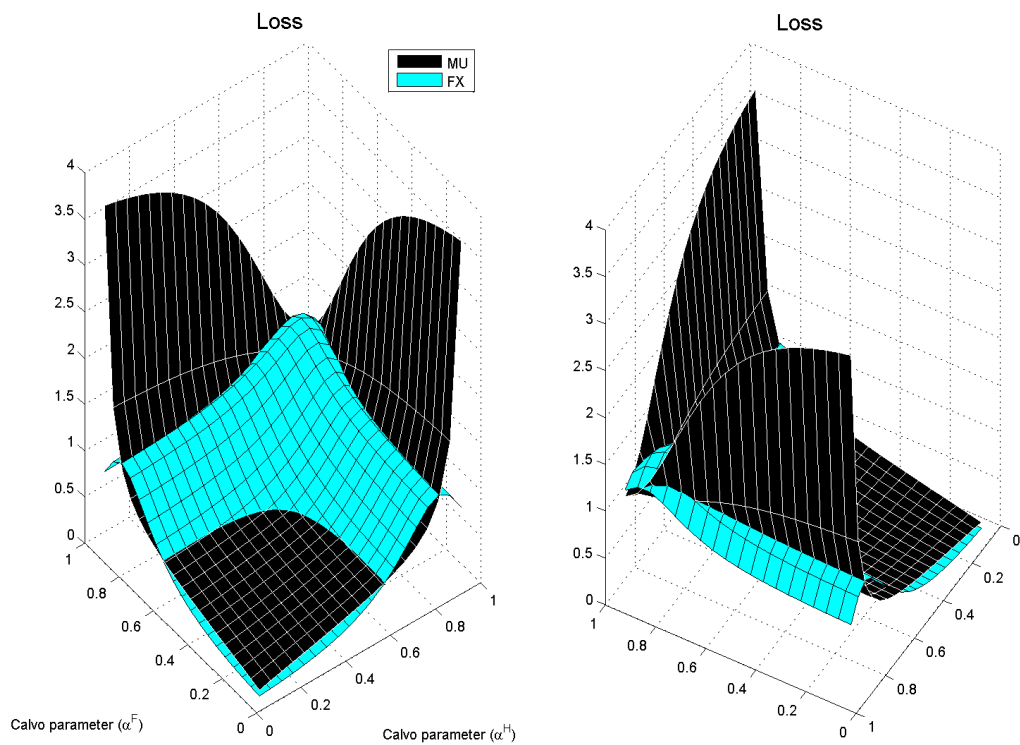
$$\begin{aligned}
 W_t = & -\frac{1}{2} \left( (\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta)n(1 - n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\
 & + \sigma(1 + \sigma\eta)n \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H\beta)} \text{var} \pi_t^H \\
 & \left. + \sigma(1 + \sigma\eta)(1 - n) \frac{\alpha^F}{(1 - \alpha^F)(1 - \alpha^F\beta)} \text{var} \pi_t^F \right) \\
 & + t.i.p. + O(\|\xi\|^3). \tag{4.4.1}
 \end{aligned}$$

It contains four components: the variance of the consumption gap ( $\hat{C}_t - \tilde{C}_t$ ), the variance of the terms-of-trade gap ( $\hat{T}_t - \tilde{T}_t$ ), the variance of inflation in country  $H$  ( $\pi_t^H$ ), and the variance of inflation in country  $F$  ( $\pi_t^F$ ).

Unless the degree of price stickiness is extremely high ( $\alpha \geq 0.9$ , which corresponds to an average duration of price contracts of at least 10 quarters), asymmetry in the degree of price stickiness does not matter for the welfare ranking between the MU and FX regime (Figure 4.11). Drawing from the anal-

ysis above, the intuition is the following. First, the inherent benefit of monetary unions of inducing history dependence is independent of country characteristics. It depends only on the fact that the nominal exchange rate is fixed and that price setters are forward-looking. Second, the nominal exchange rate inherits the stickiness of goods prices from both countries. It does not matter if the stickiness is equally present in both countries or if the stickiness comes primarily from one country. Thus, as long as the aggregate degree of price stickiness in the world as a whole is sufficiently high, the MU regime continues to be beneficial.

**Figure 4.11.:** *World welfare loss as a function of the degree of price stickiness in country H ( $\alpha^H$ ) and country F ( $\alpha^F$ ), from two different angles*



## 4.5. Contrast to traditional OCA theory

This paper has shown that in the standard two-country New Keynesian DSGE model, countries benefit from forming a monetary union when prices are rel-

actively sticky but do not when prices are relatively flexible. This finding is clearly at odds with Friedman (1953)'s case for flexible exchange rates and with the traditional OCA theory.<sup>34</sup> First, note that in the model employed in this paper the need for macroeconomic adjustment is triggered by an asymmetric *temporary* change in *productivity*, whereas the traditional OCA analysis usually assumes a *permanent* shift in *demand* from the products of one country to the products of the other.

Second, and more importantly, the role of expectations differs quite substantially between the New Keynesian and the Old Keynesian framework. In the Old Keynesian framework, in which the key predictions of the traditional OCA theory were developed, expectations of economic agents were treated as exogenous. In contrast, in New Keynesian models private-sector expectations are treated as endogenous. As shown, it is precisely the expectations channel that renders inflation rates more stable under the monetary union regime. This channel is naturally missing in models without such an expectational feedback mechanism.<sup>35</sup>

## 4.6. Conclusion

The main finding of this paper is that in the basic two-country New Keynesian DSGE model, in which monetary policy is conducted via Taylor-type interest rate rules, forming a monetary union is welfare-improving when prices are relatively sticky. In this case, the costs of higher output-gap and terms-of-trade-gap instability are outweighed by the benefit of higher inflation stability. Two endogenous effects are responsible for this. First, the stabilizing property of a flexible nominal exchange rate declines as prices become stickier. Second, fixing the exchange rate entails the inherent benefit that it stabilizes inflation expectations by inducing inertia into the economy.

The paper has also shown that whether forming a monetary union is beneficial or not depends heavily on the way monetary policy is conducted. When monetary policy responds to inflation aggressively or when it implements a high degree of interest rate smoothing, maintaining a flexible exchange rate is superior. In contrast, monetary policy mistakes (such as a reaction to output) are

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<sup>34</sup>De Grauwe (2012) summarizes the key insights from the traditional OCA theory.

<sup>35</sup>See King (1993) for a critical assessment of the Old Keynesian, IS-LM models with respect to their treatment of expectations.

## CHAPTER 4. MONETARY UNION AND MACROECONOMIC STABILIZATION

more harmful under a flexible exchange rate. In this sense, a monetary union provides a protective mechanism against monetary policy mistakes.

These findings suggest that the conventional view of the costs and benefits of forming a monetary union in terms of macroeconomic stabilization may need to be revised, at least to the extent that real-world monetary policy is not optimal.

## B. Appendix

### B.1. Monetary union regime

This appendix contains the full derivation of the model under the monetary union regime. The world, which consists of two countries labeled  $H$  and  $F$ , is populated by a continuum of agents on the interval  $[0, 1]$ . The population on the segment  $[0, n)$  lives in country  $H$ , the population on the segment  $[n, 1]$  lives in country  $F$ . Thus,  $n$  measures the population size as a fraction of world population. An agent is both consumer and producer. He produces a single differentiated good and consumes all the goods produced in both countries.

#### Consumer problem

Agent  $j$  in country  $i = H, F$  derives positive utility from consumption  $C^j$  and negative utility from producing the differentiated good  $y^j$ . The present discounted value of lifetime utility  $U^j$  is given by<sup>36</sup>

$$U^j = E_0 \sum_{t=0}^{\infty} \beta^t [U(C_t^j) - V(y_t^j, z_t^i)], \quad (\text{B.1})$$

where  $E$  denotes the expectations operator and  $\beta$  the discount factor.

$V$  is an increasing, convex function of agent  $j$ 's supply of his product  $y_t^j$  and a decreasing convex function of productivity  $z_t^i$ , which is common to all agents in country  $i$ . One can think of  $V$  as the combination of the agent's disutility from working and the production function. If the disutility from working is given by  $g(N_t^j)$ , where  $N_t^j$  is the number of hours worked, and the production function is given by  $y_t^j = f(N_t^j, z_t^i)$ , then  $V = g(f^{-1}(y_t^j, z_t^i))$ .

$U$  is an increasing, concave function of consumption  $C_t^j$ . The agent consumes both a bundle of differentiated goods from country  $H$  and from country  $F$  with a preference structure of the Cobb-Douglas type, so that

$$C_t^j = \left( \frac{C_{H,t}^j}{n} \right)^n \left( \frac{C_{F,t}^j}{1-n} \right)^{1-n}, \quad (\text{B.2})$$

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<sup>36</sup>In Benigno (2004), the agent derives utility also from holding money. However, money in the utility function is not necessary if monetary policy is conducted via interest rates rules.

where the bundles of differentiated goods are given by aggregators according to Dixit and Stiglitz (1977):

$$\begin{aligned} C_{H,t}^j &= \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c_t^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \\ C_{F,t}^j &= \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c_t^j(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}. \end{aligned} \quad (\text{B.3})$$

These preferences imply (1) that the elasticity of substitution between differentiated goods  $c_t^j$  from one country is  $\sigma$ , which is assumed to be greater than one and equal across countries, (2) that the elasticity of substitution between the bundles of goods from the two countries  $C_{H,t}$  and  $C_{F,t}$  is one and equal across countries, and (3) that the share of the expenditures for a bundle of goods from one country in the overall consumption expenditures of an agent coincides with the country's share in world population, i.e., there is no home bias in consumption.

Accordingly, the aggregate price index in country  $i$  is given by

$$P_t^i = (P_{H,t}^i)^n (P_{F,t}^i)^{1-n}, \quad (\text{B.4})$$

where the price indices for the bundles of differentiated goods in each country are defined by

$$\begin{aligned} P_{H,t}^i &= \left[ \frac{1}{n} \int_0^n p_t^i(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}} \\ P_{F,t}^i &= \left[ \frac{1}{1-n} \int_n^1 p_t^i(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}}. \end{aligned} \quad (\text{B.5})$$

In their role as producers, agents charge only one price for their good irrespective of whether the good remains in the country or is exported (no price discrimination). Furthermore, exporting does not entail transportation costs. These assumptions imply that a single good has the same price in both countries, i.e.,  $p_t^H(h) = p_t^F(h)$  and  $p_t^H(f) = p_t^F(f)$ . Given identical consumption preferences across countries, this immediately leads purchasing power parity to hold, so  $P_t^H = P_t^F = P_t$ . Consequently, the superscript  $i$  can be dropped from all the price indices.<sup>37</sup>

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<sup>37</sup>Note that  $P_t$  can be interpreted as a consumer price index,  $P_{H,t}$  and  $P_{F,t}$  as producer price indices.

Agent  $j$  takes three decisions with respect to his consumption choices. First, he decides on the overall level of consumption  $C_t^j$ .<sup>38</sup> Second, given  $C_t^j$  the agent optimally allocates expenditures between the bundles of differentiated goods  $C_{H,t}^j$  and  $C_{F,t}^j$  by minimizing total expenditure  $P_t C_t^j$  subject to (B.2). As a result, demand for these bundles is given by

$$C_{H,t}^j = n \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t^j, \quad C_{F,t}^j = (1-n) \left( \frac{P_{F,t}}{P_t} \right)^{-1} C_t^j. \quad (\text{B.6})$$

Third, given  $C_{H,t}^j$  and  $C_{F,t}^j$  the agent optimally allocates expenditures between the differentiated goods by minimizing  $P_{H,t} C_{H,t}^j$  and  $P_{F,t} C_{F,t}^j$  subject to equations (B.3). This yields

$$c_t^j(h) = \frac{1}{n} \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} C_{H,t}^j, \quad c_t^j(f) = \frac{1}{1-n} \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} C_{F,t}^j. \quad (\text{B.7})$$

Combining (B.6) and (B.7) yields

$$c_t^j(h) = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} \left( \frac{P_{H,t}}{P_t} \right)^{-1} C_t^j, \quad c_t^j(f) = \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} \left( \frac{P_{F,t}}{P_t} \right)^{-1} C_t^j. \quad (\text{B.8})$$

The terms of trade are defined from the perspective of country  $F$ , i.e., the ratio of the price of the bundle of goods produced in country  $F$  to the price of the bundle of goods imported from country  $H$ :

$$T_t = \frac{P_{F,t}}{P_{H,t}}. \quad (\text{B.9})$$

Equations (B.8) can then be expressed in terms of the terms of trade as

$$c_t^j(h) = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} T_t^{1-n} C_t^j, \quad c_t^j(f) = \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} T_t^{-n} C_t^j, \quad (\text{B.10})$$

where the terms of trade were inserted by rearranging the aggregate price equation (B.4) and by using the definition of the terms of trade (B.9).

Aggregating over all agents living in both countries, world demand for the differentiated goods can be written as

$$y_t(h) = \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} T_t^{1-n} C_t^W, \quad y_t(f) = \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} T_t^{-n} C_t^W, \quad (\text{B.11})$$

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<sup>38</sup>As shown below,  $C_t^j$  is determined by the usual Euler consumption equation (B.14).

where world consumption is given by

$$C_t^W = \int_0^1 C_t^j dj. \quad (\text{B.12})$$

There are two types of assets agents can trade in. Within each country, agents can insure against all possible states of nature by holding a portfolio of contingent, one-period securities whose real value (denominated in units of the consumption-based price index) is denoted by  $B_t^{i,j}$  and whose vector of prices is denoted by  $q_t^i$ . Across countries, agents can trade in a non-contingent, one-period bond whose nominal value (denominated in the currency of the union) is denoted by  $B_t^j$  and whose nominal interest rate is denoted by  $R_t$ . Thus, asset markets are incomplete across countries, but complete within countries. The intertemporal budget constraint of agent  $j$  in country  $i$  is then given by

$$C_t^j + q_t^i B_t^{i,j} + \frac{B_t^j}{P_t(1+R_t)} = B_{t-1}^{i,j} + \frac{B_{t-1}^j}{P_t} + (1-\tau^i) \frac{p_t(j)y_t(j)}{P_t}, \quad (\text{B.13})$$

The agent's income stems also from sales revenues  $p_t(j)y_t(j)$  net of a proportional, country-specific tax  $\tau^i$ .<sup>39</sup>

All contingent securities and non-contingent bonds are assumed to be in zero supply in the initial period, so  $B_0^{i,j} = B_0^j = 0$  for all  $i$  and  $j$ . Together with the facts that agents have identical preferences and that asset markets are complete within countries, this assumption implies perfect risk sharing of consumption within each country. Therefore, it is possible to analyze the consumer problem from the viewpoint of the representative agent of country  $H$  and country  $F$ .

The representative agent in country  $i$  maximizes his lifetime utility (B.1) subject to the budget constraint (B.13). By combining the resulting first order conditions with respect to consumption and bond holdings, the usual Euler consumption equation is then given by

$$U_C(C_t^i) = (1+R_t)\beta E_t \left\{ U_C(C_{t+1}^i) \frac{P_t}{P_{t+1}} \right\}. \quad (\text{B.14})$$

One important implication of the Cobb-Douglas type consumption preferences given by (B.2) together with the initial condition  $B_0^H = B_0^F = 0$  is that risk sharing is perfect across countries as well despite incomplete asset markets

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<sup>39</sup>The tax will turn out to be a subsidy to exactly offset the distortion caused by monopolistic competition.



at the international level, in the sense that<sup>40</sup>

$$C_t^H = C_t^F = C_t. \quad (\text{B.15})$$

To gain intuition, first note that, similar to (B.3), aggregate output in the two countries can be expressed as

$$\begin{aligned} Y_t^H &= \left[ \left( \frac{1}{n} \right) \int_0^n y_t(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \\ Y_t^F &= \left[ \left( \frac{1}{1-n} \right) \int_n^1 y_t(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}. \end{aligned} \quad (\text{B.16})$$

Then, applying (B.16) to (B.11) and using (B.15) yields

$$Y_t^H = T_t^{1-n} C_t, \quad Y_t^F = T_t^{-n} C_t. \quad (\text{B.17})$$

Making use of the definition of the terms of trade (B.9) and the aggregate price equation (B.4), this can be rearranged to

$$P_{H,t} Y_t^H = P_t C_t, \quad P_{F,t} Y_t^F = P_t C_t. \quad (\text{B.18})$$

Finally, the ratio of the two equations is given by

$$\frac{P_{F,t} Y_t^F}{P_{H,t} Y_t^H} = T_t \frac{Y_t^F}{Y_t^H} = 1. \quad (\text{B.19})$$

Nominal output equals nominal consumption in both countries at all times, as can be seen from (B.18). Thus, current accounts are always balanced. The reason is that any variation in the terms of trade is accompanied by an exact proportional variation in relative output across countries, as shown by (B.19). Agents shift consumption from the good that has become relatively expensive to the good that has become relatively cheap (expenditure switching effect) in such a way that a one percent increase in the relative price (terms of trade) leads to a one percent decrease in relative quantities. This is ultimately due to the fact that the elasticity of substitution between the bundles of goods from the two countries is one (Cobb-Douglas preferences).

As a result, relative nominal output and therefore relative income between the two countries are constant at all times. Thus, there are no gains from asset

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<sup>40</sup>For a proof, see Benigno (2003), Appendix A.

trade across countries, and the internationally traded bond becomes redundant ( $B_t^H = B_t^F = 0 \quad \forall t$ ).<sup>41</sup>

### Producer problem

In their role as producers, agents act in an environment of monopolistic competition, in which they dispose of some degree of market power. Furthermore, prices are sticky in the sense that the agent is able to change his price in a given period with a fixed probability, as in Calvo (1983). The probability of being able to change the price may differ across countries and is given by  $1 - \alpha^i$ .

Agent  $j$  in country  $i$  maximizes expected, discounted profits by choosing the price  $\tilde{p}_t(j)$  taking into account that demand depends on the chosen price and that the price may remain unchanged for some periods. Formally, the agent maximizes

$$E_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k \left[ \lambda_{t+k} (1 - \tau^i) \tilde{p}_t(j) \tilde{y}_{t,t+k}(j) - V(\tilde{y}_{t,t+k}(j), z_{t+k}^i) \right] \quad (\text{B.20})$$

subject to the demand function

$$\tilde{y}_{t,t+k}(h) = \left( \frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} T_{t+k}^{1-n} C_{t+k} \quad (\text{B.21})$$

if the agent lives in country  $H$  or

$$\tilde{y}_{t,t+k}(f) = \left( \frac{\tilde{p}_t(f)}{P_{F,t+k}} \right)^{-\sigma} T_{t+k}^{-n} C_{t+k}, \quad (\text{B.22})$$

if the agent lives in country  $F$ , where  $\tilde{y}_{t,t+k}(j)$  denotes total demand of good  $j$  at time  $t + k$  if the price  $\tilde{p}_t(j)$  prevails. Profits are expressed in utility units. Therefore, nominal sales revenues net of taxes  $(1 - \tau^i) \tilde{p}_t(j) \tilde{y}_{t,t+k}(j)$  are converted into utility units using the marginal utility of nominal revenues  $\lambda_{t+k} = \frac{U_C(C_{t+k})}{P_{t+k}}$ , which is the same for all agents in both countries due to perfect risk sharing

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<sup>41</sup>The result that, under Cobb-Douglas preferences, the terms of trade provide perfect insurance against output variations was already shown by Cole and Obstfeld (1991). Note that the result does not hinge upon the specification in which the expenditure share in the Cobb-Douglas function coincides with the population size  $n$  (a feature also common to the Obstfeld and Rogoff (2001) model). If the expenditure share does not coincide with the population size, as in Corsetti and Pesenti (2001), relative consumption across countries as well as relative income across countries are still constant over time. However, they are not equal to one, as in (B.15) and (B.19) respectively. Consumption and nominal output, then, differ across countries.

within and across countries and due to purchasing power parity. The cost of production expressed in utility units is given by the function  $V$ .

The first order condition yields the optimal price

$$\tilde{p}_t(j) = \frac{\sigma}{(\sigma - 1)(1 - \tau^i)} \frac{E_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k V_y(\tilde{y}_{t,t+k}(j), z_{t+k}^i) \tilde{y}_{t,t+k}(j)}{E_t \sum_{k=0}^{\infty} (\alpha^i \beta)^k \lambda_{t+k} \tilde{y}_{t,t+k}(j)}, \quad (\text{B.23})$$

where  $V_y$  denotes the derivative of function  $V$  with respect to output  $\tilde{y}(j)$ . All agents that live in the same country and are able to reset their price in a certain period will set the same price, since they share identical preferences (function  $V$ ) and face the same demand curves, which depend only on aggregate variables such as  $P_H$ ,  $P_F$ ,  $T$ , and  $C$ , and the common elasticity of substitution  $\sigma$ . Hence, in a given period, a fraction  $1 - \alpha^i$  of agents will set the same optimal price, while for a fraction  $\alpha^i$  of agents the price from the previous period remains effective:

$$\begin{aligned} P_{H,t} &= [\alpha^H P_{H,t-1}^{1-\sigma} + (1 - \alpha^H) \tilde{p}_t(h)^{1-\sigma}]^{\frac{1}{1-\sigma}} \\ P_{F,t} &= [\alpha^F P_{F,t-1}^{1-\sigma} + (1 - \alpha^F) \tilde{p}_t(f)^{1-\sigma}]^{\frac{1}{1-\sigma}}. \end{aligned} \quad (\text{B.24})$$

When prices are flexible, the optimal price equation (B.23) for country  $H$  simplifies to

$$T_t^{n-1} = \frac{\sigma}{(\sigma - 1)(1 - \tau^H)} \frac{V_y(y_t^H, z_t^H)}{U_C(C_t)}, \quad (\text{B.25})$$

and for country  $F$  to

$$T_t^n = \frac{\sigma}{(\sigma - 1)(1 - \tau^F)} \frac{V_y(y_t^F, z_t^F)}{U_C(C_t)}. \quad (\text{B.26})$$

Note that the closed-economy counterpart is given by

$$1 = \frac{\sigma}{(\sigma - 1)(1 - \tau)} \frac{V_y(y_t, z_t)}{U_C(C_t)}. \quad (\text{B.27})$$

Moreover, variations in the marginal disutility of production of one country relative to the other country are reflected in variations in the terms of trade. Dividing (B.26) by (B.25) yields

$$T_t = \frac{1 - \tau^H}{1 - \tau^F} \frac{V_y(y_t^F, z_t^F)}{V_y(y_t^H, z_t^H)}. \quad (\text{B.28})$$

### Terms of trade

It is necessary to express the terms-of-trade equation (B.9) in changes, since the model will only contain price changes (i.e., inflation) rather than price levels.

Thus

$$\frac{T_t}{T_{t-1}} = \frac{P_{F,t}}{P_{F,t-1}} \frac{P_{H,t-1}}{P_{H,t}}. \quad (\text{B.29})$$

### Log-linearization

In the following, the model equations will be log-linearized. Given a variable  $X_t$ , the following definitions will be used:

$$X_t^W = nX_t^H + (1 - n)X_t^F \quad (\text{B.30})$$

$$X_t^R = X_t^F - X_t^H \quad (\text{B.31})$$

Furthermore, deviations of the logarithm of a variable  $X_t$  from its steady state are denoted by  $\tilde{X}_t$  under flexible prices and by  $\hat{X}_t$  under sticky prices.

**Flexible-price equilibrium** Under flexible prices, prices are set as a markup over marginal costs, monetary policy is neutral, and consumption, output, and the terms of trade are driven by productivity shocks only. Accordingly, consumption, world output, and the terms of trade evolve as follows:<sup>42</sup>

$$\tilde{C}_t = \frac{\eta}{\rho + \eta} \bar{Y}_t^W \quad (\text{B.32})$$

$$\tilde{T}_t = -\frac{\eta}{1 + \eta} \bar{Y}_t^R \quad (\text{B.33})$$

$$\tilde{Y}_t^W = \frac{\eta}{\rho + \eta} \bar{Y}_t^W. \quad (\text{B.34})$$

The first equation is derived by log-linearizing (B.25) and (B.26) and taking the weighted average with weight  $n$ . The second equation is derived by subtracting the log-linear approximation of (B.25) from the log-linear approximation of (B.26). The third equation is derived by inserting the first two equations into the weighted average of the log-linear approximations of equations (B.17).

The following definitions were used:

$$\rho = -\frac{U_{CC}\bar{C}}{U_C} \quad (\text{B.35})$$

<sup>42</sup>In contrast to Benigno (2004), I abstract from fiscal policy shocks.

denotes the inverse of the intertemporal elasticity of substitution in consumption,

$$\eta = \frac{V_{yy}\bar{C}}{V_y} \quad (\text{B.36})$$

denotes the inverse of the elasticity of producing the differentiated good, and finally

$$\bar{Y}_t^i = -\frac{V_{yz}}{V_{yy}\bar{C}}\hat{z}_t^i \quad (\text{B.37})$$

reparameterizes the productivity shock in country  $i$ .

The productivity shock in country  $i$  follows an AR(1) process of the form

$$\bar{Y}_t^i = \rho_i \bar{Y}_{t-1}^i + \nu_t^i, \quad (\text{B.38})$$

where  $\nu_t^i$  is a white noise process with  $\text{var } \nu_t^i = 1$ .

