

ESSAYS ON ECONOMIC GROWTH AND BUSINESS CYCLE DYNAMICS

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List of Acronyms

Acronym	Description
ACE	Agent-based Computational Economics
DSGE	Dynamic Stochastic General Equilibrium
FCM	Functional Coefficient Model
FDI	Foreign Direct Investment
FRED	Federal Reserve Economic Data
GDP	Gross Domestic Product
HIV	Human Immunodeficiency Virus
HP-Filter	Hodrick-Prescott Filter
IPC	International Patent Classification
MATLAB	Matrix Laboratory (brand name)
NK	New Keynesian
NKPC	New Keynesian Phillips Curve
OECD	Organization for Economic Cooperation and Development
OLS	Ordinary Least Squares
PATSTAT	Worldwide Patent Statistics Database
PPP	Purchasing Power Parity
PRO	Public Research Organization
PWT	Penn World Tables
RV	Random Variable
SQL	Structured Query Language
WDI	World Development Indicators
ZEW	Zentrum für Europäische Wirtschaftsforschung

CHAPTER 1

Outline

This dissertation consists of five articles. All covered topics are related to growth or business cycle dynamics to some extent, although applied methodologies differ more or less strongly between them. Chapters 2, 4 and 5 are considered the main part. Chapter 6 is attached as a complementary reference bearing witness to my efforts in the field of innovation research in the years as a doctoral candidate.

Chapter 2: Agent-based model of Growth and Business Cycles

The first article describes a simulated monetary macro model with different types of interacting agents. As such, it is assigned to the field of agent-based computational economics (ACE), where agents become virtual objects in a computer simulation. The ACE model core with labor market and goods market interaction between households and firms is adopted from *Lengnick (2013)*¹, whereas production technology and technological progress of firms are adopted from the neoclassical *Solow (1956)* model. Therefore, long-run economic growth on aggregate level is determined by an exogenous growth rate. Nominal interest rates are set in accordance with the *Taylor (1993)* principle, characterized by strong responses of monetary policy to deviations from inflation target. Although inflation desirably follows lagged output in a pro-cyclical manner, the dynamic system allows for long-run stability of inflation rates.

Firms on aggregate level endogenously generate waves of higher and lower investment. A recurrent cyclical movement of aggregate economic activity, in particular demand, employment and inflation, is transmitted from these waves of investment activity, so model dynamics are in line with the reasoning of *Keynes (1936)* and *Kalecki (1937)* on business cycles. Aggregate consumption also develops pro-cyclically, but is less volatile compared to investment. Cyclical patterns of boom and bust emerge with a frequency of approximately seven years just like

¹ References cited in this introductory outline are listed at the end of the respective chapters.

Juglar-type cycles. Moreover, the model generates a short-run *Phillips*-curve relationship, long-run neutrality of monetary policy and business cycle patterns similar to the *Goodwin* (1967) model. Fiscal stabilization policy is shown to dampen macroeconomic fluctuations, thus allowing for a higher level of average employment. Calibration of model parameters is conducted to generate realistic orders of magnitude of important macroeconomic proportions. The newly developed model is a combination of ideas from different economic perspectives and contributes to macroeconomic model-building under the paradigm of agent-based computational economics. The article demonstrates the usefulness of ACE model-building in general and of the presented demand-led growth model in particular.

This chapter is based on my single-authored paper “The Agent-Based Solow Growth Model with Endogenous Business Cycles”.

Chapter 3: Stabilization Policy

A topic closely related to business cycle dynamics is macroeconomic stabilization policy. To determine the possible welfare gains from such policies to society, we adapt the approach of *Lucas* (2003) with a risk-averse representative agent. However, instead of using a General Equilibrium model, we apply an agent-based macro model, thus relaxing the assumption of perfect ex ante coordination brought about by the *Walrasian* auctioneer. We find that the ACE macro model allows for a higher average level of employment, if economic activity is stabilized. As a result, the welfare gain of a perfect stabilization at full employment level is by orders of magnitude higher compared to *Lucas*' estimate, who implicitly assumes that stabilization means dampening both, booms and recessions, in a symmetrical way. The welfare gain under the ACE paradigm consists not only of reduced variability, but also in a higher average level of consumption and output.

This chapter is based on a joint article with *Matthias Lengnick*. The initial idea was developed by *Matthias*, whereas my contribution consisted of programming the adjustment of the ACE model, as well as application and analysis of 1000 replications of both model types. The (short) article was written by both authors in collaborative work.

Chapter 4: Business Climate

Certainly, there are *real* causes beyond psychology that originally cause macroeconomic fluctuations, like stochastic exogenous shocks in real business cycle theory; or endogenous aggregate dynamics emerging in the interaction of economic agents as described in chapter 2. These real causes may create some variability of demand and output in the first place, but undoubtedly, upward and downward movements are reinforced by expectations. Confidence and expectations of consumers and firms play an important role in the generation and dynamics of business cycles. For example, a firm expecting demand to increase is more likely to invest, thus contributing even more to the increase of aggregate demand. Moreover, expectations of agents are influenced by other agents' expectations, thus linking the formation of expectations

with regard to economic prospects closely to the pattern and intensity of actually observable business cycles.

Processes of social opinion formation might be dominated by a set of closely connected agents who constitute the cohesive ‘core’ of a network and have a higher influence on the overall outcome of the process than those agents in the more sparsely connected ‘periphery’. In this chapter, it is explored whether such a perspective could shed light on the dynamics of a well known economic sentiment index (ZEW survey). To this end, it was hypothesized that the respondents of the survey under investigation form a core-periphery network. In a discrete setting, those agents defining the core were identified, whereas in a continuous setting, the proximity of each agent to the core was determined. As it turns out, there is significant correlation between the so identified cores of different survey questions. Both the discrete and the continuous cores allow an almost perfect replication of the original series with a reduced data set of core members or weighted entries according to core proximity. Using a monthly time series on industrial production in Germany, experts’ predictions with the real economic development were also compared. The core members identified in the discrete setting showed significantly better prediction capabilities than those agents assigned to the periphery of the network.

This chapter is based on a joint paper with *Thomas Lux* published under the title “Identification of a Core-Periphery Structure Among Participants of a Business Climate Survey – An Investigation Based on the ZEW Survey Data” in *European Physical Journal B* 84 (2011), pp. 521 – 533. *Prof. Lux* came up with the initial idea to apply a core-periphery network analysis to ZEW data, gave valuable advice and suggestions throughout the process. My contribution to this paper consisted of handling and utilizing the raw data set, application and adjustment of algorithms and discussion and determination of further steps in coordination with *Prof. Lux*. The draft of the article was also written by me.

Chapter 5: Empirical Growth Analysis

The article “Growth Determinants Across Time and Space – A Semiparametric Panel Data Approach” empirically analyzes worldwide data on economic growth and its determinants. A panel data set covering 145 countries between 1960 and 2010 has been investigated closely by using models of parameter heterogeneity. The Functional Coefficient Model (FCM) introduced by *Cai, Fan and Yao* (2000) allows estimated parameters of growth determinants to vary as functions of one or two status variables. As a status variable, coefficients depend on the level of development, measured by initial per capita GDP. In a two-dimensional setting, time is used as an additional status variable. At first, the analysis is restricted to bivariate relationships between growth and only one of its determinants, dependent on one or both status variables in a local estimation. Afterwards, the *Solow* (1956) model serves as a core setting of theory-based control variables, while functional dependence of additional explanatory variables is investigated. While some constraints of this modeling approach have to be kept in mind, functional specifications are a promising tool to investigate growth relationships. Moreover,

it contributes to an assessment of robustness and sensitivity of uncovered determinants of growth to alterations in the covered period and in the composition of countries in cross-country growth regressions. At the end of this chapter, a simple derivation of FCM called local mean values provides a suitable way to visualize macroeconomic or demographic patterns across two dimensions in a descriptive diagram.

This chapter is based on my single-authored paper “Growth Determinants Across Time and Space – A Semiparametric Panel Data Approach” published as *Economics Working Paper* 2014-11, University of Kiel, September 2014. *Helmut Herwartz* provided initial help and some valuable comments and suggestions at intermediate stages of the article creation.

Chapter 6: Innovation and Patent Analysis

A major source of technological progress in models of endogenous growth is innovation. In addition, the private sector’s ability to pursue profitable and sustainable innovation activity is supplemented and supported by public research efforts, which uncover basic knowledge or base technologies and provide education as a prerequisite for constructive innovation and enhancements. However, the economic significance of ideas and products generated by innovation is sometimes difficult to measure sharply. Patenting activity as an observable subset of overall innovation activity can be illuminated by analyzing patent databases.

In a recent research project at the institute of innovation research (University of Kiel) in cooperation with the German federal ministry for education and research (BMBF), the PATSTAT world wide patent database was searched for patents assigned to publicly financed research organizations (PROs) in Germany. Prof. Walter and his research team, including myself, investigated the characteristics and diffusion of knowledge uncovered by German PROs, in particular the diffusion of patented knowledge and its economic relevance to the private sector. A shortened part from the research projects’ final report (Walter, A. (ed., 2014): "Wirtschaftliche Bedeutung des patentierten grundlagenorientierten Wissens in Deutschland", Hannover: Technische Informationsbibliothek Hannover, pp. 74–104), written in German language, is attached as chapter 6, covering descriptive statistics of the patent portfolio of German PROs and investigating knowledge diffusion by utilization of data on patent citations. My contribution to this part of the final report is particularly important. It required several months of sometimes automated, sometimes manual work to recognize and determine about 700 different institutions as German PROs as well as firms from numerous countries in the raw patent data with all the inaccuracy and weakness attached to it, by means of SQL database search. This time-consuming determination of a subset of worldwide patenting activity was the basis also for more elaborate statistical analyses within the research project. The excerpt presented in chapter 6 was drafted by myself and consists of a descriptive analysis of the determined PRO patent data set.

A main finding is that public research appears to generate a stronger economic impact in Germany, if the respective research area overlaps with technological fields, that are characterized by comparative advantages and research focuses of the private sector in Germany.

Appendices

Cited references are listed at the end of each chapter, since the overlap between the chapters with regard to literature is rather limited. Similarly, lists of symbols that have been introduced in an article are positioned at the end of the respective chapter. However, such lists of symbols are available only for chapters 2, 3 and 5, where a non-negligible number of symbols is used.

The Agent-Based Solow Growth Model with Endogenous Business Cycles

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Abstract: This article describes a simulated monetary macro model with different types of interacting agents. As such, it is assigned to the field of agent-based computational economics (ACE), where agents become virtual objects in a computer simulation. The ACE model core with labor market and goods market interaction between households and firms is adopted from *Lengnick* (2013), whereas production technology and technological progress of firms are adopted from the neoclassical *Solow* (1956) model. Nominal interest rates are set in accordance with the *Taylor* (1993) principle, characterized by strong responses of monetary policy to deviations from inflation target. Although inflation desirably follows lagged output in a pro-cyclical manner, the dynamic system allows for long-run stability of inflation rates. Firms on aggregate level endogenously generate waves of higher and lower investment. A recurrent cyclical movement of aggregate economic activity, in particular demand, employment and inflation, is transmitted from these waves of investment activity. Cyclical patterns of boom and bust emerge with a frequency of approximately seven years just like *Juglar*-type cycles. Moreover, the model generates a short-run *Phillips*-curve relationship, long-run neutrality of monetary policy and business cycle patterns similar to the *Goodwin* (1967) model. Fiscal stabilization policy is shown to dampen macroeconomic fluctuations, thus allowing for a higher level of average employment. Calibration of model parameters is conducted to generate realistic orders of magnitude of important macroeconomic proportions. The newly developed model is a combination of ideas from different economic perspectives and contributes to macroeconomic model-building under the paradigm of agent-based computational economics.

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2.1 Introduction

Agent-Based Computational economics (ACE) developed recently as a new branch of macroeconomic modeling, which falls in the paradigm of complex adaptive systems (*Tesfatsion, 2003*). Economic agents become artificial objects in a computer simulation, which interact according to behavioral rules (*Page, 2008*). A newly developed ACE macro model is outlined in this article.

A key advantage of the ACE approach should be underscored here, which is that analytic tractability is not a key requirement for model equations. There is no need to solve them for an equilibrium equation or set of equations, thereby forcing assumptions to be overly simple, well behaved and tractable. Instead, model outcome is analyzed by observation of emergent time series. As a result, modeling of agent behavior becomes less restricted and allows to incorporate and combine research findings from neighboring fields such as experimental economics and behavioral economics. Another advantage of agent-based models in general is that macro results may occur that differ completely from disaggregate micro level agent behavior. A famous generally understandable example for such an emergent property on aggregate level is *Schellings (1969)* model of racial segregation in cities, emerging from rather open-minded individual attitudes with regard to mixed neighborhoods.

There are some disadvantages to acknowledge: First, it requires a good deal of effort to get familiar with programming and typical challenges of calibrating these models. *Oeffner (2009)* provides a detailed and comprehensible introduction to the virtues and challenges of ACE model building. Secondly, there is no analytic solution of equilibrium or model dynamics, so people used to it may feel a lack of mathematical certainty. According to *Page (2008)*, ACE models “occupy a middle ground between stark, dry rigorous mathematics and loose, possibly inconsistent, descriptive accounts”. Thirdly, freedom concerning modeling of agent behavior is accompanied by a growing level of complexity, which complicates understanding and interpretation of results. The term “wilderness of bounded rationality” points to the difficulty of transferring certain findings about non-rational agent behavior to functional forms. It is accompanied by the problem of (too) many parameters in large models (*Sims, 1980*). As *Lengnick (2013)* argues, ACE modelers are “tempted to over-increase the level of complexity”.

Until recently, macroeconomic model building is rather dominated by Dynamic Stochastic General Equilibrium (DSGE) models, which became the standard models of monetary policy analysis (cf. *Woodford, 2003; Clarida et al., 1999*). However, DSGE modeling is subject to ongoing criticism (*Mankiw, 2006; Solow, 2010; Colander et al., 2009*). Some standard paradigms appear questionable, especially in the light of possibilities offered by the ACE approach.

To begin with, most General Equilibrium models assume existence of a representative agent, who optimizes utility with infinite horizons. *Kirman (1993)* argues why the assumption of a representative agent is questionable. *Solow (2010)* challenges the idea “that the whole economy can be thought of if it were a single [...] person carrying out a rationally designed, long-term plan, occasionally disturbed by unexpected shocks, but adapting to them in a

rational, consistent way.” He also argues that DSGE models by construction provide no reasonable way to cope with involuntary unemployment, since the representative agent only rationally chooses to substitute work with leisure a little more. Moreover, there may also occur the need to analyze consequences of certain policies with regard to the income distribution; or to analyze how the distribution of wealth influences growth (e.g. *Alesina and Rodrick, 1994; Dynan, Skinner and Zeldes, 2004; Galor, 2009*). The ACE approach allows agents to be modeled as heterogeneous individuals, who partly can become involuntarily unemployed. Topics related to wealth and income distribution can also be tackled.

Another assumption in general equilibrium models is that relative prices are simply set optimally, thus allowing for permanent market clearing as long as no frictions are imposed. The concept of permanent fulfillment of an equilibrium condition, brought about by a fictitious *Walrasian* auctioneer, who calculates an optimal vector of relative prices in meta time, is not convincing (*Ackerman, 2002; Gaffeo et al., 2008; Kirman, 2006*). In ACE models, the process and coordination of relative price adjustments can be modeled explicitly, possibly resulting in a temporary equilibrium situation. Finally, the famous *Lucas (1976)* critique argues that relations between macroeconomic variables may change with policy, because agents incorporate new policies into economic decision making. This idea induced not only the spread of microfoundations as an acknowledged requirement for macro models, it also made the rational expectations hypothesis a key ingredient of model building. As long as the consequence derived from *Lucas’* critique concerns the requirement of microfounded agent behavior, ACE models allow for a much more complex and realistic set of assumptions applicable to simulated heterogeneous agents. However, an application of rational expectations to ACE models appears inappropriate, since it implies that (simulated) agents are able to fully understand a complex dynamic system of agent interactions and to calculate expected values for aggregate outcomes. Not even the designer of the dynamic ACE system is able to calculate an accurate probability distribution of possible outcomes. A more general critique of rational expectations in macro models is provided by *Syll (2012)*.

It should be noted that former arguments do not disqualify DSGE models to analyze and estimate real aggregate economic behavior in a valuable way. *Friedman (1951)* argues that accuracy of predictions derived from a theory is more important than the underlying assumptions, as long as they are consistent. Nevertheless, the goal of economic theory is not restricted to prediction and data-fitting; it also consists of providing convincing explanations of real-world phenomena and processes, so contrary to *Friedmans* claim assumptions actually *do* matter. The ACE approach offers a new way for macroeconomic model building, facilitating more realistic designs of agent behavior.

Point of departure for the newly developed ACE macro model is the baseline model of *Lengnick (2013)*, later referred to as *L13* model.¹ It is a simple model of a closed economy, which partly draws on former models of *Dosi et al. (2008)* and *Gaffeo et al. (2008)*. The

¹ Java Source code of the *Lengnick* model was adopted, which was very helpful and facilitated the start. Program output such as macroeconomic time series were analyzed with MATLAB. The source code of the extended model described here is available upon request.

simulated economy consists of households and firms interacting on a goods market and on a labor market, where each type of agent follows simple adaptive rules. The model generates endogenous business cycles and some desired characteristics on aggregate level.² The basic structure of the model has been adopted, such as the sequence of activities, connections between agents and the organization of the goods market and of the labor market.

Certain restrictions of the *Lengnick* model prompted several changes and extensions: (1) Labor is sole production factor input in *L13*, while in the extended model each firm is endowed with a capital stock, which is subject to depreciation. Technological progress in combination with *Cobb-Douglas* production technology is incorporated, so that firms become customers of other firms by purchasing investment goods. This extension simply forces production technology and architecture of the *Solow-Swan* growth model (*Solow*, 1956; *Swan*, 1956) on single firms of an agent-based framework.³ (2) In *L13*, firm ownership remains unspecified. Here, households are shareholders of firms, so that profit is paid out as a return rate of firm shares. (3) In *L13*, firms are frequently unable to meet customer demand, followed by a loss of customers. To prevent a constant flow of restricted, disappointed customers between firms, firms are modeled with excess production capacity in the new model. (4) A fixed quantity of money is circling between agents in *L13*. Instead, endogenous money is introduced with credit, savings, interest rates and a monetary policy rule. Availability of credit also avoids frequent emergency wage cuts of firms, if they run out of money in *L13*. (5) Finally, consumption paths of unemployed households in *L13* are unrealistic, since consumption almost immediately drops to low one-digit percentages of former levels. The new model introduces a government, providing for unemployment benefits and collecting taxes.

The presented *monetary* model incorporates elements from different economic schools: It is *Keynesian*, since economic activity of firms is strictly demand-driven and features involuntary unemployment, while there is no hypothetical lower wage that allows for market-clearing. *Say's law* does not apply, when firms are designed to have excess production capacities, thus allowing for unsold quantities. Fluctuating consumption and investment, combined with unknown household savings and availability of credit weaken the link between production and aggregate demand even more. In the context of this model, the notion of "equilibrium" may be understood as a situation when aggregate demand fluctuates around a target percentage of production capacity. Business cycles are created endogenously, generated by higher or lower firm investment as the leading determinant of economic dynamics. Both *Keynes* (1936) and *Kalecki* (1937) also uncovered investment as a major source of periodical macroeconomic fluc-

² Crises in the *L13* emerge as follows: After a period of abundant aggregate demand, firm inventories decline to critical lower values. Firms are permanently unable to acquire more workers for production enlargement, thus inducing rising wages and correspondingly decreasing profits. As soon as profits reach a lower bound, price increases are triggered, by which the money supply is devalued in a system with a fixed quantity of money. Finally, the reduced value of real money in the system causes a drop in firm sales, which induces firms to fire workers.

³ To name a few major differences of the *Solow* model: It only captures the *aggregate* level and assumes all factors of production to be constantly employed; there is no role for money; savings are generated as a fixed proportion of output and are directly invested.

tuations. As it turns out, firm profits depend on aggregate investment, but not on investment of the one single firm. On the other hand, firm investment in turn depends on past firm profits. A reinforcing spiral of investment, demand and profits is an emergent property on aggregate level, both upwards and downwards. As it turns out, the emergent cyclical pattern resembles dynamics of the *Goodwin* (1961) model. Elements from *neoclassical* theory are incorporated, namely the *Solow-Swan* growth model from which firm technology and technological progress are adopted on firm level. Monetary policy is conducted in accordance with the *New Macroeconomic Consensus* where the *Taylor* principle is applied to automatically adjust nominal interest rates as a strong response to deviations from inflation target. A short-run *Phillips curve* emerges from aggregate model dynamics, similar to *New Keynesian* models.

Naturally, there have been other approaches to ACE macro modeling before. *Lengnick* (2013) distinguishes two categories of such models: The first category models the economy in considerable detail and complexity such as the EURACE project (*Dawid et al.*, 2011) with even a spatial structure. The second category abstracts from reality to a larger degree, which is where *L13* and the presented model belong to. *Dosi et al.* (2008) develop a model of investment with R&D, where aggregate demand and output are driven by lumpy investment. They obtain macro behavior in line with a number of stylized facts. However, contrary to the approach followed here, neither a labor market nor a goods market are modeled, instead firms are simply assigned a proportion of aggregate demand. Moreover, households fully consume their income, which also differs from the more general consumption decision applied here. *Gaffeo et al.* (2008) explicitly model the goods market, but do not capture the capital side of the economy, since their production function employs labor as the sole input. *Oeffner* (2009) develops an ACE macro model characterized by demand-driven economic activity, business cycles amplified by firm investment, Cobb-Douglas technology, growth and inflation. It resembles in many ways the model presented here. However, it simulates three firm sectors instead of one: Two consumption goods sectors (with a capital stock) and a capital goods sector (modeled without a capital stock); each of them populated by a fixed number of firms. Another major difference is that firm employment is fixed in *Oeffners* model, so there is no analysis of unemployment.

The ACE model presented here is the first to combine an explicit modeling of goods market and labor market, firm capital stock, investment, growth, inflation and endogenously created business cycles. Contrary to numerous other models, it encompasses the demand-led character of firm decisions on employment and price setting. This article provides a comprehensive description of the model structure and analyzes simulation behavior. As will be shown, the model reproduces a number of stylized facts and is calibrated to generate realistic proportions on aggregate level. To further demonstrate the usefulness of ACE models in general and of this extended model in particular, the response to a monetary shock is analyzed. Additionally, three different fiscal policy regimes are analyzed: As it turns out, a policy aiming at demand stabilization performs best with respect to average employment. Yet this result, admittedly, is a rather expectable property of a demand-led macro model.

The remainder of this article is organized as follows: Section 2.2 explains the building blocks of the agent-based economy step-by-step, such as the basic structure, banking system, agent classes of households, firms and the government as well as the conduct of monetary policy. Section 2.3 describes the model behavior in the running simulation. Time series for individual firms (2.3.1) are followed by macroeconomic variables and analysis of business cycle dynamics (2.3.2). In addition, Phillips curves, monetary policy shocks (2.3.3), fiscal policy (2.3.4), calibration issues (2.3.5) and limitations (2.3.6) are also treated in this chapter. Section 2.4 concludes.

2.2 Model

2.2.1 Structure and Timing

The sequence of activities consists of in two different time intervals (cf. *Lengnick, 2013*). All relevant decisions take place on a monthly basis, as well as payment of wages, profits, interest, taxes and unemployment benefits. On daily basis, goods are only produced and sold to customers. Figure 2.1 provides an overview of the sequence of important activities.

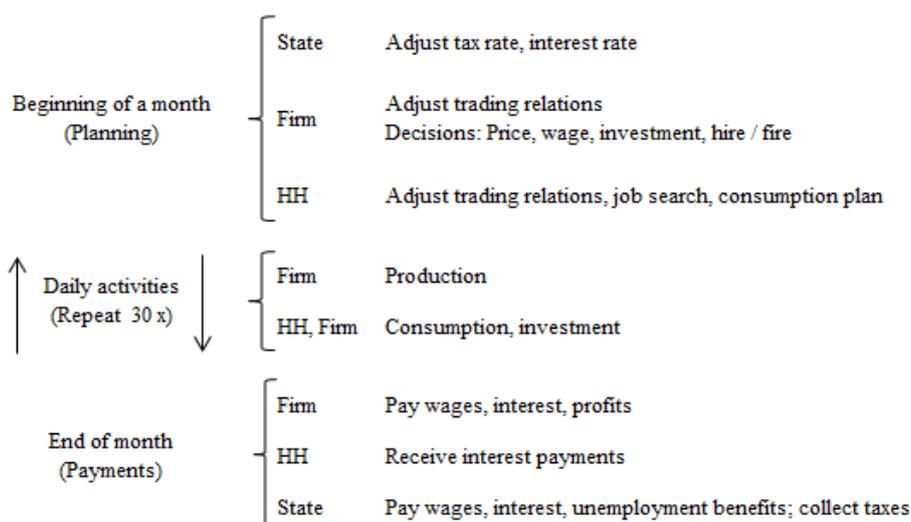


Figure 2.1: Sequence of monthly events.

The developed *monetary* model distinguishes carefully between real variables, which are counted in natural units (goods), and nominal measures counted in currency units. Regarding notation, all nominal measures are written consequently with a preceding letter n in order to avoid confusion, while real measures are written without this preceding letter.⁴ The model internally calculates on a monthly basis, so that inflation rates, interest rates, the depreciation rate and return rates are actually very small values. In this article, however, corresponding values are presented as annualized percentages in order to simplify understanding. In the

⁴ The only exception is the *nominal* interest rate i_t , which is measured as a percentage, so it is actually not a nominal variable with respect to its unit of measurement.

following sections, variables are in monthly notation with a subscript t with months $t = 1, \dots, T$.

There are three types of agents: households, firms, and a government sector (state). The number of households ($N^{hh} = 2000$) and firms ($N^{fi} = 100$) is fixed in order to exclude demographic aspects as well as firm entry and exit. A central bank is also present, but has no other function but to set the nominal interest rate. Figure 2.2 depicts the model structure.

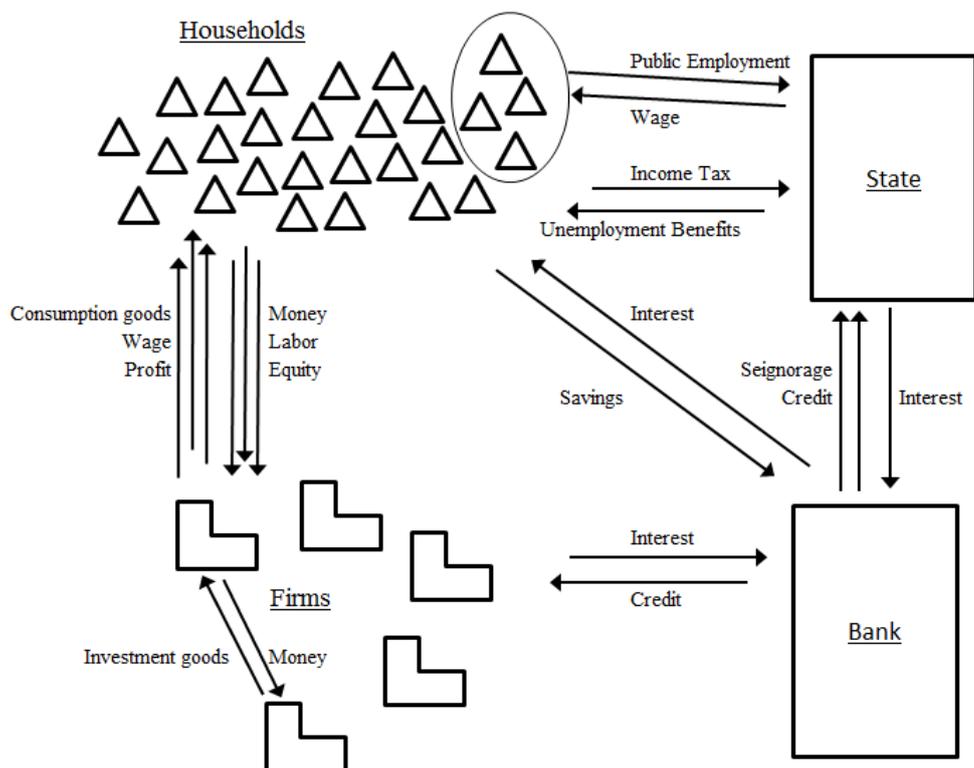


Figure 2.2: Model structure with financial and real flows between agents.

2.2.2 Banking System

All households, firms and the government are endowed with a bank account. There is only one bank representing the banking system. However, it is abstracted from administration costs, employees and the objective to generate profits. Each bank account contains a money account, which is used for payments⁵, and a savings account, which can be used as an interest-bearing financial asset; and which can also become a credit account, if the balance is negative. The money account is restricted to positive values and it is interest-free. The savings account can be positive and negative, so it is either a financial asset or a credit account.

All agents are free to transfer arbitrary amounts from the money account to the savings account and back, so there is no credit restriction and no credit risk evaluation. Money

⁵ The model abstracts from cash payments, so deposits of the money account are transferred between agents in order to execute payments. The two-staged structure of real-world banking systems with a role for central bank money is also not captured.

is created endogenously once an agent demands for additional liquidity, thereby increasing aggregate money. The created monetary amount is used for payments, thus circling between agents. Once it is transferred to a savings account, the aggregate quantity of money is reduced respectively.⁶ At the start of each month, agents decide about their liquidity need, so they decide to hold transaction money based on their past monthly cost. Spare money is moved to the savings/credit account, either to gain interest income (households) or to avoid unnecessary cost (firms).

