

News Shocks, Monetary Policy, and Amplification Effects in New Keynesian Macroeconomics

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List of acronyms

AR	Autoregressive (model)
ARMA	Autoregressive moving average (model)
ABSPP	Asset-Backed Securities Purchase Program
BRE	Boundedly rational expectations
CB	Central bank
CE	Competition effect
CES	Constant elasticity of substitution
CPI	Consumer price index
CVPP	Covered Bond Purchases Program
DSGE	Dynamic stochastic general equilibrium
ECB	European Central Bank
FA	Full anticipation
GDP	Gross domestic product
MA	Moving average (model)
MM	Money multiplier
NKM	New Keynesian Macroeconomics
OMT	Outright Monetary Transaction
PA	Partial anticipation
RE	Rational expectations
SMP	Security Markets Program
UIP	Uncovered interest parity
U.S.	United States
VE	Variety effect
VAR	Vector autoregression

Chapter 1

Introduction

This thesis contributes to the existing theoretical and empirical literature on New Keynesian Macroeconomics in the field of news shocks and internal amplification effects in conjunction with and without (optimal) monetary policy. It is based on five articles of which three are published and two are currently under review. The thesis is organized in two parts. The first part (chapters 2 to 4) deals with anticipated disturbances (i.e. news shocks) and their volatility implications. The second part (chapters 5 and 6) deals with two amplification mechanisms that amplify the effects of monetary and non-monetary disturbances.

Part I – News shocks and their volatility implications

Several empirical studies emphasize the importance of news shocks for business cycle fluctuations. These shocks materialize in the future, but their size and maturity time is anticipated in advance by the agents. Most prominently, Schmitt-Grohé and Uribe (2012)¹ find in an estimated real business cycle model that about 50 percent of economic fluctuations can be attributed to anticipated disturbances. A theoretical branch of the literature indicates that news shocks destabilize the economy, i.e. lead to a higher volatility than unanticipated shocks of the same form. Fève et al. (2009) demonstrate in a purely rational expectations model that news shocks increase the volatility with increasing length of anticipation.

In light of these findings, this part begins to deal with the nonfundamentalness problem that is associated with news shocks in chapter 2. The remainder of this part studies the (de)stabilizing effects of anticipated disturbances in case of optimal monetary policy and bounded rationality (chapter 3) and in case of anticipated monetary disturbances that are not fully anticipated (chapter 4).

¹The list of references for each article is attached at the end of each chapter right after the corresponding conclusions.

Chapter 2 – News shocks, nonfundamentalness, and volatility

Rational expectations models with news shocks may generate moving average (MA) representations that are nonfundamental, i.e. some roots of the MA determinant lie inside the unit circle. Such representations cannot be obtained from the inversion of estimated vector autoregression (VAR). In fact, given an autocovariance structure, there exist infinitely many MA representations compatible with it, where fundamental representations present only a thin subset.² The nonfundamentalness typically arises from the lag polynomial that is associated with news shocks.

This chapter contributes to the existing literature in three ways: First, it derives an analytical solution formula for this special type of polynomial and discusses its main properties. Our solution formula may be used to convert a nonfundamental representation into a fundamental one by means of a Blaschke-type transformation matrix that flips the roots of the determinant of the moving average process. Second, we find that the roots do not change with increasing length of anticipation. We, therefore, conclude that the destabilizing effects of news shocks are exclusively due to its anticipation characteristic. Finally, as a terminological remark, we relate the lag polynomial associated with news shocks to the class of cyclotomic polynomials. The literature maintains that news shock polynomial is cyclotomic. We argue that this is not the case.

This chapter is based on the paper entitled “News shocks, nonfundamentalness, and volatility” which is joint work with Hans-Werner Wohltmann and has been published 2013 in *Economics Letters* 119(1), 17-19. The research question whether the destabilizing effects of news shocks are related to the problem of nonfundamentalness was initially raised by a referee to the paper of Winkler and Wohltmann (2012). Prof. Wohltmann further developed this idea and provided most of the analytical analysis. My contribution consists of linking the lag polynomial to the class of cyclotomic polynomials, the simulations, and most of the writing.

Chapter 3 – Volatility effects of news shocks in (B)RE models with optimal monetary policy

This chapter studies the volatility implications of anticipated cost-push shocks in a New Keynesian model under optimal unrestricted monetary policy with forward-looking rational expectations (RE) and backward-looking boundedly rational expectations (BRE). Bounded rationality assumes that agents have cognitive limitations and use simple heuristics (rule of thumbs) to guide their behavior. We analyze how the relative volatility results of news shocks change if rational expectations are replaced by boundedly rational expectations.

We find that the (volatility) effects of news shocks on the economy and, thus, their

²See Lippi and Reichlin (1994).

importance for business cycle fluctuations depend on the assumption of forward-looking rational expectations. If the degree of backward-looking price setting behavior is sufficiently small (large), anticipated cost-push shocks lead to a higher (lower) volatility in the output gap and in the central bank's loss than an unanticipated shock of the same size. The inversion of the volatility effects of news shocks between rational and boundedly rational expectations follows from the inverse relation between the price-setting behavior and the optimal monetary policy. By contrast, if the central bank does not optimize and follows a standard Taylor-type rule and the price setters are purely (forward-) backward-looking, the volatility of the economy is (increasing with) independent of the anticipation horizon. The volatility results for the inflation rate are ambiguous.

This chapter is based on the paper entitled "Volatility effects of news shocks in (B)RE models with optimal monetary policy" which is joint work with Hans-Werner Wohltmann and has been published 2015 as *Economics Working Paper 2015-07*, Christian-Albrechts-University of Kiel, Department of Economics. The initial idea to study news shock under bounded rationality was developed by myself and further exploited in collaboration with Prof. Wohltmann. My contribution further consists of substantial parts of the analytical analysis, the simulations, and most of the writing.

Chapter 4 – Partially anticipated monetary policy shocks – Are they stabilizing or destabilizing?

This chapter studies the volatility effects of anticipated monetary disturbances which are not fully anticipated by the public. So far, the literature has only considered two extreme cases of anticipation. Either the public has perfect information and fully anticipates the shock process (as in chapters 2 and 3) or the public is completely uninformed and does not anticipate the shock process at all. This paper introduces an intermediate scenario of partial anticipation, which covers both extreme scenarios as special cases. Under partial anticipation, the public has imperfect information about the magnitude, the start, and/or the end of the future monetary policy intervention. We use a dynamic Dornbusch-type model framework of a small open economy as model framework.

Our main results are as follows: First, partially anticipated monetary policy shocks may be stabilizing, i.e. lead to a lower volatility than a fully anticipated monetary policy shock of the same form. Second, we typically obtain a trade off in volatilities such that a simultaneous stabilization of inflation and output is not possible. If the public underestimates (overestimates) the size of the shock, output (inflation) may be stabilized. Our results imply that the central bank may have an incentive to withhold information from the public about the true central bank's intentions.

This chapter is based on the paper entitled "Partially anticipated monetary policy shocks – Are they stabilizing or destabilizing?" which is joint work with Hans-Werner Wohltmann and has been accepted for publication in the *Journal of Economics and*

Statistics. The research idea to study partially anticipated disturbances was developed by myself and further exploited in collaborations with Prof. Wohltmann who's initial idea was to study risk premium shocks. My contribution further consists of the simulations, substantial parts of the analytical analysis, and most of the writing.

Part II – Amplification channels of monetary and non-monetary disturbances

This part of the thesis considers two extensions of the standard business cycle models. Both extensions give rise to an internal amplification effect that may amplify the effects of exogenous shocks. Chapter 6 introduces a money and credit market into the baseline New Keynesian model which gives implicitly rise to a credit channel that may amplify monetary disturbances. Chapter 7 estimates a medium-scale real business cycle model with endogenous firm or product entry in order to quantify the amplification mechanism associated with endogenous firm entry.

Chapter 5 – Money and credit in the New Keynesian model

This chapter introduces a money and loan market into a static approximation of the New Keynesian framework. The demand side of the money and loan market follows from a money-and-credit-in-the-utility approach, where real balances and borrowing contribute to the household's utility. Contrarily to the baseline New Keynesian model, the central bank has no direct control over the bond rate. Instead, the central bank's instrument variables are the monetary base, the refinancing rate, and the central bank's inflation target. The introduced money-and-credit-in-the-utility approach implicitly gives rise to a credit channel as an additional transmission mechanism of monetary shocks in which the (current and future) bond and loan rate directly affect current goods demand.

Our main results are as follows: First, the central bank's influence on the bond rate and on a broader money aggregate is reduced. Changes in the refinancing rate and in the monetary base lead to less than proportional changes in the bond rate and in broad money, respectively. Accompanied by a contractionary credit supply shock, the effects of a monetary expansion are further reduced and may lead to a nearly complete neutralization. Second, in line with Bernanke and Blinder (1988), the credit channel amplifies the (output) effects of isolated monetary disturbances. Third, a rise in inflation (bond and loan rate) expectations decrease (increase) the effectiveness of expansionary monetary policy. Hence, if the monetary impulse is accompanied by a sufficiently large adjustment in inflation expectations in the same direction, the credit channel dampens the output effects. Fourth, in a dynamic version of our model, in which expectations are formed endogenously, we find that the credit channel amplifies output responses.

This chapter is based on the article entitled "Money and credit in the New Keynesian

model” which is joint work with Hans-Werner Wohltmann and has been published 2014 in *Review of Economics* 65(1), 253-280. The initial research idea to use a money-and-credit-in-the-utility approach to model the demand side of the money and loan market and most of the analytical analysis was provided by Prof. Wohltmann. My contribution consists of the simulations (including the calibration), most of the writing, and developing the dynamic version of the model.

Chapter 6 – Endogenous firm entry in an estimated model of the U.S. business cycle

This chapter studies the empirical importance of endogenous firm entry as amplification mechanism for business cycle fluctuations. To this end, we use the firm entry model by Bilbiie et al. (2012), extend it with several real frictions and estimate the model on U.S. data with Bayesian methods. In this model, the amplification mechanism of firm entry works through a competition and a variety effect. The variety effect describes the productivity gains from additional varieties. An increase in the number of firms increases output more than proportional due to increasing returns to specialization. The so-called competition effect captures the inverse relation between the number of producers and price mark-ups. An increase in the number of producers erodes market power. Price mark-ups fall which in turn boosts aggregate demand. To quantify the amplification mechanism and to disentangle the competition and the variety effect, we specify two counterfactual model frameworks, where either the competition effect or both the competition and the variety effect are switched off. We measure the amplification as percentage volatility difference across the three model variants.

We find that the amplification mechanism associated with firm entry substantially amplifies fluctuations in output and consumption, but dampens fluctuations in investment. For output, the total increase is given by 8.5 percent. The competition effect accounts for most of the amplification, amplifying output by 7 percent, whereas the increase through the variety effect only amounts to 1.5 percent. Both effects are statistically significant. If we consider each structural shock in isolation, the results are mixed. On the one hand, the competition and the variety effect amplify the impacts of labor productivity and wage mark-up shocks on output. This follows from the fact that for these shocks, output and firm entry are positively correlated. On the other hand, the competition and the variety effect dampen the output effects of aggregate demand and investment-specific technology shocks, for which the conditional correlation between firm entry and output is negative.

This chapter is based on the paper entitled “Endogenous firm entry in an estimated model of the U.S. business cycle” which is joint work with Roland Winkler and has been published 2015 as *Economics Working Paper* 2015-06, Christian-Albrechts-University of Kiel, Department of Economics. The initial research idea to quantify the importance of firm entry was developed by Roland Winkler. He also provided most of the theoretical

model derivation. My contribution consists of the estimation and most of the programming and simulations. Both authors contributed equally to the writing.

Part I

News shocks and their volatility implications

Chapter 2

News shocks, nonfundamentalness, and volatility

Coauthored by: Hans-Werner Wohltmann

Published in: Economics Letters, April 2013, vol. 119(1), pp. 17-19

Abstract

Rational expectations models with news shocks may generate moving average representation that are nonfundamental. The nonfundamentalness typically arises from the lag polynomial associated with news shocks. This paper provides an exact solution formula for this special type of polynomial and discusses its main properties. In the presence of news shocks, the solutions may be used to convert a nonfundamental moving average representation into a fundamental one and vice versa. From the properties of these solutions, we conclude that the destabilizing effects of news shocks are exclusively due to its anticipation characteristic.

JEL classification: E32, C22

Keywords: News shock, Nonfundamentalness, Cyclotomic polynomial

The full article can be downloaded via

<http://dx.doi.org/10.1016/j.econlet.2013.01.004>

Chapter 3

Volatility effects of news shocks in (B)RE models with optimal monetary policy

Coauthored by: Hans-Werner Wohltmann

Published in: Economics Working Papers 2015-07, Christian-Albrechts-University of
Kiel, Department of Economics

Abstract

This paper studies the volatility implications of anticipated cost-push shocks (i.e. news shocks) in a New Keynesian model under optimal unrestricted monetary policy with forward-looking rational expectations (RE) and backward-looking boundedly rational expectations (BRE). If the degree of backward-looking price setting behavior is sufficiently small (large), anticipated cost-push shocks lead to a higher (lower) volatility in the output gap and in the central bank's loss than an unanticipated shock of the same size. The inversion of the volatility effects of news shocks between rational and boundedly rational expectations follows from the inverse relation between the price-setting behavior and the optimal monetary policy. By contrast, if the central bank does not optimize and follows a standard Taylor-type rule and the price setters are purely (forward-) backward-looking, the volatility of the economy is (increasing with) independent of the anticipation horizon. The volatility results for the inflation rate are ambiguous.

JEL classification: E32, E52

Keywords: Anticipated shocks, Optimal monetary policy, Bounded rationality, Volatility

3.1 Introduction

Several empirical studies emphasize the importance of news shocks for business cycle fluctuations. These shocks materialize in the future, but their size and maturity time is anticipated in advance by the agents. Most prominently, Schmitt-Grohé and Uribe (2012) find in an estimated real business cycle model that about 50 percent of economic fluctuations can be attributed to anticipated disturbances.¹

A theoretical branch of the literature indicates that news shocks destabilize the economy, i.e. lead to a higher volatility than unanticipated shocks of the same form. Fève et al. (2009) demonstrate in a purely forward-looking rational expectations model that news shocks increase the volatility with increasing length of anticipation. With both backward- and forward-looking expectations, the volatility results are ambiguous as it is shown by Winkler and Wohltmann (2012) in an univariate model. However, they find that the anticipation of cost shocks – as considered here – greatly amplifies the volatility of all key macroeconomic variables in the estimated model of Smets and Wouters (2003).²

These (empirical and theoretical) findings rely on the assumption of forward-looking rational expectations. By contrast, under purely backward-looking boundedly rational expectations, the volatility is independent of the anticipation horizon.³ Bounded rationality assumes that agents have cognitive limitations and use simple heuristics (rule of thumbs) to guide their behavior and are recently under growing investigation.⁴

In light of these findings, our paper contributes to the existing literature in three ways: First, we combine the theory of news shocks and optimal monetary policy in a New Keynesian framework. Second, we study the (de)stabilizing effects of anticipated cost shocks in a multivariate environment. Third, we analyze how the relative volatility results of news shocks change if rational expectations are replaced by boundedly rational expectations. We introduce bounded rationality by assuming that a fraction of price setters have static expectations as in Leitemo (2008). We provide analytical results for

¹Their finding is supported by several VAR-based studies including Beaudry and Lucke (2010) and Barsky and Sims (2011). Beaudry and Portier (2006) and Jaimovich and Rebelo (2009) demonstrate that news shocks may help to explain recessions without relying on technological regress. However, there is no consensus about the importance of news shocks. Studies that find that news shocks only play a minor role include Fujiwara et al. (2011) and Forni et al. (2014). Kahn and Tsoukalas (2012) find in a structural DSGE model that news shocks account for less than 15 percent of the variance in output growth, but explain more than 60 percent in hours worked and inflation. For an extensive literature review on news shocks, readers are referred to Barsky and Portier (2013).

²Further related to this branch of literature is the paper by Offick and Wohltmann (2013), who study the properties of the lag polynomial associated with news shocks.

³To see this, consider the model $y_t = \rho y_{t-1} + \varepsilon_{t-q}$, where $\varepsilon_{t-q} \sim N(0, \sigma^2)$ is an i.i.d. news shock that is anticipated q periods in advance. Assuming stationarity, the variance of this model is given by $Var(y_t) = \sigma^2 / (1 - \rho^2)$, i.e. independent of q .

⁴De Grauwe (2012) e.g. combines boundedly rational expectations with the theory of discrete choice, which allows agents to choose between a set of heuristics. His model is able to create non-normally distributed movements in output growth. Lengnick and Wohltmann (2014) use a similar approach in a New Keynesian model with financial markets.

the limit case of purely forward- and purely backward-looking price setting behavior.

So far, optimal monetary policy has been studied almost exclusively in the presence of unanticipated disturbances.⁵ One exception is the study of Winkler and Wohltmann (2011), who analyze optimal simple interest rules. They find that the inclusion of forward-looking elements in an instrument rule is welfare enhancing in the case of anticipated shocks.⁶ However, they focus on purely forward-looking private expectations and the resulting welfare effects. By contrast, we study the relation between news shocks, volatility, optimal unrestricted monetary policy, and (boundedly) rational expectations.

3.2 News shocks and optimal monetary policy

We assume that the inflation rate is governed by a standard hybrid New Keynesian Phillips curve of the form

$$\pi_t = \beta(1 - \phi_\pi)E_t\pi_{t+1} + \beta\phi_\pi\pi_{t-1} + \kappa x_t + \varepsilon_{t-q} \quad (3.1)$$

where π_t and x_t are the inflation rate and the output gap measured as percentage deviations from the steady state, respectively. ϕ_π measures the degree to which price setters are boundedly rational and have backward-looking expectations. For $\phi_\pi = 0$ ($\phi_\pi = 1$), the price-setting behavior is purely forward-looking (backward-looking). ε_{t-q} is a white noise cost-push shock with unit variance which is anticipated q periods in advance.⁷ The shock is unanticipated for $q = 0$.

For convenience, we assume that the central bank aims to minimize the weighted sum of variance of the inflation rate and the output gap. The central bank's loss is given by

$$Loss_q = \lambda_1 Var_q(\pi_t) + \lambda_2 Var_q(x_t) \quad (3.2)$$

As in Leitemo (2008), the optimal targeting rule then includes forward- and backward-looking elements and reads as

$$\pi_t = -\frac{\lambda_2}{\lambda_1\kappa}(x_t - x_{t-1}) - \frac{\lambda_2}{\lambda_1\kappa}\phi_\pi x_{t-1} + \frac{\lambda_2}{\lambda_1\kappa}\beta^2\phi_\pi E_t x_{t+1} \quad (3.3)$$

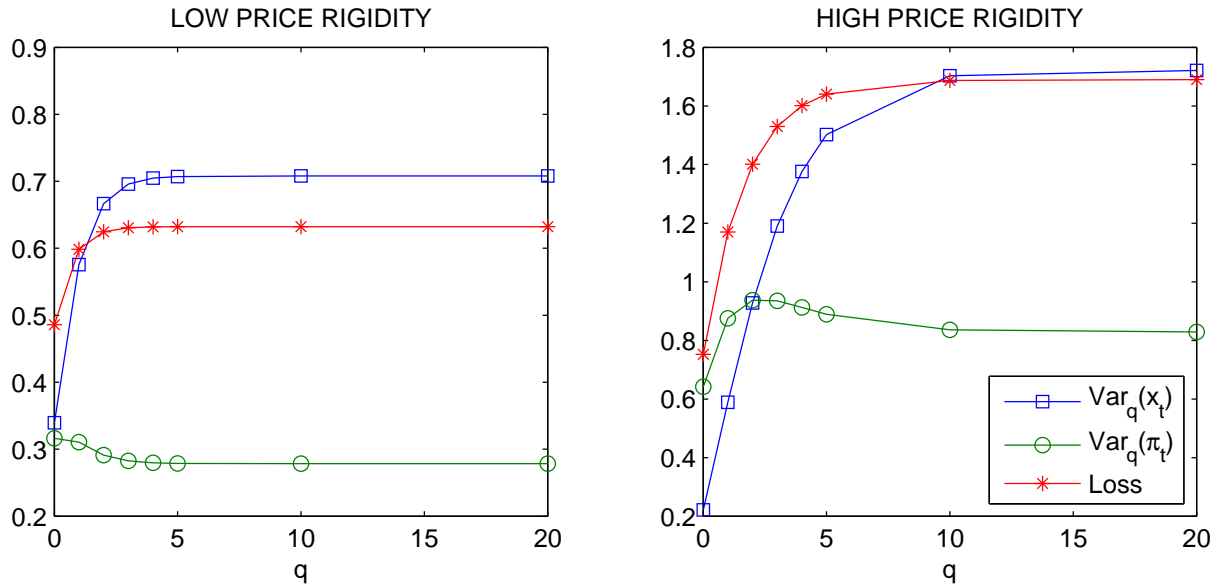
The central bank optimization is independent of the form of the IS equation and of the

⁵This includes Leitemo (2008), who finds an inverse relation between the private pricing behavior and the optimal monetary strategy. If the private sector is backward-looking, monetary policy should be forward-looking, and vice versa. This general result also holds for news shocks.

⁶Further noteworthy is the paper by Winkler and Wohltmann (2009), who show how to solve rational expectations models with news shock under optimal monetary policy.

⁷Note that we limit our discussion to cost-push shocks for which the central bank faces a trade off between output and inflation stabilization even without instrument target as considered here. This type of shock is also found to be highly relevant for business cycle fluctuations, see e.g. Schmitt-Grohé and Uribe (2012).

Figure 3.1: Loss and variances in the case of purely forward-looking price setting



Note: Parameters are set to $\beta = 0.99$, $\sigma = \eta = 2$, $\kappa = (\sigma + \eta)(1 - \omega)(1 - \omega\beta)/\omega$, $\lambda_1 = 1$, $\lambda_2 = 0.5$. Under low (high) price rigidity, the Calvo parameter ω is set to 0.7 (0.8), implying $\kappa = 0.2$ ($\kappa = 0.53$). Assuming continuity, the maximum in the inflation variance is reached in $q^* = 0.08$ ($q^* = 2.0$).

lead time q . Equations (3.1) and (3.3) fully describe the dynamics of the output gap and the inflation rate.

Before we turn to the general case of hybrid private price-setting behavior, we discuss the limit case of purely forward-looking price setting. Note that in both limit cases ($\phi_\pi = 0$ and $\phi_\pi = 1$) the system remains hybrid. This is due to the inverse relation between the price-setting behavior and the optimal monetary strategy as described in Leitemo (2008).

3.2.1 Purely forward-looking price setters

For $\phi = 0$, the system can be reduced to an univariate hybrid equation of the form

$$x_t = aE_t x_{t+1} + bx_{t-1} + c\varepsilon_{t-q} \quad (3.4)$$

with $a = \beta b$, $b = \lambda_2/(\lambda_2(1+\beta) + \lambda_1\kappa^2)$, and $c = -\lambda_1\kappa/(\lambda_2(1+\beta) + \lambda_1\kappa^2)$. Since $1 > \beta > 0$, $\text{sgn}(a) = \text{sgn}(b)$. This implies that the variance of x_t is unambiguously increasing in q as it is shown by Winkler and Wohltmann (2012).

The volatility of the inflation rate, on the other hand, may also be decreasing in q . Its variance is given by

$$\text{Var}(\pi_t) = \frac{2\beta_0^2}{(1+\alpha)(1+\delta)(1-\alpha\delta)} \left(\frac{\lambda_2}{\lambda_1\kappa} \right)^2 \left[1 - \frac{1-\alpha\delta}{\alpha-\delta} \delta^{2(q+1)} + \frac{(1-\alpha)(1+\delta)\delta\alpha}{\alpha-\delta} (\alpha\delta)^q \right] \quad (3.5)$$

where $|\alpha| < 1$ is the stable root of $\alpha_{1,2} = (1 \pm \sqrt{1-4ab})/(2a)$, $\beta_0 = c/(1-a\alpha)$, and

$\delta = a/(1 - a\alpha)$.⁸ An unanticipated shock may generate a higher inflation volatility than a cost-push shock that is anticipated in the infinite past:

$$Var_{q=0}(\pi_t) > Var_{q \rightarrow \infty}(\pi_t) \quad \text{if} \quad \frac{\lambda_1 \kappa^2}{\lambda_2} > \sqrt{1 + 4\beta} - (1 + \beta) \quad (\lambda_2 > 0) \quad (3.6)$$

The reason for the ambiguity in the inflation volatility are two opposing effects: On the one hand, the longer the length of anticipation, the higher is the variance of the output gap, which – in isolation – also leads to a higher variance in inflation. On the other hand, the response of the output gap becomes smoother, i.e. x_t is more autocorrelated, with increasing q . Since the inflation rate depends via the targeting rule on the change in the output gap, this reduces – in isolation – the variance of inflation.⁹ Condition (3.6) does not imply that an anticipated shock gives a lower inflation volatility for all anticipation horizons. That is, the inflation variance may not be monotonic in q . The maximum is reached in $q = \max(q^*, 0)$ where¹⁰

$$q^* = \frac{1}{\log \alpha - \log \delta} \left\{ \log \frac{2\delta(1 - \alpha\delta)}{(1 - \alpha)\alpha(1 + \delta)} + \log \frac{\log \delta}{\log \alpha\delta} \right\} \quad (3.7)$$

Despite the fact that the variance of inflation may be decreasing in q , the loss (3.2) is always increasing in q . Only under strict inflation targeting ($\lambda_2 = 0$) does the central bank perfectly stabilize the inflation rate and the loss is zero, independently from q .

Figure 3.1 illustrates the above results for high and low price rigidity. Under low (high) price rigidity, the Phillips curve parameter κ is relatively large (small) such that condition (3.6) is (not) satisfied.

3.2.2 Hybrid price-setting behavior

If we allow for backward-looking price-setting behavior (i.e. $\phi_\pi > 0$), the results under purely forward-looking price setting of the previous subsection may be reversed. This reversion can be seen in figures 3.2 and 3.3. Figure 3.2 shows the differences in the loss and in the volatilities of the output gap and the inflation rate between an anticipated and an unanticipated cost shock for different degrees of hybridity and anticipation horizons. If ϕ_π – the degree of backward-lookingness – is sufficiently large, all three differences are negative for arbitrary anticipation horizons. Contrarily to the output gap, the volatility in inflation may not be monotonic in q for $\phi_\pi > 0$.

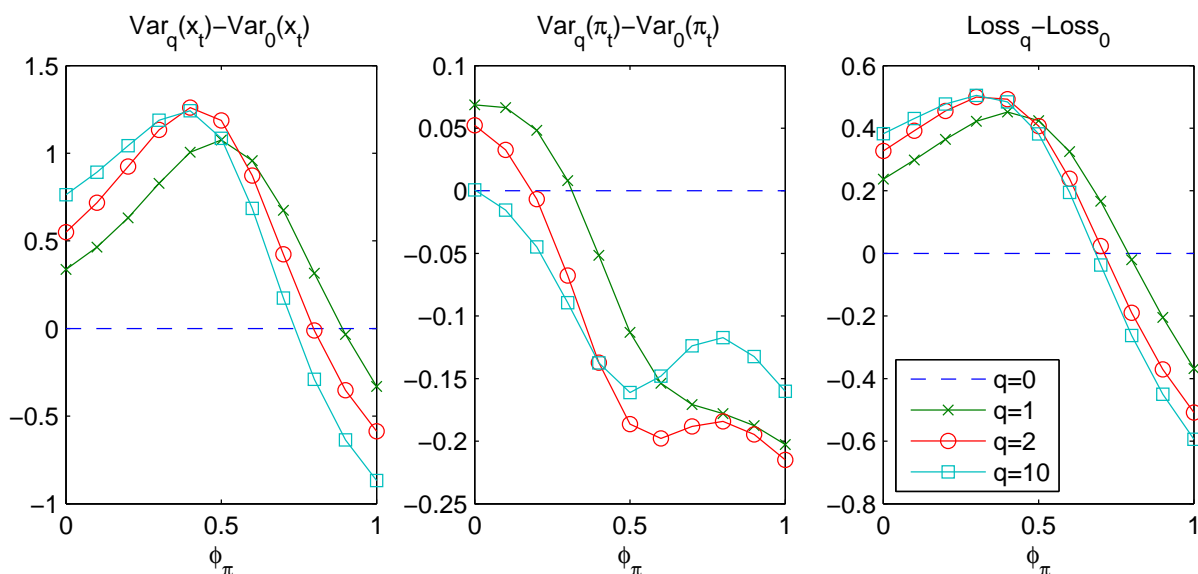
Figure 3.3 compares the volatilities and the loss of an unanticipated ($q = 0$) and an

⁸Note that the output gap can be written as an ARMA(1,q) process of the form $x_t = \alpha x_{t-1} + \sum_{k=0}^q \delta^k \beta_0 \varepsilon_{t+k-q}$. A stable solution requires $|\alpha| < 1$. For a full derivation of the results under purely forward-looking price setting, see appendices 3.A and 3.B.

⁹The two opposing effects can be directly seen by taking the variance of the targeting rule: $Var(\pi_t) = 2\lambda_2^2/(\lambda_1 \kappa)^2 [Var(x_t) - E(x_t x_{t-1})]$, where both $Var(x_t)$ and $E(x_t x_{t-1})$ are increasing in q .

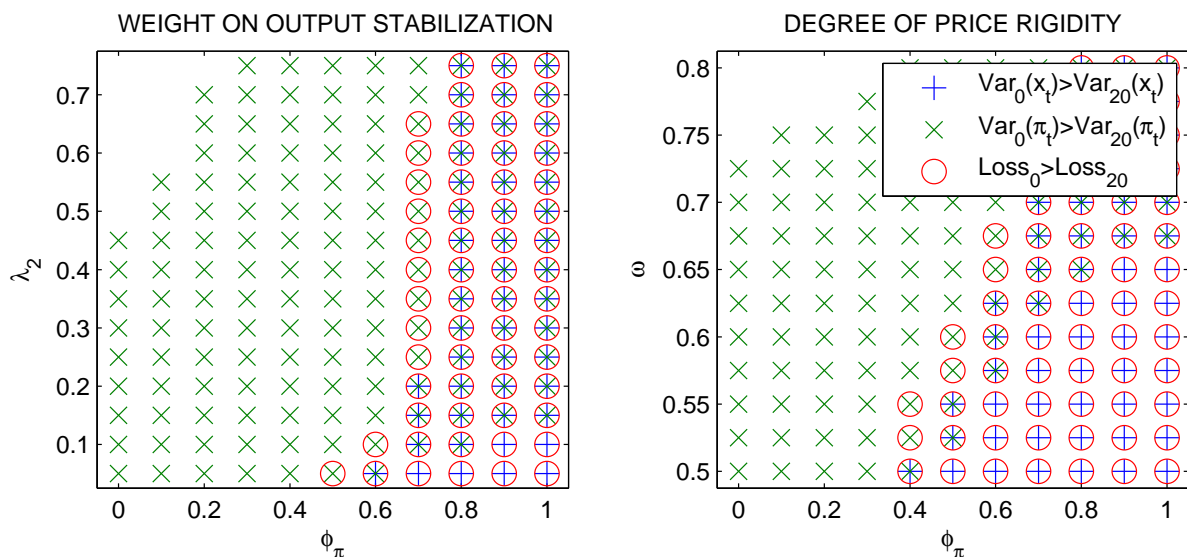
¹⁰Note that equation (3.7) assumes that q is continuous.

Figure 3.2: Loss and variances for different degrees of hybridity



Parameter calibration: $\beta = 0.99$, $\sigma = \eta = 2$, $\omega = 0.75$, $\kappa = (\sigma + \eta)(1 - \omega)(1 - \omega\beta)/\omega$, $\lambda_1 = 1$, $\lambda_2 = 0.5$.

Figure 3.3: Parameter sensitivity



Note: In left plot we set $\omega = 0.75$ and change the central bank's weight of output stabilization λ_2 ranging from 0.05 to 0.75. In the right plot we set $\lambda_2 = 0.5$ and change the degree of price rigidity ω ranging from 0.5 to 0.8. The remaining parameters are calibrated as follows: $\beta = 0.99$, $\sigma = \eta = 2$, $\kappa = (\sigma + \eta)(1 - \omega)(1 - \omega\beta)/\omega$, $\lambda_1 = 1$. Note that \oplus means that both $Var_{20}(x_t)$ and $Loss_{20}$ are smaller, \otimes means that both $Var_{20}(\pi_t)$ and $Loss_{20}$ are smaller, and the combination of all three symbols means that both variances and the loss are smaller for $q = 20$ than for $q = 0$.

anticipated ($q = 20$) shock – additionally to ϕ_π – for different degrees of price rigidity ω and for different weights λ_2 the central bank puts on output stabilization. We find that the volatility in the output gap and the loss are less variant to changes in λ_2 and ω . In case of purely backward-looking price setting, the volatility in output and the loss are decreasing in q for all parameter constellations under consideration. Contrarily, the volatility results for the inflation rate are ambiguous for both limit cases.¹¹

In summary, if ϕ_π is sufficiently large, it holds that: (i) The variance of the output gap and the loss decrease monotonically with increasing lead time q . (ii) The variance of the inflation rate is decreasing (increasing) in q if the weight λ_2 and/or the degree of price rigidity ω are sufficiently large (small). The reason for this inversion of volatility results is the inverse relation between the private pricing behavior and the optimal monetary policy strategy as described in Leitemo (2008).

3.3 Concluding remarks

This paper studies the volatility implications of anticipated cost-push shocks in a hybrid New Keynesian model with forward- and backward-looking price setters and optimal (unrestricted) monetary policy response. In particular, it is analyzed how the relative volatility results of news shocks under optimal monetary policy change if rational expectations are replaced by boundedly rational expectations.

We find that the destabilizing effects of anticipated cost-push shocks crucially depend on the type of private expectations. Under purely forward-looking rational expectations, the volatility in the output gap and the central bank's loss are unambiguously increasing with increasing anticipation horizon. Contrarily, under bounded rationality, we obtain the reversed result: If the degree of backward-looking price setting behavior is sufficiently large, the anticipation of cost-push shocks leads to a stabilization of the output gap and the central bank's loss. If – in addition – the central bank's weight on output stabilization and/or the degree of price rigidity is sufficiently large, we also obtain a stabilization of the inflation rate.

The inversion of the volatility effects of news shocks between rational and boundedly rational expectations follows from the optimization of the central bank. This optimization leads to an inverse relation between the price-setting behavior and the optimal monetary policy. By contrast, if the central bank follows an ad hoc or optimized standard Taylor-type rule and the price setters are purely (forward-) backward-looking, the volatility of the economy is (increasing with) independent of the anticipation horizon.¹²

Two remarks on the robustness of our results in order: First, without instrument target

¹¹ Analytical results for this limit case can be found in appendix 3.C.

¹² Specifically, the Taylor-type rule must not contain any (backward-) forward-looking element for this to hold.

in the loss function of the central bank, the form of the targeting rule is independent of the form of the dynamic IS equation. Hence, our volatility results also hold for non-separable utility functions as in Jaimovich and Rebelo (2009) and Schmitt-Grohé and Uribe (2012). Second, we argue that our results also hold for more complex backward-looking price-setting behavior as in De Grauwe (2012). For reasons of space, we model boundedly rational expectations only as static expectations.¹³

¹³We also studied the (de)stabilization effects of news shocks in a boundedly rational model with switching, in which the price setters are able to choose from a set of backward-looking expectations heuristics. The model setup is taken from Lengnick and Wohltmann (2014). Contrarily to the model in De Grauwe (2012) and Lengnick and Wohltmann (2014), we include forward-looking rational expectations through the optimal monetary strategy. We do not obtain qualitative changes in comparison to the model without purely backward-looking model without switching. Results are available upon request.

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3.A Appendix: Hybrid univariate model

A hybrid univariate model of the form

$$y_t = aE_t y_{t+1} + by_{t-1} + c\varepsilon_{t-q} \quad (3.8)$$

with $\varepsilon_t \stackrel{i.i.d.}{\sim} N(0, \sigma^2)$ can be written as MA(∞) of the form

$$y_t = \sum_{s=0}^{\infty} \alpha^s \sum_{k=0}^q \delta^k \beta_0 \varepsilon_{t-s+k-q} = \sum_{s=0}^{\infty} \alpha^s h_{t-s} \quad \text{with} \quad h_t = \sum_{k=0}^q \delta^k \beta_0 \varepsilon_{t+k-q} \quad (3.9)$$

where $\alpha = (1 - \sqrt{1 - 4ab}) / (2a)$, $\beta_0 = c / (1 - a\alpha)$, and $\delta = a / (1 - a\alpha)$. The variance of y_t can be derived as follows:

$$Var(y_t) = \sum_{s=0}^{\infty} \sum_{\bar{s}=0}^{\infty} \alpha^s \alpha^{\bar{s}} \sum_{k=0}^q \sum_{\bar{k}=0}^q \beta_0^2 \delta^k \delta^{\bar{k}} E(\varepsilon_{t-s+k-q} \varepsilon_{t-\bar{s}+\bar{k}-q}) \quad (3.10)$$

$$= \beta_0^2 \sum_{s=0}^{\infty} \alpha^{2s} \sum_{k=0}^q \delta^{2k} \sigma^2 + 2\beta_0^2 \sum_{s=0}^{\infty} \sum_{j=0}^{q-1} \sum_{k=0}^{q-1-j} \alpha^{2s+j+1} \delta^{2k+j+1} \sigma^2 \quad (3.11)$$

$$= \beta_0^2 v_t \sigma^2 + 2\beta_0^2 w_t \sigma^2 \quad (3.12)$$

v_t and w_t can be simplified to

$$v_t = \sum_{s=0}^{\infty} \alpha^{2s} \sum_{k=0}^q \delta^{2k} = \frac{1}{1 - \alpha^2} \frac{1 - \delta^{2(q+1)}}{1 - \delta^2} \quad (3.13)$$

$$w_t = \sum_{s=0}^{\infty} \sum_{j=0}^{q-1} \sum_{k=0}^{q-1-j} \alpha^{2s+j+1} \delta^{2k+j+1} \quad (3.14)$$

$$= \frac{\alpha\delta}{1 - \delta^2} \sum_{s=0}^{\infty} \alpha^{2s} \sum_{j=0}^{q-1} (\alpha\delta)^j - \frac{\delta^{2(q+1)}}{1 - \delta^2} \sum_{s=0}^{\infty} \alpha^{2s} \sum_{j=0}^{q-1} \left(\frac{\alpha}{\delta}\right)^{j+1} \quad (3.15)$$

$$= \frac{\alpha\delta}{1 - \delta^2} \frac{1}{1 - \alpha^2} \frac{1 - (\alpha\delta)^q}{1 - \alpha\delta} - \frac{\alpha\delta^{2(q+1)}}{1 - \delta^2} \frac{1}{1 - \alpha^2} \frac{1 - \left(\frac{\alpha}{\delta}\right)^q}{\delta - \alpha} \quad (3.16)$$

In summary, the variance of y_t is given by $Var(y_t) = V(q)$ where

$$V(q) = \frac{\beta_0^2}{(1 - \alpha^2)(1 - \delta^2)} \left\{ 1 - \delta^{2(q+1)} + 2 \frac{\alpha\delta}{1 - \alpha\delta} [1 - (\alpha\delta)^q] + 2 \frac{\alpha}{\alpha - \delta} [\delta^{2(q+1)} - \delta^2(\alpha\delta)^q] \right\} \sigma^2 \quad (3.17)$$

Note that $V(q)$ can also be written as

$$V(q) = \frac{1}{1 - \alpha^2} [Var(h_t) + 2\alpha Cov(x_{t-1}, h_t)] \quad (3.18)$$

where

$$Var(h_t) = \frac{\beta_0^2}{1 - \delta^2} (1 - \delta^{2(q+1)}) \sigma^2 = \beta_0^2 \sigma^2 \sum_{k=0}^q \delta^{2k} \quad (3.19)$$

$$Cov(x_{t-1}, h_t) = \frac{\beta_0^2}{1 - \delta^2} \left\{ \frac{\delta}{1 - \alpha\delta} [1 - (\alpha\delta)^q] + \frac{1}{\alpha - \delta} [\delta^{2(q+1)} - \delta^2(\alpha\delta)^q] \right\} \sigma^2 \quad (3.20)$$

$$= \beta_0^2 \sigma^2 \delta \sum_{j=0}^{q-1} (\alpha\delta)^j \sum_{k=0}^{q-1-j} \delta^{2k} \quad (3.21)$$

3.B Appendix: Purely forward-looking price setting

The model (3.1) and (3.3) in case of purely forward-looking price setting reads

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + \varepsilon_{t-q} \quad (3.22)$$

$$\pi_t = -\frac{\lambda_2}{\lambda_1 \kappa} (x_t - x_{t-1}) \quad (3.23)$$

The output gap x_t can be written as hybrid univariate model equation of the form (3.8) with

$$a = \beta b \quad (3.24)$$

$$b = \frac{\lambda_2}{\lambda_2(1 + \beta) + \lambda_1 \kappa^2} \quad (3.25)$$

$$c = -\frac{\lambda_1 \kappa_2}{\lambda_2(1 + \beta) + \lambda_1 \kappa^2} \quad (3.26)$$

Hence, the variance of x_t is given by $Var(x_t) = V(q)$, where

$$\delta = \frac{2a}{1 + \sqrt{1 - 4ab}} \quad (3.27)$$

$$\alpha\delta = \frac{1 - \sqrt{1 - 4ab}}{1 + \sqrt{1 - 4ab}} \quad (3.28)$$

$$1 - 4ab = \frac{(1 - \beta)^2 + 2(1 + \beta)z + z^2}{(1 + \beta + z)^2} \quad (3.29)$$

$$z = \frac{\lambda_1 \kappa^2}{\lambda_2} \quad (3.30)$$

Since $dVar_q(h_t)/dq > 0$ and $dCov_q(x_{t-1}, h_t)/dq > 0$, it holds $dVar_q(x_t)/dq > 0$.

The variance of the inflation rate can be deduced from the targeting rule (3.23):

$$Var(\pi_t) = 2 \left(\frac{\lambda_2}{\lambda_1 \kappa} \right)^2 (Var(x_t) - E(x_t x_{t-1})) \quad (3.31)$$

$$= \frac{2}{1 + \alpha} \left(\frac{\lambda_2}{\lambda_1 \kappa} \right)^2 [Var_q(h_t) - (1 - \alpha)Cov_q(x_{t-1}, h_t)] \quad (3.32)$$

$$= \frac{2}{1 + \alpha} \left(\frac{\lambda_2}{\lambda_1 \kappa} \right)^2 \beta_0^2 \sigma^2 \left[\sum_{k=0}^q \delta^{2k} - (1 - \alpha)\delta \sum_{j=0}^{q-1} (\alpha\delta)^j \sum_{k=0}^{q-1-j} \delta^{2k} \right] \quad (3.33)$$

To derive the condition for $Var_{q=0}(\pi_t) > Var_{q \rightarrow \infty}(\pi_t)$, note that

$$Var_{q=0}(h_t) = \beta_0^2 \sigma^2 \quad (3.34)$$

$$Cov_{q=0}(x_{t-1}, h_t) = 0 \quad (3.35)$$

$$Var_{q \rightarrow \infty}(h_t) = \frac{\beta_0^2}{1 - \delta^2} \sigma^2 \quad (3.36)$$

$$Cov_{q \rightarrow \infty}(x_{t-1}, h_t) = \frac{\beta_0^2}{1 - \delta^2} \frac{\delta}{1 - \alpha\delta} \sigma^2 \quad (3.37)$$

Using the definitions (3.24) to (3.26), $Var_{q=0}(\pi_t) > Var_{q \rightarrow \infty}(\pi_t)$ is equivalent to

$$(1 - \alpha)Cov_{q \rightarrow \infty}(x_{t-1}, h_t) > Var_{q \rightarrow \infty}h_t - Var_{q=0}h_t \Leftrightarrow \frac{1 - \alpha}{1 - \alpha\delta} > \delta \Leftrightarrow \quad (3.38)$$

$$1 - \beta^{-1}a - 2\beta^{-1}a^2 > [(2 + \beta^{-1})a - 1]\sqrt{1 - 4\beta^{-1}a^2} \Leftrightarrow \quad (3.39)$$

$$(\beta^2 - \beta) + (1 + 2\beta)\frac{\lambda_1 \delta^2}{\lambda_2} + \frac{\lambda_1^2 \kappa^4}{\lambda_2^2} > \left(\beta - \frac{\lambda_1 \kappa^2}{\lambda_2} \right) \sqrt{(1 - \beta)^2 + 2(1 + \beta)\frac{\lambda_1 \kappa^2}{\lambda_2} + \frac{\lambda_1^2 \kappa^4}{\lambda_2^2}} \quad (3.40)$$

Let $z = \lambda_1 \kappa^2 / \lambda_2$, then inequality 3.40 can be simplified to

$$z^2 + 2(1 + \beta)z + \beta(\beta - 2) > 0 \quad (3.41)$$

and holds if

$$z = \frac{\lambda_1 \kappa^2}{\lambda_2} > \sqrt{1 + 4\beta} - (1 + \beta) \quad (3.42)$$

Although the variance of the inflation rate may decrease with increasing anticipation horizon q , it can be shown that the loss

$$Loss_q = \lambda_1 Var_q(\pi_t) + \lambda_2 Var_q(x_t) \quad (3.43)$$

is always smaller for $q = 0$ than for $q \rightarrow \infty$. It holds:

$$Loss_{q \rightarrow \infty} = \left\{ \lambda_1 \frac{2}{1+\alpha} \left(\frac{\lambda_2}{\lambda_1 \kappa} \right)^2 \frac{1-\delta}{1-\alpha\delta} \frac{1}{1-\delta^2} + \lambda_2 \frac{1}{1-\alpha^2} \frac{1}{1-\delta^2} \frac{1+\alpha\delta}{1-\alpha\delta} \right\} \beta_0^2 \sigma^2 \quad (3.44)$$

$$Loss_{q=0} = \left\{ \lambda_1 \frac{2}{1+\alpha} \left(\frac{\lambda_2}{\lambda_1 \kappa} \right)^2 + \lambda_2 \frac{1}{1-\alpha^2} \right\} \beta_0^2 \sigma^2 \quad (3.45)$$

Then $J_{q \rightarrow \infty} \stackrel{!}{>} J_{q=0}$ is equivalent to

$$\frac{2\lambda_2}{\lambda_1 \kappa} [1 - \alpha(1 + \delta)] < \frac{2\alpha + \delta(1 - \alpha\delta)}{(1 - \alpha)(1 - \delta)} \Leftrightarrow 2b[\sqrt{1 - 4ab} - b] < b + b(1 + \beta)\sqrt{1 - 4ab} \quad (3.46)$$

Since $b = 1/[1 + \beta + z]$, $1 - 4ab = 1 - 4\beta/[1 + \beta + z]^2$, (3.46) is equivalent to

$$0 < 4\beta(1 - \beta)^2 + \beta(2 - \beta)(1 + \beta + z) + 4(1 + \beta + z) \quad (3.47)$$

This inequality is always satisfied since $z = \lambda_1 \kappa^2 / \lambda_2 > 0$.