**Sticky-price equilibrium** Under sticky prices, the system of equations is given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho}(\hat{R}_t - E_t \pi_{t+1}^W) \quad (\text{B.39})$$

$$\hat{Y}_t^H = (1-n)\hat{T}_t + \hat{C}_t \quad (\text{B.40})$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t \quad (\text{B.41})$$

$$\pi_t^H = (1-n)k_T^H(\hat{T}_t - \tilde{T}_t) + k_C^H(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^H \quad (\text{B.42})$$

$$\pi_t^F = -nk_T^F(\hat{T}_t - \tilde{T}_t) + k_C^F(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^F \quad (\text{B.43})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H \quad (\text{B.44})$$

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1-\phi_R)(\phi_\pi \pi_t^W + \phi_Y \hat{Y}_t^W). \quad (\text{B.45})$$

Equation (B.39) is the log-linear approximation of the Euler consumption equation (B.14), where  $C_t^i = C_t$  and  $\pi_t = \ln(P_t/P_{t-1})$ . Recall that, due to perfect risk sharing, consumption is the same across countries, which implies that there is only one Euler equation. Equations (B.40) and (B.41) are log-linear approximations of the equations for aggregate demand (B.17).

Equations (B.42) and (B.43) represent the New Keynesian Phillips curves for country  $H$  and country  $F$  respectively, where  $\pi_t^H = \ln(P_{H,t}/P_{H,t-1})$  and  $\pi_t^F = \ln(P_{F,t}/P_{F,t-1})$ . They are derived by combining the log-linear approximation of the optimal price (B.23) with the log-linear approximation of (B.24) for each country separately. The parameters in front of the terms-of-trade gap ( $\hat{T}_t - \tilde{T}_t$ )

and the consumption gap ( $\hat{C}_t - \tilde{C}_t$ ) are defined as follows (for  $i = H, F$ ):

$$k_C^i = \frac{(1 - \alpha^i \beta)(1 - \alpha^i)}{\alpha^i} \frac{\rho + \eta}{1 + \sigma \eta} \quad (\text{B.46})$$

$$k_T^i = \frac{(1 - \alpha^i \beta)(1 - \alpha^i)}{\alpha^i} \frac{1 + \eta}{1 + \sigma \eta} \quad (\text{B.47})$$

Equation (B.44) is the log-linear approximation of the terms-of-trade equation (B.29). Finally, equation (B.45) represents the Taylor-type interest rate rule, according to which the common monetary policy reacts to union-wide inflation and to union-wide output (measured as the weighted average of country-specific inflation and output, respectively) with coefficients  $\phi_\pi$  and  $\phi_Y$  and engages in interest rate smoothing with coefficient  $\phi_R$ .

## B.2. Flexible exchange rate regime

The main difference to the MU regime, of course, is that both countries possess their own currency and independent monetary policy. Notwithstanding, the model structure is to a large extent identical. The behavior of output, consumption, and the terms of trade under flexible prices is given by equations (B.32) through (B.34).

Under sticky prices, the system of equations is given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( n(\hat{R}_t^H - E_t \pi_{t+1}^H) + (1-n)(\hat{R}_t^F - E_t \pi_{t+1}^F) \right) \quad (\text{B.48})$$

$$\hat{Y}_t^H = (1-n)\hat{T}_t + \hat{C}_t \quad (\text{B.49})$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t \quad (\text{B.50})$$

$$\pi_t^H = (1-n)k_T^H(\hat{T}_t - \tilde{T}_t) + k_C^H(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^H \quad (\text{B.51})$$

$$\pi_t^F = -nk_T^F(\hat{T}_t - \tilde{T}_t) + k_C^F(\hat{C}_t - \tilde{C}_t) + \beta E_t \pi_{t+1}^F \quad (\text{B.52})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^F - \pi_t^H + \Delta \hat{S}_t \quad (\text{B.53})$$

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t^H - \hat{R}_t^F \quad (\text{B.54})$$

$$\hat{R}_t^H = \phi_R \hat{R}_{t-1}^H + (1-\phi_R)(\phi_\pi \pi_t^H + \phi_Y \hat{Y}_t^H) \quad (\text{B.55})$$

$$\hat{R}_t^F = \phi_R \hat{R}_{t-1}^F + (1-\phi_R)(\phi_\pi \pi_t^F + \phi_Y \hat{Y}_t^F). \quad (\text{B.56})$$

Given the same assumptions as in the MU regime on the set of assets agents can trade in, on the agents' preferences, and on the law of one price, the result of perfect risk sharing carries over to the FX regime.<sup>43</sup> Therefore, consumption is described by one Euler equation (B.48). In contrast to the MU regime, the Euler equation contains two interest rates, since monetary policy is country-specific.

The equations for aggregate demand (B.49) and (B.50) as well as the New Keynesian Phillips curves (B.51) and (B.52) are the same as under the MU regime. Agents are assumed to set their price in the currency of their country (producer currency pricing). The assumption of no price discrimination and no transportation costs implies that the law of one price holds, which in turn implies that exchange rate pass-through is complete. The law of one price

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<sup>43</sup>See Obstfeld and Rogoff (2001). By adopting the assumption from the MU regime that asset markets are incomplete across countries I deviate from Benigno and Benigno (2008), who assume asset markets across countries to be complete. With identical Cobb-Douglas preferences, however, risk sharing is perfect regardless of whether asset markets are complete or not.

together with identical consumption preferences implies that purchasing power parity holds as well.

The terms of trade are now given by

$$T_t = \frac{S_t P_{F,t}}{P_{H,t}}, \quad (\text{B.57})$$

where  $P_{H,t}$  denotes the price of the bundle of differentiated goods produced in country  $H$  denominated in country  $H$ 's currency,  $P_{F,t}$  denotes the price of the bundle of differentiated goods produced in country  $F$  denominated in country  $F$ 's currency, and  $S_t$  is the nominal exchange rate defined as the price of country  $F$ 's currency in terms of country  $H$ 's currency. First-differencing and log-linearizing the definition of the terms of trade yields (B.53).

Equation (B.54) represents the uncovered interest parity condition, which can be obtained by subtracting the log-linearized Euler equation of country  $F$  from the one of country  $H$ , using the fact that purchasing power parity holds. Thus, the expected change in the nominal exchange rate corresponds to the interest rate differential across countries. Finally, equations (B.55) and (B.56) represent the Taylor-type interest rate rules, according to which monetary policy reacts to country-specific inflation and output with coefficients  $\phi_\pi$  and  $\phi_Y$  and engages in interest rate smoothing with coefficient  $\phi_R$ .



### B.3. Welfare loss function

This appendix contains the full derivation of the world welfare loss function. The world welfare loss function is the discounted value of a weighted average across countries of the average utility flow of agents using a second-order Taylor series expansion in the spirit of Woodford (2003).<sup>44</sup>

The average utility among agents in country  $H$  is given by

$$w_t^H = U(C_t) - \frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh, \quad (\text{B.58})$$

and average utility among agents in country  $F$  is given by

$$w_t^F = U(C_t) - \frac{1}{1-n} \int_{1-n}^1 V(y_t(f), z_t^F) df. \quad (\text{B.59})$$

The discounted value of the weighted average of the two flows is then given by

$$\tilde{W}_t = E_t \sum_{k=0}^{\infty} \beta^k (n w_{t+k}^H + (1-n) w_{t+k}^F). \quad (\text{B.60})$$

Each term of the utility function is treated separately.

#### The term $U(C_t)$

Taking a second-order linear expansion of  $U(C_t)$  around the steady state value  $\bar{C}$  yields

$$U(C_t) = U(\bar{C}) + U_C(C_t - \bar{C}) + \frac{1}{2} U_{CC}(C_t - \bar{C})^2 + O(\|\xi\|^3), \quad (\text{B.61})$$

where the term  $O(\|\xi\|^3)$  groups all the terms that are of third or higher order in the deviations of the various variables from their steady state.

Furthermore, a second-order Taylor expansion to  $C_t$  yields

$$\frac{C_t - \bar{C}}{\bar{C}} = \hat{C}_t + \frac{1}{2} \hat{C}_t^2 + O(\|\xi\|^3) \Leftrightarrow C_t - \bar{C} = \bar{C} \hat{C}_t + \frac{1}{2} \bar{C} \hat{C}_t^2 + O(\|\xi\|^3), \quad (\text{B.62})$$

where  $\hat{C}_t = \ln(C_t) - \ln(\bar{C})$ .

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<sup>44</sup>The derivation follows Benigno (2003), Appendix D. Here, I do not abstract from exogenous government expenditures. The loss function without government expenditure shocks is identical.

Inserting (B.62) into (B.61) yields

$$\begin{aligned}
 U(C_t) &= \underbrace{U(\bar{C})}_{=t.i.p.} + U_C(\bar{C}\hat{C}_t + \frac{1}{2}\bar{C}\hat{C}_t^2) + \frac{1}{2}U_{CC}(\bar{C}\hat{C}_t + \frac{1}{2}\bar{C}\hat{C}_t^2)^2 + O(\|\xi\|^3) \\
 &= U_C\bar{C}\hat{C}_t + \frac{1}{2}U_C\bar{C}\hat{C}_t^2 + \frac{1}{2}U_{CC}(\bar{C}^2\hat{C}_t^2 + \underbrace{\bar{C}^2\hat{C}_t^3 + \frac{1}{4}\bar{C}^2\hat{C}_t^4}_{=O(\|\xi\|^3)}) + t.i.p. + O(\|\xi\|^3) \\
 &= U_C\bar{C}\hat{C}_t + \frac{1}{2}U_C\bar{C}\hat{C}_t^2 + \frac{1}{2}U_{CC}\bar{C}^2\hat{C}_t^2 + t.i.p. + O(\|\xi\|^3) \\
 &= U_C\bar{C} \left[ \hat{C}_t + \frac{1}{2}\hat{C}_t^2 + \frac{1}{2} \underbrace{\frac{U_{CC}\bar{C}}{U_C}}_{=-\rho} \hat{C}_t^2 \right] + t.i.p. + O(\|\xi\|^3) \\
 &= U_C\bar{C} \left[ \hat{C}_t + \frac{1}{2}(1 - \rho)\hat{C}_t^2 \right] + t.i.p. + O(\|\xi\|^3), \tag{B.63}
 \end{aligned}$$

where the term *t.i.p.* collects all the terms that are independent of monetary policy and independent of whether the two countries form a monetary union or not.

**The term**  $\frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh$

A second-order Taylor expansion of the second term in (B.58) around a steady state, where  $y_t(h) = \bar{Y}^H$  for all  $h$  and  $t$ , and where  $z_t^H = 0$  for all  $t$  yields

$$\begin{aligned}
 V(y_t(h), z_t^H) &= V(\bar{Y}^H, 0) + V_y(y_t(h) - \bar{Y}^H) + V_z z_t^H + \frac{1}{2}V_{yy}(y_t(h) - \bar{Y}^H)^2 \\
 &\quad + V_{yz}(y_t(h) - \bar{Y}^H) z_t^H + \frac{1}{2}V_{zz} z_t^{H2} + O(\|\xi\|^3). \tag{B.64}
 \end{aligned}$$

World demand for a differentiated good produced in country  $H$  (including demand from government expenditures  $G^H$ ) can be expressed by

$$\begin{aligned}
 y(h) &= \left( \frac{p(h)}{P_H} \right)^{-\sigma} [T^{1-n}C^W + G^H] \\
 &= \underbrace{\left( \frac{p(h)}{P_H} \right)^{-\sigma} T^{1-n}C^W}_{=y^d(h)} + \underbrace{\left( \frac{p(h)}{P_H} \right)^{-\sigma} G^H}_{=y^g(h)} \\
 &= y^d(h) + y^g(h). \tag{B.65}
 \end{aligned}$$

A second-order Taylor expansion to  $y_t^d(h)$  yields

$$y_t^d(h) - \bar{Y}^H = \bar{Y}^H \hat{y}_t^d(h) + \frac{1}{2} \bar{Y}^H \hat{y}_t^d(h)^2 + O(\|\xi\|^3), \quad (\text{B.66})$$

where  $\hat{y}_t^d(h) = \ln(y_t^d(h)) - \ln(\bar{Y}^H)$ .

A second-order Taylor expansion to  $y_t^g(h)$  yields

$$y_t^g(h) = \bar{Y}^H \hat{y}_t^g(h) + \frac{1}{2} \bar{Y}^H \hat{y}_t^g(h)^2 + O(\|\xi\|^3). \quad (\text{B.67})$$

Combining (B.65), (B.66), and (B.67) gives

$$\begin{aligned} y_t(h) - \bar{Y}^H &= y_t^d(h) + y_t^g(h) - \bar{Y}^H \\ &= \bar{Y}^H \hat{y}_t^d(h) + \frac{1}{2} \bar{Y}^H \hat{y}_t^d(h)^2 + \bar{Y}^H \hat{y}_t^g(h) + \frac{1}{2} \bar{Y}^H \hat{y}_t^g(h)^2 \\ &= \bar{Y}^H \left( \hat{y}_t^d(h) + \frac{1}{2} \hat{y}_t^d(h)^2 + \hat{y}_t^g(h) + \frac{1}{2} \hat{y}_t^g(h)^2 \right). \end{aligned} \quad (\text{B.68})$$

Inserting into (B.64) and simplifying yields

$$\begin{aligned}
 V(y_t(h), z_t^H) &= \underbrace{V(\bar{Y}^H, 0)}_{=t.i.p.} + V_y \bar{Y}^H \left( \hat{y}_t^d(h) + \frac{1}{2} \hat{y}_t^d(h)^2 + \underbrace{\hat{y}_t^g(h) + \frac{1}{2} \hat{y}_t^g(h)^2}_{=t.i.p.} \right) \\
 &+ \underbrace{V_z z_t^H}_{=t.i.p.} + \frac{1}{2} V_{yy} \bar{Y}^{H^2} \underbrace{\left( \hat{y}_t^d(h) + \frac{1}{2} \hat{y}_t^d(h)^2 + \hat{y}_t^g(h) + \frac{1}{2} \hat{y}_t^g(h)^2 \right)^2}_{=\hat{y}_t(h)^2 + t.i.p. + O(\|\xi\|^3)} \\
 &+ V_{yz} \bar{Y}^H \underbrace{\left( \hat{y}_t^d(h) + \frac{1}{2} \hat{y}_t^d(h)^2 + \hat{y}_t^g(h) + \frac{1}{2} \hat{y}_t^g(h)^2 \right) z_t^H}_{=\hat{y}_t^d(h) z_t^H + t.i.p. + O(\|\xi\|^3)} + \underbrace{\frac{1}{2} V_{zz} z_t^{H^2}}_{=t.i.p.} \\
 &+ O(\|\xi\|^3) \\
 &= V_y \bar{Y}^H \left( \hat{y}_t^d(h) + \frac{1}{2} \hat{y}_t^d(h)^2 \right) + \frac{1}{2} V_{yy} \bar{Y}^{H^2} \hat{y}_t(h)^2 + V_{yz} \bar{Y}^H \hat{y}_t^d(h) z_t^H \\
 &+ t.i.p. + O(\|\xi\|^3) \\
 &= V_y \bar{Y}^H \left( \hat{y}_t^d(h) + \frac{1}{2} \hat{y}_t^d(h)^2 + \frac{1}{2} \underbrace{\frac{V_{yy} \bar{Y}^H}{V_y}}_{\equiv \eta} \hat{y}_t(h)^2 + \frac{V_{yz} \hat{y}_t^d(h) z_t^H}{V_y} \right) \\
 &+ t.i.p. + O(\|\xi\|^3) \\
 &= V_y \bar{Y}^H \left( \hat{y}_t^d(h) + \frac{1}{2} \hat{y}_t^d(h)^2 + \frac{\eta}{2} \hat{y}_t(h)^2 + \underbrace{\frac{V_{yy} \bar{Y}^H}{V_y}}_{=\eta} \underbrace{\frac{V_{yz} z_t^H}{V_{yy} \bar{Y}^H}}_{\equiv -\bar{Y}_t^H} \hat{y}_t^d(h) \right) \\
 &+ t.i.p. + O(\|\xi\|^3) \\
 &= V_y \bar{Y}^H \left( \hat{y}_t^d(h) + \frac{1}{2} \hat{y}_t^d(h)^2 + \frac{\eta}{2} \hat{y}_t(h)^2 - \eta \hat{y}_t^d(h) \bar{Y}_t^H \right) \\
 &+ t.i.p. + O(\|\xi\|^3) \tag{B.69}
 \end{aligned}$$

Next, a relationship between  $V_y$  and  $U_C$  will be derived. In the steady state, equations (B.25) and (B.26) can be expressed as

$$(1 - \tau^H) U_C(\bar{C}) = \frac{\sigma}{\sigma - 1} \bar{T}^{1-n} V_y(\bar{T}^{1-n} \bar{C}, 0) \tag{B.70}$$

$$(1 - \tau^F) U_C(\bar{C}) = \frac{\sigma}{\sigma - 1} \bar{T}^{-n} V_y(\bar{T}^{-n} \bar{C}, 0), \tag{B.71}$$

which can be rearranged to

$$(1 - \Phi^H)U_C(\bar{C}) = \bar{T}^{1-n}V_y(\bar{T}^{1-n}\bar{C}, 0) \quad (\text{B.72})$$

$$(1 - \Phi^F)U_C(\bar{C}) = \bar{T}^{-n}V_y(\bar{T}^{-n}\bar{C}, 0) \quad (\text{B.73})$$

with

$$(1 - \Phi^H) = (1 - \tau^H)\frac{\sigma - 1}{\sigma} \quad (\text{B.74})$$

$$(1 - \Phi^F) = (1 - \tau^F)\frac{\sigma - 1}{\sigma}. \quad (\text{B.75})$$

The analysis must be restricted to the case in which distortions from the efficient steady state are small, i.e., the deviations of  $\Phi^H$  and  $\Phi^F$  are at least of order  $O(\|\xi\|)$ . Furthermore, for reasons of tractability, it is assumed that  $\Phi^H = \Phi^F$ . If  $\tau^H = \tau^F$ , it follows that  $\bar{T} = 1$  and  $\bar{Y}^H = \bar{Y}^F = \bar{C}$ . Then, equation (B.72) yields

$$\begin{aligned} (1 - \Phi^H)U_C(\bar{C}) &= \bar{T}^{1-n}V_y(\bar{T}^{1-n}\bar{C}, 0) \\ &= V_y(\bar{Y}^H, 0) \\ &= V_y. \end{aligned} \quad (\text{B.76})$$

Plugging into (B.69) yields

$$\begin{aligned} V(y_t(h), z_t^H) &= (1 - \Phi^H)U_C(\bar{C})\bar{Y}^H \left( \hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \frac{\eta}{2}\hat{y}_t(h)^2 \right. \\ &\quad \left. - \eta\hat{y}_t^d(h)\bar{Y}_t^H \right) + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{B.77})$$

With  $\bar{Y}^H = \bar{C}$  and  $U_C(\bar{C}) = U_C$  together with the small distortion assumption, i.e., the product of  $\Phi^H$  with second-order terms can be neglected, the last equation can be written as

$$\begin{aligned} V(y_t(h), z_t^H) &= U_C\bar{C} \left( (1 - \Phi^H)\hat{y}_t^d(h) + \frac{1}{2}\hat{y}_t^d(h)^2 + \frac{\eta}{2}\hat{y}_t(h)^2 \right. \\ &\quad \left. - \eta\hat{y}_t^d(h)\bar{Y}_t^H \right) + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{B.78})$$

Integrating across agents belonging to country H yields

$$\begin{aligned}
 \frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh &= U_C \bar{C} \left( (1 - \Phi^H) \frac{1}{n} \int_0^n \hat{y}_t^d(h) dh + \frac{1}{2} \frac{1}{n} \int_0^n \hat{y}_t^d(h)^2 dh \right. \\
 &\quad \left. + \frac{\eta}{2} \frac{1}{n} \int_0^n \hat{y}_t(h)^2 dh - \eta \frac{1}{n} \int_0^n \hat{y}_t^d(h) dh \bar{Y}_t^H \right) \\
 &\quad + t.i.p. + O(\|\xi\|^3) \\
 &= U_C \bar{C} \left( (1 - \Phi^H) E_h \hat{y}_t^d(h) + \frac{1}{2} E_h \hat{y}_t^d(h)^2 + \frac{\eta}{2} E_h \hat{y}_t(h)^2 \right. \\
 &\quad \left. - \eta E_h \hat{y}_t^d(h) \bar{Y}_t^H \right) + t.i.p. + O(\|\xi\|^3). \tag{B.79}
 \end{aligned}$$

Recall the basic relationship

$$\text{var}(X) = E(X^2) - (E(X))^2 \Leftrightarrow E(X^2) = \text{var}(X) + (E(X))^2. \tag{B.80}$$

Thus,

$$\begin{aligned}
 \frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh &= U_C \bar{C} \left( (1 - \Phi^H) E_h \hat{y}_t^d(h) + \frac{1}{2} \left( \text{var}_h \hat{y}_t^d(h) + [E_h \hat{y}_t^d(h)]^2 \right) \right. \\
 &\quad \left. + \frac{\eta}{2} \left( \text{var}_h \hat{y}_t(h) + [E_h \hat{y}_t(h)]^2 \right) - \eta E_h \hat{y}_t^d(h) \bar{Y}_t^H \right) \\
 &\quad + t.i.p. + O(\|\xi\|^3). \tag{B.81}
 \end{aligned}$$

### Expanding $Y_t^H$

Recall the aggregator

$$Y_t^H = \left\{ \frac{1}{n} \int_0^n y_t(h)^{\frac{\sigma-1}{\sigma}} dh \right\}^{\frac{\sigma}{\sigma-1}}. \tag{B.82}$$

I conduct a second-order Taylor series expansion of both sides of the equation. Note that the more general case of (B.62) is given by

$$\frac{C_t^a - \bar{C}^a}{\bar{C}^a} = a \hat{C}_t + \frac{1}{2} a^2 \hat{C}_t^2 + O(\|\xi\|^3). \tag{B.83}$$

Thus, approximating  $y_t(h)^{\frac{\sigma-1}{\sigma}}$  up to second-order yields

$$y_t(h)^{\frac{\sigma-1}{\sigma}} = \bar{Y}^H \frac{\sigma-1}{\sigma} \left[ 1 + \frac{\sigma-1}{\sigma} \hat{y}_t(h) + \frac{1}{2} \left( \frac{\sigma-1}{\sigma} \right)^2 \hat{y}_t(h)^2 \right] + O(\|\xi\|^3). \tag{B.84}$$

Inserting into (B.82) yields

$$\begin{aligned}
 Y_t^{H \frac{\sigma-1}{\sigma}} &= \frac{1}{n} \int_0^n \bar{Y}^{H \frac{\sigma-1}{\sigma}} \left[ 1 + \frac{\sigma-1}{\sigma} \hat{y}_t(h) + \frac{1}{2} \left( \frac{\sigma-1}{\sigma} \right)^2 \hat{y}_t(h)^2 \right] dh + O(\|\xi\|^3) \\
 &= \bar{Y}^{H \frac{\sigma-1}{\sigma}} \left\{ \frac{1}{n} \int_0^n 1 dh + \frac{\sigma-1}{\sigma} \frac{1}{n} \int_0^n \hat{y}_t(h) dh + \frac{1}{2} \left( \frac{\sigma-1}{\sigma} \right)^2 \frac{1}{n} \int_0^n \hat{y}_t(h)^2 dh \right\} \\
 &\quad + O(\|\xi\|^3) \\
 &= \bar{Y}^{H \frac{\sigma-1}{\sigma}} \left\{ 1 + \frac{\sigma-1}{\sigma} E_h \hat{y}_t(h) + \frac{1}{2} \left( \frac{\sigma-1}{\sigma} \right)^2 E_h \hat{y}_t(h)^2 \right\} + O(\|\xi\|^3).
 \end{aligned} \tag{B.85}$$

A second-order Taylor expansion to  $Y_t^{H \frac{\sigma-1}{\sigma}}$  yields

$$Y_t^{H \frac{\sigma-1}{\sigma}} = \bar{Y}^{H \frac{\sigma-1}{\sigma}} \left\{ 1 + \frac{\sigma-1}{\sigma} \hat{Y}_t^H + \frac{1}{2} \left( \frac{\sigma-1}{\sigma} \right)^2 \hat{Y}_t^{H^2} \right\} + O(\|\xi\|^3). \tag{B.86}$$

Equating the previous two equations yields

$$\hat{Y}_t^H + \frac{1}{2} \frac{\sigma-1}{\sigma} \hat{Y}_t^{H^2} = E_h \hat{y}_t(h) + \frac{1}{2} \frac{\sigma-1}{\sigma} E_h \hat{y}_t(h)^2 + O(\|\xi\|^3). \tag{B.87}$$