All Money and savings accounts sum up to zero at all times.⁷

$$0 = \underbrace{nM_t^{hh} + nM_t^{fi}}_{\text{transaction money}} + \underbrace{nSC_t^{hh}}_{+} + \underbrace{nSC_t^{fi}}_{-} + \underbrace{nSC_t^{st}}_{-}, \quad (2.1)$$

Aggregate money and savings/credit accounts of the household sector and firm sector are simple aggregates: $nM_t^{hh} = \sum_{h=1}^{N^{hh}} nM_{t,h}^{hh}$, $nM_t^{fi} = \sum_{i=1}^{N^{fi}} nM_{t,i}^{fi}$, $nSC_t^{hh} = \sum_{h=1}^{N^{hh}} nSC_{t,h}^{hh}$ and $nSC_t^{fi} = \sum_{i=1}^{N^{fi}} nSC_{t,i}^{fi}$. The government does not hold any transaction money at trading days. Therefore, $nM_t^{st} = 0$.

The interest rate is adjusted monthly by a central bank. It is imposed on all savings/credit account balances and is paid by debtors and received by creditors of the bank. For simplicity, the *current* nominal interest rate is valid for the whole stock of savings and credit.⁸

2.2.3 Household

Job Market

Job market decisions take place at the beginning of a month. Each household is connected to one employer unless he is unemployed, while a firm is able to employ an arbitrary number of employees. Households offer inelastically one unit of labor per month, so that wage payment is monthly labor income. Each firm pays the same wage to all of its employees, though there may be differences between firms.

If a household is fired, it will remain employed during the current month and becomes unemployed at the beginning of the following month. A reservation wage, which is the minimum

⁶ Concerning double bookkeeping, money is created in a credit contract, once an indebted firm asks for more credit, which is a balance sheet extension. If a household transfers an amount of money from its savings account to the money account, it is a mere asset swap. Both ways, aggregate money is increased. On the other hand, the quantity of money is reduced if a firm transfers it to the savings/credit account in order to pay back debt, which is a balance sheet contraction. A household transferring money to its stock of savings is experiencing an asset swap again. The aggregate quantity of money is decreased in both cases.

⁷ Firms and the government are usually indebted, so they are debtors of the banking system. Households accumulate savings, so they are creditors of the banking system. Money holders (households and firms) are also creditors.

⁸ In a more complex setting, savings and credit are modeled as financial contracts featured by a contract duration of a fixed number of months. This is actually more plausible, since firms consider investments dependent on the current nominal interest rate and expected inflation. An investment decision financed by a fixed credit contract at least preserves decisive credit conditions.

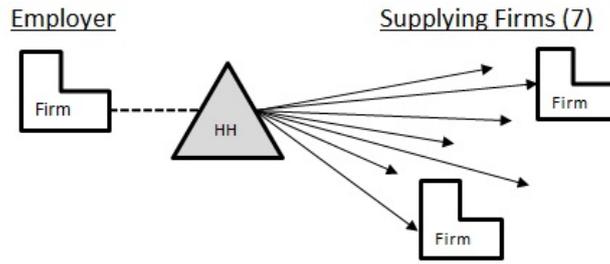


Figure 2.3: Household Connections: Seven firms as trading partners for consumption goods (arrows) and one employer (dashed line).

wage for acceptance of a new job, is set at its latest wage payment. Each month of unsuccessful job search reduces the reservation wage by 5%, which is effectively a small obstacle to employment. The household contacts up to five firms per month to ask for a job. As soon as a firm offers a job, for which the wage exceeds the current reservation wage, the household is employed instantly. With a probability of 10%, an employed household will also contact one firm to ask for a better-paid job. These job offers are refused directly, if the new wage is lower than the current one. If the offered wage is higher, the job is accepted with a probability dependent on the wage difference according to $Prob(Accept) = 1 - e^{-\gamma_w \cdot \ln(2) \cdot \Delta nW}$ (see Figure 2.4), which ensures a restricted level of wage competition.

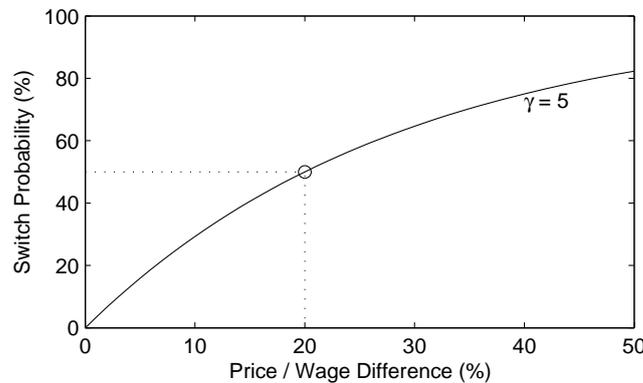


Figure 2.4: Household decision: Probability to change employer (supplier) dependent on wage (price) difference.

Income and Taxes

All monthly payments to households will be carried out at the end of a month, thus determining household income for the following month. Primary income consists of up to three sources: (1) wage from employer i , (2) paid-out profit of firms the household is shareholder of and (3) nominal interest payment on nominal savings/credit account $nSC_{t,h}$. If a household is indebted, $nSC_{t,h}$ and interest payments are negative.

$$nInc_{t,h}^{prim} = nW_{t,i} + n\Pi_{t,h}^{paid} + i_t \cdot nSC_{t,h} \tag{2.2}$$

A tax rate τ_t is imposed equally on all sorts of primary income: $nTax_{t,h} = \tau_t \cdot nInc_{t,h}^{prim}$. Unemployment benefits (nUB_t) are paid if the household was unemployed during the running month. Instead of wage, it receives unemployment benefits, which is 50% of net average wage:

$$nUB_t = 0.5 \cdot (1 - \tau_t) \cdot nWI_{t-1}, \quad (2.3)$$

where nWI_{t-1} is the average wage (or the wage index), calculated as a mean value of previous month' firm wages, weighted by the employment share of each firm.⁹ Household h is left with net income

$$nInc_{t,h}^{net} = \begin{cases} \left(nW_{t,i} + n\Pi_{t,h}^{paid} + i_t \cdot nSC_{t,h} \right) \cdot (1 - \tau_t) & \text{if employed} \\ \left(n\Pi_{t,h}^{paid} + i_t \cdot nSC_{t,h} \right) \cdot (1 - \tau_t) + nUB_t & \text{if unemployed} \end{cases} \quad (2.4)$$

Goods Market Trading Partners

Each household maintains a fixed number of seven connections to firms (cf. *Lengnick, 2013*). Firms, on the other hand, are not limited in the number of connections to customers. Connections to supplying firms are adjusted slowly and infrequently, thus expressing loyalty of customers and stability of trading relations. Nevertheless, each household adapts its list of firm connections monthly due to price consideration, customer restrictions and randomly.

1. With a fixed probability of $p_p = 25\%$, households search for cheaper trading partners. One existing (old) connection is chosen randomly, and one other (new) firm is also chosen randomly. If the new price is higher than the old price, the existing connection is kept. Otherwise, the price difference is translated into a probability to replace the existing connection: $Prob(Switch) = 1 - e^{-\gamma_p \cdot \ln(2) \cdot \Delta P}$ (see Figure 2.4). This way, imperfect price competition is established, but customers do not react strongly when price differences are negligible.
2. If a firm is sold out, it is unable to satisfy further customer requests. In the respective trading day, restricted customers simply buy from the next firm. However, being restricted more often by the same firm induces households to replace the respective trading connection. Again, with a probability $p_r = 25\%$ one (restricted) connection is reconsidered and eventually replaced by a random new firm. The probability for a switch is dependent on the severeness of the restriction compared to consumption plans, while negligible restrictions will have no consequences.¹⁰
3. Finally, trading connections are exchanged randomly with a low percentage of $p_s = 2\%$. With a sum of $2,000 \cdot 7 = 14,000$ firm connections, random rearrangement concerns 40

⁹ $nWI_{t-1} = \sum_{i=1}^{N_f^i} nW_{t-1,i} \cdot \frac{L_{t-1,i}}{\sum_{j=1}^{N_f^i} L_{t-1,j}}$

¹⁰ Restrictions of demand are measured in daily consumption packages: RD_t . Probability to change a connection depends on last month' restriction: $Prob(Switch) = 1 - e^{-\gamma_r \cdot \ln(2) \cdot RD_{t-1}}$. Restrictions by more than one firm will induce replacement of one connection dependent on the relation of the restriction between the two (or more) restricting firms.

connections per months. This random customer redistribution ensures that small firms do not run out of customers but are stabilized at some point.

Consumption

Simulated households plan consumption based on current income, not taking into consideration the stock of savings, which may be a multitude of income. However, it is assumed that households offset expected real devaluation of savings stocks ($nSC_{t,h} \cdot \pi_t^e$) by directly reinvesting the respective amount, where π_t^e is expected inflation (as defined below in section 2.2.4). Therefore, consumption-relevant net income is reduced, so that interest income is only relevant for consumption, as long as it is generated by the *real* interest rate r_t .¹¹ A personal price index $nPI_{t,h}^c$, which is the average price of current trading partners, is calculated to determine the purchasing power of individual income. Consumption-relevant real net income is therefore

$$RNI_{t,h} = \frac{nInc_{t-1,h}^{net} - nSC_{t,h} \cdot \pi_t^e}{nPI_{t,h}^c} \quad (2.5)$$

All households are assigned a common intercept parameter \bar{C}_t and a common marginal rate of consumption c .¹² Consumption also depends negatively on the expected real interest rate $r_t = i_t - \pi_t^e$, which is not only plausible, but also derived from optimizing behavior of representative agents in DSGE models like *Woodford* (2003).

Households plan real consumption at the beginning of each month.¹³ It is strongly affected by latest consumption, so it adjusts only gradually to a new income level, modeled with a parameter of consumption inertia $\lambda_c = 0.9$. Therefore, a household adjusts at a monthly rate of $(1 - \lambda_c) = 10\%$ to a new income level, for example if there is a change in employment status. Planned real consumption of household h is:

$$C_{t,h} = \lambda_c \cdot C_{t-1,h} + (1 - \lambda_c) \cdot (\bar{C}_t + c \cdot e^{-r_t} \cdot RNI_{t,h}) \quad (2.6)$$

¹¹ *Oeffner* (2009) went further and made interest payments reinvested completely, not only the part of nominal interest that offsets inflationary devaluation. He argues at length why reinvestment of interest payments is a crucial stability condition for the simulated economy. Otherwise, a rising interest rate would increase household income and induce higher consumption expenditure, so that economic activity were counter-factually stimulated by “tight” monetary policy.

¹² A simple linear Keynesian consumption function is adopted here as explained in *Mankiw* (2000, p. 480). Parameter $c = 0.85$ reflects the marginal propensity to consume, whereas the intercept parameter is set at a small share of previous month’ net average wage: $\bar{C}_t = 0.18 \cdot (1 - \tau_t) \cdot nWI_{t-1}$, therefore the intercept grows at the same rate as the entire economy.

¹³ Usually, *planned* real consumption becomes *actual* real consumption. If one of the randomly chosen trading partners is sold out, the household simply chooses randomly the next firm out of 7 (see section 2.2.3) to satisfy its daily demand. Since all firms provide excess production capacities, it practically never happens that a household is restricted completely by all seven firms, even if the case is a theoretical possibility.

2.2.4 Firm

Technology

Firms employ *Cobb-Douglas* production technology with factor inputs capital $K_{t,i}$, labor $L_{t,i}$ and technology parameter A_t . Daily production capacity of firm i with capital exponent $\alpha = 0.2$ is: $Y_{t,i}^{c,day} = K_{t,i}^\alpha \cdot (A_t \cdot L_{t,i})^{1-\alpha}$. Thus, monthly production capacity multiplies the former equation by 30 days per month:

$$Y_{t,i}^c = 30 \cdot K_{t,i}^\alpha \cdot (A_t \cdot L_{t,i})^{1-\alpha} \tag{2.7}$$

Technology parameter A_t is identical for all firms and grows at a constant exogenous rate of labor-enhancing technological progress $g_A = 1.2\%$ annually (or 0.1% per month), so $A_t = A_{t-1} \cdot e^{g_A}$ (cf. *Barro and Sala-i-Martin*, 2003, chapter 3).

Capital Stock Development

In contrast to the original *Lengnick* (2013) model, each firm not only acts as a supplier but also as a customer of goods and services, when it invests in real capital. There is no distinction between consumption goods and investment goods.¹⁴ Investing firms and consuming households contribute to a unique demand flow. To purchase goods or services for investment, each firm maintains a limited number of seven connections to other supplying firms. These supplier connections are reconsidered monthly equivalent to household connections in section 2.2.3 with regard to prices, restrictions and randomly. At the beginning of each month, connections are reconsidered and replaced with some probability.

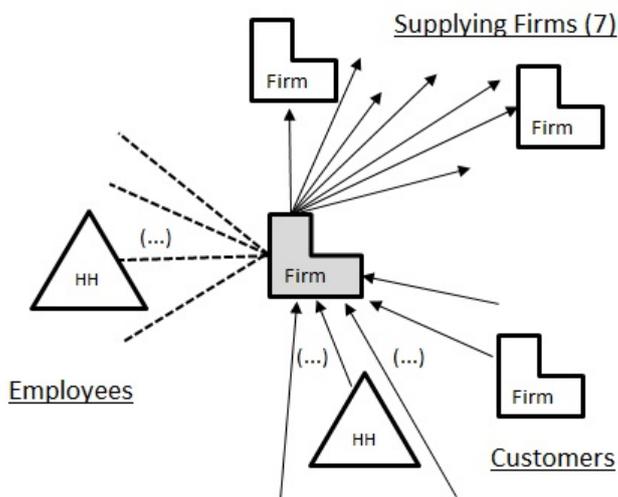


Figure 2.5: Firm Connections: Number of trading partners for investment goods supply is limited to 7, number of employees (households) and customers (households and firms) is not limited.

¹⁴The same implicit assumption is part of the *Solow* (1956) model, where aggregate output is split into investment ($s \cdot Y$) and consumption.

Each firm maintains a stock of real capital goods, which is devalued by parameter ρ at the end of each month at an annual rate of 9.6% (or monthly 0.8%). A firm invests according to its monthly gross investment plan, which is discussed later (see section 2.2.4). On a daily basis, each firm purchases goods from other firms to accomplish its investment plan. As soon as the month has passed, the sum of newly purchased investment goods is added to the capital stock. It is ready for productive use in the following month. Firm capital evolves according to

$$K_{t,i} = K_{t-1,i} + \underbrace{I_{t-1,i} - \rho \cdot K_{t-1,i}}_{I_{t-1,i}^{net}} \quad (2.8)$$

Key Data for Firm Decisions

Firm decisions are built upon few key variables, some of which are explained here:

Utilization: Capacity utilization is defined as real sales divided by production capacity:

$$U_{t,i} = \frac{Y_{t,i}^{sales}}{Y_{t,i}^c} \quad (2.9)$$

In order to prevent supply shortages with a likely loss of trading relations to customers, firms provide for excess capacity. The target value U^* is 85%, permitting firms to accommodate demand fluctuations.¹⁵ Prices are set such that they offset cost of temporarily idle resources. For decision-making, firms determine a short-run weighted average level of utilization of last $T_u = 6$ months, where weights are highest for the most current month and decline linearly:

$$\bar{U}_{t,i} = \sum_{s=1}^{T_u} U_{t-s,i} \cdot \frac{(T_u + 1 - s)}{0.5 \cdot (T_u \cdot (T_u + 1))} \quad (2.10)$$

Expected Inflation: Inflation is measured by the price index PI_t which is a mean price of all $N^{fi} = 100$ firms in month t , weighted by their (real) market shares:

$$nPI_t = \sum_{i=1}^{N^{fi}} nP_{t,i} \cdot \frac{Y_{t,i}^{sales}}{\sum_{i=1}^{N^{fi}} Y_{t,i}^{sales}} \quad (2.11)$$

Annual inflation is the logarithmic difference of the price index with 12 months lag: $\pi_t = (\ln(nPI_t) - \ln(nPI_{t-12}))$. Annualized *monthly* inflation is the logarithmic difference of neighboring values of the price index: $\pi_t^m = 12 \cdot (\ln(nPI_t) - \ln(nPI_{t-1}))$.

For simplicity, inflation expectations are homogeneous among all agents. It is assumed that medium-run expected inflation is adaptive based on the last $T^\pi = 24$ monthly inflation rates as a weighted mean value with linearly declining weights. It is further assumed that central

¹⁵ Idle resources are quite normal: Think of restaurants, empty shops, hotel rooms, or food production. Apparently, the degree of utilization of available resources is not 100% in many branches, but on average well below, particularly in the service sector. Moreover, consumption goods are often short-lived (food), go out of fashion or out of date (clothes, electronics) and are costly to store. Respective firms will have to deal with idle resources and emergent cost of produced goods that can not be sold immediately.

bank announcements of the inflation target π^* influence expectations directly to a some extent with $\lambda_\pi = 0.1$ reflecting central bank credibility. Expected inflation is

$$\pi_t^e = \lambda_\pi \cdot \pi^* + (1 - \lambda_\pi) \cdot \sum_{s=1}^{T^\pi} \pi_{t-s}^m \cdot \frac{(T^\pi + 1 - s)}{0.5 \cdot T^\pi \cdot (T^\pi + 1)} \quad (2.12)$$

Profit Rate: Let return on capital (RoC , elsewhere also termed h , e.g. in *Hein and Schoder*, 2011) relate profit with capital stock value. In order to assess profitability per unit of invested capital, nominal interest payments on firm debt are left out of the calculation. Therefore, return on capital is calculated by subtracting costs of wages and capital depreciation from firm turnover, and to divide it by the current capital stock, valued by the mean price $nPI_{t,i}^K$ of currently connected suppliers of investment goods:

$$RoC_{t,i} = \frac{nP_{t,i} \cdot Y_{t,i}^{sales} - nW_{t,i} \cdot L_{t,i} - \rho \cdot K_{t,i} \cdot PI_{t,i}^K}{K_{t,i} \cdot nPI_{t,i}^K} \quad (2.13)$$

The profit rate of last 12 months $\overline{RoC}_{t,i}$ is calculated as a simple mean value of monthly profit rates. Firm decisions for investment also consider the development of the profit rate, in particular the difference between $\overline{RoC}_{t,i}$ and the average profit rate 12 months before, which is $\overline{RoC}_{t-12,i}$.

Decision: Price

Firms adjust their prices only infrequently as a result of menu costs. The current firm market price $nP_{t,i}$ is accompanied by a target price nP_i^* which reflects the exact price the firm would be willing to choose in the absence of menu costs. If the target price deviates by more than (an arbitrary threshold of) 1.5% from the current market price, the firm sets the current target price as a new market price.

$$nP_{t,i} = \begin{cases} nP_{t-1,i} & \text{if } \frac{nP_{t,i}^* - nP_{t,i}}{nP_{t,i}} \in (0.985; 1.015) \\ nP_{t,i}^* & \text{else} \end{cases} \quad (2.14)$$

Similar to the *Calvo* (1983) model, this price setting behavior ensures that only a fraction of firms changes its price in a certain period. However, unlike the *Calvo* approach, where firms are forced to wait for a random event (the “*Calvo* fairy”) to finally being allowed to adjust the price, firms decide freely about the timing of price adjustments here.

The target price evolves monthly with expected inflation and capacity utilization: With a high level of capacity utilization, a firm is more likely and willing to increase the price, considering itself in a strong market position. Deviations below target utilization trigger price drops, utilization above target leads to a rising target price. The probability of an additional target price movement is modeled with a random decision, whose probability is given by a reversed bell curve with standard deviation $\sigma = 0.14$ as shown in Figure 2.6. The

price change decision is given by:

$$DP_{t,i} = \begin{cases} 1 & \text{if } (\bar{U}_{t,i} - U^* \geq 0) \text{ with } Prob = 1 - e^{-\left(\frac{\bar{U}_{t,i} - U^*}{\sigma}\right)^2} \\ -1 & \text{if } (\bar{U}_{t,i} - U^* < 0) \text{ with } Prob = 1 - e^{-\left(\frac{\bar{U}_{t,i} - U^*}{\sigma}\right)^2} \\ 0 & \text{with } Prob = e^{-\left(\frac{\bar{U}_{t,i} - U^*}{\sigma}\right)^2} \end{cases} \quad (2.15)$$

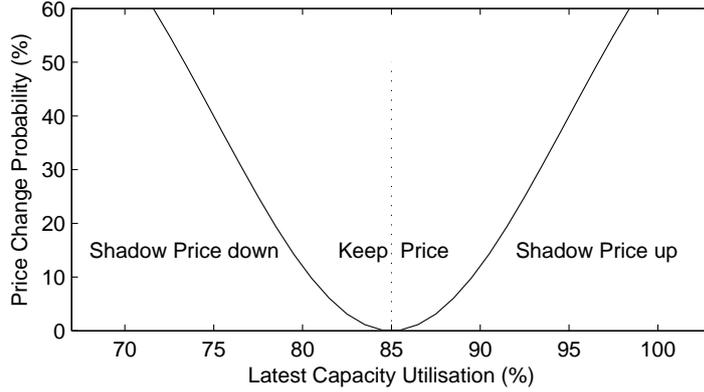


Figure 2.6: Probability to change the firm shadow price. When capacity utilization is low, price is likely decreased, for high utilization, price is likely increased.

Then, the target price is actually moved up or down by ε_p , which again is a random variable that follows a uniform distribution between 0% and 1.5%.¹⁶ Therefore, utilization below target implies a likely reduction (or slower increase) of the target price. Utilization above target level implies a higher probability of additional increases. In the absence of such acceleration of the slow inflationary price drift, an annual inflation of approximately 1.2% leaves the market price unchanged for more than 12 months on average, before price adjustments are triggered.

$$nP_{t,i}^* = nP_i^* \cdot (1 + \pi_t^e + DP_{t,i} \cdot \varepsilon_p), \quad \varepsilon_p \sim U(0, 0.015), \quad (2.16)$$

Please note that deviation of firm utilization from target, $\bar{U}_{t,i} - U^*$, is a similar concept as the output gap in *New Keynesian* (NK) models. Price setting of an individual firm depends on this “utilization gap” and is effectively a similar mechanism as the *New Keynesian Phillips Curve* (cf. *Woodford*, 2003). Target utilization at 85% of production capacity equates to production at 100% of the so-called production potential of a representative firm. In NK models, the output gap directly affects inflation with an estimated parameter. Here, deviation from target utilization of a single firm generates price movements only with a certain probability, while on aggregate this stochastic element reliably generates inflation dynamics following economic activity. One key difference is that NK models assume rational expectations, while the presented ACE model applies adaptive inflation expectations. However, NK models often use

¹⁶Since latest utilization is correlated with neighboring values, a high degree of latest utilization will likely be followed by another one, so the target price may rise several months in a row.

hybrid *Phillips* curves which incorporate forward-looking and also backward-looking inflation (hybrid NKPC).

Decision: Hire or Fire

Short-run fluctuations of demand and utilization will be accommodated within a corridor (U^{low}, U^{up}) around the target value $U^* = 0.85$, without any adjustments to production. However, if latest utilization rises to levels above $U^{up} = 0.91$, the firm will create an open position by increasing the employment target $L_{t,i}^*$. A household asking for a job in that month is employed immediately, if the firms' offered wage is high enough for the household to accept it. On the other hand, if latest utilization falls below $U^{low} = 0.78$, the employment target is decreased so that a random worker is fired at the beginning of the following month:

$$\text{Firm Decision: } \begin{cases} L_{t,i}^* = L_{t-1,i}^* - 1 \text{ (Fire)} & \bar{U}_{t,i} < U^{low} \\ L_{t,i}^* = L_{t-1,i}^* + 1 \text{ (Hire)} & \bar{U}_{t,i} > U^{up} \\ L_{t,i}^* = L_{t-1,i}^* & U^{low} \leq \bar{U}_{t,i} \leq U^{up} \end{cases} \quad (2.17)$$

However, if employment of the firm has changed during the last $T_u = 6$ months, the firm has to recalculate its degree of latest utilization $\bar{U}_{t,i}$ with production capacity values based on current employment. Then, firm employment decisions are based on hypothetical values of past utilization. For example, if the firm just fired a worker, past sales are compared to now reduced production capacity.

Decision: Wage

A firm-specific wage contract is fixed for several months. Similar to price setting behavior, a target wage develops permanently, while the actual market wage is adjusted only infrequently. When a new wage contract is due at time s , the current target wage is set as new firm wage.

$$nW_{t,i} = nW_{t,i}^* \quad \text{if } t = s \quad (2.18)$$

The new wage contract runs until month $s = t + 10 + \nu$, while ν follows a discrete uniform distribution between 0 and 4, so duration for the new contract is a random number between 10 and 14 months. Only a part of firms adjusts its wage in each month; average contract duration is 12 months.

Owners of the capital stock and workers struggle for their proper share of generated value added. The development of firm target wages depends on (1) expected inflation, (2) labor productivity growth, (3) latest utilization (4) the deviation of current markup (of price over unit wage cost) from its target value and (5) whether the firm was able to fulfill its employment target lately. The target wage is adjusted monthly:

1. It is increased by expected monthly inflation, so $nW_{t,i}^*$ is multiplied by $(1 + \pi_t^e)$.
2. The rate of labor productivity growth is a natural part of wage negotiations. The target wage is therefore multiplied by: $(1 + g_A)$

3. High utilization increases firm profit, so negotiation power of employees increases with high utilization. Monthly deviations from utilization target are translated with factor $a_u = 0.05$ to changes in the target wage. nW^* is multiplied by $(1 + a_u \cdot (\bar{U}_{t,i} - U^*))$.
4. The firm compares wage cost per unit of production with its current market price. Assuming a target value of $m^* = 60\%$ for a markup of price over unit wage cost, the current markup $m_{t,i}$ is closing the gap to its target value m^* at a rate of $a_m = 3\%$ per month. The target wage is multiplied monthly by $(1 + a_m \cdot \ln(\frac{m_{t,i}}{m^*}))$.
5. If the firm was unable to fulfill its employment target $L_{t,i}^*$ in the last month, for example when no household was willing to get hired, the shadow wage is increased by a random variable ε_w , which follows a uniform distribution between 0 and 0.01. If the firm was able to fulfill its employment target for the past $T_w = 6$ months, the shadow wage is decreased by that random variable. Therefore, the target wage is changed additionally, if $DW_{t,i}$ is different from 0:

$$DW_{t,i} = \begin{cases} 1 & \text{if } L_{t,i}^* > L_{t,i} \\ -1 & \text{if } L_{t-s,i}^* = L_{t-s,i}, \quad s = 0, 1, \dots, T_w - 1 \\ 0 & \text{else} \end{cases} \quad (2.19)$$

The target wage is multiplied by $(1 + DW_{t,i} \cdot \varepsilon_w)$ with $\varepsilon_w \sim U(0, 0.01)$. Note that this last part of the wage setting mechanism is adopted from *Lengnick* (2013).

In addition, the target wage evolves according to:

$$nW_{t,i}^* = nW_{t-1,i}^* \cdot \left(1 + \pi_t^e + g_A + a_u \cdot (\bar{U}_{t,i} - U^*) + a_m \cdot \ln\left(\frac{m_{t,i}}{m^*}\right) + DW_{t,i} \cdot \varepsilon_w \right) \quad (2.20)$$

Decision: Investment

In the context of investment decisions, hypothetical profit $n\Pi_{t,i}^{hyp}$ is maximized, which ignores *nominal* interest payments, but optimizes capital input with respect to the *real* interest rate:

$$n\Pi_{t,i}^{hyp} = nP_{t,i} \cdot Y_{t,i}^{Sales} - nW_{t,i} \cdot L_{t,i} - (r_t + \rho) \cdot K_{t,i} \cdot nPI_{t,i}^K \quad (2.21)$$

$nPI_{t,i}^K$ is the mean price of current trading partners for capital investment (which is close to the general price index nPI_t), $r_t = i_t - \pi_t^e$ is the expected real interest rate. The firm calculates with output at target utilization U^* :

$$Y_{t,i}^{Sales} \approx U^* \cdot Y_{t,i}^c = U^* \cdot 30 \cdot K_{t,i}^\alpha \cdot (A_t \cdot L_{t,i})^{1-\alpha} \quad (2.22)$$

The capital stock is optimal when the marginal productivity of capital equals its marginal running cost, i.e. capital depreciation and real interest payments.

$$\frac{\partial n\Pi_{t,i}^{hyp}}{\partial K_{t,i}} = 0 = nP_{t,i} \cdot U^* \cdot \frac{\partial Y^c}{\partial K} - (i_t - \pi_t^e + \rho) \cdot nPI_{t,i}^K \quad (2.23)$$

Rearranging yields the target capital stock:

$$K_{t,i}^* = \left(\frac{nP_{t,i}}{nP_{t,i}^K} \cdot \frac{U^* \cdot 30 \cdot \alpha}{(i_t - \pi_t^e + \rho)} \right)^{\frac{1}{1-\alpha}} \cdot A_t \cdot L_{t,i} \quad (2.24)$$

All terms in brackets of equation (2.24) are constant or stable in the long run. Therefore, we see that the target capital depends linearly on labor input $L_{t,i}$ and technology parameter A_t . Once a firm hires a worker, marginal productivity of capital rises, so that K^* rises proportionally. When employment is constant, A_t grows at a constant rate g_A , so K^* also grows at that rate. For the whole economy, aggregate capital also grows at g_A .

Investment of a firm depends on (1) capital depreciation, (2) distance to the target capital stock $K_{t,i}^*$, (3) last years average profit rate $\overline{RoC}_{t,i}$ and (4) the change in the average profit rate.

1. The base level of gross investment is set by real capital depreciation $\rho \cdot K_{t,i}$.
2. It is multiplied by target capital divided by current capital. The resulting convergence to target capital also brings about a dependence of current investment on employment $L_{t,i}$, current capital $K_{t,i}$ and real interest rates r_t). Constant growth of A_t also determines average net investment to be positive.
3. The return rate to the capital stock of the last 12 months influences investment, since capital is invested where it is most productive and profitable. Gross investment is multiplied by $(1 + a_h \cdot \overline{RoC}_{t,i})$.
4. The development of profit also influences investment, as it was claimed by *Kalecki* as described by *Dobb* (1973, p.222). Investment is multiplied by $(1 + a_{\Delta h} \cdot (\overline{RoC}_{t,i} - \overline{RoC}_{t-12,i}))$. As it turns out, this term strongly determines business cycle dynamics on aggregate level.