3.C Appendix: Purely backward-looking price setting

The model (3.1) and (3.3) in case of purely backward-looking price setting reads

$$\pi_t = \beta\pi_{t-1} + \kappa x_t + \varepsilon_{t-q} \quad (3.48)$$

$$\pi_t = -\frac{\lambda_2}{\lambda_1 \kappa} (x_t - \beta^2 E_t x_{t+1}) \quad (3.49)$$

The inflation rate can be written as a hybrid univariate equation of the form

$$\pi_t = aE_t \pi_{t+1} + b\pi_{t-1} + c(\varepsilon_{t-q} - \beta^2 E_t \varepsilon_{t-q+1}) \quad (3.50)$$

with $c = \varphi/(1 + \varphi + \varphi\beta^3)$, $b = \beta c$, $a = \beta^2 c$, and $\varphi = \lambda_2/(\lambda_1 \kappa^2)$. The system can again be written as

$$\pi_t = \sum_{s=0}^{\infty} \alpha^s h_{t-s} \quad (3.51)$$

where $\alpha = (1 - \sqrt{1 - 4ab})/2a$ and

$$h_t = \sum_{k=0}^q \delta^k \beta_0 \varepsilon_{t+k-q} - \beta^2 \sum_{k=0}^{q-1} \delta^k \beta_0 \varepsilon_{t+k-q+1} \quad (3.52)$$

The variance of the inflation rate can be derived as follows:

$$Var(\pi_t) = E \left(\sum_{s=0}^{\infty} \alpha^s \sum_{k=0}^q \delta^k \beta_0 \varepsilon_{t-s+k-q} - \beta^2 \sum_{s=0}^{\infty} \alpha^s \sum_{k=0}^{q-1} \delta^k \beta_0 \varepsilon_{t-s+k-q+1} \right)^2 \quad (3.53)$$

$$= V(q) - 2\beta^2 Z + \beta^4 V(q-1) \quad (3.54)$$

where $V(\cdot)$ is given by (3.17) and

$$Z = E \left[\left(\sum_{s=0}^{\infty} \alpha^s \sum_{k=0}^q \delta^k \beta_0 \varepsilon_{t-s+k-q} \right) \left(\sum_{s=0}^{\infty} \alpha^s \sum_{k=0}^{q-1} \delta^k \beta_0 \varepsilon_{t-s+k-q+1} \right) \right] \quad (3.55)$$

$$= E \left[\left(\sum_{s=0}^{\infty} \alpha^s \beta_0 \varepsilon_{t-s-q} \right) \left(\sum_{s=0}^{\infty} \alpha^s \sum_{k=0}^{q-1} \delta^k \beta_0 \varepsilon_{t-s+k-q+1} \right) \right] + \delta V(q-1) \quad (3.56)$$

$$= \frac{\alpha}{1-\alpha^2} \frac{1-(\alpha\delta)^q}{1-\alpha\delta} \beta_0^2 \sigma^2 + \delta V(q-1) = \frac{\alpha}{1-\alpha^2} \beta_0^2 \sigma^2 \sum_{j=0}^{q-1} (\alpha\delta)^j + \delta V(q-1) \quad (3.57)$$

Then $Var_q(\pi_t)$ can be written as

$$Var(\pi_t) = V(q) - 2\beta^2 \frac{\alpha}{1-\alpha^2} \frac{1-(\alpha\delta)^q}{1-\alpha\delta} \beta_0^2 \sigma^2 + (\beta^4 - 2\beta^2\delta)V(q-1) \quad (3.58)$$

3.D Appendix: Hybrid price-setting behavior

The model (3.1) and (3.3) in case of both forward- and backward-looking price setting can be written in matrix form

$$\Phi s_{t+1} = \Psi s_t + g \varepsilon_{t+1} \quad (3.59)$$

where $s_{t+1} = (\tilde{\eta}_{t+1}^{(q)}, \tilde{x}_{t+1}, \tilde{\pi}_{t+1}, E_t x_{t+1}, E_t \pi_{t+1})'$, $\tilde{\eta}_{t+1}^{(q)} = (\eta_{t+1}^{(0)}, \eta_{t+1}^{(1)}, \dots, \eta_{t+1}^{(q-1)}, \eta_{t+1}^{(q)})'$ with $\eta_t^{(j)} = \varepsilon_{t-j} \forall j = 0, \dots, q$ and $g = (1, 0, \dots, 0)'$, and

$$\Phi = \begin{pmatrix} I_{q+3} & 0_{2 \times (q+3)} \\ 0_{(q+3) \times 2} & \Phi_{22} \end{pmatrix} \quad (3.60)$$

$$\Psi = \begin{pmatrix} \Psi_{11} & 0_{(q+1) \times 5} \\ 0_{4 \times q} & \Psi_{22} \end{pmatrix} \quad (3.61)$$

with $0_{n \times m}$ as $(n \times m)$ -dimensional zero matrix, I_n as n -dimensional identity matrix, and

$$\Phi_{22} = \begin{pmatrix} \frac{\lambda_2 \beta^2 \phi_\pi}{\lambda_1 \kappa} & 0 \\ 0 & \beta(1 - \phi_\pi) \end{pmatrix} \quad (3.62)$$

$$\Psi_{11} = \begin{pmatrix} 0_{1 \times q} \\ I_q \end{pmatrix} \quad (3.63)$$

$$\Psi_{22} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -(1 - \phi_\pi) \frac{\lambda_2}{\lambda_1 \kappa} & 0 & \frac{\lambda_2}{\lambda_1 \kappa} & 1 \\ -1 & 0 & -\beta \phi_\pi & -\kappa & 1 \end{pmatrix} \quad (3.64)$$

Let $w_{t+1} = (\tilde{\eta}_{t+1}^{(q)}, \tilde{x}_{t+1}, \tilde{\pi}_{t+1})'$ contain the backward-looking variables. The variance-covariance matrix $Cov(w_t) = \Sigma_w$ in vectorized form is given by

$$vec(\Sigma_w) = (I_{(q+3)^2} - M \otimes M)^{-1} vec(gg') \sigma^2 \quad (3.65)$$

where $M = Z_{11} S_{11}^{-1} T_{11} Z_{11}^{-1}$. According to Söderlind (1999) Z_{11} , S_{11} , and T_{11} follow from the Generalized Schur decomposition $\Phi = \bar{Q}' S \bar{Z}'$ and $\Psi = \bar{Q}' T \bar{Z}'$ with

$$S = \begin{pmatrix} S_{11} & S_{12} \\ 0 & S_{22} \end{pmatrix}, \quad T = \begin{pmatrix} T_{11} & T_{12} \\ 0 & T_{22} \end{pmatrix}, \quad Z = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \quad (3.66)$$

\bar{Q} and \bar{Z} are the complex-conjugates of Q and Z , respectively. The $(q + 3 \times q + 3)$ -dimensional matrices S_{11} and T_{11} contain the stable eigenvalues of the system (3.59).

Chapter 4

Partially anticipated monetary policy shocks – Are they stabilizing or destabilizing?

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Abstract

This paper uses a dynamic framework of a small open economy to study the volatility effects of partially anticipated monetary policy shocks in which the public has imperfect information about the size and/or the timing of the future expansionary policy intervention. Our two main results are as follows: (i) Partially anticipated monetary policy shocks may be stabilizing, i.e. lead to a lower volatility than a fully anticipated monetary policy shock of the same form. (ii) However, we typically obtain a trade off in volatilities such that a simultaneous stabilization of inflation and output is not possible. If the public underestimates (overestimates) the size of the shock, output (inflation) may be stabilized. Our results imply that the central bank may have an incentive to withhold information from the public about the true central bank's intention.

JEL classification: E32, E52

Keywords: Anticipated shocks, Optimal monetary policy, Bounded rationality, Volatility

4.1 Introduction

This paper studies the volatility effects of monetary policy disturbances, which are not fully anticipated by the public. So far, the literature has only considered two extreme cases of anticipation. Either the public has perfect information and fully anticipates the shock process or the public is completely uninformed and does not anticipate the shock process at all. This paper introduces an intermediate scenario of partial anticipation, which covers both extreme scenarios as special cases. Under partial anticipation, the public has partially correct and partially incorrect expectations about the exact evolution (i.e. about size and timing) of the monetary shock process. To the best of our knowledge, we are the first to study this kind of partially anticipated shocks.¹

The importance of anticipated shocks in general (like pre-announced future monetary policy interventions), also known as news shocks, for business cycle fluctuations is confirmed by several empirical studies. Most prominently, Schmitt-Grohé and Uribe (2012) find in an estimated real business cycle model that about 50 percent of economic fluctuations can be attributed to fully (or possibly partially) anticipated disturbances.² Milani and Treadwell (2012) focus on anticipated monetary policy. They find that anticipated monetary policy shocks have a larger impact on output fluctuations than unanticipated monetary policy shocks.³ They conclude that the central bank's communication can be an effective monetary policy tool.⁴

Central banks may not be able or not willing to communicate the exact timing and/or size of a (future) monetary intervention in advance so that the public needs to form expectations about it. For example, in July 2012 at the Global Investment Conference in London, the President of the European Central Bank, Mario Draghi, signaled further purchases of government bonds by stating that “the ECB is ready to do whatever it

¹The terminology of partial anticipation was already introduced by Fischer (1979). However, he defines partially anticipated shocks as fully anticipated shocks with reduced length of anticipation. Contrarily, we assume that partially and fully anticipated shocks have the same length of anticipation. The public may, however, have wrong expectations about the length of anticipation.

²A partially anticipated shock can also be viewed as a combination of a fully anticipated and a non-anticipated shock, where the non-anticipated shock reflects the correction in expectations. Let ε_t^i be a random innovation that is anticipated in t , but materializes in $t+i$ (notation of Schmitt-Grohé and Uribe (2012)). Then, if ε_t^0 (non-anticipated shock) and ε_{t-i}^i are from the same source, this is an indication of partial anticipation. $\varepsilon_t^0 > 0$ and $\varepsilon_{t-i}^i > 0$ may not be two distinct shocks, but instead may be one partially anticipated shock where the public initially underestimates the shock size in $t-i$ and corrects its expectations in t . The difference $\varepsilon_t^0 - \varepsilon_{t-i}^i$ could be interpreted as expectations bias. Note that the variances of $\{\varepsilon_{t-i}^i\}$ can be identified separately for different values of i , see Schmitt-Grohé and Uribe (2012).

³Further empirical studies include Beaudry and Portier (2006), Beaudry and Lucke (2010), Barsky and Sims (2011), Fujiwara et al. (2011), and Khan and Tsoukalas (2012). The main bulk of the literature, however, emphasizes the role of anticipated technology shocks as in Schmitt-Grohé and Uribe (2012) or of anticipated fiscal policy shocks as in Leeper et al. (2008) and Mertens and Ravn (2010). The idea of these shocks playing an important role in driving business cycle fluctuations goes back to Pigou (1927) and was revived by Beaudry and Portier (2004).

⁴For a survey on central bank communication, see Blinder et al. (2008).

takes to preserve the euro” to bring down risk premiums on government bonds. However, Draghi is mute about the exact threshold of risk premiums at which the ECB is planning to intervene. This limited information strategy leaves room for public misperceptions such that the monetary intervention may only be partially anticipated.⁵

The main aim of this paper is twofold: First, we aim to study the (de)stabilizing effects of partially anticipated monetary disturbances on inflation and output fluctuations, where we define stabilization as follows: Partial anticipation (de)stabilizes a particular variable if the variable’s volatility under partial anticipation is (larger) smaller than under fully correct anticipation of the same shock process. Second, we aim to derive the optimal central bank’s communication strategy. Is it possible to obtain a lower central bank’s loss by either directly deceiving the public or withholding information about the true monetary policy intentions?

To this end, we consider several partial anticipation scenarios, in which the public initially has incorrect expectations about the size and/or the timing of the monetary disturbances. With interest rates at the zero lower bound, central banks are forced to use unconventional policy instruments to stimulate the economy. In line with this change in policy, we model monetary policy interventions as (temporary) increases in the money growth rate. To discuss the limited information strategy of the ECB during the European sovereign debt crisis, we consider increases in the money growth rate not only in isolation but also as response to increasing risk premiums on government bonds, where the public may have incorrect expectations about the start of the monetary intervention.⁶

As model framework, we use a continuous-time Dornbusch-type⁷ model of a small open economy. This framework has been used in several papers to study the dynamic impacts of (fully) anticipated shocks. Early studies include Turnovsky (1986a,b). More recent studies include Clausen and Wohltmann (2005), who study anticipated and unanticipated monetary and fiscal policy in an asymmetric monetary union and Clausen and Wohltmann (2013), who study anticipated oil price shocks in a similar model of a small open monetary union. Recently, the continuous-time formulation also has gained some attention in the New Keynesian literature. Posch et al. (2011) formulate and solve the New Keynesian model in continuous time.

Our two main results are as follows: (i) Partially anticipated monetary policy shocks may stabilize inflation and output fluctuations, i.e. lead to a lower volatility than a fully anticipated monetary policy shock of the same form. (ii) However, we typically obtain a

⁵As a contrary example to the ECB’s communication policy, the recent decision of the Swiss National Bank in January 2015 to discontinue the minimum exchange rate of CHF 1.20 per euro was not announced at all and came as a complete surprise to the market participants. As a result, the Swiss Franc sharply appreciated by roughly 20 percent.

⁶Recently, the European Central Bank (ECB) indicated to oppose the deflationary risk in the European Union by expanding the ECB’s Outright Monetary Transactions (OMT) program. The Federal Reserve, on the other hand, is starting to cut down its asset-purchasing program since December 2013.

⁷The model is a variant of the famous overshooting model introduced by Dornbusch (1976).

trade off in volatilities such that a simultaneous stabilization of output and inflation is not possible. If the public underestimates (overestimates) the size of the shock, output (inflation) may be stabilized.

Our results are in line with the literature: The existence of a trade off in volatilities of inflation and output is well known and already described in Taylor (1979) and revisited in Taylor (1994). He finds the trade off in volatilities – in contrast to the trade off in levels – to be stable in the long run for the U.S. economy. Further related to this paper is the literature on news shocks which studies the potential destabilizing effects of completely anticipated shocks. Fève et al. (2009) show in a purely forward-looking discrete-time framework with rational expectations that anticipated shocks destabilize the economy, i.e. lead to a higher volatility than non-anticipated shocks of the same size. The volatility increases with increasing length of anticipation. This result does not hold unambiguously for the hybrid case with backward-looking elements as it is shown by Winkler and Wohltmann (2012).⁸ They find the same trade off in volatilities of inflation and output in the estimated Euro area model of Smets and Wouters (2003). With increasing anticipation horizon, output volatility increases, but inflation volatility decreases. Our paper may help to explain why this trade off in volatilities occurs and why anticipated shocks may lead to a (de)stabilization of the economy.

For the aforementioned results of this paper, we implicitly assume a stable relation between base and broad money such that the central bank can perfectly control the money stock. However, since the outburst of the financial crisis in 2008, such a stable relation in the Euro area is questionable as e.g. De Grauwe and Ji (2013) demonstrate. We, therefore, also study partially anticipated changes in the monetary base that have no effect on the money stock. We find that changes in the monetary base may still have real effects on the economy and may impose cyclical adjustment movements even if the relation between base and broad money is non-existent. This requires, however, that the public indeed believes in a stable relation.

The remainder of the paper is organized as follows: Section 4.2 describes the model framework. Section 4.3 introduces our (partial) anticipation scenarios and studies the responses to a temporary increase in the money growth rate. Section 4.4 introduces our volatility measure and discusses the (de)stabilizing effects of partially anticipated changes in the monetary growth rate for different degrees of expectation biases. Section 4.5 introduces two communication strategies in which the central bank either deceives the public or withhold information from the public to obtain a lower central bank's loss. As a digression, section 4.6 discusses the responses to partially anticipated increases in the monetary base in the presence of an unstable relation between base and broad money. Section 4.7 discusses six modifications including a simultaneous increase in the risk premium on

⁸Also related to this literature is the paper by Offick and Wohltmann (2013), who investigate the sources of these destabilizing effects.

government bonds and in the money growth rate. The last section concludes.

4.2 Model framework

As model framework we use a dynamic continuous-time Dornbusch-type model for a small open economy. The economy is described by the following set of log-linearized equations:

$$y = (a_0 + a_1y - a_2(i - E(\dot{p}))) + g + (b_0 - b_1y + b_2y^* - b_3\tau) \quad (4.1)$$

$$\tau = p - (p^* + e) \quad (4.2)$$

$$m - p = l_0 + l_1y - l_2i \quad (4.3)$$

$$i = i^* + E(\dot{e}) + s \quad (4.4)$$

$$\dot{p} = \pi + \delta(y - \bar{y}) \quad (4.5)$$

$$\pi = \dot{m} \quad (4.6)$$

$$\bar{y} = f_0 + f_1\bar{\tau} \quad (4.7)$$

All variables, except for the (nominal and real) interest rate and the inflation rate, are in logarithm. The notation is as follows: y = real output, \bar{y} = natural output level, i = nominal interest rate, $i - E\dot{p}$ = real interest rate, τ = terms of trade, g = government spending, p = price level, e = exchange rate, m = nominal money stock, \dot{p} = inflation rate, π = augmentation term of the Phillips curve, s = risk premium shock. Foreign variables (i^* , y^* , p^*) are denoted by a superscript star. A dot above a variable (\dot{p} , \dot{e} , \dot{m}) stands for the time derivative (differentiated from the right) of that variable, a bar above a variable (\bar{y} , $\bar{\tau}$) stands for its long-run value, and E is the expectations operator. We assume rational expectations. In a deterministic framework this implies $E\dot{p} = \dot{p}$ and $E\dot{e} = \dot{e}$. Depending on the assumed anticipation scenario, expectations on the exogenous evolution of the money growth may deviate from the true evolution.⁹ Further details will be provided in the subsequent sections.

Equation (4.1) is a standard IS equation, determining the short-run development of output. The first term in brackets stands for real private absorption depending on real income and the real interest rate. The second term in brackets stands for the trade balance depending on domestic and foreign income and the terms of trade. The terms of trade are defined in equation (4.2). Equation (4.3) represents the money market equilibrium and is a traditional LM curve. Equation (4.4) is the uncovered interest rate parity (UIP), and equation (4.5) represents a Phillips-type inflation equation. Equation (4.6) specifies the augmentation term in the Phillips curve, which we set equal to the expected long-term rate of inflation. According to monetaristic theory, we assume that inflation is solely determined

⁹Turnovsky (1977) labels these expectations as structural where the public has correct information about the structure of the model but false information about the underlying exogenous processes.

by the money growth rate in the long run.¹⁰ In the short to medium run, inflation might temporarily deviate from its long-term rate. However, inflation is, as we will see in the subsequent sections, rather tied to the money growth rate.¹¹ For completeness, the last equation describes the long-run relation between output and the terms of trade. Since changes in the money growth rate do not alter the steady state of output and the terms of trade, we can neglect this equation until section 4.7, where we also consider changes in the risk premium on government bonds.¹²

The model can be reduced to a two-dimensional system of ordinary differential equations with the terms of trade τ and the real money stock $m^r = m - p$ as state variables. For the parameter calibration given in table 4.1, the reduced model exhibits one stable and one unstable eigenvalue. The system then describes a saddle point system. To ensure stability of the system, we assume that the terms of trade are forward-looking and the real money stock is backward-looking such that the number of unstable eigenvalues equals the number of forward-looking variables.¹³

Table 4.1: Parameter calibration

Parameter	Value	Definition
a_1	0.7	Income elasticity of private consumption
a_2	0.3	Real interest rate (semi-)elasticity of private absorption
b_1	0.2	Income elasticity of the trade balance
b_3	0.1	Terms of trade elasticity of the trade balance
l_1	1.0	Income elasticity of money demand
l_2	4.0	Interest rate (semi-)elasticity of money demand
δ	0.2	Slope of the Phillips curve

In the subsequent simulations, we use the calibration given in table 4.1.¹⁴ We focus on deviations from the initial steady state. Therefore, we do not need to specify a_0 , b_0 , and b_2 . For the remaining parameters, we broadly follow the textbook calibration given in Galí (2008) and Walsh (2010) and the estimates from Moons et al. (2007), who estimate a stylized open-economy New Keynesian model for the euro area.

The income elasticity and the interest rate semi-elasticity of the money demand are set to $l_1 = 1$ and $l_2 = 4$, respectively, which are the values proposed by Galí (2008). He derives the microfounded money demand equation from a money-in-the-utility approach,

¹⁰See Friedman (1977).

¹¹As a modification, we assume in section 4.7 that the augmentation term is alternatively given by the expected inflation rate of the consumer price index, i.e. $\pi = \dot{p}_c$ with $p_c = \gamma p + (1 - \gamma)(p^* + e)$. $1 - \gamma$ measures the degree of openness.

¹²For details on the model equation and the underlying assumptions, see Clausen and Wohltmann (2005) and Clausen and Wohltmann (2013). They study monetary and fiscal policy in a continuous-time model of an asymmetric monetary union that is of a similar form as the above model.

¹³For details on the model reduction and solution for fully anticipated, partially anticipated, and non-anticipated changes in the money growth rate and in the risk premium, see appendix 4.B.

¹⁴All computations have been performed by using MATLAB. The codes are available from the authors upon request.

Table 4.2: Evolution of the exogenous increase in the money growth rate

$0 \leq t < T$	$T \leq t < t_1$	$t > t_1$
$\dot{m} = 0$	$\dot{m} = c$	$\dot{m} = 0$

which implicitly gives rise to an income elasticity of unity. Estimates of l_1 reported in Walsh (2010) suggest values greater than unity, whereas Ball (2001) finds a value of 0.5. Estimates of l_2 reported in Walsh (2010) range from 1 to 10, which is in line with the estimate of 5 found by Ball (2001).¹⁵ The income elasticity of private consumption is set to $a_1 = 0.7$, which we implicitly derived from an income tax rate of 0.3, a consumption output ratio of 0.7, and a consumption rate of 0.7.¹⁶ The income elasticity of the trade balance is set to 0.2.¹⁷ Then, the net effect of the real interest rate and of the terms of trade on goods demand are given by $a_2/(1 - a_1 + b_1) = 0.6$ and $b_3/(1 - a_1 + b_1) = 0.2$, respectively. Both values match the estimates given in Moons et al. (2007). The former is close to the values given in Galí (2008) and Walsh (2010) and is also in line with Smets and Wouters (2003), who find a mean intertemporal elasticity of substitution of 0.7 with a 90 percent probability band ranging from 0.52 to 1.05.¹⁸ The slope of the Phillips curve is set to $\delta = 0.2$ proposed by Galí (2008) and which is also close to the estimate given in Moons et al. (2007).¹⁹ In section 4.7, we investigate how our results change for different parameter values for δ , l_2 , a_2 , and b_3 .

4.3 Anticipation scenarios and responses to a monetary shock

This section introduces our anticipation scenarios and discusses the responses to a temporary increase in the money growth rate in the above model framework.

The realized but not necessarily correctly anticipated shock process is the same across all anticipation scenarios. The evolution of the shock process is described in table 4.2. The increase in the money growth is implemented at a constant rate c over the implementation

¹⁵Note that the interest rate is – contrarily to output – not logarithmized. Given that the steady state interest rate is relatively small, the interest rate semi-elasticity is typically much larger than the income elasticity of money demand ($l_2 > l_1$).

¹⁶More details on the derivation of a_1 can be found in appendix 4.A.

¹⁷Note that $a_1 > b_1$ is not a necessary condition for stability. For stability, it is sufficient that $l_2 < l_1/\delta$ or $a_2 < (1 - a_1 + b_1)/\delta$ hold. Further details are given in appendix 4.B.

¹⁸Taking habit formation into account, Smets and Wouters (2003) find that the impact of the real interest rate on consumption is reduced to 0.3.

¹⁹From microfoundation, the slope of the Phillips curve can be expressed as $\delta = \frac{(1-\theta)(1-\beta\theta)}{\theta}(\sigma + \eta)$. Assuming a quarterly interest rate of 1 percent (implying a discount factor of $\beta = 0.99$), a Calvo parameter of $\theta = 0.75$, an inverse elasticity of substitution of $\sigma = 1.5$, and a Frisch elasticity of labor supply of $\eta = 1.5$, we obtain a slope of $\delta = 0.25$. Estimates in Smets and Wouters (2003) and Smets and Wouters (2007) for θ , σ , and η suggest that δ may vary between 0.05 and 0.5. Differences in δ mainly result from differences in the Calvo parameter, which ranges from 0.7 to 0.9.

Table 4.3: Set of anticipation scenarios

		FA	NA	PA-MAG	PA-START	PA-EXIT
Magnitude	$E(c)$	c	0	$\neq c$	c	c
Start	$E(T)$	T	-	T	$\neq T$	T
Exit	$E(t_1)$	t_1	-	t_1	t_1	$\neq t_1$
Length	$E(t_1 - T)$	$t_1 - T$	-	$t_1 - T$	$\neq t_1 - T$	$\neq t_1 - T$
Size	$E(c(t_1 - T))$	$c(t_1 - T)$	-	$\neq c(t_1 - T)$	$\neq c(t_1 - T)$	$\neq c(t_1 - T)$
Breakpoint	t^*	-	T	T	$\min[E(T), T]$	$\min[E(t_1), t_1]$

period $T < t < t_1$ and is temporary in the sense that $\dot{m} = 0$ for $t > t_1$. In $t = 0$, the increase is fully or partially anticipated, or non-anticipated at all by the public. We, therefore, refer to the time period $0 < t < T$ as anticipation period.

In the long run, a temporary increase in the money growth rate does not alter the steady state of the real variables.²⁰ The steady state of the nominal money stock changes according to

$$d\bar{m} = \int_0^\infty \dot{m}(z) dz = c(t_1 - T) \quad (4.8)$$

which implies a change in the steady-state values of the price level and the nominal exchange rate of equal size, i.e. $\frac{d\bar{p}}{d\bar{m}} = \frac{d\bar{\epsilon}}{d\bar{m}} = 1$. In the following, we refer to the expression $c(t_1 - T)$ as the size of the shock process. c is the magnitude, T is the start, and t_1 is the end or exit of the shock process.

We consider five anticipation scenarios: one full anticipation scenario in which the public correctly anticipates the full monetary policy intervention (denoted as FA), three partial anticipation regimes in which the public has partially correct and partially incorrect expectations (denoted as PA), and one non-anticipation scenario in which the policy intervention completely comes as a surprise (denoted as NA). Table 4.3 summarizes the complete set of anticipation scenarios.²¹ In the three scenarios of partially correctly anticipated shocks, the public forms incorrect expectations either about the magnitude c (scenario PA-MAG), about the starting point T (scenario PA-START), or about the exit point t_1 (scenario PA-EXIT) of the increase in the money growth rate. Note that in all three partial anticipation scenarios, the public has incorrect expectations about the size of the shock.

Since we aim to study only temporary and not permanent anticipation errors, we

²⁰This nominal neutrality follows from the long-run stability condition that the real money stock and the terms of trade do not change in the long run. In case of a permanent increase in the money growth rate, the steady state of the real money stock changes, but the steady state of most other real variables (including output, terms of trade, and the real interest rate) remains unchanged.

²¹Note that in scenario NA, the public does not anticipate the increase in the money growth rate at all and therefore has no expectations on the start of the implementation. This is highlighted by a minus sign in table 4.3.

have to define how the public switches to correct expectations. For simplicity, we assume that the switch from partially incorrect to fully correct expectations occurs at once at some particular breakpoint t^* .²² Hence, the public has correct expectations for $t > t^*$ and may have incorrect expectations for $t < t^*$ about the shock process. In line with non-anticipated shocks, we assume that the switch to correct expectations occurs when the public realizes for the first time that the expected evolution of the shock process deviates from the true one. In scenarios NA and PA-MAG, the public's expectations deviate from the true shock process for the first time at the start of implementation, i.e. $t^* = T$. In scenario PA-START, the public has incorrect expectations about the start of the monetary intervention. If the public expects an earlier start, i.e. $E(T) < T$, the public already switches in $t^* = E(T)$ to fully correct expectations. If the public expects a later start, i.e. $E(T) > T$, the public switches in $t^* = T$. In scenario PA-EXIT, the public correctly expects the start (T) and the magnitude (c), but has incorrect expectations about the end (t_1) of the shock process. Since $E(t_1) > T$, the public does not switch to correct expectations before T . If the public expects an earlier end of the shock process, i.e. $E(t_1) < t_1$, the public switches in $t^* = E(t_1)$, where $t^* > T$. If the public expects a later end of the shock process, i.e. $E(t_1) > t_1$, the public switches in $t^* = t_1$.

In the following, we subsequently study the responses to the above temporary increase in the money growth rate under the partial anticipation scenarios PA-MAG, PA-START, and PA-EXIT in comparison to the full anticipation scenario FA.

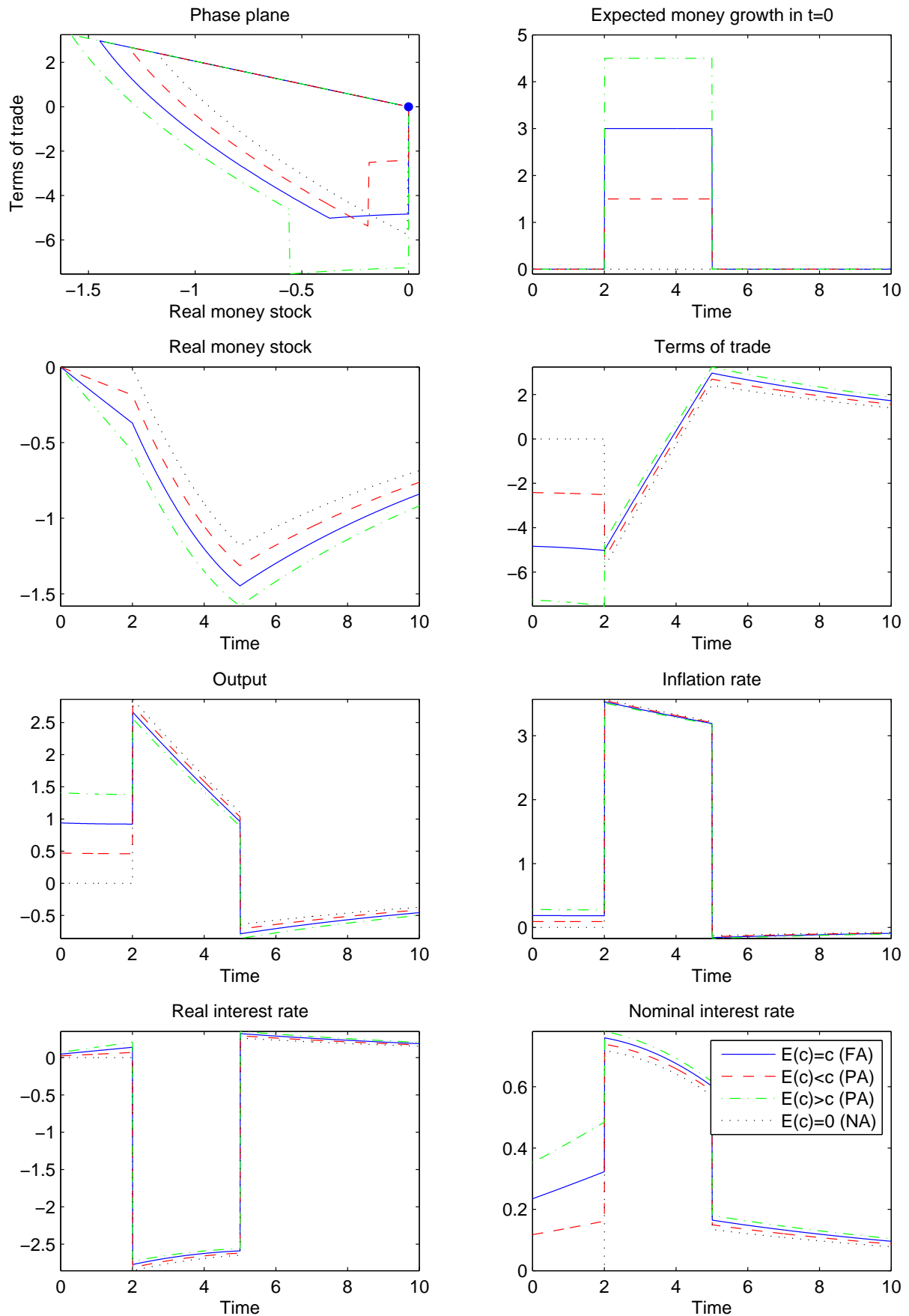
4.3.1 Scenario PA-MAG and NA

Figure 4.1 depicts the responses to a temporary increase in the money growth rate under the full anticipation scenario FA and the partial anticipation scenario PA-MAG, where the public either underestimates ($E(c) < c$) or overestimates ($E(c) > c$) the magnitude of the shock. As a special case of PA-MAG, the figure also includes the non-anticipation scenario NA, where the public does not expect the increase at all ($E(c) = 0$). The first (upper-left) plot displays the development of the terms of trade and the real money stock in the phase plane. The remaining plots show the responses in the time domain. The second (upper-right) plot displays the initially expected evolution of the money growth based on the information set in $t = 0$. Note that in the FA scenario, the anticipated evolution of the money growth is equal to the realized money growth. We set $c = 3$, $T = 2$, and $t_1 = 5$.

To start with, the adjustment process in the anticipation scenario FA can be described as follows: In $t = 0$, the increase in the money growth rate is announced and correctly anticipated by the public. The anticipation of a future expansionary monetary shock leads to an immediate (real) devaluation of the home currency (fall in the terms of trade).

²²In section 4.7, we modify the mechanism with which the public switches to correct expectations and assume that the public sequentially adapts its expectations in several steps.

Figure 4.1: Responses to a monetary shock in scenario PA-MAG



Since prices are assumed to be sluggish and do not change on impact, this is represented by a vertical downward adjustment in the phase. The real devaluation continues until the start of the implementation T . Simultaneously, the real money stock continuously declines, which is equivalent to a continuous upward adjustment of prices. The devaluation of the home currency leads via the UIP to a rise in the nominal interest rate. The inflation response on impact and during the anticipation phase is relatively small such that the real interest rate rises. Despite the contractionary real interest rate effect, output unambiguously stays above its steady state value on impact and during the anticipation phase. This immediately follows from the inverse Phillips curve

$$y = \bar{y} - \frac{1}{\delta} \dot{m}^r \quad (4.9)$$

which determines output by the change in the real money stock.

In $t = T = 2$, the money growth rate increases as expected. Inflation expectations shoot up and induce a sharp rise in the inflation rate, which overshoots the rise in the money growth rate and in the nominal exchange rate. The overshooting continues over the whole implementation phase leading to a further decline in the real money stock and to a continuous real devaluation. Note that the terms of trade are assumed to react discontinuously only to new information. In the FA scenario, the shock process is completely known by the public in $t = 0$ such that the terms of trade behave continuously for the remaining course of adjustment ($t > 0$). Due to the fall in the real interest rate, output shoots up in T , but continuously decreases over the implementation phase.

After the implementation phase ($t > t_1 = 5$), we observe reverse adjustments and all shown variables return to their initial steady state. The real money stock starts to increase and converges from below to its initial steady state. The terms of trade start to decrease and converge from above to their initial steady state.

Note that the impact and the anticipation reaction of inflation are relatively small compared to the inflation reaction during the implementation phase. This is mainly due to our assumption that long-term inflation expectations are exclusively driven by changes in the money growth rate.

In the following, we denote this FA scenario as benchmark scenario and compare the responses of the remaining three anticipation scenarios to this benchmark scenario. Since we only change the nature of anticipation and leave the realized shock process unchanged, differences to the full anticipation case mainly occur on impact and during the anticipation period. After the occurrence of the shock, differences to the benchmark scenario are less visual.

In the anticipation scenario PA-MAG, the public has incorrect expectations about the magnitude c , but is correct about the start and the end of the shock process. In case the public underestimates the magnitude ($E(c) = 1.5 < c = 3$), the public implicitly

underestimates the size $c(t_1 - T)$ as well. On impact and during the anticipation phase, the economy is driven by expectations. The underestimation of the size of the shock, therefore, leads to a smaller impact and anticipation reaction for all variables in comparison to the benchmark scenario. In $t = T = 2$, the shock occurs with larger magnitude than (originally) expected. We, therefore, assume that the public switches in $t^* = T$ to correct expectations and correctly anticipates the remaining evolution of the shock process. Since the real money stock is assumed to be predetermined, all other variables and, hence, the system as whole are not able to jump on the solution path of the benchmark scenario.²³ The decline in the real money stock is, however, steeper than in the benchmark scenario, converging towards the FA solution path. The terms of trade are, on the other hand, allowed to react discontinuously to this new information. To compensate for the sluggishness of the real money stock, the terms of trade undershoot its benchmark value.²⁴ Likewise, output and inflation overshoot and the real interest rate undershoots their benchmark values. During the implementation phase, the real money stock, output, inflation and the nominal interest rate stay above, and the terms of trade and the real interest rate stay below the benchmark responses.

If the public overestimates the magnitude (and the size) of the shock ($E(c) = 4.5 > c = 3$) until $T = 2$, we see reverse adjustments. During the anticipation phase, the system responds more strongly than in the FA scenario. In $t^* = T$, the public switches to fully correct expectations. The terms of trade overshoot the benchmark value, but the real money stock stays below the FA scenario.

Scenario NA, where the public does not anticipate the increase in the money growth at all, is equivalent to the special case $E(c) = 0$ in scenario PA-MAG. Until T , all variables remain constant. In $t = T$, the policy intervention completely comes as a surprise. Therefore, we neither have an impact nor an anticipation reaction.

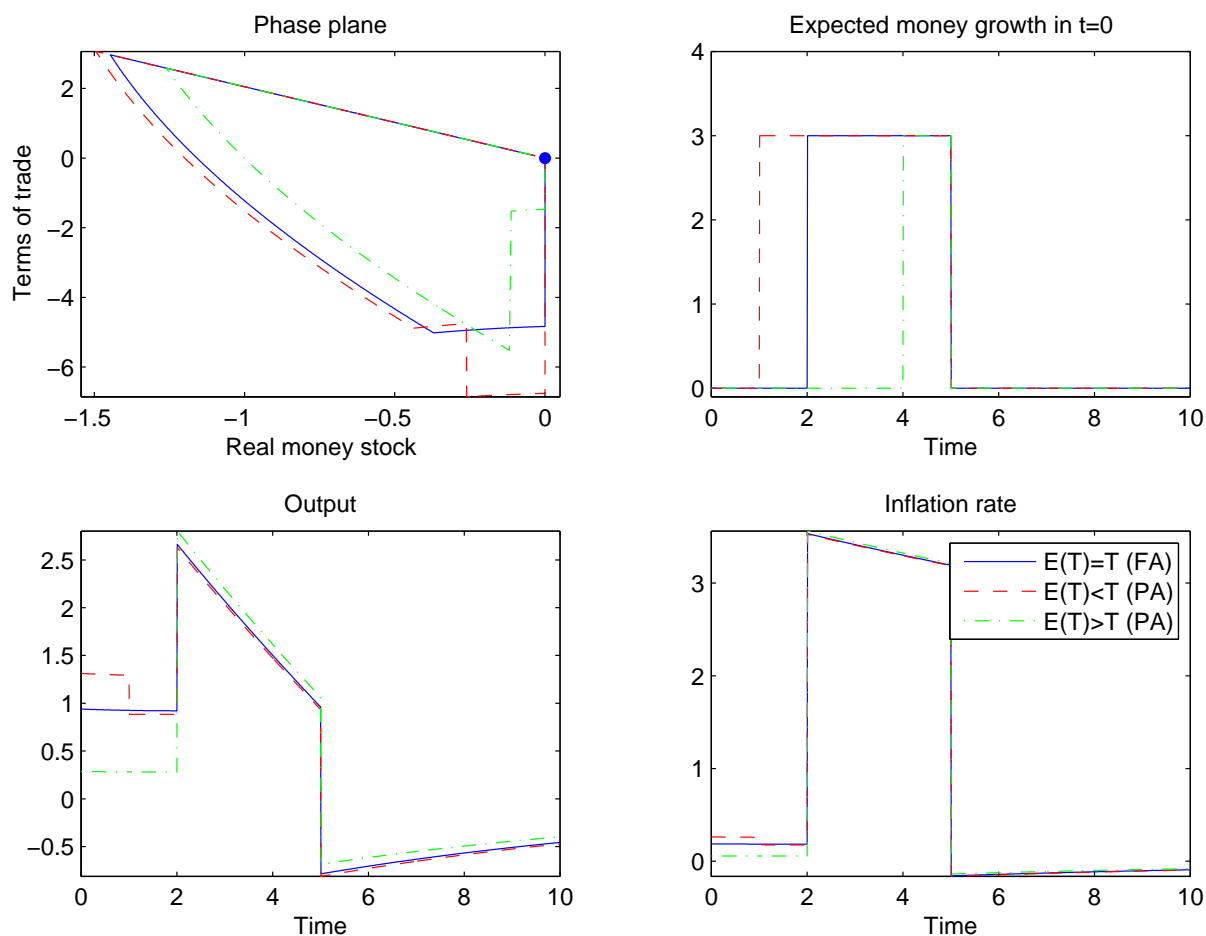
4.3.2 Scenarios PA-START and PA-EXIT

Figure 4.2 depicts the responses for scenario PA-START. For reference purposes, we again include the benchmark scenario FA, where the public has fully correct expectations. In scenario PA-START, the public has incorrect expectations about the start of implementation. The end t_1 and the magnitude c of the shock process are, on the other hand,

²³To illustrate this, consider the LM equation and note that output y and the interest rate i are non-predetermined, whereas the real money stock m^r is predetermined. Let the subscript PA (FA) denote the solution under partial (full) anticipation. According to the LM equation, the difference in the real money stock between the PA and FA scenario in t^* is then given by $m_{PA}^r(t^*) - m_{FA}^r(t^*) = l_1[y_{PA}(t^*) - y_{FA}(t^*)] - l_2[i_{PA}(t^*) - i_{FA}(t^*)]$. If the non-predetermined output and the interest rate jump on the FA solution path in t^* , i.e. $y_{PA}(t^*) = y_{FA}(t^*)$ and $i_{PA}(t^*) = i_{FA}(t^*)$, this would imply that the real money stock jumps on the FA solution path as well (i.e. $m_{PA}^r(t^*) = m_{FA}^r(t^*)$). This, however, would contradict that the real money stock is predetermined.

²⁴Without price stickiness, the real money stock would be able to react discontinuously to new information. The system then would jump in t^* directly onto the solution path under full anticipation.

Figure 4.2: Responses to a monetary shock in scenario PA-START



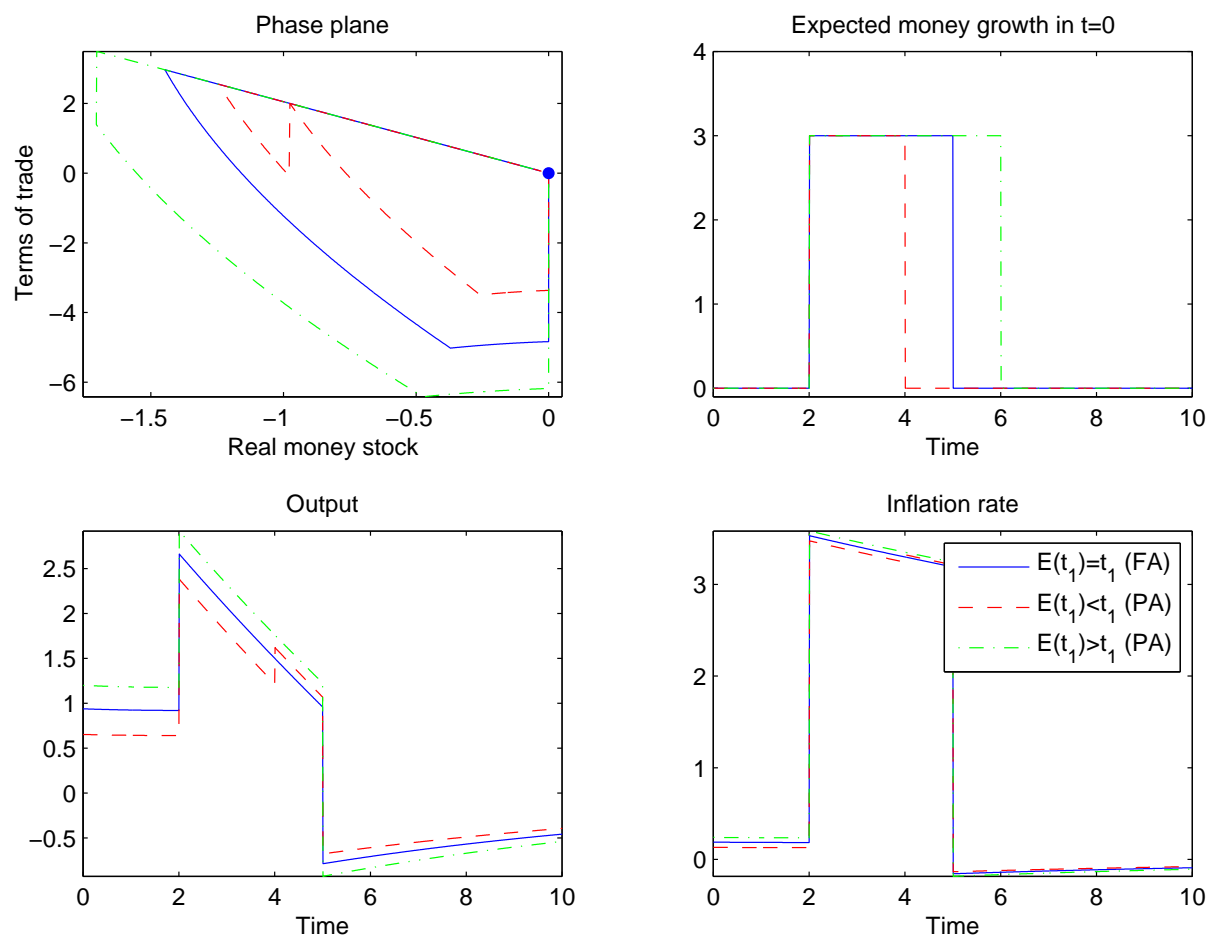
correctly anticipated. This implies that the public overestimates (underestimates) the size of the shock $c(t_1 - T)$ if the public expects an earlier (later) start of the monetary intervention.²⁵

Consequently, if the public expects a later start of the monetary policy shock ($E(T) = 4 > T = 2$), the impact and the anticipation reaction are smaller than in the FA scenario. In $t^* = T < E(T)$, the shock occurs earlier than expected and the public immediately switches to correct expectations. As in scenario PA-MAG, the terms of trade react discontinuously to this change in expectations, jumping on a lower trajectory. To compensate for the sluggishness of prices, the terms of trade undershoot the benchmark response of scenario FA, which leads to an overshooting of output and inflation.