This expression raised to the power of two gives

$$\hat{Y}_t^{H^2} = (E_h \hat{y}_t(h))^2 + O(\|\xi\|^3). \tag{B.88}$$

Inserting back into (B.87) and simplifying yields

$$\begin{aligned}
 \hat{Y}_t^H + \frac{1}{2} \frac{\sigma-1}{\sigma} (E_h \hat{y}_t(h))^2 &= E_h \hat{y}_t(h) + \frac{1}{2} \frac{\sigma-1}{\sigma} E_h \hat{y}_t(h)^2 + O(\|\xi\|^3) \\
 \hat{Y}_t^H &= E_h \hat{y}_t(h) + \frac{1}{2} \frac{\sigma-1}{\sigma} [E_h \hat{y}_t(h)^2 - (E_h \hat{y}_t(h))^2] + O(\|\xi\|^3) \\
 \hat{Y}_t^H &= E_h \hat{y}_t(h) + \frac{1}{2} \frac{\sigma-1}{\sigma} \text{var}_h \hat{y}_t(h) + O(\|\xi\|^3).
 \end{aligned} \tag{B.89}$$

Analogously,

$$\hat{Y}_t^{H,d} = E_h \hat{y}_t^d(h) + \frac{1}{2} \frac{\sigma-1}{\sigma} \text{var}_h \hat{y}_t^d(h) + O(\|\xi\|^3). \tag{B.90}$$

Using the previous two equations to substitute out  $E_h \hat{y}_t(h)$  and  $E_h \hat{y}_t^d(h)$  in (B.81) gives

$$\begin{aligned}
 & \frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh \\
 &= U_C \bar{C} \left( (1 - \Phi^H) \left[ \hat{Y}_t^{H,d} - \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) \right] \right. \\
 &+ \frac{1}{2} \left( \text{var}_h \hat{y}_t^d(h) + \left[ \hat{Y}_t^{H,d} - \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) \right]^2 \right) \\
 &+ \frac{\eta}{2} \left( \text{var}_h \hat{y}_t(h) + \left[ \hat{Y}_t^H - \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t(h) \right]^2 \right) \\
 &- \eta \left[ \hat{Y}_t^{H,d} - \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) \right] \bar{Y}_t^H \Big) + t.i.p. + O(\|\xi\|^3) \\
 &= U_C \bar{C} \left( (1 - \Phi^H) \hat{Y}_t^{H,d} - \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) + \underbrace{\Phi^H \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h)}_{=O(\|\xi\|^3)} \right. \\
 &+ \frac{1}{2} \left( \text{var}_h \hat{y}_t^d(h) + \hat{Y}_t^{H,d^2} - \underbrace{\hat{Y}_t^{H,d} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) + \frac{1}{4} \left( \frac{\sigma - 1}{\sigma} \right)^2 (\text{var}_h \hat{y}_t^d(h))^2}_{=O(\|\xi\|^3)} \right) \\
 &+ \frac{\eta}{2} \left( \text{var}_h \hat{y}_t(h) + \hat{Y}_t^{H^2} - \underbrace{\hat{Y}_t^H \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t(h) + \frac{1}{4} \left( \frac{\sigma - 1}{\sigma} \right)^2 (\text{var}_h \hat{y}_t(h))^2}_{=O(\|\xi\|^3)} \right) \\
 &- \eta \hat{Y}_t^{H,d} \bar{Y}_t^H + \underbrace{\frac{\eta}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) \bar{Y}_t^H}_{=O(\|\xi\|^3)} \Big) + t.i.p. + O(\|\xi\|^3) \\
 &= U_C \bar{C} \left( (1 - \Phi^H) \hat{Y}_t^{H,d} + \frac{1}{2} \hat{Y}_t^{H,d^2} + \frac{\eta}{2} \hat{Y}_t^{H^2} - \eta \hat{Y}_t^{H,d} \bar{Y}_t^H \right. \\
 &- \left. \frac{1}{2} \frac{\sigma - 1}{\sigma} \text{var}_h \hat{y}_t^d(h) + \frac{1}{2} \text{var}_h \hat{y}_t^d(h) + \frac{\eta}{2} \text{var}_h \hat{y}_t(h) \right) + t.i.p. + O(\|\xi\|^3). \quad (\text{B.91})
 \end{aligned}$$

Note that since  $\hat{y}_t(h) = \hat{y}_t^d(h)$ ,  $\text{var}_h \hat{y}_t(h) = \text{var}_h \hat{y}_t^d(h)$ . Therefore, the previous expression can be simplified to

$$\begin{aligned}
 \frac{1}{n} \int_0^n V(y_t(h), z_t^H) dh &= U_C \bar{C} \left( (1 - \Phi^H) \hat{Y}_t^{H,d} + \frac{1}{2} \hat{Y}_t^{H,d^2} + \frac{\eta}{2} \hat{Y}_t^{H^2} - \eta \hat{Y}_t^{H,d} \bar{Y}_t^H \right. \\
 &+ \left. \frac{1}{2} (\sigma^{-1} + \eta) \text{var}_h \hat{y}_t(h) \right) + t.i.p. + O(\|\xi\|^3). \quad (\text{B.92})
 \end{aligned}$$



**Combining the results**

Inserting (B.92) and (B.63) into (B.58) yields

$$\begin{aligned}
 w_t^H = U_C \bar{C} & \left( \hat{C}_t + \frac{1}{2}(1 - \rho)\hat{C}_t^2 - (1 - \Phi^H)\hat{Y}_t^{H,d} - \frac{1}{2}\hat{Y}_t^{H,d^2} - \frac{\eta}{2}\hat{Y}_t^{H^2} + \eta\hat{Y}_t^{H,d}\bar{Y}_t^H \right. \\
 & \left. - \frac{1}{2}(\sigma^{-1} + \eta) \text{var}_h \hat{y}_t(h) \right) + t.i.p. + O(\|\xi\|^3). \tag{B.93}
 \end{aligned}$$

Average utility among agents living in country  $F$  is derived completely analogously. Thus,

$$\begin{aligned}
 w_t^F = U_C \bar{C} & \left( \hat{C}_t + \frac{1}{2}(1 - \rho)\hat{C}_t^2 - (1 - \Phi^F)\hat{Y}_t^{F,d} - \frac{1}{2}\hat{Y}_t^{F,d^2} - \frac{\eta}{2}\hat{Y}_t^{F^2} + \eta\hat{Y}_t^{F,d}\bar{Y}_t^F \right. \\
 & \left. - \frac{1}{2}(\sigma^{-1} + \eta) \text{var}_f \hat{y}_t(f) \right) + t.i.p. + O(\|\xi\|^3). \tag{B.94}
 \end{aligned}$$

World welfare consists of the linear combination of country  $H$ 's and country  $F$ 's welfare with weight  $n$  and  $1 - n$ :

$$\begin{aligned}
 w_t & = nw_t^H + (1 - n)w_t^F \\
 & = U_C \bar{C} \left( \underbrace{n\hat{C}_t + (1 - n)\hat{C}_t}_{=\hat{C}_t} + \frac{1}{2}(1 - \rho) \underbrace{(n\hat{C}_t^2 + (1 - n)\hat{C}_t^2)}_{=\hat{C}_t^2} \right. \\
 & \quad - n(1 - \Phi^H)\hat{Y}_t^{H,d} - (1 - n)(1 - \Phi^F)\hat{Y}_t^{F,d} - \frac{1}{2} \left( n\hat{Y}_t^{H,d^2} + (1 - n)\hat{Y}_t^{F,d^2} \right) \\
 & \quad - \frac{\eta}{2} \left( n\hat{Y}_t^{H^2} + (1 - n)\hat{Y}_t^{F^2} \right) + \eta \left( n\hat{Y}_t^{H,d}\bar{Y}_t^H + (1 - n)\hat{Y}_t^{F,d}\bar{Y}_t^F \right) \\
 & \quad \left. - \frac{1}{2}(\sigma^{-1} + \eta) [n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) + t.i.p. + O(\|\xi\|^3). \tag{B.95}
 \end{aligned}$$

Inserting the expressions

$$\hat{Y}_t^H = (1 - n)\hat{T}_t + \hat{C}_t + g_t^H \tag{B.96}$$

$$\hat{Y}_t^F = -n\hat{T}_t + \hat{C}_t + g_t^F \tag{B.97}$$

$$\hat{Y}_t^{H,d} = (1 - n)\hat{T}_t + \hat{C}_t \tag{B.98}$$

$$\hat{Y}_t^{F,d} = -n\hat{T}_t + \hat{C}_t \tag{B.99}$$

and simplifying yields

$$\begin{aligned}
 w_t &= U_C \bar{C} \left( \hat{C}_t + \frac{1}{2}(1 - \rho) \hat{C}_t^2 \right. \\
 &\quad - n(1 - \Phi^H)[(1 - n)\hat{T}_t + \hat{C}_t] - (1 - n)(1 - \Phi^F)[-n\hat{T}_t + \hat{C}_t] \\
 &\quad - \frac{1}{2} \left( n[(1 - n)\hat{T}_t + \hat{C}_t]^2 + (1 - n)[-n\hat{T}_t + \hat{C}_t]^2 \right) \\
 &\quad - \frac{\eta}{2} \left( n[(1 - n)\hat{T}_t + \hat{C}_t + g_t^H]^2 + (1 - n)[-n\hat{T}_t + \hat{C}_t + g_t^F]^2 \right) \\
 &\quad + \eta \left( n[(1 - n)\hat{T}_t + \hat{C}_t] \bar{Y}_t^H + (1 - n)[-n\hat{T}_t + \hat{C}_t] \bar{Y}_t^F \right) \\
 &\quad - \frac{1}{2} (\sigma^{-1} + \eta) [n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \\
 &\quad + t.i.p. + O(\|\xi\|^3) \tag{B.100}
 \end{aligned}$$

$$\begin{aligned}
 &= U_C \bar{C} \left( \hat{C}_t + \frac{1}{2}(1 - \rho) \hat{C}_t^2 \right. \\
 &\quad - (1 - \Phi^H)n(1 - n)\hat{T}_t - n\hat{C}_t + n\Phi^H \hat{C}_t + (1 - \Phi^F)n(1 - n)\hat{T}_t - (1 - n)\hat{C}_t + (1 - n)\Phi^F \hat{C}_t \\
 &\quad - \frac{1}{2} \left( n(1 - n)^2 \hat{T}_t^2 + 2n(1 - n)\hat{T}_t \hat{C}_t + n\hat{C}_t^2 + (1 - n)n^2 \hat{T}_t^2 - 2n(1 - n)\hat{T}_t \hat{C}_t + (1 - n)\hat{C}_t^2 \right) \\
 &\quad - \frac{\eta}{2} \left( n(1 - n)^2 \hat{T}_t^2 + n\hat{C}_t^2 + \underbrace{ng_t^{H^2}}_{=t.i.p.} + 2n(1 - n)\hat{T}_t \hat{C}_t + 2n(1 - n)\hat{T}_t g_t^H + 2\hat{C}_t g_t^H \right. \\
 &\quad \left. + (1 - n)n^2 \hat{T}_t^2 + (1 - n)\hat{C}_t^2 + \underbrace{(1 - n)g_t^{F^2}}_{=t.i.p.} - 2n(1 - n)\hat{T}_t \hat{C}_t - 2n(1 - n)\hat{T}_t g_t^H + 2\hat{C}_t g_t^F \right) \\
 &\quad + \eta \left( n(1 - n)\hat{T}_t \bar{Y}_t^H + n\hat{C}_t \bar{Y}_t^H - (1 - n)n\hat{T}_t \bar{Y}_t^F + (1 - n)\hat{C}_t \bar{Y}_t^F \right) \\
 &\quad - \frac{1}{2} (\sigma^{-1} + \eta) [n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \\
 &\quad + t.i.p. + O(\|\xi\|^3). \tag{B.101}
 \end{aligned}$$

Further simplification gives

$$\begin{aligned}
 w_t = & U_C \bar{C} \left( \hat{C}_t + \frac{1}{2}(1 - \rho) \hat{C}_t^2 \right. \\
 & - \underbrace{\left( (1 - \Phi^H) - (1 - \Phi^F) \right)}_{= 0 \text{ if } \Phi^H = \Phi^F} n(1 - n) \hat{T}_t \underbrace{- n \hat{C}_t - (1 - n) \hat{C}_t}_{= -\hat{C}_t} + \hat{C}_t \left[ n \Phi^H + (1 - n) \Phi^F \right] \\
 & - \frac{1}{2} \left( \hat{C}_t^2 + \underbrace{\left[ n(1 - n)^2 + (1 - n)n^2 \right]}_{= n(1 - n)} \hat{T}_t^2 \right) \\
 & - \frac{\eta}{2} \left( \hat{C}_t^2 + n(1 - n) \hat{T}_t^2 + 2n(1 - n) \hat{T}_t \underbrace{\left[ g_t^H - g_t^F \right]}_{= -g_t^R} + 2\hat{C}_t \underbrace{\left[ n g_t^H + (1 - n) g_t^F \right]}_{= g_t^W} \right) \\
 & + \eta \left( \hat{C}_t \underbrace{\left[ n \bar{Y}_t^H + (1 - n) \bar{Y}_t^F \right]}_{= \bar{Y}_t^W} + n(1 - n) \hat{T}_t \underbrace{\left[ \bar{Y}_t^H - \bar{Y}_t^F \right]}_{= -\bar{Y}_t^R} \right) \\
 & - \frac{1}{2} (\sigma^{-1} + \eta) [n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \\
 & + t.i.p. + O(\|\xi\|^3). \tag{B.102}
 \end{aligned}$$

This yields<sup>45</sup>

$$\begin{aligned}
 w_t = & U_C \bar{C} \left( \hat{C}_t \left[ n \Phi^H + (1 - n) \Phi^F \right] + \frac{1}{2}(1 - \rho) \hat{C}_t^2 \right. \\
 & + \eta \left( \hat{C}_t \bar{Y}_t^W - n(1 - n) \hat{T}_t \bar{Y}_t^R \right) \\
 & - \frac{1}{2} \left( \hat{C}_t^2 + n(1 - n) \hat{T}_t^2 \right) \\
 & - \frac{\eta}{2} \left( \hat{C}_t^2 + n(1 - n) \hat{T}_t^2 + 2\hat{C}_t g_t^W - 2n(1 - n) \hat{T}_t g_t^R \right) \\
 & - \frac{1}{2} (\sigma^{-1} + \eta) [n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \\
 & + t.i.p. + O(\|\xi\|^3). \tag{B.103}
 \end{aligned}$$

<sup>45</sup>This equation corresponds to equation (E.21) in Benigno (2003), Appendix D, except for a typo: There must be a minus sign in front of  $n(1 - n) \hat{T}_t \bar{Y}_t^R$ .

Factoring out a minus sign yields

$$\begin{aligned}
 w_t = & -U_C \bar{C} \left( -\hat{C}_t [n\Phi^H + (1-n)\Phi^F] - \frac{1}{2}(1-\rho)\hat{C}_t^2 \right. \\
 & - \eta (\hat{C}_t \bar{Y}_t^W - n(1-n)\hat{T}_t \bar{Y}_t^R) \\
 & + \frac{1}{2} (\hat{C}_t^2 + n(1-n)\hat{T}_t^2) \\
 & + \frac{\eta}{2} (\hat{C}_t^2 + n(1-n)\hat{T}_t^2 + 2\hat{C}_t \hat{g}_t^W - 2n(1-n)\hat{T}_t \hat{g}_t^R) \\
 & \left. + \frac{1}{2} (\sigma^{-1} + \eta) [n \text{var}_h \hat{y}_t(h) + (1-n) \text{var}_f \hat{y}_t(f)] \right) \\
 & + t.i.p. + O(\|\xi\|^3). \tag{B.104}
 \end{aligned}$$

Expanding gives

$$\begin{aligned}
 w_t = & -U_C \bar{C} \left( -\hat{C}_t [n\Phi^H + (1-n)\Phi^F] - \frac{1}{2}(1-\rho)\hat{C}_t^2 \right. \\
 & - \eta \hat{C}_t \bar{Y}_t^W + \eta n(1-n)\hat{T}_t \bar{Y}_t^R \\
 & + \frac{1}{2}\hat{C}_t^2 + \frac{1}{2}n(1-n)\hat{T}_t^2 \\
 & + \frac{\eta}{2}\hat{C}_t^2 + \frac{\eta}{2}n(1-n)\hat{T}_t^2 + \eta \hat{C}_t \hat{g}_t^W - \eta n(1-n)\hat{T}_t \hat{g}_t^R \\
 & \left. + \frac{1}{2} (\sigma^{-1} + \eta) [n \text{var}_h \hat{y}_t(h) + (1-n) \text{var}_f \hat{y}_t(f)] \right) \\
 & + t.i.p. + O(\|\xi\|^3). \tag{B.105}
 \end{aligned}$$

Rearranging yields

$$\begin{aligned}
 w_t &= -U_C \bar{C} \left( -\hat{C}_t [n\Phi^H + (1-n)\Phi^F] \right. \\
 &\quad + \frac{1}{2}(\rho + \eta) \hat{C}_t^2 - \underbrace{\eta [\bar{Y}_t^W - g_t^W]}_{=(\rho+\eta)\tilde{C}_t} \hat{C}_t \\
 &\quad + \frac{1}{2}(1 + \eta)n(1 - n) \hat{T}_t^2 - n(1 - n) \underbrace{\eta [g_t^R - \bar{Y}_t^R]}_{=(1+\eta)\tilde{T}_t} \hat{T}_t \\
 &\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
 &\quad + t.i.p. + O(\|\xi\|^3) \tag{B.106}
 \end{aligned}$$

$$\begin{aligned}
 &= -U_C \bar{C} \left( -\hat{C}_t [n\Phi^H + (1 - n)\Phi^F] \right. \\
 &\quad + \frac{1}{2}(\rho + \eta) [\hat{C}_t^2 - 2\tilde{C}_t \hat{C}_t] \\
 &\quad + \frac{1}{2}(1 + \eta)n(1 - n) [\hat{T}_t^2 - 2\tilde{T}_t \hat{T}_t] \\
 &\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
 &\quad + t.i.p. + O(\|\xi\|^3) \tag{B.107}
 \end{aligned}$$

$$\begin{aligned}
 &= -U_C \bar{C} \left( -\hat{C}_t [n\Phi^H + (1 - n)\Phi^F] \right. \\
 &\quad + \frac{1}{2}(\rho + \eta) \underbrace{[\hat{C}_t^2 - 2\tilde{C}_t \hat{C}_t + \tilde{C}_t^2]}_{=[\hat{C}_t - \tilde{C}_t]^2} - \underbrace{\frac{1}{2}(\rho + \eta)\tilde{C}_t^2}_{=t.i.p.} \\
 &\quad + \frac{1}{2}(1 + \eta)n(1 - n) \underbrace{[\hat{T}_t^2 - 2\tilde{T}_t \hat{T}_t + \tilde{T}_t^2]}_{=[\hat{T}_t - \tilde{T}_t]^2} - \underbrace{\frac{1}{2}(1 + \eta)n(1 - n)\tilde{T}_t^2}_{=t.i.p.} \\
 &\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
 &\quad + t.i.p. + O(\|\xi\|^3) \tag{B.108}
 \end{aligned}$$

$$\begin{aligned}
 &= -U_C \bar{C} \left( -\hat{C}_t [n\Phi^H + (1 - n)\Phi^F] \right. \\
 &\quad + \frac{1}{2}(\rho + \eta) [\hat{C}_t - \tilde{C}_t]^2 + \frac{1}{2}(1 + \eta)n(1 - n) [\hat{T}_t - \tilde{T}_t]^2 \\
 &\quad \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\
 &\quad + t.i.p. + O(\|\xi\|^3). \tag{B.109}
 \end{aligned}$$

The difference between steady-state consumption under the presence of the monopolistic distortion  $\bar{C}$  and the efficient level of consumption  $C^*$  (situation without the distortion) is given by

$$\bar{c} = -\ln\left(\frac{\bar{C}}{C^*}\right) = \frac{n\Phi^H + (1-n)\Phi^F}{\rho + \eta}. \quad (\text{B.110})$$

Inserting yields

$$\begin{aligned} w_t = & -U_C \bar{C} \left( -\hat{C}_t(\rho + \eta)\bar{c} \right. \\ & + \frac{1}{2}(\rho + \eta) [\hat{C}_t - \tilde{C}_t]^2 + \frac{1}{2}(1 + \eta)n(1 - n) [\hat{T}_t - \tilde{T}_t]^2 \\ & \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\ & + t.i.p. + O(\|\xi\|^3) \end{aligned} \quad (\text{B.111})$$

$$\begin{aligned} = & -U_C \bar{C} \left( \frac{1}{2}(\rho + \eta) \left( [\hat{C}_t - \tilde{C}_t]^2 - 2\hat{C}_t\bar{c} \right) + \frac{1}{2}(1 + \eta)n(1 - n) [\hat{T}_t - \tilde{T}_t]^2 \right. \\ & \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\ & + t.i.p. + O(\|\xi\|^3) \end{aligned} \quad (\text{B.112})$$

$$\begin{aligned} = & -U_C \bar{C} \left( \frac{1}{2}(\rho + \eta) \left( \underbrace{[\hat{C}_t - \tilde{C}_t]}_{\equiv c_t} - \bar{c} \right)^2 + \frac{1}{2}(1 + \eta)n(1 - n) [\hat{T}_t - \tilde{T}_t]^2 \right. \\ & \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{B.113})$$

Thus,

$$\begin{aligned} w_t = & -U_C \bar{C} \left( \frac{1}{2}(\rho + \eta) [c_t - \bar{c}]^2 + \frac{1}{2}(1 + \eta)n(1 - n) [\hat{T}_t - \tilde{T}_t]^2 \right. \\ & \left. + \frac{1}{2}(\sigma^{-1} + \eta)[n \text{var}_h \hat{y}_t(h) + (1 - n) \text{var}_f \hat{y}_t(f)] \right) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{B.114})$$

Next,  $\text{var}_h \hat{y}_t(h)$  can be linked to inflation  $\pi_t^H$  and  $\text{var}_f \hat{y}_t(f)$  to  $\pi_t^F$ . Note that

$$\text{var}_h \hat{y}_t(h) = \text{var}_h y_t(h) = \sigma^2 \text{var}_h \hat{p}_t(h) = \sigma^2 \text{var}_h p_t(h). \quad (\text{B.115})$$

Then, the following relationship is derived in a completely analogous way as in Woodford (2003):

$$\sum_{t=0}^{\infty} \beta^t \text{var}_h p_t(h) = \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H \beta)} \sum_{t=0}^{\infty} \beta^t \pi_t^{H^2} + t.i.p. + O(\|\xi\|^3). \quad (\text{B.116})$$

Finally, calculating the discounted value of all future utility flows yields

$$\begin{aligned} \tilde{W}_t &= E_t \sum_{k=0}^{\infty} \beta^k w_{t+k} \\ &= E_t \sum_{k=0}^{\infty} \beta^k (-U_C \bar{C}) \left( \frac{1}{2} (\rho + \eta) [c_{t+k} - \bar{c}]^2 + \frac{1}{2} (1 + \eta) n(1 - n) [\hat{T}_{t+k} - \tilde{T}_{t+k}]^2 \right. \\ &\quad \left. + \frac{1}{2} (\sigma^{-1} + \eta) [n \text{var}_h \hat{y}_{t+k}(h) + (1 - n) \text{var}_f \hat{y}_{t+k}(f)] \right) \\ &\quad + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{B.117})$$

Thus,

$$\begin{aligned} \tilde{W}_t &= -\frac{1}{2} U_C \bar{C} E_t \sum_{k=0}^{\infty} \beta^k \left( (\rho + \eta) [c_{t+k} - \bar{c}]^2 + (1 + \eta) n(1 - n) [\hat{T}_{t+k} - \tilde{T}_{t+k}]^2 \right. \\ &\quad \left. + \sigma(1 + \sigma\eta) n \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H \beta)} \pi_{t+k}^{H^2} \right. \\ &\quad \left. + \sigma(1 + \sigma\eta)(1 - n) \frac{\alpha^F}{(1 - \alpha^F)(1 - \alpha^F \beta)} \pi_{t+k}^{F^2} \right) \\ &\quad + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{B.118})$$

This expression is equivalent to equation (26) in Benigno (2004) with  $\bar{c} = 0$ , i.e., the monopolistic distortion is perfectly neutralized by an appropriate subsidy, and with  $c_t = y_t^W$ .

Dividing both sides by  $U_C \bar{C}$ , letting  $\beta \rightarrow 1$ , and with  $\bar{c} = 0$ , the loss function can be written as

$$\begin{aligned} W_t &= -\frac{1}{2} \left( (\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta) n(1 - n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\ &\quad \left. + \sigma(1 + \sigma\eta) n \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H \beta)} \text{var} \pi_t^H \right. \\ &\quad \left. + \sigma(1 + \sigma\eta)(1 - n) \frac{\alpha^F}{(1 - \alpha^F)(1 - \alpha^F \beta)} \text{var} \pi_t^F \right) \\ &\quad + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{B.119})$$

This equation corresponds to equation (4.2.25) in the main text.

**Special case:**  $\alpha^H = \alpha^F$

When prices are equally rigid in the two countries ( $\alpha^H = \alpha^F$ ), the world welfare loss function can be simplified further in a useful way.