Finally, since investment decisions are often carried out with some lag, actual gross investment is adjusted slowly with an investment inertia parameter $\lambda_I = 0.9$. In sum, planned gross investment is:¹⁷

$$I_{t,i} = \lambda_I \cdot I_{t-1,i} + (1 - \lambda_I) \cdot \left(\rho \cdot K_{t,i} \cdot \frac{K_{t,i}^*}{K_{t,i}} \cdot (1 + a_h \cdot \overline{RoC}_{t,i} + a_{\Delta h} \cdot (\overline{RoC}_{t,i} - \overline{RoC}_{t-12,i})) \right),$$

which boils down to

$$I_{t,i} = \lambda_I \cdot I_{t-1,i} + (1 - \lambda_I) \cdot \rho \cdot K_{t,i}^* \cdot (1 + (a_h + a_{\Delta h}) \cdot \overline{RoC}_{t,i} - a_{\Delta h} \cdot \overline{RoC}_{t-12,i}). \quad (2.25)$$

¹⁷Usually, *planned* real gross investment becomes *actual* real gross investment. If one of the randomly chosen trading partners is sold out, the firm simply randomly chooses the next supplier (out of 7) to satisfy its daily investment demand. Since all firms have excess production capacities, it practically never happens that a firm is restricted completely by all seven trading partners.

Decision: Profit Payout

Payout decisions are based on realized profit $n\Pi_{t,i}$. It is different from hypothetical profit (as explained above in section 2.2.4), since realized profit is determined by calculating nominal sales subtracted by realized cost for wages, *nominal* interest payments and capital depreciation:

$$n\Pi_{t,i} = nP_{t,i} \cdot Y_{t,i}^{sales} - w_{t,i} \cdot L_{t,i} + i_t \cdot SC_{t,i} - \rho \cdot K_{t,i} \cdot nPI_{t,i}^K \quad (2.26)$$

Each firm decides about paid-out profit, which is distributed equally among shareholders. Overall profit is either paid out or kept in to increase equity, i.e. to reduce the debt ratio: $n\Pi_{t,i} = n\Pi_{t,i}^{paid} + n\Pi_{t,i}^{kept}$. If realized profit is negative, no profit is paid out. Otherwise, as long as the debt ratio is lower than 50%, all profits are paid out to shareholders. If the debt ratio increases to levels above 50%, only half of the profit is paid out.

$$n\Pi_{t,i}^{paid} = \begin{cases} 0 & n\Pi_{t,i} \leq 0 \\ 0.5 \cdot n\Pi_{t,i} & (n\Pi_{t,i} > 0) \& \left(\frac{nSC_{t,i}}{nPI_{t,i}^K \cdot K_{t,i}} < -0.5 \right) \\ n\Pi_{t,i} & (n\Pi_{t,i} > 0) \& \left(\frac{nSC_{t,i}}{nPI_{t,i}^K \cdot K_{t,i}} \geq -0.5 \right) \end{cases} \quad (2.27)$$

2.2.5 Government

Ten per cent of all households are employed by public authorities at private sectors average wage. Randomly chosen workers are publicly employed at the start of the simulation and never change their employer. Public employment does not play a vital role in the model: Employees simply receive monthly wages and do not produce any goods.¹⁸

The fiscal surplus (or deficit) nFS_t is calculated as public revenue minus public cost. Revenue is composed of taxes and seignorage gain (see below). Cost includes unemployment benefits, wages for public employment $L^{st} = 200$ and interest on public debt:

$$nFS_t = \underbrace{nTax_t + nSG_t}_{\text{revenue}} - \underbrace{UP_t \cdot nUB_t - L^{st} \cdot nWI_{t-1} + i_t \cdot nSC_t^{st}}_{\text{public cost}}, \quad (2.28)$$

where monthly tax revenue $nTax_t = \tau_t \cdot nInc_t$ depends on the tax base, which is current household income $nInc_t$. Public employees receive wage payments as high as previous month' wage index nWI_{t-1} . All unemployed persons UP_t receive monthly unemployment benefits UB_t (see section 2.2.3). There is also a rather small seignorage gain nSG_t from the banking system, which is transferred to the treasury and is therefore a public revenue. It emerges because money, as a liability of the banking system, is interest-free, while the opposing credit on the banks' balance sheet is met with interest payments. Interest payments *to the bank* by debtors outweigh interest payments *by the bank* to creditors. The difference is $nSG_t = i_t \cdot (nM_t^{hh} + nM_t^{fi})$.

¹⁸ Public employment was introduced to permanently generate a considerable amount of public cost, so that the income tax rate ranges around 10% rather than 0.5%.

The sum of average public cost subtracted by the seignorage gain represents the financing requirement of the general income tax. The respective tax base is composed of all sorts of primary income, on which the general income tax is applied. The tax rate is set to balance long-run public cost and long-revenue so that public debt remains stable. It is calculated as an average long-run financing requirement. However, the government approves a small budget deficit on average, since nominal output is constantly growing. The tax rate is set so that it generates revenue as high as 95% of financing requirements *on average*, adjusting very slowly.

$$\tau_t = \frac{1}{300} \cdot \sum_{s=t-300}^{t-1} 0.95 \cdot \frac{UP_t \cdot nUB_t + L^{st} \cdot nWI_{t-1} - i_t \cdot nSC_t^{st} - nSG_s}{nInc_s} \quad (2.29)$$

In this long-run perspective, the tax rate remains stable across business cycles and serves as an automatic stabilization mechanism. In a recession, for example, when public revenue stagnates and public cost rises, a stable tax rate ensures that the fiscal deficit (and public debt) rises, so that aggregate demand is dampened automatically (see section 2.3.4).

2.2.6 Monetary Policy

The central bank aims at price stability by employing the nominal interest rate i_t as its sole instrument. Price stability is accomplished if medium-run inflation remains close to its target value of $\pi^* = 1.2\%$ annually (0.1% per month). The central bank influences inflation rates directly via inflation expectations (cf. equation (2.12)). Moreover, the aggregate level of economic activity is also influenced by i_t . It is the sum of aggregate consumption and investment.

$$\begin{aligned} \sum_{i=1}^{Nfi} Y_{t,i}^{sales} &= \sum_{h=1}^{Nhh} C_{t,h} + \sum_{i=1}^{Nfi} I_{t,i} \\ Y_t^{Sales} &= C_t + I_t. \end{aligned} \quad (2.30)$$

Firm investment as well as household consumption both depend negatively on the real interest rate. Therefore, monetary policy transmission runs along a consumption channel and an investment channel. For comparison, baseline *New Keynesian* models apply an IS-curve without investment activity, which is derived from a representative consumer, optimizing the level of economic activity dependent on real interest rates. Both cases share negative dependence of economic activity on r_t . In the ACE model, consumption also depends strongly on current income and past consumption.

The nominal interest rate is set by the central bank according to the *Taylor* (1993) rule, except that the output gap is not part of the equation.

$$i_t = r^* + \pi^* + 1.5 \cdot (\pi_{t,i} - \pi^*), \quad (2.31)$$

where r^* is the long-run real interest rate that is considered neutral with respect to monetary policy (*Blinder*, 1999), $\pi^* \approx 1.2\%$ is the annual inflation target (0.1% monthly) and $\pi_{t,i}$ is

inflation during last 12 months.¹⁹ In the simulation, r^* is adjusted if the central bank does not meet its inflation target over a long period. Deviation of measured inflation above (below) target are responded by more than proportionate raises (declines) of the nominal interest rate, thus dampening (stimulating) economic activity by raising (lowering) the real interest rate. The *Taylor* principle is expressed in the more than proportionate policy reaction with a factor of 1.5 to deviations from target.

2.3 Simulation and Model Behavior

The simulation is started with initialization. At the beginning, all virtual agents are created and randomly connected with respect to employment and trading relations. 100 shares per firm are also randomly distributed equally to 2,000 households. Then, reasonable starting values for capital stock and household consumption are calculated based on model parameters. Firms are initially indebted by 50% of the capital stock value, thus firms are partly financed externally.²⁰ To ensure that all monetary accounts sum up to zero, the sum of firm credit is accompanied by the transfer of an identical amount of deposits to bank accounts of households, both as transaction money and savings. Household savings are also distributed equally, rendering the initial wealth distribution completely equal with small random fluctuations. Finally, the simulation starts and requires some "burn-in" adjustment time to adjust initial firm values and random connections to stable proportions.

2.3.1 Single Firm

This section covers a closer investigation of individual firm behavior. Key firm variables across business cycles are analyzed. The left side of figure 2.7 shows how price setting behavior of

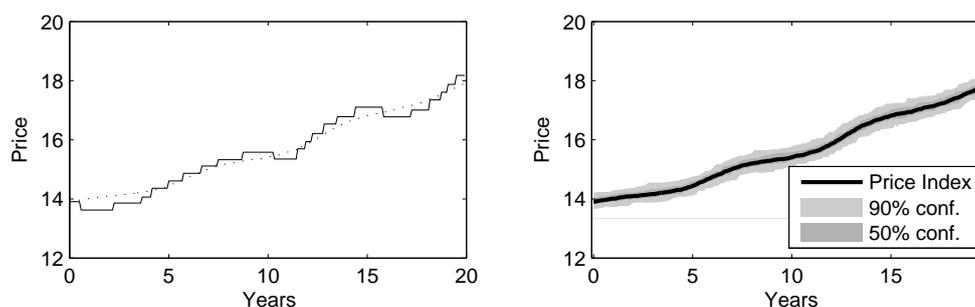


Figure 2.7: Firm price (left) with price index (dotted) and distribution of firm prices across business cycles (right).

¹⁹To be precise, the central bank uses a weighted average of inflation rates of different horizons. It consists of 3-month-inflation (weight 0.25), 6-month-inflation (weight 0.25) and annual inflation (weight 0.5) in order to respond directly and properly to short-run developments.

²⁰Given that firms pay out profits as explained in section 2.2.4, simulation shows that firm debt ratios are stable at around 50%, as long as the simulated economy does not experience strong movements of the price level like a deflation.

firms works in the simulated economy. On average, the firm adjusts its price less than once in a year, while in times of recessions, price adjustment is delayed even more due to reduced pressure on prices. For comparison, the dotted line depicts the current aggregate price index. A considerable deviation of a firm price from competing prices affects firm sales strongly, so price competition prevents firm prices from diverging. The graph on the right side depicts the aggregate price index surrounded by shaded areas, which indicate the (narrow) distribution of firm prices. Competition for customer connections keeps prices on a comparable level.

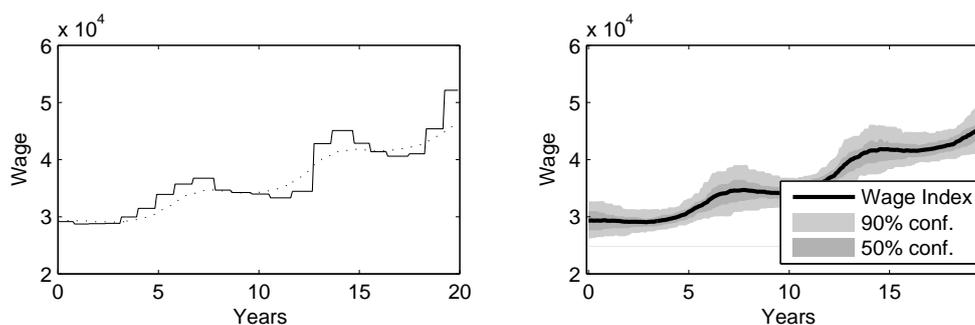


Figure 2.8: Firm wage (left) with wage index (dotted) and distribution of firm wages across business cycles (right).

Similarly, the diagram on the left side of figure 2.8 shows how wage contracts are actually fixed across business cycles. On average, firm wage is adjusted once every 12 months, while sometimes there are bigger “jumps”, when employees are in a good position for negotiation. For comparison, the dotted line depicts the current aggregate wage index. A considerable deviation of a firm wage from competing wages has consequences for the firm’s ability to fulfill its employment target. Since wage competition is less intense than price competition in the presented setting, wages fluctuate more than prices. The diagram on the right side depicts the wage index surrounded by shaded areas, indicating the distribution of firm wages. In times of recession, firm wages deviate stronger from average, while increasing competition for employees during boom induces wages to converge.

More firm-specific variables are shown in Figure 2.9, such as monthly real firm sales (solid line) together with production capacity (dotted line) in the upper left. Excess capacity accommodates short-run fluctuations of monthly demand. Below, an internal variable of the modeled firm is shown, namely the number of trading connections to customers (mostly households, but also firms). Actual sales fluctuate, but follow on average the number of connections, whereas there is an upward trend of sales due to rising labor productivity and therefore rising income. Please note that the number of connections tends to rise whenever the firm price is below average and vice versa (compare to Figure 2.7, left side). The bottom graph on the left is latest utilization as a weighted average of last six months, which is a key variable for firm decisions as explained above. Whenever average utilization leaves the corridor around its target, firm employment is adjusted, which is shown on the upper right. Employment of this firm ranges between 22 and 29 in the shown time period, whereas changes in employment

are triggered by latest utilization.²¹

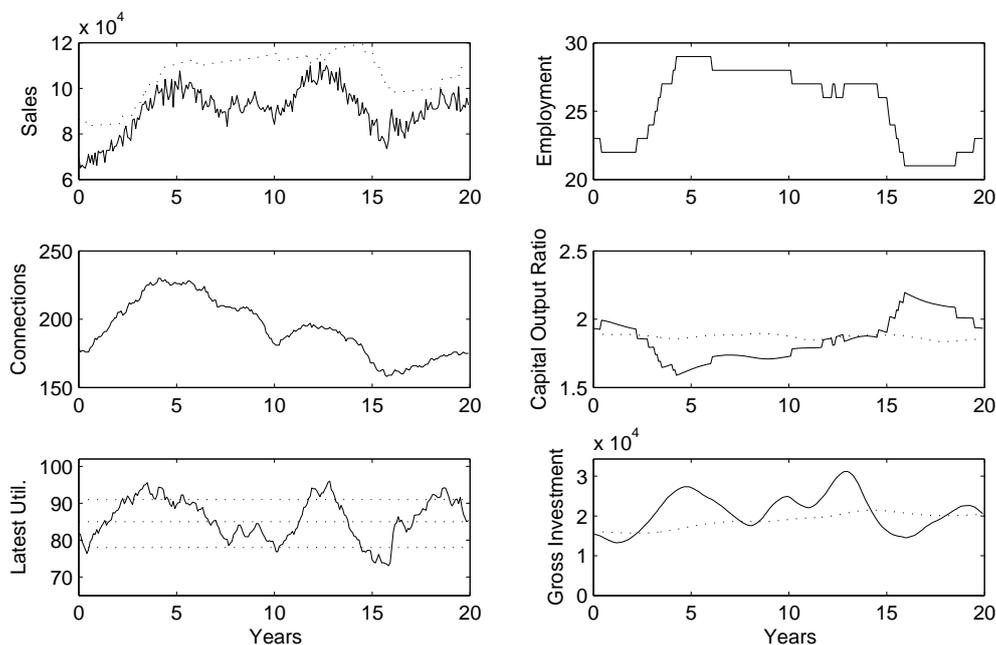


Figure 2.9: Firm-specific variables across business cycles: Demand, customer connections, average utilization (left), employment, capital-output ratio, gross investment (right).

Firms' capital-output ratio is depicted below employment. Whenever a worker is hired or fired, there is a jump in this ratio. Please note the dotted line, which is the average capital-output-ratio of all firms (≈ 1.85). If a firm's capital output ratio deviates from the average value, there is a tendency of convergence back to the dotted line. Since investment activity depends on the optimal capital stock, which itself depends on employment, hiring and firing decisions are followed by investment decisions that support the new level of employment. Gross investment is depicted as solid line in the bottom diagram on the right side, together with a dotted line, indicating the amount of capital depreciation. When the firm reduces employment, net investment may become even negative, while hiring decisions are followed by growing investment activity. Therefore, investment of a firm clearly follows employment²², although it also depends on past profits and the current interest rate.

Figure 2.10 shows monthly utilization of a firm (left) and the distribution of firm utilization (right). Firm sales and corresponding utilization degrees are fluctuating from month to month. Therefore, the distribution of utilization values is scattered with considerable variance. If monthly utilization leaves the corridor bounded by dotted lines only once in a while, the firm does not immediately hire or fire workers. Yet if the short-run average of utilization (left side of figure 2.11) leaves the respective corridor, employment decisions are triggered immediately.

²¹ Firm employment is 17 – 18 on average: 200 households are public employees, so that 100 firms employ up to 1,800 households.

²² The *Solow* (1956) growth model predicts this dependency on aggregate level, i.e. population growth induces additional capital accumulation.

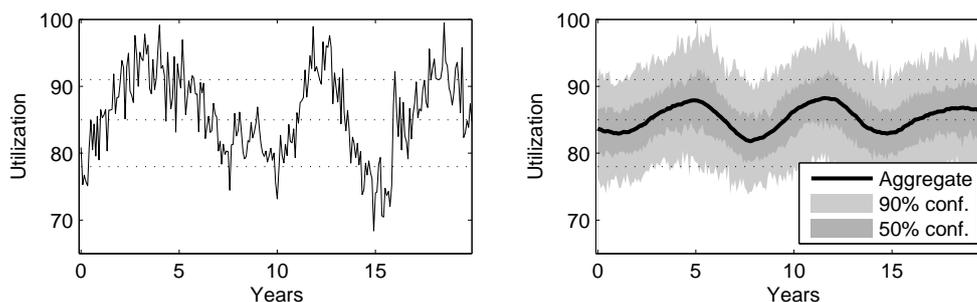


Figure 2.10: Distribution of firm utilization across business cycles.

The right side shows the distribution of latest utilization, whose variance is considerably lower compared to monthly utilization, since fluctuations of firm sales from one month to the next are smoothed away to a large extent. The business cycle is clearly visible, with a higher percentage of firms beyond the upper corridor line in booms and a higher percentage beyond the lower dotted line in times of recession.

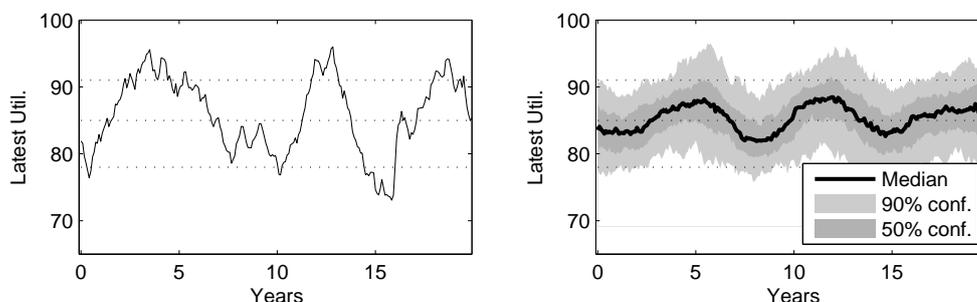


Figure 2.11: Distribution of short-run average firm utilization across business cycles.

A firm's return on capital (RoC) (as defined in equation (2.13)) is shown in the left diagram of figure 2.12, whereas the graph on the right side shows the distribution of return rates to capital across business cycles. Please compare to firm utilization in figure 2.10 to recognize that monthly returns are strongly correlated with sales and utilization. The rate of return to capital of most firms ranges between 0% and 15%, while average returns are approximately 5% during recessions and 8% during booms. However, return rates of individual firms comprise a continuum of values waving up and down with aggregate economic activity.

Finally, Figure 2.13 plots a histogram of return rates as they occur during 20 years. In the upper graph, a histogram of return rates to capital of 100 firms with 240 monthly observations is shown. Below, return on equity is shown for comparison, which is defined as realized firm profit, which is already subtracted by nominal interest payments, divided by equity. Note that equity owners also experience capital gains when nominal firm debt is devalued by inflation, which is also considered. Firm equity is calculated by subtracting firm debt from the capital stock value. The return rate of shareholders is:

$$RoE_{t,i} = \frac{nP_{t,i} \cdot Y_{t,i}^{sales} - nW_{t,i} \cdot L_{t,i} - \rho \cdot K_{t,i} \cdot nPI_{t,i}^K + (i_t - \pi_t) \cdot nSC_{t,i}^{fi}}{K_{t,i} \cdot nPI_{t,i}^K + nSC_{t,i}^{fi}} \quad (2.32)$$

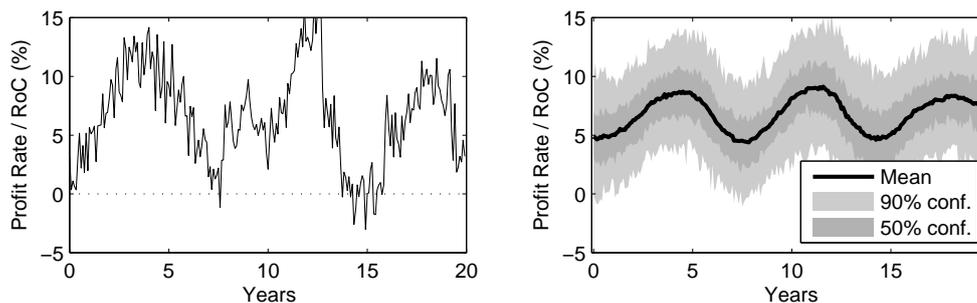
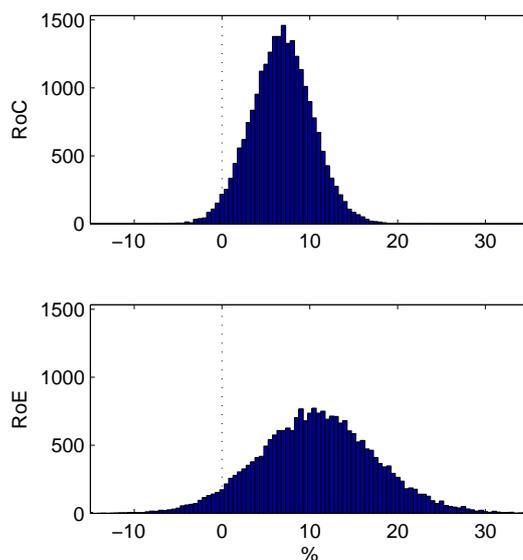


Figure 2.12: Monthly return on capital (RoC) distribution across business cycles.


 Figure 2.13: Return distributions of 100 firms across business cycles. Upper graph: Return on capital stock, lower graph: Return on equity ($\approx 50\%$ of capital stock).

Equity investment is risky, so variance of the RoE distribution is considerably larger. Moreover, since the real interest rate is lower than corresponding return on capital in the current model setting, the mean of the RoE distribution (of approximately 10 – 12%) is considerably higher compared to the RoC distribution (6 – 7%).²³

A closer look at firms marginal cost of production provides an argument why firm production is actually limited by demand. In the short run, production is adjusted by hiring or firing employees, while capital depreciation and interest payments are fixed cost components. Therefore, marginal cost of production $nMC_{t,i}$ is given by wage cost of one produced good:

$$nMC_{t,i} = \frac{\partial(nW_{t,i} \cdot L(Y^c, K))}{\partial Y^c} \Big|_{(K_{t,i}, Y_{t,i}^c)} = \frac{nW_{t,i}}{A_t \cdot (1 - \alpha) \cdot 30^{\frac{1}{1-\alpha}}} \cdot \left(\frac{K_{t,i}}{Y_{t,i}^c} \right)^{\frac{-\alpha}{1-\alpha}} \quad (2.33)$$

²³ Return rates seem to be normally distributed. The reason is that households and firms split monthly spending to 30 daily consumption packages and choose one random supplier per day from their list of suppliers. As a result, this stochasticity of individual demand decisions generates fluctuations in firm sales, utilization and profit. The law of large numbers applies to both distributions, rendering the sum of random choices to follow a normal distribution.

Similarly, marginal productivity of labor $nMPL_{t,i}$ is calculated by evaluating the partial derivative of monthly production capacity with regard to labor input at current values for capital and labor and multiplying it with the current price:

$$nMPL_{t,i} = P_{t,i} \cdot \frac{\partial Y^c(K, L)}{\partial L} \Bigg|_{(K_{t,i}, L_{t,i})} = P_{t,i} \cdot 30 \cdot K_{t,i}^\alpha \cdot A_t^{1-\alpha} \cdot (1 - \alpha) \cdot L_{t,i}^{-\alpha} \quad (2.34)$$

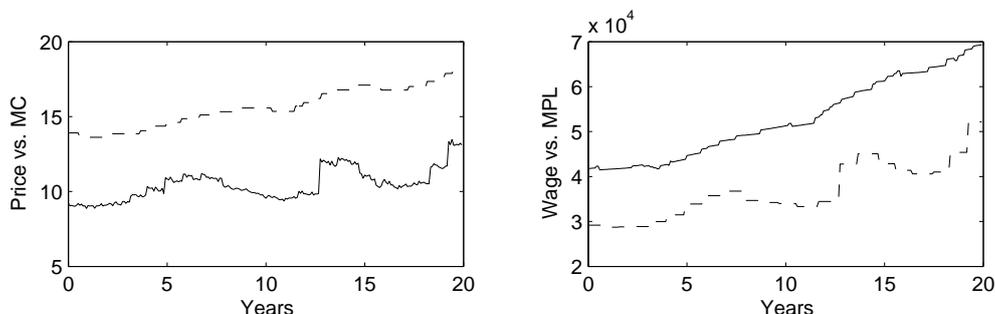


Figure 2.14: Left: Marginal cost (solid) compared to price (dashed), right: Marginal productivity of labor (solid) compared to wage (dashed).

The left side of Figure 2.14 shows that marginal cost of production is always below current firm price, with certain “jumps” occurring when the firm wage is adjusted. The right side indicates that marginal productivity of labor is always considerably higher than the current wage. Again, there are small “jumps” in marginal productivity occurring when the firm’s market price is adjusted. Both graphs suggest that firm production is effectively limited by demand, since each additional unit sold increases profit. Therefore, firms in general would enjoy to increase output by large amounts, if there were the possibility to sell the additionally produced units.

2.3.2 Macro Behavior: The Business Cycle

Figure 2.15 shows aggregate model behavior and emergent business cycle dynamics by plotting six macroeconomic variables over 240 months. Endogenous business cycles are generated by recurrent waves of investment activity as shown in the upper left diagram, with a frequency of about 7 years.

The second graph on the left depicts aggregate firm sales (solid) with investment as a component of it. With a small lag, aggregate consumption (dashed line) also shows a small cyclical movement, since increasing sales are accompanied by increasing household incomes. Production capacity (dotted line) rises with technology growth, but experiences cyclical periods of slightly higher and lower growth. Slope differences in the development of production capacity result from different levels of employment and net investment. Aggregate capacity utilization is depicted in the lower graph on the left with dotted lines of target utilization and the corridor around it determining employment decisions. The business cycle is clearly visible also in this measure of aggregate economic activity, when aggregate utilization falls below target in recessions and rises above in times of boom.

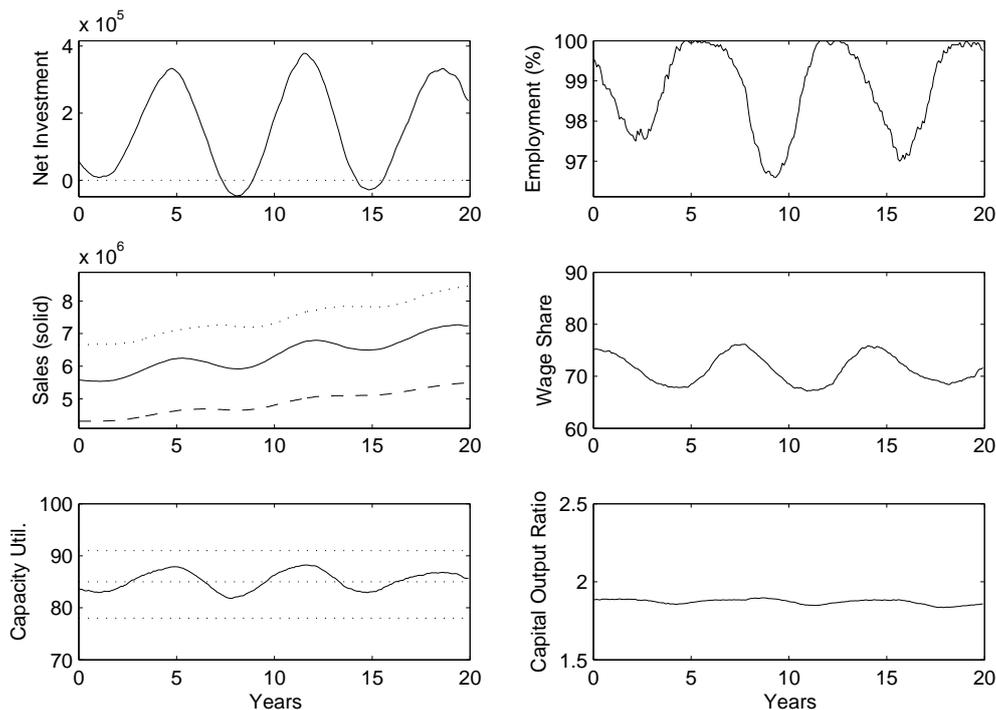


Figure 2.15: Model behavior on macro level across business cycles. Left side: Net investment, aggregate sales (GDP), utilization. Right side: employment, wage share as a percentage of GDP, capital output ratio. Second graph on the left (Sales, solid line) also shows production capacity (dotted) and consumption (dashed).

The graph on top right side of figure 2.15 shows how employment develops during business cycles. The economy is near full employment, since no relevant labor market frictions are modeled and labor is a universal factor input, thus rendering each worker to match in all firms. During recession, unemployment rises by about three per cent, unless recovering investment activity increases utilization levels above target again. Then, more firms decide to hire workers, instead of firing them. The second graph on the right shows the sum of wage income compared to GDP, which rises up to 75% during recession, accompanied by a drop in firm profits. During boom, it falls slightly below 70% when firm profit is at its peak. Finally, the lower graph on the right depicts the capital-output ratio, calculated as real capital stock divided by *annual* production capacity, which is constant in a steady-state equilibrium in the *Solow* (1956) model. Except from small variations over the business cycles, resulting from variations in aggregate employment, it is actually stable at about 185% of annual production capacity.