If the public expects an earlier start of the shock process ($E(T) = 1 < T = 2$), which implies a larger expected shock size, the system overreacts until t^* . This time, however, the switch to correct expectations already occurs during the anticipation phase in $t^* = E(T) < T$, which is the time the public originally expected an increase in the

²⁵In section 4.7, we also consider the scenario PA-ST/EX where we change $E(T)$ and $E(t_1)$ simultaneously such that the length and the size of the shock are correctly anticipated. The responses and the relative volatilities are very similar to scenario PA-START.

Figure 4.3: Responses to a monetary shock in scenario PA-EXIT



money growth rate, but no change in the money growth rate occurred. Consequently, output and inflation undershoot the benchmark responses already during the anticipation phase.

As a last scenario, figure 4.3 shows the responses under scenario PA-EXIT. In this scenario, the public has incorrect expectations about the end of the monetary intervention. The start and the magnitude of the shock process are, on the other hand, correctly anticipated. This implies that the public overestimates (underestimates) the size of the shock $c(t_1 - T)$ if it expects a later (an earlier) end of the monetary intervention implying a stronger (smaller) reaction on impact and during the anticipation phase.

The main difference to the other two partial anticipation scenarios is that the switch to correct expectations now occurs during the implementation phase and not during the anticipation phase. Since the start and the magnitude are correctly anticipated, the public expectations about the shock process and the true shock process do not deviate from one another until $t^* = \min(E(t_1), t_1) > T$. Hence, if the public expects a later end of the shock process ($E(t_1) = 6 > t_1 = 5$), output and inflation stay above the benchmark response over the whole anticipation and implementation phase. If the public expects an earlier end ($E(t_1) = 4 < t_1 = 5$), output and inflation stay below the benchmark responses until

$$t^* = E(t_1) > T.$$

4.4 Measuring the (de)stabilization effects

In order to study the (de)stabilizing effects of partially anticipated monetary policy interventions, we use a relative volatility measure, which relates the volatility under partial anticipation to the volatility under full anticipation. The relative volatility for $x \in \{y, \pi\}$ in scenario $S \in \{\text{PA-MAG, PA-START, PA-EXIT}\}$ is defined as

$$RV(x|S) = \frac{V(x|S)}{V(x|FA)} \quad (4.10)$$

where $V(x|S)$ and $V(x|FA)$ measure the quadratic deviations of x from its initial steady state \bar{x}_0 over the time domain D given the expectation assumptions of scenario S and FA , respectively.²⁶

$$V(x) = \int_{t \in D} (x(t) - \bar{x}_0)^2 dt \quad (4.11)$$

Figure 4.4 shows the relative volatility of output and inflation as defined in (4.10) for all three partial anticipation scenarios for different degrees of expectation biases. We compute the relative volatility for $E(c)$ ranging from -1 to 6 in scenario PA-MAG, for $E(T)$ ranging from 0.5 to 3.5 in scenario PA-START, and for $E(t_1)$ ranging from 2.5 to 7 in scenario PA-EXIT. With correct expectations ($E(c) = c = 3$, $E(T) = T = 2$, and $E(t_1) = t_1 = 5$), the relative volatility of output and inflation intersect at unity. Values greater (smaller) than unity means that the volatility in the partial anticipation scenario is larger (smaller) than in the FA scenario. The four columns of figure 4.4 correspond to four different phases. The first column shows the overall relative volatility over the whole adjustment process ($0 < t < \infty$). The remaining columns show the relative volatility in the three distinct phases: (i) the anticipation phase ($0 < t < T$), (ii) the implementation phase ($T < t < t_1$), and (iii) the return phase ($t > t_1$).

Let us first have a look at the overall relative volatility over the whole adjustment process. In scenarios PA-MAG and PA-START, we have a trade off between output and inflation stabilization. If the public underestimates the size of the shock $c(t_1 - T)$ (either by $E(c) < c$ or $E(T) > T$), the volatility in inflation is higher and the volatility in output may be smaller than in the FA scenario. If the public overestimates the size of the shock (either by $E(c) > c$ or $E(T) < T$), inflation is stabilized and output is destabilized. Contrarily, in scenario PA-EXIT, both inflation and output may be stabilized simultaneously if the

²⁶Let the time domain D be defined over the interval $[a, b]$. Then, we can approximate (4.11) by $V(x) \approx \frac{1}{n} \sum_{j=1}^{bn} (x(j/n) - \bar{x}_0)^2 - \frac{1}{n} \sum_{j=1}^{an} (x(j/n) - \bar{x}_0)^2$ for a sufficiently small step length $1/n$. During the computation, we set $n = 100$.

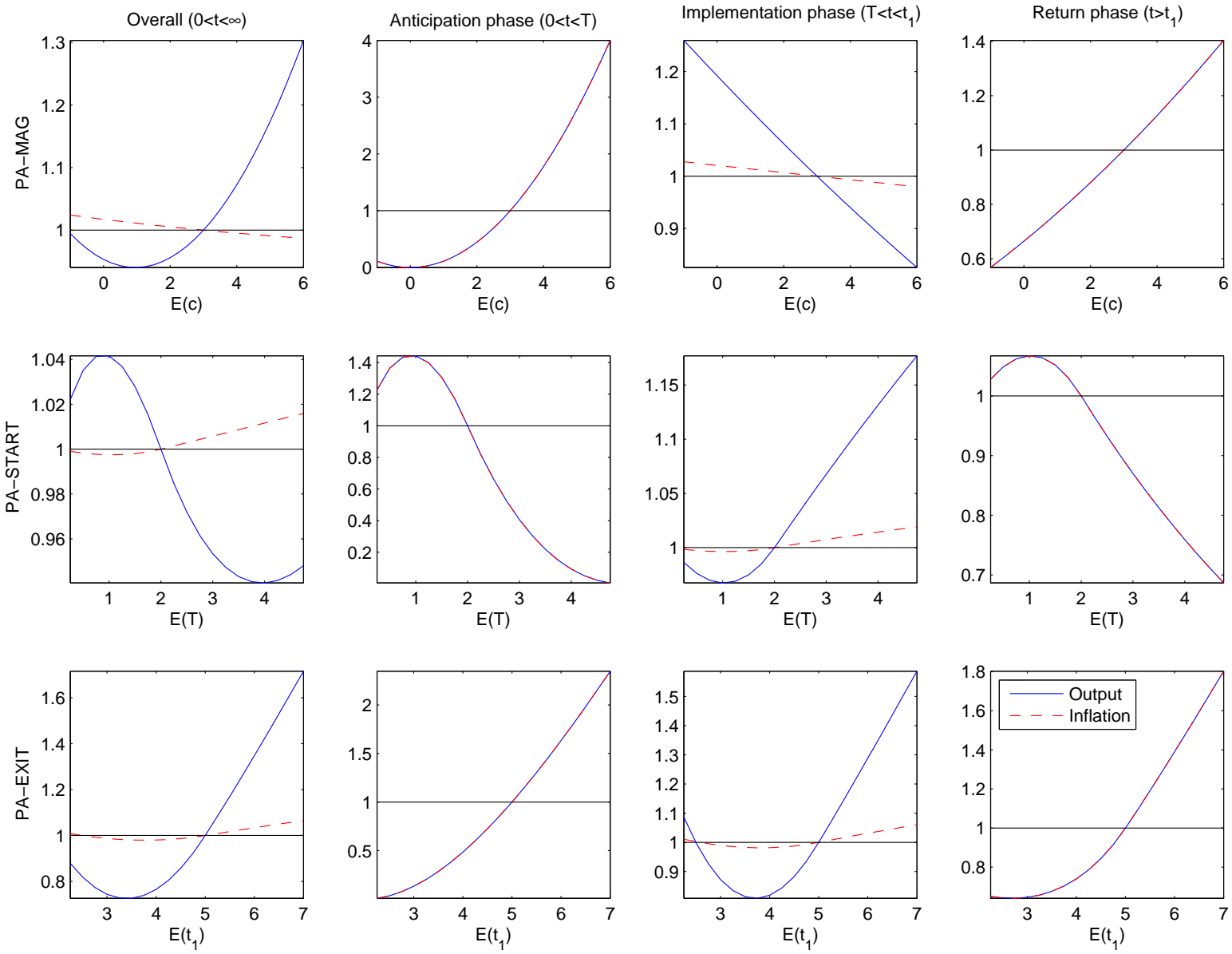


Figure 4.4: Relative volatilities in scenarios PA-MAG, PA-START, and PA-EXIT

public expects an earlier end (implying a smaller expected shock size).

Next, we consider the relative volatility during the three subperiods. Two points are common in all three scenarios: First, the trade off in volatilities that we obtain based on the overall adjustment process in scenarios PA-MAG and PA-START is not present if we consider the three phases in isolation. In all three subperiods, output and inflation are both either stabilized or destabilized. Second, during the anticipation and return phase, the relative volatility in output and inflation is even identical.²⁷ Differences in the relative volatility only occur during the implementation phase. This follows from the structure of the Phillips curve (4.5), where the expected future inflation is pinned down by the money growth rate. During the anticipation and return phase, the money growth rate is at its steady state level such that changes in the inflation rate are proportional to changes in output.²⁸ During the implementation phase ($T < t < t_1$) the money growth is different from its steady state and, therefore, we obtain different relative volatilities in inflation and output.

During the anticipation phase ($0 < t < T$), the system is driven by expectations and, hence, in all three scenarios, the volatility in inflation and output is reduced if the public underestimates the shock size $c(t_1 - T)$ (either by $E(c) < c$, $E(T) > T$, or $E(t_1) < t_1$). Contrarily, the volatility in output and inflation is enhanced during the anticipation phase if the public overestimates the shock size.

During the implementation phase ($T < t < t_1$), the underestimation of the shock size leads to a destabilization of inflation and output in scenarios PA-MAG and PA-START. Recall from the previous section that the non-predetermined variables overreact for $t > t^*$ (i.e. do not jump on the FA solution path) to compensate for the sluggishness in prices. In scenarios PA-MAG and PA-START, a smaller (stronger) reaction during the anticipation phase causes the system to respond more strongly (less strongly) during the implementation phase.

This trade off between stabilizing the system during the anticipation (and return) phase and during the implementation phase is not present in scenario PA-EXIT. The main reason is that in this scenario the switch to correct expectations occurs much later during the implementation phase. If the public underestimates the shock size ($E(t_1) < t_1$), not only the reaction on impact and during the anticipation phase is smaller, but also partly during the implementation period.²⁹

²⁷Graphically, the dashed and solid curves completely overlap such that only one solid graph is visual during the implementation and return phase.

²⁸Formally, the volatility of inflation over the time domain D can be written as

$$V(\pi) = \int_{t \in D} (\pi(z) - \bar{\pi}_0)^2 dz = \int_{t \in D} [\dot{m}(z) - \bar{m}_0 + \delta(y(z) - \bar{y}_0)]^2 dz$$

For $0 < t < T$ and $t > t_1$, $\dot{m}(t) = \bar{m}_0$ such that $V(\pi) = \delta^2 V(y)$ and $RV(\pi) = RV(y)$.

²⁹If we let the switch already occur in $t^* = T$, the same trade off occurs as in the other two scenarios. See next section 4.5 for details.

During the return phase ($t < t_1$), the volatility results are qualitatively the same as during the anticipation phase, which follows from the fact that responses are reversed compared to the anticipation phase and no new information is revealed such that the terms of trade behave continuously for $t > t_1$.

The question arises, why do we face in scenarios PA-MAG and PA-START a trade off between inflation and output over the whole adjustment process, but not in any of the three subperiods separately. This trade off in overall volatilities results from the combination of the following two arguments: First, as described above, output and inflation can not be stabilized in all three subperiods simultaneously. A smaller (stronger) reaction during the anticipation phase causes the system to respond more strongly (less strongly) during the implementation phase. Second, inflation strongly responds to realizations in the money growth rate during the implementation phase, whereas the anticipation effect on inflation is relatively small. The difference between the anticipation and the implementation reaction is less pronounced for output. Under fully correct expectations, the volatility share of the anticipation phase contributing to overall volatility only amounts to 0.2 percent for inflation and to almost 12 percent for output.³⁰ Therefore, we find that the anticipation effect is dominant for output, whereas the opposite implementation effect is dominant for inflation.

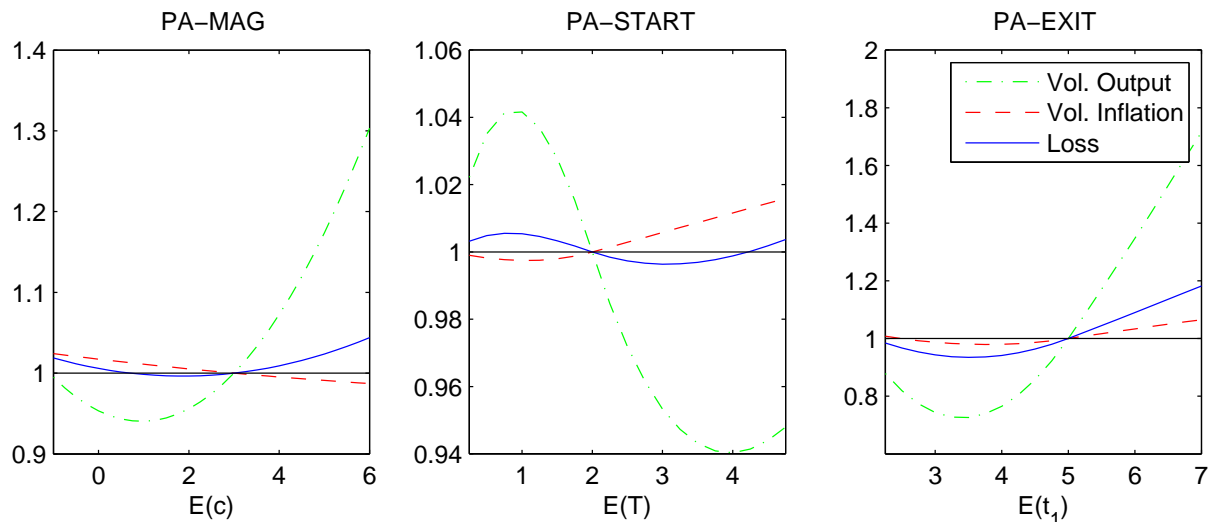
4.5 Two communication strategies

In the last section, we have shown that the volatility in inflation and output under partial information can be reduced below the volatility under full anticipation, although not necessarily simultaneously. This section discusses the policy implication of partially anticipated monetary shocks. We introduce two communication strategies and show how these strategies may improve the central bank's loss compared to the FA scenario. The first communication strategy presumes that the central bank has a sufficiently strong influence on private expectations and is, thereby, able to control the expectations $E(c)$, $E(T)$, and $E(t_1)$ directly. Since this strategy involves to create biased news about the future monetary intervention, we refer to this strategy as deception strategy.

In the second communication strategy, the central bank does not create, but is confronted with biased expectations about its future monetary intervention. The central bank now controls the breakpoint, at which the central bank is revealing the true evolution of the monetary shock and the public switches to fully correct expectations. We denote this breakpoint as t_{CB}^* to make clear that it is now set exogenously by the central bank and to distinguish it from the breakpoint t^* , at which the public (independently from the

³⁰The volatility share of the implementation phase contributes with 99 percent to overall inflation volatility and with 69 percent to overall output volatility. For more details, see section 4.7 and table 4.4 in appendix 4.C.

Figure 4.5: Optimal private expectation bias (deception strategy)



central bank) realizes its expectation biases by itself. This strategy presumes that the central bank is able to correctly monitor the private expectation biases. In this strategy the central bank does not actively deceive the public, but (only) withhold information from the public. We, therefore, refer to this strategy as withholding strategy.

The central bank aims to stabilize inflation and output. In particular, we assume that the central bank's loss function is given by

$$L_0 = \int_{t=0}^{\infty} \{(\pi(z) - \bar{\pi}_0)^2 + \alpha(y(z) - \bar{y}_0)^2\} dz = V(\pi) + \alpha V(y) \quad (4.12)$$

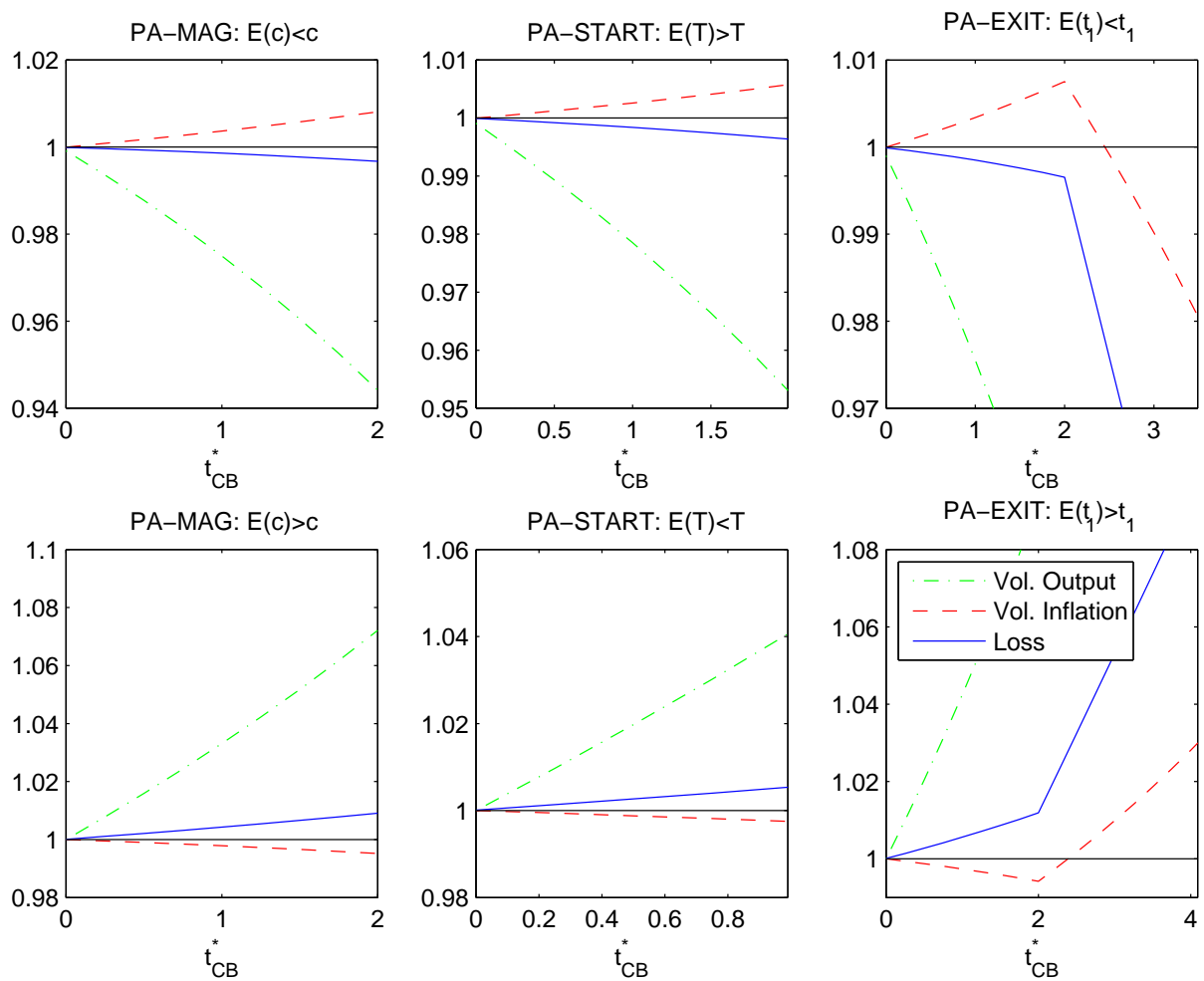
During the simulation, we set $\alpha = 0.5$. That is, the central bank's main objective is the stabilization of inflation (flexible inflation targeting).³¹ To compare the loss between partially and fully anticipated increases in the money growth rate, we compute the relative loss, which is the ratio of the loss under partial anticipation and under full anticipation.

We start with the deception strategy. Figure 4.5 shows the relative loss for different values of $E(c)$, $E(T)$, and $E(t_1)$ in scenarios PA-MAG, PA-START, and PA-EXIT, respectively. Values smaller (greater) than unity mean that the loss under partial anticipation is lower (higher) than in the FA scenario. We further add the (overall) relative volatilities of output and inflation that were already shown in the last section.

In all three partial anticipation scenarios, it is possible to improve the central bank's loss in comparison to scenario FA. The lowest loss is obtained in scenario PA-EXIT if the public expects an earlier end (lower size) of the shock process, i.e. $E(t_1) < t_1 = 5$. This is not surprising since in scenario PA-EXIT the volatility in inflation and output can be reduced simultaneously (cf. figure 4.4). Contrarily, in scenarios PA-MAG and PA-START, the central bank faces a trade off between output and inflation stabilization, which may, however, be more favorable than in the FA scenario. Although inflation stabilization is

³¹See e.g. Svensson (1999).

Figure 4.6: Optimal time of expectations correction (withholding strategy)



assumed to be the primary central bank's objective ($0 < \alpha < 1$), we find that the central bank can improve its loss if the public expects a smaller shock size (either by $E(c) < c = 3$ or $E(T) > T = 2$) and, thereby, can reduce the volatility in output at the cost of a higher volatility in inflation.

Next, we discuss the withholding strategy. Until now, we have assumed that the public switches to fully correct expectations in the very last possible moment in t^* , i.e. when the public realizes for the first time that the expected evolution of the shock process deviates from the true one. We now discuss how the central bank's loss and the volatility in inflation and output change if the switch to correct expectations occurs earlier than assumed so far ($t_{CB}^* \leq t^*$). Figure 4.6 shows the relative loss and the relative volatility for output and inflation for different values of t_{CB}^* for all three scenarios. The earliest possible time to switch is $t_{CB}^* = 0$, which is equivalent to the FA scenario. The latest possible time to switch (which we have used so far) depends on the anticipation scenario.³² In the top three plots of figure 4.6, the central bank is confronted with a public that initially

³²We have used $t^* = T$ in scenario PA-MAG, $t^* = \min(E(T), T)$ in scenario PA-START, and $t^* = \min(E(t_1), t_1)$ in scenario PA-EXIT (cf. table 4.3 of section 4.4).

underestimates the shock size (either by $E(c) < c$, $E(T) > T$, or $E(t_1) < t_1$). In the lower three plots, the central bank is confronted with a public that initially overestimates the shock size (either by $E(c) > c$, $E(T) < T$, or $E(t_1) > t_1$).

If the central bank reveals the true shock process during the anticipation phase ($0 < t_{CB}^* < T$), we have a trade off between output and inflation stabilization in all three anticipation scenarios including scenario PA-EXIT. If the public underestimates the shock size (either by $E(c) < c$, $E(T) > T$, or $E(t_1) < t_1$), output is stabilized and inflation is destabilized. If the public overestimates the shock size (either by $E(c) > c$, $E(T) < T$, or $E(t_1) > t_1$), output is destabilized and inflation is stabilized. The difference between output and inflation volatility decreases with decreasing length of withholding the true shock process (with decreasing t_{CB}^*). In scenario PA-EXIT, the switch to correct expectations may also occur during the implementation phase ($T < t_{CB}^* < t_1$). For a sufficiently late switch (t_{CB}^* sufficiently large), the trade off between output and inflation stabilization vanishes.

Under the loss function (4.12), the best communication strategy is as follows: If the public overestimates the size of the shock (lower three plots), the FA scenario produces the best outcome, i.e. the best central bank's policy is to inform the public as soon as possible about the true evolution. If, on the other hand, the public underestimates the size of the shock (upper three plots), the best central bank's policy is to inform the public as late as possible. Note that this communication strategy typically stabilizes output, but destabilizes inflation (unless $t_{CB}^* > T$) and, therefore, is only optimal if the central bank's concern about output stabilization is sufficiently strong. If e.g. the central bank's only objective is to achieve inflation stability (strict inflation targeting), this strategy typically does not achieve an optimal outcome.³³

This section has shown that the central bank may have the incentive to improve the central bank's loss by either actively deceiving private expectations (deception strategy) or by withholding information about the true evolution of the shock process (withholding strategy). However, this section should not be understood as a policy advice since both strategies may involve drawbacks that have not been mentioned so far, including the following: First, both strategies, particularly the deception strategy, may involve reputational costs by reducing the central bank's credibility in future periods. Second, the central bank has to be able to correctly monitor the expectations bias. Withholding information about the true shock process may, therefore, lead to a higher central bank's loss if the public is biased in the opposite direction.

³³Under strict inflation targeting ($\alpha = 0$), the central bank's best communication strategy is typically reversed to the strategy under flexible inflation targeting: If the public overestimates the size of the shock (lower three plots), the best strategy is to inform the public in $t_{CB}^* = T$. If the public overestimates the size of the shock (upper three plots), the best strategy is to inform the public as soon as possible (unless $t_{CB}^* > T$ is possible).

4.6 Unstable money multiplier

Until now, we have assumed that the central bank can perfectly control the money growth rate, which requires a stable relation between the (adjusted) monetary base and broad money. This stable relation implies that the asset purchases from central banks – without neutralization – lead to increases in the money stock. Since the financial crisis in 2008, we do not observe such a stable relation between base and broad money in the euro zone.³⁴ Therefore, we consider in this section the case $c = 0$, which implies that the money growth rate and the money stock do not change (see table 4.2 in section 4.3).

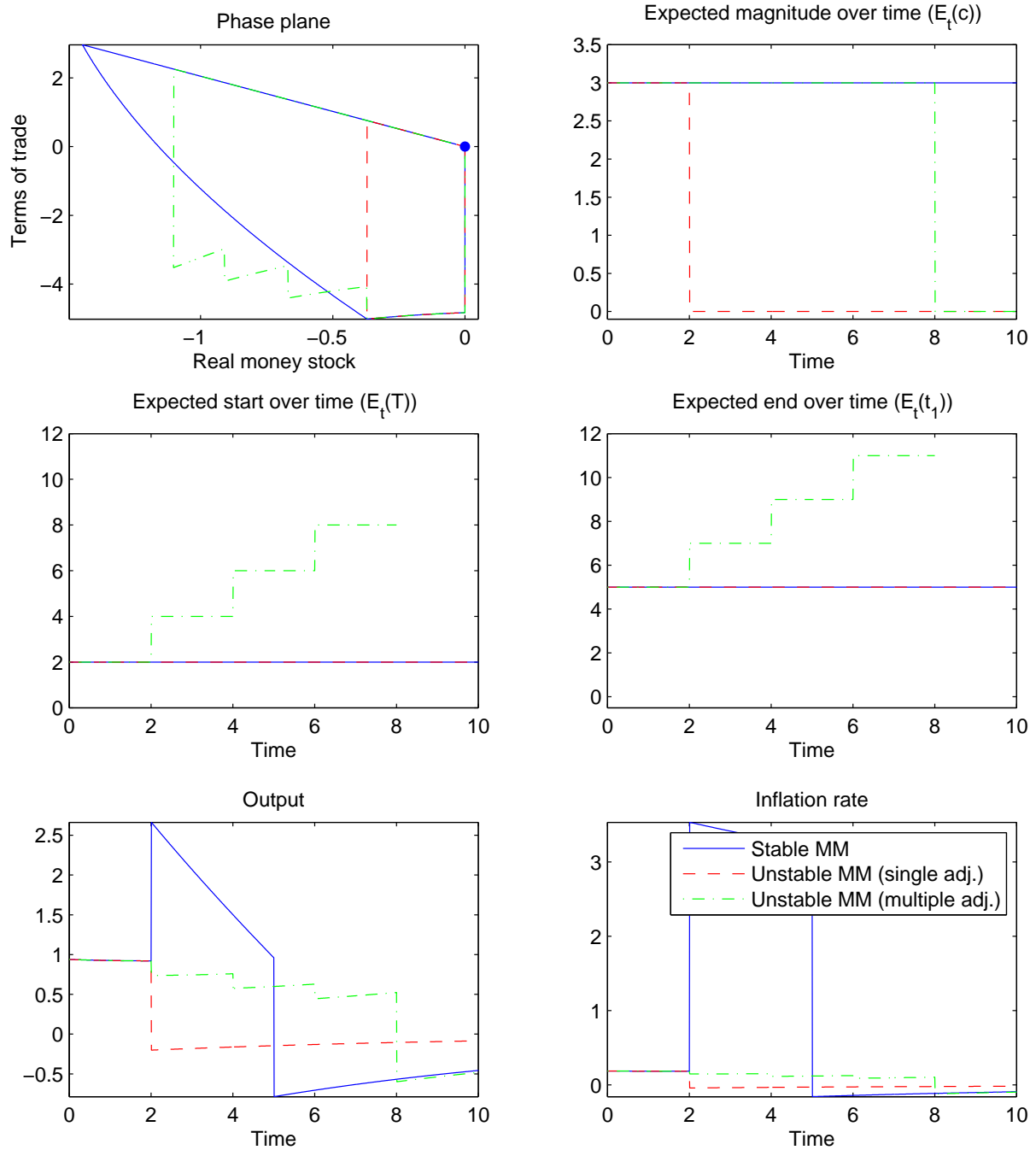
Central bank interventions (e.g. asset purchases) that lead to an expansion of the monetary base then have no effect on the economy if the public correctly anticipates this unstable relation. Figure 4.7 shows two scenarios in which expansions in the monetary base have real effects even without a stable money multiplier. In both scenarios, we presume that the public initially believes in a stable money supply multiplier and expects in $t = 0$ that the central bank interventions will indeed lead to a monetary expansion at a particular future time $T = 2$. In the first scenario, the public immediately switches to fully correct expectations in a single step after its expectations failed for the first time. In the second scenario with multiple expectations adjustments, the public sequentially updates its expectations and expects a later start (and end) of the increase in the money stock before it switches to fully correct expectations. For reference purposes, we also include the responses to a fully anticipated increase in base money with stable money multiplier from figure 4.1 in section 4.3.

In the following, we discuss the two scenarios in more detail. On impact and during the anticipation phase, the two scenarios produce the same responses as under a stable money multiplier since the initial expectations on the increase in the money growth rate are the same. In $T = 2$, the public realizes that – contrarily to its expectations – no increase in the money stock occurred. In the first scenario (single expectations adjustment), the public, therefore, immediately switches in T to fully correct expectations and correctly expects no change in the money growth rate (i.e. $E(c) = 0$ for $t > T$). In the phase plane, we see an immediate vertical jump upwards onto the original saddle path and a subsequent adjustment from above along the saddle path to the old and new steady state. Similarly, output and inflation jump downward in T and converge from below to the old steady state.

In the second scenario (multiple expectations adjustment), the public believes in T still in a stable money multiplier and sequentially expects a later start of the increase in the money stock. Note that we assume that the public also sequentially updates its expectations on the end of the increase in the money growth rate t_1 such that the expec-

³⁴See e.g. De Grauwe and Ji (2013), who finds a stable relation prior to 2008 and an unstable relation thereafter.

Figure 4.7: Unstable money multiplier (MM) with single and multiple expectations adjustments



tations on the size of the shock remain the same. In $T = 2$, the public expects a start in $T_1 = 4 > T$. In T_1 , no change in the money stock occurred and the public expects a start in $T_2 = 6 > T_1$ and so forth. This sequential updating of expectations leads to a cyclical adjustment path. Whenever the public updates its expectations (in T, T_1, T_2, \dots), the system jumps on a higher trajectory in the phase plane, which corresponds with an immediate output contraction. During two contractions, output gradually increases in anticipation of the expansionary increase in the money stock. Only after several expectations adjustments (in $t = 8$) does the public realize that no change in the money stock will occur and switches to fully correct expectations. Not until then does the system jump on the initial stable saddle path and converges from above towards the initial steady state.

To sum up, this section has shown that changes in the monetary base may have real effects on the economy and may impose cyclical adjustment movements even if a stable relation between the monetary base and a broader money aggregate is non-existent. This requires, however, that the public indeed believes in a stable relation between base and broad money and expects a future increase in the money growth rate.

4.7 Modifications

In this section, we apply six modifications. First, we change our parameter calibration. Second, we modify the length of anticipation relative to the length of the implementation phase. Third, we consider a further partial anticipation scenario PA-ST/EX which is an intermediate scenario of PA-START and PA-EXIT. Fourth, we change the mechanism with which the private expectations switch to correct expectations. Fifth, we modify the augmentation term in the Phillips curve. Finally, we consider a simultaneous increase in the risk premium s and in the money growth rate m . Figures and tables to which we refer in this section can be found in the appendices below.

1. Parameter calibration: To check the robustness of our volatility results, we simulate our model for different parameter calibrations for scenario PA-MAG, where the public has incorrect expectations about the magnitude c of the increase in the money growth rate. We consider the following alternative parameter specifications: We use $l_2 = \{1, 10\}$ for the interest rate semi-elasticity of the money demand, $\delta = \{0.1, 0.5\}$ for the slope of the Phillips curve, $a_2 = \{0.1, 0.6\}$ for the interest rate semi-elasticity of private absorption, and $b_3 = \{0.03, 0.6\}$ for the terms of trade elasticity of the trade balance.³⁵ Figures 4.8 and 4.9 show the relative volatility of output and inflation. Figures 4.10 and 4.11 show the corresponding responses to a fully anticipated increase in the money growth

³⁵In particular, we consider nine different parameter sets. In eight of these nine sets, we only change one parameter at a time and keep the remaining parameters at their baseline values. In the ninth set, we change all four parameters simultaneously.

rate. Table 4.4 gives the volatility share of the three subperiods on overall inflation and output volatility in the full anticipation scenario.

The volatility results for the alternative parameter specifications are as follows: First, we do not find any qualitative change in the relative volatility during the three subperiods. During the anticipation phase, the differences are so small that they are not visual. Second, for all parameter specifications, we find that an isolated stabilization of output and inflation is possible while a simultaneous stabilization is not possible. Indeed, we obtain the same trade off in overall volatility as in the baseline calibration. This trade off vanishes only when the anticipation effect of output is sufficiently small, i.e. the volatility share of the anticipation phase has to be at least below 1.3 percent.³⁶ Third, the anticipation effect of inflation remains very small for all parameter sets under consideration. Therefore, we obtain no qualitative change in the overall inflation volatility.

2. Anticipation length: In this modification, we change the length of the anticipation phase relative to the length of the implementation phase in scenario PA-MAG. Until now, we have assumed that the anticipation and implementation period are of similar length, where we set the anticipation length to $T = 2$ and the implementation length to $t_1 - T = 3$. We now consider two different length of anticipation. In figure 4.12 and figure 4.13, we set the relative length of anticipation to 1/10th of the length of implementation phase (i.e. $T = 0.3$ and $t_1 = 3.3$). In figure 4.14 and figure 4.15, we set the relative length anticipation to 10 times the length of the implementation phase (i.e. $T = 30$ and $t_1 = 33$). We find that the volatility differences increase with increasing length of anticipation. Our volatility results, however, do not change qualitatively. For $T = 0.3$ and $T = 30$, we obtain the same trade off in volatilities as for $T = 2$.

3. Scenario PA-ST/EX: In this modification, we consider the partial anticipation scenario PA-ST/EX, which is a combination of scenarios PA-START and PA-EXIT. So far, in each of the three partial anticipation scenarios, the public has implicitly incorrect expectations about the size of the shock. In scenario PA-ST/EX, the public has incorrect expectations about the start and the end of the monetary shock, but is correct about the length and the size. That is, the expectations bias on the start and the end of the shock has to be the same.³⁷ Figure 4.16 depicts the responses to a temporary increase in the money growth rate and figure 4.17 shows the relative volatility in this scenario. Both, the responses and the relative volatility are very similar to scenario PA-START.

³⁶For a volatility share of 1.3 percent, the trade off in volatilities still remains. Only at the border of the parameter space (e.g. if we simultaneously set $l_2 = 1$, $\delta = 0.5$, $b_3 = 0.03$, and $a_2 = 0.6$) do we find that the anticipation effect of output is small enough (below 0.1 percent) so that output and inflation may be stabilized simultaneously.

³⁷The same assumption is made in the previous section in the scenario with multiple expectations adjustments.

Consequently, not only the expected size of the shock, but also the expected timing of the shock matters.

4. Sequential correction of expectations: So far, we have assumed that the switch to correct expectations occurs immediately at one particular point in time t^* . We now assume that the public sequentially adapts its expectations over time in several steps. With each step the public gains more information about the true evolution of the shock process. For simplicity, we assume that the information gain is equally distributed over time. Figure 4.18 shows the response of the three anticipation scenarios (PA-MAG with $E(c) < c$, PA-START with $E(T) < T$, and PA-EXIT with $E(t_1) < t_1$) for three different degrees of frequency, i.e. number of expectations adjustments: (i) With frequency one, which is equivalent to the one-step adjustment of section 4.3, (ii) with frequency two, and (iii) with a frequency of 200, which gives a quasi-continuous expectations adjustment. To save space, we only show the responses in the phase plane.

The more frequent the public adjusts its expectations, the smaller is the discontinuous adjustment in the non-predetermined variables for $t < t^*$ and the smoother is the adjustment path during the anticipation phase. In the limit case of a continuous adjustment of expectations, the non-predetermined variables behave continuously after the impact during the anticipation phase ($0 < t < T$).

For all three scenarios, we compute the relative volatility using continuous expectations adjustments instead of the single adjustment frequency of section 4.3. Figure 4.19 summarizes our results. We find no notable differences to the relative volatility analysis from section 4.4. In section 4.5, we have seen that our volatility results crucially depend on whether the switch to correct expectations occurs during the anticipation or during the implementation phase. However, this modification has shown that the pace with which the switch occurs is somewhat irrelevant.

5. Inflation expectations based on consumer price index: Until now, we have assumed that the augmentation term in the Phillips curve is given by the trend rate of inflation $\pi = \dot{m}$. Figures 4.20 to 4.23 show the responses and the relative volatilities if the formation of inflation expectations in the Phillips curve is based on the (short-run) consumer price index p_c for scenarios PA-MAG, PA-START, and PA-EXIT. The augmentation term then reads as

$$\pi = \dot{p}_c = \gamma \dot{p} + (1 - \gamma)(\dot{p}^* + \dot{e}) \quad (4.13)$$

where $1 - \gamma$ measures the degree of openness and γ is set equal to 0.6.

We do not observe a qualitative change in the relative volatility during the three subperiods. Furthermore, an isolated reduction in the overall volatility of output and

inflation is possible. Again, a simultaneous stabilization of output and inflation is not possible, even in scenario PA-EXIT. However, this trade off in volatilities is reversed compared to our baseline model: If the public underestimates (overestimates) the size of the shock, output (inflation) may be destabilized, i.e. not stabilized as in our baseline model.

The reason for this reversed trade off in volatilities is as follows: First, inflation responds now much more strongly during the anticipation phase than in our baseline model. Inflation expectations in the Phillips curve are not anchored anymore to the exogenous money growth, which does not change until T . Instead, inflation expectations that are based on CPI inflation already change during the anticipation phase. The last row of table 4.4 shows that more than 50 percent of overall inflation volatility is accumulated during the anticipation phase in case $\pi = \dot{p}_c$ (in contrast to 0.2 percent in our baseline model). Therefore, the relative volatility during the anticipation phase contributes much more to the overall volatility and dominates the volatility effects during the implementation phase.

Second, the volatility share of the anticipation phase also increases for output, i.e. output reacts more strongly during the anticipation phase. However, the relative output volatility during the implementation phase is now much more sensitive to anticipation errors than in our baseline model. Therefore, the implementation effect is now dominant for output and overall output volatility may only be stabilized if it is stabilized during the implementation phase.

6. Risk premium shock: In the sixth and last modification, we apply our volatility analysis to the recent developments during the European sovereign debt crisis, where several (southern) European countries are suffering from increasing risk premiums on government bonds. To oppose these risk premiums, the President of the ECB, Mario Draghi, signaled in July 2012 further purchases of government bonds at the Global Investment Conference in London. He is, however, mute about the exact threshold \bar{s} of risk premiums at which the ECB is willing to intervene. This leaves room for private misperceptions such that the public may only partially anticipate the size and the timing of the announced monetary intervention. We aim to study the consequences of this limited information strategy with respect to the volatility of inflation and output.

To this end, we consider a simultaneous increase in the risk premium and in the money growth rate. The true, but not necessarily correctly anticipated evolution of the increase in the risk premium and in the money growth rate is summarized in table 4.5: In $t = t_0$, the risk premium starts to gradually increase. In $t = T$, the risk premium reaches the threshold \bar{s} at which the monetary authority starts to purchase government bonds at a constant rate c . Without neutralization and stable money supply multiplier, this is equivalent to a temporary increase in the money growth rate and a permanent increase in the money stock. We assume that this monetary intervention leads quasi-endogenously

to a gradual decline in the risk premium until it reaches its initial level in $t = t_1$.

In $t = 0$, the public starts to form expectations about both the risk premium and the monetary policy intervention. The public correctly anticipates and observes the increase in the risk premium, but may have incorrect expectations about the start T of the monetary intervention and, therefore, on the start of the decline in the risk premium. Contrarily to an isolated monetary shock in scenario PA-START, the public overestimates (underestimates) the size of the risk premium shock and the monetary intervention if the public expects a later (an earlier) monetary intervention. Table 4.6 summarizes the expectation biases under partial anticipation.

The responses under full and partial anticipation are shown in figure 4.24. Figures 4.25 and 4.26 summarize our volatility results: If the public expects an earlier intervention of the central bank to bring down the risk premiums on government bonds, the volatility in output and inflation may be reduced. Under flexible inflation targeting, the best central bank's communication policy then is to withhold information about the true shock process as long as possible. If, on the other hand, the public expects a later monetary intervention, output is and inflation may be destabilized. Under flexible inflation targeting, the fully correct anticipation scenario then typically gives the lowest central bank's loss. Hence, the best policy is to inform the public as soon as possible about the true intentions of the central bank.³⁸

4.8 Conclusion

In this paper, we use a continuous-time Dornbusch-type model of a small open economy to study the (de)stabilizing effects of fully anticipated, fully non-anticipated, and partially anticipated increases in the money growth rate. Under partial anticipation, the public has either imperfect information about the magnitude, the start, and/or the end of the future monetary policy intervention, and, therefore, has implicitly imperfect information about the size of the shock.

Our main results are as follows: (i) Partially anticipated monetary policy shocks may stabilize inflation and output fluctuations, i.e. lead to a lower volatility than a fully anticipated monetary policy shock of the same form. (ii) However, we typically obtain a trade off in volatilities of output and inflation over the whole adjustment process such

³⁸If the size of the monetary intervention is sufficiently small, inflation and output may be simultaneously stabilized. The reason is that the anticipation and implementation reaction of the inflation rate in response to an isolated increase in the risk premium is of similar magnitude. If the monetary intervention is sufficiently large compared to the increase in the risk premium, we obtain a trade off between inflation and output stabilization. During the simulation, we set $\bar{s} = 3$, $c = 1$, $t_0 = 2$, $T = 5$, and $t_1 = 8$, implying that the change in the money stock is given by $dm = 3$. Note that the interest rate and the risk premium shock are not logarithmized. Assuming a interest rate steady state of $\bar{i}_0 = 0.01$ implies that the risk premium shock is 100 times larger than the monetary shock in terms of percentage deviations from the steady state.

that a simultaneous stabilization of output and inflation is not possible. If the public underestimates (overestimates) the size of the shock, output (inflation) may be stabilized. (iii) This trade off in volatilities does typically not exist during the three subperiods (anticipation phase, implementation phase, and return phase) separately. If the public underestimates (overestimates) the size of the shock, both output and inflation are stabilized (destabilized) during the anticipation phase and are destabilized (stabilized) during the implementation phase. (iv) The volatility gain/loss from partial anticipation is (much) larger for output than for inflation. Under flexible inflation targeting, the best central bank's communication strategy, therefore, is typically to stabilize output fluctuations. If the public underestimates the size of the shock, the central bank then has an incentive to withhold information from the public about the true central bank's future policy intentions.