When  $\alpha^H = \alpha^F = \alpha$ , it immediately follows that

$$\begin{aligned} \tilde{W}_t = & -\frac{1}{2}U_C\bar{C}E_t \sum_{k=0}^{\infty} \beta^k \left( (\rho + \eta) [c_{t+k} - \bar{c}]^2 + (1 + \eta)n(1 - n) [\hat{T}_{t+k} - \tilde{T}_{t+k}]^2 \right. \\ & \left. + \sigma(1 + \sigma\eta) \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)} [n\pi_{t+k}^{H^2} + (1 - n)\pi_{t+k}^{F^2}] \right) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{B.120})$$

The last term in square brackets can be modified in the following way:

$$\begin{aligned} & n\pi_t^{H^2} + (1 - n)\pi_t^{F^2} \\ & = n \underbrace{(n\pi_t^{H^2} + (1 - n)\pi_t^{H^2})}_{=\pi_t^{H^2}} + (1 - n) \underbrace{(n\pi_t^{F^2} + (1 - n)\pi_t^{F^2})}_{=\pi_t^{F^2}} \\ & = n^2\pi_t^{H^2} + n(1 - n)\pi_t^{H^2} + n(1 - n)\pi_t^{F^2} + (1 - n)^2\pi_t^{F^2}. \end{aligned} \quad (\text{B.121})$$

Adding  $2n(1 - n)\pi_t^H\pi_t^F - 2n(1 - n)\pi_t^H\pi_t^F$  and simplifying yields

$$\begin{aligned} n\pi_t^{H^2} + (1 - n)\pi_t^{F^2} & = n^2\pi_t^{H^2} + 2n(1 - n)\pi_t^H\pi_t^F + (1 - n)^2\pi_t^{F^2} \\ & \quad + n(1 - n)\pi_t^{F^2} - 2n(1 - n)\pi_t^H\pi_t^F + n(1 - n)\pi_t^{H^2} \\ & = \underbrace{n^2\pi_t^{H^2} + 2n(1 - n)\pi_t^H\pi_t^F + (1 - n)^2\pi_t^{F^2}}_{(n\pi_t^H + (1 - n)\pi_t^F)^2} \\ & \quad + n(1 - n) \underbrace{(\pi_t^{F^2} - 2\pi_t^H\pi_t^F + \pi_t^{H^2})}_{(\pi_t^F - \pi_t^H)^2} \\ & = \underbrace{(n\pi_t^H + (1 - n)\pi_t^F)^2}_{\pi_t^{W^2}} + n(1 - n) \underbrace{(\pi_t^F - \pi_t^H)^2}_{\pi_t^{R^2}} \\ & = \pi_t^{W^2} + n(1 - n)\pi_t^{R^2}. \end{aligned} \quad (\text{B.122})$$



The world welfare loss function is, then, given by

$$\begin{aligned} \tilde{W}_t = & -\frac{1}{2}U_C\bar{C}E_t \sum_{k=0}^{\infty} \beta^k \left( (\rho + \eta) [c_{t+k} - \bar{c}]^2 + (1 + \eta)n(1 - n) [\hat{T}_{t+k} - \tilde{T}_{t+k}]^2 \right. \\ & \left. + \sigma(1 + \sigma\eta) \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)} [\pi_{t+k}^W + n(1 - n)\pi_{t+k}^R] \right) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{B.123})$$

Dividing both sides by  $U_C\bar{C}$ , letting  $\beta \rightarrow 1$ , and with  $\bar{c} = 0$ , the loss function can be written as

$$\begin{aligned} W_t = & -\frac{1}{2} \left( (\rho + \eta) \text{var}(\hat{C}_t - \tilde{C}_t) + (1 + \eta)n(1 - n) \text{var}(\hat{T}_t - \tilde{T}_t) \right. \\ & \left. + \sigma(1 + \sigma\eta) \frac{\alpha}{(1 - \alpha)(1 - \alpha\beta)} [\text{var} \pi_t^W + n(1 - n) \text{var} \pi_t^R] \right) \\ & + t.i.p. + O(\|\xi\|^3). \end{aligned} \quad (\text{B.124})$$

This equation corresponds to equation (4.2.26) in the main text.

### Version containing country-specific output gaps

The welfare loss function can be expressed alternatively in terms of the country-specific output gaps instead of the consumption gap. This makes the analogy to the closed-economy counterpart, which is expressed in terms of the output gap as well, more obvious.

Inserting the gap version of the equations for aggregate demand

$$\hat{Y}_t^H - \tilde{Y}_t^H = (1 - n)(\hat{T}_t - \tilde{T}_t) + \hat{C}_t - \tilde{C}_t \quad (\text{B.125})$$

$$\hat{Y}_t^F - \tilde{Y}_t^F = -n(\hat{T}_t - \tilde{T}_t) + \hat{C}_t - \tilde{C}_t \quad (\text{B.126})$$

into the weighted average of the squared output gaps yields

$$n(\hat{Y}_t^H - \tilde{Y}_t^H)^2 + (1 - n)(\hat{Y}_t^F - \tilde{Y}_t^F)^2 = n(1 - n)(\hat{T}_t - \tilde{T}_t)^2 + (\hat{C}_t - \tilde{C}_t)^2. \quad (\text{B.127})$$

Solving this equation for  $(\hat{C}_t - \tilde{C}_t)^2$  and inserting the resulting expression into equation (B.118) with  $\bar{c} = 0$  yields

$$\begin{aligned}
 \tilde{W}_t = & -\frac{1}{2}U_C\bar{C}E_t\sum_{k=0}^{\infty}\beta^k\left((\rho+\eta)\left[n(\hat{Y}_{t+k}^H-\tilde{Y}_{t+k}^H)^2+(1-n)(\hat{Y}_{t+k}^F-\tilde{Y}_{t+k}^F)^2\right]\right. \\
 & + (1-\rho)n(1-n)\left[\hat{T}_{t+k}-\tilde{T}_{t+k}\right]^2 \\
 & + \sigma(1+\sigma\eta)n\frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)}\pi_{t+k}^{H^2} \\
 & \left. + \sigma(1+\sigma\eta)(1-n)\frac{\alpha^F}{(1-\alpha^F)(1-\alpha^F\beta)}\pi_{t+k}^{F^2}\right) \\
 & + t.i.p. + O(\|\xi\|^3). \tag{B.128}
 \end{aligned}$$

Expressed in variances, the welfare loss function is then given by

$$\begin{aligned}
 W_t = & -\frac{1}{2}\left((\rho+\eta)\left[n\text{var}(\hat{Y}_t^H-\tilde{Y}_t^H)+(1-n)\text{var}(\hat{Y}_t^F-\tilde{Y}_t^F)\right]\right. \\
 & + (1-\rho)n(1-n)\text{var}(\hat{T}_t-\tilde{T}_t) \\
 & + \sigma(1+\sigma\eta)n\frac{\alpha^H}{(1-\alpha^H)(1-\alpha^H\beta)}\text{var}\pi_t^H \\
 & \left. + \sigma(1+\sigma\eta)(1-n)\frac{\alpha^F}{(1-\alpha^F)(1-\alpha^F\beta)}\text{var}\pi_t^F\right) \\
 & + t.i.p. + O(\|\xi\|^3). \tag{B.129}
 \end{aligned}$$

This welfare loss function closely resembles those in Benigno and Benigno (2006, eq. 21) as well as in Corsetti, Dedola, and Leduc (2011, eq. 40).

## B.4. Variances

To obtain analytical expressions for the variance of the terms-of-trade gap and of the inflation differential under each monetary regime, I first derive the recursive laws of motion (RLOM). Then I set up the corresponding vector autoregressive (VAR) model of the system of equations. Finally, since the matrix algebra is very extensive, I use MATLAB Symbolic Math Toolbox to obtain the expressions of interest from the variance-covariance matrix.

The derivations in this Appendix are only valid if the degree of price stickiness and the persistence of productivity shocks are identical across countries ( $\alpha^H = \alpha^F$  and  $\rho_H = \rho_F$ ) and if monetary policy does not engage in interest rate smoothing ( $\phi_R = 0$ ). As shown in Section 4.3, the variables consumption and world inflation need not be considered as they behave identically across monetary regimes and independently of the variables terms of trade, nominal exchange rate and inflation differential.

### Monetary union regime

The number of equations can be reduced by subtracting the New Keynesian Phillips curve of country  $H$  (B.42) from the one of country  $F$  (B.43). As a result, the consumption gap vanishes due to  $k_C^H = k_C^F$ . The second equation is given by the terms-of-trade identity (B.44). The resulting system of equations is, then, given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \quad (\text{B.130})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R. \quad (\text{B.131})$$

The general form of the corresponding RLOM is given by

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \quad (\text{B.132})$$

$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t \quad (\text{B.133})$$

$$\tilde{T}_t = \rho_H \tilde{T}_{t-1} - \frac{\eta}{1 + \eta} \nu_t^R, \quad (\text{B.134})$$

where  $\nu_t^R = \nu_t^F - \nu_t^H$ . Equation (B.134) is obtained by inserting the country-specific shock processes (B.38) into the equation of the flexible-price terms of trade (B.33).

To obtain the unknown coefficients as functions of the deep parameters of the model, I use the method of undetermined coefficients. First, inserting equations (B.132) through (B.134) into equations (B.130) and (B.131) and rearranging yields

$$b_2\hat{T}_{t-1} + c_2\tilde{T}_t = [-k_T b_1 + \beta b_2 b_1]\hat{T}_{t-1} \quad (\text{B.135})$$

$$+ [-k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H]\tilde{T}_t$$

$$b_1\hat{T}_{t-1} + c_1\tilde{T}_t = [1 + b_2]\hat{T}_{t-1} + c_2\tilde{T}_t. \quad (\text{B.136})$$

Setting  $\hat{T}_{t-1} = 1, \tilde{T}_t = 0$  and  $\hat{T}_{t-1} = 0, \tilde{T}_t = 1$  respectively gives the following four conditions for the four unknown coefficients:

$$b_2 = -k_T b_1 + \beta b_2 b_1 \quad (\text{B.137})$$

$$c_2 = -k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H \quad (\text{B.138})$$

$$b_1 = 1 + b_2 \quad (\text{B.139})$$

$$c_1 = c_2. \quad (\text{B.140})$$

Straightforward manipulation yields the quadratic equation

$$0 = \beta b_1^2 - (1 + k_T + \beta)b_1 + 1 \quad (\text{B.141})$$

and therefore two solutions for  $b_1$ . Only one solution fulfills the requirement for a stable equilibrium, i.e.,  $|b_1| < 1$ . Using  $b_1$  immediately yields the other coefficients. Thus, the coefficients of the RLOM take the following form:

$$b_1 = \frac{1 + k_T + \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta} \quad (\text{B.142})$$

$$b_2 = \frac{1 + k_T - \beta - \sqrt{(1 + k_T + \beta)^2 - 4\beta}}{2\beta} \quad (\text{B.143})$$

$$c_1 = c_2 = c = \frac{k_T}{1 + k_T + \beta(1 - \rho_H - b_1)}. \quad (\text{B.144})$$

The corresponding VAR model can be written as follows<sup>46</sup>:

$$\begin{pmatrix} \pi_t^R \\ \hat{T}_t \\ \tilde{T}_t \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & b_2 & \rho_H c \\ 0 & b_1 & \rho_H c \\ 0 & 0 & \rho_H \end{pmatrix}}_{\equiv A} \begin{pmatrix} \pi_{t-1}^R \\ \hat{T}_{t-1} \\ \tilde{T}_{t-1} \end{pmatrix} - \underbrace{\begin{pmatrix} c \frac{\eta}{1+\eta} \\ c \frac{\eta}{1+\eta} \\ \frac{\eta}{1+\eta} \end{pmatrix}}_{\equiv B} \nu_t.$$

A closed-form solution of the variance-covariance matrix  $\Sigma$  can be obtained in terms of the *vec* operator as follows:<sup>47</sup>

$$\text{vec}(\Sigma) = (I - A \otimes A)^{-1} \text{vec}(B), \quad (\text{B.145})$$

where  $I$  denotes the identity matrix.

Since matrix  $A$  is of dimension  $3 \times 3$ , matrix  $A \otimes A$  is of dimension  $9 \times 9$ . Although matrix  $A \otimes A$  is triangular, calculating the inverse of that matrix is very cumbersome. Therefore, I resort to MATLAB Symbolic Math Toolbox. Further simplification of the resulting expressions finally yields

$$\text{var}_{MU}(\hat{T}_t - \tilde{T}_t) = \left[ \frac{(1 + \rho_H b_1) c^2}{(1 - b_1^2)(1 - \rho_H b_1)} - \frac{2c}{1 - \rho_H b_1} + 1 \right] \text{var } \tilde{T}_t \quad (\text{B.146})$$

$$\text{var}_{MU} \pi_t^R = \frac{2c^2}{(1 + b_1)(1 - \rho_H b_1)(1 + \rho_H)} \text{var } \tilde{T}_t \quad (\text{B.147})$$

$$\text{var } \tilde{T}_t = \frac{1}{1 - \rho_H^2} \left( \frac{\eta}{1 + \eta} \right)^2 \left[ \text{var } \nu_t^H + \text{var } \nu_t^F - 2 \text{cov}(\nu_t^H, \nu_t^F) \right]. \quad (\text{B.148})$$

### Flexible exchange rate regime

The derivation of the variances under the FX regime follows the exact same steps as under the MU regime. The number of equations can be reduced by subtracting the New Keynesian Phillips curves from each other. Furthermore, the expected change in the nominal exchange rate can be expressed as a function of the inflation differential and the terms of trade by inserting the interest rate rules (B.55) and (B.56) as well as the equations for aggregate demand (B.49) and (B.50) into the uncovered interest parity condition (B.54). The resulting

<sup>46</sup>The order of variables was chosen as to render matrix  $A$  and therefore matrix  $A \otimes A$  triangular. This facilitates the calculation of the determinant considerably, since, in that case, the determinant is simply given by the product of the diagonal elements.

<sup>47</sup>See, e.g., Hamilton (1994).

system of equations is, then, given by

$$\pi_t^R = -k_T(\hat{T}_t - \tilde{T}_t) + \beta E_t \pi_{t+1}^R \quad (\text{B.149})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_t^R + \Delta \hat{S}_t \quad (\text{B.150})$$

$$E_t \Delta \hat{S}_{t+1} = -\phi_\pi \pi_t^R + \phi_Y \hat{T}_t. \quad (\text{B.151})$$

The general form of the corresponding RLOM is given by

$$\hat{T}_t = b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t \quad (\text{B.152})$$

$$\pi_t^R = b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t \quad (\text{B.153})$$

$$\Delta \hat{S}_t = b_3 \hat{T}_{t-1} + c_3 \tilde{T}_t \quad (\text{B.154})$$

$$\tilde{T}_t = \rho_H \tilde{T}_{t-1} - \frac{\eta}{1 + \eta} \nu_t^R. \quad (\text{B.155})$$

Inserting equations (B.152) through (B.155) into equations (B.149) through (B.150) and rearranging yields

$$b_2 \hat{T}_{t-1} + c_2 \tilde{T}_t = [-k_T b_1 + \beta b_2 b_1] \hat{T}_{t-1} \quad (\text{B.156})$$

$$+ [-k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H] \tilde{T}_t$$

$$b_1 \hat{T}_{t-1} + c_1 \tilde{T}_t = [1 + b_2 + b_3] \hat{T}_{t-1} + [c_2 + c_3] \tilde{T}_t \quad (\text{B.157})$$

$$b_1 b_3 \hat{T}_{t-1} + [b_3 c_1 + c_3 \rho_H] \tilde{T}_t = [-\phi_\pi b_2 + \phi_Y b_1] \hat{T}_{t-1} \quad (\text{B.158})$$

$$+ [-\phi_\pi c_2 + \phi_Y c_1] \tilde{T}_t.$$

Setting  $\hat{T}_{t-1} = 1, \tilde{T}_t = 0$  and  $\hat{T}_{t-1} = 0, \tilde{T}_t = 1$  respectively gives the following six conditions for the six unknown coefficients:

$$b_2 = -k_T b_1 + \beta b_2 b_1 \quad (\text{B.159})$$

$$c_2 = -k_T(c_1 - 1) + \beta b_2 c_1 + \beta c_2 \rho_H \quad (\text{B.160})$$

$$b_1 = 1 + b_2 + b_3 \quad (\text{B.161})$$

$$c_1 = c_2 + c_3 \quad (\text{B.162})$$

$$b_1 b_3 = -\phi_\pi b_2 + \phi_Y b_1 \quad (\text{B.163})$$

$$b_3 c_1 + c_3 \rho_H = -\phi_\pi c_2 + \phi_Y c_1. \quad (\text{B.164})$$

Straightforward manipulation yields the quadratic equation

$$0 = \beta b_1^2 - [1 + k_T + (1 + \phi_Y)\beta] b_1 + (1 + \phi_\pi k_T + \phi_Y). \quad (\text{B.165})$$

In this case, there are either two real or two imaginary solutions for  $b_1$ , depending on the realizations of the deep parameters. However, neither solution fulfills the requirement for a stable equilibrium. Yet,  $b_1 = 0$  is another solution to the above system of equations, and it implies a stable equilibrium, since  $|b_1| < 1$ . Given  $b_1 = 0$ , the coefficients of the RLOM take the following form:

$$b_1 = 0 \quad (\text{B.166})$$

$$b_2 = 0 \quad (\text{B.167})$$

$$b_3 = -1 \quad (\text{B.168})$$

$$c_1 = \frac{(\phi_\pi - \rho_H)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)} \quad (\text{B.169})$$

$$c_2 = \frac{(1 - \rho_H + \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)} \quad (\text{B.170})$$

$$c_3 = \frac{(\phi_\pi - 1 - \phi_Y)k_T}{(\phi_\pi - \rho_H)k_T + (1 - \rho_H + \phi_Y)(1 - \beta\rho_H)}. \quad (\text{B.171})$$

The corresponding VAR model can be written as follows<sup>48</sup>:

$$\begin{pmatrix} \Delta \hat{S}_t \\ \hat{T}_t \\ \pi_t^R \\ \tilde{T}_t \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -1 & 0 & \rho_H c_3 \\ 0 & 0 & 0 & \rho_H c_1 \\ 0 & 0 & 0 & \rho_H c_2 \\ 0 & 0 & 0 & \rho_H \end{pmatrix}}_{\equiv A} \begin{pmatrix} \Delta \hat{S}_{t-1} \\ \hat{T}_{t-1} \\ \pi_{t-1}^R \\ \tilde{T}_{t-1} \end{pmatrix} - \underbrace{\begin{pmatrix} c_3 \frac{\eta}{1+\eta} \\ c_1 \frac{\eta}{1+\eta} \\ c_2 \frac{\eta}{1+\eta} \\ \frac{\eta}{1+\eta} \end{pmatrix}}_{\equiv B} \nu_t.$$

A closed-form solution of the variance-covariance matrix  $\Sigma$  can be obtained in terms of the *vec* operator as follows:<sup>49</sup>

$$\text{vec}(\Sigma) = (I - A \otimes A)^{-1} \text{vec}(B), \quad (\text{B.172})$$

where  $I$  denotes the identity matrix.

Since matrix  $A$  is of dimension  $4 \times 4$ , matrix  $A \otimes A$  is of dimension  $16 \times 16$ . Although matrix  $A \otimes A$  is triangular, calculating the inverse of that matrix is very cumbersome. Therefore, I resort to MATLAB Symbolic Math Toolbox.

<sup>48</sup>The order of variables was chosen as to render matrix  $A$  and therefore matrix  $A \otimes A$  triangular. This facilitates the calculation of the determinant considerably, since, in that case, the determinant is simply given by the product of the diagonal elements.

<sup>49</sup>See, e.g., Hamilton (1994).

Further simplification of the resulting expressions finally yields

$$\text{var}_{FX}(\hat{T}_t - \tilde{T}_t) = (c_1 - 1)^2 \text{var } \tilde{T}_t \quad (\text{B.173})$$

$$\text{var}_{FX} \pi_t^R = c_2^2 \text{var } \tilde{T}_t \quad (\text{B.174})$$

$$\text{var } \tilde{T}_t = \frac{1}{1 - \rho_H^2} \left( \frac{\eta}{1 + \eta} \right)^2 \left[ \text{var } \nu_t^H + \text{var } \nu_t^F - 2 \text{cov}(\nu_t^H, \nu_t^F) \right]. \quad (\text{B.175})$$



# Monetary Policy as an Optimum Currency Area Criterion

## Abstract

Whether countries benefit from forming a monetary union depends critically on the way monetary policy is conducted. This is mainly because monetary policy determines whether and to what extent a flexible nominal exchange rate fosters or hampers macroeconomic stabilization, even if monetary policy does not target the nominal exchange rate explicitly.

Keywords: Monetary union; macroeconomic stabilization; welfare analysis; optimum currency area theory; trade openness.

JEL classification: F33, F41, E52.

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## 5.1. Introduction

Over the decades since its initiation by Mundell (1961), the Optimum Currency Area (OCA) literature has identified numerous criteria that are considered important in determining whether countries benefit from forming a monetary union. These OCA criteria typically comprise the economic characteristics of the countries involved, such as the degree of labor mobility, the degree of price and wage flexibility, or the degree of trade openness.<sup>1</sup>

In this study, I argue that one OCA criterion has not yet been emphasized sufficiently, although it is absolutely critical for the welfare implications of monetary unification: the conduct of monetary policy. In particular, using a standard two-country New Keynesian dynamic stochastic general equilibrium (DSGE) model, I systematically demonstrate how and why different types of rule-based interest rate policies lead to different welfare rankings between a monetary union and a flexible exchange rate regime. These interest rate policies differ in terms of both the choice of the target variables and the strength of the response to these variables. Both dimensions can be decisive in determining whether countries benefit from forming a monetary union.<sup>2</sup>

When monetary policy in each country responds to inflation aggressively or implements a high degree of interest rate smoothing, forming a monetary union, where the common monetary policy continues to follow the same interest rate policy, tends to make countries worse off in terms of welfare by decreasing macroeconomic stability. However, when monetary policy responds to inflation only modestly or implements a low degree of interest rate smoothing, forming a monetary union tends to make countries better off by increasing macroeconomic stability. When monetary policy responds to output, forming a monetary union is almost always beneficial.

The conduct of monetary policy is not only an independent OCA criterion in itself, but it can also modify the nature of other OCA criteria, such as the degree of trade openness. Whether an increase in the degree of trade openness

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<sup>1</sup>Summarizing the vast OCA literature is beyond the scope of this study. Excellent surveys are Mongelli (2002), Dellas and Tavlas (2009), Beetsma and Giuliodori (2010), and De Grauwe (2012).

<sup>2</sup>The present study builds on my previous research (Groll, 2013) and elaborates on the importance of monetary policy for the welfare implications of monetary unification. Also, the present study employs a more general model that allows, among other things, for trade imbalances, deviations from purchasing power parity (i.e., real exchange rate fluctuations), and more variation in the conduct of monetary policy.

increases the likelihood of a monetary union being beneficial, as proposed by the vast bulk of previous OCA studies, depends critically on whether monetary policy targets producer price inflation or consumer price inflation. If monetary policy targets producer price inflation, it is also possible that an increase in the degree of trade openness decreases the likelihood of a monetary union being beneficial.

The most important reason why monetary policy is crucial for the welfare implications of monetary unification is as follows. Monetary policy determines whether and to what extent a flexible nominal exchange rate fosters or hampers macroeconomic stabilization, even if monetary policy does not target the nominal exchange rate explicitly. The flexibility of the nominal exchange rate renders monetary policy more potent under a flexible exchange rate regime in the sense that monetary policy affects all welfare-relevant variables directly. By contrast, in a monetary union, the influence of monetary policy is limited, especially with respect to relative prices such as the terms of trade and the real exchange rate.

The fact that monetary policy is more potent under a flexible exchange rate regime is a double-edged sword. When the interest rate policy is specified such that the nominal exchange rate moves in the “right” direction (which will be defined precisely), forming a monetary union generally reduces macroeconomic stability and thus welfare, because it eliminates the stabilizing effects of the nominal exchange rate. This is especially true if monetary policy can implement the optimal (i.e., welfare-maximizing) policy, as is assumed frequently in the literature.

However, the information that is required to implement such an optimal policy is usually not available in practice. Even the specification of a simple interest rate rule that is expected to move the nominal exchange rate in the right direction is not always obvious *ex ante*. For example, an interest rate policy that induces the nominal exchange rate to move in the right direction in a rather closed economy can induce it to move in the wrong direction in a very open economy.

When the interest rate policy is specified such that the nominal exchange rate moves in the wrong direction, forming a monetary union increases macroeconomic stability and thus welfare, because it eliminates the destabilizing effects of the nominal exchange rate. Essentially, the nominal exchange rate does not compensate for monetary policy mistakes, but instead it reinforces them. In

this sense, a monetary union provides a protective mechanism against monetary policy mistakes.

Certain interest rate policies render a monetary union beneficial, although they push the nominal exchange rate in the right direction in a flexible exchange rate regime. In these cases, countries benefit from monetary unification due to higher inflation stability. One example is a very modest response to inflation.<sup>3</sup> In practice, there are many temptations to implement such a lax policy. In times with a (private or public) debt overhang, monetary policy might let the inflation rate overshoot the inflation target for a prolonged period of time, with the aim of reducing the real debt burden and lowering borrowing costs. In times of high unemployment, allowing inflation to temporarily overshoot the target might also seem attractive, with the aim of reducing real wages in the presence of fixed-term nominal wage contracts, thereby increasing the demand for labor.