Concerning long-run growth of aggregate sales, it is known that technology parameter A_t grows at 1.2%. In section 2.2.4 it was argued that the firm capital stock grows at the same rate. With constant returns to scale, this implies that firm production capacity also grows at 1.2% in the long run, as long as employment is stable:

$$Y^c(\lambda \cdot K, \lambda \cdot A, L) = 30 \cdot (\lambda \cdot K)^\alpha \cdot (\lambda \cdot A \cdot L)^{1-\alpha} = \lambda \cdot Y^c(K, A, L), \quad (2.35)$$

where λ is an arbitrary factor. If a single firm (with constant employment) grows at a certain

rate, the aggregate of all firms with employment on a constant level will also grow at that rate. As it turns out, aggregate real firm turnover (GDP) actually grows at an average annual rate of approximately 1.2% when it is calculated for a long horizon of 1,000 months or more, thus capturing several cycles. Therefore, aggregate long-run growth is determined by the exogenous rate of technological progress, similar to the *Solow* (1956) model.

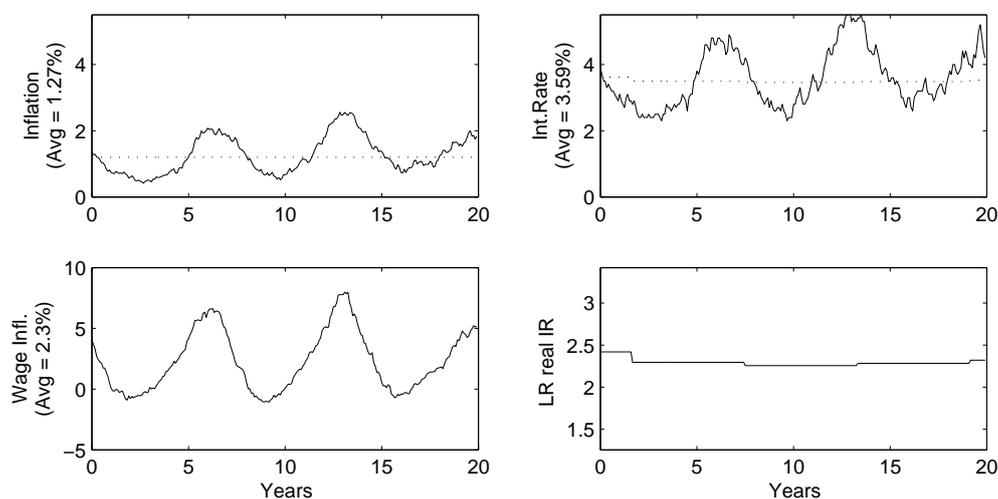


Figure 2.16: Monetary variables across business cycles. Left side: inflation, wage inflation. Right side: nominal interest rate, long-run real interest rate.

Figure 2.16 shows the development of monetary variables. On the left, (annual) inflation and wage inflation both follow economic activity along the business cycle. On average, inflation is quite close to its target value (dotted line). Wage inflation fluctuates stronger than inflation and is higher on average, because it additionally incorporates labor productivity growth. Over long horizons, average growth of wages converges to 2.4%, which is the sum of labor productivity growth and the inflation target. On the top right of Figure 2.16, the nominal interest rate set by the central bank is depicted. Below, the almost invariant long run real interest rate is shown, which is relevant for monetary policy decisions. The average nominal interest rate is approximately the sum of average inflation and the long run real rate (dotted line). Note that the interest rate time series amplifies fluctuations of inflation, which reflects over-proportionate reactions of monetary policy to deviations from inflation target (*Taylor* principle). In its presented setting, the model runs at a long-run real interest rate of about 2.3% on average.²⁴

As it turns out, dynamics of the business cycle in the ACE model resembles *Goodwin* (1967) cycles. Figure 2.17 depicts these Goodwin cycles in a scatterplot of employment level and the

²⁴ A drop in household's consumption propensity, however, would induce the model dynamics to converge to a lower real interest rate. This mechanism is known as "paradox of thrift" and was described by *Keynes* (1936, chapter 23). An increase in the savings rate (thrift) induces adverse effects on aggregate demand and other macroeconomic variables. In the adjustment process, the simulated ACE economy faces a drop in aggregate consumption, demand and employment. The central bank will eventually recognize that the current value of r^* generates average inflation rates below target, so that r^* is reduced.

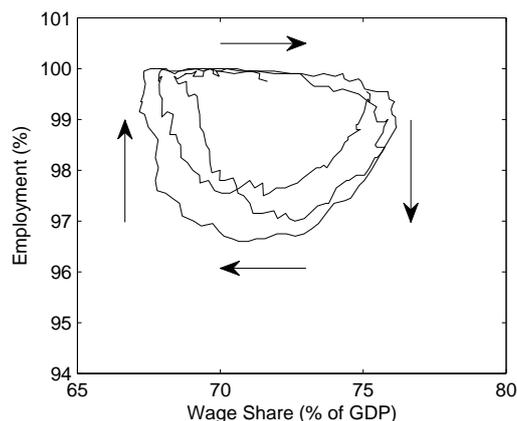


Figure 2.17: Cyclical behavior of employment and the wage share similar to *Goodwin* cycles.

wage share. The mechanism behind the cyclical process visible in ACE time series is described in the following. As soon as the economy reaches a level near full employment, competition for workers and high utilization induces firms to raise wages faster than prices. Subsequently, profits fall and the share of wages is increased. Investment, which is strongly influenced by past profits, is reduced as a consequence. Aggregate demand is stagnating or even falling, firm utilization declines. Eventually, some of the firms fire workers, thereby reducing consumption and aggregate demand even more. The downward spiral ends when profits stop falling and availability of unemployed workers allows for moderate wage contracts. Now, a declining wage share with corresponding rise in profits induces a new wave of investment with increasing consumption and output. Increasing demand and utilization induces firms to hire more workers, thus triggering even more investment to endow additional workers with capital. Soon, the business cycle returns to the point where it started.

2.3.3 (Non-)Neutrality of Money

A major discussion of macroeconomic theory of the past concerns the question of neutrality of money. The classical dichotomy claims that real variables and monetary variables may be analyzed separately, because economic agent's decisions are claimed not to be affected by changes in nominal units of scale. Therefore, changes in the money supply are claimed not to affect real variables (*Patinkin*, 1987). In this context, the quantity theory of money is based on the equation of exchange. Translated to the notation of simulated ACE aggregates, it is:

$$nM_t \cdot v_t = nPI_t \cdot Y_t^{sales} \quad (2.36)$$

The left diagram of figure 2.18 shows the equation of exchange, solved for velocity v_t . Apparently, the velocity of money turnover is a constant relation between economic activity and money. A monetarist might interpret this graph by claiming that an increase of the quantity of money $M_t = M_t^{hh} + M_t^{fi}$ per unit of output Y_t^{sales} causes an increase in price level PI_t . However, since money is created endogenously in this ACE model, with agents that decide

freely about the desired amount of transaction money, such an interpretation is invalid. Actually, agents hold transaction money as high as previous month's individual cost, plus a liquidity buffer of 20%. As a result, velocity of monthly money holding is slightly higher than 1, when compared to monthly turnover. Causality runs from price level to the desired and realized amount of transaction money, not vice versa.

Economists in the tradition of *Keynes* argue that money actually affects economic activity. Recent *New Keynesian* DSGE models incorporate frictions of price and wage developments that render short-run economic activity influenceable by monetary policy., However, in the long run, the classical dichotomy holds (see, for example, *Benchimol and Fourçans*, 2012). The *New Keynesian Phillips curve* marks the short run efficiency of monetary policy.

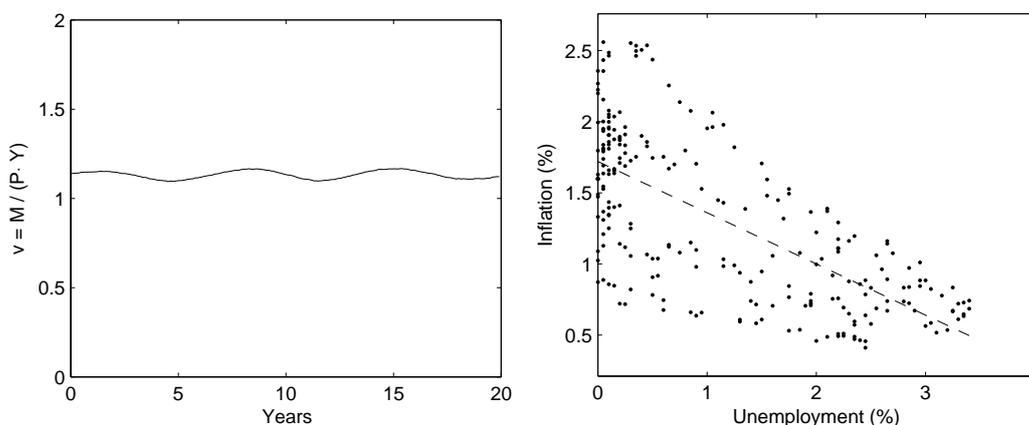


Figure 2.18: Left: Velocity of money turnover, Right: Phillips curve at $\pi^* = 1.2\%$ of 240 subsequent monthly observations. Dashed line depicts OLS regression.

The presented ACE model also generates a *Phillips* curve relationship, which will be derived here: Inflation is measured as a growth rate of the price index PI_t , which itself depends on price decisions of each single firm (cf. equations (2.11), (2.12) and (2.16) above). Now, firm prices are driven by a common value for expected inflation π_t^e , therefore aggregate inflation is also directly influenced by it. Moreover, firms accelerate or slow down their individual target price development with capacity utilization. On aggregate, there is a continuum of firm capacity utilization levels. As the right diagram of figure 2.11 indicates, the positioning of this continuum shifts with aggregate capacity utilization U_t :

$$U_t = \frac{\sum_{i=1}^{N^{fi}} Y_{t,i}^{Sales}}{\sum_{i=1}^{N^{fi}} Y_{t,i}^c} \quad (2.37)$$

If U_t increases (decreases), a larger number of firms faces high (low) levels of capacity utilization, thus inflation accelerates (slows down). On aggregate, we can state that inflation is driven by aggregate utilization, or rather its deviation from target:

$$\pi_t \approx \pi_t^e + f(U_t - U^*) \quad (2.38)$$

Please note the similarity to a *New Keynesian Phillips curve* (*Woodford*, 2003), except that the output gap has been replaced by a “utilization gap” as another measure for aggregate

economic activity. Another difference is that inflation expectation π_t^e is purely adaptive, so there is no rational expectations hypothesis employed. Moreover, no stochastic term is explicitly given, although there are stochastic elements incorporated in the ACE simulation, for example the number of firms that actually adjusts its price in a certain month.

In the ACE economy, a *Phillips curve* is clearly visible in the right diagram of figure 2.18, where annual inflation is plotted against current unemployment, indicating non-neutrality of money at least in the short run. Due to adaptive expectations, a central bank that tries to reduce unemployment by increasing its inflation target will increase demand in the short run. However, there is no trade-off in a way that higher inflation *causes* low unemployment, as was implicitly assumed in the “rules vs. discretion” debate.²⁵

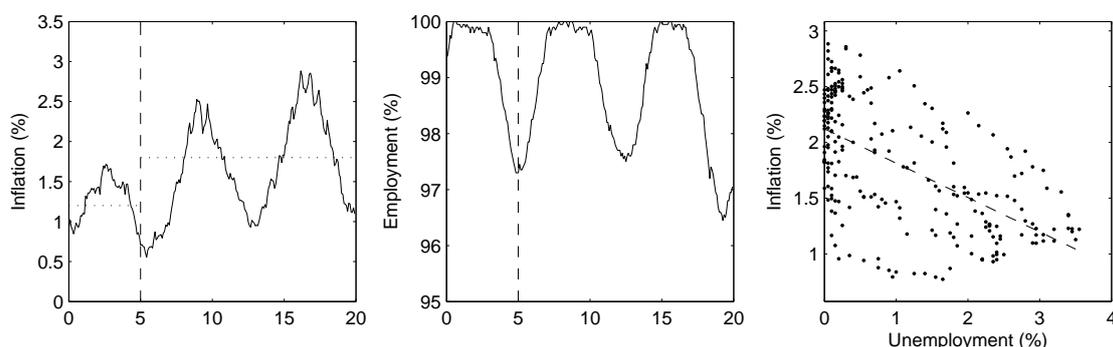


Figure 2.19: Adjustment after shock to inflation target from 1.2% to 1.8%. Left: Inflation, Middle: Employment, Right: Phillips curve 240 monthly observations right after shock month.

The direction of causality runs strictly from economic activity to rising prices in this model. Nevertheless, monetary policy is not necessarily inefficient: A sudden increase of the inflation target induces monetary policy to lower the nominal interest rate, which moves economic activity upward in the short run. Adaptive expectations subsequently adjust to the new level of inflation and afterwards, the central bank faces a shifted Phillips curve that emerges from a higher inflation target. See Figure 2.19 to see the adjustment process after the annual inflation target is increased to $\pi^* = 1.8\%$ at year 5. Inflation almost immediately switches to cyclical fluctuation on a higher average level of inflation. Employment also continues in its cyclical pattern. The graph on the right side depicts the *Phillips curve* relationship for 240 monthly observations after the shock month. Clearly, the new *Phillips curve* relation is simply shifted upward. Therefore, monetary policy is neutral with respect to real variables in the medium and long run.

²⁵ See *Kydland and Prescott (1982)*. According to this view, a representative agent balances leisure and consumption. Surprise inflation, in turn, induces the agent to work more, as a result of erroneously over-estimating the real purchasing power of nominal wages. This is a good example for the limited explanatory value of microfoundations, if assumptions on individual behavior are questionable. In the real world, *most* households determine their level of work time by a mixture of labor contracts with fixed hours (as a convention of society) and the need to satisfy their employer, who might ask for temporary overtime to accommodate demand variations.

2.3.4 Stabilization policy

The tax regime of “automatic stabilization” as described above (section 2.2.5, equation (2.29)) is applied in all diagrams throughout this article, except partly for this section. Its mechanism is shown in the left side of figure 2.20. The upper graph shows public cost and revenue; the corresponding tax rate is depicted in the middle and the bottom graph depicts aggregate employment. Please note that the tax rate remains stable slightly above 9%. In recessions, tax revenue is stagnating, while public cost for unemployment benefits is increasing strongly. Therefore, the fiscal deficit is increasing during economic downturn, so that we have an effect of automatic stabilization of aggregate demand: Public debt growth is increased in times of recession.²⁶ Fluctuations of demand are dampened automatically by the state, which is the reason for the term “Automatic Stabilization” assigned to this tax regime. A second tax

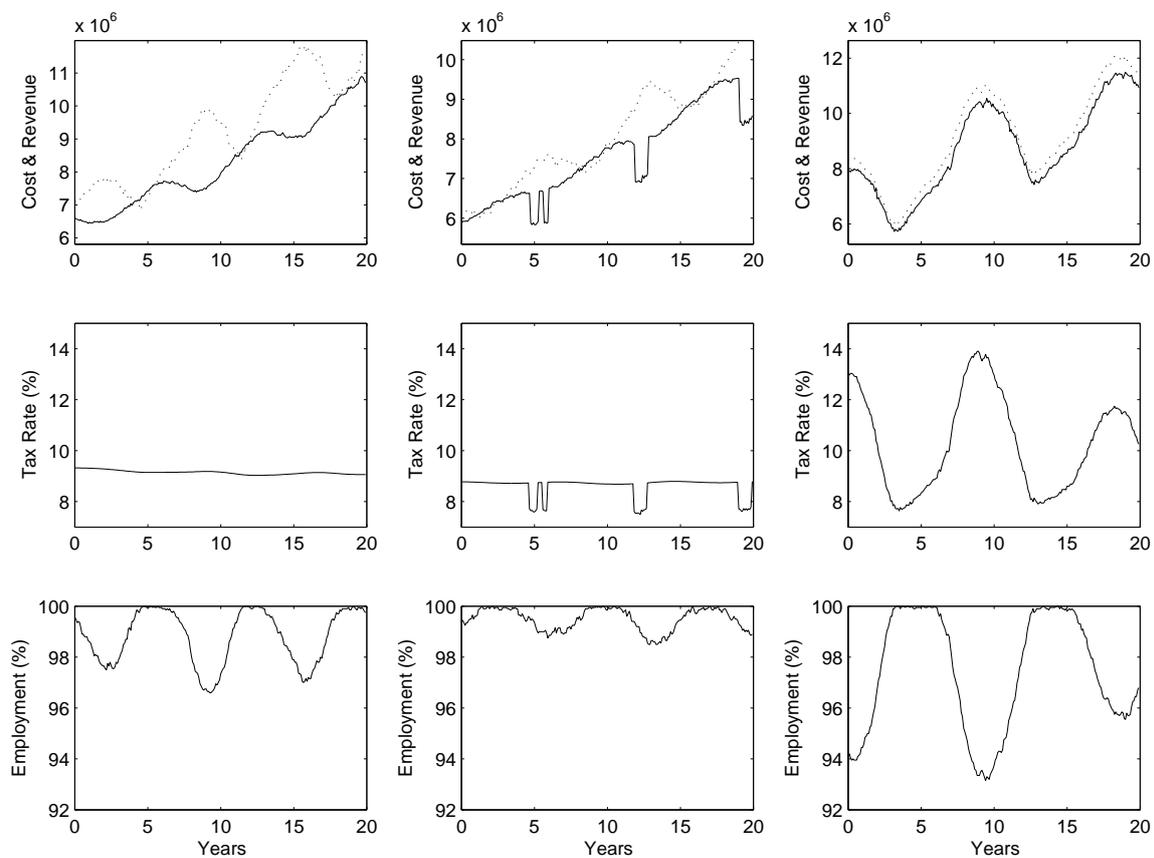


Figure 2.20: Fiscal Policy Regimes: Automatic Stabilization (left side), Deficit Spending (middle) and Pro-Cyclical (right). All columns show: Upper diagram: Public cost (dotted) and revenue (solid), middle: Tax rate, bottom: Employment.

regime is termed “Deficit Spending”, which is shown in the middle column of figure 2.20. Since this is a *Keynesian* model characterized by demand-led economic activity, it is not surprising that fiscal policy aiming at demand stabilization actually works. The government in this ACE

²⁶Public debt varies between 15 and 20% of the capital stock value in the presented setting. When compared to annual turnover, the public debt burden is stable at 25-35% of GDP.

model does not directly buy goods and services. Instead, aggregate demand is influenced by adjusting the general income tax. Whenever aggregate utilization falls considerably below target, the state reacts with a tax cut in order to stimulate consumption expenditure. Otherwise, the tax rate is adjusted according to the regime of “Automatic Stabilization”, defined as τ_t in equation (2.29). The deficit-spending tax rate is adjusted according to

$$\tau_t^{\text{Def}} = \begin{cases} \tau_t & \text{if } U_t - U^* \geq -0.01 \\ \tau_t + (U_t - U^*) & \text{if } U_t - U^* < -0.01 \end{cases} \quad (2.39)$$

The second column of diagrams in figure 2.20 depicts the fiscal regime of deficit spending. As the employment plot (bottom diagram) shows, business cycles are dampened considerably in this *Keynesian* regime of demand stabilization. As a result, there is less unemployment on average, so that financing requirements of the general income tax are even reduced. Therefore, the average tax rate is slightly lower at approximately 8.5–9%. Public debt is also lower in this regime, because high deficits in times of recession are reduced in frequency and magnitude.²⁷

In principle, the ACE framework allows for stabilization of aggregate demand with a higher average level of output. In General Equilibrium frameworks such as *Lucas* (2003) as well as *Gali, Gertler and López-Salido* (2007), deviations from potential output are symmetric and can only be dampened in both directions. However, the modeled stabilization policy of deficit spending shows how it becomes possible in ACE models to “fill in troughs without shaving off peaks” (*DeLong and Summers*, 1988, p. 434). The employment plot in the middle column of figure 2.20 shows the reduction of unemployment in recessions and on average. Therefore, stabilization policy in an ACE framework offers additional welfare gains compared to General Equilibrium models as applied by *Lucas* (2003). Risk-averse individuals will not only appreciate reduced variability of consumption and employment, they will capture additional welfare gains from a higher level of average consumption.²⁸

Finally, a third tax regime is termed “pro-cyclical”, which is shown in the right column of figure 2.20. Again, the government conducts fiscal policy in this ACE model by adjusting the general income tax. Here, the state aims at balancing of public finance requirements and collected taxes permanently throughout business cycles. To this end, a recession, which is accompanied by a stagnating tax base and rising cost, is answered with an increasing tax rate, thereby dampening aggregate consumption even more. The respective tax rate is given by:

$$\tau_t^{\text{Pro}} = 0.95 \cdot \frac{UP_{t-1} \cdot nUB_{t-1} + L^{st} \cdot nWI_{t-1} - i_t \cdot nSC_t^{st} - nSG_{t-1}}{nInc_{t-1}} \quad (2.40)$$

²⁷Public debt in the deficit spending regime varies between 7 and 11% of the capital stock value, given the same model parametrization. When compared to annual turnover, the debt burden is stable at 15-20% of GDP. That is, public debt is considerably lower than in the benchmark regime of automatic stabilization. Compared to real-world experience with *Keynesian* fiscal policy in the twentieth century, this result appears counter-factual.

²⁸Moreover, we might determine additional gains from analyzing welfare on disaggregate level. Income and consumption paths of individual heterogeneous agents are considerably less volatile, because a smaller proportion of agents becomes unemployed at all. Therefore, welfare gains of stabilization are even higher.

Observe the parallel development of cost and revenue in the upper diagram of the right column of figure 2.20, thus indicating a stable budget deficit as was intended by this policy regime. However, the tax rate is increased pro-cyclically whenever demand and utilization is low already, thus dampening consumption even more. In the other extreme, the tax rate is lowered when the economy is at full employment levels and aggregate demand is at its peak, thus stimulating economic activity even more. As a result, business cycles are reinforced and amplified, with considerably higher unemployment burdens and longer duration. Therefore, fiscal policy aiming at permanently balancing the state deficit is accompanied by high cost to society.²⁹

Keynes (1936) showed that anti-cyclical fiscal policy is effective when the goal is to dampen recessions. It should have become public knowledge by now that pro-cyclical fiscal policy will have strongly adverse effects on the real economy, as shown in former analysis. However, in the current Euro crisis, economic policy of austerity is forced on southern Euro-zone countries. When investment activity and aggregate demand in general are low already, accompanied by lower tax revenue and increasing cost, governments in these countries are “encouraged” to balance the budget pro-cyclically. As a result, southern Euro-zone countries face high unemployment rates and deflationary tendencies.³⁰ In a system of double-entry bookkeeping, it can be shown that it is not possible to reduce public debt and firm sector debt at the same time, if there is no other group of agents which is willing to spend considerably more.³¹

2.3.5 Calibration

The model is calibrated to produce a number of desired properties and proportions on aggregate level. Business cycles are of 7 years frequency, which corresponds quite well to *Juglar* cycles with a length between 7 and 11 years as reported by *Korotayev and Tsirel* (2010). However, in the absence of exogenous shocks in the simulation, endogenous simulated cycles are more predictable and more even, when compared to real-world business cycle occurrences.

Figure 2.21 (again) depicts real values for production capacity (dotted line), firm sales (solid) and consumption (dashed) per capita (as shown before). Please note the upward trend of shown measures. Even production capacity fluctuates around a trend, since firms reduce employment and capital accumulation during recessions. Note that consumption is considerably less volatile than investment, given as distance between sales and consumption, which is in line with stylized facts (cf. *Stock and Watson*, 1999). The annual growth rate of aggregate firm sales (real GDP) varies approximately between -1.5% in economic downturn and $+3.5\%$ in times of boom. All real variables grow annually at 1.2% in the long run. Over

²⁹Public debt in the pro-cyclical regime varies between 8 and 9% of the capital stock value, given the same model parametrization. When compared to annual turnover, the debt burden is stable at 15-20% of GDP.

³⁰At this point, it is still unclear whether the strategy will at least accomplish the goal of an internal devaluation of countries in a crisis of competitiveness in the end. The future will show whether imbalances within the Euro zone can be reduced sufficiently, before pro-cyclical policy is perceived too costly to society.

³¹See proof in appendix 2.B.

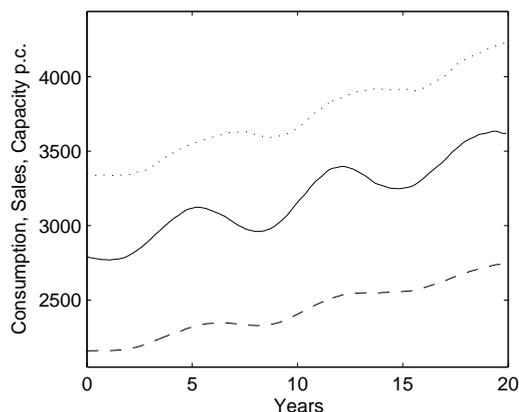


Figure 2.21: Per capita real production capacity (dotted line), real firm sales (solid) and real consumption (dashed).

20 years, exponential growth of 0.1% per month amounts to an increase of 27.1%.

The annual depreciation rate of productive firm capital is set at 9.6%. *Madsen et al.* (2012) distinguish average depreciation rates of firm capital into structures (buildings) with a depreciation rate of 1.75% and machinery and equipment with a depreciation rate of 17.6%. So the chosen depreciation rate is a compromise between both components of firms capital stocks. *Chiarella, Flaschel and Franke* (2005, p. 85) refer to detailed data on the US economy and calculate with a similar capital depreciation rate (9.5%) as this model. The ACE model produces a capital-output ratio of approximately 185%. This amounts to a reciprocal value of output to firm capital Y/K of 0.54 and is a reasonable order of magnitude for developed economies. *Madsen et al.* (2012) estimate a long-run average for 16 OECD countries for Y/K and find it to converge to levels of about 0.5 since 1980. *Chiarella et al.* (2005, p.85) calculate with 0.7 for the Y/K ratio referring to US data.

A wage share of approximately 70–75% is slightly higher than respective proportions in real-world developed economies. According to OECD data (stats.oecd.org) for the total economy in Germany and the US, the wage share ranged between 65% and 70% recently. However, unlike the simulated economy, in real world economies there are considerable amounts of capital in the form of real estate: returns of invested capital in real estate also contribute to overall GDP. Closely related, the average profit rate of simulated firms ranges between 5% in recessions and 8% in booms, so we have an average return rate to invested capital of approximately 6 – 7%. The average return rate to (debt-free) equity is considerably higher at 10 – 12%.³²

Finally, figure 2.22 depicts the correlation between detrended GDP and price level for 24 months backward and forward. For comparison, respective lags (on a quarterly basis) for US data between 1947 and 2007 are shown as reported by *Lengnick* (2013). Apparently, the pattern is rather similar, so the model qualitatively fits real data quite well.

³²The implied distribution of simulated GDP (100%): Wages (70-75%), Depreciation (17-18%), Profit (9-15%). Multiplying profit (as percentage of GDP) by simulated output-capital ratio ($Y/K = 0.54$) yields a profit rate ranging between 5 and 8 per cent again.

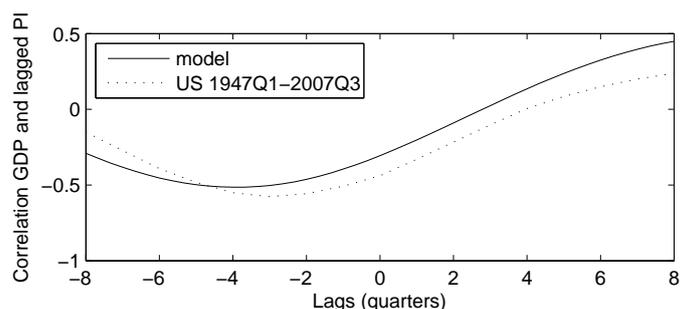


Figure 2.22: Correlation of detrended GDP and detrended Price Index from ACE model (solid line) and with data for the US economy between 1947 and 2007 (dotted line).

2.3.6 Limitations and Parameter Sensitivity

Since the developed model does not perfectly mimic real-world behavior on individual level, consider this (not necessarily exhaustive) list of limitations: Concerning firms, (1) competition between them is reduced to price competition (for customers) and wage competition (for employees), which does not even closely capture the complexity of actual firm decisions. To name a few, innovation, marketing, product quality and corporate governance are left out. (2) The model abstracts from firm entry and exit. (3) The firm size distribution as depicted in Figure 2.23 is skewed to the right, which is a stylized fact (*Delli Gatti et al.*, 2010). However, the range of firm sizes is very limited. Realism in this respect is beyond the scope of this article. (4) Firms are behaving according to fixed rules instead of adapting their strategies in a process of learning. (5) Technological progress is exogenous instead of being the product of innovative activities; a criticism that also applies to the *Solow* model, which induced more recent developments of modern growth theory.

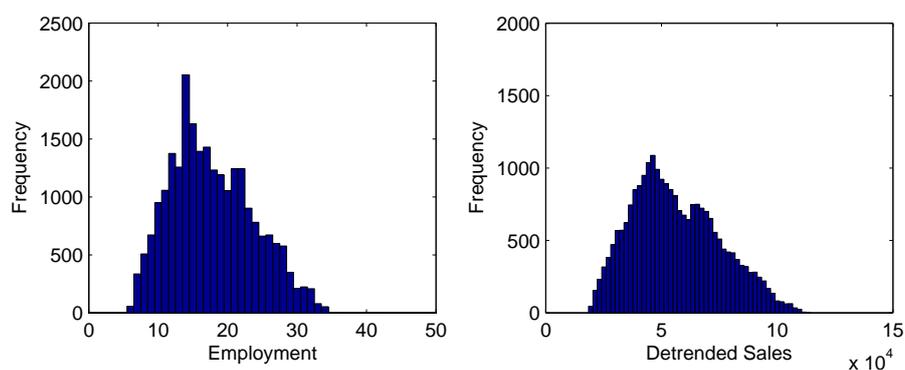


Figure 2.23: Distributions of monthly firm size (100 firms, 20 years): Employees (left) and detrended sales (right).