The aforementioned results can be explained as follows: First, during the anticipation phase the economy is driven by expectations. If the public overestimates (underestimates) the shock size, both output and inflation respond more strongly (less strongly) than under fully correct expectations. Under price stickiness, the economy is not able to jump on the solution path of fully correct expectations. To compensate for this price stickiness, the system (including output and inflation) overreacts when the true shock process is revealed and typically leads to smaller (larger) reaction of output and inflation during the implementation phase. This leads to the opposite volatility pattern during the anticipation and the implementation phase as described in result (iii). Second, we find that the anticipation response of inflation is relatively small compared to the anticipation response of output. Therefore, the volatility share of the anticipation contributing to overall volatility is smaller for inflation than for output. In combination with result (iii), this gives rise to an overall trade off in output and inflation volatility.

We find two exceptions in which results (ii) and (iii) do not or only partially hold: First, when the public underestimates the shock for a sufficiently long time (i.e. the expectations are biased also during the implementation of the shock), overall output and inflation may be stabilized in all three subperiods simultaneously. Therefore, a simultaneous stabilization of output and inflation over the whole adjustment process is possible and the overall trade off in volatilities vanishes. Second, if the anticipation effect of inflation (output) is sufficiently large (small), result (ii) may be reversed. That is, inflation (output) may be stabilized if the public underestimates (overestimates) the shock.

We further study partially anticipated monetary interventions in the presence of an unstable money multiplier. We find that changes in the monetary base may have real effects on the economy and may impose cyclical adjustment movements even if the relation between the monetary base and a broader money aggregate is non-existent. This requires, however, that the public indeed believes in a stable relation between base and broad money and expects a future increase in the money growth rate.

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4.A Appendix: Calibration

The income elasticity a_1 of private consumption takes the tax rate into account. Let us assume that consumption follows $C = \gamma_0 + \gamma_1(1-t)Y$, where $\gamma_0 > 0$, $0 < \gamma_1 < 1$, and t is the tax rate. In growth rates, the consumption equation reads

$$\frac{dC}{C} = \gamma_1(1-t)\frac{\bar{Y}}{C}\frac{dY}{\bar{Y}} \Leftrightarrow \frac{dC/\bar{C}}{dY/\bar{Y}} = \frac{\gamma_1(1-t)}{C/\bar{Y}} = a_1 \quad (4.14)$$

We assume that the consumption share on output (\bar{C}/\bar{Y}) is 0.7, the tax rate (t) is 0.3, and the marginal propensity of consumption is set to $\gamma_1 = 0.7$. These values imply that $a_1 = 0.7$.

4.B Appendix: Model solution

In this section, we reduce the model to a two-dimensional system of differential equations, apply the Jordan decomposition, and solve the model under fully and partially anticipated increases in the money growth rate and in the risk premium.

Model reduction

Using the terms of trade τ and the real money stock $m^r = m - p$ as state variables, we can simplify the model to a two-dimensional system:

$$y = a_0 + a_1y - a_2(i - E(\dot{p})) + g + b_0 - b_1y + b_2y^* - b_3\tau \quad (4.15)$$

$$\Leftrightarrow \lambda y = (a_0 + b_0 + g + b_2y^*) - a_2r - b_3\tau \quad (4.16)$$

where $r = i - \dot{p}$ and $\lambda = 1 - a_1 + b_1$.

$$\dot{p} = \pi + \delta(y - \bar{y}) \Leftrightarrow y = \bar{y} - \frac{1}{\delta}\dot{m}^r \quad (4.17)$$

with $\bar{y} = f_0 + f_1\bar{\tau}$. Since $\tau = p - (p^* + e)$ and $\dot{p}^* = 0$ (implying $\dot{\tau} = \dot{p} - \dot{e}$),

$$i = i^* + E(\dot{e}) + s \Rightarrow r = i^* + E\dot{e} - \dot{p} + s = i^* + E\dot{\tau} - \dot{p} + s \quad (4.18)$$

Replacing y and r in the IS equation gives:

$$\lambda \left(f_0 + f_1\bar{\tau} - \frac{1}{\delta}\dot{m}^r \right) = (a_0 + b_0 + g + b_2y^*) - a_2(i^* + E\dot{\tau} - \dot{p} + s) - b_3\tau \quad (4.19)$$

$$\Leftrightarrow \lambda(f_0 + f_1\bar{\tau}) - (a_0 + b_0 + g + b_2y^*) + a_2(i^* + s) + b_3\tau = a_2\dot{\tau} + \frac{\lambda}{\delta}\dot{m}^r \quad (4.20)$$

Replacing y and i in the LM equation $m^r = l_0 + l_1 y - l_2 i$ gives:

$$m^r = l_0 + l_1 \left(f_0 + f_1 \bar{\tau} - \frac{1}{\delta} \dot{m}^r \right) - l_2 (i^* + E(\dot{e}) + s - \dot{p} + \dot{p} - \dot{m} + \dot{m}) \quad (4.21)$$

$$m^r - l_0 - l_1 (f_0 + f_1 \bar{\tau}) + l_2 (i^* + s) + l_2 \dot{m} = l_2 \dot{\tau} + \left(l_2 - \frac{l_1}{\delta} \right) \dot{m}^r \quad (4.22)$$

In matrix form

$$\underbrace{\begin{pmatrix} a_2 & \lambda/\delta \\ l_2 & l_2 - l_1/\delta \end{pmatrix}}_A \begin{pmatrix} \dot{\tau} \\ \dot{m}^r \end{pmatrix} = \underbrace{\begin{pmatrix} b_3 & 0 \\ 0 & 1 \end{pmatrix}}_B \begin{pmatrix} \tau \\ m^r \end{pmatrix} + \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \quad (4.23)$$

where

$$c_1 = \lambda f_0 + \lambda f_1 \bar{\tau} - a_0 - g - b_2 y^* + a_2 (i^* + s - \dot{p}^*) \quad (4.24)$$

$$c_2 = -l_0 - l_1 f_0 - l_1 f_1 \bar{\tau} + l_2 (i^* + s - \dot{p}^*) + l_2 \dot{m} \quad (4.25)$$

and $\Delta = \det A = a_2(l_2 - l_1/\delta) - l_2\lambda/\delta$. $\Delta < 0$ if δ is sufficiently small such that either $l_2 < l_1/\delta$ or $a_2 < \lambda/\delta$. In the steady, implying $\dot{\tau} = \dot{m}^r = 0$, we have

$$\bar{\tau} = \frac{1}{b_3 + \lambda f_1} [-\lambda f_0 + a_0 - b_0 + g + b_2 y^* - a_2 (i^* + s - \dot{p}^*)] \quad (4.26)$$

$$\bar{m}^r = l_0 + l_1 f_0 + l_1 f_1 \bar{\tau} - l_2 (i^* + s - \dot{p}^*) - l_2 \dot{m} \quad (4.27)$$

Let \bar{x}_0 denote the initial steady state of $x \in \{\tau, m^r, y, i, r, e, p, m\}$. Since the risk premium s and the money growth \dot{m} are the only exogenous variables that we consider to change over time, differences between the current and the initial steady state are given by

$$\begin{pmatrix} \bar{\tau} - \bar{\tau}_0 \\ \bar{m}^r - \bar{m}_0^r \end{pmatrix} = - \begin{pmatrix} \frac{a_2}{b_3 + \lambda f_1} s \\ \left(l_2 + \frac{l_1 f_1 a_2}{b_3 + \lambda f_1} \right) s + l_2 \dot{m} \end{pmatrix} = - \underbrace{\begin{pmatrix} \frac{a_2}{b_3 + \lambda f_1} & 0 \\ \left(l_2 + \frac{l_1 f_1 a_2}{b_3 + \lambda f_1} \right) & l_2 \end{pmatrix}}_{-D} \begin{pmatrix} s \\ \dot{m} \end{pmatrix} \quad (4.28)$$

assuming for simplicity $s_0 = \dot{m}_0 = 0$. System (4.23) then can be written as

$$\begin{pmatrix} \dot{\tau} \\ \dot{m}^r \end{pmatrix} = A^{-1} B \begin{pmatrix} \tau - \bar{\tau}_0 \\ m^r - \bar{m}_0^r \end{pmatrix} + A^{-1} B D \begin{pmatrix} s \\ \dot{m} \end{pmatrix} \quad (4.29)$$

where

$$A^{-1} B = \frac{1}{\Delta} \begin{pmatrix} l_2 - l_1/\delta & -\lambda/\delta \\ -l_2 & a_2 \end{pmatrix} \begin{pmatrix} b_3 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{\Delta} \begin{pmatrix} (l_2 - l_1/\delta)b_3 & -\lambda/\delta \\ -l_2 b_3 & a_2 \end{pmatrix} \quad (4.30)$$

The above two dimensional system can be solved with the Jordan decomposition method as we will show in the subsequent section. Given a solution for $\tau - \bar{\tau}_0$ and $m^r - \bar{m}_0^r$, the solutions of the remaining variables can be computed as follows:

- Output:

$$y - \bar{y}_0 = \bar{y} - \bar{y}_0 - \frac{1}{\delta} \dot{m}^r \quad (4.31)$$

where

$$\bar{y} - \bar{y}_0 = f_1(\bar{\tau} - \bar{\tau}_0) = -\frac{f_1 a_2}{b_3 + \lambda f_1} s \quad (4.32)$$

- Real interest rate:

The UIP and the definition of the terms of trade in differentiated form give

$$r - \bar{r}_0 = i - \dot{p} - \bar{r}_0 = r^* - \dot{\tau} + s - \bar{r}_0 = -\dot{\tau} + s \quad (\text{with } \bar{r}_0 = r^* = 0) \quad (4.33)$$

- Inflation rate:

$$\dot{p} - \bar{p}_0 = \dot{p} = \dot{m} - \dot{m}^r \quad (\text{with } \bar{p}_0 = 0) \quad (4.34)$$

- Nominal interest rate

$$i - \bar{i}_0 = (r - \bar{r}_0) + (\dot{p} - \bar{p}_0) \quad (4.35)$$

Jordan decomposition

By applying the Jordan decomposition, we can decompose the system into two independent subsystems. The Jordan decomposition of $A^{-1}B$ in (4.29) gives

$$A^{-1}B = H \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix} H^{-1} \quad (4.36)$$

where

$$H = \begin{pmatrix} h_{11} & h_{12} \\ 1 & 1 \end{pmatrix} \quad (4.37)$$

is the matrix of eigenvectors, and r_1 and r_2 are the corresponding eigenvalues. Let us define the auxiliary variables

$$\begin{pmatrix} x \\ v \end{pmatrix} = H^{-1} \begin{pmatrix} \tau - \bar{\tau}_0 \\ m^r - \bar{m}_0^r \end{pmatrix} \quad (4.38)$$

implying

$$\tau(t) - \bar{\tau}_0 = h_{11}x(t) + h_{12}v(t) \quad (4.39)$$

$$m^r(t) - \bar{m}_0^r = x(t) + v(t) \quad (4.40)$$

Then we can rewrite the system (4.29) as two independent differential equations (premultiply with H^{-1}):

$$\begin{pmatrix} \dot{x} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix} \begin{pmatrix} x \\ v \end{pmatrix} + \underbrace{\begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix}}_F \begin{pmatrix} s \\ \dot{m} \end{pmatrix} \quad (4.41)$$

where $F = H^{-1}A^{-1}BD$.

General solution

The general solution of the differential equations (4.41) are

$$x(t) = e^{r_1 t} K_1 + \int_0^t e^{r_1(t-z)} d_1(z) dz \quad (4.42)$$

$$v(t) = e^{r_2 t} K_2 + \int_0^t e^{r_2(t-z)} d_2(z) dz \quad (4.43)$$

with

$$d_1(z) = f_{11}s(z) + f_{12}\dot{m}(z) \quad (4.44)$$

$$d_2(z) = f_{21}s(z) + f_{22}\dot{m}(z) \quad (4.45)$$

The evolution of the money growth is of the following form:

$$\dot{m}(t) = \begin{cases} \dot{m}_0 = 0 & 0 < t < t_0 \\ \dot{m}_0 = 0 & t_0 < t < T \\ \dot{m}_1 = c & T < t < t_1 \\ \dot{m}_2 = 0 & t > t_1 \end{cases} \quad (4.46)$$

If the increase in the money growth rate is temporary, then $\dot{m}_2 = 0$.

The evolution of the risk premium is of the following form:

$$s(t) = \begin{cases} s_0 = 0 & 0 < t < t_0 \\ s_0 + \beta_0(t - t_0) & t_0 < t < T \\ \bar{s} - \beta_1(t - T) & T < t < t_1 \\ s_1 = \bar{s} - \beta_1(t_1 - T) & t > t_1 \end{cases} \quad (4.47)$$

where $\bar{s} = s_0 + \beta_0(T - t_0)$. Note that the risk premium shock is permanent for $\beta_1 = 0$. In order to mute the risk premium shock during the simulation where we consider isolated increases in the money growth, we set $\beta_0 = \beta_1 = 0$. Note that for $\beta_0 = \beta_1$ and $T - t_0 = t_1 - T$, the risk premium shock is symmetric and temporary (i.e. $s_1 = s_0 = 0$).

Solution under FA scenario

First, we derive the solution in the case of fully correct expectations (FA scenario). If the above shock process is known to the public, the solution path for the auxiliary variables reads as

$$x(t) = \begin{cases} e^{r_1 t} K_{11} + \int_0^t e^{r_1(t-z)} d_1(z) dz & 0 < t < t_0 \\ e^{r_1 t} K_{12} + \int_{t_0}^t e^{r_1(t-z)} d_1(z) dz & t_0 < t < T \\ e^{r_1 t} K_{13} + \int_T^t e^{r_1(t-z)} d_1(z) dz & T < t < t_1 \\ e^{r_1 t} K_{14} + \int_{t_1}^t e^{r_1(t-z)} d_1(z) dz & t > t_1 \end{cases} \quad (4.48)$$

and

$$v(t) = \begin{cases} e^{r_2 t} K_{21} + \int_0^t e^{r_2(t-z)} d_2(z) dz & 0 < t < t_0 \\ e^{r_2 t} K_{22} + \int_{t_0}^t e^{r_2(t-z)} d_2(z) dz & t_0 < t < T \\ e^{r_2 t} K_{23} + \int_T^t e^{r_2(t-z)} d_2(z) dz & T < t < t_1 \\ e^{r_2 t} K_{24} + \int_{t_1}^t e^{r_2(t-z)} d_2(z) dz & t > t_1 \end{cases} \quad (4.49)$$

where

$$d_1(z) = \begin{cases} f_{11} s_0 & 0 < z < t_0 \\ f_{11} [s_0 + \beta_0(z - t_0)] & t_0 < z < T \\ f_{11} [\bar{s} - \beta_1(z - T)] + f_{12} \dot{m}_1 & T < z < t_1 \\ f_{11} s_1 + f_{12} \dot{m}_2 & z > t_1 \end{cases} \quad (4.50)$$

and

$$d_2(z) = \begin{cases} f_{21}s_0 & 0 < z < t_0 \\ f_{21}[s_0 + \beta_0(z - t_0)] & t_0 < z < T \\ f_{21}[\bar{s} - \beta_1(z - T)] + f_{22}\dot{m}_1 & T < z < t_1 \\ f_{21}s_1 + f_{22}\dot{m}_2 & z > t_1 \end{cases} \quad (4.51)$$

Solving the integrals for the solution of x_t

Solving the integrals in (4.48) gives:

- For $0 < t < t_0$:

$$\int_0^t e^{r_1(t-z)} d_1(z) dz = -\frac{f_{11}s_0}{r_1} [1 - e^{r_1 t}] \quad (4.52)$$

- For $t_0 < z < T$:

$$\int_{t_0}^t e^{r_1(t-z)} d_1(z) dz = f_{11}(s_0 - \beta_0 t_0) \int_{t_0}^t e^{r_1(t-z)} dz + f_{11}\beta_0 \int_{t_0}^t e^{r_1(t-z)} z dz \quad (4.53)$$

with

$$\int_{t_0}^t e^{r_1(t-z)} dz = \frac{1}{r_1} [e^{r_1(t-t_0)} - 1] \quad (4.54)$$

and

$$\int_{t_0}^t e^{r_1(t-z)} z dz \stackrel{!}{=} \int_{t_0}^t u(z)v'(z) dz = u(t)v(t) - u(t_0)v(t_0) - \int_{t_0}^t u'(z)v(z) dz \quad (4.55)$$

where $u(z) = z$, $u'(z) = 1$, $v'(z) = e^{r_1(t-z)}$ and $v(z) = -\frac{1}{r_1}e^{r_1(t-z)}$. Hence

$$\int_{t_0}^t e^{r_1(t-z)} z dz = -t\frac{1}{r_1} + t_0\frac{1}{r_1}e^{r_1(t-t_0)} - \int_{t_0}^t -\frac{1}{r_1}e^{r_1(t-z)} dz \quad (4.56)$$

$$= -t\frac{1}{r_1} + t_0\frac{1}{r_1}e^{r_1(t-t_0)} - \left[\frac{1}{r_1^2} - \frac{1}{r_1^2}e^{r_1(t-t_0)} \right] \quad (4.57)$$

$$= \frac{1}{r_1} [t_0e^{r_1(t-t_0)} - t] + \frac{1}{r_1^2} [e^{r_1(t-t_0)} - 1] \quad (4.58)$$

such that

$$\begin{aligned} \int_{t_0}^t e^{r_1(t-z)} d_1(z) dz &= \frac{f_{11}}{r_1} \left(s_0 - \beta_0 t_0 + \frac{\beta_0}{r_1} \right) [e^{r_1(t-t_0)} - 1] \\ &\quad + \frac{f_{11}\beta_0}{r_1} [t_0e^{r_1(t-t_0)} - t] \end{aligned} \quad (4.59)$$

- For $T < t < t_1$: Analogously,

$$\int_T^t e^{r_1(t-z)} d_1(z) dz = \int_T^t e^{r_1(t-z)} \{f_{11}[\bar{s} - \beta_1(z - T)] + f_{12}\dot{m}_1\} dz \quad (4.60)$$

$$\begin{aligned} &= [f_{11}\bar{s} + f_{11}\beta_1 T + f_{12}\dot{m}_1] \int_T^t e^{r_1(t-z)} dz \\ &\quad - f_{11}\beta_1 \int_T^t e^{r_1(t-z)} z dz \end{aligned} \quad (4.61)$$

where

$$\int_T^t e^{r_1(t-z)} dz = \frac{1}{r_1} [e^{r_1(t-T)} - 1] \quad (4.62)$$

and

$$\int_T^t e^{r_1(t-z)} z dz = \frac{1}{r_1} [Te^{r_1(t-T)} - t] + \frac{1}{r_1^2} [e^{r_1(t-T)} - 1] \quad (4.63)$$

such that

$$\begin{aligned} \int_T^t e^{r_1(t-z)} d_1(z) dz &= \frac{f_1^*}{r_1} [e^{r_1(t-T)} - 1] - \frac{f_{11}\beta_1}{r_1} [Te^{r_1(t-T)} - t] \\ &\quad - \frac{f_{11}\beta_1}{r_1^2} [e^{r_1(t-T)} - 1] \end{aligned} \quad (4.64)$$

$$= \left(\frac{f_1^*}{r_1} - \frac{f_{11}\beta_1}{r_1^2} \right) [e^{r_1(t-T)} - 1] - \frac{f_{11}\beta_1}{r_1} [Te^{r_1(t-T)} - t] \quad (4.65)$$

with $f_1^* = f_{11}\bar{s} + f_{11}\beta_1 T + f_{12}\dot{m}_1$ and $\bar{s} = s_0 + \beta_0(T - t_0)$.

- For $t > t_1$:

$$\int_{t_1}^t e^{r_1(t-z)} d_1(z) dz = \frac{f_{11}s_1 + f_{12}\dot{m}_2}{r_1} [e^{r_1(t-t_1)} - 1] \quad (4.66)$$

with $s_1 = \bar{s} - \beta_1(t_1 - T)$.

Solving the integrals for the solution of v_t

Analogously, the solution of the integrals in (4.49) is given by

- For $0 < t < t_0$:

$$\int_0^t e^{r_2(t-z)} d_2(z) dz = -\frac{f_{21}s_0}{r_2} [1 - e^{r_2 t}] \quad (4.67)$$

- For $t_0 < t < T$:

$$\int_{t_0}^t e^{r_2(t-z)} d_2(z) dz = \frac{f_{21}}{r_2} \left(s_0 - \beta_0 t_0 + \frac{\beta_0}{r_2} \right) [e^{r_2(t-t_0)} - 1] + \frac{f_{21}\beta_0}{r_2} [t_0 e^{r_2(t-t_0)} - t] \quad (4.68)$$

- For $T < t < t_1$:

$$\int_T^t e^{r_2(t-z)} d_2(z) dz = \left(\frac{f_2^*}{r_2} - \frac{f_{21}\beta_1}{r_2^2} \right) [e^{r_2(t-T)} - 1] - \frac{f_{21}\beta_1}{r_2} [T e^{r_2(t-T)} - t] \quad (4.69)$$

with $f_2^* = f_{21}\bar{s} + f_{21}\beta_1 T + f_{22}\dot{m}_1$ and $\bar{s} = s_0 + \beta_0(T - t_0)$

- For $t > t_1$:

$$\int_{t_1}^t e^{r_2(t-z)} d_2(z) dz = \frac{f_{21}s_1 + f_{22}\dot{m}_2}{r_2} [e^{r_2(t-t_1)} - 1] \quad (4.70)$$

with $s_1 = \bar{s} - \beta_1(t_1 - T)$.

Determining the constants for the solution of x_t

In order to obtain a unique and stable solution, we assume that the real money stock m^r is continuous over the whole adjustment process, and the terms of trade τ are continuous for $t > 0$, but may behave discontinuously in $t = 0$. Denote

$$q(t-) = \lim_{\varepsilon \rightarrow 0} q(t - \varepsilon) \quad (4.71)$$

$$q(t+) = \lim_{\varepsilon \rightarrow 0} q(t + \varepsilon) \quad (4.72)$$

with $\varepsilon > 0$ for $q \in \{x, v\}$. From our continuity assumptions on m^r and τ , we can specify the constant K_{11} , K_{12} , and K_{13} as follows:

$$x(t_0-) \stackrel{!}{=} x(t_0+) \quad (4.73)$$

$$\Leftrightarrow K_{11} = K_{12} - \frac{f_{11}s_0}{r_1} [1 - e^{-r_1 t_0}] \quad (4.74)$$

$$x(T-) \stackrel{!}{=} x(T+) \quad (4.75)$$

$$\Leftrightarrow K_{12} = K_{13} - \frac{f_{11}}{r_1} \left(s_0 - \beta_0 t_0 + \frac{\beta_0}{r_1} \right) [e^{-r_1 t_0} - e^{-r_1 T}] - \frac{f_{11}\beta_0}{r_1} [t_0 e^{-r_1 t_0} - T e^{-r_1 T}] \quad (4.76)$$

$$x(t_1-) \stackrel{!}{=} x(t_1+) \quad (4.77)$$

$$\Leftrightarrow K_{13} = K_{14} - \left(\frac{f_1^*}{r_1} - \frac{f_{11}\beta_1}{r_1^2} \right) [e^{-r_1 T} - e^{-r_1 t_1}] + \frac{f_{11}\beta_1}{r_1} [T e^{-r_1 T} - t_1 e^{-r_1 t_1}] \quad (4.78)$$

K_{14} follows from the stability condition of x . For $t > t_1$:

$$x(t) = e^{r_1 t} K_{14} + \int_{t_1}^t e^{r_1(t-z)} d_1(z) dz \quad (4.79)$$

$$= e^{r_1 t} \left[K_{14} + \frac{f_{11}s_1 + f_{12}\dot{m}_2}{r_1} e^{-r_1 t_1} \right] - \frac{f_{11}s_1 + f_{12}\dot{m}_2}{r_1} \quad (4.80)$$

Since $r_1 > 0$, convergence of $x(t)$ for $t \rightarrow \infty$ requires

$$K_{14} = - \frac{f_{11}s_1 + f_{12}\dot{m}_2}{r_1} e^{-r_1 t_1} \quad (4.81)$$

Determining the constants for the solution of v_t

Analogously, we can determine the constant K_{21} , K_{22} , K_{23} , and K_{24} . Continuity of $m^r(t)$ in $t = 0$ implies

$$m^r(0+) - m_0^r = x(0) + v(0) \stackrel{!}{=} 0 \quad (4.82)$$

Since $x(0) = K_{11}$ and $v(0) = K_{21}$, we have $K_{21} = -K_{11}$.

$$v(t_0-) \stackrel{!}{=} v(t_0+) \quad (4.83)$$

$$e^{r_2 t_0} K_{21} + \int_0^{t_0} e^{r_2(t_0-z)} dz = e^{r_2 t_0} K_{22} \quad (4.84)$$

$$e^{r_2 t_0} K_{21} - \frac{f_{21}s_0}{r_2} [1 - e^{r_2 t_0}] = e^{r_2 t_0} K_{22} \quad (4.85)$$

$$K_{22} = K_{21} - \frac{f_{21}s_0}{r_2} (e^{-r_2 t_0} - 1) \quad (4.86)$$

$$v(T-) \stackrel{!}{=} v(T+) \quad (4.87)$$

$$e^{r_2 T} K_{22} + \int_{t_0}^T e^{r_2(T-z)} d_2(z) dz = e^{r_2 T} K_{23} \quad (4.88)$$

$$\begin{aligned} K_{23} = K_{22} + \frac{f_{21}}{r_2} \left(s_0 - \beta_0 t_0 + \frac{\beta_0}{r_2} \right) [e^{-r_2 t_0} - e^{-r_2 T}] \\ + \frac{f_{21}\beta_0}{r_2} [t_0 e^{-r_2 t_0} - T e^{-r_2 T}] \end{aligned} \quad (4.89)$$

$$v(t_1-) \stackrel{!}{=} v(t_1+) \quad (4.90)$$

$$e^{r_2 t_1} K_{23} + \int_T^{t_1} e^{r_2(t_1-z)} d_2(z) dz = e^{r_2 t_1} K_{24} \quad (4.91)$$

$$\begin{aligned} K_{24} = K_{23} + \left(\frac{f_2^*}{r_2} - \frac{f_{21}\beta_1}{r_2^2} \right) [e^{-r_2 T} - e^{-r_2 t_1}] \\ - \frac{f_{21}\beta_1}{r_2} [T e^{-r_2 T} - t_1 e^{-r_2 t_1}] \end{aligned} \quad (4.92)$$

Note that constants K_{11}, \dots, K_{14} and K_{21}, \dots, K_{24} need to be solved recursively, starting with K_{14} .

Solution under scenarios NA and PA

In the anticipation scenarios NA and PA, the public has incorrect expectations on the parameters of the shock process. In particular, we assume that the public only may have incorrect expectations on T , t_1 , s_1 , \dot{m}_1 , \dot{m}_2 , and β_1 . We denote expected values that may deviate from the corresponding true parameter by a superscript *ant*. At the breakpoint t^* , the public switches to fully correct expectations. That is, the public has correct expectations about all shock parameters for $t > t^*$ and may have incorrect expectations for $t < t^*$. In the following, we assume that $t_0 < t^* < T$.³⁹

Solution of x_t

The solution for x_t under incorrect expectations, then reads

$$x(t) = \begin{cases} e^{r_1 t} K_{11}^{PA} + \int_0^t e^{r_1(t-z)} d_1(z) dz & 0 < t < t_0 \\ e^{r_1 t} K_{12}^{PA} + \int_{t_0}^t e^{r_1(t-z)} d_1(z) dz & t_0 < t < t^* \\ e^{r_1 t} \tilde{K}_{12}^{PA} + \int_{t^*}^t e^{r_1(t-z)} d_1(z) dz & t^* < t < T \\ e^{r_1 t} \tilde{K}_{13}^{PA} + \int_T^t e^{r_1(t-z)} d_1(z) dz & T < t < t_1 \\ e^{r_1 t} \tilde{K}_{14}^{PA} + \int_{t_1}^t e^{r_1(t-z)} d_1(z) dz & t > t_1 \end{cases} \quad (4.93)$$

For $t < t^*$, the solution of x_t is exclusively based on (incorrect) private expectations. Hence, the constants K_{11}^{PA} and K_{12}^{PA} are of the same form as the constants K_{11} and K_{12} under fully correct expectations. The only difference is that the constants are based on the anticipated values β_1^{ant} , s_1^{ant} , and t_1^{ant} , instead of the true values β_1 , s_1 , and t_1 . This

³⁹Note that in the discussed scenario PA-EXIT, the switch to correct expectations occurs during the implementation of the monetary shock ($T < t^* < t_1$). An explicit solution for scenario PA-EXIT is not shown here, but could be derived analogously.

is highlighted by a superscript PA. The constants can be computed recursively;

$$K_{11}^{PA} = K_{12}^{PA} - \frac{f_{11}s_0}{r_1} [1 - e^{-r_1 t_0}] \quad (4.94)$$

$$K_{12}^{PA} = K_{13}^{PA} - \frac{f_{11}}{r_1} \left(s_0 - \beta_0 t_0 + \frac{\beta_0}{r_1} \right) [e^{-r_1 t_0} - e^{-r_1 T_{\bar{s}}^{ant}}] \\ - \frac{f_{11}\beta_0}{r_1} [t_0 e^{-r_1 t_0} - T_{\bar{s}}^{ant} e^{-r_1 T}] \quad (4.95)$$

$$K_{13}^{PA} = K_{14}^{PA} - \left(\frac{f_1^{*,ant}}{r_1} - \frac{f_{11}\beta_1^{ant}}{r_1^2} \right) [e^{-r_1 T_{\bar{s}}^{ant}} - e^{-r_1 t_1^{ant}}] \\ + \frac{f_{11}\beta_1^{ant}}{r_1} [T_{\bar{s}}^{ant} e^{-r_1 T_{\bar{s}}^{ant}} - t_1^{ant} e^{-r_1 t_1^{ant}}] \quad (4.96)$$

$$K_{14}^{PA} = -\frac{f_{11}s_1^{ant} + f_{12}\dot{m}_2^{ant}}{r_1} e^{-r_1 t_1^{ant}} \quad (4.97)$$

with

$$f_1^{*,ant} = f_{11}\bar{s}^{ant} + f_{11}\beta_1 T_{\bar{s}}^{ant} + f_{12}\dot{m}_1^{ant} \quad (4.98)$$

$$\bar{s}^{ant} = s_0 + \beta_0(T^{ant} - t_0) \quad (4.99)$$

The remaining constants \tilde{K}_{12}^{PA} , \tilde{K}_{13}^{PA} , and \tilde{K}_{14}^{PA} follow from the continuity of $x(t)$ in T and t_1 and from the stability condition.

$$\tilde{K}_{12}^{PA} = \tilde{K}_{13}^{PA} - \frac{f_{11}}{r_1} \left(s_0 - \beta_0 t_0 + \frac{\beta_0}{r_1} \right) [e^{-r_1 t^*} - e^{-r_1 T}] \\ - \frac{f_{11}\beta_0}{r_1} [t^* e^{-r_1 t^*} - T e^{-r_1 T}] \quad (4.100)$$

$$\tilde{K}_{13}^{PA} = \tilde{K}_{14}^{PA} - \left(\frac{f_1^*}{r_1} - \frac{f_{11}\beta_1}{r_1^2} \right) [e^{-r_1 T} - e^{-r_1 t_1}] + \frac{f_{11}\beta_1}{r_1} [T e^{-r_1 T} - t_1 e^{-r_1 t_1}] \quad (4.101)$$

$$\tilde{K}_{14}^{PA} = -\frac{f_{11}s_1 + f_{12}\dot{m}_2}{r_1} e^{-r_1 t_1} \quad (4.102)$$

Note, that we assume that the terms of trade also behave discontinuously to the change in expectations in $t = t^*$, i.e. $\tau(t^*-) \neq \tau(t^+)$. This also holds if t^* coincides with t_0 or T .

Solution of v_t

Analogously, the solution of v_t in the anticipation scenarios FA and NA is given by

$$v(t) = \begin{cases} e^{r_2 t} K_{21}^{PA} + \int_0^t e^{r_2(t-z)} d_2(z) dz & 0 < t < t_0 \\ e^{r_2 t} K_{22}^{PA} + \int_{t_0}^t e^{r_2(t-z)} d_2(z) dz & t_0 < t < t^* \\ e^{r_2 t} \tilde{K}_{22}^{PA} + \int_{t^*}^t e^{r_2(t-z)} d_2(z) dz & t^* < t < T \\ e^{r_2 t} \tilde{K}_{23}^{PA} + \int_T^t e^{r_2(t-z)} d_2(z) dz & T < t < t_1 \\ e^{r_2 t} \tilde{K}_{24}^{PA} + \int_{t_1}^t e^{r_2(t-z)} d_2(z) dz & t > t_1 \end{cases} \quad (4.103)$$

Again, K_{21}^{PA} and K_{22}^{PA} are of the same form as in the FA scenario, but they depend on different expectation values through its dependence on K_{11}^{PA} :

$$K_{21}^{PA} = -K_{11}^{PA} \quad (4.104)$$

$$K_{22}^{PA} = K_{21}^{PA} - \frac{f_{21} s_0}{r_2} (e^{-r_2 t_0} - 1) \quad (4.105)$$

The remaining constants follow from the continuity of m^r in t^* , T and t_1 :

$$m^r(t^* -) = m^r(t^* +) \quad (4.106)$$

$$x(t^* -) + v(t^* -) = x(t^* +) + v(t^* +) \quad (4.107)$$

$$x(t^* -) + v(t^* -) = e^{r_1 t^*} \tilde{K}_{12} + e^{r_2 t^*} \tilde{K}_{22} \quad (4.108)$$

$$\tilde{K}_{22}^{PA} = e^{-r_2 t^*} (x(t^* -) + v(t^* -) - e^{r_1 t^*} \tilde{K}_{12}^{PA}) \quad (4.109)$$

where $x(t^* -)$ and $v(t^* -)$ are the last values of the anticipated solution path.

$$\tilde{K}_{23}^{PA} = \tilde{K}_{22}^{PA} + \frac{f_{21}}{r_2} \left(s_0 - \beta_0 t_0 + \frac{\beta_0}{r_2} \right) [e^{-r_2 t^*} - e^{-r_2 T}] + \frac{f_{21} \beta_0}{r_2} [t^* e^{-r_2 t^*} - T e^{-r_2 T}] \quad (4.110)$$

$$\tilde{K}_{24}^{PA} = \tilde{K}_{23}^{PA} + \left(\frac{f_2^*}{r_2} - \frac{f_{21} \beta_1}{r_2^2} \right) [e^{-r_2 T} - e^{-r_2 t_1}] - \frac{f_{21} \beta_1}{r_2} [T e^{-r_2 T} - t_1 e^{-r_2 t_1}] \quad (4.111)$$

Note that in the NA scenario, the public has the following expectations: $\beta_0^{ant} = \beta_1^{ant} = \dot{m}_1^{ant} = \dot{m}_2^{ant} = 0$.

4.C Appendix: Modifications

Table 4.4: Absolute volatilities in scenario FA for different parameter and model specifications

Modification		Overall	Ant. phase	Impl. phase	Return phase
Baseline	Output	14.91	1.73 (11.6 %)	10.35 (69.4 %)	2.84 (19.0 %)
	Inflation	34.04	0.07 (0.2 %)	33.86 (99.5 %)	0.11 (0.3 %)
$l_2 = 1$	Output	5.03	0.21 (4.1 %)	3.69 (73.3 %)	1.13 (22.6 %)
	Inflation	31.08	0.01 (0.0 %)	31.02 (99.8 %)	0.05 (0.1 %)
$l_2 = 10$	Output	21.86	3.52 (16.1 %)	14.87 (68.0 %)	3.48 (15.9 %)
	Inflation	35.60	0.14 (0.4 %)	35.32 (99.2 %)	0.14 (0.4 %)
$\delta = 0.1$	Output	15.48	2.02 (13.1 %)	11.63 (75.1 %)	1.82 (11.8 %)
	Inflation	30.61	0.02 (0.1 %)	30.57 (99.9 %)	0.02 (0.1 %)
$\delta = 0.5$	Output	15.19	1.41 (9.3 %)	8.59 (56.6 %)	5.19 (34.2 %)
	Inflation	44.71	0.35 (0.8 %)	43.06 (96.3 %)	1.30 (2.9 %)
$b_3 = 0.03$	Output	9.68	0.13 (1.3 %)	8.50 (87.8 %)	1.05 (10.8 %)
	Inflation	33.41	0.01 (0.0 %)	33.36 (99.9 %)	0.04 (0.1 %)
$b_3 = 0.6$	Output	89.71	33.67 (37.5 %)	30.64 (34.1 %)	25.41 (28.3 %)
	Inflation	38.70	1.35 (3.5 %)	36.33 (93.9 %)	1.02 (2.6 %)
$a_2 = 0.1$	Output	7.09	2.63 (37.2 %)	3.19 (45.0 %)	1.26 (17.8 %)
	Inflation	30.57	0.11 (0.3 %)	30.42 (99.5 %)	0.05 (0.2 %)
$a_2 = 0.6$	Output	36.16	0.92 (2.6 %)	28.49 (78.8 %)	6.74 (18.7 %)
	Inflation	39.36	0.04 (0.1 %)	39.05 (99.2 %)	0.27 (0.7 %)
$l_2 = 1, b_3 = 0.03,$ $\delta = 0.5, a_2 = 0.6$	Output	8.82	0.00 (0.0 %)	5.70 (64.7 %)	3.12 (35.3 %)
	Inflation	40.92	0.00 (0.0 %)	40.14 (98.1 %)	0.78 (1.9 %)
$\pi = \dot{p}_c$	Output	14.18	6.20 (43.7 %)	4.60 (32.4 %)	3.39 (23.9 %)
	Inflation	11.48	6.01 (52.3 %)	5.22 (45.5 %)	0.25 (2.2 %)

Note: Percentage figures in brackets give the volatility share of each phase contributing to overall volatility. Due to rounding figures may not add up to 100 percent.

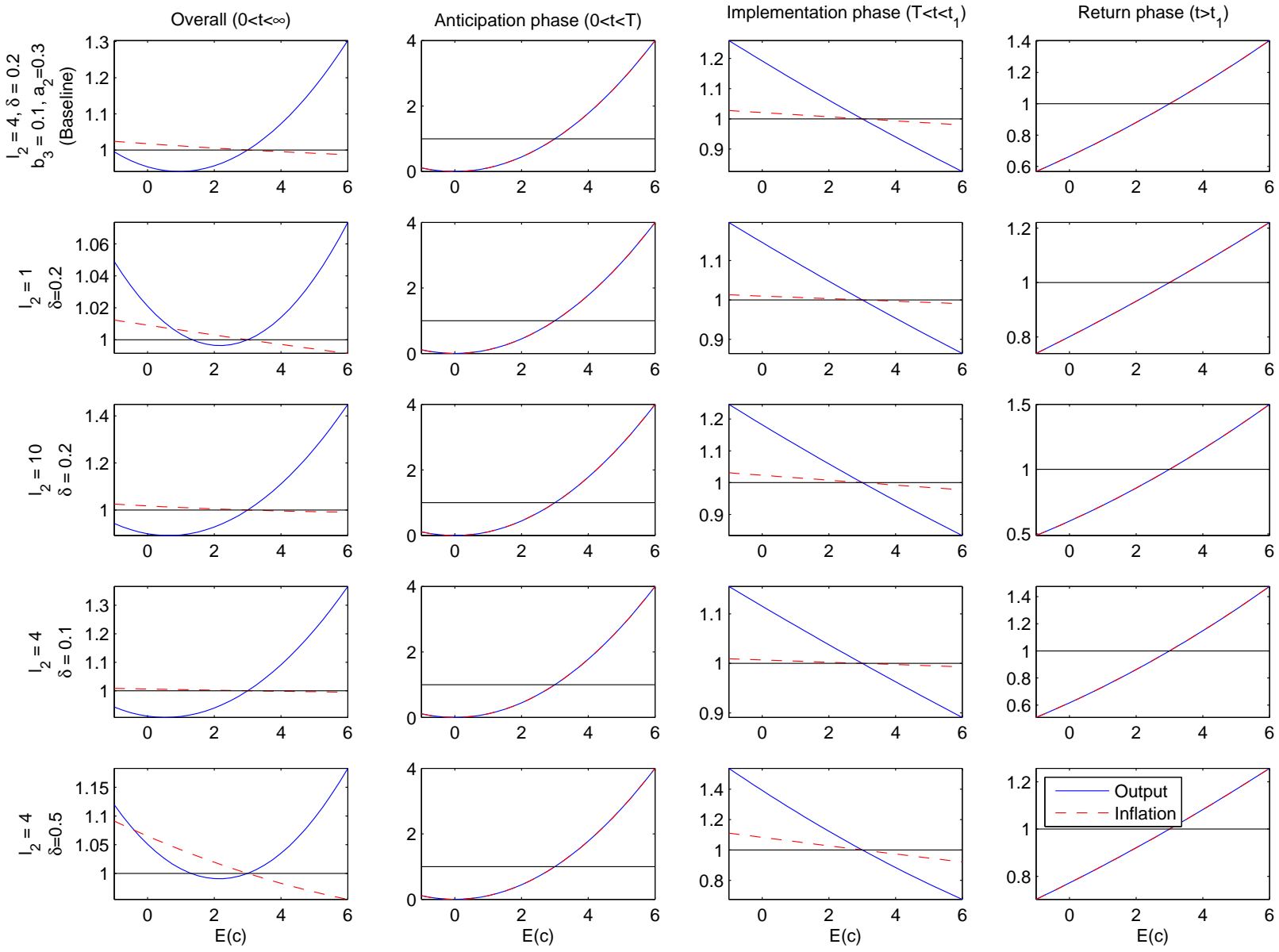


Figure 4.8: Relative volatility for scenario PA-MAG for different values of l_2 and δ

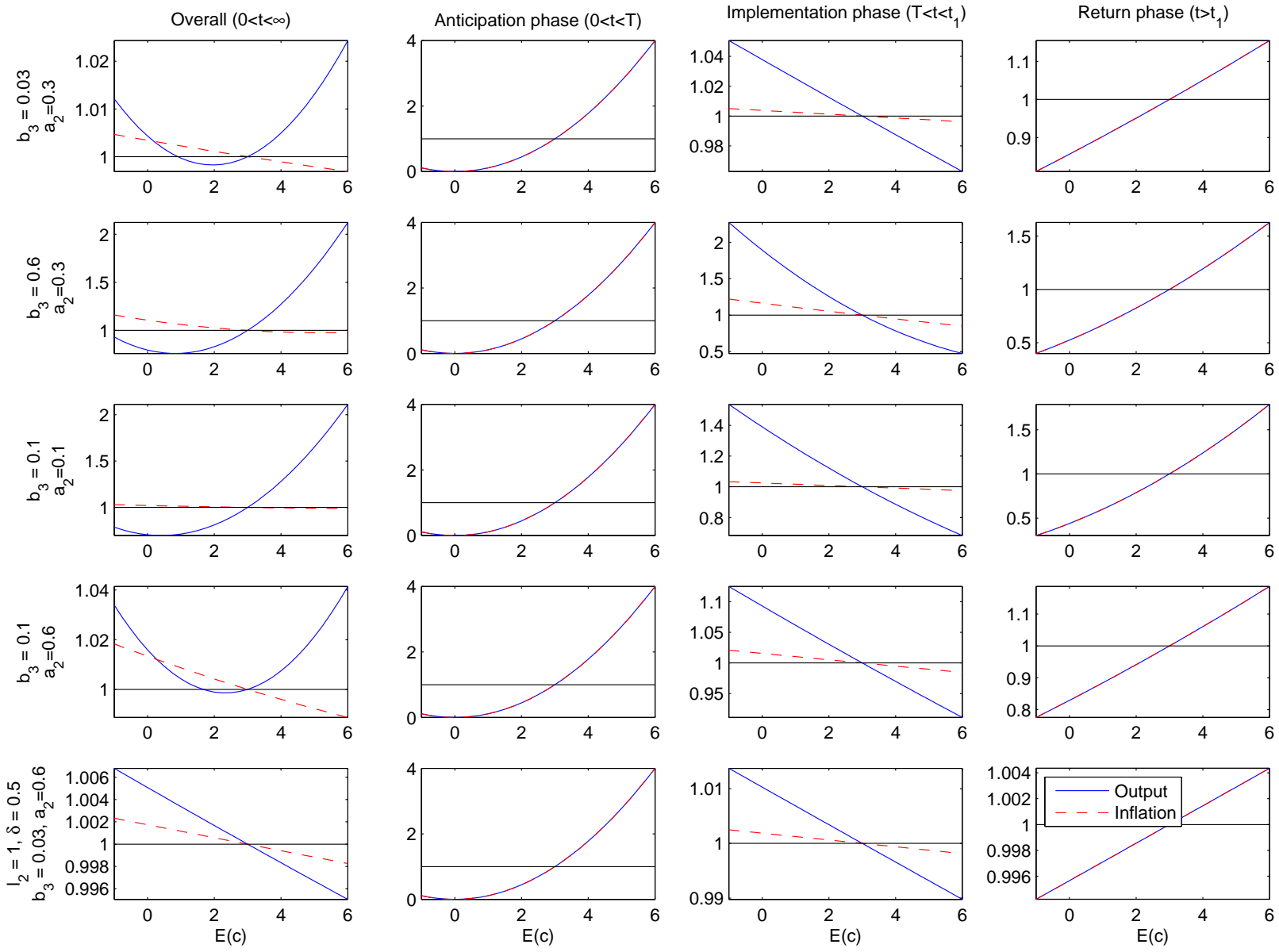


Figure 4.9: Relative volatility for scenario PA-MAG for different values of b_3 and a_2

Figure 4.10: Responses to a fully anticipated monetary shock (scenario FA) for different values of l_2 and δ