These temptations lead to a related problem, which is also less severe in a monetary union. A country that is unable to credibly commit itself to withstand these types of temptations suffers from a persistently high level of inflation. If it forms a monetary union with a country that does not suffer from such commitment problems, it benefits from a lower level of inflation by eliminating the so-called inflation bias (Alesina and Barro, 2002).<sup>4</sup>

However, this point differs from the point made in the present study in at least two important respects. First, the benefit of eliminating an inflation bias is related to the *level* of inflation, whereas the benefit described in the present study is related to the *stability* of inflation. Second, the benefit of eliminating an inflation bias is not inherent to a monetary union because it is only obtained if the monetary policy after monetary unification is more credible than that before monetary unification. By contrast, the benefit described in the present study is inherent to a monetary union because it is obtained even if the monetary policy after monetary unification continues to make the same mistake it made before monetary unification. This is because the same policy mistake is less harmful in a monetary union.

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<sup>3</sup>That is, the inflation coefficient in the interest rate rule is above but close to one.

<sup>4</sup>Note that the “currency union” in Alesina and Barro (2002) refers to a situation where a client country unilaterally adopts the currency of an anchor country—a situation also known as dollarization. Nevertheless, the benefit of eliminating an inflation bias also exists if the client and anchor country form a monetary union where the common monetary policy inherits the credibility of the anchor country. In Giavazzi and Pagano (1988), the “advantage of tying one’s hands” follows the same logic, although they refer to the former European Monetary System (1979–1999).

One reason for the inability to withstand the temptations mentioned above is a lack of “discipline and institutions that can provide a firm domestic commitment to a monetary policy that is dedicated to price stability” (Alesina and Barro, 2001, p. 382). It seems very likely that such weak institutions not only lead to a high level of inflation, but also to a poor stabilization of inflation. If this is the case, the trade-off that countries face when forming a monetary union between relinquishing national monetary policy as a macroeconomic stabilization device and gaining credibility to reduce undesired inflation, as stressed by Alesina and Barro (2002), does not exist.<sup>5</sup>

The remainder of this paper is organized as follows. Section 5.2 briefly outlines the structure of the model. Section 5.3 shows how different interest rate policies lead to different welfare rankings between a monetary union and a flexible exchange rate regime. Section 5.4 shows how different interest rate policies change the nature of the traditional OCA criterion of “trade openness.” Section 5.5 concludes this study.

## 5.2. Model

The model I use is a standard two-country New Keynesian DSGE model, and thus I provide only a very brief description. The model features two international monetary regimes:

1. A monetary union (MU) regime: Both countries share the same currency. A common monetary policy governs the common nominal interest rate.
2. A flexible exchange rate (FX) regime: Each country maintains its national currency and conducts its own, independent monetary policy. Nominal interest rates are country-specific. The nominal exchange rate between the two currencies is flexible.

The FX version of the model, including the microfounded, quadratic welfare measure, is described in Corsetti, Dedola, and Leduc (2011). The MU version of the model is largely identical (see, e.g., Benigno, 2004). The model economy features two countries of equal size (labeled  $H$  and  $F$ ), where there is trade in

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<sup>5</sup>Indeed, empirical evidence suggests that countries with higher levels of inflation also tend to experience a higher variability of inflation (early examples are Okun, 1971; Taylor, 1981). The latter study also presents a theoretical model where a “policy under-reaction” to inflation causes high and variable inflation. Similarly, Friedman (1977) establishes a link between weak institutions and high and variable inflation.

consumption goods (as opposed to trade in intermediate goods). The consumption baskets are allowed to differ among countries, so purchasing power parity does not necessarily hold. International asset markets are complete, i.e., risk sharing is perfect across countries. Producers act in an environment of monopolistic competition. The only factor of production is labor, which is immobile between countries. The only rigidity is the nominal price rigidity in the spirit of Calvo (1983).

Under the FX regime, prices are set in the currency of the producer's country ("producer currency pricing"), i.e., the producer does not discriminate the price between countries. The law of one price holds and exchange rate pass-through is complete. Complete asset markets imply that uncovered interest parity holds.

In both regimes, monetary policy is conducted via Taylor-type interest rate rules. Importantly, I assume that monetary policy is not able to observe the flexible-price equilibrium of the economy, particularly the flexible-price interest rate and flexible-price output, because this information is usually not available in practice. Thus, monetary policy responds to inflation (either producer price inflation or consumer price inflation) and to output (deviation from the steady state), rather than to the output gap (deviation from flexible-price output).

### 5.2.1. Model equations

The equations of the complete log-linearized model are shown below (for the full derivation, see Appendices C.1 and C.2). Deviations of the logarithm of a variable  $X_t$  from its steady state are denoted by  $\hat{X}_t$  if prices are sticky and by  $\tilde{X}_t^{fb}$  if prices are flexible and markups are neutralized (efficient allocation). The variables and parameters are defined in Tables 5.1 and 5.2, respectively.

CHAPTER 5. MONETARY POLICY AS AN OCA CRITERION

**Table 5.1.:** *Variables*

$C_t, C_t^*$	Consumption in country $H$ and $F$ , respectively
$Y_{H,t}, Y_{F,t}$	Output in country $H$ and $F$ , respectively
$\pi_{H,t}, \pi_{F,t}$	Producer price inflation in country $H$ and $F$ , respectively
$\pi_t, \pi_t^*$	Consumer price inflation in country $H$ and $F$ , respectively
$\pi_t^{MU}$	Union-wide inflation (average of country-specific inflation)
$R_t, R_t^*$	Nominal interest rate in country $H$ and $F$ , respectively
$R_t^{MU}$	Nominal interest rate in monetary union
$T_t$	Terms of trade
$S_t$	Nominal exchange rate
$Q_t$	Real exchange rate
$\zeta_{Y,t}, \zeta_{Y,t}^*$	Productivity shock in country $H$ and $F$ , respectively
$\zeta_{C,t}, \zeta_{C,t}^*$	Consumption preference shock in country $H$ and $F$ , respectively
$\mu_t^H, \mu_t^F$	Cost-push (or markup) shock in country $H$ and $F$ , respectively

**Table 5.2.:** *Parameters*

$\rho$	Inverse of elasticity of intertemporal substitution in consumption
$\beta$	Discount factor
$\eta$	Inverse of elasticity of producing the differentiated good
$a$	Home bias/degree of trade openness
$\sigma$	Elasticity of substitution between differentiated goods within countries
$\theta$	Elasticity of substitution between goods across countries
$\alpha^i$	Probability of not being able to reset the price in country $i = H, F$
$\phi_\pi$	Inflation coefficient in interest rate rule
$\phi_Y$	Output coefficient in interest rate rule
$\phi_R$	Interest rate smoothing coefficient in interest rate rule
$\rho_i$	Persistence of productivity shock in country $i = H, F$
$k_Y^i$	$k_Y^i = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i} \frac{\rho+\eta}{1+\sigma\eta}$
$k_T^i$	$k_T^i = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i} \frac{\rho\theta-1}{1+\sigma\eta}$
$k_\mu^i$	$k_\mu^i = \frac{(1-\alpha^i\beta)(1-\alpha^i)}{\alpha^i} \frac{1}{1+\sigma\eta}$

**Sticky-price equilibrium under the FX regime**

Under sticky prices, the model equations for the FX regime are given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}_t - E_t \pi_{t+1} + E_t \hat{\zeta}_{C,t+1} - \hat{\zeta}_{C,t} \right) \quad (5.2.1)$$

$$\hat{Q}_t = \rho \left( \hat{C}_t - \hat{C}_t^* \right) + \left( \hat{\zeta}_{C,t}^* - \hat{\zeta}_{C,t} \right) \quad (5.2.2)$$

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t - \hat{R}_t^* \quad (5.2.3)$$

$$\hat{Q}_t = (2a - 1) \hat{T}_t \quad (5.2.4)$$

$$\hat{Y}_{H,t} = 2a(1 - a) \theta \hat{T}_t + a \hat{C}_t + (1 - a) \hat{C}_t^* \quad (5.2.5)$$

$$\hat{Y}_{F,t} = -2a(1 - a) \theta \hat{T}_t + (1 - a) \hat{C}_t + a \hat{C}_t^* \quad (5.2.6)$$

$$\pi_{H,t} = k_Y^H \left( \hat{Y}_{H,t} - \tilde{Y}_{H,t}^{fb} \right) - 2a(1 - a) k_T^H \left( \hat{T}_t - \tilde{T}_t^{fb} \right) + k_\mu^H \hat{\mu}_t^H + \beta E_t \pi_{H,t+1} \quad (5.2.7)$$

$$\pi_{F,t}^* = k_Y^F \left( \hat{Y}_{F,t} - \tilde{Y}_{F,t}^{fb} \right) + 2a(1 - a) k_T^F \left( \hat{T}_t - \tilde{T}_t^{fb} \right) + k_\mu^F \hat{\mu}_t^F + \beta E_t \pi_{F,t+1}^* \quad (5.2.8)$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t}^* - \pi_{H,t} + \Delta \hat{S}_t \quad (5.2.9)$$

$$\pi_t = a \pi_{H,t} + (1 - a) (\pi_{F,t}^* + \Delta \hat{S}_t) \quad (5.2.10)$$

$$\pi_t^* = (1 - a) (\pi_{H,t} - \Delta \hat{S}_t) + a \pi_{F,t}^* \quad (5.2.11)$$

Following standard Taylor-type interest rate rules, monetary policy in each country responds to output and some measure of inflation. If monetary policy responds to producer price inflation, the interest rate rules for each country are given by

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R) (\phi_\pi \pi_{H,t} + \phi_Y \hat{Y}_{H,t}) \quad (5.2.12)$$

$$\hat{R}_t^* = \phi_R \hat{R}_{t-1}^* + (1 - \phi_R) (\phi_\pi \pi_{F,t}^* + \phi_Y \hat{Y}_{F,t}). \quad (5.2.13)$$

If monetary policy responds to consumer price inflation, the interest rate rules are given by

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R) (\phi_\pi \pi_t + \phi_Y \hat{Y}_{H,t}) \quad (5.2.14)$$

$$\hat{R}_t^* = \phi_R \hat{R}_{t-1}^* + (1 - \phi_R) (\phi_\pi \pi_t^* + \phi_Y \hat{Y}_{F,t}). \quad (5.2.15)$$



**Sticky-price equilibrium under the MU regime**

Under sticky prices, the model equations for the MU regime are given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}_t^{MU} - E_t \pi_{t+1} + E_t \hat{\zeta}_{C,t+1} - \hat{\zeta}_{C,t} \right) \quad (5.2.16)$$

$$\hat{Q}_t = \rho \left( \hat{C}_t - \hat{C}_t^* \right) + \left( \hat{\zeta}_{C,t}^* - \hat{\zeta}_{C,t} \right) \quad (5.2.17)$$

$$\hat{Q}_t = (2a - 1) \hat{T}_t \quad (5.2.18)$$

$$\hat{Y}_{H,t} = 2a(1 - a) \theta \hat{T}_t + a \hat{C}_t + (1 - a) \hat{C}_t^* \quad (5.2.19)$$

$$\hat{Y}_{F,t} = -2a(1 - a) \theta \hat{T}_t + (1 - a) \hat{C}_t + a \hat{C}_t^* \quad (5.2.20)$$

$$\pi_{H,t} = k_Y^H \left( \hat{Y}_{H,t} - \tilde{Y}_{H,t}^{fb} \right) - 2a(1 - a) k_T^H \left( \hat{T}_t - \tilde{T}_t^{fb} \right) + k_\mu^H \hat{\mu}_t^H + \beta E_t \pi_{H,t+1} \quad (5.2.21)$$

$$\pi_{F,t}^* = k_Y^F \left( \hat{Y}_{F,t} - \tilde{Y}_{F,t}^{fb} \right) + 2a(1 - a) k_T^F \left( \hat{T}_t - \tilde{T}_t^{fb} \right) + k_\mu^F \hat{\mu}_t^F + \beta E_t \pi_{F,t+1}^* \quad (5.2.22)$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t}^* - \pi_{H,t} \quad (5.2.23)$$

$$\pi_t = a \pi_{H,t} + (1 - a) \pi_{F,t}^* \quad (5.2.24)$$

$$\pi_t^* = (1 - a) \pi_{H,t} + a \pi_{F,t}^*. \quad (5.2.25)$$

The common monetary policy responds to union-wide inflation (average country-specific inflation) and union-wide output (average country-specific output). However, whether the common monetary policy responds to producer price inflation or consumer price inflation does not make a difference. Using equations (5.2.24) and (5.2.25), it is straightforward to show that the average of consumer price inflation rates is equal to the average of producer price inflation rates:

$$\frac{\pi_t + \pi_t^*}{2} = \frac{\pi_{H,t} + \pi_{F,t}^*}{2} \equiv \pi_t^{MU}. \quad (5.2.26)$$

Accordingly, the interest rate rule of the common monetary policy can be written in either case as

$$\hat{R}_t^{MU} = \phi_R \hat{R}_{t-1}^{MU} + (1 - \phi_R) \left( \phi_\pi \pi_t^{MU} + \phi_Y \frac{\hat{Y}_{H,t} + \hat{Y}_{F,t}}{2} \right). \quad (5.2.27)$$

**Efficient allocation under both regimes**

The following equations describe the first-best (*fb*) or efficient allocation, where prices are fully flexible and markups are neutralized at all times with an appro-

private subsidy ( $\mu_t^i = 0$ ). This efficient allocation provides a useful benchmark for assessing the welfare implications of the two international monetary regimes.

The efficient output in each country is given by

$$\begin{aligned}
 (\rho + \eta)\tilde{Y}_{H,t}^{fb} = & 2a(1-a)(\rho\theta - 1)\tilde{T}_t^{fb} \\
 & - (1-a)(\hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^*) + \hat{\zeta}_{C,t} + \eta\hat{\zeta}_{Y,t}
 \end{aligned} \tag{5.2.28}$$

$$\begin{aligned}
 (\rho + \eta)\tilde{Y}_{F,t}^{fb} = & -2a(1-a)(\rho\theta - 1)\tilde{T}_t^{fb} \\
 & + (1-a)(\hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^*) + \hat{\zeta}_{C,t}^* + \eta\hat{\zeta}_{Y,t}^*.
 \end{aligned} \tag{5.2.29}$$

The efficient terms of trade can be written as

$$[4a(1-a)\rho\theta + (2a-1)^2]\tilde{T}_t^{fb} = \rho(\tilde{Y}_{H,t}^{fb} - \tilde{Y}_{F,t}^{fb}) - (2a-1)(\hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^*). \tag{5.2.30}$$

### 5.2.2. Model description

Consumption is described by standard Euler equations, which are given by equations (5.2.1) and (5.2.16) in the case of country  $H$ . The difference between these two Euler equations is that the nominal interest rate is country-specific under the FX regime, whereas it is common to both countries under the MU regime. The risk sharing condition, which describes the link between consumption across countries, is identical across regimes and it is given by (5.2.2) and (5.2.17), respectively. Purchasing power parity does not hold, i.e., the real exchange rate is not constant, unless consumption and consumption preference shocks are perfectly correlated across countries. Under the FX regime, perfect risk sharing implies that the uncovered interest parity (5.2.3) holds, i.e., the expected change in the nominal exchange rate corresponds to the interest rate differential across countries.<sup>6</sup> This equation is obsolete under the MU regime because both countries share the same currency and a common nominal interest rate.

The link between the real exchange rate and the terms of trade is described by equations (5.2.4) and (5.2.18), respectively. Accordingly, the correlation

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<sup>6</sup>Combining the Euler consumption equation for country  $H$ , the risk sharing condition, and the uncovered interest parity condition yields the Euler consumption equation for country  $F$ , which is therefore redundant. Alternatively, the model can be specified by including both country-specific Euler consumption equations and the risk sharing condition, while omitting the uncovered interest parity condition. The model specification I employ includes the uncovered interest parity condition because it more clearly illustrates the economic mechanisms that underlie the results of this study.

between the real exchange rate and the terms of trade can be positive, zero, or negative, depending on the degree of trade openness between the two countries. Aggregate demand in each country depends on consumption in both countries and the terms of trade and is given by equations (5.2.5), (5.2.6), (5.2.19), and (5.2.20), respectively. The country-specific New Keynesian Phillips curves are also identical across regimes and they are given by (5.2.7), (5.2.8), (5.2.21), and (5.2.22), respectively. In contrast to a closed-economy framework, not only the output gap but also the terms-of-trade gap (the difference between the sticky-price and the efficient terms of trade) affect producer price inflation.

The terms-of-trade identity is given by equation (5.2.9) under the FX regime and by equation (5.2.23) under the MU regime, which differ due to the presence of the nominal exchange rate in the former. Equations (5.2.10), (5.2.11), (5.2.24), and (5.2.25) describe the relationship between the consumer price inflation rate and the producer price inflation rates in each country. Likewise, these equations only differ across regimes in terms of the presence of the nominal exchange rate.

Monetary policy is conducted via Taylor-type interest rate rules, according to which it responds to output and some measure of inflation. Under the FX regime, the interest rate rules for each country are given by equations (5.2.12) and (5.2.13) if monetary policy responds to producer price inflation, and by equations (5.2.14) and (5.2.15) if monetary policy responds to consumer price inflation. Under the MU regime, whether the common monetary policy responds to producer price inflation or consumer price inflation does not make a difference. In both cases, the interest rate rule is given by equation (5.2.27).<sup>7</sup>

Under flexible prices, monetary policy is neutral and real variables are driven only by productivity shocks and consumption preference shocks. Thus, the efficient allocation, which is given by equations (5.2.28) through (5.2.30), is the same under both international monetary regimes.

### 5.2.3. Welfare loss function

The welfare analysis follows the logic of the familiar linear-quadratic approach, where log-linear model equations are used to evaluate a quadratic welfare loss measure (Woodford, 2003). The world welfare loss function is given by the

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<sup>7</sup>Throughout this study, I only consider interest rate rules with a contemporaneous response to output and inflation. However, forward-looking and backward-looking interest rate rules do not change the main conclusions of this study.

discounted value of a weighted average across countries of the average utility flow of agents using a second-order Taylor series expansion.<sup>8</sup> It is assumed that the distortion induced by monopolistic competition is offset by an appropriate subsidy, thereby ensuring efficiency in the steady state. Thus,

$$\begin{aligned}
 W_t = & -\frac{1}{2} \left( (\rho + \eta) \text{var}(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^{fb}) + (\rho + \eta) \text{var}(\hat{Y}_{F,t} - \tilde{Y}_{F,t}^{fb}) \right. \\
 & - 2a(1-a) \frac{\rho\theta - 1}{\rho} \left[ 4a(1-a)\rho\theta + (2a-1)^2 \right] \text{var}(\hat{T}_t - \tilde{T}_t^{fb}) \\
 & + \sigma(1 + \sigma\eta) \frac{\alpha^H}{(1 - \alpha^H)(1 - \alpha^H\beta)} \text{var} \pi_{H,t} \\
 & \left. + \sigma(1 + \sigma\eta) \frac{\alpha^F}{(1 - \alpha^F)(1 - \alpha^F\beta)} \text{var} \pi_{F,t}^* \right) \\
 & + t.i.p. + O(\|\xi\|^3). \tag{5.2.31}
 \end{aligned}$$

As in the closed economy, the welfare loss depends on the producer price inflation rate and the output gap. In the open economy, the welfare loss also depends on the terms-of-trade gap. Intuitively, when the terms of trade deviate from their efficient level, the resulting allocation of production across countries is inefficient due to the presence of price stickiness. The weights in front of each component of the welfare loss function are functions of the deep parameters of the model. The term *t.i.p.* contains all the terms that are independent of monetary policy and the international monetary regime. The term  $O(\|\xi\|^3)$  contains third and higher order terms, which can be neglected provided that the model equations are log-linear, i.e., first-order approximations of the non-linear equilibrium conditions.

#### 5.2.4. Calibration

Unless stated otherwise, the parameters of the model are calibrated to the values displayed in Table 5.3 (see also Benigno, 2004). For the sake of simplicity, the two countries are assumed to be symmetric. A value of 0.99 for the discount factor  $\beta$  implies a steady-state real interest rate of around 4.1 percent annually. A value of 7.66 for the elasticity of substitution between differentiated goods  $\sigma$  implies a steady-state markup of prices over marginal costs of 15 percent. A

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<sup>8</sup>Computing country-specific welfare would complicate the calculations significantly because more accurate approximations of the non-linear model equations would be necessary (Benigno and Woodford, 2005), which is beyond the scope of this study.

value of 0.75 for the probability of not being able to reset the price  $\alpha^i$  implies an average duration of price contracts of 4 quarters.

**Table 5.3.:** *Baseline calibration*

$\rho$	1/6	Inverse of elasticity of intertemporal substitution in consumption
$\beta$	0.99	Discount factor
$\eta$	0.67	Inverse of elasticity of producing the differentiated good
$\epsilon_{wy}$	0.5	Production elasticity of average real wage
$\gamma$	0.75	Labor income share
$a$	0.75	Home bias/degree of trade openness
$\sigma$	7.66	Elasticity of substitution between differentiated goods within countries
$\theta$	2	Elasticity of substitution between goods across countries
$\alpha^i$	0.75	Probability of not being able to reset the price in country $i = H, F$
$\phi_\pi$	1.5	Inflation coefficient in interest rate rule
$\phi_Y$	0	Output coefficient in interest rate rule
$\phi_R$	0	Interest rate smoothing coefficient in interest rate rule
$\rho_i$	0.9	Persistence of productivity shock in country $i = H, F$

The degree of trade openness  $a$  is calibrated to 0.75, which corresponds to a steady-state share of home-produced goods in the consumption basket of 75 percent in each country (i.e., a home bias in consumption) and a steady-state trade-to-GDP ratio of 50 percent.<sup>9</sup> This roughly equals the average trade-to-GDP ratio across OECD countries. Following Rotemberg and Woodford (1998) and Benigno (2004), the inverse of the elasticity of producing the differentiated good  $\eta$  is calculated as

$$\eta = \epsilon_{wy} - \rho + \frac{1 - \gamma}{\gamma}, \quad (5.2.32)$$

where  $\epsilon_{wy}$  denotes the elasticity of the average real wage with respect to production and  $\gamma$  denotes the labor income share.

Under the baseline calibration, monetary policy responds to inflation ( $\phi_\pi = 1.5$ ), but it does not respond to output ( $\phi_Y = 0$ ) and does not engage in interest rate smoothing ( $\phi_R = 0$ ). All interest rate rule coefficients are assumed to be identical across countries and regimes. Finally, the persistence of the productivity shock is 0.9 in each country and the cross-country correlation is zero. I abstract from consumption preference shocks and cost-push shocks because they do not change the main conclusions of this study.

<sup>9</sup>The trade-to-GDP ratio expressed in percent is given by  $2(1 - a) \times 100$ .

### 5.3. Monetary policy as an OCA criterion

In the following, I use the theoretical model described in the previous section to show that the conduct of monetary policy is a critical criterion for the welfare implications of the MU regime relative to the FX regime. The conduct of monetary policy is modified in two dimensions: (1) the coefficients in the interest rate rules that determine the response of monetary policy to inflation (Section 5.3.1), to output (Section 5.3.2), and to past realizations of the interest rate (Section 5.3.3); and (2) the measure of inflation to which monetary policy responds, i.e., producer price inflation (henceforth PPI inflation targeting) or consumer price inflation (henceforth CPI inflation targeting).

#### 5.3.1. Response to inflation

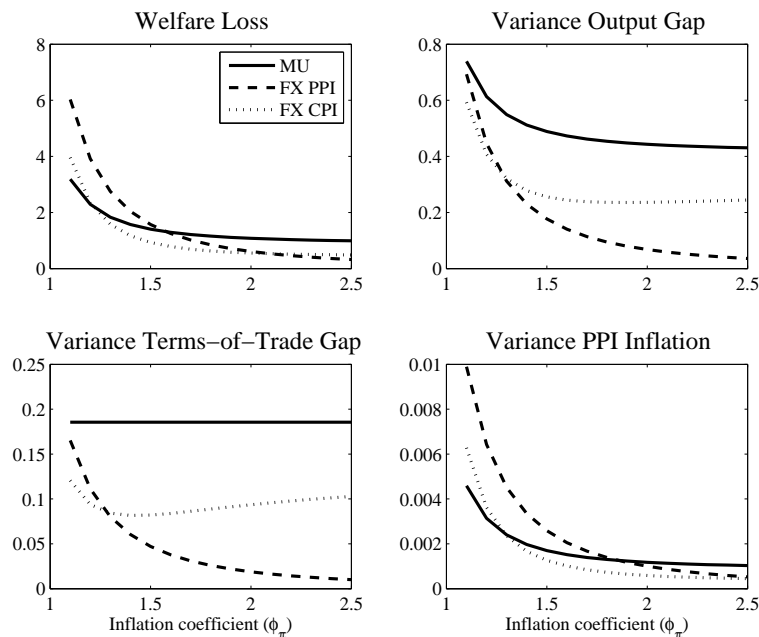
The aggressiveness of monetary policy in its response to inflation is an important criterion that determines whether countries are better off under the MU regime or under the FX regime (Figure 5.1, upper left panel). If the response to inflation is relatively modest (i.e., low values of  $\phi_\pi$ ), the two countries are better off under the MU regime. If the response to inflation is relatively strong, the two countries are better off under the FX regime. The threshold value of  $\phi_\pi$  beyond which the FX regime becomes superior in terms of welfare depends on whether monetary policy targets PPI inflation or CPI inflation.<sup>10</sup> Under CPI inflation targeting, the threshold value for  $\phi_\pi$  is lower than that under PPI inflation targeting, i.e., the two countries are less likely to be better off under the MU regime.