Workers, on the other hand, (1) are modeled with identical labor supply, so there are no individual differences in capabilities, human capital, specialization or productivity. (2) Household agents are infinitely living. (3) The distribution of firm shares is fixed from the beginning, so there is no market to trade shares and no market determining asset prices. (4)

Savings and asset prices are not subject to agent behavior, for example asset prices inflation might increase household consumption. (5) ACE modeling permits investigation of topics related to wealth and income distribution. However, the current model specification is not calibrated with respect to actual distributions.

Lorenz (1905) curves are depicted in figure 2.24, which are useful to visualize the concentration of wealth (left) and income (right) in the population. It was measured at the end of the period presented throughout the diagrams above. Starting from an initially equal wealth distribution, even after more than 100 years wealth inequality did not increase to extreme levels. Moreover, the income distribution is almost egalitarian. The reason is that households are modeled homogeneous with respect to the value of individual labor supply, which renders wages between households almost identical. Since about 80% of national income consist of wages, the income distribution would be quite close to equality, even if the wealth distribution reflected a maximum level of inequality. Moreover, the modeled income tax applies equally to all levels of wage and capital income. It appears to be a fruitful area of future research to apply realistic patterns for a wealth and income distribution.

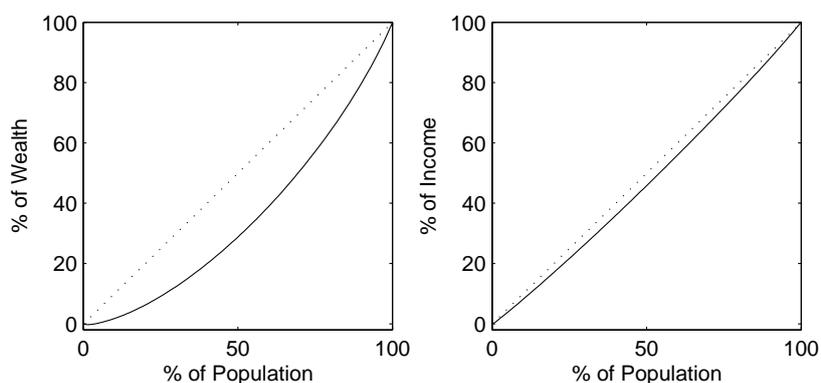


Figure 2.24: Lorenz curve of wealth (left) and income distribution (right).

Regarding sensitivity of model parameters, it should be distinguished between two different goals: Stability and establishment of business cycles. First, in order to render the simulated economy somewhat stable, hyper inflation or a deflationary depression with increasing debt burdens should be avoided. To this end, some parameters are crucial: (1) Model parameters of the consumption function have to be chosen, so that the sum of consumption and investment is sufficiently utilizing firm capacities. Aggregate demand needs to be in a range that allows the central bank to stabilize aggregate demand at a level, that corresponds to firms' target capacity utilization. (2) Another important parameter is λ_π , which allows the central bank to directly influence expected inflation. In the current setting it is 0.1, however, if it is 0, adaptive inflation expectations become much more likely to destabilize the economy in a deflationary or inflationary spiral. (3) Upper and lower boundaries for acceptable firm utilization levels should not be too narrow. Otherwise, employment becomes volatile with small variations in aggregate demand. This becomes critical, when fired workers reduce consumption, so that aggregate demand is lowered even more, thus triggering firing decisions of other firms. (4)

Limited variability of firm investment is a desired property, but it should also remain reliably stable. If, for example, a firm tried to invest as much to directly “jump” to the target capital stock in a single month, aggregate investment might become extremely volatile. In that situation, firms might suddenly be unable to satisfy demand on aggregate scale, rendering the central bank unable to influence aggregate output sufficiently.

Secondly, the establishment of a business cycle pattern as presented in this article is quite sensible to model parameters determining firm investment. Small variations may influence the shape and intensity of endogenously created business cycles strongly, with the possible result, that variability vanishes almost completely. Among the most important parameters for emergence of business cycles are: (1) Parameter $a_{\Delta h}$ which determines the dependence of investment on changes in profits is particularly critical. (2) Parameter a_h which determines the degree to which investment is influenced by past profits. (3) The depreciation rate, which determines the level of gross investment as a major component of aggregate demand. (4) The frequency of business cycles is strongly affected by investment inertia λ_I . However, seemingly unrelated parameters in their interplay may also influence the pattern to some extent. The presented set of parameter values was determined in a process of trial and error by using common-sensical reasoning. It is a characteristic feature of agent-based computational models with a considerable degree of complexity, that aggregate dynamic model behavior can not be traced back to a single parameter. Therefore, it is a feature of ACE models that can hardly be displaced.

2.4 Conclusion

“Yet another model” might be a comprehensible first comment on a newly developed macro model. There have been numerous attempts to establish new models and new variants of existing models. These models differ with respect to the underlying paradigm, thus highlighting different mechanisms or key variables. Surely it constitutes a sound request to ask whether a new model actually contributes to this large reservoir of ideas and elaborate model constructions.

First, agent-based computational economics is a fairly new and yet densely populated field of research, compared to widespread paradigms like general equilibrium models. The mere number of economists employing this way of model-building is limited, so chances are that yet undiscovered ideas emerge when conducting research off the beaten track. Secondly, the presented model combines modeling of economic growth and business cycle dynamics in a comprehensive approach, where even unemployment is modeled explicitly, instead of simply relying on the link of output and employment. Usually, growth and business cycle dynamics are analyzed separately, although they are related closely. Thirdly, the newly presented model is capable of incorporating elements and ideas from different perspectives and economic schools. It encompasses *neoclassical* technology and investment decisions; *Keynesian* ideas enter in multiple ways such as demand-led output determination, involuntary unemployment, income-dependent consumption and business cycles, that are created and reinforced by in-

vestment activity as the main engine of market economies. Monetary policy is conducted in accordance with the *Taylor* rule. Moreover, it is designed as a *monetary* model with double-entry bookkeeping, allowing for close supervision of monetary flows between agents and groups of agents.

The model simulates households and firms as artificial objects in a computer simulation, who interact according to behavioral rules on a goods market and on a labor market. A state provides for unemployment benefits and collects taxes, a central bank adjusts the nominal interest rate in order to stabilize inflation close to a target level. Firm investment is modeled such that it is reinforced by rising profit, whereas firm profit in turn depends on aggregate investment activity. This way, a virtuous circle (or a vicious circle) in the interplay of investment and profit generates recurrent waves of increasing and declining economic activity on aggregate level. Business cycles are created endogenously from investment activity, whose fluctuations are transmitted to firm demand, capacity utilization, employment and price setting behavior. Model parameters are calibrated in order to mimic reasonable macroeconomic proportions like cycle length, the capital-output ratio and the wage share.

As it turns out, the model is capable to generate *Goodwin* cycles and a *Phillips* curve relationship. After a monetary policy shock with a higher inflation target, the Phillips curve is simply shifted upward, so money neutrality holds in the long run. Demand stabilization policy is shown to dampen business cycles considerably, while on the contrary, a pro-cyclical fiscal regime is characterized by business cycles of larger amplitude and cycle length.

In addition, it appeals to common-sense, when modeled firm agents resemble important characteristics of real-world firms. Decisions are constructed to actually depend on the amount of goods or services, that a firm is able to sell. This explicit demand-led property of firm behavior is absent in many other models. As a consequence, production decisions are generally not limited by restricted availability of resources or rising marginal cost of production, which is counter-factually assumed in many models (e.g. *Lindbeck and Snower*, 1994). Instead, modeled firms are restricted by demand and would be glad to sell as many units as possible. As *Sraffa* (1926) puts it: “*The chief obstacle against which [business men] have to contend when they want gradually to increase their production does not lie in the cost of production – which, indeed, generally favors them in that direction – but of the difficulty of selling the larger quantity of goods without reducing the price [...]*”

Avenues for future research are plenty in the yet densely populated world of ACE modeling. For example, starting from this presented model, imposing a more realistic distribution of income and wealth among households would be required to investigate suppositions about relationships of certain macro variables with measures of wealth concentration. Another example would be the design of herd behavior among firms concerning economic outlook, combined with firm investment depending on this level of sentiment. Waves of high and low economic sentiment index values might thus be transmitted to real variables, so that business cycles emerge from a process of social opinion formation (*Lux and Stolzenburg*, 2011).

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2.A Appendix: List of Symbols and Model Parameters

Symbol	Type	Unit/Value	Description
i	subscript	#	Firm number
h	subscript	#	Household number
t	subscript	#	Month
a_h	Parameter	7	Dependence of firm investment on profit rate
$a_{\Delta h}$	Parameter	5	Dependence of firm investment on change in profit rate
a_u	Parameter	0.03	Dependence of firm wage setting on utilization
a_m	Parameter	0.05	Dependence of firm wage setting on price markup
α	Parameter	0.2	Cobb-Douglas capital exponent
c	Parameter	0.85	Marginal rate of consumption
g_A	Parameter	(annually) 1.2%	Technology growth rate
γ_p	Parameter	5	Price competition intensity
γ_r	Parameter	0.5	Restriction aversion
γ_w	Parameter	2.5	Wage competition intensity
λ_I	Parameter	0.9	Investment inertia
λ_C	Parameter	0.9	Consumption inertia
L^{st}	Parameter	200	Public Employment (fixed)
m^*	Parameter	60%	Target price markup over unit wage cost
N^{hh}	Parameter	2,000	Number of households
N^{fi}	Parameter	100	Number of firms
p_p	Parameter	25%	Probability to search for a cheaper supplying firm
p_r	Parameter	25%	Probability to reconsider 1 connection to restricting firm(s)
p_s	Parameter	2%	Probability to replace 1 connection randomly
π^*	Parameter	(annually) 1.2%	Inflation target
ρ	Parameter	(annually) 9.6%	Capital stock depreciation rate
σ_p	Parameter	0.14	Standard deviation of bell curve (price setting behavior)
T	Parameter	3,000	Number of simulated months
T^π	Parameter	24	Backward-looking months of adaptive inflation expectation
T^u	Parameter	6	Backward-looking months of short run utilization average
T^w	Parameter	6	Backward-looking months of employment target fulfillment
U^*	Parameter	85%	Firm target utilization
\underline{U}^*	Parameter	78%	Firm lower bound utilization
\bar{U}^*	Parameter	91%	Firm upper bound utilization
A_t	Variable	real number	Technology parameter (growing)
C_t	Variable	goods	Aggregate real consumption
$C_{t,h}$	Variable	goods	Real household consumption
\bar{C}_t	Variable	goods	Autonomous real household consumption
dur	Variable	#	Wage contract duration (month number)
$DP_{t,i}$	Variable	$\in \{-1, 0, 1\}$	Decision target price change (Price setting behavior)
$DW_{t,i}$	Variable	$\in \{-1, 0, 1\}$	Decision target wage change (Wage setting behavior)
ε_p	RV	$\in (0, 0.015)$	Stochastic price change
ε_w	RV	$\in (0, 0.01)$	Stochastic wage change
i_t	Variable	%	Nominal interest rate
I_t	Variable	goods	Aggregate real gross investment
$I_{t,i}$	Variable	goods	Firm real gross investment
$I_{t-1,i}^{net}$	Variable	goods	Firm real net investment
$K_{t,i}$	Variable	goods	Firm real capital stock

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Table – Continued

Symbol	Type	Unit/Value	Description
$K_{t,i}^*$	Variable	goods	Firm target capital stock
$L_{t,i}$	Variable	#	Firm employees
$L_{t,i}^*$	Variable	#	Firm employment target
$m_{t,i}$	Variable	%	Firm markup over unit wage cost
ν	RV	$\in \{0, 1, \dots, 4\}$	Stochastic wage contract duration (+10)
π_t	Variable	%	Inflation rate during last 12 months
π_t^e	Variable	%	Expected inflation rate
π_t^m	Variable	%	Monthly inflation rate (annualized)
r_t	Variable	%	Expected real interest rate
r^*	Variable	%	Long run real interest rate
$RNI_{t,h}$	Variable	goods	Real net disposable household income
$RoC_{t,i}$	Variable	%	Profit rate (Return on capital stock)
$\overline{RoC}_{t,i}^{12}$	Variable	%	Profit rate (RoC) during last 12 months
$RoE_{t,i}$	Variable	%	Return on equity
τ_t	Variable	%	Tax rate in default fiscal policy regime “Automatic Stabil.”
τ_t^{Def}	Variable	%	Tax rate in fiscal policy regime “Deficit spending”
τ_t^{Pro}	Variable	%	Tax rate in fiscal policy regime “Pro-cyclical”
$U_{t,i}$	Variable	%	Firm utilization
U_t	Variable	%	Aggregate utilization
UP_t	Variable	#	Unemployed persons
$Y_{t,i}^c$	Variable	goods	Firm real production capacity
$Y_{t,i}^{\text{sales}}$	Variable	goods	Firm real sales
Y_t^{sales}	Variable	goods	Firm turnover / Aggregate sales
$nC_{t,h}$	Variable	currency	Nominal household consumption expenditure
nFS_t	Variable	currency	Fiscal surplus / deficit
$nI_{t,i}$	Variable	currency	Firm nominal investment expenditure
$nInc_{t,h}^{\text{prim}}$	Variable	currency	Primary household income
$nInc_{t,h}^{\text{net}}$	Variable	currency	Net household income (secondary)
nM_t	Variable	currency	Aggregate transaction money
nM_t^{hh}	Variable	currency	Aggregate household sector money
$nM_{t,h}^{\text{hh}}$	Variable	currency	Household money
nM_t^{fi}	Variable	currency	Aggregate firm sector money.
$nM_{t,i}^{\text{fi}}$	Variable	currency	Firm money
nM_t^{st}	Variable	currency	State money
$nMC_{t,i}^c$	Variable	currency	Marginal cost of production (1 good)
$nMPL_{t,i}$	Variable	currency	Marginal productivity of labor
$nO_{t,i}$	Variable	currency	Firm nominal operative result
nO_t	Variable	currency	Aggregate firm nominal operative result
$nP_{t,i}$	Variable	currency	Firm price
$nP_{t,i}^*$	Variable	currency	Firm target price
nPI_t	Variable	currency	Price index
$nPI_{t,h}^C$	Variable	currency	Average price of supplying firms (HH consumption)
$nPI_{t,i}^K$	Variable	currency	Average price of supplying firms (firm investment)
$n\Pi_{t,i}$	Variable	currency	Realized firm profit
$n\Pi_{t,i}^{\text{hyp}}$	Variable	currency	Hypothetical firm profit (for optimal investment)
$n\Pi_{t,i}^{\text{kept}}$	Variable	currency	Accumulated kept-in part of firm profit
$n\Pi_{t,i}^{\text{paid}}$	Variable	currency	Firm profit paid-out to shareholders

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Table – Continued

Symbol	Type	Unit/Value	Description
$nS_{t,h}$	Variable	currency	Nominal household savings
nS_t	Variable	currency	Aggregate nominal household savings
nSC_t^{hh}	Variable	currency	Aggregate household sector savings
$nSC_{t,h}^{hh}$	Variable	currency	Household Savings/Credit account
nSC_t^{fh}	Variable	currency	Aggregate firm sector savings (debt)
$nSC_{t,h}^{fh}$	Variable	currency	Firm Savings/Credit account
nSG_t	Variable	currency	Seignorage gain (public revenue)
$nTax_t$	Variable	currency	Tax revenue
$nW_{t,i}$	Variable	currency	Firm wage
$nW_{t,i}^*$	Variable	currency	Firm target wage
nWI_t	Variable	currency	Wage index
nUB_t	Variable	currency	Unemployment benefit

2.B Appendix: National Accounting

The monetary ACE model features double-entry bookkeeping, that is, each movement of an amount of money has a counterpart on another agents' bank account. Agent-Based model simulation allows a closer look on aggregate and disaggregate movement of economic variables. By defining aggregate monetary flows of sectors, macroeconomic relations are derived that allow for economic policy consequences.

1. Let nominal household savings $nSC_{t,h}$ be calculated as monthly net income of a household $nInc_{t,h}^{net}$, subtracted by consumption expenditures $nC_{t,h}$, which equals the sum of changes in households money account $\Delta nM_{t,h}$ and savings/credit account $\Delta nSC_{t,h}$, both individually and on aggregate.³³

$$nS_{t,h} = nInc_{t,h} - nC_{t,h} = \Delta nM_{t,h} + \Delta nSC_{t,h} \quad (2.41)$$

$$\sum_{h=1}^{N^{hh}} nS_{t,h} = \sum_{h=1}^{N^{hh}} (nInc_{t,h} - nC_{t,h}) = \sum_{h=1}^{N^{hh}} (\Delta nM_{t,h} + \Delta nSC_{t,h})$$

$$nS_t = \Delta nM_t^{hh} + \Delta nSC_t^{hh} \quad (2.42)$$

2. Let nominal firm operating result $nO_{t,i}$ be calculated as firm nominal turnover, subtracted by all monetary outflows (nominal wages, interest payments, gross investment expenditure³⁴ and profit payout):

$$nO_{t,i} = nP_{t,i} \cdot Y_{t,i}^{sales} - nW_{t,i} \cdot L_{t,i} + i_t \cdot nSC_{t,i}^{fi} - n\Pi_{t,i}^{paid} - nI_{t,i}, \quad (2.43)$$

³³ Due to random choice of trading partners, nominal consumption expenditures are only translated approximately to real consumption: $nC_{t,h} \approx PI_{t,h}^c \cdot C_{t,h}$.

³⁴ Translation of nominal investment expenditures to real investment holds only approximately due to random choice of trading partners: $nI_{t,i} \approx PI_{t,i}^K \cdot I_{t,h}$.

$nO_{t,i}$ equals changes in firms money and savings accounts, both individually and on aggregate.

$$nO_{t,i} = \Delta nM_{t,i} + \Delta nSC_{t,i} \quad (2.44)$$

$$\begin{aligned} \sum_{i=1}^{N^{fi}} (nO_{t,i}) &= \sum_{i=1}^{N^{fi}} (\Delta nM_{t,i} + \Delta nSC_{t,i}) \\ nO_t &= \Delta nM_t^{fi} + \Delta nSC_t^{fi} \end{aligned} \quad (2.45)$$

3. The fiscal surplus/deficit is public revenue minus cost for unemployment benefits, public employment $L_t^{st} = 200$ and interest on public debt:

$$nFS_t = \underbrace{nTax_t + nSG_t}_{\text{revenue}} - \underbrace{UP_t \cdot nUB_t - L_t^{st} \cdot nWI_{t-1} + i_t \cdot nSC_t^{st}}_{\text{public cost}} \quad (2.46)$$

with tax revenue $nTax_t$, seignorage gain nSG_t , public wage nWI_{t-1} and UP_t the number of unemployed households. The fiscal surplus also equals the change in the states' wealth position (public debt):

$$nFS_t = \Delta nSC_{t,i}^{st} + \underbrace{\Delta M_{t,i}^{st}}_{=0} \quad (2.47)$$

As introduced in section 2.2.2, positive and negative monetary balances of all agents sum up to zero (deposit equals debt) at all times:

$$0 = \underbrace{nM_t^{hh} + nM_t^{fi}}_{\text{transaction money}} + \underbrace{nSC_t^{hh}}_{\text{Savings}} + \underbrace{nSC_t^{fi} + nSC_t^{st}}_{\text{Credit}} \quad (2.48)$$

By taking differences and rearranging, we have:

$$0 = \underbrace{\Delta nM_t^{hh} + \Delta nSC_t^{hh}}_{\text{nominal HH savings}} + \underbrace{\Delta nM_t^{fi} + \Delta nSC_t^{fi}}_{\text{firm sector operating result}} + \underbrace{\Delta nSC_t^{st}}_{\text{fiscal deficit}} \quad (2.49)$$

Therefore, the sum of nominal household savings, nominal firm operating result, and state surplus/deficit is zero, as the running simulation confirms every month:

$$0 = nS_t + nO_t + nFS_t \quad (2.50)$$

That is, improvement of aggregate financial position of one sector³⁵ is possible if and only if someone else's financial position is worsened, given a closed economy as is modeled here. For example, if the household sector increases savings, either firms or the state will have to take a higher debt burden with mathematical certainty. Put differently, if firms on aggregate and the state both try to reduce their nominal debt burden, they will only succeed if households on aggregate are willing reduce the aggregate stock of savings.

³⁵Sum of all money and savings/credit accounts.

Additional Welfare Gains From Stabilization Policy

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Abstract: To determine the possible welfare gains from macroeconomic stabilization policy, we adapt the approach of *Lucas* (2003) with a risk-averse representative agent. However, instead of using a General Equilibrium model, we apply an agent-based macro model, thus relaxing the assumption of perfect ex ante coordination brought about by the *Walrasian* auctioneer. We find that the ACE macro model allows for a higher average level of employment when economic activity is stabilized. Therefore, the welfare gain of perfect stabilization at an optimal level of production is approximately 16 times higher than Lucas estimated. It consists of reduced variability and also in a higher level of average consumption.

3.1 Introduction

One of the topics most heavily debated among macroeconomists is the question whether stabilization policy (monetary or fiscal) should be conducted and to what extent. *Lucas* (2003) has made a very simple but strong argument that additional stabilization of aggregate variables will not improve welfare of a representative consumer considerably, thereby repeating a provocative exercise (*Lucas*, 1987).

Others have argued against it, for example *Barlevy* (2004) develops a model where output smoothing today increases the long-run growth rate tomorrow. Also, *Gali, Gertler and Lopez-Salido* (2007) argue within the framework of a *New Keynesian* General Equilibrium model, that welfare gains of output above potential are not symmetric to welfare losses of identical deviations below output. According to their argument, the welfare gain of additional job creation for the marginal employee in good times is outweighed by the bigger welfare loss during recession. *Krusell and Smith* (1998) investigate welfare effects for heterogeneous consumers, some of whom experience considerable welfare losses during crisis. While the average welfare loss of business cycles is moderate, it is not for individual consumers, which are heterogeneous with respect to employment status, preferences and wealth. *Yellen* (2007) refers to research findings from behavioral economics suggesting a higher level of employment on average in a stabilized economy.

In recent years, a new methodology is becoming popular among macroeconomists, which is agent-based computational economics (ACE). This methodology defines different types of interacting agents, in order to generate households and firms as artificial objects in a computer simulation (*Page*, 2008). Models of this type apply no equilibrium assumption, but instead have to deal with adjustment of relative prices and quantities as well as coordination between interacting agents. On the contrary, DSGE models assume perfect ex ante coordination: The *Walrasian* auctioneer computes equilibrium prices *before* trade takes place (*Kirman* (2006), *Gaffeo et. al.* (2007, 2008)). Real markets, however, operate the other way around: Prices are a result of the market mechanism and not its precondition. For example, under changing conditions, a new equilibrium price vector is not simply set by some auctioneer, but has to be uncovered by agents themselves.¹

Lucas' argument on the negligible benefit of stabilization policy is questionable, since it strongly depends on the seemingly innocent assumption of general equilibrium. Once the unrealistic assumption of the *Walrasian* auctioneer (or GE) is relaxed, stabilization policy performs systematically better. We demonstrate this point within the model of *Lengnick* (2013), which is developed under the ACE paradigm and leaves the coordination problem to the agents instead of a fictitious auctioneer. Stabilization policy in this model (as well as in *Lucas*, 2003) reduces output volatility. Reduced volatility, in turn, also implies fewer changes of the market environment and thus a reduced need for coordination between agents:

¹ According to Austrian theory, one of the most important properties of real world economies is that they are so complex that no individual possesses the computational power to determine the efficient (equilibrium) state. Instead, it is the market mechanism that has to *work out* the equilibrium over time. Compare *Hayek* (1945).

The economy as a whole is capable of producing closer to its efficient level. Therefore, by supporting agents in the reduction of required coordination, stabilization policy not only reduces volatility, but also increases the average level of output. On the contrary, general equilibrium models implicitly assume that stabilization means dampening both, recessions as well as booms.

In this article, we oppose *Lucas* (2003) by showing that stabilization policy actually performs better in ACE models. Dampening output variability allows for additional welfare gains, emerging from an increase in average employment, instead of simply smoothing output on an average level. We apply the model of *Lengnick* (2013), which was developed under the ACE paradigm and show that the the argument of *Lucas* (2003) does not hold in the given model, i.e. it does not survive relaxation of the assumptions of perfect coordination and existence of a representative agent. In the context of this framework with interacting agents, we will show how stabilization policy actually increases average employment and consumption.

In section 3.2 we repeat the argument put forward by *Lucas* (2003). In section 3.3 we briefly sketch the ACE model and explain the way stabilization policy is implemented to it. In section 3.4 we evaluate stabilization policy with respect to attainable welfare gains and compare it to the results of *Lucas*. Section 3.5 concludes.

3.2 The Argument of Robert Lucas

Lucas (2003) assumes the utility of a representative agent to depend on the consumption stream $\{c_t\}_{t=0}^{\infty}$ in the following way:²

$$U(\{C_t\}) = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \log(C_t) \right] \quad (3.1)$$

The above utility function implies risk aversion, that is, a representative consumer prefers smooth consumption paths over variations. Let the consumption stream follow the law of motion

$$C_t = A_0 \cdot e^{\mu t + \eta_t}, \quad \eta_t \sim N(0, \sigma^2) \quad (3.2)$$

which fluctuates around the trend:

$$C_t^{\text{trend}} = A_0 \cdot e^{\mu t} \quad \forall t = 0, 1, 2, \dots \quad (3.3)$$

Estimates of log-linear trend consumption (3.2) with real annual per capita US consumption between 1947 and 2013³ yield $A_0 = 8197$, $\mu = 0.023$ and $\sigma = 0.038$ (see Figure 3.1(a)). In

² *Lucas* (2003) also mentions a more general version of the risk-averse utility function, given by $U(\{C_t\}) = \mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma} \right]$, where γ is the coefficient of risk aversion that has been estimated to lie in the range of $\gamma \in (1, 4)$ as reported by *Lucas* (2003, p. 4). We follow him and henceforth assume $\lim \gamma \rightarrow 1$. After subtracting $\frac{1}{1-\gamma}$ from the utility function (a linear transformation), we take the limit for which the utility function approaches equation (3.1).

³ Data source: FRED, Federal reserve bank of St. Louis.

order to simplify comparison, we use the same period as *Lucas* for further investigation, which is 1947 through 2001. Then, estimation of (3.2) yields $A_0 = 8076$, $\mu = 0.0237$ and $\sigma = 0.0299$.

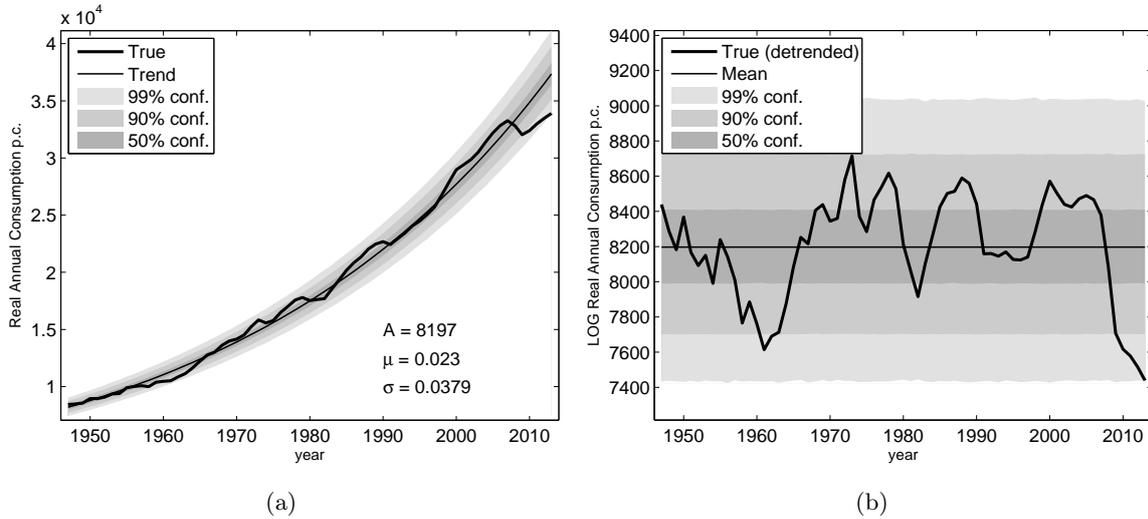


Figure 3.1: (a) US consumption per capita with estimated trend and (b) detrended time series.

Dynamic general equilibrium models return to their steady states if they are not hit by exogenous shocks. Put differently: Once exogenous noise is turned off, their state corresponds to the *potential level of output*, i.e. the maximum *sustainable* level of economic activity.⁴ The term is still misleading, since the economy is capable of producing at a higher level, at least temporarily. However, this level of production beyond “potential output”, which is attainable with available productive resources, is not an equilibrium of dynamically optimizing behavior: Representative agents are modeled to favor an allocation with a different balance of work and leisure. Therefore, it can be considered a *decision* of rational agents, to produce at a lower level than the true production potential.⁵ From a theoretical perspective, the so-called “potential output” in the context of general equilibrium models is actually below the true production potential of the economy. Empirically, a medium-run average level of output, often determined ex-post by HP-filtering, is simply claimed to represent potential output. Then, it is assumed that deviations above and below average are symmetric, so that stabilization policy implies dampening both, recessions as well as booms.

As a typical representative of the GE paradigm, Lucas interprets C_t^{trend} as potential output and answers the hypothetical question of how much a representative agent would value a *perfect* smoothing of all business cycle fluctuations, by solving

$$U(\{(1 + \lambda) \cdot C_t\}) = U(\{C_t^{\text{trend}}\}) \tag{3.4}$$

⁴ For definitions of potential output, consider *Gibbs* (1995) or *Borio* (2013).