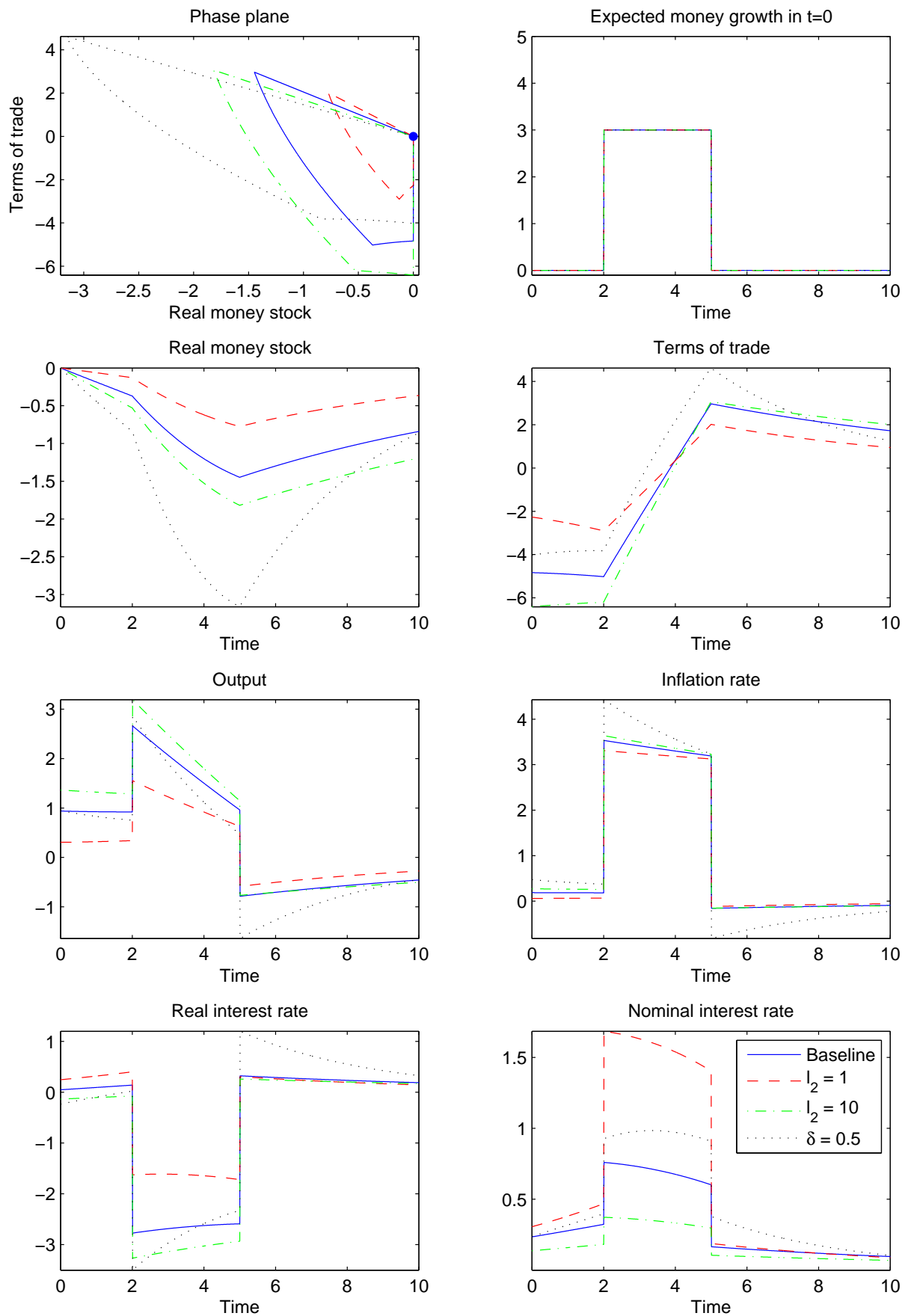


Figure 4.11: Responses to a fully anticipated monetary shock (scenario FA) for different values of b_3 and a_2

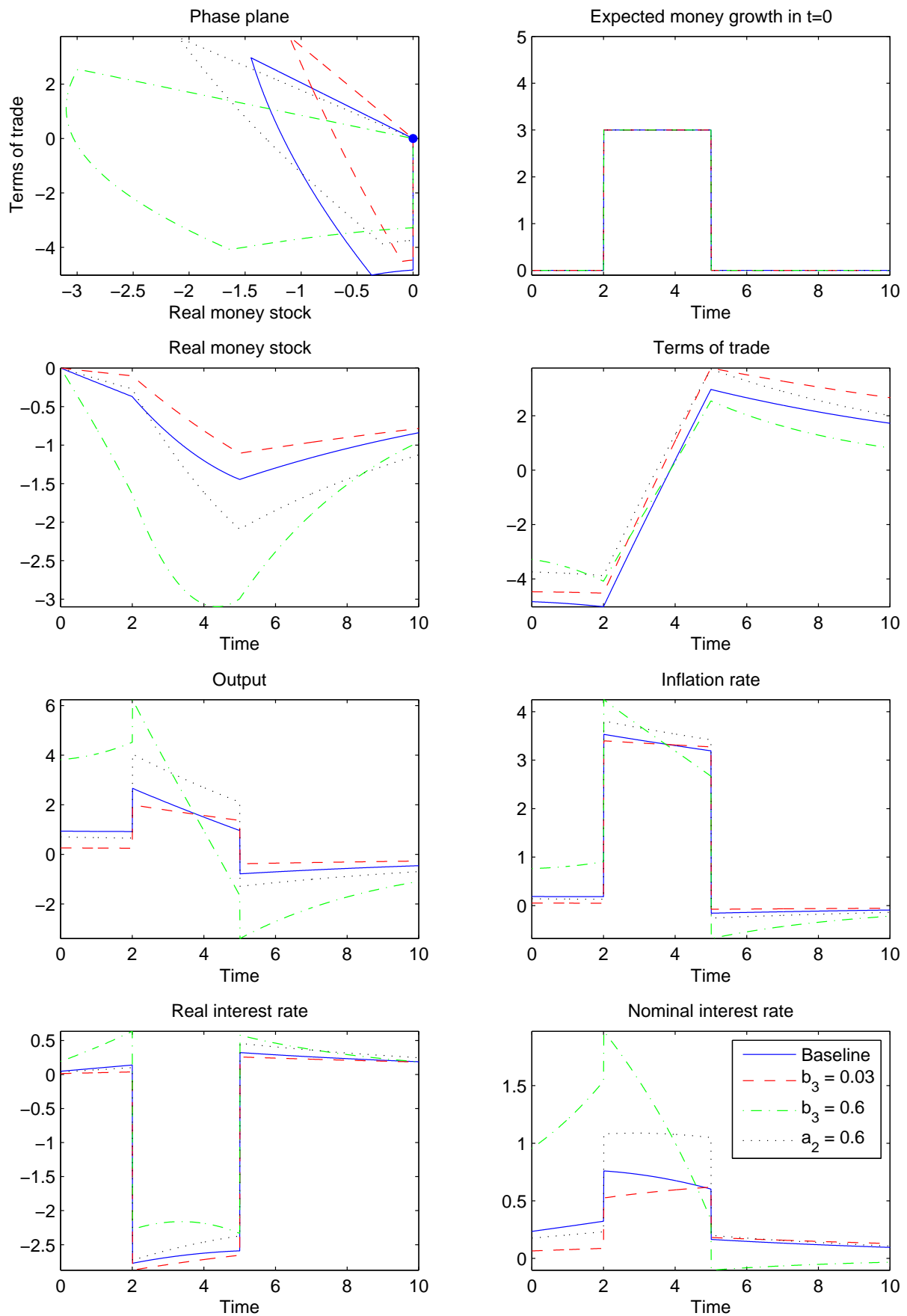


Figure 4.12: Responses to a monetary shock in scenario PA-MAG with short anticipation horizon ($T = 0.3$)

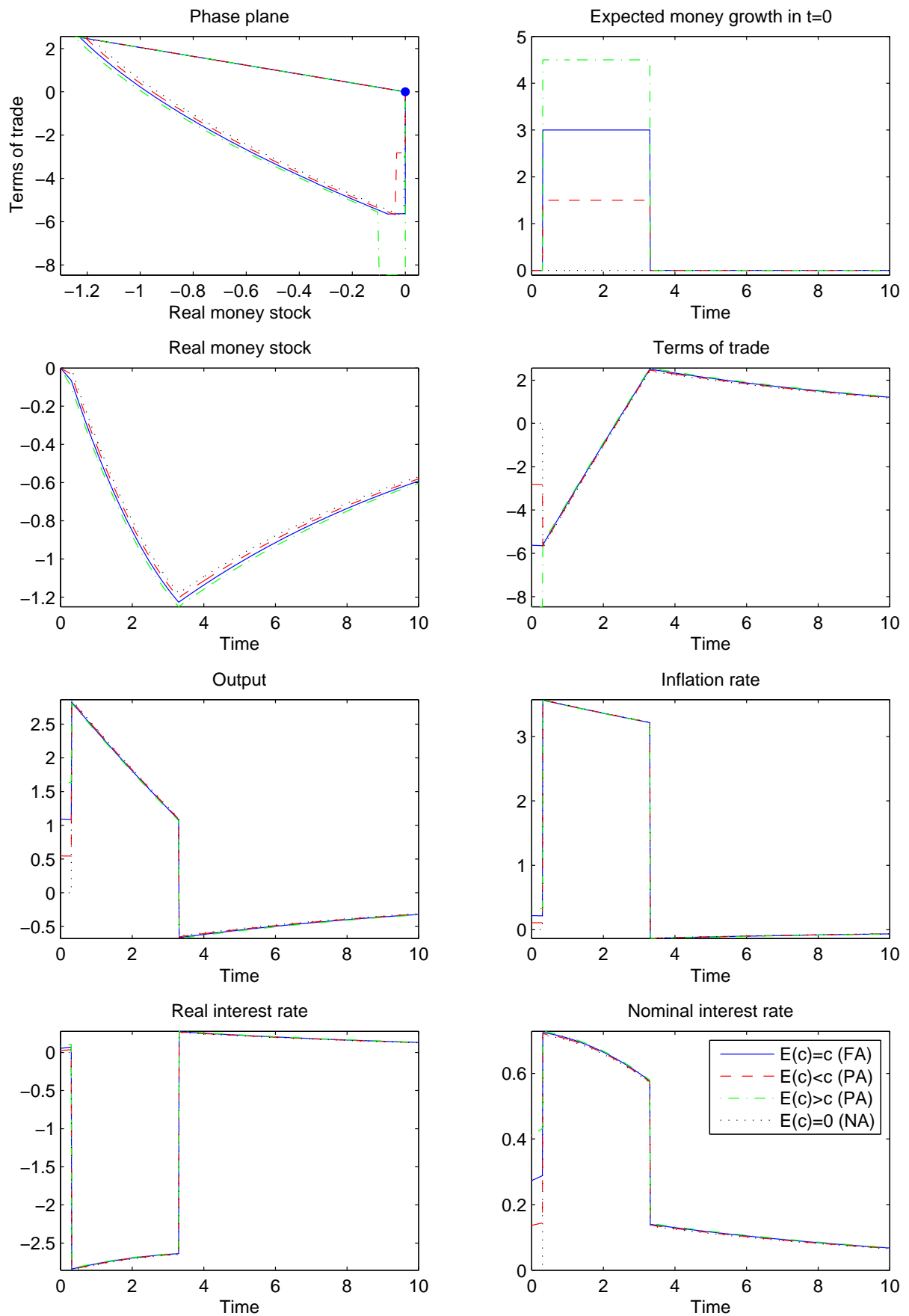


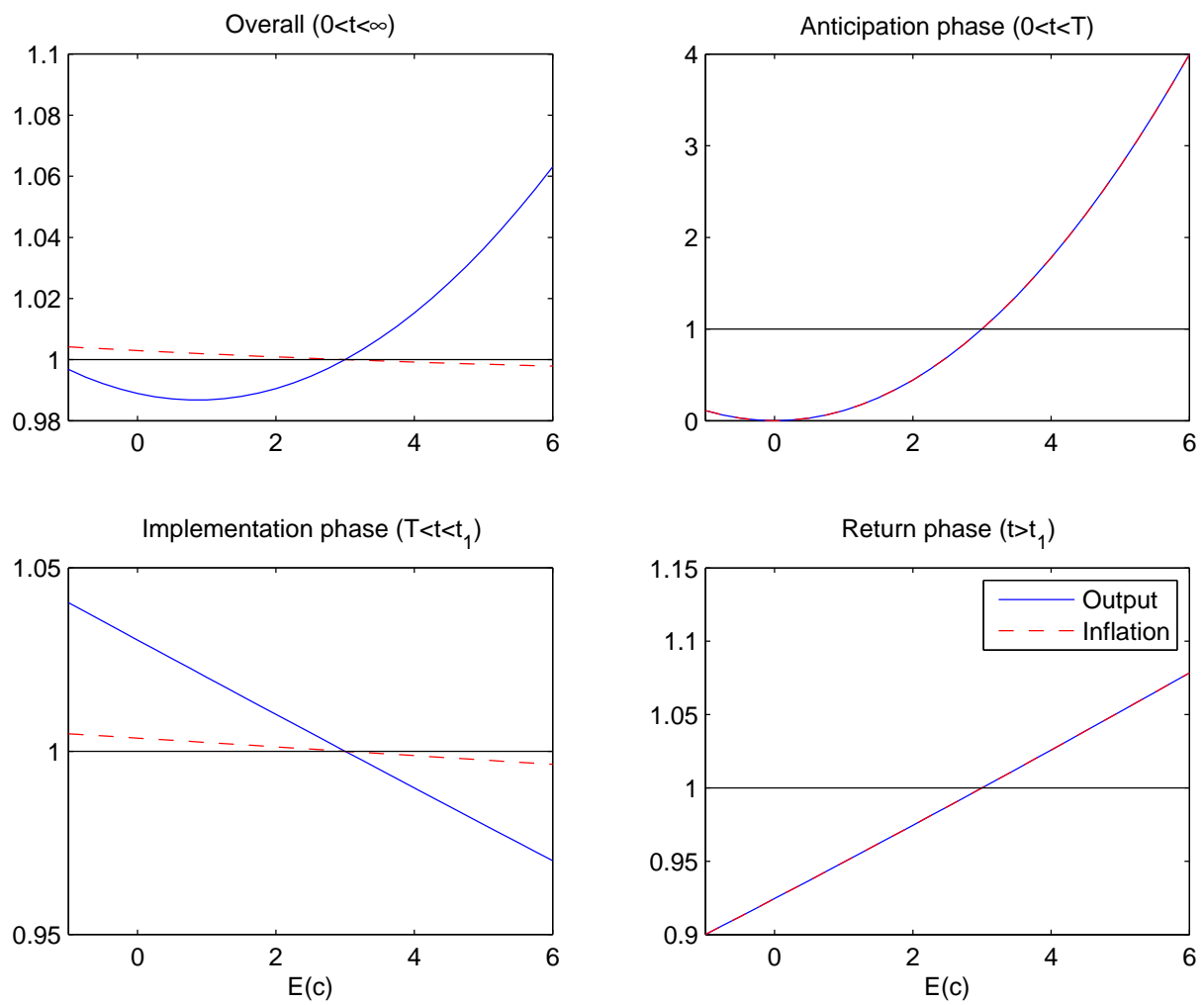
Figure 4.13: Relative volatility for scenario PA-MAG with short anticipation horizon ($T = 0.3$)

Figure 4.14: Responses to a monetary shock in scenario PA-MAG with long anticipation horizon ($T = 30$)

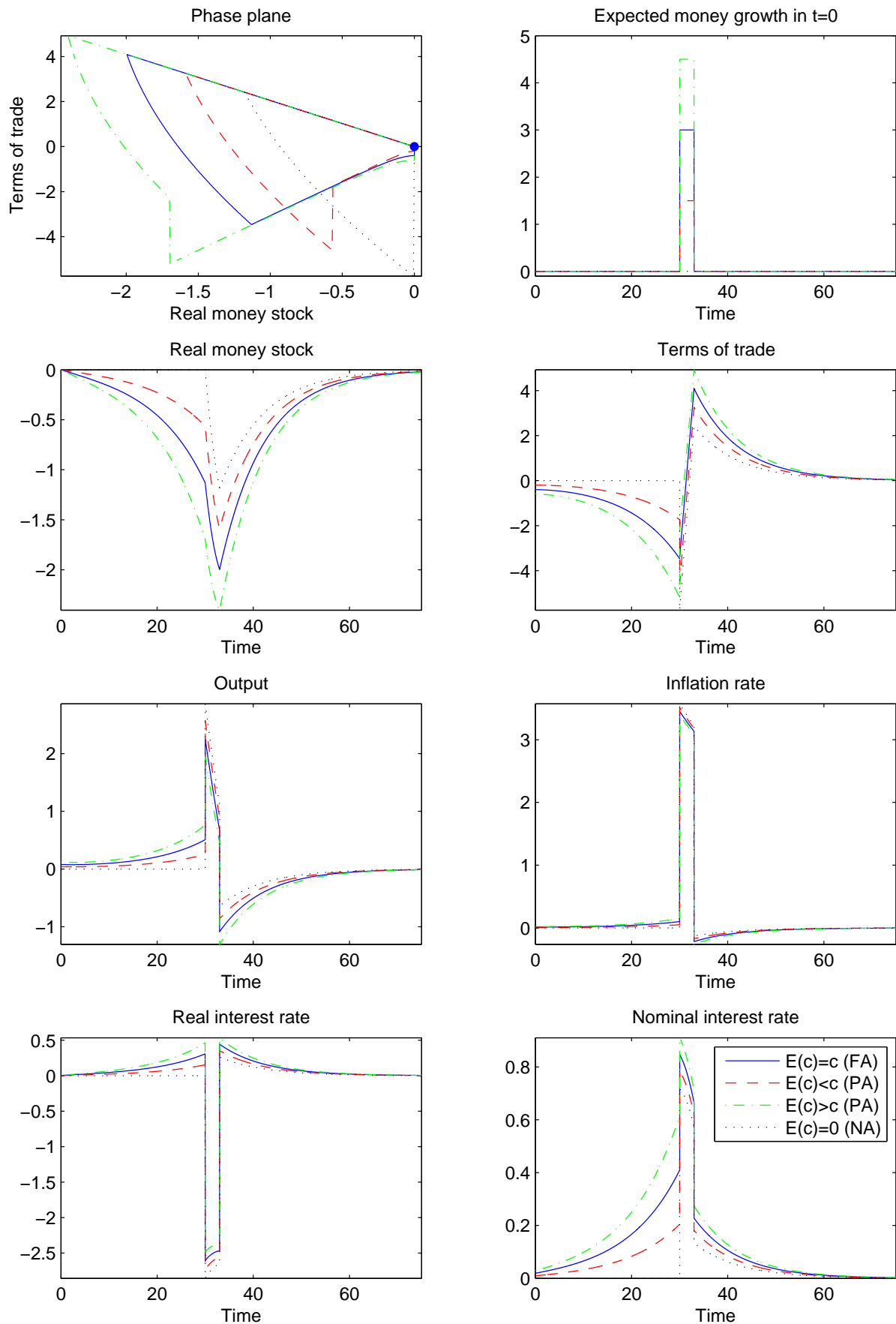


Figure 4.15: Relative volatility for scenario PA-MAG with long anticipation horizon ($T = 30$)

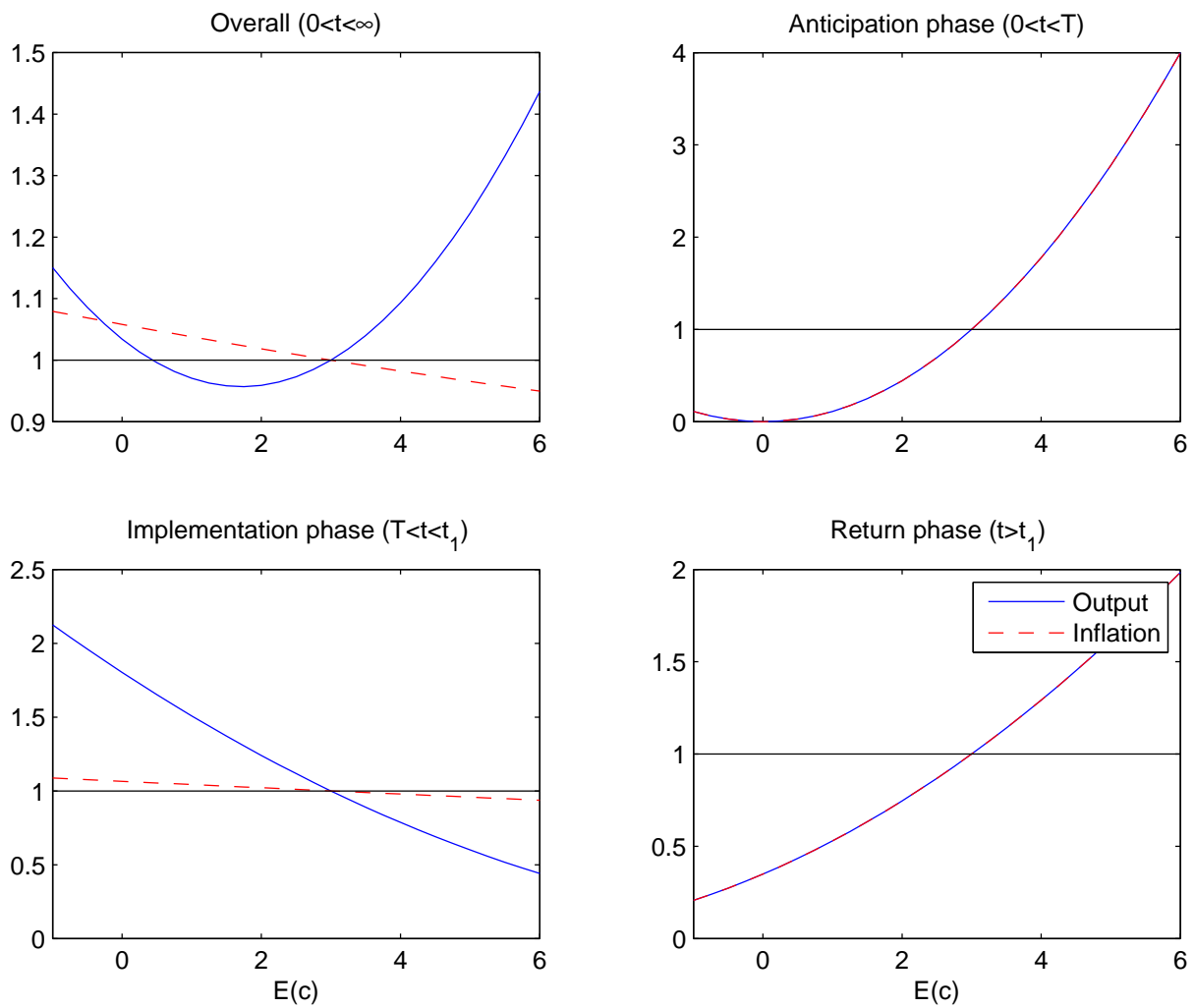


Figure 4.16: Responses to a monetary shock in scenario PA-ST/EX

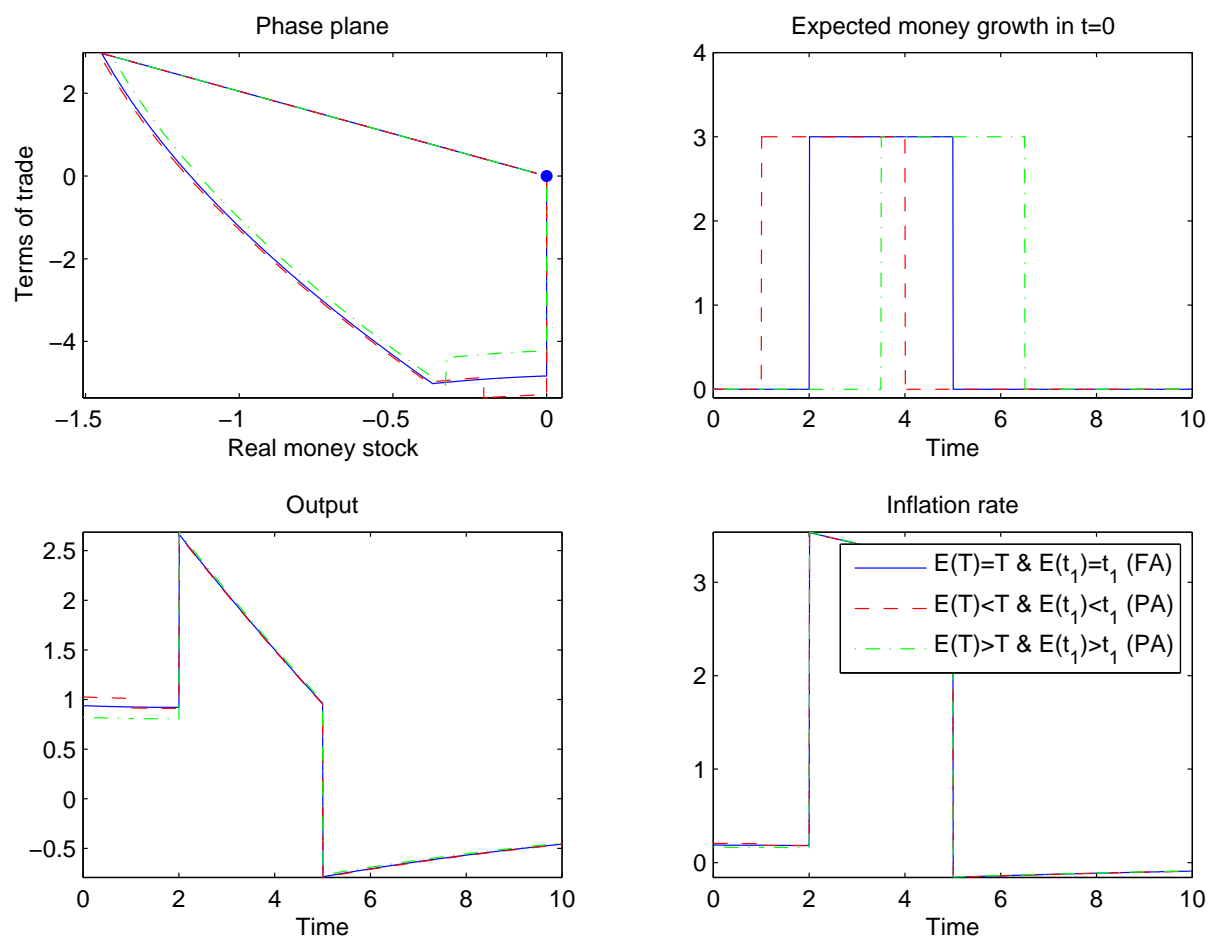


Figure 4.17: Relative volatility in scenario PA-ST/EX

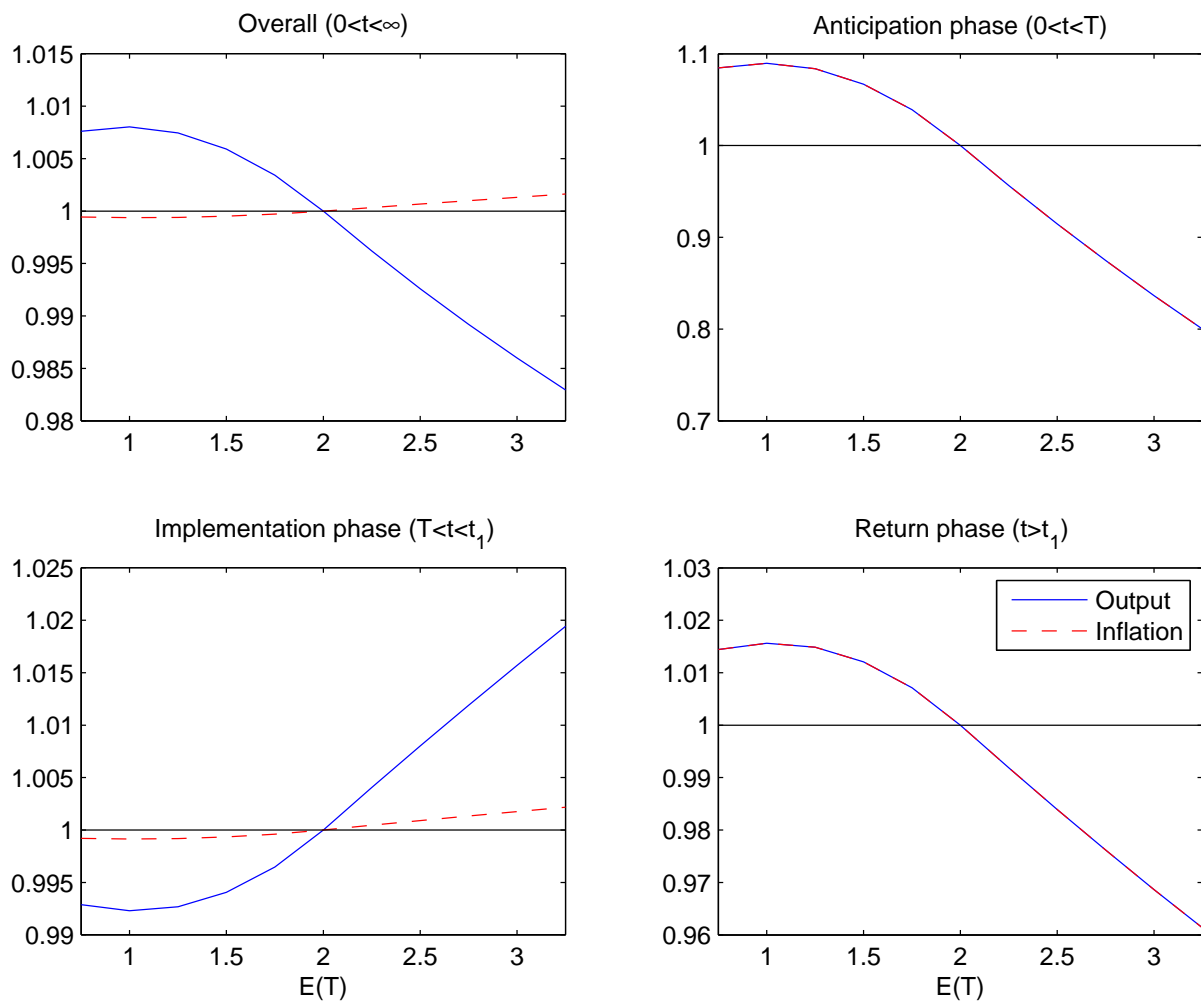
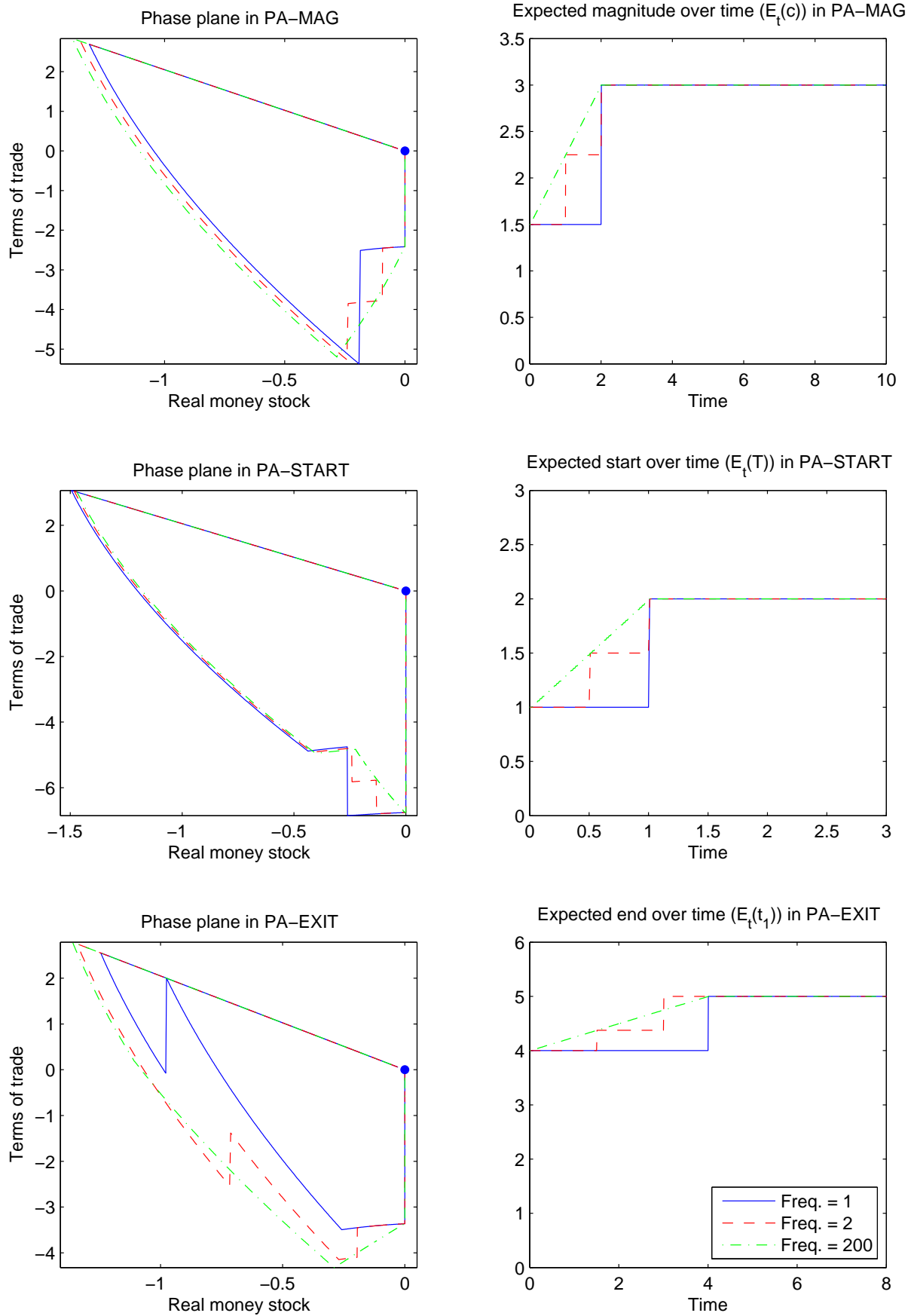


Figure 4.18: Responses to a monetary shock under various frequencies of expectations adjustments



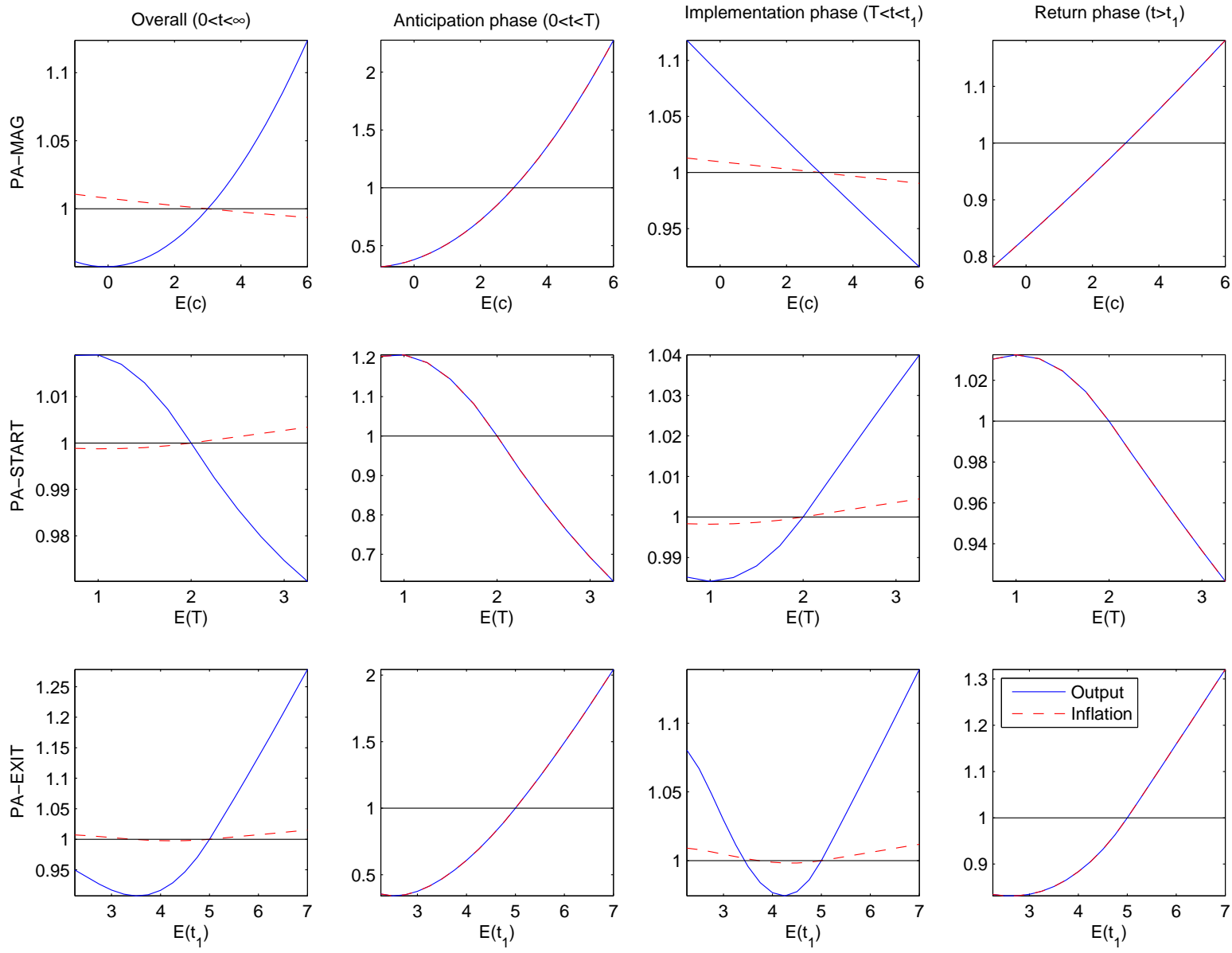


Figure 4.19: Relative volatility under continuous expectations adjustment

Figure 4.20: Responses to a monetary shock in scenario PA-MAG with inflation expectations of the form $\pi = \dot{p}_c$

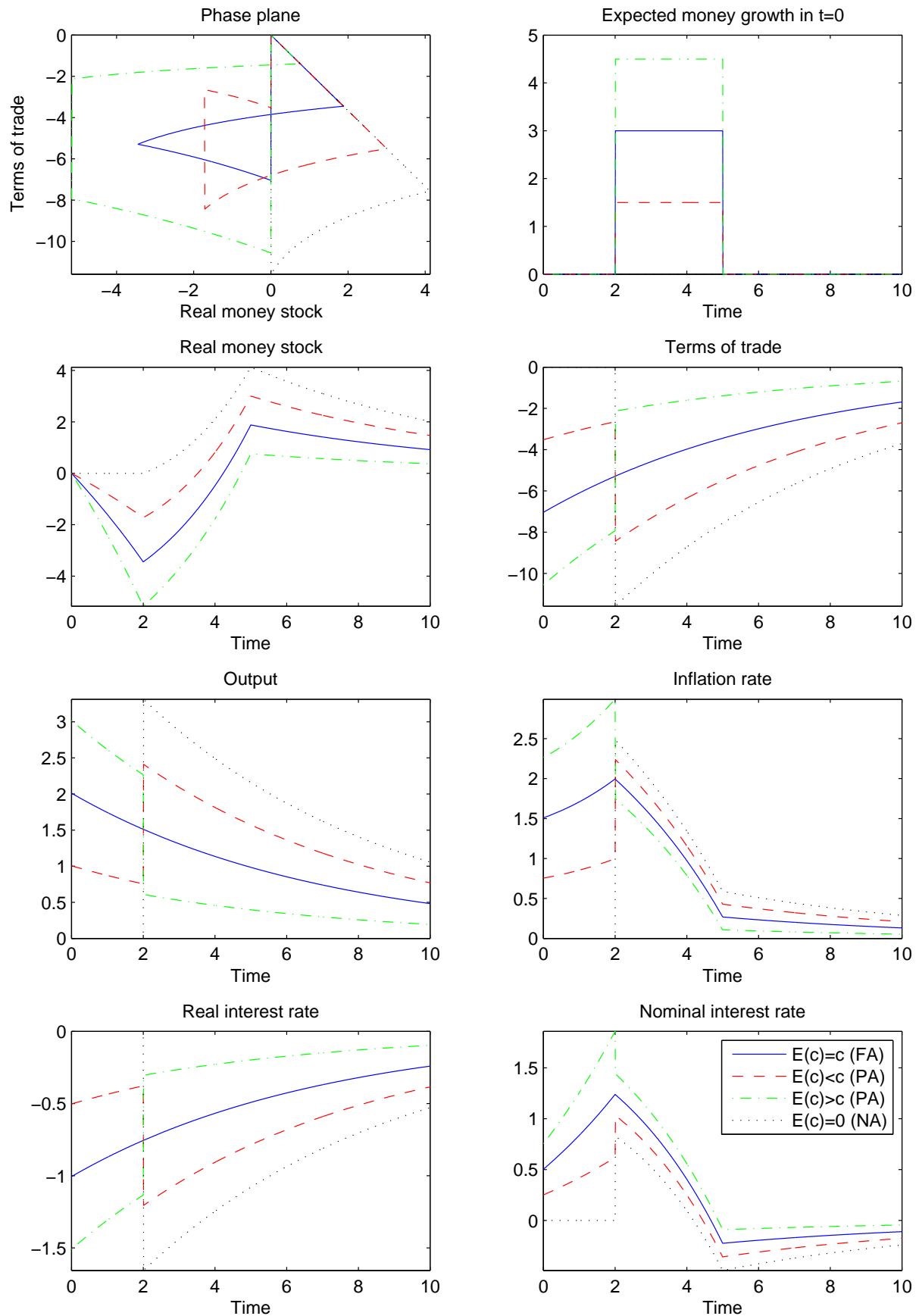


Figure 4.21: Responses to a monetary shock in scenario PA-START with inflation expectations of the form $\pi = \dot{p}_c$

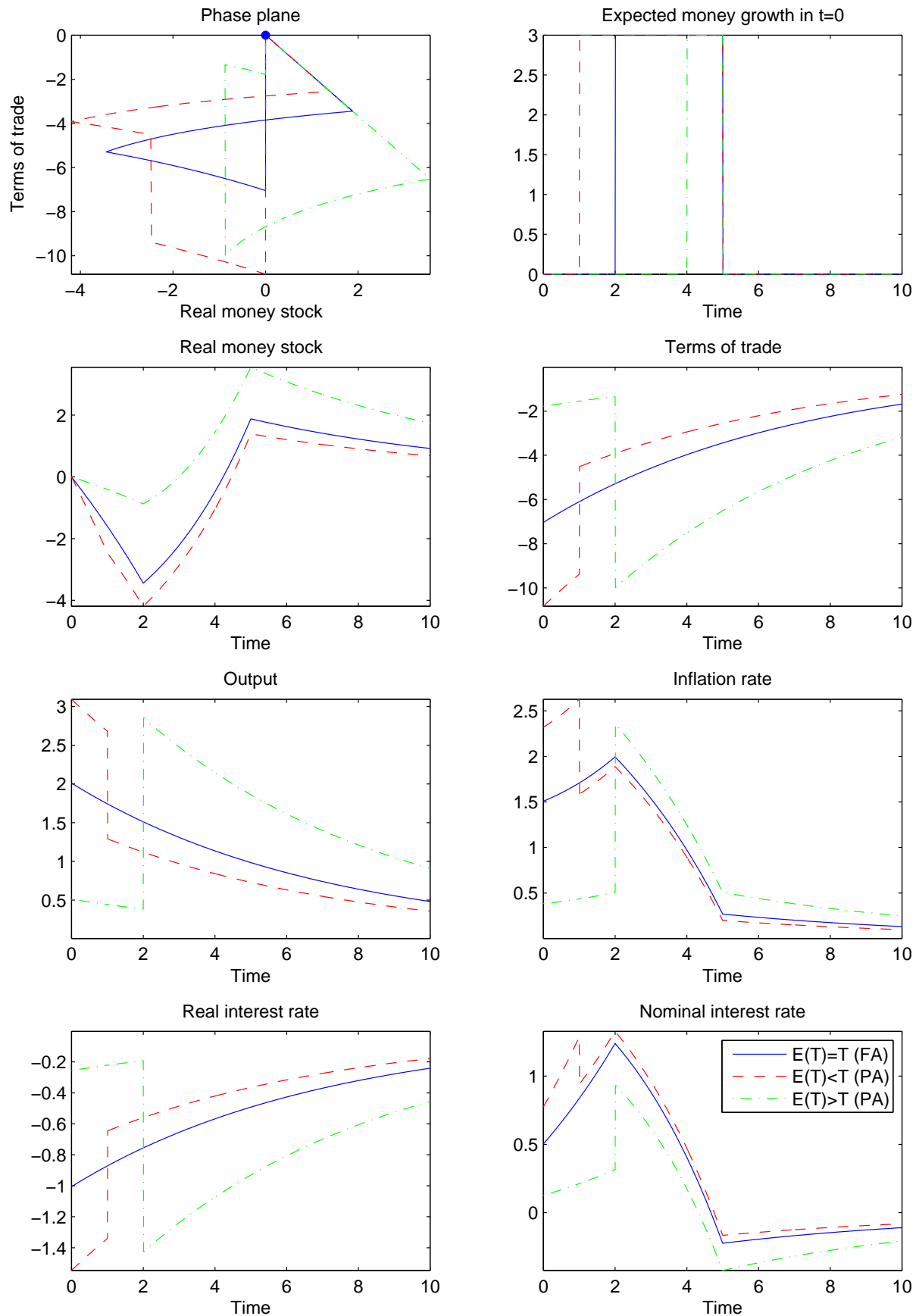


Figure 4.22: Responses to a monetary shock in scenario PA-EXIT with inflation expectations of the form $\pi = \dot{p}_c$