The welfare ranking between the MU and the FX regime is driven almost exclusively by the variance of PPI inflation, which exhibits the same pattern with respect to  $\phi_\pi$  as the welfare loss (Figure 5.1, lower right panel).<sup>11</sup> This is because agents attach by far the highest weight to inflation, which is traditionally the case in microfounded welfare measures derived from New Keynesian models. Accordingly, the cost of a higher variance of the output gap and of the terms-of-trade gap under the MU regime (Figure 5.1, upper right and lower left panel) may be outweighed by the benefit of a lower variance of PPI infla-

<sup>10</sup>Under the MU regime, PPI inflation targeting and CPI inflation targeting are the same policy (recall equation 5.2.27).

<sup>11</sup>Although the welfare loss depends on the output gap and the PPI inflation rate of both countries, Figure 5.1 shows only one of each because the variances are identical in both countries due to the assumption of symmetric countries.

**Figure 5.1.:** *World welfare loss and variances of welfare-relevant variables as a function of the inflation coefficient ( $\phi_\pi$ )*



tion. This is the case for low values of  $\phi_\pi$ , i.e., a relatively modest response of monetary policy to inflation.<sup>12</sup>

The two countries are better off under the FX regime for a sufficiently strong response of monetary policy to inflation (either PPI or CPI inflation) because monetary policy is more potent under the FX regime than under the MU regime. This becomes clear if the number of policy instruments is compared to the number of welfare-relevant distortions in the economy. Under the FX regime, there are as many policy instruments as distortions in the economy. These distortions are due to monopolistic competition and to sticky prices.<sup>13</sup>

The distortion due to monopolistic competition induces an inefficiently low level of aggregate output. This distortion can be eliminated by an appropriate subsidy in each country. The distortion due to sticky prices induces inefficient markup fluctuations, which lead to inefficiently low or high levels of aggregate output, and an inefficient dispersion of prices in the presence of inflation, which causes an inefficient dispersion of output across the producers of differentiated goods *within* each country. This distortion can be mitigated or even eliminated

<sup>12</sup>A brief explanation of why PPI inflation may be more stable under the MU regime is given in Section 5.3.3. A detailed explanation is given in Groll (2013).

<sup>13</sup>Both distortions are common to the closed-economy framework (see, e.g., Woodford, 2003, for details).

by monetary policy in each country by using the nominal interest rate to reduce the fluctuations of inflation around zero as far as possible.

By contrast, under the MU regime, there are less policy instruments than distortions. First, monetary policy sets the nominal interest rate for both countries and thus it can no longer target inflation in each country separately, thereby losing one policy instrument. Second, in addition to the two distortions described above, the *de facto* fixed nominal exchange rate combined with sticky prices induces an additional distortion into the economy, namely an intrinsic inertia in the terms of trade (Benigno, 2004; Pappa, 2004).<sup>14</sup> This causes an inefficient dispersion of aggregate output *across* the two countries.

Given that there are as many policy instruments as distortions under the FX regime but less policy instruments than distortions under the MU regime, monetary policy is more potent under the FX regime, which is shown clearly in Figure 5.1. The “leverage” of monetary policy is higher under the FX regime than under the MU regime in the sense that a given increase in the aggressiveness of monetary policy towards inflation (measured by  $\phi_\pi$ ) leads to a larger reduction in the variance of each welfare-relevant variable. In fact, under PPI inflation targeting, monetary policy can reduce the variances of all welfare-relevant variables to zero (if  $\phi_\pi \rightarrow \infty$ ). This is impossible under the MU regime.<sup>15</sup>

The limitations of monetary policy under the MU regime apply in particular with regard to the terms-of-trade gap (Figure 5.1, lower left panel). Monetary policy has no effect whatsoever on the terms of trade and thus on the terms-of-trade gap. Since both countries face the same nominal interest rate, any interest rate adjustment by the common monetary policy has the same initial effect on both countries. If the degree of price stickiness is identical across the two countries, such an interest rate adjustment propagates through both economies in exactly the same way. In these circumstances, the influence of monetary policy on the terms of trade is zero. If the degree of price stickiness is not identical across the two countries, the influence of monetary policy on the terms of trade is not exactly zero, but it is close to zero.

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<sup>14</sup>Intrinsic inertia is defined as follows: Consider a one-off (i.e., non-persistent) productivity shock in one country. Under the MU regime, several periods are required before the terms of trade return to the steady state after the shock has vanished. Thus, the terms of trade are said to be intrinsically persistent or inertial. Under the FX regime, the terms of trade return to the steady state immediately after the shock has vanished. In this case, the terms of trade are not intrinsically inertial.

<sup>15</sup>See Groll (2013) for the analytical proof of the special case where  $a = 1/2$  and  $\theta = 1$ . The proof in the case with no restrictions on  $a$  and  $\theta$  is completely analogous.

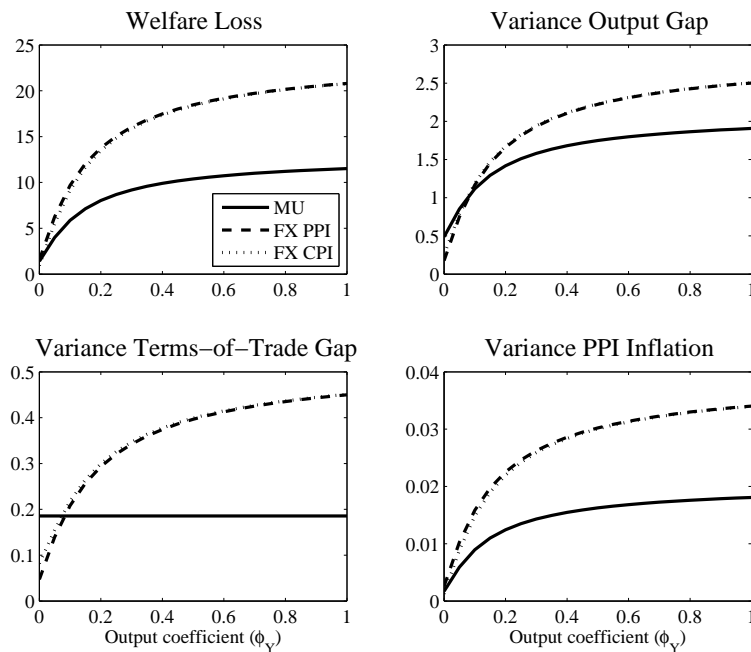


### 5.3.2. Response to output

As shown above, monetary policy is more potent under the FX regime than under the MU regime due to the flexibility of the nominal exchange rate. However, there is a flipside to this greater potency, which becomes clear when considering the case where monetary policy responds to output in addition to inflation.<sup>16</sup>

If monetary policy responds to output in addition to inflation, the two countries are generally better off under the MU regime (Figure 5.2, upper left panel). Moreover, the welfare gain of the MU regime relative to the FX regime increases with the strength of the response to output (measured by  $\phi_Y$ ). Beyond a certain value for  $\phi_Y$ , all welfare-relevant variables are more stable under the MU regime (Figure 5.2, upper right, lower left, and lower right panel). All of this holds irrespective of whether monetary policy targets PPI or CPI inflation rates. Under the FX regime, these two policies are virtually identical (in the figure, the ‘FX PPI’ lines and the ‘FX CPI’ lines basically coincide). Under the MU regime, they are identical anyway.

**Figure 5.2.:** *World welfare loss and variances of welfare-relevant variables as a function of the output coefficient ( $\phi_Y$ )*



The key to understanding these results is the role played by the nominal exchange rate in stabilizing the terms-of-trade gap. Combining equations (5.2.28)

<sup>16</sup>The basic conclusions of this section also hold if monetary policy responds to the *change* in output instead. The corresponding graphs are available upon request.

to (5.2.30) and focusing on productivity shocks in country  $H$  yields the following relationship between the efficient terms of trade  $\tilde{T}_t^{fb}$  and the productivity shock  $\hat{\zeta}_{Y,t}$ :

$$\tilde{T}_t^{fb} = \frac{\rho\eta}{4a(1-a)\rho(1+\eta\theta) + (\rho+\eta)(2a-1)^2} \hat{\zeta}_{Y,t}. \quad (5.3.1)$$

The term in front of the productivity shock is unambiguously positive. Accordingly, the terms of trade would increase in response to a positive productivity shock in country  $H$  if prices were perfectly flexible. However, because prices are sticky, the increase in the terms of trade is actually smaller. In these circumstances, an increase in the nominal exchange rate would help to close the gap between the actual response of the terms of trade and its efficient counterpart, thereby stabilizing the terms-of-trade gap and reducing the welfare loss. But whether the nominal exchange rate stabilizes or destabilizes the terms-of-trade gap depends crucially on the conduct of monetary policy and, in particular, on whether and how strongly monetary policy responds to output. This is because the nominal exchange rate is linked directly via the uncovered interest parity condition (5.2.3) to the interest rates governed by monetary policy in each country.<sup>17</sup>

If monetary policy does not respond to output ( $\phi_Y = 0$ ), the impact response of the nominal exchange rate to a positive productivity shock in country  $H$  is positive, i.e., country  $H$ 's currency depreciates (Figure 5.3, dashed line).<sup>18</sup> Thus, the nominal exchange rate pushes the sticky-price terms of trade in the same direction as the efficient terms of trade, thereby stabilizing the terms-of-trade gap to some extent. This holds under both PPI and CPI inflation targeting.<sup>19</sup>

However, as monetary policy starts to respond to output and becomes more aggressive towards output (i.e.,  $\phi_Y$  becomes positive and increases), the impact response of the nominal exchange rate becomes smaller and it is already negative for very small values of  $\phi_Y$  (Figure 5.3, solid and dotted line).<sup>20</sup> A negative

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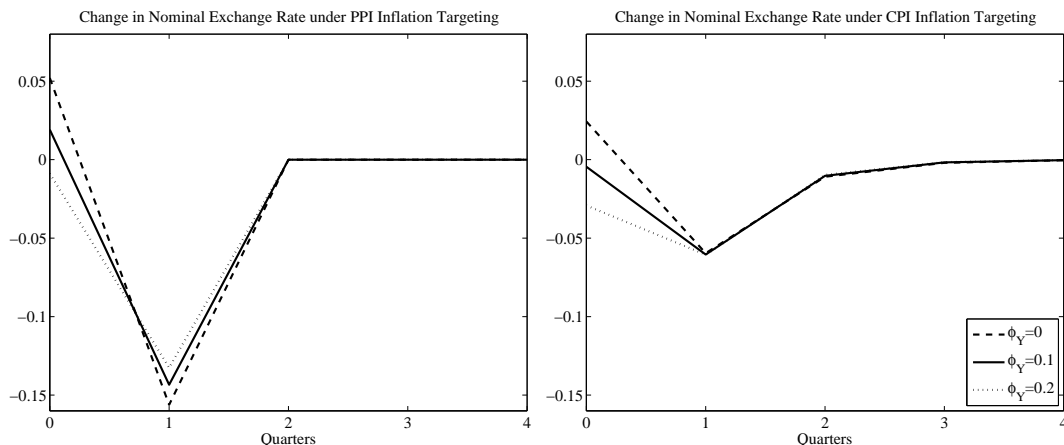
<sup>17</sup>Importantly, this link also exists when asset markets are incomplete. In this case, the uncovered interest parity condition merely contains an additional term that describes the net asset positions of countries.

<sup>18</sup>The degree of price stickiness was set low ( $\alpha = 0.2$ ) to ensure that the differences in the impulse responses are clearly visible. The differences for higher degrees of price stickiness are smaller, but they are qualitatively the same.

<sup>19</sup>Under CPI inflation targeting, this is sensitive to the degree of trade openness  $a$ , which is analyzed in detail in Section 5.4.

<sup>20</sup>The threshold value of  $\phi_Y$  at which the impact response of the nominal exchange rate becomes negative also depends on other model parameters. Under PPI inflation targeting, it is feasible to derive the exact analytical expression, i.e., the impact response is negative if  $\phi_Y > \frac{(\phi_\pi - 1)\rho}{4a(1-a)\rho\theta + (2a-1)^2}$ . This expression is obtained in the following way (for details, see

**Figure 5.3.:** *Impulse response of the change in the nominal exchange rate ( $\Delta\hat{S}_t$ ) to a positive one-off productivity shock in country  $H$  for three different values of the output coefficient ( $\phi_Y$ ), with  $\alpha = 0.2$ . Left panel: PPI inflation targeting. Right panel: CPI inflation targeting.*



impact response means that the nominal exchange rate *destabilizes* the terms-of-trade gap by pushing the sticky-price terms of trade away from the efficient terms of trade. As a result, not only the terms-of-trade gap, but also the output gap and the PPI inflation rate are destabilized, thereby increasing the welfare loss. In these circumstances, a fixed nominal exchange rate would make the countries better off because this is neither destabilizing nor stabilizing.

Importantly, the nominal exchange rate only amplifies a detrimental effect that is already present, but it does not cause the detrimental effect. In a closed economy, a response of monetary policy to output is also detrimental to welfare (see, e.g., Galí, 2008, Ch. 4.4). It is not the deviation of output from the steady state that is welfare-relevant, it is the deviation from the flexible-price counterpart (i.e., the output gap). For example, a positive productivity shock in country  $H$  induces an increase in output, but a decrease in the output gap, because the increase in output is lower than the increase in flexible-price output.

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Groll, 2013, who presents a completely analogous derivation for a special case of the model). Derive the coefficient  $c_3$  of the recursive law of motion for the reduced system of equations given by  $\hat{T}_t = b_1\hat{T}_{t-1} + c_1\tilde{T}_t^{fb}$ ,  $\hat{\pi}_{F,t}^* - \hat{\pi}_{H,t} = b_2\hat{T}_{t-1} + c_2\tilde{T}_t^{fb}$ , and  $\Delta\hat{S}_t = b_3\hat{T}_{t-1} + c_3\tilde{T}_t^{fb}$ , using the method of undetermined coefficients. Recall that  $\tilde{T}_t^{fb}$  is a function of the productivity shock  $\hat{\zeta}_{Y,t}$ , see equation (5.3.1). Then, the impact response of the nominal exchange rate to the productivity shock is given by the coefficient  $c_3$ , which is negative if  $\phi_Y$  is larger than the expression given above. This procedure involves the solution of a full quadratic equation, but applying the same procedure under CPI inflation targeting requires the solution of a full cubic equation. This derivation is too cumbersome and its solution is too complex to be useful, which is why I chose a numerical approach.

A welfare-oriented response of monetary policy would require a reduction in the interest rate due to the negative output gap. Instead, monetary policy raises the interest rate due to the rise in output. This is detrimental to welfare.

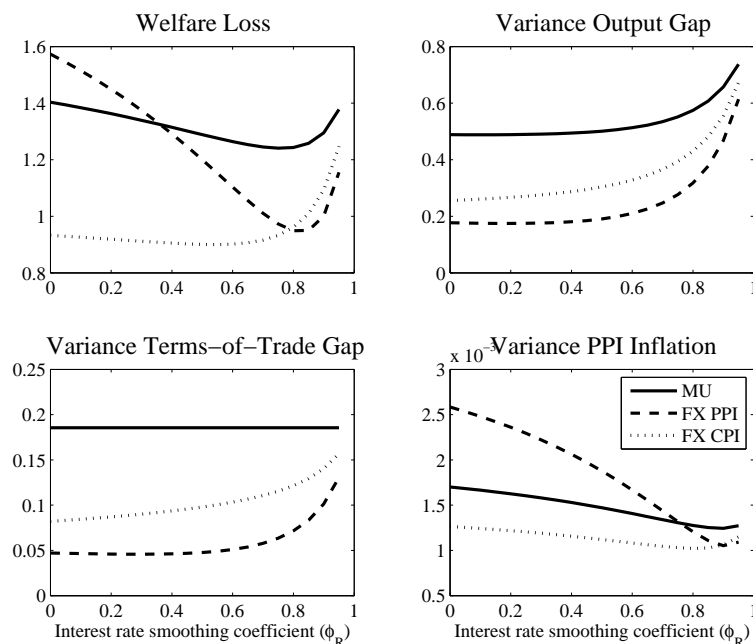
For these reasons, a response of monetary policy to output is detrimental under both the FX regime and the MU regime (in Figure 5.2, the welfare loss increases in  $\phi_Y$  under both regimes). However, and this is of the utmost importance, the detrimental effect is larger under the FX regime due to amplification by the nominal exchange rate. The same “bad” policy is more harmful under the FX regime. As described in the previous section, monetary policy is more potent under the FX regime than under the MU regime in terms of macroeconomic stabilization because of the flexibility of the nominal exchange rate. The flipside of this is that monetary policy can do more harm in terms of macroeconomic stabilization when it is not conducted properly. Essentially, the nominal exchange rate does not compensate for monetary policy mistakes, it reinforces them. In this sense, the MU regime provides a protective mechanism against monetary policy mistakes.

### 5.3.3. Interest rate smoothing

Finally, whether countries are better off under the MU regime or under the FX regime also depends on the degree of interest rate smoothing implemented by monetary policy, which is particularly true under PPI inflation targeting (Figure 5.4, solid and dashed lines). Starting with very low degrees of interest rate smoothing (i.e., low values of  $\phi_R$ ), the two countries are better off under the MU regime. As the degree of interest rate smoothing increases (i.e., an increase in  $\phi_R$ ), the welfare loss decreases faster under the FX regime than under the MU regime. At some point, the welfare ranking changes and the two countries are better off under the FX regime.

As described in Section 5.3.1, the MU regime entails the cost of higher instability of both the output gap and the terms-of-trade gap, but the benefit of higher stability of the PPI inflation rate (except for high values of  $\phi_R$ ). The higher stability of the PPI inflation rate under the MU regime is explained as follows (see Groll, 2013, for details). The nominal exchange rate is fixed under the MU regime. As a result, the terms of trade exhibit an inertial or history-dependent behavior, even if monetary policy does not smooth interest rates. Since the terms of trade are an important determinant of marginal costs, this

**Figure 5.4.:** *World welfare loss and variances of welfare-relevant variables as a function of the interest rate smoothing coefficient ( $\phi_R$ )*



history dependence has the advantage of stabilizing private-sector expectations about future PPI inflation, thereby stabilizing actual PPI inflation.<sup>21</sup>

If monetary policy targets PPI inflation and does not smooth interest rates under the FX regime, there is no such history dependence. This regime suffers from a kind of stabilization bias. As a result, PPI inflation is less stable under the FX regime. However, if monetary policy starts to smooth interest rates, it induces history dependence into the economy, with the same advantageous effect on inflation expectations. This effect strengthens as the degree of interest rate smoothing increases. If the degree of interest rate smoothing is sufficiently high, PPI inflation is more stable under the FX regime.

Under CPI inflation targeting, the degree of interest rate smoothing does not have such an important effect on the welfare ranking between the MU and FX regime (Figure 5.4, solid and dotted lines).<sup>22</sup> This is because, like the MU regime, the FX regime also features history dependence, even if monetary policy does not smooth interest rates. As a result, engaging in interest rate smoothing,

<sup>21</sup>The assumptions in Groll (2013) are somewhat more restrictive ( $a = 1/2$  and  $\theta = 1$ ), but this benefit is also found in the more general model employed in the present study (i.e., no restrictions on  $a$  and  $\theta$ ).

<sup>22</sup>Under the baseline calibration, the welfare ranking does not change with the degree of interest rate smoothing. However, parameter constellations exist where this is the case, e.g., if  $\phi_\pi = 1.2$  instead of 1.5.

thereby inducing greater history dependence into the economy, does not change the relative welfare performance of the FX and MU regimes dramatically.

## 5.4. Monetary policy and trade openness

The conduct of monetary policy is not only an independent OCA criterion in itself, as illustrated in Section 5.3, but it can also modify the nature of other OCA criteria. This is demonstrated in the following using the degree of trade openness as an example. But first, I briefly summarize how the relationship between trade openness and the costs and benefits of a monetary union has been described in previous studies.

### 5.4.1. Trade openness in OCA theory

The degree of trade openness or trade integration is one of the oldest and most prominent OCA criteria. Most studies have established a positive link between trade openness and the likelihood of a monetary union being beneficial. More precisely, the more open economies are, the smaller are the costs and the larger are the benefits associated with monetary unification.

McKinnon (1963) first proposed trade openness as an OCA criterion. He argued that with an increasingly open economy, the effects of exchange rate fluctuations on consumer prices via import prices become greater, thereby making it more difficult for monetary policy to maintain (consumer) price stability. Thus, the costs of giving up monetary independence decrease with the degree of trade openness.

One of the main costs attributed to monetary unification is the loss of the ability to react to asymmetric (i.e., country-specific) shocks via monetary policy and the nominal exchange rate. However, there are conflicting views on whether the incidence of country-specific shocks decreases or increases with the degree of trade openness. This depends on whether trade between countries is characterized predominantly by intra-industry trade or inter-industry trade. In the former case, industry-specific shocks affect countries symmetrically, thus an increase in the degree of trade openness reduces the cost of giving up monetary independence (see Ch. 6.2 in the ‘One Market, One Money’ report by the European Commission in Emerson et al., 1992). In the latter case, industry-specific shocks affect countries asymmetrically, thus an increase in the degree of trade

openness raises the cost of giving up monetary independence (Krugman, 1991, p. 82).<sup>23</sup>

The benefits traditionally associated with monetary unification are usually considered to increase with the degree of trade openness, such as the elimination of transaction costs when exchanging currencies, the increase in price transparency across countries, or the elimination of exchange rate risk (e.g., De Grauwe, 2012, Ch. 3.8). The latter point is also made by Kollmann (2004) using a New Keynesian DSGE model similar to that employed in the present study. He concludes that if the nominal exchange rate is subject to shocks, a monetary union is welfare-improving because the cost of giving up monetary independence is overcompensated by the benefit of eliminating exchange rate risk. This benefit increases with the degree of trade openness because exchange rate shocks are more harmful to macroeconomic stability as economies become more open.

Another benefit of a monetary union also increases with the degree of trade openness, as shown by Pappa (2004) using a similar model. Compared to a flexible exchange rate regime where the monetary authorities do not cooperate to maximize domestic welfare, forming a monetary union eliminates the possibility of strategic terms-of-trade manipulations. This benefit increases with the degree of trade openness because terms-of-trade movements have larger effects on macroeconomic stability as economies become more open.

#### **5.4.2. Monetary policy and the nature of trade openness as an OCA criterion**

The preceding, brief overview shows that OCA theory mainly establishes a favorable relationship between the degree of trade openness and the costs and benefits of a monetary union. Next, I show that this relationship is highly sensitive to the way monetary policy is conducted.

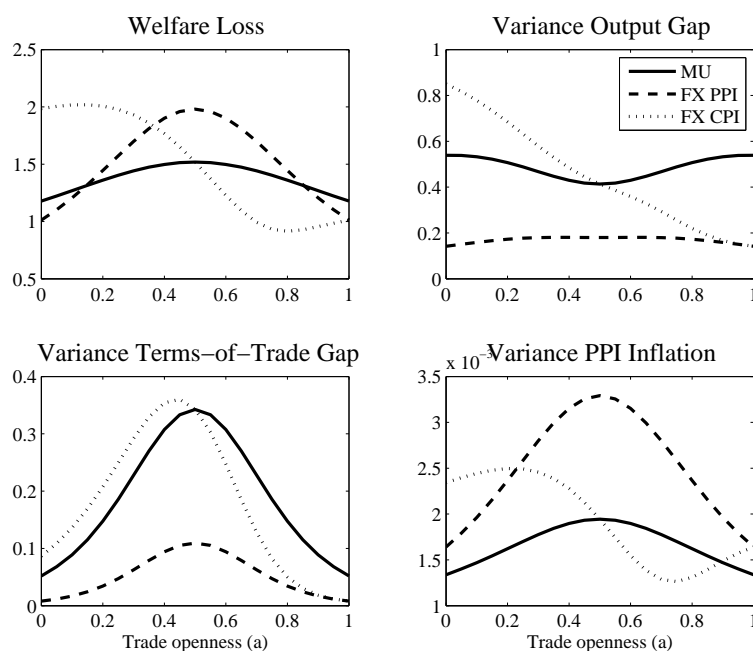
The influence of monetary policy on the nature of the degree of trade openness as an OCA criterion is particularly clear when distinguishing between PPI inflation targeting and CPI inflation targeting. First consider the case of PPI inflation targeting (Figure 5.5, solid and dashed lines). Two observations are noteworthy. First, under both the MU and the FX regime, the relationship between the welfare loss (and each of its components) and the degree of trade

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<sup>23</sup>See De Grauwe (2012, Ch. 2.1) for a more detailed description and assessment of the “European Commission view” and the “Krugman view.”

openness is symmetric around a trade-to-GDP ratio of 100 percent ( $a = 1/2$ ). Second, the two countries are better off under the FX regime if they are either relatively closed ( $a$  close to one) or very open to trade ( $a$  close to zero), but better off under the MU regime for intermediate values. Thus, the likelihood of the MU regime being beneficial increases initially and then decreases with the degree of trade openness.