⁵ In a simplified two-good world, the production-possibility frontier (or product transformation curve) marks the maximum level of attainable production levels. An economy that operates below this production-possibility frontier is considered inefficient. Cf. *Varian* (2006), p. 605.

for λ . The result is interpreted as the necessary (relative) increase of the stochastic consumption path (3.2) that yields indifference between stochastic and trend consumption. *Lucas* (2003) finds that the representative agent values a perfect smoothing of consumption by only one-twentieth of one per cent ($\lambda = 0.0005$).⁶ This surprisingly low estimate suggests that additional efforts of macroeconomic stabilization policy will not even remotely allow for considerable welfare gains.⁷

The welfare loss of fluctuations around trend consumption λ identified by (3.4) is independent of A and μ , i.e. the growth trend.⁸ For simplicity, we can therefore use the detrended (stationary) time series

$$\log(C_t) - \mu t = \log(A_0) + \eta_t \sim N(\log(A_0), \sigma^2) \quad (3.5)$$

3.3 Stabilization Policy in ACE Model

The agent-based macro model of *Lengnick* (2013) generates stationary time series of aggregate consumption. It simulates 1000 households along with 100 firms interacting according to simple adaptive rules. No state and therefore no stabilization policy is present in the baseline version of the model. Each firm sets its own price and wage, competing for customers and employees. Households receive wages and decide individually about consumption and savings. The model endogenously generates business cycles, i.e. waves of higher and lower economic activity. Figure 3.2(a) depicts a typical pattern of aggregate employment with regular business cycle downturns and recoveries at a frequency of approximately 7 years, 3.2(b) shows corresponding aggregate consumption. Temporary consumption levels above 100 (Full Production) are accomplished only because firms are modeled with inventories. Note that whenever consumption drops, aggregate production (dotted line) follows with a lag.

The maximum level of output is accomplished when all 1000 households are employed, which happens only temporarily due to the necessity of coordination between households (workers) and firms (employers). Theoretically, this level is sustainable: If an auctioneer calculated the general equilibrium and enforced the corresponding price vector, full employment would become a permanent state and consumption would be given by $C^{max}=100$, i.e. potential output. However, the absence of an enforced permanent equilibrium prevents the economy from settling down at potential output. Instead, the model is subject to endogenous business cycles created by individual interactions.

Analogous to *Lucas* (2003) we evaluate the welfare gain from stabilization policy by comparing the baseline model with two different settings. First, we compare it to the theoretical

⁶ Using newer data for US consumption between 1947 and 2013 (instead of 2001) with $\sigma = 0.038$ (instead of $\sigma = 0.032$), *Lucas*' estimate would have been $\lambda = 0.00076$.

⁷ However, *Lucas* rather confirms that $\log(1+x) \approx x$ for small x . The sum of deviations from average consumption $C_t^{trend} \cdot (1 + \hat{\eta}_t)$ is zero by construction, since it captures deviations from log-linear trend consumption: $\sum_{t=1}^T \hat{\eta}_t = E[\eta_t] = 0$. The accumulated utility loss resulting from small variations of C^{trend} is therefore: $\sum_{t=1}^T \log(1 + \hat{\eta}_t) \approx \sum_{t=1}^T \hat{\eta}_t = 0$. Seen this way, it becomes obvious that calculated welfare gains from stabilization policy are negligibly low.

⁸ See proof 1 in appendix.

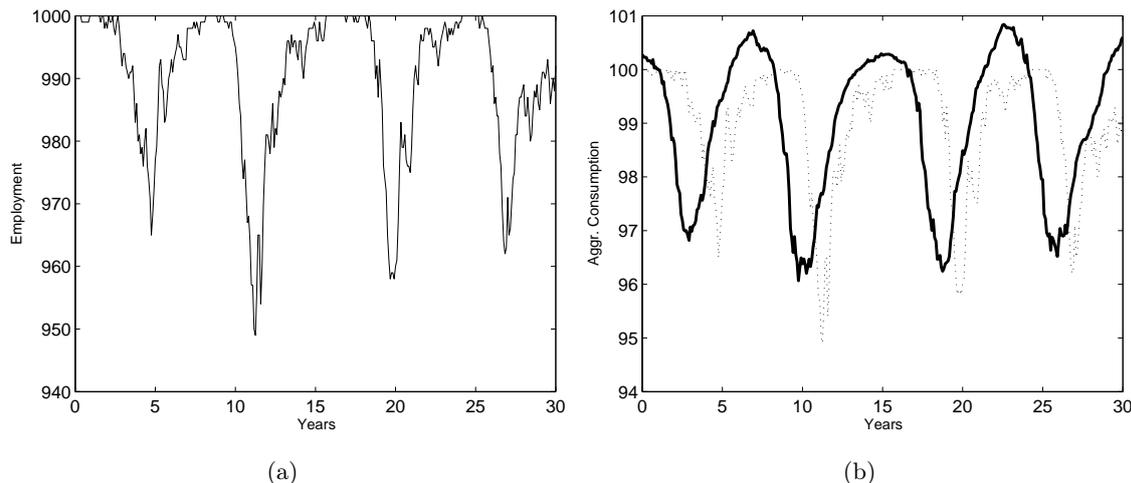


Figure 3.2: (a) Monthly Employment and (b) aggregate consumption in ACE model.

case where a “perfect stabilization” policy brings production in line with (true) potential output. In this case consumption equals C^{max} for all t , which is full employment output. The result could be interpreted as the upper bound of welfare gains that *Lucas* claimed to assess.

In a second experiment we compare our result with a more realistic policy: Unemployment benefits (UB). The *Lengnick* (2013) model is extended by a state that provides for UB amounting to 25% of the average wage. Taxes are collected from employed households in order to finance public expenses for UB.⁹ During crisis, cost of unemployment benefits outweighs tax revenue, such that public debt increases, while in times of full employment collected taxes pay back public debt.

Figure 3.3(a) depicts the distribution of aggregate consumption in the original *Lengnick* model, figure 3.3(b) shows the distribution of consumption in the adjusted model with UB, calculated from $S=1000$ Monte-Carlo replications of both models. Aggregate household consumption is dampened strongly by the described stabilization policy. Note that not only the variability of consumption is reduced, but there is also an increase in the average level of consumption. Stabilization policy reduces the need for coordination among agents, thereby increasing the average level. See table 3.1 below for details.

We have already shown qualitatively, that if the coordination problem is not assumed away by imposing equilibrium from the outset, stabilization policy might give rise to both a decrease in volatility and also an increase in average consumption. In the next section we compute the welfare gains λ of the two experiments and compare the results to *Lucas*.

⁹ The tax rate is adjusted smoothly with respect to the long-run financing requirements of unemployment benefits. This assumption reflects the fact that tax rates do not encounter large jumps over the course of a business cycle.

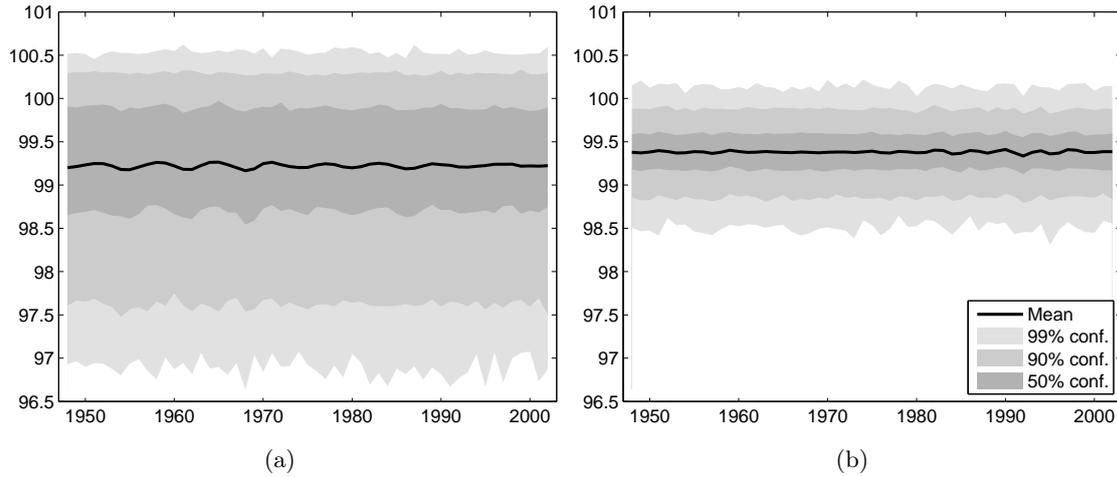


Figure 3.3: Distribution of aggregate consumption as a percentage of full production (a) from original model and (b) from adjusted model with UB using $S=1000$ replications.

3.4 Model comparison

We compare real consumption paths from the original model $C_{t,s}^{orig}$ to those from the adjusted model $C_{t,s}^{UB}$; and to perfectly smoothed consumption at full production C_t^{max} , with years $t = 1, 2, \dots, T$ and random seeds $s = 1, 2, \dots, S$. However, measured deviations of real-world consumption from an estimated long-run log-linear trend as shown in Figure 3.1(a) happen to be higher than medium-run consumption variability generated by the baseline ACE model. In order to ensure comparability, we enforce an identical standard deviation. To this end, we scaled the standard deviation of annual ACE consumption time series up.¹⁰

To assess the welfare gains of stabilization policy λ , we apply the same approach as above (compare eq. (3.4) and proof 1 in appendix):

$$U(\{(1 + \lambda) \cdot C_{t,s}^{orig}\}_{t=1}^T) = U(\{C_{t,s}^{UB}\}_{t=1}^T) \quad (3.6)$$

$$E \left[\sum_{t=0}^T \beta^t \cdot (1 + \lambda) \cdot C_{t,s}^{orig} \right] = E \left[\sum_{t=0}^T \beta^t \cdot C_{t,s}^{UB} \right] \quad (3.7)$$

To approximate the expectational value we have to apply a Monte Carlo Simulation simulation method over S replications:

$$\frac{1}{S} \sum_{s=1}^S \left[\sum_{t=1}^T \beta^t \cdot (1 + \lambda) \cdot C_{t,s}^{orig} \right] = \frac{1}{S} \sum_{s=1}^S \left[\sum_{t=1}^T \beta^t \cdot C_{t,s}^{UB} \right] \quad (3.8)$$

The benchmark value for λ (*Lucas'* estimate) is calculated by choosing trend consumption for comparison, so that the right side of equation (3.8) is replaced by the mean value of consumption $\sum_{t=1}^T \beta^t \cdot \bar{C}^{orig}$. In this case, $\lambda = 0.00049$ represents the welfare gain from

¹⁰To this end, each annualized consumption path is subtracted by its mean. Then, it is multiplied by the relation of desired standard deviation and average standard deviation of all ACE consumption paths. Finally, the mean value of the consumption path is added again.

smoothing consumption on the average level. Finally, comparison of consumption $C_{t,s}^{orig}$ with a perfectly smooth consumption path at full production is conducted when the right side of equation (3.8) is replaced by $\sum_{t=1}^T \beta^t \cdot C^{max}$.

Calculations replicate *Lucas*' (approximated) estimate of λ in the upper row of table 3.1 as a benchmark case, thus providing an estimate for the welfare gain of eliminating fluctuations at average consumption.¹¹ With $S = 1000$ replications, $T = 55$ years (to apply the same time span from 1947 to 2001) and $\beta = 1$ to apply equal weights to all simulated consumption values, λ is estimated at 0.0016, which is 3.4 times higher than the benchmark estimate of *Lucas* (see table 3.1, second row). Note that the estimated welfare gain can be decomposed into the gain from higher average consumption ($\lambda_a \approx 0.0012$) and that from reduced variability ($\lambda_v \approx 0.0004$).

$C_{t,s}^{orig}$ compared to...	welfare gain	<i>Lucas</i> "error"
Mean (no variation): \bar{C}^{orig}	$\lambda = 0.000487$	(1.0)
Adjusted Model: $C_{t,s}^{UB}$	$\lambda = 0.00163$	3.4
Full Production (no variation): C^{max}	$\lambda = 0.00787$	16.2

Table 3.1: Welfare gain λ between original model consumption and competing setups.

In the case of perfect coordination between agents, where all workers are constantly employed, consumption would be permanently at potential output C^{max} . Comparing this case with $C_{t,s}^{orig}$ yields a welfare gain of $\lambda = 0.0079$. This value is 16.2 times as high as *Lucas*' estimate for the upper bound of welfare gains from stabilization, whereas the major part of it results from a higher value of average consumption (approx. $\lambda_a = 0.0074$). The absence of coordination problems (i.e. GE) is not an innocent assumption, if effects of stabilization policy are analyzed. As soon as the coordination problem is taken into account, such policies might perform much better because they "fill the troughs without shaving the peaks" (*DeLong and Summers* (1988)).

3.5 Conclusion

We have argued that the common assumptions of perfect coordination and existence of a representative agent in general equilibrium models give rise to systematically biased (i.e. undervalued) estimates of the attainable welfare gain from macroeconomic stabilization policies. The application of such policies in a baseline ACE macro model induced a reduction of coordination necessities between interacting agents and thus a slightly higher average level of employment and consumption, thus confirming *DeBelle and Laxton* (1997) in their finding of a dependence between volatility in unemployment and mean unemployment. Given the

¹¹ *Lucas*' estimate was $\lambda = 0.0005$ for data between 1947 and 2001, which is almost reproduced here. Newer data of aggregate US consumption until 2013 yields $\lambda = 0.00076$, since the financial crisis after 2007 induced a strong deviation from long-run trend.

adapted methodology of *Lucas* (2003), the welfare gain of perfect stabilization at an optimal level of production in the ACE framework is 16.2 times higher than the *Lucas* estimate.

However, the order of magnitude is still very low. Even the disappearance of all variability of per capita consumption in the ACE model at potential output is valued according to former analysis by only 0.79% of average consumption. This result appears utterly unrealistic and it principally challenges the adapted methodology of measuring welfare gains. For the sake of completeness, we like to add some additional arguments why the methodology does not capture welfare implications of business cycles properly: To begin with, the chosen utility function does not incorporate the true uncertainty of unemployment, of unknown future income and consumption, that real-world households face particularly during crises.¹² Secondly, the idea of a representative consumer who deliberately chooses or decides to enjoy more leisure time does not capture welfare losses of actually involuntarily unemployed individuals. According to behavioral research, people's happiness is inversely correlated with unemployment (*Yellen*, 2007). *Wolfers* (2003) exploits subjective measures of life satisfaction and finds that volatility of unemployment has adverse effects on overall well-being. Finally, aggregate consumption is rather stable across business cycles, whereas investment, industrial production and income are more affected by macroeconomic fluctuations. Households are able to smooth consumption paths, both on aggregate and on individual level, but variability of other macroeconomic quantities also affects welfare.

¹²The distinction between true uncertainty and risk, understood as an unknown outcome of a known distribution was introduced by *Knight* (1921).

3.6 References

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3.A Appendix: List of Symbols

Symbol	Description
A_0	Base level of trend consumption stream
C_t	Consumption stream
$C_{t,s}^{orig}$	Consumption stream from original model
\bar{C}^{orig}	Average consumption in original model
C_t^{trend}	Consumption trend
$C_{t,s}^{UB}$	Consumption stream from adjusted model
C^{max}	Consumption level of full production
η_t	Normally distributed random variable
$\hat{\eta}_t$	Estimated deviation from trend consumption
β	time preference factor ($\beta < 1$)
γ	risk aversion parameter
λ	Welfare gain between two consumption streams
μ	Growth rate of consumption trend
σ	Standard deviation of fluctuations around trend consumption
s	Random seed
S	Number of random seeds / replications
t	Discrete time period (year)
T	Number of years
$U(\dots)$	Utility function

3.B Appendix: Proof

Proof 1. The welfare gain of perfect stabilization λ , which assures indifference between the consumption path (3.2) and its mean is identified by:

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \log \left((1 + \lambda) \cdot A_0 e^{\mu t + \eta_t} \right) \right] = \sum_{t=0}^{\infty} \beta^t \log (A_0 e^{\mu t}) \quad (3.9)$$

Taking logs

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \{ \log (1 + \lambda) + \log (A_0) + \mu t + \eta_t \} \right] = \sum_{t=0}^{\infty} \beta^t \{ \log A_0 + \mu t \} \quad (3.10)$$

and cancelling non-stochastic elements on both sides:

$$\mathbb{E} \left[\sum_{t=0}^{\infty} \beta^t \{ \log (1 + \lambda) + \eta_t \} \right] = 0 \quad (3.11)$$

The welfare gain λ is thus independent of A_0 und μ . □

Identification of a Core-Periphery Structure
Among Participants of a Business Climate Survey –
An Investigation Based on the ZEW Survey Data

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&

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Abstract: Processes of social opinion formation might be dominated by a set of closely connected agents who constitute the cohesive ‘core’ of a network and have a higher influence on the overall outcome of the process than those agents in the more sparsely connected ‘periphery’. Here we explore whether such a perspective could shed light on the dynamics of a well known economic sentiment index. To this end, we hypothesize that the respondents of the survey under investigation form a core-periphery network, and we identify those agents that define the core (in a discrete setting) or the proximity of each agent to the core (in a continuous setting). As it turns out, there is significant correlation between the so identified cores of different survey questions. Both the discrete and the continuous cores allow an almost perfect replication of the original series with a reduced data set of core members or weighted entries according to core proximity. Using a monthly time series on industrial production in Germany, we also compared experts’ predictions with the real economic development. The core members identified in the discrete setting showed significantly better prediction capabilities than those agents assigned to the periphery of the network.

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Growth Determinants Across Time and Space – A Semiparametric Panel Data Approach

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Abstract: A panel data set covering 145 countries between 1960 and 2010 has been investigated closely by using models of parameter heterogeneity. The Functional Coefficient Model (FCM) introduced by *Cai, Fan and Yao* (2000) allows estimated parameters of growth determinants to vary as functions of one or two status variables. As a status variable, coefficients depend on the level of development, measured by initial per capita GDP. In a two-dimensional setting, time is used as an additional status variable. At first, the analysis is restricted to bivariate relationships between growth and only one of its determinants, dependent on one or both status variables in a local estimation. Afterwards, the well-known *Solow* (1956) model serves as a core setting of control variables, while functional dependence of additional explanatory variables is investigated. While some constraints of this modeling approach have to be kept in mind, functional specifications are a promising tool to investigate growth relationships, as well as their robustness and sensitivity. Finally, a simple derivation of FCM called local mean values provides a suitable way to visualize macroeconomic or demographic patterns over time in a descriptive diagram.

Also available as: *Economics Working Papers* 2014-11, University of Kiel.

5.1 Introduction

Improved data availability and computing power gave birth to large empirical studies searching for determinants of growth during the last 30 years. A popular method turned out to be cross-country growth regressions, trying to uncover relationships between per-capita growth rates and supposedly growth-relevant measures in a multiple regression model. Explanatory variables in these regressions covered macroeconomics, demographic and geographic characteristics, education measures, political and social indicators and even the prevalence of certain diseases. A numerous but still incomplete list of variables that have been applied in growth regressions is provided by *Durlauf, Johnson and Temple (2005)*.

Without mentioning, most of these empirical studies assume parameter homogeneity, that is, a certain explanatory variable is expected to have a homogeneous influence on economic growth across countries and years. *McCartney (2006)* denotes this procedure the ‘assumption of universalism’ - which exposes the view that each statistical observation will shed light on a universal underlying economic relationship, no matter when and where it has been observed.

Some authors deny this assumption of universal growth determinants. For example, *Sala-i-Martin and Artadi (2004)* rank countries according to their definition of competitiveness, which supposedly influences future growth prospects. According to this concept, there are three different stages of development in which different input factors are most relevant for a countries’ growth prospects.¹ Put differently, certain economic variables are of changing importance for growth, depending on the stage of development. *Aghion and Howitt (2006)* argue that countries with highly developed technology will accomplish further improvements of their productivity only by innovation, while less developed countries primarily need to adapt available technologies in order to enhance growth prospects. Therefore, economic policy and the design of institutions should depend on a countries’ stage of development. As a result, capital accumulation and the adaption of available technologies serve as a powerful growth engine as long as the country is somewhat distant to the technological frontier. The closer an economy converges to the technological frontier, the more important becomes, for example, attainment of higher education for further growth prospects. Again, changing key factors fostering economic growth are highlighted dependent on a countries’ development stage. With regard to growth regressions, this implies coefficients for mentioned variables to be dependent on the stage of development.

A suitable way to investigate functional dependencies of estimated coefficients is provided by *Cai, Fan und Yao’s (2000)* Functional Coefficient Model (FCM), which was applied in a different context by *Herwartz and Xu (2007)*. It is a multivariate version of the non-parametric

¹ The underlying idea is that economic development is following three subsequent stages: In the *factor-driven* stage, basic economic requirements are considered crucial for development: Quality of institutions, infrastructure, macroeconomic stability, personal security and basic human capital. In the second, *efficiency-driven* stage, a growing importance is attached to advanced human capital, efficiency of goods markets, labor markets and financial markets, ‘technological readiness’ and access to markets. During the *innovation-driven* stage of development, capability to innovate and business sophistication are considered most important. See *Snowdon (2006)* for more details.

*Nadaraya-Watson-Estimator*² and allows for local estimation depending on one (or more) economic status variables. FCM is considered a semi-parametric approach, since a linear (parametric) model is estimated locally by a nonparametric method, which is Kernel regression (cf. *Härdle*, 1990). For instance, in the light of arguments of *Aghion and Howitt* (2006) the status variable indicates the stage of development of an economy. Using FCM, estimated functional parameters can be plotted against the underlying status variable in a suitable diagram. This way, we are able to investigate whether an economic relationship changes during the stages of economic development, but we may also confirm that it remains rather stable as is implicitly assumed in conventional growth regressions. The aim of this study is an investigation of functional dependence of coefficients in empirical growth models. Thereby, we weaken the restrictive assumption of universalism, i.e. we allow estimated parameters of growth determinants to vary across development stages. In a setting with two status variables, we also allow coefficients to change over time.

The remainder of this paper is structured as follows: Section 5.1 provides a brief overview on existing literature modeling economic growth with parameter heterogeneity. Section 5.2 describes briefly the composition of our panel dataset, followed by descriptions of statistical methods and diagrams used throughout the paper. The paper presents selected results with regard to a bivariate setting (section 5.3), a multivariate model with partial least squares prefiltering (Section 5.4), followed by some examples of two-dimensional local mean values, which provide a useful visualization of macroeconomic and demographic panel data (Section 5.5). Section 5.6 concludes.

Related Literature: A Brief Overview

Explicit modeling of varying parameters has been conducted by several studies. *Durlauf and Johnson* (1995) use average growth rates between 1960 and 1985 and divide their sample of 96 countries into four groups according to output level and literacy in order to generate four subsamples. Estimated coefficients in each regression differ considerably in sign, magnitude and p-value, so *Durlauf and Johnson* conclude that there are considerable differences between the groups which may be explained with a perspective of different steady states for each subsample. At the same time, countries of similar initial conditions may obey the same linear model.

Liu and Stengos (1999) use a semiparametric partially linear approach for the augmented *Solow* model from *Mankiw et al.* (1992), in which they allow the coefficients of initial GDP and of initial human capital investment to vary for different values of the respective explanatory variable. Data covers 86 countries from 1960 to 1990 in three 10-year-averages for all included variables. They find that there is a threshold effect, such that a growth-dampening effect of high initial GDP is only valid if initial GDP exceeds \$ 1800. Similarly, results suggest that secondary school enrollment (as a measure of human capital formation) fosters growth only for values beyond 15 %. Similarly to *Durlauf and Johnson* (1995), findings of *Liu and Stengos*

² See *Nadaraya* (1964) and *Watson* (1964).

(1999) suggest that there might be different growth regimes and different equilibria.

Durlauf, Kourtellos and Minkin (2001) allow for parameter heterogeneity also in the augmented *Solow* model, while they use initial GDP per capita as a status variable. Explanatory variables are population growth, investment and initial human capital, such that they use a theory-based small setting for their cross-country growth regression model. Data covers 98 countries with average growth rates over the whole period between 1960 and 1985. They conclude that there appears to be a considerable degree of parameter heterogeneity in the data, without providing an explicit interpretation of locally estimated parameters. Instead, a general finding is that growth rates, the magnitude of local parameters as well as estimated residuals vary considerably more for the poorest countries. Accordingly, local measures for goodness-of-fit reach a much higher level for countries with a high initial GDP. *Kourtellos* (2002) confirms the finding of parameter heterogeneity using the same smooth coefficient approach. Instead of initial GDP, he uses initial literacy and initial life expectancy as status variables for dependent local parameters. Additional studies investigating parameter heterogeneity and finding considerable nonlinearities with regard to economic growth regressions are *Ketteni, Mamuneas and Stengos* (2007), *Mamuneas, Savides and Stengos* (2006) as well as *Vaona and Schiavo* (2007).

5.2 Model and Data

5.2.1 Data Sources

The data set is combined from two sources. From ‘Penn World Tables’ Version 7.1 (*Heston, Summers and Aten*, 2012) annual data on per capita growth rates, GDP levels, investment and government shares between 1960 and 2010 are obtained. The second source is World Banks ‘World Development Indicators’ (World Bank, 2012) providing other macroeconomic measures, demographic variables and data on educational attainment.³

Measurement quality of real per capita GDP is not convincing throughout the whole dataset, since we observe considerable fluctuations of real GDP values that can hardly be explained by regular growth dynamics. Annual growth rate “jumps” of more than 10 percentage points, up or down, are recorded in 1150 cases out of about 7500 valid observations. Annual growth rates of more than 10 per cent, up or down, can be observed in 839 observations. A few extreme values may reasonably be explained by growth spurts (for example in East Asia) or economic and political turmoil. Nonetheless, there appears to be a considerable amount of measurement uncertainty in the data. Since too much additional uncertainty may jeopardize our goal of explaining economic growth rates properly, extreme fluctuations have been reduced

³ Data availability: Time series from PWT are rather complete, except for countries of former Yugoslavia and Soviet Union and some developing countries. As a result, no more than about 10-15% of values are missing for measures drawn from Penn World Tables between 1960 and 2010. Data availability for WDI measures is rather mixed: While demographic variables offer almost complete time series for most countries and years, time series of macroeconomic variables and some of the educational attainment measures are incomplete more often with missing values in 20-25% of the cases.

to some extent. To this end, a standard HP-Filter (*Hodrick and Prescott, 1997*) was employed on annual observations of real GDP per capita. Resulting trend growth rates, computed from HP-filtered GDP values, are used in subsequent growth regressions.⁴

Throughout this paper, per capita growth rates and explanatory variables were transformed to 5-year averages in order to focus on medium-run growth. Former HP-Pre-filtering ensures that discussed fluctuations of annual GDP values at the beginning and at the end of 5-year episodes do not affect the dependent variable too much. Each country in the data set consists of up to 10 growth observations between 1960 and 2010.

Logarithm of real GDP per capita (measured in constant PPP-adjusted prices) is used as status variable. It is expected to capture and to rank the development stage of each country in the data set at any point in time. Thus, the status variable contains initial GDP of each of the ten growth episodes. However, due to our choice, we exclude a few heavily oil- and gas-producing small countries from estimation, because growth rates are fluctuating strongly and GDP levels may not reflect their true stage of development.⁵ We also exclude countries with a population of less than one million in order to exclude tax havens and independent island groups whose economies may work differently. From 185 countries available both in the WDI and PWT data sets we use 145 countries in the following analysis.⁶

5.2.2 Local Estimation in One Dimension

Model

A model equation for the growth rate $y_{i,t}$ is specified as follows:

$$y_{i,t} = \tilde{x}'_{i,t} \cdot \tilde{\beta} + e_{i,t}, \quad i = 1, \dots, N, t = 1, \dots, T, \quad (5.1)$$

where N and T are the cross sectional and time dimension number of observations, respectively, $\tilde{x}_{i,t}$ is the vector of explanatory variables for country i at time t , $\tilde{\beta}$ is a vector containing K corresponding coefficients and $e_{i,t}$ is the stochastic error term satisfying $E[e_{i,t}] = 0$.

The model equation for a one-dimensional FCM is

$$y_{i,t} = \tilde{x}'_{i,t} \cdot \tilde{\beta}(u_{i,t}) + e_{i,t}. \quad (5.2)$$

⁴ HP-filtered time series of real GDP per capita for each country were computed by using a (small) smoothing parameter of 10, in order to smooth away only strong short-term fluctuations. Then, first and last values of the smoothed time series were replaced by actual values, such that a countries' overall growth record remains to be built upon observed values only. Trend growth rates are computed by computing logarithmic differences.

⁵ Annual GDP levels of these countries are influenced strongly by concurrent international oil prices and oil production quotas. Therefore growth rates as the dependent variable is influenced strongly by it, which is not the focus of our study. Moreover, some of these countries have very high average income levels, not capturing their real development stage. This might render the chosen status variable unsuitable, since these countries were to be found among the highly developed countries.

⁶ Exclusion of countries has been made by using arbitrary thresholds of at least one million people (which excludes 34 countries) and an average share of fuel exports of at most 80% of all merchandise exports in combination with a logarithm of GDP per capita of more than 9.5 (about US-\$ 13.300) which excludes additional 6 countries). Arbitrary thresholds were not chosen in order to influence results in any way.