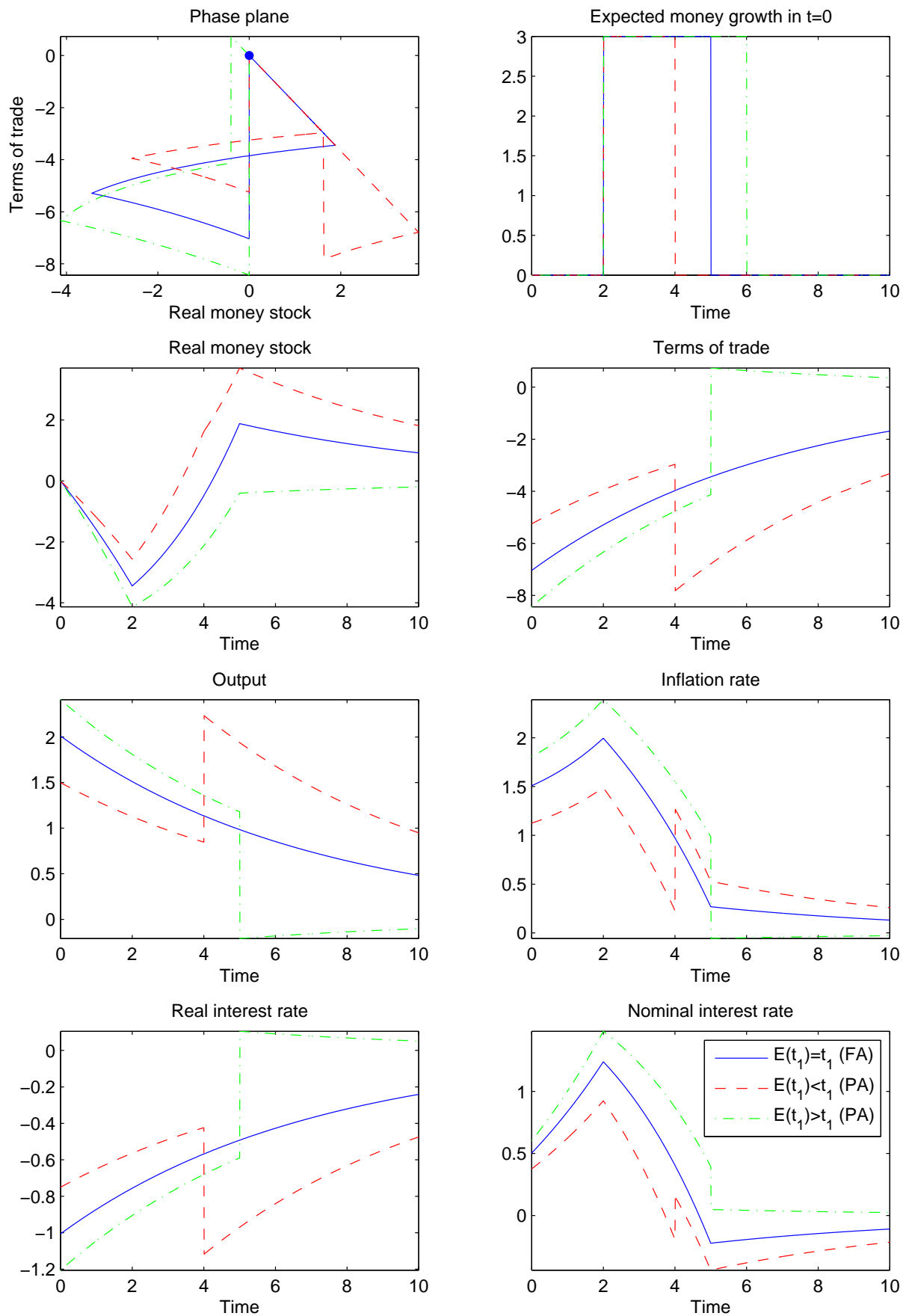


Figure 4.23: Relative volatilities in scenarios PA-MAG, PA-START, and PA-EXIT with inflation expectations of the form $\pi = \dot{p}_c$

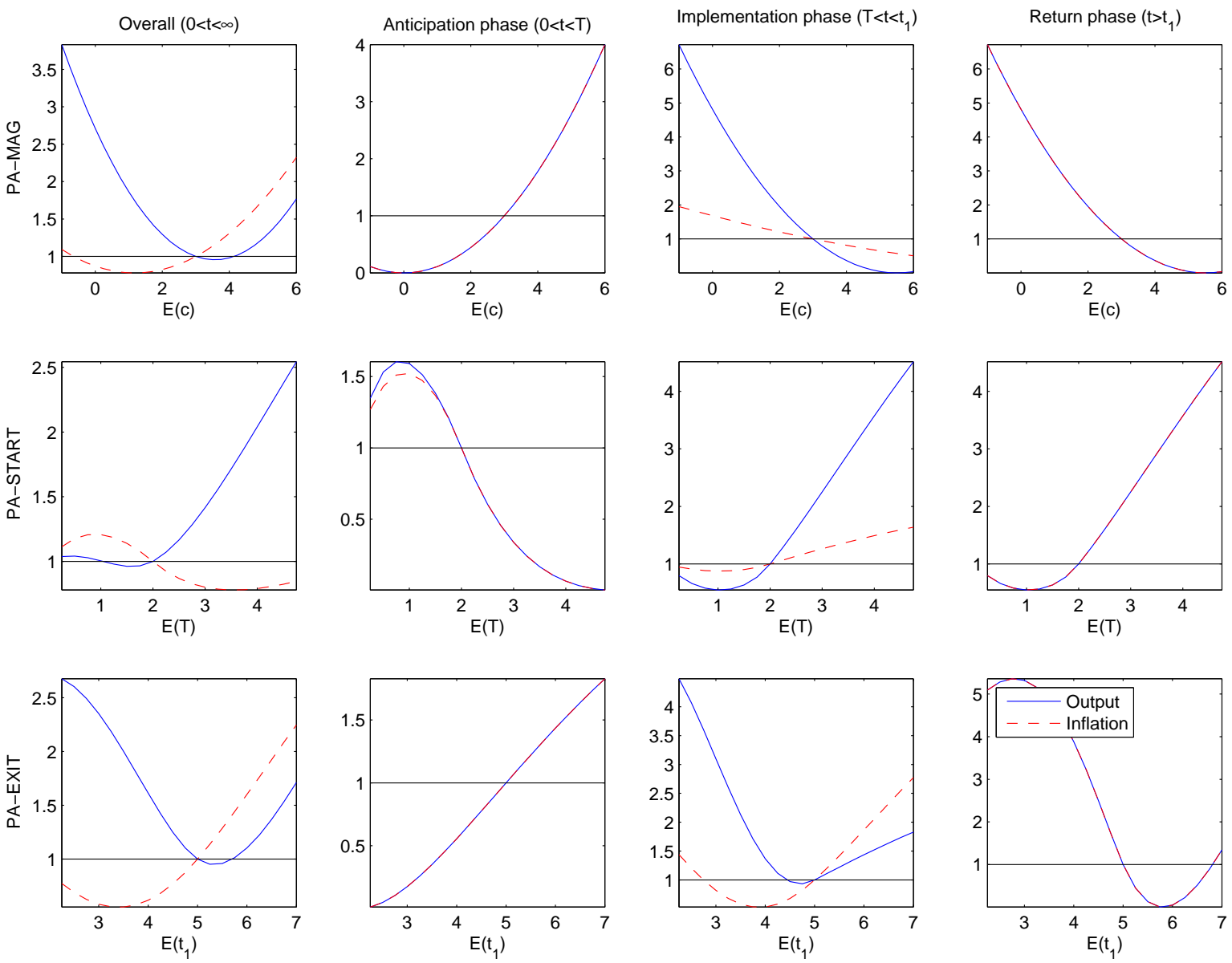


Table 4.5: Evolution of the risk premium and monetary policy shock

	$0 \leq t < t_0$	$t_0 \leq t < T$	$T \leq t < t_1$	$t > t_1$
Money growth	$\dot{m} = 0$	$\dot{m} = 0$	$\dot{m} = c$	$\dot{m} = 0$
Risk premium	$s = 0$	$s = \beta_0(t - t_0)$	$s = \bar{s} - \beta_1(t - T)$	$s = 0$

Table 4.6: Expectation (biases) on the risk premium shock and on the monetary response

		Scenario FA	Scenario PA
<i>Risk premium shock:</i>			
Start	$E(t_0)$	$t_0 = 2$	t_0
End	$E(t_1)$	$t_1 = 8$	$\neq t_1$
Pace of the increase	$E(\beta_0)$	$\beta_0 = 1$	β_0
Pace of the decline	$E(\beta_1)$	$\beta_1 = 1$	β_1
Size/Threshold	$E(\bar{s})$	$\bar{s} = 3$	$\neq \bar{s}$
<i>Monetary response:</i>			
Magnitude	$E(c)$	$c = 1$	c
Start	$E(T)$	$T = 5$	$\neq T$
End	$E(t_1)$	$t_1 = 8$	$\neq t_1$
Length	$E(t_1 - T)$	$t_1 - T = 3$	$\neq t_1 - T$
Size	$E(c(t_1 - T))$	$c(t_1 - T) = 3$	$\neq c(t_1 - T)$
Breakpoint	t^*	-	$\min(T, E(T))$

Figure 4.24: Responses to a simultaneous increase in the risk premium and the money growth rate

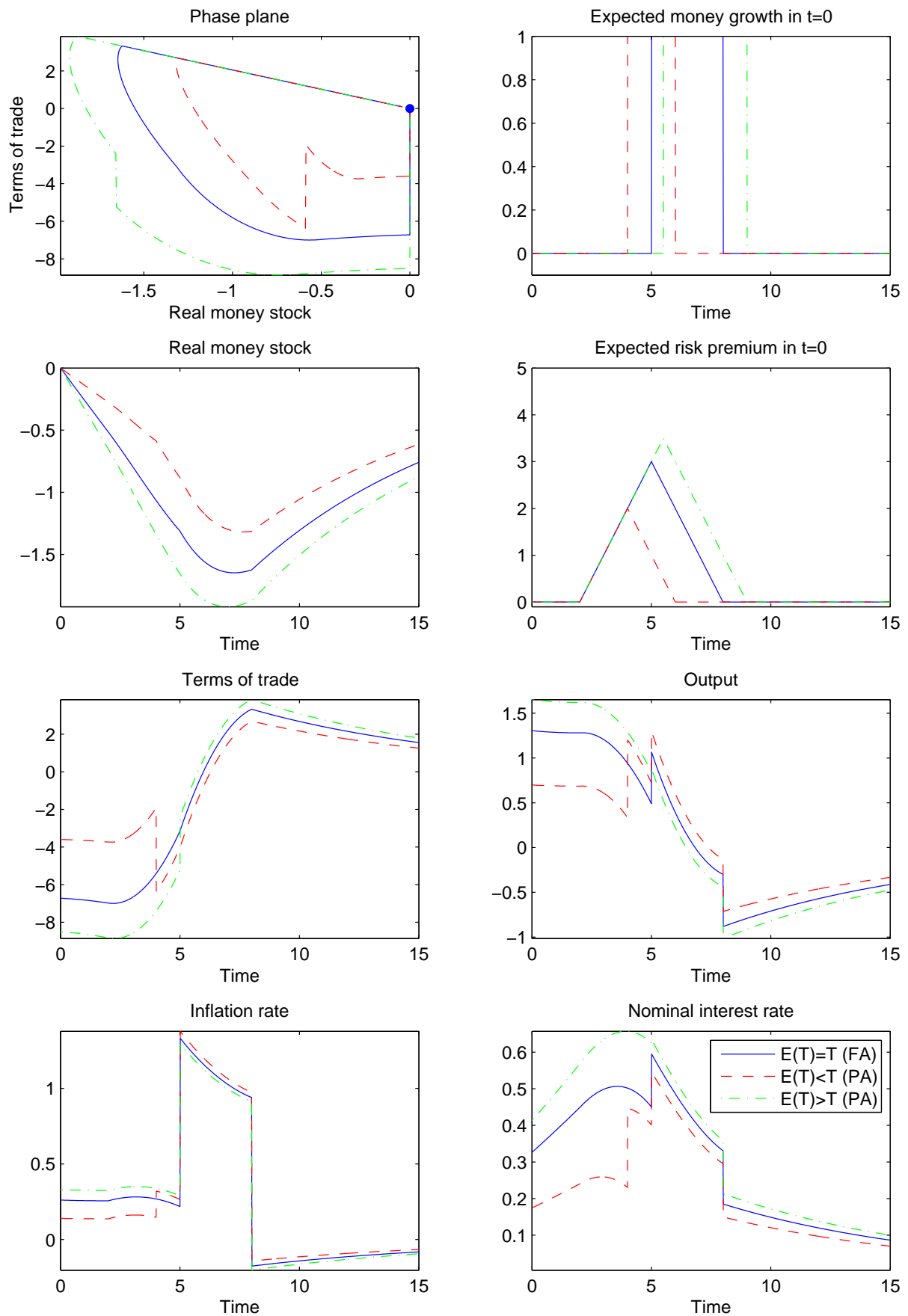


Figure 4.25: Relative volatility (simultaneous increase in the risk premium and the money growth rate)

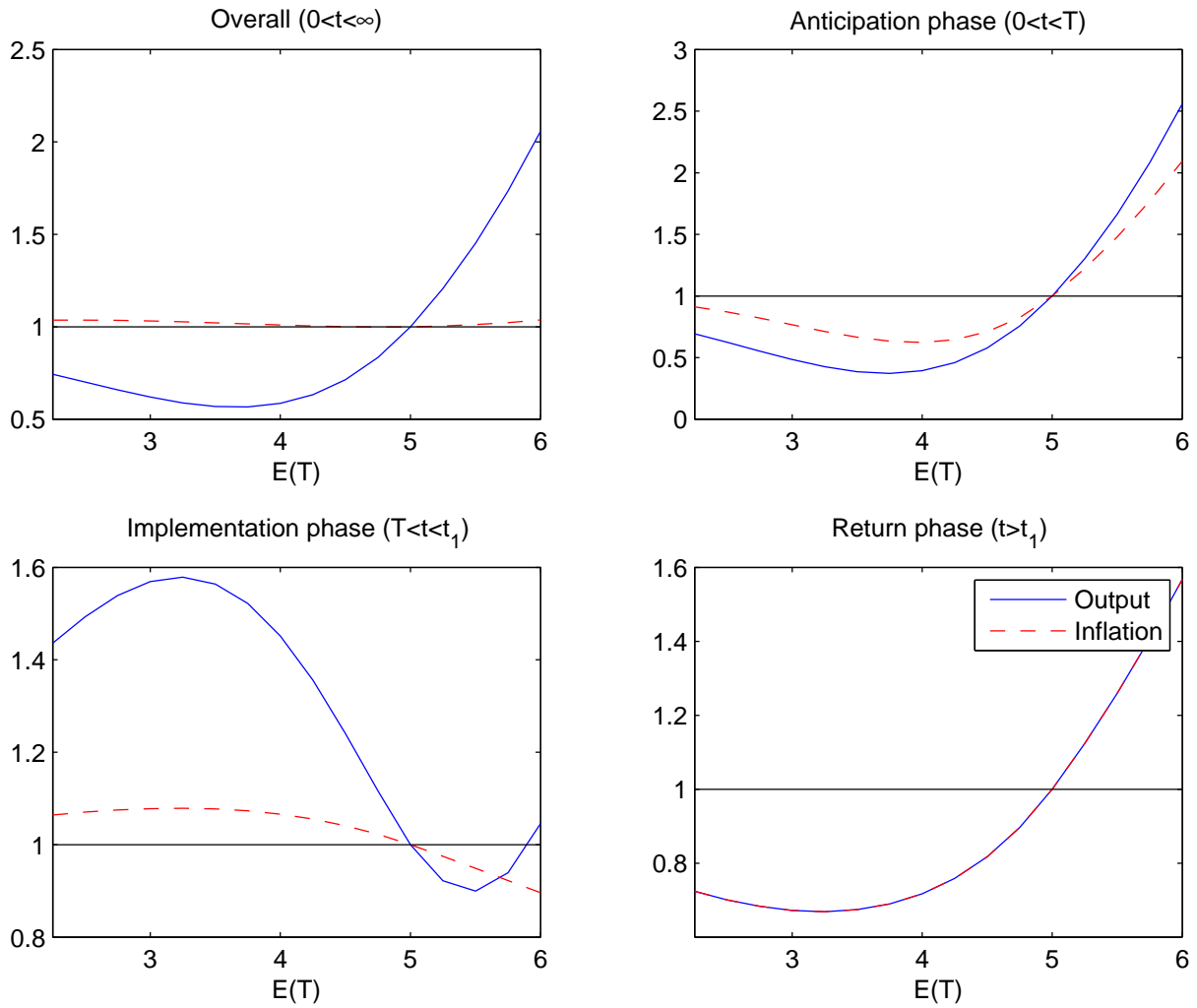
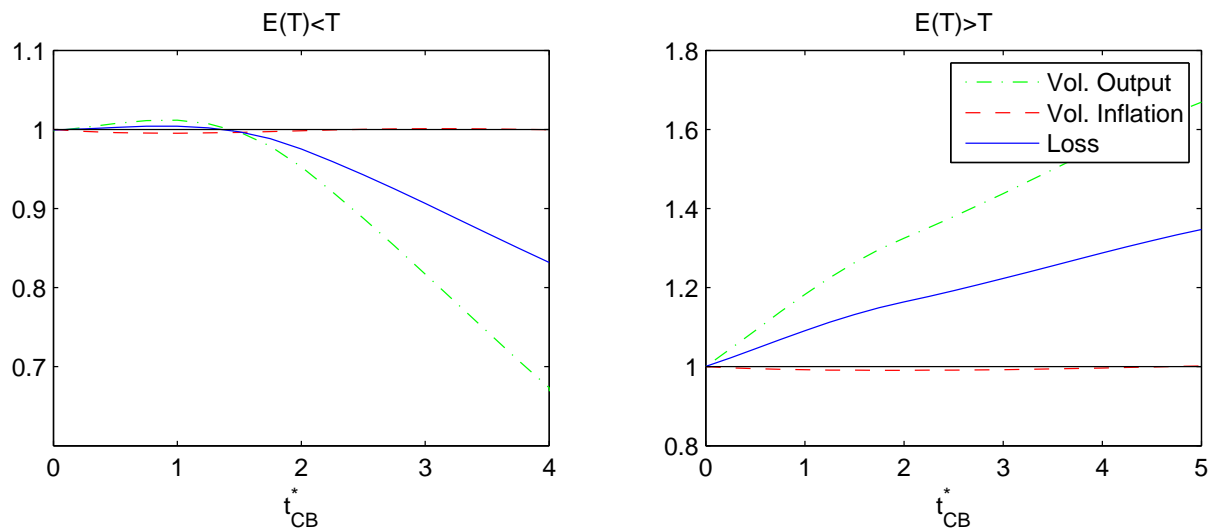


Figure 4.26: Optimal withholding strategy (simultaneous increase in the risk premium and the money growth rate)



Part II

Amplification effects in business cycles

Chapter 5

Money and credit in the New Keynesian model

Coauthored by: Hans-Werner Wohltmann

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Abstract

This paper integrates a money and credit market into a static approximation of the baseline New Keynesian model based on a money-and-credit-in-the-utility approach, in which real balances and borrowing contribute to the household's utility. In this framework, the central bank has no direct control over the interest rate on bonds. Instead, the central bank's instrument variables are the monetary base and the refinancing rate, i.e. the rate at which the central bank provides loans to the banking sector. Our approach gives rise to a credit channel, in which current and expected future interest rates on the bond and loan market directly affect current goods demand. The credit channel amplifies the output effects of isolated monetary disturbances. Taking changes in private (inflation and interest rate) expectations into account, we find that – contrarily to Bernanke and Blinder (1988) – the credit channel may also dampen the output effects of monetary disturbances. The expansionary effects of a monetary expansion may be substantially diminished if the monetary disturbance is accompanied by a contractionary credit shock. In a dynamic version of our model, in which expectations are formed endogenously, we find that the credit channel amplifies output responses.

JEL classification: A20, E51, E52

Keywords: Money, Loan, Money-and-credit-in-the-utility, Credit channel, New Keynesian model, Monetary policy

The full article can be downloaded via

<http://ideas.repec.org/a/lus/reveco/v65y2014i3p253-280.html>

Chapter 6

Endogenous firm entry in an estimated model of the U.S. business cycle

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Abstract

A recent theoretical literature highlights the role of endogenous firm entry as an internal amplification mechanism of business cycle fluctuations. The amplification mechanism works through the competition and the variety effect. This paper tests the significance of this amplification mechanism, quantifies its importance, and disentangles the competition and the variety effect. To this end, we estimate a medium-scale real business cycle model with firm entry for the U.S. economy. The competition and the variety effect are estimated to be statistically significant. Together, they amplify the volatility of output by 8.5 percent relative to a model in which both effects are switched off. The competition effect accounts for most amplification, whereas the variety effect only plays a minor role.

JEL classification: E20, E32

Keywords: Bayesian estimation; Business cycles, Competition effect, Entry, Mark-ups, Variety effect

6.1 Introduction

Standard dynamic stochastic general equilibrium (DSGE) models used for forecasting and policy analysis have a fixed range of products and firms and neglect the effect of firm entry on business cycle fluctuations.¹ Empirical evidence on firm and product entry, however, suggests otherwise. First, the number of firms varies substantially over the business cycle and is strongly procyclical. Second, the opening of establishments explains around 20 percent of quarterly job gains. Third, product creation (at new and existing firms) accounts for almost 50 percent of output in a 5 year interval.²

In light of these findings, a recent theoretical branch of the literature has started to study the role of endogenous firm (or product) entry in business cycle fluctuations.³ This literature identifies endogenous entry as an important amplification mechanism for business cycle fluctuations. This amplification mechanism works through two channels. The variety effect describes the productivity gains from additional varieties. An increase in the number of firms, equivalent to an increase in the number of varieties, increases output more than proportional due to increasing returns to specialization.⁴ The so-called competition effect captures the inverse relation between the number of producers and price mark-ups. An increase in the number of producers erodes market power. Price mark-ups fall which in turn boosts aggregate demand.

This paper aims to test the significance of this amplification mechanism, to quantify its importance, and to disentangle the competition and the variety effect in a medium-scale business cycle model. To this end, we use the firm entry model by Bilbiie et al. (2012), extend it with several real frictions, and estimate the model on U.S. data with Bayesian methods. We investigate the capacity of the model in fitting the data, study how firm entry affects the estimates of structural model parameters, and explore the amplification mechanism embedded in the firm entry model for transitory supply and demand shocks.

The model is characterized by sunk entry costs and a translog final goods production technology as proposed by Feenstra (2003).⁵ The number of firms is endogenously determined by a free entry condition that equates expected future profits with entry costs. Under the translog technology both the competition and the variety effect are present. The competition effect is demand-side driven and stems from the fact that the substitutability between different varieties, and hence the price elasticity of demand, is increasing in the

¹See e.g. Smets and Wouters (2007) and Christiano et al. (2005).

²The empirical evidence is based on US data. The procyclicality of firm entry is demonstrated by Chatterjee and Cooper (1993), Devereux et al. (1996), or Etro and Colcagio (2010). Davis et al. (1998) investigate the role of firm turnover for job flows. Bernard et al. (2010) compute the contribution of product creation for aggregate output.

³For a detailed overview on the existing literature, see section 5 in Bilbiie et al. (2012).

⁴Note that this effect is equivalent to the well-known love of variety effect, where households 'love' varieties and gain utility from an increasing set of consumption goods.

⁵Note that Bilbiie et al. (2012) aggregate products through the consumers' intratemporal optimization and therefore refer to a translog expenditure function instead. However, both concepts are equivalent.

number of varieties.

We find the competition effect and the variety effect to be statistically significant. To quantify the strength of the amplification mechanism and to disentangle the competition and the variety effect, we specify two counterfactual model frameworks with a constant elasticity of substitution (CES) production technology, where either the competition effect or both effects are switched off. We measure the amplification as the percentage volatility difference in GDP, consumption, and investment across the three model variants. By conducting counterfactual simulations, we find that the competition and the variety effect substantially amplify fluctuations in output and consumption, but dampen fluctuations in investment. For output, the total increase in volatility is given by 8.5 percent. The competition effect accounts for most of the amplification, amplifying output by 7 percent, whereas the increase through the variety effect only amounts to 1.5 percent. If we consider each structural shock in isolation, the results are mixed. On the one hand, the competition and the variety effect amplify the impacts of labor productivity and wage mark-up shocks on output. This follows from the fact that for these shocks, output and firm entry are positively correlated. On the other hand, the competition and the variety effect dampen the output effects of aggregate demand and investment-specific technology shocks, for which the conditional correlation between firm entry and output is negative.

The evidence on the cyclicity of mark-ups does not speak with a single voice. A large body of literature finds evidence for countercyclical mark-ups, for example, Bils (1987) and Rotemberg and Woodford (1999), while there is competing evidence of procyclicality, see Nekarda and Ramey (2013). Countercyclical responses of mark-ups to technology shocks and monetary policy shocks have recently been documented by Colcagio and Etro (2010) and Lewis and Poilly (2012). However, mark-ups are not directly unobservable. The literature uses relations from structural models to construct a measure of mark-ups from observable variables. In our framework, a fully model-consistent construction of a mark-up series is not possible. We therefore exclude a mark-up series from our baseline estimation and treat the mark-up as an unobserved state in our estimation procedure.⁶ The implied cyclicity of price mark-ups, using our estimates, is shock-dependent. Productivity and wage mark-up shocks entail a countercyclical response of mark-ups. In response to shocks to aggregate demand and to investment-specific technology, mark-ups behave procyclical.

This article is among the first attempts to bring a business cycle model with firm entry to the data. Lewis and Poilly (2012) study the role of firm entry for the monetary transmission mechanism by minimizing the distance between the impulse responses to a monetary policy shock generated by a sticky price entry model and those obtained from a VAR. Lewis and Stevens (2015) estimate – as we do – a business cycle with firm entry using Bayesian methods. However, they consider a monetary DSGE model and focus

⁶We show in robustness exercise, that all our results go through when we include a mark-up proxy in the estimation.

mainly on the role of firm entry for inflation dynamics. Our primary focus is on output dynamics and on the amplification mechanism embedded in the endogenous entry model.⁷

Closely related to our paper is the work of Jaimovich and Floetotto (2008). Based on a calibrated and – apart from firm entry – standard real business cycle model, they show that amplification effect associated with firm entry amplifies the impacts of technology shocks on output by 64 to 158 percent, depending on the exact specification of their model. This paper confirms the qualitative results in Jaimovich and Floetotto (2008) but finds the quantitative impact of firm entry to be less dominant. Our paper extends their work in a number of ways. First, we carry out a full-fledged estimation of a medium-scale real business cycle model, that already accounts for a large fraction of economic fluctuations. This approach enables us to extract the net amplification effect associated with firm entry. Moreover, we provide an estimate of the strength of the amplification mechanism. Second, we consider several shocks and demonstrate that the role of firm entry in aggregate fluctuations depends on the nature of the shock. Third, in the model of Jaimovich and Floetotto (2008), the variety effect is turned off, and the sole focus lies on the competition effect. The latter is supply-side driven and stems from the strategic interaction between oligopolistic firms.⁸ However, Lewis and Poilly (2012) find that a model with strategic interactions cannot generate an empirically relevant competition effect. Therefore, we consider a demand-side driven competition effect based on a translog production technology.

The remainder of the paper is organized as follows. Section 6.2 presents the model. Section 6.3 shows analytically the effects that the competition and the variety effects have on the model's dynamics. Section 6.4 describes the data and the estimation procedure. Section 6.5 discusses the estimation results. Section 6.6 quantifies the amplification mechanism. Section 6.7 tests the robustness of our results. Section 6.8 concludes.

6.2 The model

This section outlines our business cycle model for the U.S. economy. The core is a medium-scale real business cycle model which is characterized by monopolistic competition on product and labor markets, habit formation in consumption, investment adjustment costs, variable capital utilization, and non-separable preferences as proposed by Jaimovich and

⁷We consider a real model. While the transmission channels through which firm entry affects the model dynamics (the competition and the variety effect) would also be at work in a sticky-price framework, the quantitative results may change. We leave a quantitative evaluation within a sticky-price model for future work.

⁸Other studies that consider a supply-side driven competition effect in business cycle models are, for example, Colcagio and Etro (2010) and Etro and Colcagio (2010). In contrast to Jaimovich and Floetotto (2008), however, these studies do not provide a quantitative evaluation of the amplification mechanism.

Rebelo (2009).⁹ To this, we add the entry model proposed by Bilbiie et al. (2012) which features a variety effect and a demand-side driven competition effect stemming from a translog production technology of final goods producers.

The model economy consists of a government, a fixed mass of final goods producers, labor bundlers, households, and a time-varying and endogenously determined mass of intermediate goods producers. Households consume, invest in physical capital and in start-ups (or new firms), hold government bonds and equity of intermediate goods producers, and supply differentiated labor types to a labor bundler under monopolistically competitive conditions. Competitive labor bundlers aggregate the differentiated labor types into a homogeneous labor input. A time-varying mass of monopolistic firms employ labor and capital to produce differentiated intermediate goods. The creation of a new product variety – equivalent to the establishment of a new firm – requires labor input. The entry of firms into the intermediate goods market is endogenously determined by a free entry condition that equates expected future profits with entry costs. Final goods producers bundle the intermediate goods to a homogenous final good used for private and government consumption and for investment in physical capital. We specify a translog production function as in Feenstra (2003) to describe how intermediate goods are combined to produce final goods. This specification gives rise to countercyclical price mark-ups and increasing returns to specialization (or love of variety). In the following, we discuss the model in more detail.

6.2.1 Final goods producers

There is a mass N_t of monopolistically competitive firms, each producing a different variety of an intermediate good, indexed by $i \in [0, N_t]$. Final goods producers buy the differentiated intermediate goods or varieties $y_{i,t}$ at a price $p_{i,t}$, bundle them to a homogenous final good Y_t^C , and sell it to the households and to the government under perfectly competitive conditions at a price P_t . A final goods producer maximizes its profits $Y_t^C P_t - \int_0^{N_t} p_{i,t} y_{i,t} di$ subject to a final goods production function that is specified using the translog cost function as proposed by Feenstra (2003). The first-order condition for profit maximization yields the demand function for variety i , given by $y_{i,t} = \frac{\partial P_t}{\partial p_{i,t}} Y_t^C$.

In a symmetric equilibrium, all firms make identical choices: $y_{i,t} = y_t$, $p_{i,t} = p_t$ and $\rho_{i,t} = \rho_t$, where $\rho_{i,t} \equiv p_{i,t}/P_t$ is the relative price of variety i . The demand function for a single variety is then given by $y_t = (\rho_t N_t)^{-1} Y_t^C$ and the price index can be written as $P_t = \exp\left(\frac{(\tilde{N} - N_t)}{(2\tilde{\sigma}\tilde{N}N_t)}\right) p_t$, where \tilde{N} is the mass of potential entrants. The price elasticity of demand (or elasticity of substitution between different varieties), ε_t , is increasing in the number of varieties: $\varepsilon_t = 1 + \tilde{\sigma} N_t$ with $\tilde{\sigma} > 0$.¹⁰ The degree of increasing

⁹The structure of our core model is based on Schmitt-Grohé and Uribe (2012). They estimate the model to assess the contribution of news shocks to business cycle fluctuations.

¹⁰See appendix 6.A for the derivation of the price index P_t and the price elasticity ε_t . For convenience,

returns to specialization (the variety effect), is captured by the elasticity of the relative price with respect to the number of firms, which is given by $\omega_t = \frac{\partial p_t}{\partial N_t} \frac{N_t}{p_t} = (2\tilde{\sigma}N_t)^{-1}$.

6.2.2 Intermediate goods producers

Each intermediate good is produced by a monopolist $i \in [0, N_t]$ that uses the amount $l_{i,t}$ of labor, the amount $k_{i,t}^s$ of capital services, and the constant returns to scale technology

$$y_{i,t} = (z_t l_{i,t})^\alpha (k_{i,t}^s)^{1-\alpha} \quad (6.1)$$

to produce its output $y_{i,t}$. z_t is a labor productivity shifter, which follows the exogenous AR(1) process $\log z_t = (1 - \rho_z) \log z + \rho_z \log z_{t-1} + \varepsilon_t^z$, where z is the steady state of z_t , and ε_t^z is *i.i.d.* $N(0, \sigma_{\varepsilon^z}^2)$. $\alpha \in (0, 1)$ denotes the share of labor in production. The firm takes the real factor prices w_t and r_t^k as given. Firm i chooses prices $p_{i,t}$ and factor inputs to maximize real profits $d_{i,t} = \frac{p_{i,t}}{P_t} y_{i,t} - w_t l_{i,t} - r_t^k k_{i,t}^s$ subject to the production technology and the demand for its variety.

At the optimum, the firm sets its real price as a mark-up, μ_t^p , over real marginal costs, mc_t :

$$\frac{p_{i,t}}{P_t} = \mu_t^p mc_t, \quad (6.2)$$

where $\mu_t^p = \frac{\varepsilon_t}{\varepsilon_t - 1}$. Inserting $\varepsilon_t = 1 + \tilde{\sigma}N_t$ yields $\mu_t^p = 1 + \frac{1}{\tilde{\sigma}N_t}$, implying that the mark-up is decreasing in the number of goods. The competition effect is captured by the (negative) elasticity of the mark-up with respect to the number of goods (or firms), which is given by $\xi_t = -\frac{\partial \mu_t^p}{\partial N_t} \frac{N_t}{\mu_t^p} = (1 + \tilde{\sigma}N_t)^{-1}$.

The demands of firm i for hours and capital are given by

$$w_t = \alpha mc_t \frac{y_{i,t}}{l_{i,t}}, \quad (6.3)$$

$$r_t^k = (1 - \alpha) mc_t \frac{y_{i,t}}{k_{i,t}^s}. \quad (6.4)$$

In a symmetric equilibrium, the aggregate production of intermediated goods is given by $N_t y_t = (z_t L_t^C)^\alpha (K_t^s)^{1-\alpha}$, where $L_t^C = N_t l_t$ and $K_t^s = N_t k_t^s$. Total profits can be expressed as $N_t d_t = (1 - 1/\mu_t^p) Y_t^C$.

6.2.3 Labor bundlers

The economy is made up by a continuum of households, indexed by $j \in [0, 1]$. Each household is a monopolistic supplier of a differentiated labor type $L_{j,t}$. Analogously to final

we denote N_t in the following as the number of firms/varieties. Note, however, that $N_t \in \mathbb{R}$ is strictly speaking the mass of firms.

goods producers, labor bundlers combine the differentiated labor types to a homogenous labor input L_t , according to $L_t = \left(\int_0^1 L_{j,t}^{1/\mu_t^w} dj \right)^{\mu_t^w}$. The wage mark-up μ_t^w is assumed to follow the ARMA(1,1) process $\log \mu_t^w = (1 - \rho_\mu) \log \mu^w + \rho_\mu \log \mu_{t-1}^w + \varepsilon_t^\mu + \nu \varepsilon_{t-1}^\mu$, where μ^w is the steady state of μ_t^w , and ε_t^μ is *i.i.d.* $N(0, \sigma_{\varepsilon^\mu}^2)$.¹¹ Profit maximization by the perfectly competitive labor bundlers yields the labor demand function

$$L_{j,t} = \left(\frac{w_{j,t}}{w_t} \right)^{-\mu_t^w / (\mu_t^w - 1)} L_t, \quad (6.5)$$

where $w_t = \left(\int_0^1 w_{j,t}^{-1/(\mu_t^w - 1)} dj \right)^{-(\mu_t^w - 1)}$ is the real wage paid for the homogenous labor input, and $w_{j,t}$ is the (real) price of labor type j .

6.2.4 Households

Each household j maximizes the following lifetime utility function proposed by Jaimovich and Rebelo (2009):

$$E_0 \sum_{t=0}^{\infty} \beta^t \chi_t \log (C_{j,t} - bC_{j,t-1} - \psi L_{j,t}^\eta S_{j,t}), \quad (6.6)$$

where $C_{j,t}$ and $L_{j,t}$ denote consumption and hours worked, respectively. $\beta \in (0, 1)$ is the discount factor, $\psi > 0$ is a scale parameter, and $b \in [0, 1)$ measures the degree of (internal) habit formation. $\chi_t > 0$ is a preference shock and follows $\log \chi_t = \log \chi + \varepsilon_t^\chi$, where χ is the steady state of χ_t , and ε_t^χ is *i.i.d.* $N(0, \sigma_{\varepsilon^\chi}^2)$. S_t is a habit-adjusted weighted average of current and past consumption, which evolves over time according to

$$S_{j,t} = (C_{j,t} - bC_{j,t-1})^\gamma S_{j,t-1}^{1-\gamma}, \quad (6.7)$$

where $\gamma \in (0, 1]$ governs the wealth elasticity of labor supply and $\theta = \eta - 1$ is the Frisch elasticity of labor supply in the limiting case $\gamma = b = 0$.

The household's period-by-period budget constraint (in units of final goods) is given by

$$C_{j,t} + I_{j,t} + \frac{B_{j,t}}{R_t} + v_t x_{j,t} + \frac{f_{E,t}}{z_t} w_t N_{E,j,t} + T_t = w_{j,t} L_{j,t} + r_t^k K_t^s + B_{j,t-1} + (1 - \delta)(v_t + d_t) \left[x_{j,t-1} + \left(1 - \frac{\kappa_E}{2} \left(\frac{N_{E,j,t-1}}{N_{E,j,t-2}} - 1 \right)^2 \right) N_{E,j,t-1} \right]. \quad (6.8)$$

The household purchases the amount $C_{j,t}$ of final goods, pays lump-sum taxes T_t , buys risk-less government bonds $B_{j,t}$ at a price $1/R_t$, and buys equity of firms operating in the

¹¹The moving average term allows the wage mark-up shock to capture high frequency movements in the wage series, see Smets and Wouters (2007).

intermediate goods market $x_{j,t}$ at a price v_t . Each bond pays one unit of the final good one period later. Each unit of equity bought at period $t - 1$ pays a (real) profit equal to $(1 - \delta)d_t$ and is worth $(1 - \delta)v_t$, where $\delta \in (0, 1)$ denotes the exogenous exit rate of firms.

The household invests into new firms $N_{E,j,t}$. Setting up a new firm (or inventing a new product) requires $f_{E,t}/z_t$ units of the composite labor input, where $f_{E,t}$ represents an entry cost shock that follows the exogenous AR(1) process $\log f_{E,t} = (1 - \rho_{f_E}) \log f_E + \rho_{f_E} \log f_{E,t-1} + \varepsilon_t^{f_E}$, where f_E is the steady state of $f_{E,t}$, and $\varepsilon_t^{f_E}$ is *i.i.d.* $N(0, \sigma_{\varepsilon^{f_E}}^2)$. Consequently, household j spends $f_{E,t}/z_t \cdot w_t N_{E,j,t}$ on investment in new firms. We assume that it takes one period before newly established firms become operational.¹² During this period, new firms are hit by the exogenous exit shock δ . In addition, we follow Lewis (2009) and model an endogenous failure rate that is an increasing function of the change in firm entry. The payoff in period t from investing in new firms in period $t - 1$ is thus given by $(1 - \delta)(v_t + d_t) \left(1 - \frac{\kappa_E}{2} \left(\frac{N_{E,j,t-1}}{N_{E,j,t-2}} - 1\right)^2\right) N_{E,j,t-1}$. The parameter κ_E serves as the counterpart of the investment adjustment cost parameter κ_I , introduced below, at the firm entry margin.

Finally, the household invests the amount $I_{j,t}$ into physical capital $K_{j,t}$, which is assumed to be owned by households. Capital evolves according to the following law of motion

$$K_{j,t} = (1 - \delta^K(u_{j,t}))K_{j,t-1} + u_t^I \left[1 - \frac{\kappa_I}{2} \left(\frac{I_{j,t}}{I_{j,t-1}} - 1\right)^2\right] I_{j,t}, \quad (6.9)$$

where $\frac{\kappa_I}{2} (I_{j,t}/I_{j,t-1} - 1)^2$ represents investment adjustment costs, and $u_t^I > 0$ is an investment-specific technology shock that follows the exogenous AR(1) process $\log u_t^I = (1 - \rho_I) \log u^I + \rho_I \log u_{t-1}^I + \varepsilon_t^I$, where u^I is the steady state of u_t^I , and ε_t^I is *i.i.d.* $N(0, \sigma_{\varepsilon^I}^2)$. The household chooses the capital utilization rate $u_{j,t}$, which transforms physical capital into capital services $K_{j,t}^s$ according to $K_{j,t}^s = u_{j,t} K_{j,t}$. We assume that an increasing utilization of capital implies a higher depreciation rate $\delta^K(u_{j,t})$, specified as

$$\delta^K(u_{j,t}) = \delta_0 + \delta_1(u_{j,t} - 1) + \frac{\delta_2}{2}(u_{j,t} - 1)^2, \quad (6.10)$$

where δ_0 is the capital depreciation rate in a deterministic steady state in which capital utilization is set to unity. The elasticity of capital utilization with respect to the rental rate of capital is given by δ_1/δ_2 . Capital services $K_{j,t}^s$ are rented to intermediate goods firms at a rental rate r_t^k .

Household j chooses $\{C_{j,t}, w_{j,t}, S_{j,t}, I_{j,t}, N_{E,j,t}, u_{j,t}, K_{j,t}, x_{j,t}, B_{j,t}\}_{t=0}^{\infty}$ taking as given $\{w_t, r_t^k, R_t, v_t, d_t, L_t, T_t, z_t, f_{E,t}, u_t^I, \chi_t, \mu_t^w\}_{t=0}^{\infty}$ and the initial conditions $B_{-1}, K_{-1}, C_{-1}, I_{-1}, N_{E,-1}, S_{-1}$ so as to maximize (6.6) subject to (6.7), (6.8), (6.9), (6.10), and (6.5).

¹²Empirically, firm entry lags GDP. See, e.g. Devereux et al. (1996).

Since all households will choose in equilibrium the same wage and quantities, we can now assume symmetry and drop the index j . Let λ_t^C , $\lambda_t^C Q_t$, λ_t^S denote Lagrange multipliers for the budget constraint, the capital accumulation equation, and the definition of S_t , respectively. The first-order conditions read as follows:

$$\lambda_t^C = \beta R_t \mathbb{E}_t \{ \lambda_{t+1}^C \}, \quad (6.11)$$

$$\lambda_t^C Q_t = \beta \mathbb{E}_t \{ \lambda_{t+1}^C (r_{t+1}^K u_{t+1} + Q_{t+1} (1 - \delta^K (u_{t+1}))) \}, \quad (6.12)$$

$$\lambda_t^C v_t = (1 - \delta) \beta \mathbb{E}_t \{ \lambda_{t+1}^C (v_{t+1} + d_{t+1}) \}, \quad (6.13)$$

$$\lambda_t^C = \left(\chi_t V_t - \gamma \lambda_t^S \frac{S_t}{C_t - bC_{t-1}} \right) - \beta b \mathbb{E}_t \left\{ \chi_{t+1} V_{t+1} - \gamma \lambda_{t+1}^S \frac{S_{t+1}}{C_{t+1} - bC_t} \right\}, \quad (6.14)$$

$$\lambda_t^S = \chi_t V_t \psi L_t^\eta + \beta (1 - \gamma) \mathbb{E}_t \left\{ \lambda_{t+1}^S \frac{S_{t+1}}{S_t} \right\}, \quad (6.15)$$

$$1 = Q_t u_t^I \left(1 - \frac{\kappa_I}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa_I \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right) + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^C}{\lambda_t^C} Q_{t+1} u_{t+1}^I \kappa \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right\}, \quad (6.16)$$

$$\frac{w_t}{z_t} f_{E,t} = v_t \left(1 - \frac{\kappa_E}{2} \left(\frac{N_{E,t}}{N_{E,t-1}} - 1 \right)^2 - \kappa_E \left(\frac{N_{E,t}}{N_{E,t-1}} - 1 \right) \frac{N_{E,t}}{N_{E,t-1}} \right) + \beta \mathbb{E}_t \left\{ \frac{\lambda_{t+1}^C}{\lambda_t^C} v_{t+1} \kappa_E \left(\frac{N_{E,t+1}}{N_{E,t}} - 1 \right) \left(\frac{N_{E,t+1}}{N_{E,t}} \right)^2 \right\}, \quad (6.17)$$

$$r_t^k = Q_t (\delta_1 + \delta_2 (u_t - 1)), \quad (6.18)$$

$$\lambda_t^C w_t = \mu_t^w \chi_t V_t \psi \eta L_t^{\eta-1} S_t, \quad (6.19)$$

where $V_t = (C_t - bC_{t-1} - \psi L_t^\eta S_t)^{-1}$.

6.2.5 Aggregate accounting and data consistency

The aggregate resource constraint

$$Y_t^C + \frac{w_t}{z_t} f_{E,t} N_{E,t} = w_t L_t + N_t d_t + r_t^k u_t K_{t-1} \quad (6.20)$$

can be obtained by combining the aggregate budget constraint of households (using $x_t = N_t$) with the government budget constraint $G_t + B_{t-1} = T_t + \frac{B_t}{R_t}$. Government consumption G_t is described by the exogenous AR(1) process $\log G_t = (1 - \rho_g) \log G + \rho_g \log G_{t-1} + \varepsilon_t^G$, where G is the steady state of G_t , and ε_t^G is *i.i.d.* $N(0, \sigma_{\varepsilon^G}^2)$.

The goods market clearing condition requires aggregate output of final goods Y_t^C to be equal to private and government consumption plus investment in physical capital, i.e. $Y_t^C = C_t + G_t + I_t$. Total investment TI_t is the sum of investment in physical capital and investment in new firms, i.e. $TI_t = I_t + \frac{w_t}{z_t} f_{E,t} N_{E,t}$. The gross domestic product Y_t is

equal to Y_t^C plus investment in new firms $\frac{w_t}{z_t} f_{E,t} N_{E,t}$.

The law of motion of the total mass of firms is given by

$$N_t = (1 - \delta)N_{t-1} + (1 - \delta) \left[1 - \frac{\kappa_E}{2} \left(\frac{N_{E,t-1}}{N_{E,t-2}} - 1 \right)^2 \right] N_{E,t-1}. \quad (6.21)$$

Every period, a fraction of incumbent firms exits the market, where the exit rate δ is assumed to be constant and exogenous. In contrast, the exit rate of newly established firms consists of the exogenous component δ and the endogenous component $\frac{\kappa_E}{2} \left(\frac{N_{E,t-1}}{N_{E,t-2}} - 1 \right)^2$.