**Figure 5.5.:** *World welfare loss and variances of welfare-relevant variables as a function of the degree of trade openness ( $a$ )*



The relationship between trade openness and the welfare ranking between the MU and FX regime changes considerably if monetary policy targets CPI inflation rates instead of PPI inflation rates (Figure 5.5, solid and dotted lines). First, the relationship between the welfare loss (and each of its components) and the degree of trade openness is no longer symmetric under the FX regime.<sup>24</sup> Second, the two countries are better off under the FX regime for trade-to-GDP ratios between 0 and 100 percent ( $1/2 < a \leq 1$ ) and better off under the MU regime for ratios between 100 and 200 percent ( $0 \leq a < 1/2$ ). Thus, the likelihood of the MU regime being beneficial increases with the degree of trade openness.

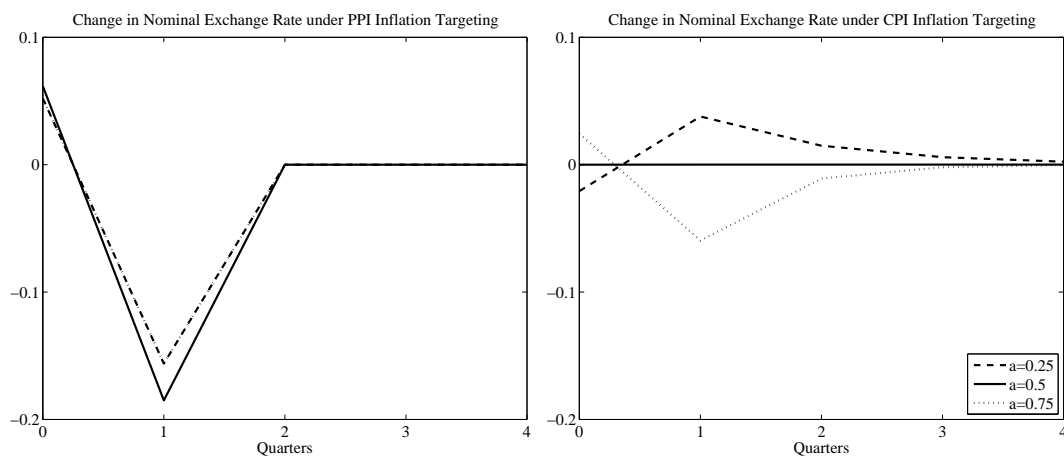
<sup>24</sup>Recall that there is no difference between PPI and CPI inflation targeting under the MU regime.



Again, the key to understanding these results is the role played by the nominal exchange rate in stabilizing the terms-of-trade gap. Consider a positive productivity shock in country  $H$ . Recall that the efficient terms of trade unambiguously increase on impact (see equation 5.3.1), thus an increase in the nominal exchange rate would help to stabilize the terms-of-trade gap, thereby reducing the welfare loss. But whether the nominal exchange rate stabilizes or destabilizes the terms-of-trade gap depends crucially on the conduct of monetary policy and, in particular, on whether monetary policy targets PPI or CPI inflation rates.

Under PPI inflation targeting, the impact response of the nominal exchange rate is positive irrespective of the degree of trade openness, i.e., country  $H$ 's currency depreciates (Figure 5.6, left panel).<sup>25</sup> Thus, the nominal exchange rate pushes the sticky-price terms of trade in the same direction as the efficient terms of trade, thereby stabilizing the terms-of-trade gap to some extent. Note that the response of the nominal exchange rate is identical for  $a = 0.25$  and  $a = 0.75$ , which explains the symmetric patterns visible in Figure 5.5.

**Figure 5.6.:** *Impulse response of the change in the nominal exchange rate ( $\Delta\hat{S}_t$ ) to a positive one-off productivity shock in country  $H$  for three different degrees of trade openness ( $a$ ), with  $\alpha = 0.2$ . Left panel: PPI inflation targeting. Right panel: CPI inflation targeting.*



By contrast, under CPI inflation targeting, the impact response of the nominal exchange rate is positive if the two countries have a trade-to-GDP ratio

<sup>25</sup>The degree of price stickiness was set low ( $\alpha = 0.2$ ) to ensure that the differences in the impulse responses are clearly visible. The differences are smaller for higher degrees of price stickiness, but they are qualitatively the same.

below 100 percent ( $a > 1/2$ ), but negative if it is above 100 percent ( $a < 1/2$ ) (Figure 5.6, right panel). Thus, the nominal exchange rate helps to stabilize the terms-of-trade gap only in the first case. In the latter case, the nominal exchange rate actually destabilizes the terms-of-trade gap by pushing the sticky-price terms of trade away from the efficient terms of trade.

This is robust with respect to the other deep parameters of the economy (except with respect to the interest rate rule coefficients, see Section 5.3).<sup>26</sup> To see this, insert the country-specific interest rate rules (5.2.14) and (5.2.15) with  $\phi_Y = 0$  and  $\phi_R = 0$  together with the definitions of the CPI inflation rates (5.2.10) and (5.2.11) and the terms-of-trade identity (5.2.9) into the uncovered interest parity condition (5.2.3) to obtain

$$\Delta \hat{S}_t = (2a - 1) \Delta \hat{T}_t + \frac{1}{\phi_\pi} E_t \Delta \hat{S}_{t+1}. \quad (5.4.1)$$

Solving forward yields

$$\Delta \hat{S}_t = (2a - 1) E_t \sum_{k=0}^{\infty} \left( \frac{1}{\phi_\pi} \right)^k \Delta \hat{T}_{t+k}. \quad (5.4.2)$$

Accordingly, the current change in the nominal exchange rate depends on the discounted sum of current and expected future changes in the terms of trade. Importantly, this relationship is unambiguously positive if  $a > 1/2$ , but negative if  $a < 1/2$ .

Under PPI inflation targeting, the analogous equations are given by

$$\Delta \hat{S}_t = \Delta \hat{T}_t + \frac{1}{\phi_\pi} E_t \Delta \hat{S}_{t+1} \quad (5.4.3)$$

and

$$\Delta \hat{S}_t = E_t \sum_{k=0}^{\infty} \left( \frac{1}{\phi_\pi} \right)^k \Delta \hat{T}_{t+k}. \quad (5.4.4)$$

In contrast to the PPI inflation targeting case, the relationship between the current change in the nominal exchange rate and the discounted sum of current

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<sup>26</sup>In particular, it does not make a difference whether  $\rho\theta$  is smaller than, equal to, or larger than 1, although this condition has important macroeconomic implications. For example, it determines whether the cross-country correlation of output is positive, zero, or negative (see, e.g., Corsetti, Dedola, and Leduc, 2011, for details). Also, if it is zero ( $\rho\theta = 1$ ), the terms-of-trade gap vanishes from the welfare loss function (5.2.31).

and expected future changes in the terms of trade is always positive, regardless of the degree of trade openness  $a$ .<sup>27</sup>

The nominal exchange rate can be destabilizing under CPI inflation targeting for the following reason. If the trade-to-GDP ratio is above 100 percent ( $a < 1/2$ ), consumer prices in one country are determined mainly by producer prices in the other country because consumers consume more imported goods than home-produced goods. If monetary policy targets consumer prices, interest rate adjustments in one country are triggered mainly by producer price changes in the other country. This pushes the nominal exchange rate, which depends on the interest rate differential between the two countries, in the wrong direction, i.e., away from the efficient terms of trade. As a result, the welfare-relevant terms-of-trade gap is destabilized by the nominal exchange rate. In these circumstances, a fixed nominal exchange rate would make the countries better off because this is neither destabilizing nor stabilizing. For this reason, the countries are better off under the MU regime for  $a < 1/2$ .

In the special case of a trade-to-GDP ratio of exactly 100 percent ( $a = 1/2$ ), the two countries are indifferent in terms of welfare between the FX and the MU regime under CPI inflation targeting. Ultimately, this is because the nominal exchange rate is constant under both regimes.<sup>28</sup> Under the MU regime, the nominal exchange rate is fixed by construction. Under the FX regime, it is fixed by coincidence. That is, by targeting CPI inflation rates, the two countries unintentionally implement a symmetric fixed exchange rate regime.<sup>29</sup> This is because consumer price changes and thus interest rate adjustments are identical in the two countries.

As mentioned above, the nominal exchange rate stabilizes the terms-of-trade gap under PPI inflation targeting regardless of the degree of trade openness. Hence, one would expect that this would always make the FX regime superior to the MU regime, where the nominal exchange rate is fixed. However, as is shown in Figure 5.5, higher instability of the terms-of-trade gap (and the output gap) may be outweighed by higher stability of the PPI inflation rate. The reason why the PPI inflation rate may be more stable under the MU regime was explained briefly in Section 5.3.3 and is explained in detail in Groll (2013). In short,

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<sup>27</sup>Note that equation (5.4.2) and equation (5.4.4) are equivalent if  $a = 1$ . In this case, there is no difference between PPI and CPI inflation targeting. This is because the consumer price index equals the producer price index if  $a = 1$ , see equations (5.2.10) and (5.2.11).

<sup>28</sup>According to equation (5.4.2),  $\Delta \hat{S}_t = 0$  if  $a = 1/2$ .

<sup>29</sup>Note that an *asymmetric* fixed exchange rate regime (i.e., one country pegs its currency to the other country's currency) is generally not equivalent to a monetary union regime.

because the nominal exchange rate is fixed under the MU regime, the terms of trade exhibit an inertial or history-dependent behavior. Since the terms of trade are an important determinant of marginal costs, this history dependence has the advantage of stabilizing private-sector expectations about future PPI inflation, thereby stabilizing actual PPI inflation.

## 5.5. Conclusion

The main point of this study is that monetary policy is one of the most important OCA criteria. That is, whether countries benefit from forming a monetary union depends critically on the way monetary policy is conducted. This point has not yet been emphasized sufficiently in previous OCA studies, if at all. In this study, I demonstrated systematically how and why different types of rule-based interest rate policies lead to different welfare rankings between a monetary union and a flexible exchange rate regime.

In general, when countries are unable or unwilling to implement monetary policy optimally, for whatever reason, they are likely to benefit from forming a monetary union. This is especially true when countries tend to make monetary policy mistakes. In all these circumstances, forming a monetary union entails the benefit of higher macroeconomic stability, particularly inflation stability.

This study only presented results based on productivity shocks and contemporaneous interest rate rules, but all of these conclusions are still valid under consumption preference shocks and cost-push shocks, as well as under backward-looking and forward-looking interest rate rules.

## C. Appendix

### C.1. Flexible exchange rate regime

This appendix contains the full derivation of the model under the flexible exchange rate regime (based on Corsetti, Dedola, and Leduc, 2011). The world, which consists of two countries labeled  $H$  and  $F$ , is populated by a continuum of agents on the interval  $[0, 1]$ . The population on the segment  $[0, n)$  lives in country  $H$ , the population on the segment  $[n, 1]$  lives in country  $F$ . Thus,  $n$  measures the population size as a fraction of world population. An agent is both consumer and producer. He produces a single differentiated good and consumes all the goods produced in both countries.

#### Consumer problem

Agent  $j$  in country  $H$  derives positive utility from consumption  $C^j$  and negative utility from producing the differentiated good  $y(h)$ . The present discounted value of lifetime utility  $U^j$  is given by<sup>30</sup>

$$U^j = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \zeta_{C,t} \frac{C_t^{j1-\rho} - 1}{1-\rho} - \zeta_{Y,t}^{-\eta} \frac{y_t(h)^{1+\eta}}{1+\eta} \right]. \quad (\text{C.1})$$

$E$  denotes the expectations operator,  $\beta$  the discount factor,  $\rho$  the inverse of the intertemporal elasticity of substitution in consumption, and  $\eta$  the inverse of the elasticity of producing the differentiated good.<sup>31</sup>  $\zeta_{Y,t}$  and  $\zeta_{C,t}$  denote shocks to productivity and to preferences in consumption, respectively. These shocks are common to all agents living in country  $H$ .

The agent consumes both a bundle of differentiated goods from country  $H$  and from country  $F$  according to the following Constant-Elasticity-of-Substitution (CES) aggregator:

$$C_t^j = \left[ a^{\frac{1}{\theta}} C_{H,t}^j \frac{\theta-1}{\theta} + (1-a)^{\frac{1}{\theta}} C_{F,t}^j \frac{\theta-1}{\theta} \right]^{\frac{\theta}{\theta-1}}, \quad (\text{C.2})$$

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<sup>30</sup>In Corsetti, Dedola, and Leduc (2011), the agent derives utility also from the liquidity services of holding money. I abstract from money in the utility function, since monetary policy is conducted via interest rate rules.

<sup>31</sup>The parameter  $\eta$  is equivalent to the inverse of the Frisch elasticity of labor supply.

where the bundles of differentiated goods are given by aggregators according to Dixit and Stiglitz (1977):

$$\begin{aligned}
 C_{H,t}^j &= \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c_t^j(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \\
 C_{F,t}^j &= \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c_t^j(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}} .
 \end{aligned} \tag{C.3}$$

These preferences imply (1) that the elasticity of substitution between differentiated goods  $c_t^j$  from one country is  $\sigma$ , which is assumed to be greater than one and equal across countries, (2) that the elasticity of substitution between the bundles of goods from the two countries  $C_{H,t}$  and  $C_{F,t}$  is  $\theta$ , which is assumed to be greater than zero and equal across countries, and (3) that the steady-state share of imported goods in overall consumption expenditures is  $1 - a$ . If  $a > 1/2$ , the agent consumes more goods from the country the agent lives in than from the other country, i.e., the agent has a home bias in consumption. This home bias is assumed to be symmetric across countries. Thus, the CES aggregator for an agent  $j$  living in country  $F$  is given by

$$C_t^{j*} = \left[ (1-a)^{\frac{1}{\theta}} C_{H,t}^{j* \frac{\theta-1}{\theta}} + a^{\frac{1}{\theta}} C_{F,t}^{j* \frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} . \tag{C.4}$$

Accordingly, the consumer price index (CPI) in country  $H$  expressed in country  $H$ 's currency is given by

$$P_t = \left[ a P_{H,t}^{1-\theta} + (1-a) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} , \tag{C.5}$$

where the producer price indices (PPI) for the bundles of differentiated goods expressed in country  $H$ 's currency are defined by

$$\begin{aligned}
 P_{H,t} &= \left[ \frac{1}{n} \int_0^n p_t(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}} \\
 P_{F,t} &= \left[ \frac{1}{1-n} \int_n^1 p_t(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}} .
 \end{aligned} \tag{C.6}$$

In their role as producers, agents charge one price for their good irrespective of whether the good is sold in their country or is exported to the other country (no price discrimination), setting the price in their country's currency (producer currency pricing). Furthermore, exporting does not entail transportation costs.

These assumptions imply that the law of one price holds, i.e., a single differentiated good has the same price in both countries if expressed in the same currency, and that exchange rate pass-through is complete:

$$p_t(h) = S_t p_t^*(h), \quad p_t(f) = S_t p_t^*(f), \quad (\text{C.7})$$

where  $p_t(h)$  denotes the price of a differentiated good  $y(h)$  produced in country  $H$  denominated in country  $H$ 's currency,  $p_t^*(h)$  denotes the price of the same good  $y(h)$  denominated in country  $F$ 's currency,  $p_t(f)$  denotes the price of a differentiated good  $y(f)$  produced in country  $F$  denominated in country  $H$ 's currency,  $p_t^*(f)$  denotes the price of the same good  $y(f)$  denominated in country  $F$ 's currency, and  $S_t$  is the nominal exchange rate defined as the price of country  $F$ 's currency in terms of country  $H$ 's currency. Given equations (C.6), it is straightforward to show that the law of one price for each differentiated good translates into the law of one price for each bundle of goods:

$$P_{H,t} = S_t P_{H,t}^*, \quad P_{F,t} = S_t P_{F,t}^*. \quad (\text{C.8})$$

The CPI in country  $F$  expressed in country  $F$ 's currency is given by

$$P_t^* = \left[ (1-a) P_{H,t}^{*1-\theta} + a P_{F,t}^{*1-\theta} \right]^{\frac{1}{1-\theta}}. \quad (\text{C.9})$$

In general, the law of one price does not translate into purchasing power parity. Thus, the real exchange rate, defined as the ratio of country-specific consumer prices

$$Q_t = \frac{S_t P_t^*}{P_t}, \quad (\text{C.10})$$

adjusts in response to changing economic conditions. Purchasing power parity ( $Q_t = 1$ ) only holds if the consumption baskets are identical across countries ( $a = 1/2$ ).

Another international relative price of interest are the terms of trade, defined from the perspective of country  $H$  as the ratio of the price of imported goods to the price of exported goods:

$$T_t = \frac{S_t P_{F,t}^*}{P_{H,t}}. \quad (\text{C.11})$$

Agent  $j$  in country  $H$  takes three decisions with respect to his consumption choices. First, he decides on the overall level of consumption  $C_t^j$ .<sup>32</sup> Second, given  $C_t^j$  the agent optimally allocates expenditures between the bundles of differentiated goods  $C_{H,t}^j$  and  $C_{F,t}^j$  by minimizing total expenditure  $P_t C_t^j$  subject to the CES aggregator (C.2). As a result, demand for these bundles is given by

$$C_{H,t}^j = a \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} C_t^j, \quad C_{F,t}^j = (1-a) \left( \frac{P_{F,t}}{P_t} \right)^{-\theta} C_t^j. \quad (\text{C.12})$$

Third, given  $C_{H,t}^j$  and  $C_{F,t}^j$  the agent optimally allocates expenditures between the differentiated goods by minimizing  $P_{H,t} C_{H,t}^j$  and  $P_{F,t} C_{F,t}^j$  subject to equations (C.3). This yields

$$c_t^j(h) = \frac{1}{n} \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} C_{H,t}^j, \quad c_t^j(f) = \frac{1}{1-n} \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} C_{F,t}^j. \quad (\text{C.13})$$

Combining (C.12) and (C.13) yields

$$\begin{aligned} c_t^j(h) &= \frac{a}{n} \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} C_t^j \\ c_t^j(f) &= \frac{1-a}{1-n} \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} \left( \frac{P_{F,t}}{P_t} \right)^{-\theta} C_t^j. \end{aligned} \quad (\text{C.14})$$

Analogously, demand equations for an agent  $j$  in country  $F$  are given by

$$\begin{aligned} c_t^{j*}(h) &= \frac{1-a}{n} \left( \frac{p_t^*(h)}{P_{H,t}^*} \right)^{-\sigma} \left( \frac{P_{H,t}^*}{P_t^*} \right)^{-\theta} C_t^{j*} \\ c_t^{j*}(f) &= \frac{a}{1-n} \left( \frac{p_t^*(f)}{P_{F,t}^*} \right)^{-\sigma} \left( \frac{P_{F,t}^*}{P_t^*} \right)^{-\theta} C_t^{j*}. \end{aligned} \quad (\text{C.15})$$

Market clearing for the differentiated goods  $y_t(h)$  and  $y_t(f)$  requires

$$\begin{aligned} y_t(h) &= \int_0^n c_t^j(h) dj + \int_n^1 c_t^{j*}(h) dj \\ y_t(f) &= \int_0^n c_t^j(f) dj + \int_n^1 c_t^{j*}(f) dj. \end{aligned} \quad (\text{C.16})$$

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<sup>32</sup>As shown below,  $C_t^j$  is determined by the usual Euler consumption equation.



Using equations (C.14) and (C.15), world demand for the differentiated goods can be expressed as

$$\begin{aligned} y_t(h) &= \frac{1}{n} \left( \frac{p_t(h)}{P_{H,t}} \right)^{-\sigma} \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} [aC_t + (1-a)Q_t^\theta C_t^*] \\ y_t(f) &= \frac{1}{1-n} \left( \frac{p_t(f)}{P_{F,t}} \right)^{-\sigma} \left( \frac{P_{F,t}}{P_t} \right)^{-\theta} [(1-a)C_t + aQ_t^\theta C_t^*], \end{aligned} \quad (\text{C.17})$$

where  $C_t \equiv \int_0^n C_t^j dj$  and  $C_t^* \equiv \int_n^1 C_t^{j^*} dj$ .

Similar to (C.3), aggregate output in the two countries can be expressed as

$$\begin{aligned} Y_{H,t} &= \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n y_t(h)^{\frac{\sigma-1}{\sigma}} dh \right]^{\frac{\sigma}{\sigma-1}} \\ Y_{F,t} &= \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 y_t(f)^{\frac{\sigma-1}{\sigma}} df \right]^{\frac{\sigma}{\sigma-1}}. \end{aligned} \quad (\text{C.18})$$

Inserting equations (C.17) into the previous equations finally yields aggregate demand

$$\begin{aligned} Y_{H,t} &= \left( \frac{P_{H,t}}{P_t} \right)^{-\theta} [aC_t + (1-a)Q_t^\theta C_t^*] \\ Y_{F,t} &= \left( \frac{P_{F,t}}{P_t} \right)^{-\theta} [(1-a)C_t + aQ_t^\theta C_t^*]. \end{aligned} \quad (\text{C.19})$$

Asset markets are assumed to be complete within and across countries. Agents can insure against all possible states of nature by holding a portfolio of state-contingent, one-period securities whose real value (denominated in units of the consumption-based price index) is denoted by  $B_t^{H,j}$  and whose vector of prices is denoted by  $q_t^H$ . In addition, agents can trade in a non-contingent, one-period bond whose nominal value (denominated in country  $H$ 's currency) is denoted by  $B_t^j$  and whose nominal interest rate is denoted by  $R_t$ . The intertemporal budget constraint of agent  $j$  in country  $H$  is then given by

$$C_t^j + q_t^H B_t^{H,j} + \frac{B_t^j}{P_t(1+R_t)} = B_{t-1}^{H,j} + \frac{B_{t-1}^j}{P_t} + (1-\tau_t^H) \frac{p_t(h)y_t(h)}{P_t}, \quad (\text{C.20})$$

The agent's income stems also from sales revenues  $p_t(h)y_t(h)$  net of a proportional, country-specific tax  $\tau_t^H$ .<sup>33</sup>

<sup>33</sup>The tax will turn out to be a subsidy to exactly offset the distortion caused by monopolistic competition.

All contingent securities and non-contingent bonds are assumed to be in zero supply in the initial period, so  $B_0^{H,j} = B_0^j = 0$  for all  $j$ . Together with the facts that, within countries, agents have identical preferences and that asset markets are complete, this implies perfect risk sharing of consumption within each country. Therefore, it is possible to analyze the consumer problem from the viewpoint of the representative agent of country  $H$  and country  $F$ .

The representative agent in country  $H$  maximizes his lifetime utility (C.1) subject to the budget constraint (C.20). By combining the resulting first order conditions with respect to consumption and bond holdings, the usual Euler consumption equation is then given by

$$U_C(C_t, \zeta_{C,t}) = (1 + R_t)\beta E_t \left\{ U_C(C_{t+1}, \zeta_{C,t+1}) \frac{P_t}{P_{t+1}} \right\}. \quad (\text{C.21})$$

The Euler consumption equation for the representative agent in country  $F$  is obtained analogously and given by

$$U_C(C_t^*, \zeta_{C,t}^*) = (1 + R_t^*)\beta E_t \left\{ U_C(C_{t+1}^*, \zeta_{C,t+1}^*) \frac{P_t^*}{P_{t+1}^*} \right\}. \quad (\text{C.22})$$

Complete asset markets across countries leads to price equalization in the state-contingent securities (expressed in country  $H$ 's currency), implying the following risk sharing condition:

$$\beta \frac{U_C(C_{t+1}, \zeta_{C,t+1})}{U_C(C_t, \zeta_{C,t})} \frac{P_t}{P_{t+1}} = \beta \frac{U_C(C_{t+1}^*, \zeta_{C,t+1}^*)}{U_C(C_t^*, \zeta_{C,t}^*)} \frac{S_t P_t^*}{S_{t+1} P_{t+1}^*} \quad (\text{C.23})$$

Inserting the country-specific Euler consumption equations yields the uncovered interest parity condition, according to which the expected change in the nominal exchange rate corresponds to the ratio of the country-specific interest rates:

$$E_t \Delta S_{t+1} = \frac{1 + R_t}{1 + R_t^*}. \quad (\text{C.24})$$

Assuming net foreign asset positions to be initially symmetric and applying the definition of the real exchange rate (C.10), the risk sharing condition takes the following form:

$$Q_t = \left( \frac{C_t^*}{C_t} \right)^{-\rho} \frac{\zeta_{C,t}^*}{\zeta_{C,t}}. \quad (\text{C.25})$$

**Producer problem**

In their role as producers, agents act in an environment of monopolistic competition, in which they dispose of some degree of market power. Furthermore, prices are sticky in the sense that the agent is able to change his price in a given period with a fixed probability, as in Calvo (1983). The probability of being able to change the price may differ across countries and is given by  $1 - \alpha^i$  for  $i = H, F$ .