A local vector $\hat{\beta}(u_{i,t})$ is determined through minimization of the weighted residual sum of squares

$$RSS = \sum_{i=1}^N \sum_{t=1}^T \left[y_{i,t} - \sum_{k=1}^K \beta_k \cdot x_{i,t,k} \right]^2 \cdot K_u(u_{i,t} - \bar{u})$$

In this study, we rely exclusively on the Gaussian pdf as a Kernel function:

$$K_u(u) = \frac{1}{h \cdot \sqrt{2\pi}} \cdot e^{-\frac{1}{2} \left(\frac{u}{h}\right)^2} \quad (5.3)$$

Local estimation at focus \bar{u} is carried out by computing

$$\hat{\beta}(\bar{u}) = \left(\sum_{i=1}^N \sum_{t=1}^T \tilde{x}_{i,t} \cdot \tilde{x}'_{i,t} \cdot K_u(u_{i,t} - \bar{u}) \right)^{-1} \cdot \sum_{i=1}^N \sum_{t=1}^T \tilde{x}_{i,t} \cdot y_{i,t} \cdot K_u(u_{i,t} - \bar{u}). \quad (5.4)$$

Note that (5.4) is reduced to the pooled least squares estimator, if multiplication of each term with Kernel function K_u is left out of the equation, that is, if we assign uniform weights to all available observations. In that case, estimates for β are no more dependent on focus \bar{u} . In other words, equation (5.4) is a ‘‘Pooled Least Squares’’ regression, augmented by a certain weighting scheme. The weights for each observation are dependent on the current focus \bar{u} of local estimation, such that observations with a status variable very close to \bar{u} are given a relatively high weight. Moving away from the estimation weight peak, weights are reduced according to the *pdf* of a Normal distribution. The local environment around \bar{u} will be given relevant weights, distant observations will be given weights close to zero, thus incapable of influencing local estimates.

Bandwidth Selection

Selection of a proper bandwidth parameter for the Kernel is crucial to allow for a useful interpretation of results. Loosely speaking, the higher bandwidth parameter h , the more smoothing will be done. A relatively high value of h ensures that more weight will be applied to more distant observations relative to the focus of local estimation. Estimated functional coefficients will be smoother if a larger value for the bandwidth parameter is chosen. On the other hand, a low value for h ensures that only observations in a very close environment to the focus will be given relevant weights in the estimation. In that case, functional coefficients are more volatile over the status variables’ support.

Local estimation can only provide reasonable results in an environment with a sufficient number of observations. Otherwise a functional coefficient estimate may be dominated by very few observations and will be characterized by strong fluctuations and weird estimates in that range. To maintain confidence with the results it is desirable to apply a slightly higher value for bandwidth parameter h when the density of observations is comparably thin at a certain local environment around \bar{u} . Therefore, local bandwidth parameters are selected dependent on the density of observations. Loosely inhabited environments of the status support will be smoothed stronger.⁷

⁷ See *Jennen-Steinmetz and Gasser (1988)* for density-dependent local bandwidth selection. A two-stage ap-

Visualization of Local Coefficients

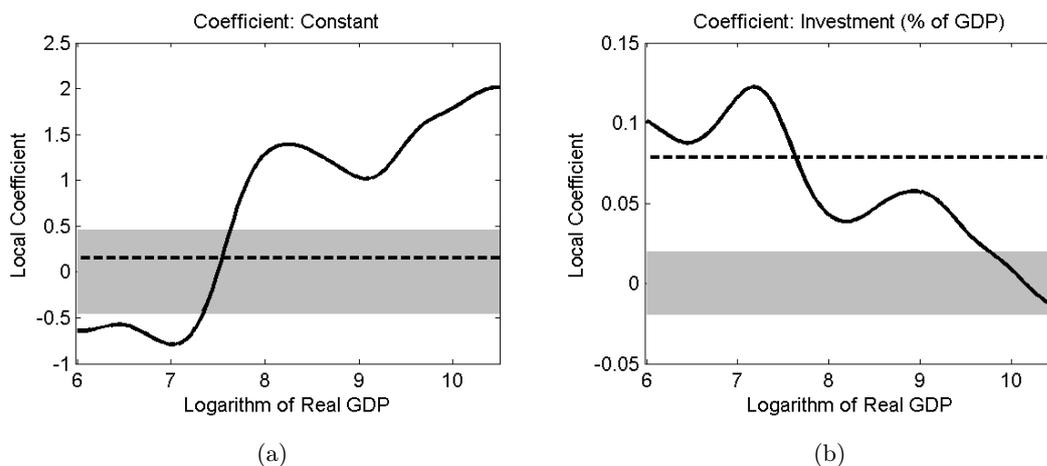


Figure 5.1: Functional coefficient estimates for 5-year growth rates between 1960 and 2010 as a dependent variable, using a constant and investment (% of GDP) as explanatory variables, $N=1267$, 1960-2010. (a) Functional coefficient of the constant, (b) Functional slope coefficient of investment in a one-dimensional FCM. The grey shaded area around 0 depicts the interval of coefficient values that are not considered significant at 5% level in pooled OLS estimation. The dashed line marks the pooled OLS point estimate.

Figures 5.1(a) and 5.1(b) depict the estimated functional coefficient values across the support of the status variable. The grey area around 0 can be considered the “area of insignificance”, i.e. coefficient values not significant at 5% confidence level in a pooled ordinary least squares (OLS) regression, while the dashed line depicts the pooled OLS point estimate of both parameters, respectively. The left figure 5.1(a) shows the intercept of the regression line in a scatter plot of growth vs. investment with OLS estimation. Since the influence of explanatory variables on growth is of primary interest, following sections will focus on slope parameters only, leaving functional estimates of the constant in the dark.

proach for the choice of appropriate local smoothing parameters along the support of the status variable has been followed here. In the first stage, a fixed value of $h = 0.35$ served to compute a preliminary weighting scheme $K_{uh}(u_{i,t} - \bar{u})$. The “effective number of observations” for each point \bar{u} of the status support is computed as follows:

$$N_{eff}(\bar{u}) = \frac{\sum_{i=1}^N \sum_{t=1}^T K_u(u_{i,t} - \bar{u})}{\max(K_u(u_{i,t} - \bar{u}))}$$

Thus, all the weights given to observations across countries and years are summed up, divided by the maximum weight of the current local weighting scheme. In a second stage, local bandwidth parameters in dependence of N_{eff} are generated via:

$$h(\bar{u}) = \frac{1}{N_{eff}(\bar{u})^{\frac{1}{4}}}$$

The bottom line is that a low number of effective observations at some point \bar{u} will result in a larger bandwidth parameter $h(\bar{u})$. Therefore, desired additional smoothing in loosely populated environments of the status support is done. Note that the bandwidth parameter of the functional coefficient model needs to be multiplied by the standard deviation of its status variable, if it is not normalized to a standard deviation of 1.

Local Mean Value

By using the weighting scheme from functional estimation, we are able to compute weighted, local mean values of explanatory variables. In fact, this procedure is identical to using the variable $x_{i,t}$ as dependent variable and to regress it on a constant using FCM along the support of the status variable $u_{i,t}$ as before. They are computed for different values of \bar{u} as follows:

$$\bar{x}(\bar{u}) = \frac{\sum_{i=1}^N \sum_{t=1}^T x_{i,t} \cdot K_u(u_{i,t} - \bar{u})}{\sum_{i=1}^N \sum_{t=1}^T K_u(u_{i,t} - \bar{u})} \quad (5.5)$$

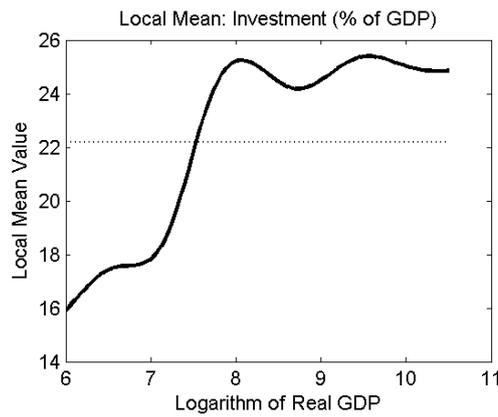


Figure 5.2: Local mean value of investment (% of GDP) over the support of the status variable, $N=1267$, 1960-2010. Dotted line depicts the non-weighted, overall mean.

Figure 5.2 shows locally weighted mean values of investment as a percentage of GDP. Average investment shares are lower for the poorest countries (16-18 %) compared to middle-income and rich countries (about 25 %). The global mean value for all 1267 observations of investment is slightly more than 22 %, which is depicted as a dotted line.

Local Correlation

To compute a local correlation coefficient between growth $y_{i,t}$ and some explanatory variable $x_{i,t}$, we have to define the local variance as follows, here shown for $x_{i,t}$:

$$\sigma_x^2(\bar{u}) = \frac{\sum_{i=1}^N \sum_{t=1}^T (x_{i,t} - \bar{x}(\bar{u}))^2 \cdot K_u(u_{i,t} - \bar{u})}{\sum_{i=1}^N \sum_{t=1}^T K_u(u_{i,t} - \bar{u})}, \quad (5.6)$$

as well as the local covariance between $x_{i,t}$ and $y_{i,t}$:

$$\sigma_{x,y}(\bar{u}) = \frac{\sum_{i=1}^N \sum_{t=1}^T (x_{i,t} - \bar{x}(\bar{u})) \cdot (y_{i,t} - \bar{y}(\bar{u})) \cdot K_u(u_{i,t} - \bar{u})}{\sum_{i=1}^N \sum_{t=1}^T K_u(u_{i,t} - \bar{u})} \quad (5.7)$$

Thus, we are able to define a local version of *Spearman's* correlation coefficient, which is the local covariance between $x_{i,t}$ and $y_{i,t}$ divided by local standard deviations at point \bar{u} respectively:

$$\rho(\bar{u}) = \frac{\sigma_{x,y}(\bar{u})}{\sigma_x(\bar{u}) \cdot \sigma_y(\bar{u})} \quad (5.8)$$

Figure 5.3 shows estimated values for a local correlation coefficient between growth and investment along the support of the status variable. It is largely related to Figure 5.1(b), but provides insights about the extent, to which investment is able to explain parts of the growth variation between countries and years. The dotted line depicts a correlation of zero. In this case the correlation between growth and investment is about 0.3 for poor countries and is decreasing with development stage. Among the richest economies, the correlation is approximately 0.

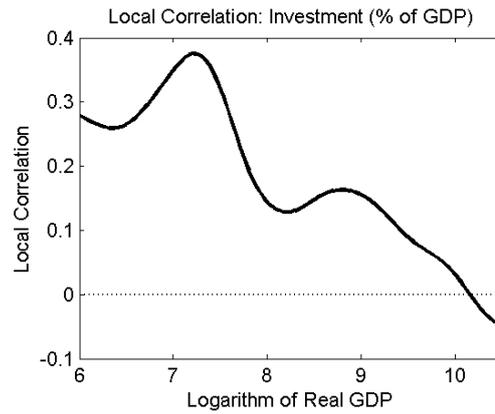


Figure 5.3: Local correlation coefficient of growth vs. investment (% of GDP) over the support of the status variable, N=1267, 1960-2010. Dotted line depicts a correlation of 0.

5.2.3 Local Estimation in Two Dimensions

Model

Instead of only using the stage of development as a status variable, time might also be included as a second dimension of status dependence. This way it becomes possible to investigate the temporal development of functional relationships, such that structural breaks in the quantitative influence of a certain variable on growth rates may be detected.

The model equation for a two-dimensional FCM is

$$y_{i,t} = \tilde{x}'_{i,t} \cdot \tilde{\beta}(u_{i,t}, w_{i,t}) + e_{i,t}. \quad (5.9)$$

Local estimation at focuses \bar{u} and \bar{w} is carried out by computing

$$\hat{\beta}(\bar{u}, \bar{w}) = \left(\sum_{i=1}^N \sum_{t=1}^T \tilde{x}_{i,t} \cdot \tilde{x}'_{i,t} \cdot K_u(u_{i,t} - \bar{u}) \cdot K_w(w_{i,t} - \bar{w}) \right)^{-1} \cdot \sum_{i=1}^N \sum_{t=1}^T \tilde{x}_{i,t} \cdot y_{i,t} \cdot K_u(u_{i,t} - \bar{u}) \cdot K_w(w_{i,t} - \bar{w}). \quad (5.10)$$

Note that the mere difference to (5.4) is a second weighting term K_w which sets an additional local estimation focus \bar{w} in the time dimension. Bandwidth parameters for both dimensions are selected equally to the one-dimensional case: In loosely populated areas of the status variables (in terms of available observations), a higher bandwidth parameter is selected to ensure additional smoothing.

Visualization of Local Coefficients

Figure 5.4(a) depicts the functional coefficient of the two-dimensional model of growth vs. investment in a surface plot. Values for the slope parameter of investment are highlighted according to the color scale on the right. Figure 5.4(b) shows the corresponding contour plot. Both figures give rise to the impression that the relationship of growth and investment is strong and rather stable for poorer countries, while this relation is of decreasing importance or strength in richer economies. The relation even turns negative in recent years.

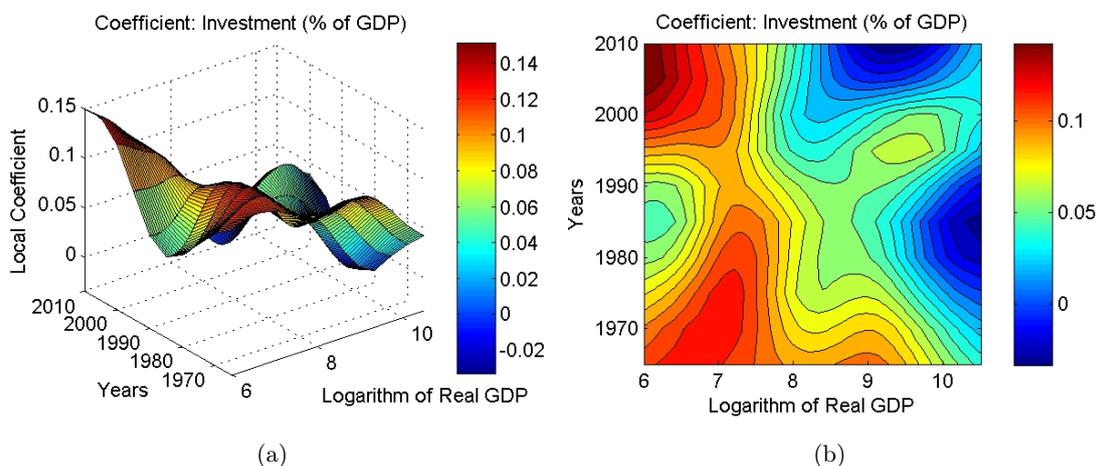


Figure 5.4: Functional coefficient estimates for 5-year growth average, using a constant and investment (% of GDP) as explanatory variables, $N=1267$, 1960-2010. (a) functional coefficient of a two-dimensional FCM as a surface plot, (b) the same functional coefficient in a two-dimensional FCM as a contour plot.

A generalization of local mean values for the two-dimensional model is discussed in section 5.5, together with a number of examples.

5.3 Bivariate Growth Analysis

5.3.1 Selection of Explanatory Variables

To reduce complexity, theoretical models of economic growth tend to emphasize one or very few aspects, but neglect other factors, that may also play an important role for real growth processes.⁸ Economic theory may contribute to the understanding of growth processes, but

⁸ The well-known *Solow* (1959) model highlights the role of factor accumulation, namely capital and labor endowment of an economy. Explicit modeling of innovation is not captured by this model, which is why modern

it does not provide a feasible and ready-to-use empirical model of all important causal relationships. *Henderson et al.* (2010) refer to this problem with the term “variable uncertainty”.

A large number of possible explanatory variables have been investigated in cross-country growth regressions. *Durlauf et al.* (2005) provide a still incomplete list of 140 such variables. However, coefficients with expected sign and significance level in a certain setting of explanatory variables cannot be regarded as sufficient evidence for a stable or even causal relationship between the measure of interest and economic growth. For most variables, it is possible to find specifications with coefficients below critical values or even with reversed sign. *Levine and Renelt* (1992) performed a robustness analysis using more than 50 explanatory variables. The results were disappointing, because almost all relationships are very sensitive to alterations in the composition of explanatory variables, and in many settings even the sign of the coefficient was changing. *Sala-i-Martin* (2007), on the other hand, used a similar robustness analysis and found some variables with rather stable results.

Anyway, intuition suggests that “true” growth determinants are not at all related in a simple linear model. Their growth contribution is rather interdependent, some relationships may be hierarchical and nonlinear. Moreover, each simple linear setting of explanatory variables gives rise to scepticism on the robustness of empirical findings with regard to the chosen combination of explanatory and control variables. This is even more true for functional estimates, because in this case there is not only a single point estimate of the corresponding coefficient, but a functional course across the support of the status variable instead. Apparently, a functional pattern for each coefficient is even more sensitive to alterations than a mere point estimate. In a two-dimensional functional coefficient approach, allowing for additional functional variation in the time dimension, we have even more reason to be doubtful about its robustness. Former approaches of parameter heterogeneity like *Liu and Stengos* (1999) and *Durlauf et al.* (2001) circumvented this difficulty and used a theory-based list of explanatory variables from *Mankiw et al.* (1993): The augmented *Solow* model which captures investment, population growth and initial educational attainment. Nevertheless, there seems to be pretty low confidence in the estimated functional coefficients, when *Durlauf et al.* (2001) interpret their results considerably careful, claiming that “there appears to be a lot of heterogeneity in the data”.

Since no theoretical model provides an exhaustive list of explanatory variables; since there is doubt whether true relationships can be approximated properly by a linear regression model; since functional estimates are even more sensitive to alterations in the list of explanatory variables than OLS results, the growth analysis in this section is simplified. Relationships of economic growth with only a single variable at a time will be considered, dependent on chosen status variables. However, a desirable quality of empirical growth analysis is a competitive test

growth models fill this gap by providing microfoundations for knowledge creation processes, resulting in a long-run growth rate dependent on human capital accumulation (*Lucas* (1988) as well as *Mankiw, Romer and Weil* (1992)), the level of research and development of new product variants (*Romer* (1986, 1990), *Grossman and Helpman*, 1991). Models of creative destruction, where newly developed products displace the old ones in an environment of temporal monopoly gains (*Aghion and Howitt*, 2006), provide another idea of the knowledge creation process and economic progress.

of certain measures in a multiple regression analysis.⁹ As *Durlauf et al.* (2005) put it, a less ambitious goal is pursued here, namely to investigate whether or not particular relationships have any support in the data at all. Patterns and systematic tendencies of the co-occurrence and correlation of certain parameters with good or bad growth records will be investigated with regard to the accompanying development stage of a country.

5.3.2 An Aghion-Howitt Model Revisited

This section covers a closer investigation of the relation between growth and the savings rate. *Aghion und Howitt* (2006) argue that middle-income countries need technological spillovers from higher developed countries to catch up with the technological frontier. Technology transfer occurs when technologically leading firms from developed economies conduct direct investments. To attract those foreign direct investments, the domestic climate for investments is crucial, i.e. infrastructure, macroeconomic stability, social peace. The governments' willingness to create this fruitful investment climate is necessary and for this purpose economy-scale savings are needed, as *Aghion and Howitt* argue. In highly developed countries, on the other hand, the ability to create savings is not that crucial, because further growth is accomplished rather by innovation, not by adaption of existing technologies. Therefore growth is not that dependent on savings any more. With regard to poor countries, which are very far from the technological frontier, total economic costs for the creation of such a favorable investment climate are relatively high, so savings are not that important for this group of countries as well. To investigate the relationship between growth and savings, *Aghion und Howitt* divide their sample into three subsamples of poor, medium-income and rich countries and regress average growth rates between 1960 and 2000 on a constant and the savings rate. In the group of poor and rich countries, they did not find the savings rate to be significantly related to growth, while the medium income group resulted in a significantly positive influence of economic savings on growth. Therefore, above mentioned hypotheses could be confirmed empirically.¹⁰

Compared to arbitrary sample division, functional estimation provides a more elegant way to estimate this relation. Again, the logarithm of real GDP per capita is used as a status variable. Average growth rates (in 5-year episodes) are regressed on a constant and the 5-year mean value of gross savings:

$$y_{i,t} = \beta_0(u_{i,t}) + \beta_s(u_{i,t}) \cdot s_{i,t} + e_{i,t}. \quad (5.11)$$

Figures 5.5(a) through 5.5(c) depict functional coefficients for the gross savings rate over the status variables' support in a one-dimensional and two-dimensional setting. Functional estimates for the constant are left out due to space considerations, since the slope parameter is of primary interest. Observe that the functional estimate shows exactly the expected pronounced hump (Inverse-U) shape, thus confirming the considerations and results of *Aghion*

⁹ This will be provided in section 5.4 to some extent.

¹⁰ See *Aghion und Howitt* (2006).

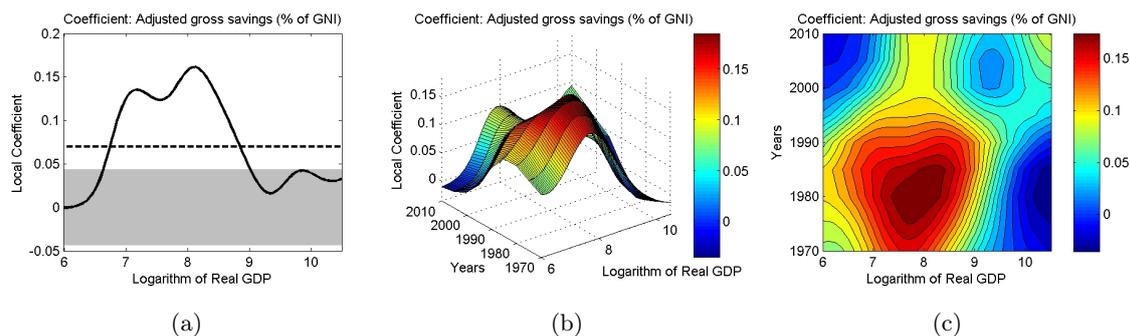


Figure 5.5: FCM estimates for rate of adjusted gross savings (% of GNI), $N=865$, 1965-2010. (a) 1-dim FCM estimate, (b) 2-dim FCM estimate as a surface plot, (c) same 2-dim FCM estimate as a contour plot. See sections 5.2.2 and 5.2.3 for a detailed description.

and Howitt (2006). The two-dimensional approach of Figures 5.5(b) and 5.5(c) shows that the expected hump shape is clearly visible over the whole sample period, even if the strength of the relationship appears to decrease since the mid-nineties.

In an alternative model setup, we replace the savings rate by a direct measure of the relationship under investigation, i.e. the net inflow of foreign direct investments as a percentage of real GDP. That way, we obtain Figures 5.6(a) through 5.6(c) as functional coefficient estimates of FDI inflow. Observe that the expected hump shape is clearly visible for the whole sample period, even if it becomes less pronounced since approximately 1995. For very rich countries as well as very poor countries the influence of foreign direct investments on growth rates is of minor importance compared to middle-income countries.

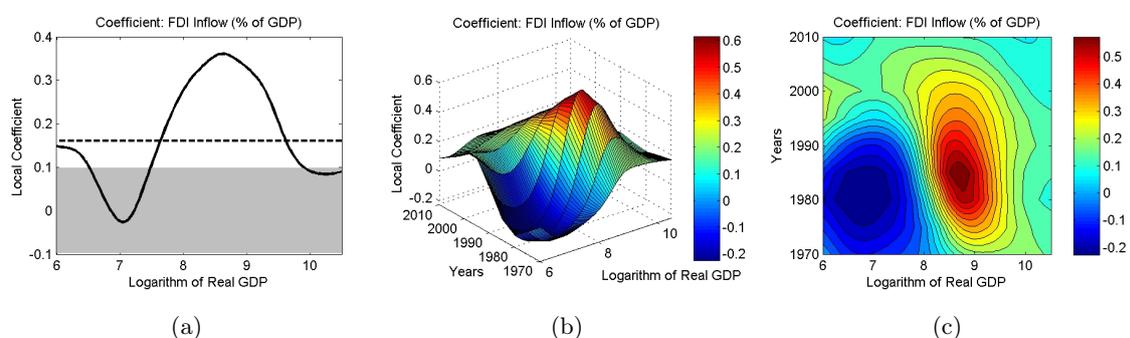


Figure 5.6: FCM estimates for FDI inflow, $N=1008$, 1965-2010. (a) 1-dim FCM estimate, (b) 2-dim FCM estimate as a surface plot, (c) same 2-dim FCM estimate as a contour plot. See sections 5.2.2 and 5.2.3 for a detailed description.

5.3.3 Macroeconomic and Demographic Measures

Figures 5.7(a) through 5.7(c) show that while the overall influence of higher government consumption seems to be negative, this relationship appears absent for very poor countries. Moreover, in recent years (2005-2010), the relationship seems to become even positive, especially in rich countries. This might be interpreted as an indication that counter-cyclical fiscal

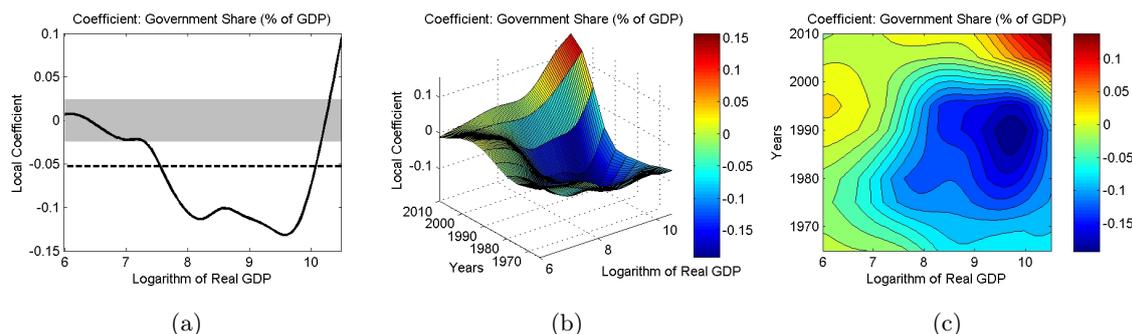


Figure 5.7: FCM estimates for government share (% of GDP), $N=1267$, 1960-2010. (a) 1-dim FCM estimate, (b) 2-dim FCM estimate as a surface plot, (c) same 2-dim FCM estimate as a contour plot. See sections 5.2.2 and 5.2.3 for a detailed description.

policy during the world financial crisis have contributed to dampen output losses.

The dotted line in Figure 5.8(a) shows that there is a positive sign for merchandise trade (% of GDP) in a regular OLS growth regression, even if this result is not significant at 5% level. It also shows that the functional coefficient is turning strongly negative for very poor countries, while only middle-income countries seem to actually benefit considerably from trade liberalization. 5.8(b) through 5.8(c) show that the result for poor countries is not stable over time. Until the seventies, possibly due to fixed exchange rates in the *Bretton-Woods-System*, the coefficient of trade was strongly positive. After 1980, the functional coefficient becomes negative, thus indicating that a higher trade share is accompanied by a very bad growth record in poor countries, at least on average. Poor open economies were probably unable to handle the uncertainty of flexible exchange rates as good as middle-income and developed economies. However, causality is not proven, so it is reasonable to remain careful.

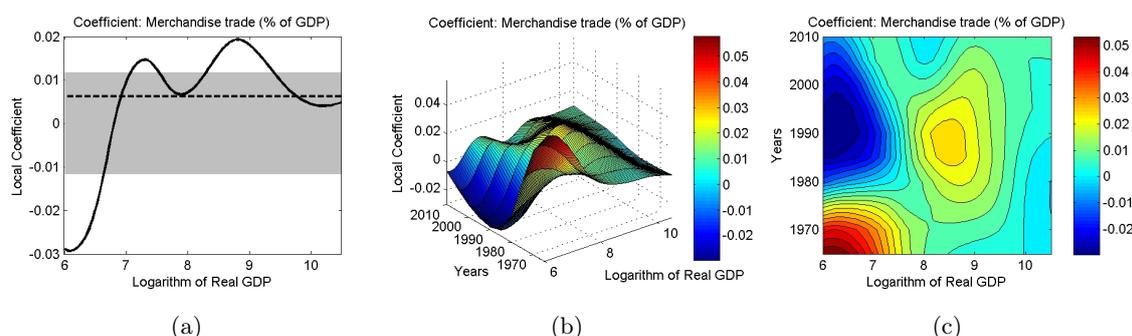


Figure 5.8: FCM estimates for share of merchandise trade (% of GDP), $N=1178$, 1960-2010. (a) 1-dim FCM estimate, (b) 2-dim FCM estimate as a surface plot, (c) same 2-dim FCM estimate as a contour plot. See sections 5.2.2 and 5.2.3 for a detailed description.

Figures 5.9(a) through 5.9(c) show functional coefficients for a growth model with (a constant and) the old age dependency ratio, which is the population share aged 65 and older divided by the working age population (15-64 years). Functional estimates show that a positive effect of this variable on growth seems to be present for poor and middle-income countries,

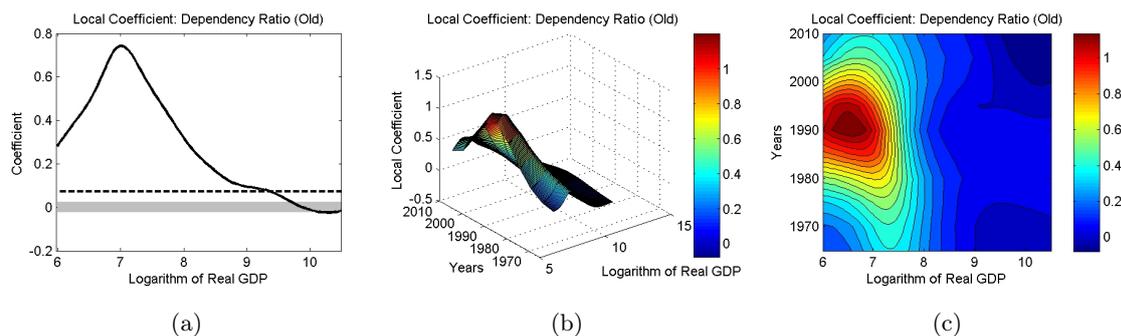


Figure 5.9: FCM estimates for old age dependency ratio, $N=1267$, 1960-2010. (a) 1-dim FCM estimate, (b) 2-dim FCM estimate as a surface plot, (c) same 2-dim FCM estimate as a contour plot. See sections 5.2.2 and 5.2.3 for a detailed description.

which may rather reflect a proxy variable for positive effects of a good health care system. In rich economies, this positive effect on growth seems to be non-existent over the whole span from 1960 through 2010. In recent years, the relationship seems to have become even negative for high-income countries reflecting the demographic burden of aging societies.