As pointed out by Ghironi and Melitz (2005), empirical measures for the price index are closer to the product price p_t than to the price index P_t for the following two reasons: First, these empirical measures do not update their product space frequently enough to fully account for changes in the number of available products. Second, the construction of these measures is likely not of the functional form present in the translog model. In order to obtain data-consistent real model variables, we thus divide the real model variables by the relative price $\rho_t = p_t/P_t$. Data-consistent real variables are denoted by a superscript r .

6.2.6 Two alternative model specifications

The above introduced model framework builds on a translog production function as in Feenstra (2003). We denote this model framework as translog model. In the translog model, the competition effect ξ and the variety effect ω , evaluated at the deterministic steady state, can be expressed in terms of the steady-state price mark-up: $\xi = 1 - \frac{1}{\mu^p}$ and $\omega = \frac{1}{2}(\mu^p - 1)$.¹³

In the following, we introduce two alternative model specifications that use a constant elasticity of substitution (CES) production function for final goods as in Benassy (1996) which is characterized by constant price mark-ups (and therefore does not feature a competition effect, $\xi = 0$) and that allows to freely parameterize the variety effect.¹⁴ In the first CES specification, the variety effect is set equal to the value under the translog specification, i.e. $\omega = \frac{1}{2}(\mu^p - 1)$. We denote this the CES-TrVE model. In the second CES specification, the variety effect is set equal to zero, i.e. $\omega = 0$. We denote this the CES-NoVE model. Table 6.1 summarizes the competition and the variety effect in all three model variants, in each case evaluated at the deterministic steady state. In all other respects, the models are identical.

In the following, we estimate the competition and the variety effect. To this end, we log-linearize the translog model around its deterministic steady state and bring the

¹³Recall that, in the steady state of the translog model, $\xi = \frac{1}{1+\bar{\sigma}N}$, $\omega = \frac{1}{2\bar{\sigma}N}$, and the price mark-up is given by $\mu^p = 1 + \frac{1}{\bar{\sigma}N}$.

¹⁴See appendix 6.A for the model equations and derivations of the final goods production sector under the CES specifications.

Table 6.1: Characteristics of model variants

Model	Competition effect $\xi = -\frac{\partial \mu^p / \mu^p}{\partial N / N}$	Variety effect $\omega = \frac{\partial \rho / \rho}{\partial N / N}$
Translog	$\xi = 1 - \frac{1}{\mu^p}$	$\omega = \frac{1}{2}(\mu^p - 1)$
CES-TrVE	$\xi = 0$	$\omega = \frac{1}{2}(\mu^p - 1)$
CES-NoVE	$\xi = 0$	$\omega = 0$

linearized model to the data.¹⁵ The two (log-linearized) CES model variants will later enable us to quantify the importance of the competition and the variety effect in amplifying business cycle fluctuations and to disentangle competition and variety effect with the help of counterfactual simulations.

6.3 Inspecting the amplification mechanism in a simplified model

Before moving to the estimation of our model and the quantification of the competition and the variety effect in terms of amplifying business cycle fluctuations, it is instructive to examine the analytics and the intuition of the amplification mechanism in our model. To this end, we consider a simplified version of our baseline model which allows us to provide analytical results. For illustration purposes, we limit our analytical analysis to labor productivity shocks. All remaining shocks are switched off, i.e., $\chi_t = f_{E,t} = u_t^I = 1$, $G_t = 0$, $\mu_t^w = \mu^w$.

The simplified model assumes instantaneous entry, full depreciation of firms each period ($\delta = 1$) and the absence of entry adjustment costs ($\kappa_E = 0$). This implies that the number of entrants is identical to the number of firms, $N_{E,t} = N_t$, and that the value of a firm equals firm's profits, $v_t = d_t$. We abstract from capital, capital investment and a varying degree of capital utilization. Setting $\alpha = 1$, the aggregate production of intermediate goods simplifies to $N_t y_t = z_t L_t^c$. Inserting the demand function for a single variety $y_t = (\rho_t N_t)^{-1} Y_t^c$, yields the aggregate production function $Y_t^c = z_t \rho_t L_t^c$. If we further abstract from government spending ($G_t = 0$), aggregate demand Y_t^c coincides with private consumption, $Y_t^c = C_t$. The aggregate resource constraint then simplifies to $C_t = w_t L_t$, where we have used the free entry condition $w_t / z_t = v_t$ together with $v_t = d_t$ and $N_t = N_{E,t}$. GDP is the sum of labor and profit income, $Y_t = w_t L_t + d_t N_t$. The simplified model further assumes no habit formation in consumption ($b = 0$) and the limiting case of GHH preferences, $\gamma = 0$, which implies a labor supply equation of the form $w_t = \mu^w \psi \eta L_t^{\eta-1}$. We proceed by log-linearizing the equilibrium conditions of the simplified model. A hatted variable denotes percentage deviations from the steady state.

¹⁵The log-linearized model equations are summarized in appendix 6.A.

Table 6.2: Linearized model equations in simplified model variant

$\hat{w}_t = (\eta - 1)\hat{L}_t$	Labor supply
$\hat{w}_t = \hat{z}_t + \hat{\rho}_t - \hat{\mu}_t$	Labor demand
$\hat{\mu}_t^p = -\xi\hat{N}_t$	Competition effect
$\hat{\rho}_t = \omega\hat{N}_t$	Variety effect
$\hat{C}_t = \hat{w}_t + \hat{L}_t$	Aggregate resource constraint
$\hat{d}_t = \hat{w}_t - \hat{z}_t$	Aggregate free entry condition
$\hat{d}_t + \hat{N}_t = \frac{1}{\mu^p - 1}\hat{\mu}_t^p + \hat{C}_t$	Aggregate profits
$\hat{Y}_t = \frac{\mu^p}{2\mu^p - 1}(\hat{w}_t + \hat{L}_t) + \frac{\mu^p - 1}{2\mu^p - 1}(\hat{d}_t + \hat{N}_t)$	GDP

Table 6.2 summarizes the model equations that jointly determine \hat{C}_t , \hat{Y}_t , \hat{w}_t , \hat{L}_t , \hat{N}_t , $\hat{\rho}_t$, $\hat{\mu}_t^p$, and \hat{d}_t , given \hat{z}_t .

We now provide an analysis of the effects the competition and the variety on the dynamics of employment, consumption and output after a rise in labor productivity \hat{z}_t . Combining labor supply with labor demand to substitute out the real wage, replacing $\hat{\mu}_t^p$ from the definition of the competition effect and $\hat{\rho}_t$ from the definition of the variety effects yields the following equation for aggregate employment:

$$\hat{L}_t = \frac{1}{\eta - 1}(\xi + \omega)\hat{N}_t + \frac{1}{\eta - 1}\hat{z}_t. \quad (6.22)$$

For a given z and since $\eta > 1$, a rise in the number of firms shifts up the labor demand schedule and raises aggregate employment if the competition effect and/or the variety effect are present ($\xi > 0$ and/or $\omega > 0$).

By inserting labor supply into the aggregate resource constraint and by replacing employment \hat{L}_t with equation (6.22), we obtain consumption as a function of the numbers of the firms and shocks to productivity: $\hat{C}_t = (\xi + \omega)\eta/(\eta - 1)\hat{N}_t + \eta/(\eta - 1)\hat{z}_t$. The data-consistent counterpart $\hat{C}_t^r = \hat{C}_t - \hat{\rho}_t$ is given by

$$\hat{C}_t^r = \frac{1}{\eta - 1}(\xi + \eta\omega)\hat{N}_t + \frac{\eta}{\eta - 1}\hat{z}_t. \quad (6.23)$$

Combining the definition of GDP with the aggregate resource constraint and the equation for aggregate profits, replacing $\hat{\mu}_t^p$ from the definition of the competition effect, and inserting the equation for \hat{C}_t from above yields $\hat{Y}_t = \omega\eta/(\eta - 1)\hat{N}_t + (2\eta(\mu^p - 1) + 1)/((\eta - 1)(2\mu^p - 1))\xi\hat{N}_t + \eta/(\eta - 1)\hat{z}_t$, or in data-consistent terms

$$\hat{Y}_t^r = \frac{1}{\eta - 1} \cdot \omega \cdot \hat{N}_t + \frac{2\eta(\mu^p - 1) + 1}{(\eta - 1)(2\mu^p - 1)} \cdot \xi \cdot \hat{N}_t + \frac{\eta}{\eta - 1}\hat{z}_t, \quad (6.24)$$

where $(2\eta(\mu^p - 1) + 1)/((\eta - 1)(2\mu^p - 1)) > 0$ since $\mu^p \geq 1$.

The amplification mechanism is evident from equations (6.22), (6.23), and (6.24). To see this, consider a positive innovation to labor productivity, i.e. $\hat{z}_t > 0$, and suppose that the number of firms increases after a rise in productivity, i.e. $\partial \hat{N}_t / \partial \hat{z}_t > 0$. If this happens, the effects of a rise in labor productivity on employment, consumption and output are amplified in the presence of the competition and/or the variety effect, i.e. if $\xi > 0$ and/or $\omega > 0$. The stronger the competition and/or the variety effect, the more does economic activity rise after a positive innovation to productivity. The rationale is that in the presence of the variety effect, an increase in the number of firms increases output more than proportional due to increasing returns to specialization. In the presence of the competition effect, an increase in the number of producers erodes market power. Price mark-ups fall which in turn boosts aggregate demand.

Notice, though, that the response of the number of firms to a labor productivity shock is ambiguous. If the number of firms drops, the effects of the productivity shock are dampened. Whether the number of firms rises or drops in response to a technology improvement is obviously an empirical question that we will address in the next section using our estimated baseline model. Moreover, our estimates will reveal whether ξ and ω are significantly different from zero and will allow us to quantify the strength of the competition and the variety effect in amplifying or attenuating shocks to the economy.

6.4 Data and estimation procedure

This section describes the data set and the estimation procedure we use to estimate the translog model. Following An and Schorfheide (2007) and Smets and Wouters (2007), we estimate a subset of the model parameters using Bayesian techniques. For the estimation, we use seven time series of U.S. quarterly data: the growth rates of real per capita GDP, consumption, and investment, the logarithm of per capita hours worked, the growth rates of two measures of real wages, and the growth rate of per capita new firms.

As empirical measure for firm entry, we use the data series of new business incorporations (NBI) from the Survey of Current Businesses published by the Bureau of Economic Analysis.¹⁶ We define consumption as consumption expenditures on non-durables and services and investment as the sum of consumption expenditures on durables, fixed private investments, and changes in private inventories. Following Justiniano et al. (2013) and Galí et al. (2012), we use two empirical wage measures: hourly compensation in the non-farm business sector and average hourly earnings of production and non-supervisory

¹⁶Alternatively, one can use the data series net business formation (NBF) published in the same survey as measure for net firm entry. We decided to use the NBI measure for two reasons: First, data on NBF is only available until 1995:Q3, whereas NBI is published until 1998:Q3. Second, we do not model firm exit endogenously. We, therefore, believe that NBI is a closer measure for firm entry than NBF for net firm entry.

employees. Both measures are taken from the Bureau of Labor Statistics. A full description of the data sources and the construction of the data series can be found in appendix 6.B. The data sample starts in 1964:Q1 and ends in 2012:Q2. Due to limited data availability, the new firm series ends in 1998:Q3.¹⁷

The measurement equations for GDP, consumption, investment, hours worked, and entry then read as follows:¹⁸

$$\begin{pmatrix} dl(\text{GDP}_t) \\ dl(\text{CONS}_t) \\ dl(\text{INV}_t) \\ l(\text{HOURS}_t) \\ dl(\text{ENTRY}_t) \end{pmatrix} = \begin{pmatrix} \Delta \hat{Y}_t^r \\ \Delta \hat{C}_t^r \\ \Delta \hat{T}I_t^r \\ \hat{L}_t \\ \Delta \hat{N}_{E,t} \end{pmatrix} \times 100$$

The functions l and dl stand for 100 times the demeaned logarithm and the demeaned log-difference, respectively. A hat denotes log-deviations from the steady state and Δ is the time-difference operator.

To include the information of both wage measures, we set up the following measurement equation:

$$\begin{pmatrix} dl\text{WAGE1}_t \\ dl\text{WAGE2}_t \end{pmatrix} = \begin{pmatrix} 1 \\ \lambda \end{pmatrix} \Delta \hat{w}_t^r + \begin{pmatrix} \varepsilon_t^{w1,me} \\ \varepsilon_t^{w2,me} \end{pmatrix},$$

where λ denotes the loading coefficient for the second wage series. Since both loadings are not separately identified, we set the first loading coefficient to unity. $\varepsilon_t^{w1,me}$ and $\varepsilon_t^{w2,me}$ are two measurement errors, which are $i.i.d.N(0, \sigma_{\varepsilon^{w1,me}}^2)$ and $i.i.d.N(0, \sigma_{\varepsilon^{w2,me}}^2)$, respectively. $\Delta \hat{w}_t^r$ can be interpreted as latent factor, which captures the common movement in both wage series. The two error terms capture the idiosyncratic fluctuations in the wage series.¹⁹

The application of seven data series requires at least seven exogenous disturbances. In total, the model is governed by eight disturbances, including innovations to government consumption ε_t^g , to labor productivity ε_t^z , to investment-specific technology ε_t^I , to entry costs ε_t^{fE} , to preferences ε_t^χ , and to the wage mark-up $\varepsilon_t^{\mu w}$, plus the two measurement errors $\varepsilon_t^{w1,me}$ and $\varepsilon_t^{w2,me}$.

A subset of parameters is calibrated as summarized in table 6.3. The discount rate β is set to 0.99, implying an annual steady-state interest rate of approximately 4 percent.

¹⁷Note that the missing observations of the new firm series are treated as an unobserved state during the Kalman filter routine. We show in a robustness exercise in section 6.7 that our estimation results do not change substantially if we limit our data sample to 1998:Q3.

¹⁸Note that we use the data-consistent measures of real variables to map the data with the model, see also section 6.2.5.

¹⁹The concept to capture the common movement of multiple time series in a few latent variables originally comes from the factor analysis. For a general discussion of estimating DSGE models in a date-rich environment, see Boivin and Giannoni (2006).

Table 6.3: Calibrated parameters

Parameter	Value	Definition
β	0.99	Discount factor
δ_0	0.025	Steady-state capital depreciation rate
δ	0.025	Steady-state firm exit rate
u	1	Steady-state capacity utilization rate
μ^w	1.2	Steady-state wage mark-up
G/Y	0.18	Steady-state ratio of government consumption to GDP
L	0.25	Steady-state of hours worked
\tilde{N}	10^9	Potential mass of firms

The steady-state value for the utilization rate u is set to unity, implying the steady-state value of the depreciation rate δ^K to be equal to δ_0 . For the latter, we choose a standard value of 0.025. Following Chugh and Ghironi (2012), the potential mass of firms in the economy \tilde{N} is assumed to be 10^9 . The exogenous firm exit rate δ is set to 0.025, as in Bilbiie et al. (2012). The steady-state values G/Y and L are set to 0.18 and 0.25, respectively. Following Galí et al. (2012), we set the steady-state wage mark-up μ^w at 20 percent.

The remaining parameters are estimated. Table 6.4 summarizes the prior distributions. Our choice of distributions is in line with the literature and mainly results from different distributional supports. The probability mass of the inverse gamma distribution is distributed over the interval $(0, \infty)$, the gamma distribution over the interval $[0, \infty)$, the beta distribution over the interval $[0, 1]$, and the normal distribution over the interval $(-\infty, \infty)$.

The standard deviation of the innovations are assumed to follow an inverse gamma distribution with mean 0.02 and standard deviation 1. For the autocorrelation parameters of the exogenous shock processes, we choose a beta distribution with mean 0.5 and standard deviation 0.2. The moving average coefficient of the wage mark-up shock is assumed to follow a normal distribution with mean zero and standard deviation 0.2.

The prior distributions for the structural parameters related to firm entry are given as follows. For the steady-state price mark-up μ^p , we use a truncated gamma distribution, where we only allow for values greater than 1.01. It has mean 1.3 and standard deviation 0.2. The 90 percent probability interval of this distribution then ranges from 1.04 to 1.68. For the entry adjustment cost parameter κ_E , we use the same prior distribution as for the investment adjustment cost parameter, i.e. a gamma distribution with mean 4.0 and standard deviation 1.0. For the prior distribution of the remaining structural parameters, we broadly follow the existing literature.

6.5 Estimation results

In this section, we first present the parameter estimates. Next, we discuss the model's predictions regarding volatility, autocorrelations, and cross-correlations of the time series included as observables. Then we discuss the contribution of each of the structural shocks to the forecast error variance of the endogenous variables at business cycle frequency. Finally, we analyze the model-implied impulse responses to the structural shocks.

6.5.1 Parameter estimates

Table 6.4 displays the estimated parameters as means of the posterior distribution and the 90 percent probability intervals obtained by the Metropolis-Hastings algorithm.

To start with, we focus on the parameters related to the entry mechanism. The gross price mark-up μ^p is estimated to be significantly different from one. The point estimate implies a steady-state mark-up of 26 percent with a probability band ranging from 16 percent to 36 percent. The point estimate of μ^p is close to the value of 1.22 reported in Lewis and Stevens (2015). However, it is significantly smaller than the value of 1.66 reported in Lewis and Poilly (2012). Regarding the competition and the variety effect, the point estimate of μ^p implies that a one percent increase in the mass of firms lowers the price mark-up by 0.21 percent (the competition effect ξ) and raises the relative price by 0.13 percent (the variety effect ω). Competition and variety effect are statistically significantly different from zero with confidence bands for ξ and ω ranging from 1.14 to 1.26 and 0.08 to 0.18, respectively.

Entry adjustment costs κ_E are estimated to be 1.5 with a probability interval ranging from 1.09 to 1.99. This is significantly lower than the 3.82 point estimate for the investment adjustment cost parameter κ_I . As discussed below, the model overestimates both the volatility of firm entry and its first-order autocorrelation. Higher entry adjustment costs would help to bring the model closer to the empirical standard deviation of firm entry but only at the cost of an even higher autocorrelation.

Turning to the other structural parameters, θ , which determines the labor supply elasticity, is estimated at 3.64 with a relatively wide probability interval ranging from 1.92 to 5.80. The point estimate of the wealth elasticity of labor supply γ is 0.79 with a probability interval ranging from 0.63 to 0.93, implying that preferences are close to those in King et al. (1988). This is in contrast to the results of Schmitt-Grohé and Uribe (2012), who estimate a near-zero wealth elasticity of labor supply in a theoretical environment that abstracts from endogenous firm entry and in which shocks feature an anticipated component. The labor share in production α is estimated at 0.85 with a probability interval ranging from 0.81 to 0.90. Notice that this estimate is not comparable to the estimates of standard DSGE models without firm entry since in our model, in which labor is utilized in the manufacturing sector and in the creation of new products, α is not

Table 6.4: Results from the Bayesian estimation including prior distribution and probability intervals

Parameter	Prior distribution			Posterior distribution	
	Type	Mean	Std	Mean	[5% , 95%]
<i>Structural parameters</i>					
Labor share in production	α	Beta	0.7	0.2	0.85 [0.81 , 0.90]
Labor utility	θ	Gamma	2.0	1.0	3.64 [1.92 , 5.80]
Wealth elast. labor supply	γ	Beta	0.5	0.2	0.79 [0.63 , 0.93]
Consumption habit	b	Beta	0.5	0.2	0.76 [0.71 , 0.80]
Investment adj. cost	κ_I	Gamma	4.0	1.0	3.82 [2.59 , 5.28]
Inv. elast. of capital util.	$\frac{\delta_2}{\delta_1}$	Igamma	1.0	1.0	0.55 [0.33 , 0.89]
Price mark-up	μ^p	Gamma	1.3	0.2	1.26 [1.16 , 1.36]
Entry adj. cost	κ_E	Gamma	4.0	1.0	1.50 [1.09 , 1.99]
<i>Autocorrelation of shock processes</i>					
Labor productivity	ρ_z	Beta	0.5	0.2	0.96 [0.94 , 0.98]
Wage mark-up	ρ_μ	Beta	0.5	0.2	0.97 [0.95 , 0.98]
Invest. spec. tech.	ρ_I	Beta	0.5	0.2	0.24 [0.12 , 0.38]
Gov. spending	ρ_G	Beta	0.5	0.2	0.92 [0.88 , 0.94]
Entry cost	ρ_{fE}	Beta	0.5	0.2	0.95 [0.91 , 0.98]
<i>Standard deviation of innovations</i>					
Labor prod.	σ_{ε^z}	Igamma	2.0	1	0.80 [0.70 , 0.90]
Wage mark-up	σ_{ε^μ}	Igamma	2.0	1	4.30 [3.00 , 5.95]
Invest. spec. tech.	σ_{ε^I}	Igamma	2.0	1	3.99 [2.39 , 6.54]
Preference	$\sigma_{\varepsilon^\chi}$	Igamma	2.0	1	1.46 [1.07 , 1.91]
Gov. spending	σ_{ε^G}	Igamma	2.0	1	1.87 [1.71 , 2.03]
Entry cost	$\sigma_{\varepsilon^{fE}}$	Igamma	2.0	1	2.43 [1.96 , 2.97]
<i>Moving average parameter and loading coefficient</i>					
Wage mark-up shock	ν	Normal	0.0	0.2	0.41 [0.26 , 0.56]
Loading coefficient	λ	Normal	1.0	2.0	0.13 [0.06 , 0.19]

Note: Using a Random Walk Metropolis Hastings algorithm, we generate 2 chains of 2 Mio. parameter draws each. For each chain, we discard the first 1 Mio. draws and use the remaining draws to compute the posterior mean and percentiles.

equal to the labor share in GDP. For the latter, our point estimate implies a standard value of 72 percent.

Regarding the exogenous shock processes, we find that shocks to labor productivity, to wage mark-ups, to government spending, and to entry costs are estimated to be highly persistent with AR(1) coefficients all above 0.9. In contrast, the persistence parameter of the investment-specific technology shock is relatively low with a value of 0.24, implying that the investment-specific technology shock explains less forecast-error variance at higher forecast-horizons.

In the following, we compute the model's predictions about the second moments, variance decomposition, impulse responses, and amplification measures at the posterior mean reported in table 6.4. To compute the corresponding probability bands, we use the last 50,000 parameter draws of the Metropolis-Hastings algorithm and compute the respective moments for each draw.

6.5.2 Second moments and variance decomposition

In order to assess the performance of the model in fitting the data, we compare the model-implied second moments to the corresponding empirical moments of the data. Table 6.5 reports standard deviations, relative standard deviations, first-order autocorrelations, and contemporaneous correlations with the growth rate of GDP. The model-implied moments are derived from simulated data, where the measurement errors are turned off during the simulation.

The empirical moments of GDP, consumption, total investment, and hours worked are matched quite well. Concerning firm entry growth, the estimated model captures the observed procyclicality of firm entry. However, the model overstates its volatility and its serial correlation. This is attributable to the fact that we model an endogenous failure rate of firms as an increasing function of the change in firm entry. This mechanism is introduced to dampen the volatility of firm entry. However, it also generates substantial persistence in firm entry. As mentioned above, this trade-off between volatility and autocorrelation explains the small point estimate of the entry adjustment cost parameter κ_E , compared to the capital adjustment cost parameter κ_I .²⁰

Table 6.6 shows the mean forecast-error variance decomposition of GDP, consumption, total investment, hours worked, wages, and firm entry at business cycle frequencies ranging from 6 to 32 quarters. Most variations are explained by labor productivity and wage mark-up shocks. Together they account for more than 75 percent of the variations in GDP, consumption, total investment, hours worked, and wages. The investment-specific technology shock explains 21 percent of the variations in total investment. The firm entry

²⁰A re-estimated model using an AR(2) process for the entry cost shock performs better in fitting the persistence of firm entry growth. However, the overestimation of the entry growth volatility is even stronger under this specification.

Table 6.5: Second moments

	X	Std (σ_X)			1st-order autocorr.		
		Data	Model		Data	Model	
			Mean	[5% , 95%]		Mean	[5% , 95%]
GDP growth	$\Delta\hat{Y}^r$	0.86	0.85	[0.80 , 0.89]	0.31	0.31	[0.27 , 0.37]
Consumption growth	$\Delta\hat{C}^r$	0.55	0.59	[0.54 , 0.63]	0.42	0.39	[0.32 , 0.46]
Investment growth	$\Delta\hat{I}^r$	3.27	3.43	[3.11 , 3.56]	0.31	0.26	[0.21 , 0.35]
Hours worked	\hat{L}	5.05	4.16	[3.40 , 5.59]	0.98	0.98	[0.97 , 0.99]
Wage growth (1st series)	$\Delta\hat{w}^r$	0.61	0.69	[0.64 , 0.75]	0.06	0.11	[0.07 , 0.15]
Wage growth (2nd series)		0.36			0.56		
Firm Entry growth	$\Delta\hat{N}_E$	3.10	4.30	[3.84 , 4.85]	-0.02	0.53	[0.47 , 0.60]

	X	Rel. std. ($\sigma_X/\sigma_{\Delta\hat{Y}^r}$)			Contemp. $corr(X, \Delta\hat{Y}^r)$		
		Data	Model		Data	Model	
			Mean	[5% , 95%]		Mean	[5% , 95%]
Consumption growth	$\Delta\hat{C}^r$	0.63	0.69	[0.64 , 0.75]	0.59	0.53	[0.49 , 0.61]
Investment growth	$\Delta\hat{I}^r$	3.80	4.01	[3.73 , 4.15]	0.86	0.84	[0.81 , 0.86]
Hours worked	\hat{L}	5.87	4.87	[4.04 , 6.57]	0.14	0.11	[0.08 , 0.13]
Wage growth (1st series)	$\Delta\hat{w}^r$	0.71	0.82	[0.76 , 0.89]	0.03	0.51	[0.44 , 0.56]
Wage growth (2nd series)		0.42			0.31		
Firm Entry growth	$\Delta\hat{N}_E$	3.43	5.03	[4.53 , 5.82]	0.20	0.27	[0.23 , 0.38]

Note: Mean values are computed by simulating the model at the posterior mean reported in table 6.4. To compute the corresponding percentiles, we only use the last 50,000 parameter draws (25,000 of each chain). For each parameter draw, we simulate the model and compute the second moments.

Table 6.6: Variance decomposition at business cycle frequency

		Labor productivity	Wage mark-up	Invest. spec. technology	Preference	Government spending	Entry cost
GDP	Y^r	31.4 [23.7 , 40.2]	62.8 [53.6 , 71.3]	4.7 [2.7 , 6.8]	0.1 [0.0 , 0.1]	0.6 [0.4 , 0.8]	0.4 [0.2 , 1.0]
Consumption	C^r	28.5 [19.0 , 40.7]	60.4 [47.3 , 72.5]	2.8 [1.3 , 4.5]	1.8 [1.0 , 2.7]	2.4 [1.2 , 4.5]	4.1 [1.5 , 6.3]
Total invest.	TI^r	25.9 [20.6 , 32.7]	48.6 [42.3 , 56.4]	15.6 [10.2 , 20.8]	0.6 [0.4 , 1.0]	1.5 [1.0 , 2.1]	7.8 [3.9 , 12.2]
Hours	L	0.5 [0.3 , 0.8]	97.4 [95.3 , 98.6]	0.7 [0.4 , 1.2]	0.0 [0.0 , 0.0]	0.3 [0.2 , 0.7]	1.0 [0.4 , 2.2]
Wages	w^r	88.1 [83.4 , 92.4]	2.1 [1.4 , 3.2]	2.3 [1.0 , 3.9]	0.2 [0.1 , 0.3]	0.2 [0.1 , 0.2]	7.2 [3.8 , 11.2]
Firm Entry	N_E	10.4 [7.5 , 14.4]	18.9 [13.6 , 25.9]	4.9 [2.6 , 7.5]	0.4 [0.2 , 0.7]	0.6 [0.3 , 0.9]	64.9 [54.1 , 74.1]

Note: Main figures are computed at the posterior mean reported in table 6.4. Figures in brackets give the corresponding fifth and ninety fifth percentiles. To compute these percentiles, we use the last 50,000 parameter draws (25,000 of each chain). For each parameter draw, we simulate the model and compute the variance decomposition. To obtain the variance decompositions at business cycle frequency, we compute variance decomposition for the forecast horizons 6 to 32 and take the mean. Shares may not add up to 100 due to rounding.

cost shock accounts for most variations in firm entry. The government spending and the preference shock are only of minor importance for the variables under consideration.

The importance of wage mark-up and labor productivity shocks in driving business cycle fluctuations is consistent with the findings in Smets and Wouters (2007) but stands in contrast to Justiniano et al. (2010), who find that most of the variations in GDP is due to shocks to investment-specific technology. The discrepancy between Smets and Wouters (2007) and Justiniano et al. (2010) stems from different definitions of investment and consumption. The latter define purchases of consumer durables and changes in inventories as part of investment, whereas the former define purchases of consumer durables as part of consumption and exclude changes in inventories from investment. Interestingly, we find that investment-specific technology shocks are minor contributors to business cycle fluctuations although we adopt the same definition of consumption and investment as Justiniano et al. (2010).²¹

We argue that the difference in the importance of the investment-specific technology shock is (at least partly) due to the endogenous firm entry mechanism which is absent in the above mentioned papers. In our framework, shocks to the efficiency with which final goods can be transformed into physical capital – in contrast to wage mark-up and labor productivity shocks – are not able to replicate the positive comovement between firm entry and other key variables in the data. On the contrary, an expansionary investment-

²¹We also re-estimate the model using data on consumption and investment as defined in Smets and Wouters (2007). In line with Justiniano et al. (2010), we find that the importance of the investment-specific technology shock in fact becomes smaller under this specification, explaining only 1.5 percent of the variations in GDP. Details on the estimation are available upon request.

specific technology shock induces a drop in investments into new firms, as discussed below. This explains why the investment-specific technology shock is estimated to be of minor importance.²²

6.5.3 Impulse responses

In this section, we analyze the model-implied impulse responses to the various structural shocks. The aim of this section is to illustrate the amplification mechanism and to visualize the impact of the competition and the variety effect on the model dynamics. To this end, we present the impulse responses for three models, the estimated translog model and two counterfactuals: the CES-TrVE model with the competition effect switched off and the CES-NoVE model with competition and variety effect switched off. For both CES models, we keep the parameter estimates from the translog model.

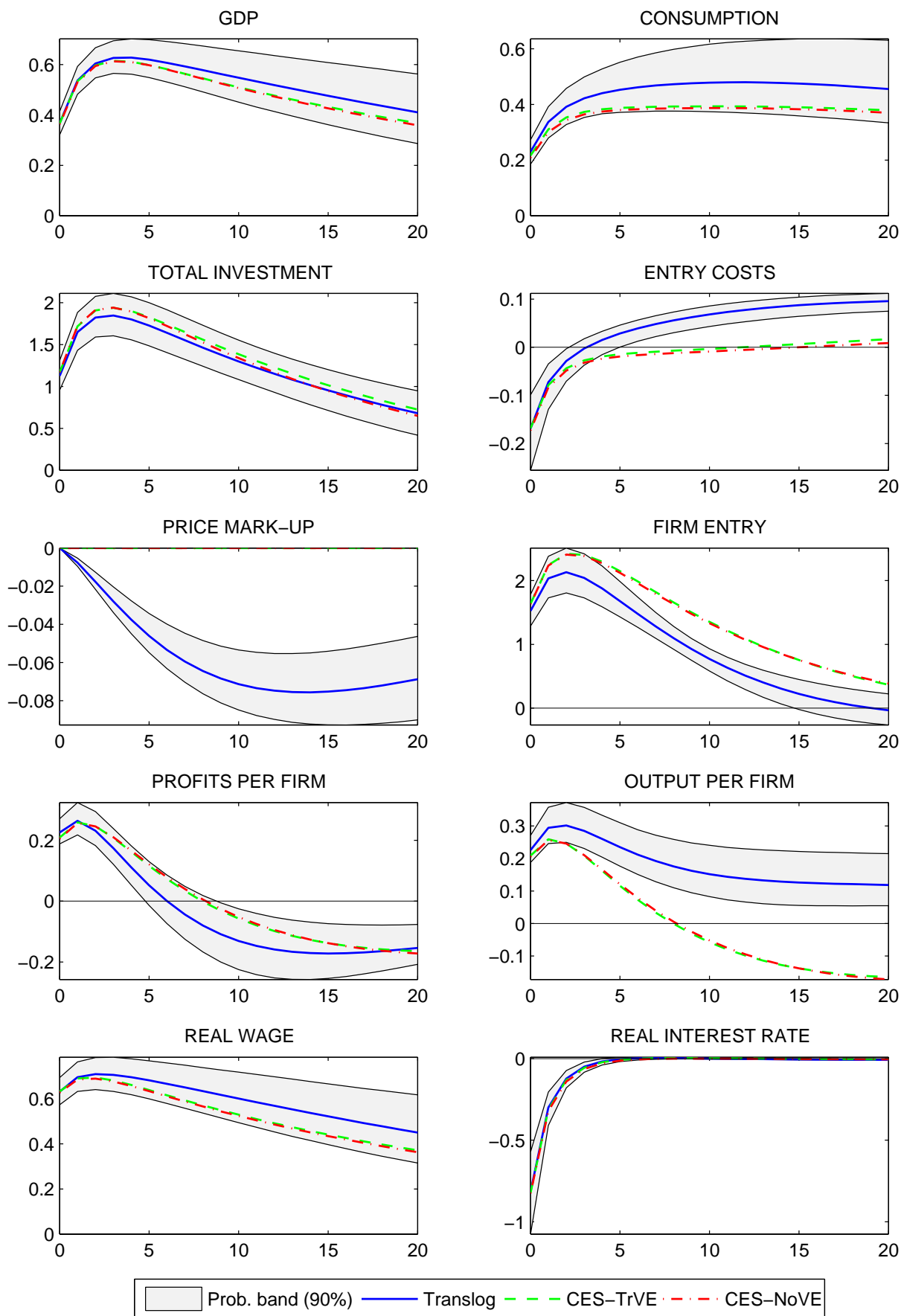
Figure 6.1 to 6.6 show the impulse responses of GDP, consumption, total investment, entry costs, the price mark-up, firm entry, profits per firm, average output of an individual firm, real wages, and the real interest rate to the six structural shocks. All real variables are shown using the data-consistent deflator p_t . Impulse responses are measured as percentage deviations from steady states. The shaded areas are the 90 percent probability bands which reflect parameter uncertainty in the translog model.

To start with, figure 6.1 plots the responses to a positive labor-augmenting technology shock z_t . This boosts GDP, consumption, as well as both components of total investment, investment in physical capital (not shown here) and in firm entry. Firm entry is fueled by rising profit opportunities of monopolistic firms due to the increase in aggregate demand as well as by the drop in entry costs. The latter decreases since the increase in z_t outweighs the rise in real wages. Hence, the mass of firms (or products) starts to increase. In the presence of the competition effect, this makes products closer substitutes and thus deteriorates market power in the monopolistic sector. This leads to a decrease in price mark-ups, which boosts aggregate demand and induces individual firms to increase their production. The increase in aggregate demand is enforced by the drop in the welfare-relevant price index if the variety effect is present.

The magnification effect is evident in the impulse response functions. The competition effect and to a much lesser extent the variety effect magnify the effects of productivity shocks on GDP and, in particular, on consumption. Total investment, however, is dampened. Since investment in physical capital is also amplified, this can only be explained by a dampening of firm entry. The latter is caused by the rise in entry costs over the medium run due to rising real wages and by the fall in price mark-ups, which, in isolation,

²²To further explore this argument, we re-estimate a version of our model in which firm entry is switched off (we achieve this by fixing the parameter governing the firm entry adjustment costs at $\kappa_E = 1000$ and exclude firm entry data from our set of observables). In fact, we find that this leads to an increase in the importance of investment-specific technology shocks, explaining 9.3 percent of the variations in GDP.

Figure 6.1: Impulse responses to a labor productivity shock



deteriorates profit opportunities of monopolistic firms. Note that the probability band does not give implications about the significance of the amplification mechanism. If the counterfactual CES responses lie inside the probability region, this does not imply that the amplification mechanism is insignificant. In order to give implications about the significance of the amplification mechanism, one needs to take into account the parameter uncertainty in all three model frameworks. This is done in the section 6.6.

Figure 6.2 shows the responses to an increase in wage mark-ups μ_t^w . The increase in wage mark-ups leads to a decline of GDP, consumption, and total investment. The drop in GDP and consumption is amplified when the competition and the variety effect are at work. The reason is that firm entry is depressed by the increase in entry costs and the drop in firm profits triggered by the initial increase in real wages. Since the mass of firms declines, the product space becomes less crowded and the elasticity of substitution declines. Consequently, the price mark-up rises, which causes aggregate demand and individual firm's production to fall. In addition to the competition effect, the rise in the welfare-relevant price index (the variety effect) puts downward pressure on aggregate demand, although this effect seems to be quantitatively small. As in the case of labor productivity shocks, there is a dampening of the response of total investment when competition and variety effect are present.

To sum up, it is mainly the competition effect that amplifies, via countercyclical price mark-ups, the impact of labor-augmenting technology and wage mark-up shocks on GDP and consumption. However, countercyclical price mark-ups dampen the responses of firm entry, which translates into a dampening of total investment.

The results are different when we consider shocks to investment-specific technology, to preferences, and to government spending. The reason is that the estimated conditional correlation between GDP and firm entry (or the number of firms) is negative for those shocks. Hence, competition and variety effect dampen the rise in GDP following positive shocks to investment-specific technology, preferences, and government spending.

Figure 6.3 shows the responses to the investment-specific technology shock u_t^I . The increase in the efficiency with which final goods can be transformed into physical capital produces a boom in capital investment and a hike in GDP. Consumption falls on impact but turns positive during the course of adjustment. Real wages and thus entry costs increase. The value of a firm v_t decreases due to the increase in the real interest rate, which outweighs the increase in individual firm's profits. Consequently, firm entry falls inducing an increases in price mark-ups. The procyclical response of firm's market power abates the impacts of investment-specific technology shocks on GDP and consumption. The response of total investment is almost identical across the models.

Figures 6.4 and 6.5 show the responses to a preference shock χ_t that induces households to consume more and to a rise in government spending G_t , respectively. Both shocks raise aggregate demand and the real interest rate. The latter lowers firm values, which induces,

Figure 6.2: Impulse responses to a wage mark-up shock

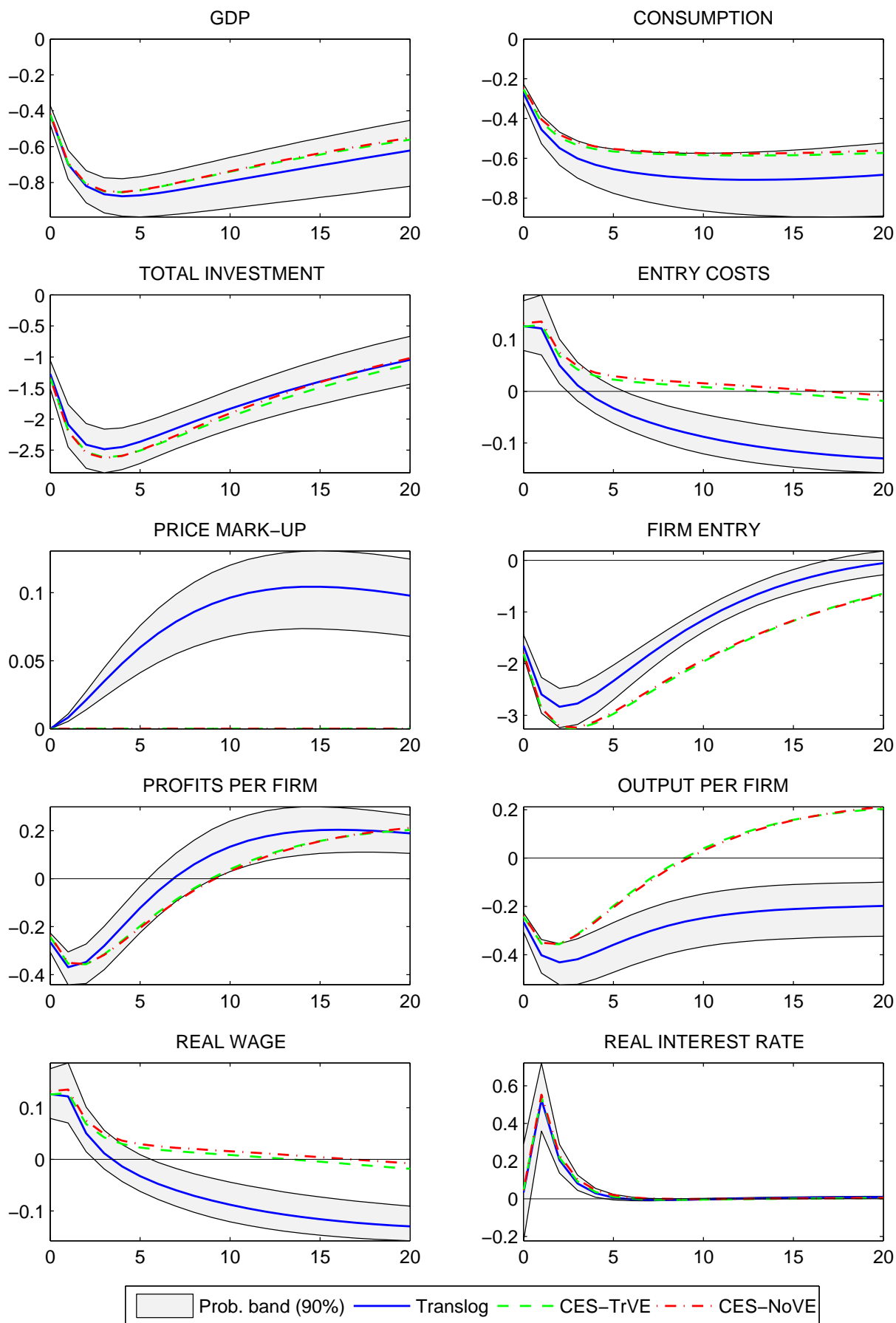


Figure 6.3: Impulse responses to an investment-specific technology shock

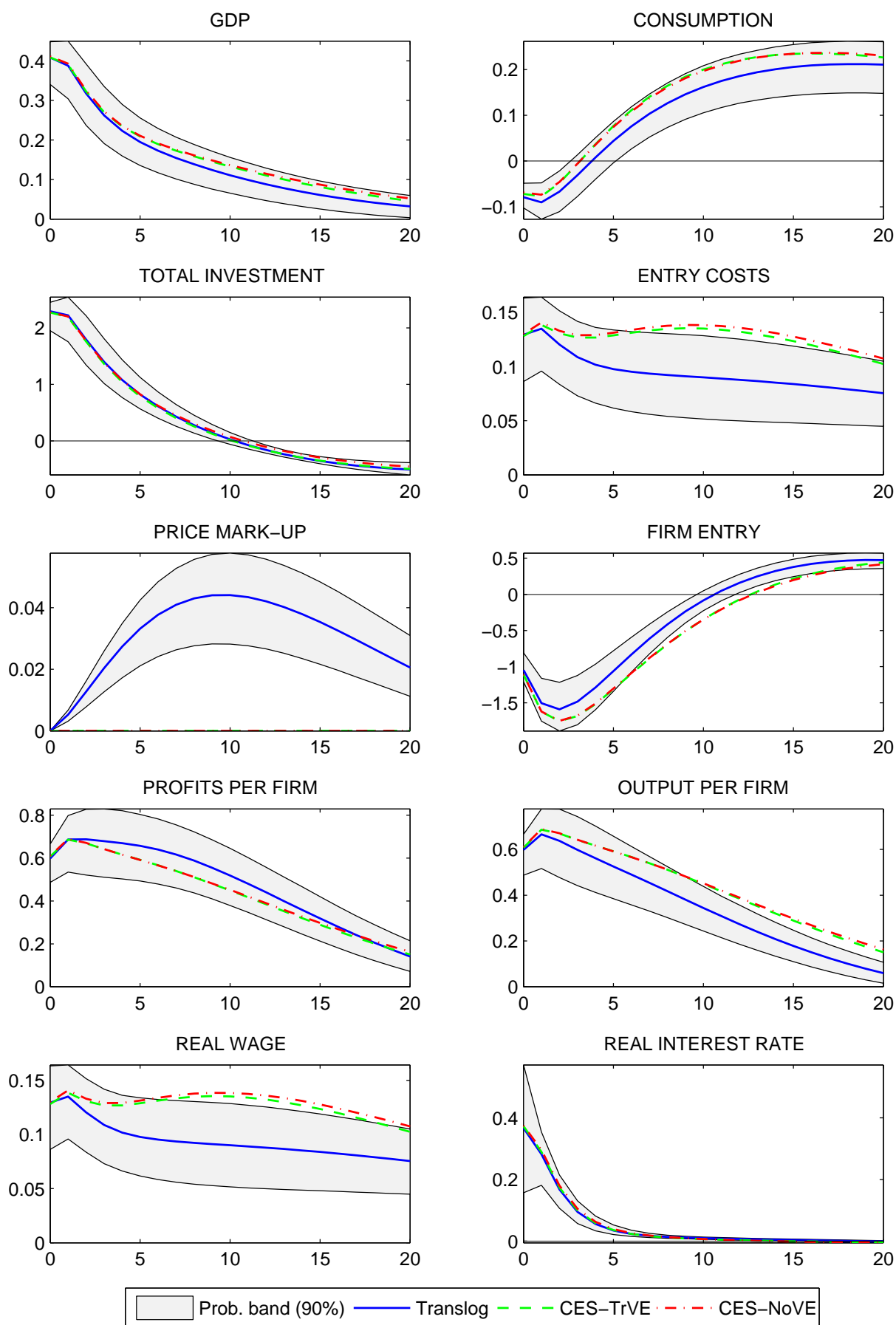
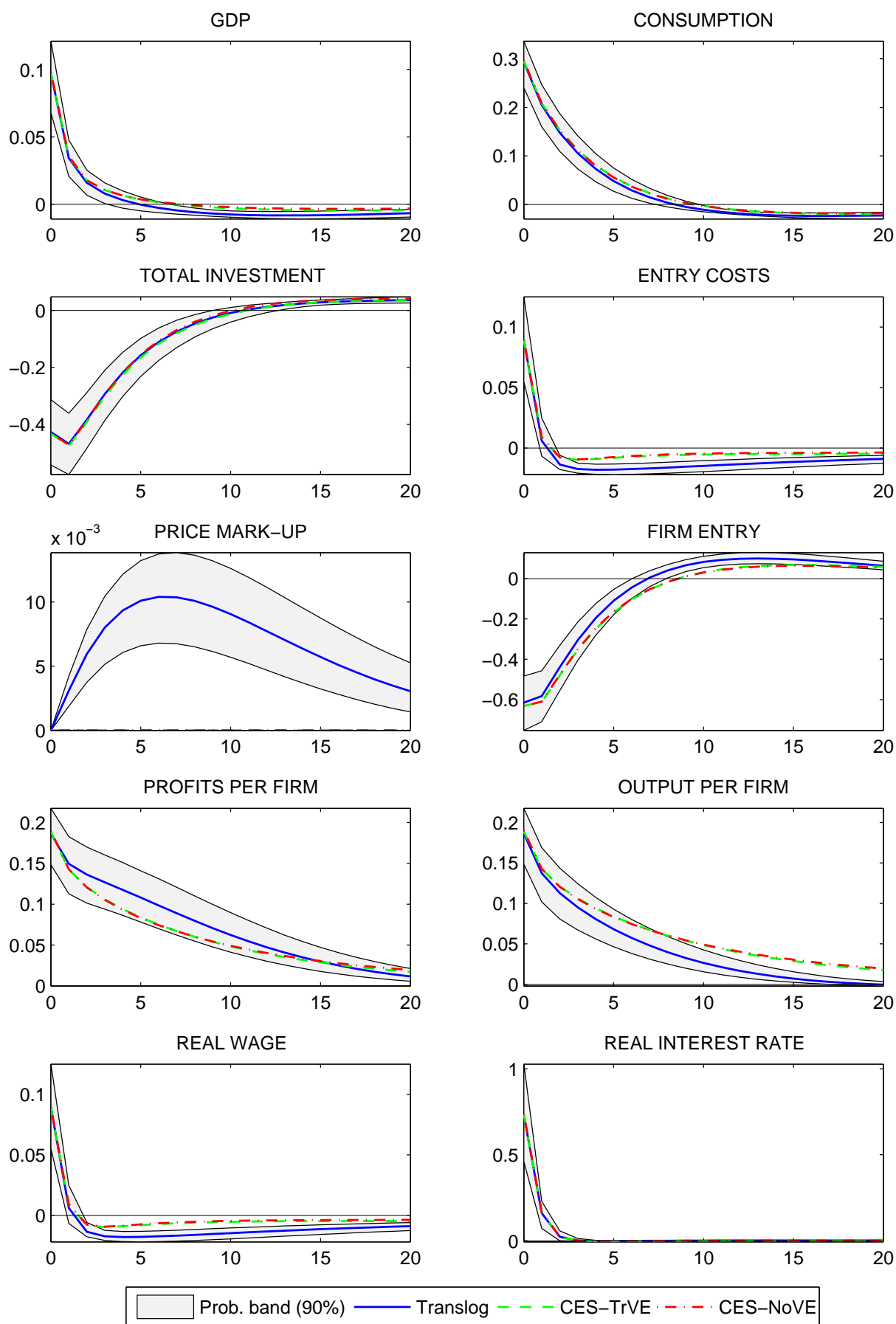


Figure 6.4: Impulse responses to a preference shock



in conjunction with the rise in entry costs, a decline in firm entry. Consequently, under translog preferences price mark-ups and the welfare-relevant price index rise, which both dampens the impacts of these aggregate demand disturbances on GDP.