Agent  $j$  in country  $H$  maximizes expected, discounted profits by choosing the price  $\tilde{p}_t(h)$  taking into account that demand depends on the chosen price and that the price may remain unchanged for some periods. Formally, the agent maximizes

$$E_t \sum_{k=0}^{\infty} (\alpha^H \beta)^k \left[ \lambda_{t+k} (1 - \tau_t^H) \tilde{p}_t(h) \tilde{y}_{t,t+k}(h) - V(\tilde{y}_{t,t+k}(h), \zeta_{Y,t+k}) \right] \quad (\text{C.26})$$

subject to the demand function

$$\tilde{y}_{t,t+k}(h) = \frac{1}{n} \left( \frac{\tilde{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} \left( \frac{P_{H,t+k}}{P_{t+k}} \right)^{-\theta} \left[ a C_{t+k} + (1 - a) Q_{t+k}^\theta C_{t+k}^* \right] \quad (\text{C.27})$$

where  $\tilde{y}_{t,t+k}(h)$  denotes total demand of good  $h$  at time  $t + k$  if the price  $\tilde{p}_t(h)$  prevails. Profits are expressed in utility units. Therefore, nominal sales revenues net of taxes  $(1 - \tau_t^H) \tilde{p}_t(h) \tilde{y}_{t,t+k}(h)$  are converted into utility units using the marginal utility of nominal revenues  $\lambda_{t+k} = \frac{U_C(C_{t+k}, \zeta_{C,t+k})}{P_{t+k}}$ . The cost of production expressed in utility units is given by the function  $V = \zeta_{Y,t}^{-\eta} \frac{y_t(h)^{1+\eta}}{1+\eta}$ .

The first order condition yields the optimal price

$$\tilde{p}_t(h) = \frac{\sigma}{(\sigma - 1)(1 - \tau_t^H)} \frac{E_t \sum_{k=0}^{\infty} (\alpha^H \beta)^k V_y(\tilde{y}_{t,t+k}(h), \zeta_{Y,t+k}) \tilde{y}_{t,t+k}(h)}{E_t \sum_{k=0}^{\infty} (\alpha^H \beta)^k \lambda_{t+k} \tilde{y}_{t,t+k}(h)}, \quad (\text{C.28})$$

where  $V_y$  denotes the derivative of function  $V$  with respect to output  $\tilde{y}(h)$ . All agents that live in the same country and are able to reset their price in a certain period will set the same price, since they share identical preferences (function  $V$ ) and face the same demand curves, which depend only on aggregate variables such as  $P_H$ ,  $P$ ,  $P^*$ ,  $S$ ,  $C$  and  $C^*$ , and the common elasticities of substitution  $\sigma$  and  $\theta$ . Hence, in a given period, a fraction  $1 - \alpha^i$  of agents will set the same optimal price, while for a fraction  $\alpha^i$  of agents the price from the previous period

remains effective:

$$\begin{aligned} P_{H,t} &= [\alpha^H P_{H,t-1}^{1-\sigma} + (1 - \alpha^H) \tilde{p}_t(h)^{1-\sigma}]^{\frac{1}{1-\sigma}} \\ P_{F,t}^* &= [\alpha^F P_{F,t-1}^{*1-\sigma} + (1 - \alpha^F) \tilde{p}_t^*(f)^{1-\sigma}]^{\frac{1}{1-\sigma}}. \end{aligned} \quad (\text{C.29})$$

When prices are flexible, the optimal-price equation (C.28) for country  $H$  simplifies to

$$\frac{P_{H,t}}{P_t} = \frac{\sigma}{(\sigma - 1)(1 - \tau_t^H)} \frac{V_y(y_t(h), \zeta_{Y,t})}{U_C(C_t, \zeta_{C,t})}, \quad (\text{C.30})$$

and for country  $F$  to

$$\frac{P_{F,t}^*}{P_t^*} = \frac{\sigma}{(\sigma - 1)(1 - \tau_t^F)} \frac{V_y(y_t(f), \zeta_{Y,t}^*)}{U_C(C_t^*, \zeta_{C,t}^*)}. \quad (\text{C.31})$$

Moreover, variations in the marginal disutility of production of one country relative to the other country are reflected in relative variations in the terms of trade and the real exchange rate. Dividing the previous two equations yields

$$\frac{T_t}{Q_t} = \frac{1 - \tau_t^H}{1 - \tau_t^F} \frac{V_y(y_t(f), \zeta_{Y,t}^*)}{V_y(y_t(h), \zeta_{Y,t})}. \quad (\text{C.32})$$

The markup that agents in country  $i = H, F$  are able to charge is defined as

$$\mu_t^i \equiv \frac{\sigma}{(\sigma - 1)(1 - \tau_t^i)}. \quad (\text{C.33})$$

### Log-linearization

Deviations of the logarithm of a variable  $X_t$  from its steady state are denoted by  $\hat{X}_t$  if prices are sticky and by  $\tilde{X}_t^{fb}$  if prices are flexible and markups are neutralized (efficient allocation).

**Sticky-price equilibrium** Under sticky prices, the system of equations is given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}_t - E_t \pi_{t+1} + E_t \hat{\zeta}_{C,t+1} - \hat{\zeta}_{C,t} \right) \quad (\text{C.34})$$

$$\hat{Q}_t = \rho \left( \hat{C}_t - \hat{C}_t^* \right) + \left( \hat{\zeta}_{C,t}^* - \hat{\zeta}_{C,t} \right) \quad (\text{C.35})$$

$$E_t \Delta \hat{S}_{t+1} = \hat{R}_t - \hat{R}_t^* \quad (\text{C.36})$$

$$\hat{Q}_t = (2a - 1) \hat{T}_t \quad (\text{C.37})$$

$$\hat{Y}_{H,t} = 2a(1 - a) \theta \hat{T}_t + a \hat{C}_t + (1 - a) \hat{C}_t^* \quad (\text{C.38})$$

$$\hat{Y}_{F,t} = -2a(1 - a) \theta \hat{T}_t + (1 - a) \hat{C}_t + a \hat{C}_t^* \quad (\text{C.39})$$

$$\pi_{H,t} = k_Y^H \left( \hat{Y}_{H,t} - \tilde{Y}_{H,t}^{fb} \right) - 2a(1 - a) k_T^H \left( \hat{T}_t - \tilde{T}_t^{fb} \right) + k_\mu^H \hat{\mu}_t^H + \beta E_t \pi_{H,t+1} \quad (\text{C.40})$$

$$\pi_{F,t}^* = k_Y^F \left( \hat{Y}_{F,t} - \tilde{Y}_{F,t}^{fb} \right) + 2a(1 - a) k_T^F \left( \hat{T}_t - \tilde{T}_t^{fb} \right) + k_\mu^F \hat{\mu}_t^F + \beta E_t \pi_{F,t+1}^* \quad (\text{C.41})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t}^* - \pi_{H,t} + \Delta \hat{S}_t \quad (\text{C.42})$$

$$\pi_t = a \pi_{H,t} + (1 - a) (\pi_{F,t}^* + \Delta \hat{S}_t) \quad (\text{C.43})$$

$$\pi_t^* = (1 - a) (\pi_{H,t} - \Delta \hat{S}_t) + a \pi_{F,t}^*. \quad (\text{C.44})$$

Equation (C.34) is the log-linear approximation of the Euler consumption equation (C.21), where  $\pi_t = \ln(P_t/P_{t-1})$ . Equation (C.35) is the log-linear approximation of the risk sharing condition (C.25), and equation (C.36) is the log-linear approximation of the uncovered interest parity condition (C.24).<sup>34</sup> Equation (C.37) describes the link between the real exchange rate and the terms of trade and is obtained by combining the log-linearized definition of the real exchange rate (C.10) with the log-linearized expressions for the country-specific CPI's (C.5) and (C.9), applying the law of one price. Equations (C.38) and (C.39) are obtained by log-linearizing the aggregate demand equations (C.19), using equation (C.37) to eliminate the real exchange rate.

Equations (C.40) and (C.41) represent the New Keynesian Phillips curves for country  $H$  and country  $F$  respectively, where  $\pi_{H,t} = \ln(P_{H,t}/P_{H,t-1})$  and  $\pi_{F,t}^* = \ln(P_{F,t}^*/P_{F,t-1}^*)$ . They are derived by combining the log-linear approximation of the optimal price (C.28) with the log-linear approximation of (C.29) for each country separately. The parameters in front of the output gap  $(\hat{Y}_{i,t} - \tilde{Y}_{i,t}^{fb})$ , the terms-of-trade gap  $(\hat{T}_t - \tilde{T}_t^{fb})$ , and the markup  $\hat{\mu}_t^i$  are defined as follows (for

<sup>34</sup>Alternatively, the model could be specified by including both country-specific Euler consumption equations next to the risk sharing condition, while omitting the uncovered interest parity condition.

$i = H, F$ ):

$$k_Y^i = \frac{(1 - \alpha^i \beta)(1 - \alpha^i)}{\alpha^i} \frac{\rho + \eta}{1 + \sigma \eta} \quad (\text{C.45})$$

$$k_T^i = \frac{(1 - \alpha^i \beta)(1 - \alpha^i)}{\alpha^i} \frac{\rho \theta - 1}{1 + \sigma \eta} \quad (\text{C.46})$$

$$k_\mu^i = \frac{(1 - \alpha^i \beta)(1 - \alpha^i)}{\alpha^i} \frac{1}{1 + \sigma \eta}. \quad (\text{C.47})$$

Equation (C.42) is the log-linear approximation of the terms of trade (C.11), expressed in first differences. Equations (C.43) and (C.44) are the log-linear approximations of the country-specific CPI's (C.5) and (C.9), applying the law of one price and expressed in first differences.

The model is closed by a Taylor-type interest rate rule for each country, according to which monetary policy responds to inflation and output with coefficients  $\phi_\pi$  and  $\phi_Y$  and engages in interest rate smoothing with coefficient  $\phi_R$ . Monetary policy responds either to PPI inflation or to CPI inflation. If monetary policy responds to PPI inflation, the interest rate rules are given by

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R)(\phi_\pi \pi_{H,t} + \phi_Y \hat{Y}_{H,t}) \quad (\text{C.48})$$

$$\hat{R}_t^* = \phi_R \hat{R}_{t-1}^* + (1 - \phi_R)(\phi_\pi \pi_{F,t}^* + \phi_Y \hat{Y}_{F,t}). \quad (\text{C.49})$$

If monetary policy responds to CPI inflation, the interest rate rules are given by

$$\hat{R}_t = \phi_R \hat{R}_{t-1} + (1 - \phi_R)(\phi_\pi \pi_t + \phi_Y \hat{Y}_{H,t}) \quad (\text{C.50})$$

$$\hat{R}_t^* = \phi_R \hat{R}_{t-1}^* + (1 - \phi_R)(\phi_\pi \pi_t^* + \phi_Y \hat{Y}_{F,t}). \quad (\text{C.51})$$

**Efficient allocation** The first-best (*fb*) or efficient allocation describes the equilibrium in which prices are fully flexible and in which markups are neutralized at all times with an appropriate subsidy ( $\mu_t^i = 0$ ). This efficient allocation provides a useful benchmark in order to assess the welfare implications of the two international monetary regimes.

Accordingly, efficient output in each country is given by

$$\begin{aligned}
 (\rho + \eta)\tilde{Y}_{H,t}^{fb} &= 2a(1-a)(\rho\theta - 1)\tilde{T}_t^{fb} & (C.52) \\
 &\quad - (1-a)(\hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^*) + \hat{\zeta}_{C,t} + \eta\hat{\zeta}_{Y,t} \\
 (\rho + \eta)\tilde{Y}_{F,t}^{fb} &= -2a(1-a)(\rho\theta - 1)\tilde{T}_t^{fb} \\
 &\quad + (1-a)(\hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^*) + \hat{\zeta}_{C,t}^* + \eta\hat{\zeta}_{Y,t}^*.
 \end{aligned}$$

The efficient terms of trade can be written as

$$[4a(1-a)\rho\theta + (2a-1)^2]\tilde{T}_t^{fb} = \rho(\tilde{Y}_{H,t}^{fb} - \tilde{Y}_{F,t}^{fb}) - (2a-1)(\hat{\zeta}_{C,t} - \hat{\zeta}_{C,t}^*). \quad (C.53)$$

The first equation is obtained by combining the risk sharing condition (C.35), equation (C.37), and the aggregate demand equation (C.38), all of which hold under flexible prices as well, with the log-linear approximation of the optimal-price equation (C.30). The second equation is derived completely analogously. The third equation is derived by subtracting the country-specific aggregate demand equations (C.38) and (C.39) from each other and by using the risk sharing condition (C.35) and equation (C.37) to eliminate country-specific consumption and the real exchange rate.

## C.2. Monetary union regime

The main difference of the monetary union regime compared to the flexible exchange rate regime, of course, is that the two countries share one currency and that the common monetary policy sets one union-wide nominal interest rate. Notwithstanding, the model structure is to a large extent identical (see, e.g., Benigno, 2004).

Under flexible prices, monetary policy is neutral, so that real variables are only driven by fundamental shocks. Thus, the efficient allocation is independent of the international monetary regime. Therefore, the behavior of efficient output and the efficient terms of trade under the monetary union regime is also described by equations (C.52) and (C.53).

Under sticky prices, the system of equations is given by

$$E_t \hat{C}_{t+1} = \hat{C}_t + \frac{1}{\rho} \left( \hat{R}_t^{MU} - E_t \pi_{t+1} + E_t \hat{\zeta}_{C,t+1} - \hat{\zeta}_{C,t} \right) \quad (\text{C.54})$$

$$\hat{Q}_t = \rho \left( \hat{C}_t - \hat{C}_t^* \right) + \left( \hat{\zeta}_{C,t}^* - \hat{\zeta}_{C,t} \right) \quad (\text{C.55})$$

$$\hat{Q}_t = (2a - 1) \hat{T}_t \quad (\text{C.56})$$

$$\hat{Y}_{H,t} = 2a(1 - a) \theta \hat{T}_t + a \hat{C}_t + (1 - a) \hat{C}_t^* \quad (\text{C.57})$$

$$\hat{Y}_{F,t} = -2a(1 - a) \theta \hat{T}_t + (1 - a) \hat{C}_t + a \hat{C}_t^* \quad (\text{C.58})$$

$$\pi_{H,t} = k_Y^H \left( \hat{Y}_{H,t} - \tilde{Y}_{H,t}^{fb} \right) - 2a(1 - a) k_T^H \left( \hat{T}_t - \tilde{T}_t^{fb} \right) + k_\mu^H \hat{\mu}_t^H + \beta E_t \pi_{H,t+1} \quad (\text{C.59})$$

$$\pi_{F,t}^* = k_Y^F \left( \hat{Y}_{F,t} - \tilde{Y}_{F,t}^{fb} \right) + 2a(1 - a) k_T^F \left( \hat{T}_t - \tilde{T}_t^{fb} \right) + k_\mu^F \hat{\mu}_t^F + \beta E_t \pi_{F,t+1}^* \quad (\text{C.60})$$

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t}^* - \pi_{H,t} \quad (\text{C.61})$$

$$\pi_t = a \pi_{H,t} + (1 - a) \pi_{F,t}^* \quad (\text{C.62})$$

$$\pi_t^* = (1 - a) \pi_{H,t} + a \pi_{F,t}^*. \quad (\text{C.63})$$

The Euler consumption equation (C.54) differs from the one under the flexible exchange rate regime only in that the nominal interest rate is given by the union-wide interest rate  $\hat{R}_t^{MU}$ . Nonetheless, real interest rates  $\hat{R}_t^{MU} - E_t \pi_{t+1}$  are generally country-specific, since CPI inflation rates usually differ across countries. Since the two countries form a monetary union, the uncovered interest parity condition is obsolete and the nominal exchange rate disappears from all relevant equations.

The model is closed by a Taylor-type interest rate rule, according to which the common monetary policy responds to union-wide inflation (average of country-



specific inflation) and union-wide output (average of country-specific output) with coefficients  $\phi_\pi$  and  $\phi_Y$  and engages in interest rate smoothing with coefficient  $\phi_R$ . If monetary policy responds to PPI inflation, the interest rate rule is given by

$$\hat{R}_t^{MU} = \phi_R \hat{R}_{t-1}^{MU} + (1 - \phi_R) \left( \phi_\pi \frac{\pi_{H,t} + \pi_{F,t}^*}{2} + \phi_Y \frac{\hat{Y}_{H,t} + \hat{Y}_{F,t}}{2} \right). \quad (\text{C.64})$$

If monetary policy responds to CPI inflation, the interest rate rule is given by

$$\hat{R}_t^{MU} = \phi_R \hat{R}_{t-1}^{MU} + (1 - \phi_R) \left( \phi_\pi \frac{\pi_t + \pi_t^*}{2} + \phi_Y \frac{\hat{Y}_{H,t} + \hat{Y}_{F,t}}{2} \right). \quad (\text{C.65})$$

Using equations (C.62) and (C.63), it is straightforward to show that the average of CPI inflation rates is equal to the average of PPI inflation rates:

$$\frac{\pi_t + \pi_t^*}{2} = \frac{\pi_{H,t} + \pi_{F,t}^*}{2} \equiv \pi_t^{MU}. \quad (\text{C.66})$$

As a result, the interest rate rule is the same irrespective of whether the common monetary policy responds to CPI or PPI inflation:

$$\hat{R}_t^{MU} = \phi_R \hat{R}_{t-1}^{MU} + (1 - \phi_R) \left( \phi_\pi \pi_t^{MU} + \phi_Y \frac{\hat{Y}_{H,t} + \hat{Y}_{F,t}}{2} \right). \quad (\text{C.67})$$

This implies that the behavior of all variables and the resulting welfare losses are the same irrespective of whether the common monetary policy responds to CPI or PPI inflation.

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# Eidesstattliche Erklärung

Ich erkläre hiermit an Eides Statt, dass ich meine Doktorarbeit “Essays on Labor Market Responses to the Great Recession and on the Effects of Monetary Unification on Macroeconomic Stability” selbständig und ohne fremde Hilfe angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen nach den mir angegebenen Richtlinien zitiert habe.

Ich versichere an Eides Statt, dass ich mich an keiner anderen Fakultät einer Doktorprüfung unterzogen habe.

Kiel, 2. September 2014

(Dominik Groll)

# DOMINIK GROLL

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## RESEARCH INTERESTS

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Macroeconomics, Open Economy Macroeconomics, Monetary Economics, Macroeconomic Policy, Economic Forecasting, Business Cycles, Labor Economics.

## CURRENT POSITION

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Economist, Forecasting Center and Research Area “Macroeconomic Policy under Market Imperfections”, Kiel Institute for the World Economy (since 2009).

## EDUCATION

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**Doctoral Programme “Quantitative Economics”**, University of Kiel, Germany (since 2007).

Supervisors: Christian Merkl (University of Erlangen-Nürnberg) and Hans-Werner Wohltmann (University of Kiel).

**Advanced Studies Program**, Kiel Institute for the World Economy, Germany (2014).

Courses: Nelson Mark (Notre Dame), Marvin Goodfriend (Carnegie Mellon), Kenneth Rogoff (Harvard), Sylvester Eijffinger (Tilburg), Philippe Aghion (Harvard), Steven Ongena (Zürich), Roberto Rigobon (MIT), Eric van Wincoop (Virginia).

**Diplom-Volkswirt (Int.)**, International Economics, University of Erlangen-Nürnberg, Germany (2007), and University of Valencia, Spain (2005).

## PREVIOUS POSITIONS

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**Kiel Institute for the World Economy**, Forecasting Center, Student Assistant (2008).

**Institute for Employment Research (IAB)**, Research Data Center (FDZ), Nürnberg (Germany), Student Assistant (2006-2007).

**University of Erlangen-Nürnberg**, International Relations Office, Student Assistant (2003-2007).

**Metropolitan Transportation Commission (MTC)**, Oakland (USA), Intern (2005).



## PUBLICATIONS

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- „Monetary Policy as an Optimum Currency Area Criterion“,  
Kiel Working Paper 1969, 2014, Kiel Institute for the World Economy.
- „Finanz- und Wirtschaftspolitik bei einer anhaltenden monetären Expansion“,  
(with Jens Boysen-Hogrefe, Klaus-Jürgen Gern, Nils Jannsen, Stefan Kooths, Martin Plödt, Tim Schwarzmüller, Björn van Roye, Joachim Scheide)  
Kieler Beiträge zur Wirtschaftspolitik 5, 2014, Kiel Institute for the World Economy.
- „Mindestlohn: Marktmacht, Preisüberwälzung und Aufstocken – eine Erwiderung“,  
(with Stefan Kooths),  
*Wirtschaftsdienst* 94(1), 2014: 45-49.
- „Monetary Union and Macroeconomic Stabilization“,  
Kiel Working Paper 1881, 2013, Kiel Institute for the World Economy.
- „Vor der Bundestagswahl: Argumente für Mindestlöhne überzeugen nicht“,  
(with Stefan Kooths),  
*Wirtschaftsdienst* 93(8), 2013: 545-551.
- „Demografie und Wachstum: Die gesamtwirtschaftlichen Effekte einer höheren  
Erwerbstätigkeit Älterer“,  
(with Ronald Bachmann, Sebastian Braun, Andreas Friedl, Matthias Giesecke, Anica Kramer, Alfredo Paloyo, Andreas Sachs),  
*Zeitschrift für Wirtschaftspolitik* 62(3), 2013: 287-310.
- „Price Competitiveness Divergence in the Euro Area: The Level Matters!“,  
(with Björn van Roye),  
Kiel Policy Brief 24, 2011, Kiel Institute for the World Economy.
- “The German Labour Market Miracle”,  
(with Jens Boysen-Hogrefe),  
*National Institute Economic Review* 214, 2010: R38-R50.
- “The Role of Labor Market Institutions in the Great Recession”,  
(with Jens Boysen-Hogrefe, Wolfgang Lechthaler, Christian Merkl),  
*Applied Economics Quarterly* 56(Supplement), 2010: 65-88.
- „Droht in Deutschland eine Kreditklemme?“,  
(with Jonas Dovern, Jens Boysen-Hogrefe, Björn van Roye),  
Kiel Discussion Paper 472/473, 2010, Kiel Institute for the World Economy.
- Co-author of more than 20 quarterly Business Cycle Reports (Konjunkturprognosen) on  
Germany and the Euro Area, published as Kiel Discussion Papers, Kiel Institute for the World  
Economy.

## PROJECTS

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- European Macroeconomic Imbalance Procedure – Experiences and proposals for improvement. *Federal Ministry of Economic Affairs and Energy, Germany*. 2014.
- Macroprudential and fiscal policy options for Germany in an expansionary monetary environment. *Federal Ministry of Finance, Germany*. 2012-2013.
- Reports in the context of the “LIME Working Group on the Methodology to assess Lisbon related structural reforms.” *Federal Ministry of Finance, Germany*. 2012-2013.
- Macroeconomic effects of an increase in the employment rate of the elderly in Germany. *Initiative Neue Soziale Marktwirtschaft (INSM), Germany*. 2012.
- Macroeconomic effects of an increase in the supply of skilled labor in Germany. *Federal Ministry of Labor and Social Affairs, Germany*. 2012.
- Medium-term Economic Plan for Dubai 2011-2015. *Government of Dubai, Department of Economic Development*. 2011-2012.
- Threat of a credit crunch in Germany during the Great Recession. *Federal Ministry of Finance, Germany*. 2009.
- Joint Economic Forecast (“Gemeinschaftsdiagnose”). *Federal Ministry of Economic Affairs and Energy, Germany*. 2009-2013.

## CONFERENCES AND SEMINARS

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- 2013      “6<sup>th</sup> Global Economic Symposium”,  
Kiel (Germany), October, Session Organizer.
- “Annual Congress of the Verein für Socialpolitik”,  
Düsseldorf (Germany), September, Presenter.
- “18<sup>th</sup> Spring Meeting of Young Economists”,  
Aarhus (Denmark), May, Presenter.
- Bundesbank Research Seminar,  
Frankfurt (Germany), April, Presenter.
- “17<sup>th</sup> Theories and Methods in Macro Conference”,  
Lyon (France), March, Presenter.
- 2012      “5<sup>th</sup> Global Economic Symposium”,  
Rio de Janeiro (Brazil), September, Session Organizer.
- “NBER Summer Institute”,  
Cambridge (USA), July, Participant.
- 2011      “4<sup>th</sup> Global Economic Symposium”,  
Kiel (Germany), September, Session Organizer.
- “4<sup>th</sup> Lindau Nobel Laureate Meeting in Economics”,  
Lindau (Germany), August, Participant.

“8<sup>th</sup> EUROFRAME Conference on Economic Policy Issues in the European Union”,  
Helsinki (Finland), June, Discussant.

2010 “73<sup>rd</sup> Scientific Conference of the ARGE Institutes”,  
Berlin (Germany), April, Presenter.

2009 “2<sup>nd</sup> Global Economic Symposium”,  
Plön (Germany), October, Session Organizer.

2008 “1<sup>st</sup> Global Economic Symposium”,  
Plön (Germany), October, Volunteer.

## **SKILLS**

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**Software:** MATLAB, Dynare, RATS, STATA, LaTeX.

**Languages:** German (native), English (fluent), Spanish (fluent), French (advanced).