It should be noted that not all variation in shown functional coefficients allows for an intuitively convincing interpretation. There is certainly a large degree of sensitivity of results to variations in the panel data set. However, even if a detailed interpretation of functional coefficients is not found reliable, there are clearly benefits from using this method, since it allows a closer investigation of heterogeneity. Similar to outlier analysis it is possible to determine whether overall results are driven systematically by a certain number of observations. In the context of growth regressions, the overall influence of a certain measure on growth may be driven by observations from a certain decade only, or by a group of very poor countries. Some generalized results may not be valid for a certain subgroup of countries. Running a functional coefficient approach may as well serve as a sensitivity analysis from which additional insights are gained. Similarly, if we are interested in the determination of a suitable subsample of a huge panel data set, that is, a subsample that allows to emphasize certain principles, the discussed method might be a helpful tool to accomplish this goal.

5.4 Functional and Constant Coefficients Combined

5.4.1 Prefiltering of Control Variables

Bivariate analysis of growth determinants from section 5.3 does not include any control variables. This could be viewed as a major shortcoming of previous analysis, since multiple regression models allow to test explanatory variables competitively. One way to include control variables is provided by the *Frisch-Waugh-Theorem* (*Greene, 2003*), by which a multivariate model can be reduced by means of a partial regression approach. The desired investigation of functional relationships with a single selected variable remains possible, while important con-

control variables can also be included into the regression model.¹¹ By using a partial regression approach it is possible to filter out control variables with assumed constant coefficients.

The model is set such that only a part $\tilde{\beta}_2$ of the explanatory variables' coefficients are characterized by functional dependence, which, in our case, is a constant and one additional slope parameter. The control variables' coefficients $\tilde{\beta}_1$ are assumed to be constant:

$$y_{i,t} = \tilde{x}'_{1,i,t} \cdot \tilde{\beta}_1 + \tilde{x}'_{2,i,t} \cdot \tilde{\beta}_2(u_{i,t}, w_{i,t}) + e_{i,t}. \quad (5.12)$$

In partial regression, the dependent variable and remaining explanatory variable(s) are regressed in separate runs on the matrix of variables to be filtered out. Then, residuals of first stage regressions are entering FCM in order to proceed with local estimation.¹²

The *Solow* (1956) growth model provides three explanatory variables suitable as a core model. Initial GDP, investment and population growth are established theory-based variables that can be filtered out via partial least squares. Logarithm of real initial GDP per capita is included to account for the effect of convergence. The coefficient is found significantly negative in most studies (see *Levine and Renelt*, 1992). Investment is a measure for physical capital accumulation, whose coefficient is expected and usually found significantly positive. Population growth accounts for a growing workforce that needs to be endowed with physical capital. Moreover, child care is time-consuming, therefore high population growth is expected to reduce per capita growth. Thus, the expected sign is negative, while existing findings in the literature are rather mixed.¹³ In the following, one variable at a time is added to the core model to investigate its functional dependence while controlling for mentioned three variables.

5.4.2 Selected Results

Figures 5.10(a) through 5.10(c) show results of a growth model containing a constant, initial GDP, investment (% of GDP), population growth and the share of government consumption (% of GDP). Initial GDP, investment and population growth, referred to as the core Solow model, are filtered out via mentioned *Frisch-Vaugh* theorem. The remaining variance of economic growth is then investigated closely for its functional dependence in one and two dimensions. Comparing the results to the model of bivariate relationships (Figures 5.7(a) through 5.7(c)), we see that its pattern is very similar. Including control variables did not

¹¹ Functional coefficients for multivariate model specifications were not found robust. See section 5.3.1 for some reasoning.

¹² In a strictly linear case of partial regression, constant coefficients of $\tilde{x}'_{1,i,t}$ can be computed afterwards by using the estimated residuals of second stage partial regression. By using that procedure in our case, i.e. using local residuals calculated from FCM regression to compute constant coefficients $\tilde{\beta}_1$ afterwards, a strictly linear least squares model would be mixed up with a highly nonlinear functional coefficient approach. That is why only functional coefficients are shown, but reasonable estimates of the constant coefficients are not available.

¹³ Long-run growth in the *Solow* model is dependent only on an external rate of technological progress, so all three variables are expected to have no effect in the long run. In the short run, however, coefficients are expected to be of mentioned sign.

change the functional dependence to a large extent.¹⁴ Again, results suggest that while the overall influence of higher government consumption seems to be negative, this relationship is absent for very poor countries. Moreover, in recent years, the relationship even seems to become positive in all development stages. A standard model of constant coefficients may claim to find a statistically significant negative influence of government consumption over the years 1960 through 2010, but FCM analysis shows how this finding is flawed for recent years.

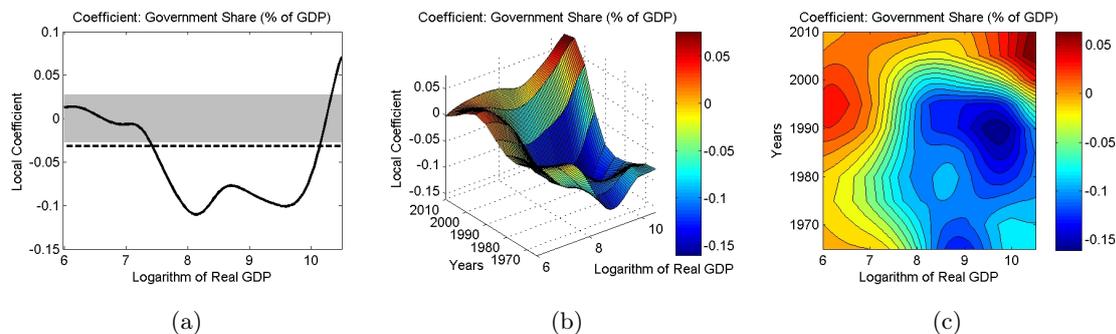


Figure 5.10: FCM estimates for government share (% of GDP), controlling for investment, population growth and initial GDP, $N=1253$, 1960-2010. (a) 1-dim FCM estimate, (b) 2-dim FCM estimate as a surface plot, (c) same 2-dim FCM estimate as a contour plot. See sections 5.2.2 and 5.2.3 for a detailed description.

Again, Figures 5.11(a) through 5.11(c) are comparable in shape to the bivariate case of Figures 5.8(a) through 5.8(c). Even if the overall effect of merchandise trade appears to be unrelated to growth, FCM analysis shows that too much trade may be detrimental to poor countries' growth prospects, at least since approximately 1980. On the other hand, middle-income countries may benefit from larger trade shares.

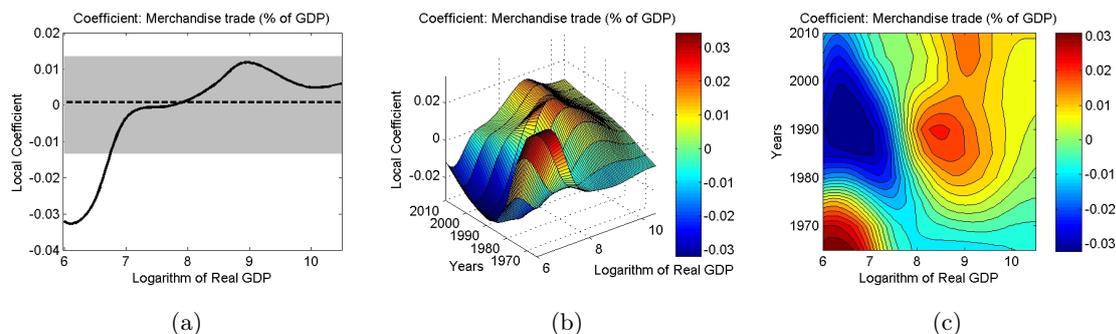


Figure 5.11: FCM estimates for share of merchandise trade (% of GDP), controlling for investment, population growth and initial GDP, $N=1166$, 1960-2010. (a) 1-dim FCM estimate, (b) 2-dim FCM estimate as a surface plot, (c) same 2-dim FCM estimate as a contour plot. See sections 5.2.2 and 5.2.3 for a detailed description.

Figures 5.12(a) through 5.12(c), compared with its bivariate counterparts of section 5.3.3 show some minor differences in detail, but qualitatively there is no big difference. The overall effect of a larger dependency ratio appears slightly positive, if anything, but this finding is only

¹⁴Note that the dotted line in Figure 5.7(a) is very close to the grey-shaded area compared to Figure 5.10(a). Therefore, the p-value for this coefficients' OLS estimate in the multivariate model is closer to 0.05.

true for poor countries (see Figure 5.12(a)). The reason is that a relatively large proportion of elderly people in a poor country reflects a good health care system to some extent, which is regarded as a prerequisite for growth. In high-income countries, there seems to be no effect until 1990. In latest years, however, a larger old age dependency ratio became a liability to growth prospects in those countries, reflected by negative local coefficients in the upper right corner of Figures 5.12(b) and 5.12(c).

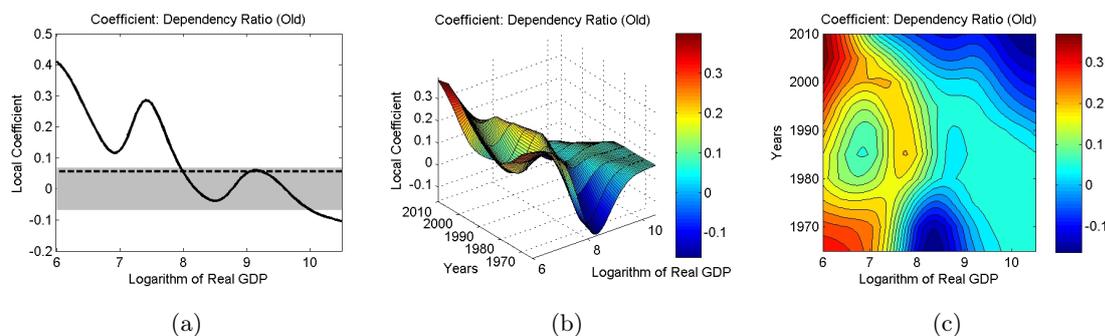


Figure 5.12: FCM estimates for old age dependency ratio, controlling for investment, population growth and initial GDP, $N=1253$, 1960-2010. (a) 1-dim FCM estimate, (b) 2-dim FCM estimate as a surface plot, (c) same 2-dim FCM estimate as a contour plot. See sections 5.2.2 and 5.2.3 for a detailed description.

Figures 5.13(a) through 5.13(c) are shown to provide rather counter-intuitive results. Combined with given control variables, this model is close to the augmented *Solow* model from *Mankiw et al. (1992)*, featuring an education variable to account for human capital accumulation. Pooled OLS provides a statistically significant positive coefficient for tertiary school enrolment, which is depicted as dotted line in Figure 5.13(a). While this overall result appears convincing at first, Figure 5.13(a) shows that the coefficient is much higher in low-income countries, while tertiary education has no effect on growth in middle-income and developed, industrial countries. This appears rather counter-intuitive. Results of the two-dimensional model suggest that tertiary education is less but still important in middle-income and high-income countries when we only focus on the years 1985 and later. Before that time, the coefficient even turns negative.

How come that the results suggest tertiary schooling to play a more important role in poor countries, while the coefficient declines with GDP level? A technical constraint of functional estimation emerges during a close investigation of results like this: Explanatory variables of FCM should not vary in their order of magnitude across the status variables' support. Otherwise, estimated local coefficients may also vary in their order of magnitude, which renders results highly questionable. In this case, tertiary school enrolment in low-income countries ranges on average between 2 and 3 per cent, while it ranges between 40 to 50 per cent in high-income countries (see figure 5.14(c)). Local estimation in an environment where most observations of the explanatory variable assume values of 2 or 3 per cent will result in much higher slope coefficients, compared to a local environment with an explanatory variable with values between 40 and 50 per cent, thus local coefficients in this case are not reliable.

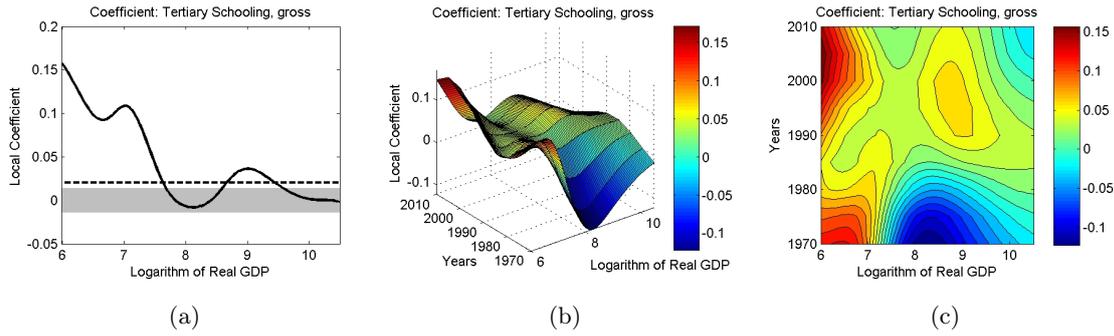


Figure 5.13: FCM estimates for tertiary school enrolment, controlling for investment, population growth and initial GDP, $N=1073$, 1965-2010. (a) 1-dim FCM estimate, (b) 2-dim FCM estimate as a surface plot, (c) same 2-dim FCM estimate as a contour plot. See sections 5.2.2 and 5.2.3 for a detailed description.

Therefore we need to be careful regarding the choice of explanatory variables. Questionable results may simply reflect a different order of magnitude of the explanatory variable at different positions of the status support. Figures 5.14(a) and 5.14(b) show that local mean values of government share and old age dependency ratio vary to a lesser extent.

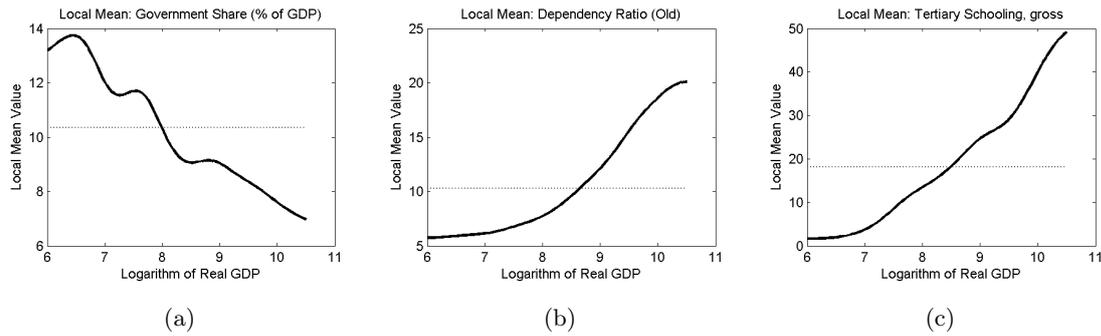


Figure 5.14: Local mean values for (a) Government share (% of GDP) (b) Old age dependency ratio, (c) Tertiary school enrolment. See section 5.2.2 for a detailed description.

5.5 Contours of Development

By using the weighting scheme from functional estimation, we are able to compute local mean values dependent on status variables. Note that this procedure is identical to regressing the variable $x_{i,t}$ on a mere constant along the support of the status variables $u_{i,t}$ and $w_{i,t}$ using functional (local) estimation. Local mean values in a two-dimensional setting at position \bar{u} and \bar{w} are computed according to:¹⁵

$$\bar{x}_{\cdot\cdot}^{local}(\bar{u}, \bar{w}) = \frac{\sum_{i=1}^N \sum_{t=1}^T x_{i,t} \cdot K_u(u_{i,t} - \bar{u}) \cdot K_w(w_{i,t} - \bar{w})}{\sum_{i=1}^N \sum_{t=1}^T K_u(u_{i,t} - \bar{u}) \cdot K_w(w_{i,t} - \bar{w})} \quad (5.13)$$

¹⁵ See section 5.2.2 for comparison with one-dimensional local mean value and its description. Notation is in accordance with earlier sections.

Contour plots of resulting local mean values across time and GDP levels provide a colorful visualization of worldwide economic and demographic development along two dimensions. Figure 5.15 is one example of how fertility (children per woman) ranges between rich and poor countries over time.

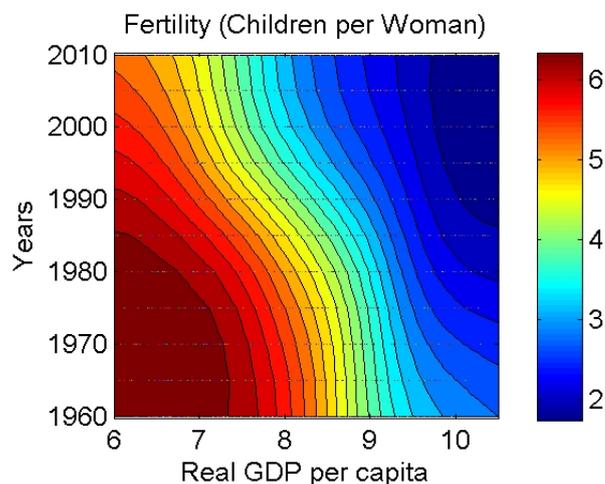


Figure 5.15: Contour plot of local mean values for 5-year fertility average, $N=1607$.

In order to provide additional information about observation density across the status support, each observation is marked in the two-dimensional contour plot, as is done in figure 5.15. Due to worldwide economic growth over the decades, observation marks are moving slightly to the right from 1960 through 2010. Observe that isoquants are moving from top-left to bottom-right, thus indicating that fertility has reduced for all income-levels over the years. In the richest economies, average fertility has reduced from approximately 3 to less than 2 children per woman, in least developed countries it declined from 6.5 to slightly more than 5 in recent years.

Selected Examples

Now that visualization of local mean values is explained, additional examples are shown in subsequent diagrams. Figure 5.16(a) features isoquants sloping downwards from top-left to bottom-right. This indicates that trade intensity has increased worldwide between 1960 and 2010, either in low-income, middle-income or in high-income countries. Moreover, trade intensity seems to be proportional to GDP level: Looking from left to right there is a rise in local means over the whole time span. The same findings are valid for life expectancy at birth (Figure 5.16(b)), but here the isoquants are steeper. Local mean values of life expectancy have increased about ten years for all per capita income levels, but the difference between poorest and richest countries is even larger. Local mean values in high-income countries moved approximately from 67 to 77 years, while poorest countries' local mean moved from roughly 45 to 55 years.

Figures 5.16(c) and 5.16(d) provide some insights regarding educational attainment. Adult

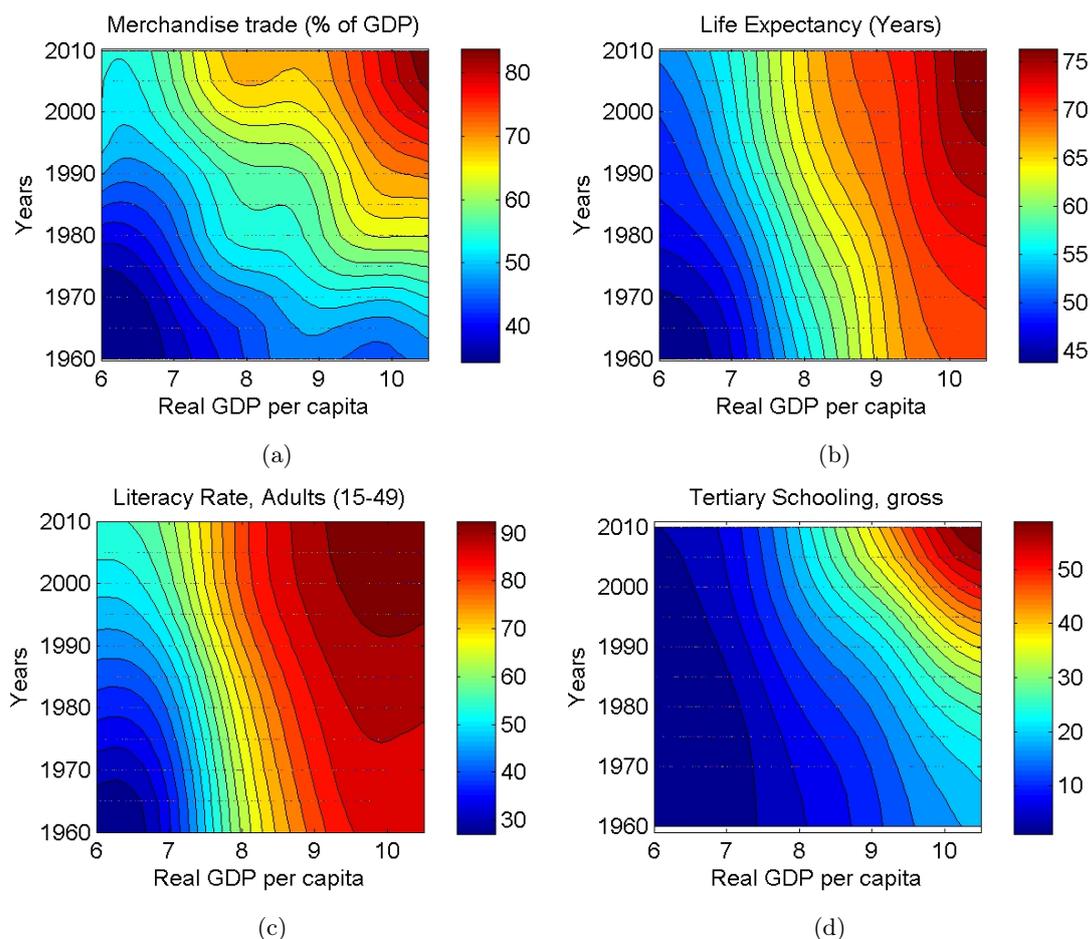


Figure 5.16: Contour plot of local mean values for (a) Trade share, $N=1463$, (b) Life expectancy at birth, $N=1608$, (c) Literacy rate of population (ages 15-49), $N=675$, (d) tertiary school enrollment ratio, $N=1216$.

literacy has been rising in all countries, but more so in poor and middle-income countries. By looking at literacy rates as a single indicator, it appears that the gap between high-income countries and the rest of the world has been narrowed over the years. Looking at tertiary school enrollment in Figure 5.16(d) as another indicator, downward sloping isoquants indicate a rise over all income-levels again. However, in this case the increase has been much larger for high-income countries. Therefore, this alternative indicator of educational attainment implies that the gap between rich and poor countries regarding higher education has widened.

Figure 5.17(a) shows that the stock of migrants is much higher and growing in high-income countries. This result is not surprising, since high-income countries appear more attractive than low- and middle-income countries when people choose to migrate. The migration stock percentage of the population in poor and middle-income countries has even been reduced over the years. Finally, Figure 5.17(b) depicts local mean values of HIV prevalence between 1990 and 2010. A maximum of infections has been recorded around the year 2000 in low-income countries. Since then, HIV casualties outweighed new infections such that the HIV prevalence in the population was reduced for the group of very poor countries. On the other hand,

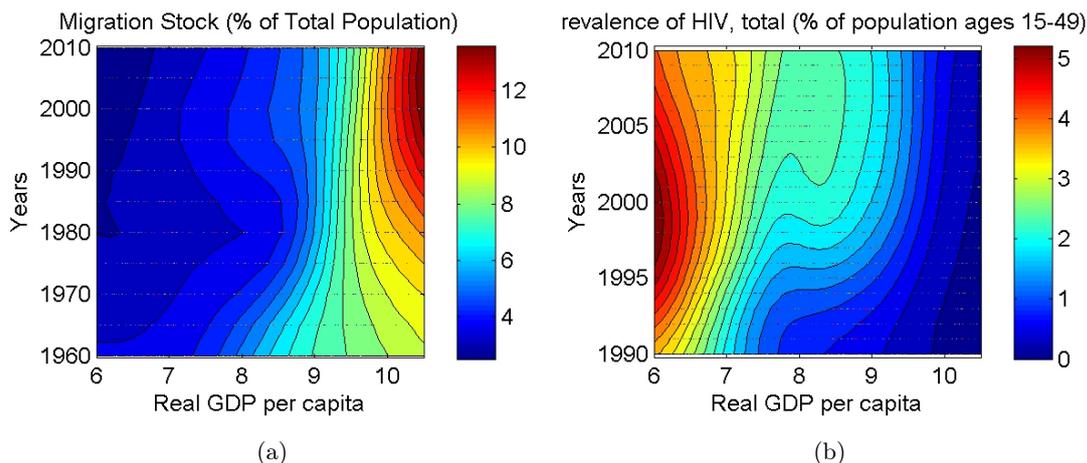


Figure 5.17: Contour plot of local mean values for (a) Migration stock (% of total population), $N=1668$, (b) HIV prevalence (% of population ages 15-49), $N=2709$.

ascending isoquants for middle-income and richer countries suggest that overall prevalence of HIV has sharply increased over time, even if average percentage levels for all income-levels remain below 4 per cent, in high-income countries even below 1 per cent. Note that these are only average levels of whole countries without considering population.

The presented local mean value procedure offers a new way to visualize panel data. It allows to investigate worldwide developments in demographic, macroeconomic and financial measures in a two-dimensional diagram.

5.6 Conclusion

This article applies functional coefficient analysis in the field of cross-country growth regressions. A panel data set of 145 countries between 1960 and 2010 was used, while focusing on medium-run growth dynamics. Functional coefficients were determined in a one-dimensional FCM with initial GDP as a status variable; and also in a two-dimensional FCM using the time dimension as a second status variable. Valuable insights can be generated from visualizations of functional coefficients of both models.

Instead of applying FCM to large multivariate models, whose results could hardly be considered robust, it was applied for the detailed investigation of bivariate relationships of economic growth. So the relationship of per capita growth rates with one variable at a time was investigated, namely investment, gross savings, foreign direct investment, government consumption, trade intensity, old age dependency ratio and tertiary educational attainment. For comparison, prefiltering of growth rates allowed to take established control variables of the *Solow* model into consideration. Estimated functional coefficients were qualitatively very similar to the bivariate case. In the light of given results, showing that locally estimated coefficients of growth determinants indeed vary strongly over time and also between poor and rich countries, it appears highly questionable to simply assume parameters to be constant as it is done in a

conventional growth regression.

Moreover, modeling economic relationships with functional coefficients allows a closer investigation of heterogeneity. It is possible to determine whether results of a constant coefficient model are primarily driven by a certain number of observations. The overall influence of a certain measure on a dependent variable may be strongly influenced by observations from a certain time period only, or by a subgroup of countries, while generalization of the finding for the whole data set may be invalid. Hence, FCM may as well provide a sensitivity analysis and robustness check for OLS regression models in general.

As a spin-off, the weighting scheme from FCM analysis can be used to compute a weighted mean value for certain variables of interest. By computing and plotting these local mean values across the support of two status variables, a diagram is generated that visualizes panel data. It is a colorful descriptive diagram, applicable for example in the field of development economics.

5.7 References

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5.A Appendix: List of Symbols

Symbol	Description
$\tilde{\beta}$	Parameter vector
$\hat{\beta}(\dots)$	Locally estimated Parameter vector
β_0	Constant
$\tilde{\beta}_1$	Vector of constant coefficient(s) (Prefiltering)
$\tilde{\beta}_2(\dots)$	Vector of functional coefficient(s) (Prefiltering)
β_k	k 'th parameter in $\tilde{\beta}$
β_s	Slope parameter for savings
$e_{i,t}$	Residual
h	bandwidth
$h(\dots)$	local bandwidth
i	Country number
k	Explanatory variable number in $\tilde{x}_{i,t}$
K	Number of explanatory variables
K_u	Kernel function (Gaussian) for $u_{i,t}$
K_w	Kernel function (Gaussian) for $w_{i,t}$
N	Number of countries
N_{eff}	Effective number of (weighted) observations
$\rho(\dots)$	Local correlation
RSS	Residual sum of squares (weighted)
$s_{i,t}$	Savings rate (explanatory variable)
$\sigma_x^2(\dots)$	Local variance of $x_{i,t}$
$\sigma_y^2(\dots)$	Local variance of $y_{i,t}$
$\sigma_{xy}(\dots)$	Local covariance between $x_{i,t}$ and $y_{i,t}$
t	Year
T	Number of years
$u_{i,t}$	Status variable
\bar{u}	Focus of local estimation in parameter space of $u_{i,t}$
$w_{i,t}$	Additional status variable (time)
\bar{w}	Focus of local estimation in parameter space of $w_{i,t}$
$x_{i,t}$	Some variable
$\tilde{x}_{i,t}$	Vector of explanatory variables
$x_{i,t,k}$	Explanatory variable in vector $\tilde{x}_{i,t}$
$\bar{x}(\dots)$	Local mean value of $x_{i,t,k}$ in one dimension
$\bar{x}^{local}(\dots)$	Local mean value of $x_{i,kt}$ in two dimensions
$\tilde{x}1_{i,t}$	Vector of explanatory variables with constant coefficients
$\tilde{x}2_{i,t}$	Vector of explanatory variables with functional coefficients
$y_{i,t}$	Dependent variable (growth rate)

Charakteristika und Diffusion deutscher PRO-Patente

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Abstract: Dieser Artikel charakterisiert die patentierten Forschungsergebnisse deutscher öffentlich finanzierter Forschungseinrichtungen (PROs) und zeigt relevante Kennzahlen, Muster und Forschungsschwerpunkte auf. Hierbei wird nach Untergruppen institutionell verbundener Forschungseinrichtungen (Forschungsfamilien) differenziert. Schließlich wird die Weiterverfolgung und Verwertung dieser Patente in unterschiedliche Länder skizziert, wiederum unter Berücksichtigung von Technologiefeldern und Forschungsfamilien.

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Eidesstattliche Erklärung

Ich erkläre hiermit an Eides Statt, dass ich meine Doktorarbeit “Essays on Economic Growth and Business Cycle Dynamics” selbständig und ohne fremde Hilfe angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen nach den mir angegebenen Richtlinien zitiert habe.

Ich versichere an Eides Statt, dass ich mich an keiner anderen Fakultät einer Doktorprüfung unterzogen habe.

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