Following a time-impatience shock, the responses of GDP, investment, and consumption are, however, not very different across the models. This is because the mark-up is only marginally affected by this type of shock. In contrast, the competition and the variety effect visibly amplify the crowding-out of private consumption after a fiscal expansion, as can be seen from figure 6.5. Private consumption falls due to the negative wealth effect of higher taxes and the rise in real interest rates. Investment is again only marginally affected by these mechanisms.

Finally, figure 6.6 shows that an exogenous increase in entry costs $f_{E,t}$ generates a strong decline in firm entry. Households that invest less in new firms raise consumption and capital investment, at least on impact. The decrease in the number of products leads to an increase in the market power of firms when the competition effect is present. The increase in price mark-ups induces a fall in capital investment and consumption (in the medium run). GDP, after a temporary rise on impact, declines substantially. In the absence of the competition effect, the decline in GDP is only short-lived and followed by a hump-shaped rise. This can be explained by the significant increase of consumption, which is otherwise depressed by the sharp increase in price mark-ups. As in the case of shocks to investment-specific technology, to preferences, and to government spending, the increase in firm's market power lowers GDP. In the case of these disturbances the increase in GDP is dampened. In the case of entry cost shocks, though, the competition effect changes the GDP response qualitatively.

Overall, the presence of the competition and the variety effect substantially amplify (dampen) the impact of labor productivity and wage mark-up (investment-specific technology) shocks on GDP and consumption. In the case of shocks to government spending and preferences, the impact of these mechanisms is only marginal. Inspecting the impulse responses suggests also marginal effects for total investment. In the case of labor productivity and wage mark-up shocks, the responses of total investment are slightly dampened. For the other disturbances, the impacts are negligible. The next section aims to quantify the amplification mechanism by comparing simulated volatilities across the three model frameworks.

6.6 Quantifying the internal amplification mechanisms

In this section, we quantify the amplification mechanism embedded in the entry model and assess the contribution of the competition and the variety effect. Following Jaimovich and Floetotto (2008), we consider overall and shock-specific amplification measures, including shocks to labor productivity, to wage mark-ups, to investment-specific technology, to

Figure 6.5: Impulse responses to a government spending shock

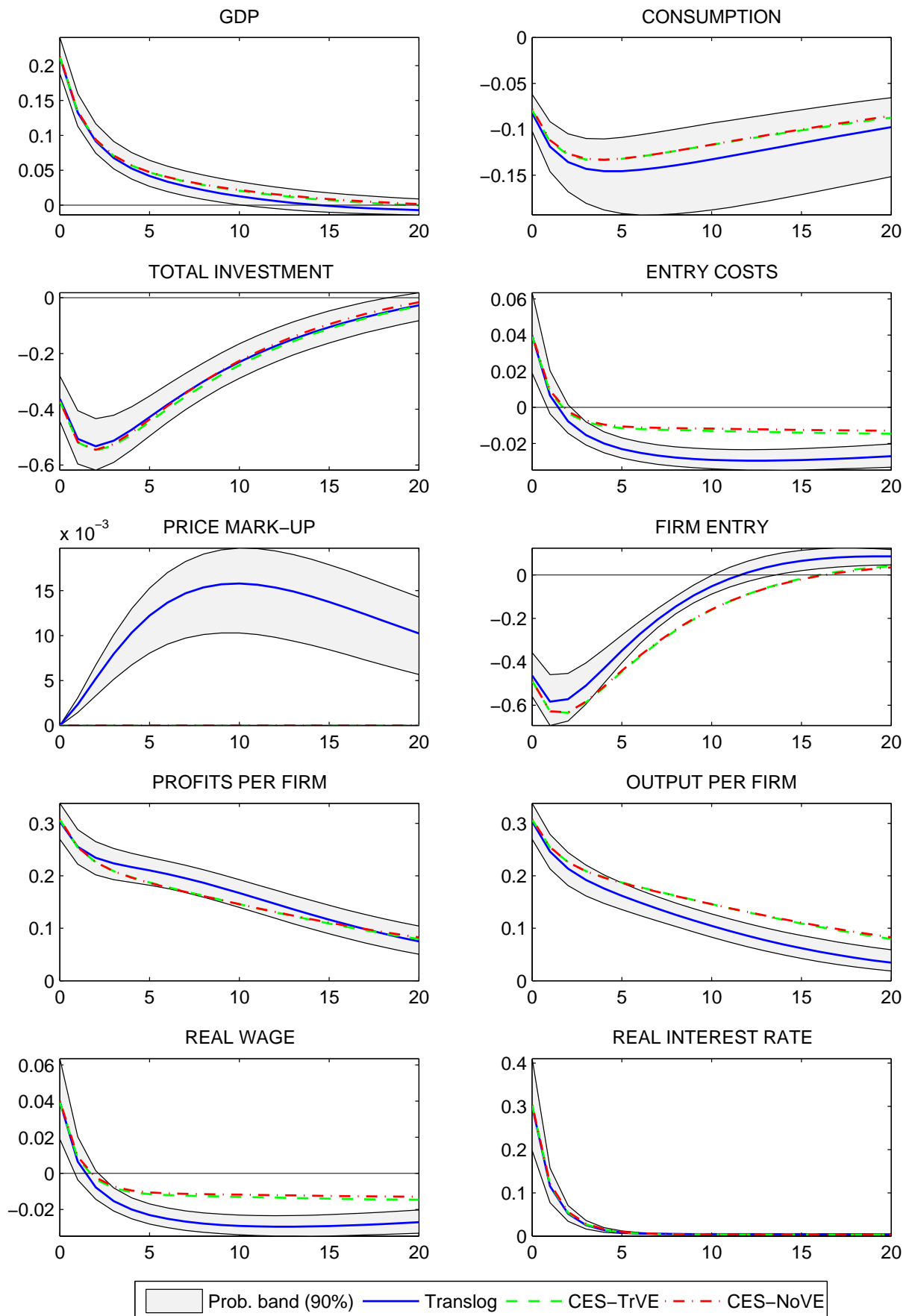
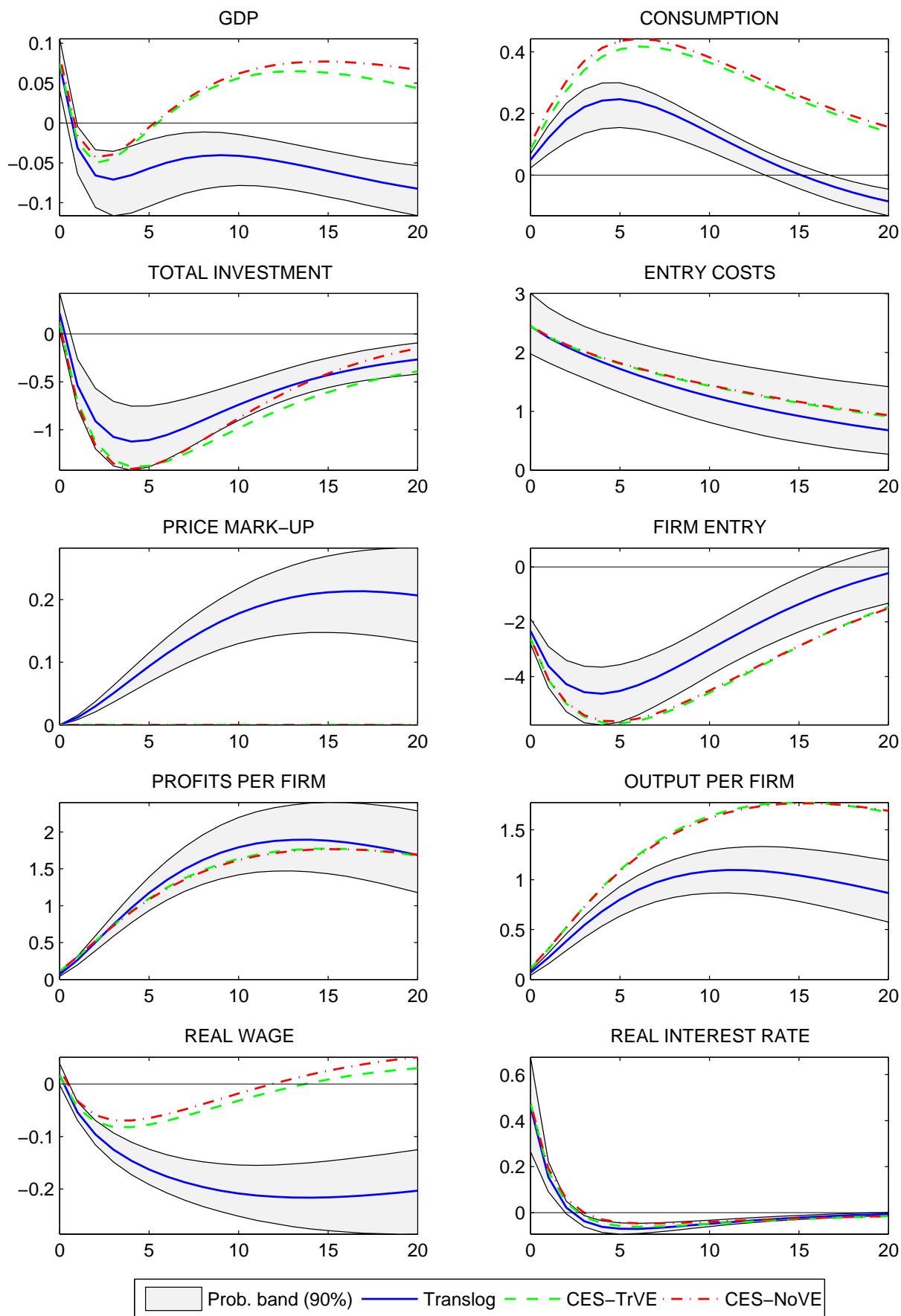


Figure 6.6: Impulse responses to an entry cost shock



preferences, and to government spending. Recall from the previous section that shocks to entry costs generate qualitative different responses across the models. We therefore exclude this shock from our amplification analysis. Including this shock would only slightly change the overall amplification results.

In order to quantify the amplification mechanism, we proceed in two steps. In the first step, we simulate the three model frameworks (translog model, CES-TrVE model, CES-NoVE model) and compute shock-specific and overall volatility measures. As in the analysis of the impulse responses, we keep the parameter estimates from the translog model when simulating the CES models.²³ Based on the simulated volatility measures, we compute, in the second step, three amplification metrics. The total amplification through both the competition and the variety effect are measured as the volatility difference, in percentage terms, between the translog model and the CES-NoVE model. The contribution of the competition effect (CE) is measured by the volatility difference, in percentage terms, between the translog model and the CES-TrVE model. Analogously, the contribution of the variety effect (VE) is computed as the volatility difference, in percentage terms, between the CES-TrVE model and the CES-NoVE model. Note that the contributions of the competition and of the variety effect do not necessarily add up to the total amplification since they are computed on different bases. Table 6.7 shows our results. The volatility and amplification measures are computed at the posterior mean reported in table 6.4. Numbers in brackets give the corresponding fifth and ninety fifth percentiles.

To start with, we focus on GDP. The results are displayed in the upper panel of table 6.7. The shock-specific and overall volatility measures are shown in columns one to three. Shock-specific volatilities are obtained by assuming that at each time only one of the above mentioned shocks is active. We then compute, for all three model frameworks and for all five shocks under consideration, the standard deviation of GDP relative to the standard deviation of the underlying shock process.²⁴ The overall volatility is measured by the absolute standard deviations of GDP when all five considered structural shocks are active. Columns four to six show the amplification ratios measured in percentage points. The last column reports the contemporaneous correlation between the number of firms and GDP.

When all five shocks are active, the volatility of GDP under translog preferences is substantially higher than under the two CES models. In total, GDP is amplified by 8.5 percent with a 90 percent probability band ranging from 6.1 to 10.1 percent. The competition effect accounts for most amplification across all shocks. Overall, the competition effect increases the volatility of GDP by 6.8 percent with a probability band ranging from 4.5 to 8.4 percent. The increase through the variety effect, on the other hand,

²³This ensures that we extract the model-specific amplification effect that exclusively results from the different model setups. In a robustness exercise in section 6.7, we estimate both CES models separately and obtain similar amplification results using the estimated CES models.

²⁴As in the previous section, GDP is deflated by the relative price ρ_t .

Table 6.7: Standard deviations, relative volatilities and amplification indicators

	Volatility			Amplification			<i>Corr</i> (X, Y^r)
	Translog	CES-TrVE	CES-NoVE	Total	CE	VE	
<i>Std of GDP relative to ... shock</i>							$X = Y^r$
labor prod.	1.0006 [0.9550, 1.0309]	0.9332 [0.8956, 0.9616]	0.9171 [0.8815, 0.9452]	9.1% [6.5%, 10.8%]	7.2% [4.8%, 9.0%]	1.8% [1.1%, 2.1%]	0.92
wage mark-up	0.1811 [0.1287, 0.2620]	0.1690 [0.1201, 0.2456]	0.1661 [0.1180, 0.2418]	9.0% [6.3%, 10.8%]	7.2% [4.7%, 9.0%]	1.8% [1.1%, 2.1%]	0.93
inv. spec. tech.	0.2166 [0.1341, 0.3413]	0.2281 [0.1420, 0.3547]	0.2317 [0.1438, 0.3605]	-6.5% [-7.9%, -4.2%]	-5.0% [-6.5%, -2.9%]	-1.6% [-1.8%, -1.0%]	-0.55
preference	0.0749 [0.0582, 0.0923]	0.0735 [0.0568, 0.0911]	0.0736 [0.0568, 0.0914]	1.8% [0.0%, 3.9%]	2.0% [0.6%, 3.5%]	-0.2% [-0.8%, 0.5%]	0.14
gov. spending	0.0627 [0.0516, 0.0757]	0.0639 [0.0530, 0.0769]	0.0643 [0.0533, 0.0772]	-2.5% [-5.1%, -0.2%]	-2.0% [-4.3%, -0.1%]	-0.6% [-1.0%, 0.0%]	-0.16
<i>Absolute std of GDP</i>							
All five shocks	0.0521 [0.0438, 0.0682]	0.0488 [0.0410, 0.0644]	0.0480 [0.0404, 0.0635]	8.5% [6.1%, 10.1%]	6.8% [4.5%, 8.4%]	1.6% [1.0%, 2.0%]	0.56
<i>Std of consumption relative to ... shock</i>							$X = C^r$
labor prod.	1.0705 [0.9284, 1.2317]	0.9287 [0.8041, 1.0797]	0.8846 [0.7667, 1.0288]	21.0% [14.0%, 26.4%]	15.3% [10.1%, 19.4%]	5.0% [3.5%, 6.1%]	0.97
wage mark-up	0.1995 [0.1416, 0.2909]	0.1741 [0.1221, 0.2584]	0.1659 [0.1160, 0.2478]	20.2% [12.7%, 27.1%]	14.6% [9.0%, 20.0%]	4.9% [3.4%, 6.1%]	0.97
inv. spec. tech.	0.2797 [0.1680, 0.4293]	0.2954 [0.1801, 0.4471]	0.3034 [0.1852, 0.4583]	-7.8% [-10.8%, -5.2%]	-5.3% [-7.9%, -3.1%]	-2.7% [-3.4%, -1.9%]	-0.49
preference	0.2915 [0.2626, 0.3215]	0.2962 [0.2670, 0.3266]	0.2948 [0.2651, 0.3255]	-1.1% [-1.8%, -0.5%]	-1.6% [-2.2%, -1.0%]	0.5% [0.3%, 0.7%]	-0.15
gov. spending	0.1461 [0.1213, 0.1741]	0.1327 [0.1090, 0.1615]	0.1297 [0.1063, 0.1584]	12.7% [7.2%, 17.6%]	10.1% [5.7%, 14.2%]	2.3% [1.4%, 3.1%]	0.95
<i>Absolute std of consumption</i>							
All five shocks	0.0577 [0.0466, 0.0802]	0.0507 [0.0406, 0.0721]	0.0486 [0.0389, 0.0693]	18.7% [12.3%, 23.3%]	13.7% [8.8%, 17.3%]	4.4% [3.2%, 5.4%]	0.55
<i>Std of total investment relative to ... shock</i>							$X = TI^r$
labor prod.	2.2082 [1.7058, 2.5469]	2.3363 [1.8696, 2.6440]	2.2656 [1.8251, 2.5616]	-2.5% [-7.5%, 1.0%]	-5.5% [-9.5%, -2.5%]	3.1% [2.0%, 3.9%]	0.67
wage mark-up	0.3743 [0.2556, 0.5564]	0.3982 [0.2728, 0.5918]	0.3865 [0.2650, 0.5731]	-3.1% [-6.7%, -0.3%]	-6.0% [-8.8%, -3.6%]	3.0% [2.1%, 3.8%]	0.68
inv. spec. tech.	1.2574 [0.7753, 2.0362]	1.2366 [0.7620, 2.0099]	1.2448 [0.7654, 2.0247]	1.0% [-0.1%, 2.2%]	1.7% [0.7%, 2.7%]	-0.7% [-1.0%, -0.3%]	-0.12
preference	0.5985 [0.5303, 0.6943]	0.6094 [0.5389, 0.7079]	0.6004 [0.5297, 0.6987]	-0.3% [-1.5%, 1.1%]	-1.8% [-2.7%, -0.6%]	1.5% [1.1%, 1.9%]	0.34
gov. spending	0.3138 [0.2423, 0.3679]	0.3231 [0.2518, 0.3759]	0.3155 [0.2463, 0.3676]	-0.5% [-3.1%, 1.7%]	-2.9% [-5.0%, -0.9%]	2.4% [1.8%, 2.9%]	0.52
<i>Absolute std of total investment</i>							
All five shocks	0.1203 [0.1079, 0.1375]	0.1261 [0.1134, 0.1436]	0.1231 [0.1109, 0.1401]	-2.3% [-5.3%, 0.0%]	-4.6% [-7.0%, -2.7%]	2.5% [1.7%, 3.1%]	0.44

Note: Volatilities and amplification measures are computed by simulating the model at the posterior mean reported in table 6.4. Figures in brackets give the corresponding fifth and ninety fifth percentiles. To compute these percentiles, we use the last 50,000 parameter draws (25,000 of each chain). For each parameter draw, we simulate the models and compute the volatility and amplification measures. The contemporaneous correlation in the last column is based on simulated data from the translog model at the posterior mean.

only amounts to 1.6 percent with a probability band ranging from 1.0 to 2.0 percent.

Turning to the shock-specific amplification ratios, we obtain the following results. Productivity shocks are amplified by 9.1 percent, wage mark-up shocks by 9.0 percent, and shocks to preferences by 1.8 percent. Contrarily, shocks to investment-specific technology are dampened by 6.5 percent, government spending shocks by 2.5 percent. For all shocks, except for shocks to preferences, the competition and the variety effect work in the same direction. Since the price mark-up μ_t^p decreases with the number of firms and the relative price ρ_t increases with the number of firms, the following rule applies: If GDP and the number of firms are sufficiently positively correlated, both effects amplify GDP. If, on the other hand, the correlation is negative, both effects dampen GDP.²⁵ The shock-specific amplification is mainly driven by the competition effect, as for total amplification. Except for shocks to preferences, we find that all amplification measures are significantly different from zero at the 5 percent level.

Next, we investigate the components of GDP. The results for consumption and investment are shown in the middle and lower panel of table 6.7, respectively. For consumption, the amplification results are qualitatively the same as for GDP, except for shocks to preferences and government spending. However, the volatility differences across the models are much more pronounced. Over all shocks, the volatility in consumption is amplified by 18.7 percent in total. For the two most important shocks to consumption, i.e. labor productivity and wage mark-ups, the total volatility increase is given by 21.0 percent and 20.2 percent, respectively. Contrarily to GDP, consumption is positively correlated with the number of firms in the presence of a government spending shock, amplifying the crowding out in consumption. For consumption, all amplification measures are significantly different from zero at the 5 percent level.

For total investment, the competition effect has two opposing effects. Recall that total investment is defined as the sum of investment in physical capital and investments in new firms. On the one hand, a lower price mark-up boosts GDP and therefore increases investment in physical capital for existing firms. On the other hand, a lower price mark-up reduces the incentive to invest into new firms. We find that the latter effect dominates such that an increase in the number of firms reduces total investment through the competition effect and outweighs the positive variety effect. Over all shocks, the volatility in total investment is dampened by -2.3 percent. However, we do not find this result to be significantly different from zero at the 5 percent level.

²⁵Note that the variety effect on data-consistent variables, as considered here, is smaller than for welfare-consistent variables since $\partial \hat{Y}_t^r / \partial \hat{\rho}_t = \partial \hat{Y}_t / \partial \hat{\rho}_t - 1$ and $\partial \hat{Y}_t / \partial \hat{\rho}_t > 0$. Except for shocks to preferences, the effect through Y_t dominates the effect which stems from deflating.

6.7 Robustness

In this section, we discuss several robustness checks for our estimation and amplification results. First, we re-estimate the translog model using a limited data sample until 1998. Second, we re-estimate the translog model using additional data on mark-ups and on profits. Finally, we estimate the two CES models and use these estimated models to quantify the amplification mechanism. Details on the results of our robustness exercises can be found in appendix 6.C.

Subsample estimation. In the first exercise, we check whether the missing observations of firm entry in our data sample considerably affect our estimation and amplification results. We therefore re-estimate the model with translog preferences using only the limited sample period until 1998:Q2, which is the latest data point available for the new firm series. Hence, all missing observations are excluded from the data sample. Everything else remains the same.

Most parameter values are not significantly different from the parameter estimates of the full sample estimation. Two differences are worth mentioning. First, the estimate of the labor share in production is given by 0.91, which is even higher than the estimate of 0.85 in the full sample estimation. However, in both estimations the values correspond roughly to the same labor share in GDP of about 70 percent. Second, the steady-state price mark-up is estimated at 1.43, which is substantially higher than the full sample estimate of 1.26. This implies a stronger competition and variety effect in this sample period. The estimate of μ^p implies a point estimate for the competition effect ξ equal to 0.3 with a confidence band ranging from 0.24 to 0.35. The implied variety effect ω is estimated at 0.22 with a confidence band from 0.16 to 0.28. Due to the larger estimates for the competition and the variety effect, the amplification measures are also substantially higher when compared to the baseline estimation. The overall amplification of GDP, consumption, and total investment through the competition and the variety effect are now given by 10.3, 24.4, and -4.5 percent, respectively. These numbers are also significantly different from zero at the 5 percent level.

Estimation using mark-up and profit data. Our analysis so far has shown that the competition effect, i.e. the effect of a change in the number of firms on mark-ups, is statistically significant and economically relevant. Therefore, one might argue that it is important to incorporate a measure of mark-ups in the estimation of the model. In our baseline estimation, we do not use a mark-up measure because mark-ups of prices over marginal costs are unobserved and the construction of a fully model-consistent mark-up measure is impossible. To see this, note that the model's definition of the price mark-up is given by $\mu_t^p = \alpha Y_t^c / (w_t L_t^c)$. Y_t^c is the aggregate output of final goods consumption, which

does not include investments into new firms. L_t^c is the corresponding labor input. Both Y_t^c and L_t^c are not observable and are different from the gross domestic product Y_t and overall labor L_t .²⁶ Nevertheless, it is important to check whether our results are robust to the inclusion of a mark-up proxy. As a proxy, we use the growth rate of the inverse labor share $Y_t/(w_t L_t)$. We construct the labor share using data on GDP, hours worked and hourly compensation in the non-farm business sector. To account for the differences between the proxy and the model-consistent definition of the mark-up, we include the error term $\varepsilon_t^{\mu,me}$ in the measurement equation, where $\varepsilon_t^{\mu,me}$ is assumed to be *i.i.d.* $N(0, \sigma_{\varepsilon_{d,me}}^2)$. This also ensures that the model is not subject to stochastic singularity. The measurement equation then reads as²⁷

$$dl(\text{MARKUP}_t) = 100\Delta\hat{\mu}_t^p + \varepsilon_t^{\mu,me} \quad (6.25)$$

The dynamics of firm entry are also strongly linked to firms' profits. Therefore, we also add the growth rate of real per capita corporate profits after taxes to our set of observables.²⁸ We map this measure to overall profits which are defined by the number of firms N_t times the (real) average profits per firm d_t^r . Note, though, that profits in the model are economics profits, while in the data profits are accounting profits. One of the main differences is that accounting profits still contain costs of capital since parts of the capital stock are equity-financed. Contrarily in the model, capital costs are fully subtracted out. To account for the difference between profits in the data and in the model, we include the measurement error $\varepsilon_t^{d,me}$ in the measurement equation, where $\varepsilon_t^{d,me}$ is assumed to be *i.i.d.* $N(0, \sigma_{\varepsilon_{d,me}}^2)$. The measurement equation for profits then reads as

$$dl(\text{PROFITS}_t) = 100(\Delta\hat{d}_t^r + \Delta\hat{N}_t) + \varepsilon_t^{d,me} \quad (6.26)$$

Re-estimating the baseline model with data on mark-ups and profits, respectively, we do not find any significant differences in the parameters estimates and the amplification measures, compared to the baseline estimation. The estimated models are able to capture the procyclicality of profits and the countercyclicality of mark-ups in the data. However, most of the variations in the mark-up and profit data are captured by the measurement errors.²⁹ We argue that this is at least partly due to the model-inconsistent construction of the mark-up data and the described differences in the concept of profits between data and model.

²⁶An alternative proxy for price mark-ups can be constructed from data on profits using the relation $N_t d_t = (1 - 1/\mu_t^p)Y_t^C$. However, this relation also contains the unobservable variable Y_t^C . Moreover, it requires to use profit data which, as we argue below, suffers from a measurement problem as well.

²⁷Recall that the function dl stands for 100 times the demeaned log-differences operator.

²⁸More details on the source of the profit data and the construction of the data series can be found in tables 6.8 and 6.9 of appendix 6.B.

²⁹This confirms the profit volatility puzzle, i.e. the inability of standard business cycle models to account for the volatility of profits. See, e.g. Lewis and Stevens (2015) or Colcagio and Etro (2010).

Estimation of CES models. In the previous section, we have used the same estimated parameter set in all three model frameworks in order to isolate the model-specific competition and variety effect that exclusively result from the different model setups. In this last exercise, we estimate the two CES models using the same data set as in our baseline estimation of the translog model. The constant price mark-up is fixed to the estimated steady-state value under the translog specification. In a second step, we compute the amplification measures using the two estimated CES models and the estimated translog model.

The parameter estimates of the CES models are not significantly different from the parameter estimates of the translog model. All estimates lie within the 90 percent probability interval of the translog estimation. Using the estimated CES models, we therefore obtain similar amplification measures. For the two most important shocks to GDP, i.e. labor productivity and wage mark-ups, the total amplification amounts to 8.7 percent and 13.2 percent, respectively. Note that we do not give overall amplification measures and probability bands. Since we estimate different shocks processes across models, differences in the absolute standard deviations do not show the strength of the amplification mechanism but rather reflect the ability of the models to capture the volatility in the data. On the contrary, shock-specific amplification measures are computed in relative terms and thus control for the different estimates of the shock processes. The computation of the corresponding probability bands would require to know the joint posterior distribution under the translog and the CES models.

6.8 Conclusion

This paper studies the empirical importance of endogenous firm entry as amplification mechanism for business cycle fluctuations. To this end, we use the firm entry model by Bilbiie et al. (2012), extend it with several real frictions and estimate the model on U.S. data with Bayesian methods. In this model, the amplification mechanism of firm entry works through a competition and a variety effect. Both effects are estimated to be statistically significant.

To quantify the strength of this amplification mechanism and to disentangle the competition and the variety effect, we also specify two model frameworks, where either the competition or both effects are switched off. We measure the amplification as the percentage volatility difference in GDP, consumption, and investment across the three model variants.

Our results support the findings of Jaimovich and Floetotto (2008) that endogenous firm entry is an important amplification mechanism for business cycle fluctuations. However, in a medium-scale environment – as considered here – the quantitative impact of firm entry is less dominant. Over all shocks, the competition and the variety effect substan-

tially amplify GDP by 8.5 percent. The impacts of the competition and the variety effect are shock-dependent. For labor productivity and wage mark-up shocks the competition and the variety effect amplify the impacts on GDP, but dampen the impacts of shocks to aggregate demand and to investment-specific technology. The competition effect accounts for most amplification, whereas the variety effect only plays a minor role.

In the theoretical framework each firm produces one differentiated product, i.e. we have an identity between the number of firms and products. In our empirical exercise we exclusively focus on firm entry dynamics by using data on new business incorporations. However, this approach neglects the effect of product creation at existing firms. Bernard et al. (2010) and Broda and Weinstein (2010) highlight the empirical importance of product creation in business cycle fluctuations. We leave it to future research to identify the entry mechanism with data on product creation.

6.9 References

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6.A Appendix: Model

The price index and the price elasticity of demand. This section derives the price index P_t and the price elasticity of demand ε_t for an individual intermediate good in the translog model. The final goods producers demand intermediate goods $y_{i,t}$ at price $p_{i,t}$ and bundle these intermediate goods to final goods Y_t^C . Under perfect competition and zero profits, total revenues equal total costs. The cost function of a final good producer then equals $P_t Y_t^C$, where P_t is the price and the cost of one final good. In the translog model, $\ln P_t$ is given by the following translog function as proposed by Feenstra (2003):

$$\begin{aligned} \ln P_t = & \alpha_0 + \frac{1}{2} \frac{\tilde{N} - N_t}{\tilde{\sigma} N_t \tilde{N}} + \frac{1}{N_t} \int_0^{N_t} \ln p_{i,t} di \\ & + \frac{1}{2} \frac{\tilde{\sigma}}{N_t} \int_0^{N_t} \int_0^{N_t} \ln p_{i,t} \ln p_{j,t} di dj - \frac{\tilde{\sigma}}{2} \int_0^{N_t} (\ln p_{i,t})^2 di \end{aligned} \quad (6.27)$$

In the following α_0 is normalized to zero.

Under symmetric intermediate goods producers, implying $p_{i,t} = p_{j,t} = p_t$, the translog function (6.27) simplifies to

$$\ln P_t = \frac{1}{2} \frac{\tilde{N} - N_t}{\tilde{\sigma} N_t \tilde{N}} + \ln p_t \quad (6.28)$$

and the relative price reads as

$$\rho_t = \frac{p_t}{P_t} = \exp \left(-\frac{1}{2} \frac{\tilde{N} - N_t}{\tilde{\sigma} N_t \tilde{N}} \right) \quad (6.29)$$

For the derivation of the price elasticity of demand for an individual good, we use the cost share $s_{i,t}$ of the i th production factor, which is defined as

$$s_{i,t} = \frac{p_{i,t} y_{i,t}}{P_t Y_t^C} \quad (6.30)$$

Taking logs of (6.30) and differentiating with respect to $\ln p_{i,t}$ gives

$$\frac{\partial \ln s_{i,t}}{\partial \ln p_{i,t}} = 1 + \frac{\partial \ln y_{i,t}}{\partial \ln p_{i,t}} \quad (6.31)$$

such that we can write the price elasticity as

$$\varepsilon_{i,t} = -\frac{\partial y_{i,t} p_{i,t}}{\partial p_{i,t} y_{i,t}} = -\frac{\partial \ln y_{i,t}}{\partial \ln p_{i,t}} = 1 - \frac{\partial \ln s_{i,t}}{\partial \ln p_{i,t}} = 1 - \frac{\partial s_{i,t}}{\partial \ln p_{i,t}} \frac{1}{s_{i,t}} \quad (6.32)$$

According to Shepard's lemma, the demand for the i th production factor is given by

$$y_{i,t} = \frac{\partial P_t Y_t^C}{\partial p_{i,t}} = Y_t^C \frac{P_t}{p_{i,t}} \frac{\partial \ln P_t}{\partial \ln p_{i,t}} \quad (6.33)$$

Replacing $y_{i,t}$ in (6.30) and differentiating (6.27) with respect to $\ln p_{i,t}$, the cost share can be written as

$$s_{i,t} = \frac{\partial \ln P_t}{\partial \ln p_{i,t}} = \frac{1}{N_t} + \frac{1}{2} \frac{\tilde{\sigma}}{N_t} \int_0^{N_t} \ln p_{i,t} di + \frac{1}{2} \frac{\tilde{\sigma}}{N_t} \int_0^{N_t} \ln p_{i,t} di - \frac{1}{2} \tilde{\sigma} \ln p_{i,t} \quad (6.34)$$

$$= \frac{1}{N_t} + \frac{\tilde{\sigma}}{N_t} \int_0^{N_t} \ln p_{i,t} di - \tilde{\sigma} \ln p_{i,t} \quad (6.35)$$

Differentiating (6.35) with respect to $\ln p_{i,t}$ gives

$$\frac{\partial s_{i,t}}{\partial \ln p_{i,t}} = \frac{\tilde{\sigma}}{N_t} - \tilde{\sigma} \quad (6.36)$$

such that

$$\varepsilon_{i,t} = 1 - \left(\frac{\tilde{\sigma}}{N_t} - \tilde{\sigma} \right) \frac{1}{s_{i,t}} \quad (6.37)$$

which reduces for large N_t to

$$\varepsilon_{i,t} = 1 + \tilde{\sigma} \frac{1}{s_{i,t}} \quad (6.38)$$

Under symmetric prices, the cost share in (6.35) reduces to

$$s_t = \frac{1}{N_t} \quad (6.39)$$

and the price elasticity is given by

$$\varepsilon_t = 1 + \tilde{\sigma} N_t \quad (6.40)$$

The final goods production sector under a CES production technology. This section presents the equations for the final goods production sector under a constant elasticity of substitution (CES) production function as in Benassy (1996). In this case, intermediate goods are combined using the production function $Y_t^C = N_t^{\zeta - \frac{1}{\varepsilon - 1}} \left(\int_0^{N_t} y_{i,t}^{\frac{\varepsilon - 1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon - 1}}$, where $\varepsilon > 1$ is the (constant) elasticity of substitution between the intermediate inputs, and the parameter $\zeta \geq 0$ captures the degree of increasing returns to specialization. The first-order condition for profit maximization yields the demand function for variety i , which is given by $y_{i,t} = \rho_{i,t}^{-\varepsilon} N_t^{\zeta(\varepsilon - 1) - 1} Y_t^C$, where $P_t = N_t^{\frac{1}{\varepsilon - 1} - \zeta} \left(\int_0^{N_t} p_{i,t}^{1 - \varepsilon} di \right)^{\frac{1}{1 - \varepsilon}}$. In a

symmetric equilibrium, the price index can be written as $P_t = N_t^{-\zeta} p_t$, implying $\rho_t = N_t^\zeta$. Thus, the degree of returns to specialization ω is equal to ζ . The two different CES-Benassy model variant differ in how we parameterize the variety effect ζ . In the CES-TrVE model, the variety effect is set equal to the value under the translog specification, i.e. $\omega = \zeta = \frac{1}{2}(\mu^p - 1)$. In the CES-NoVE model, $\omega = \zeta = 0$.

The log-linear model. This section presents the log-linearized model equations. A variable without time index denotes its steady-state value. A hat above a variable denotes the percentage deviation from its steady state.

- Consumption Euler equation:

$$\hat{\lambda}_t^C = E_t \hat{\lambda}_{t+1}^C + \hat{R}_t \quad (6.41)$$

- Shares Euler equation:

$$\hat{v}_t = E_t \left\{ \hat{\lambda}_{t+t}^C - \hat{\lambda}_t^C + \beta(1 - \delta)\hat{v}_{t+1} + (1 - \beta(1 - \delta))\hat{d}_{t+1} \right\} \quad (6.42)$$

- Capital Euler equation:

$$\hat{Q}_t = E_t \left\{ \hat{\lambda}_{t+t}^C - \hat{\lambda}_t^C + \beta(1 - \delta^K)\hat{Q}_{t+1} + (1 - \beta(1 - \delta^K))\hat{r}_{t+1}^K \right\} \quad (6.43)$$

- Lagrange multiplier associated with the household's budget constraint:

$$\begin{aligned} \hat{\lambda}_t^C = & \lambda_1 \left(\hat{V}_t + \hat{\chi}_t - b\beta E_t \{ \hat{V}_{t+1} + \hat{\chi}_{t+1} \} \right) - \lambda_2 \left(\hat{\lambda}_t^S + \hat{S}_t - b\beta E_t \{ \hat{S}_{t+1} + \hat{\lambda}_{t+1}^S \} \right) \\ & + \lambda_3 \left(\hat{C}_t - b\hat{C}_{t-1} - b\beta E_t \{ \hat{C}_{t+1} - b\hat{C}_t \} \right), \end{aligned} \quad (6.44)$$

where

$$\begin{aligned} \lambda_1 &= \frac{C(1-b)(1-\beta(1-\gamma))}{(1-b\beta)[(1-\beta(1-\gamma))C(1-b) - \gamma\psi SL^\eta]} \\ \lambda_2 &= \frac{\gamma\psi SL^\eta}{(1-b\beta)[(1-\beta(1-\gamma))C(1-b) - \gamma\psi SL^\eta]} \\ \lambda_3 &= \frac{\gamma\psi SL^\eta}{(1-b)(1-b\beta)[(1-\beta(1-\gamma))C(1-b) - \gamma\psi SL^\eta]} \end{aligned}$$

and the auxiliary variable

$$\hat{V}_t = -\frac{C}{C(1-b) - \psi L^\eta S} \left(\hat{C}_t - b\hat{C}_{t-1} \right) + \frac{\psi L^\eta S}{C(1-b) - \psi L^\eta S} \left(\eta \hat{L}_t + \hat{S}_t \right) \quad (6.45)$$

- Dynamics of S_t

$$\hat{S}_t = (1 - \gamma)\hat{S}_{t-1} + \frac{\gamma}{1-b}\hat{C}_t - \frac{\gamma b}{1-b}\hat{C}_{t-1} \quad (6.46)$$

- Lagrange multiplier associated with S_t :

$$\hat{\lambda}_t^S = \beta(1 - \gamma) E_t \left\{ \hat{\lambda}_{t+1}^S + \hat{S}_{t+1} - \hat{S}_t \right\} + (1 - \beta(1 - \gamma)) \left(\eta \hat{L}_t + \hat{V}_t + \hat{\chi}_t \right) \quad (6.47)$$

- Labor supply:

$$\hat{w}_t = \hat{\mu}_t^w + \hat{V}_t + \theta \hat{L}_t + \hat{S}_t - \hat{\lambda}_t^C + \hat{\chi}_t \quad (6.48)$$

- Optimal pricing equation:

$$\hat{\rho}_t = \hat{\mu}_t^p + \hat{m}c_t \quad (6.49)$$

- Price mark-up:

$$\hat{\mu}_t^p = -\xi \hat{N}_t, \quad (6.50)$$

where $\xi = 1 - \frac{1}{\mu^p}$ in the translog model and $\xi = 0$ in the CES models.

- Relative price:

$$\hat{\rho}_t = \omega \hat{N}_t, \quad (6.51)$$

where $\omega = \frac{1}{2}(\mu^p - 1)$ in the translog model, $\omega = \frac{1}{2}(\mu^p - 1)$ in the CES-TrVE model and $\omega = 0$ in the CES-NoVE model.

- Factor demand equation:

$$\hat{w}_t = \hat{Y}_t^C - \hat{L}_t^C - \hat{\mu}_t^p \quad (6.52)$$

$$\hat{r}_t^K = \hat{Y}_t^C - (\hat{K}_{t-1} + \hat{u}_t) - \hat{\mu}_t^p \quad (6.53)$$

- Total profit income:

$$\hat{D}_t \equiv \hat{N}_t + \hat{d}_t = \frac{1}{\mu^p - 1} \hat{\mu}_t^p + \hat{Y}_t^C \quad (6.54)$$

- Firm entry:

$$\hat{N}_{E,t} = \frac{\beta}{1+\beta} \mathbf{E}_t \hat{N}_{E,t+1} + \frac{1}{1+\beta} \hat{N}_{E,t-1} + \frac{1}{(1+\beta)\kappa_E} (\hat{v}_t - (\hat{w}_t - \hat{z}_t + \hat{f}_{E,t})) \quad (6.55)$$

- Investment in new firms:

$$\hat{I}_{E,t} = \hat{w}_t - \hat{z}_t + \hat{f}_{E,t} + \hat{N}_{E,t} \quad (6.56)$$

- Firm dynamics:

$$\hat{N}_t = (1 - \delta) \hat{N}_{t-1} + \delta \hat{N}_{E,t-1} \quad (6.57)$$

- Investment in physical capital:

$$\hat{I}_t = \frac{\beta}{1+\beta} \mathbf{E}_t \hat{I}_{t+1} + \frac{1}{1+\beta} \hat{I}_{t-1} + \frac{1}{\kappa(1+\beta)} \hat{Q}_t + \hat{u}_t^I \quad (6.58)$$

- Capital accumulation equation:

$$\hat{K}_t = (1 - \delta^K) \hat{K}_{t-1} + \delta^K \hat{I}_t - r^K \hat{u}_t + \delta^K \kappa(1 + \beta) \hat{u}_t^I \quad (6.59)$$

- Capital utilization:

$$\hat{u}_t = \frac{\delta_1}{\delta_2} (\hat{r}_t^K - \hat{Q}_t) \quad (6.60)$$

- Total investment:

$$\hat{T}I_t = \frac{I}{\hat{T}I} \hat{I}_t + \frac{vN_E}{\hat{T}I} \hat{I}_{E,t} \quad (6.61)$$

- Labor in entry:

$$\hat{L}_t^E = \hat{f}_{E,t} + \hat{N}_{E,t} - \hat{z}_t \quad (6.62)$$

- Aggregate production function:

$$\hat{Y}_t^C = \hat{\rho}_t + \alpha (\hat{z}_t + \hat{L}_t^C) + (1 - \alpha) (\hat{u}_t + \hat{K}_{t-1}) \quad (6.63)$$

- Gross domestic product:

$$\hat{Y}_t = \frac{Y^C}{Y} \hat{Y}_t^C + \frac{vN_E}{Y} (\hat{w}_t - \hat{z}_t + \hat{f}_{E,t} + \hat{N}_{E,t}) \quad (6.64)$$

- Goods market clearing:

$$\hat{Y}_t^C = \frac{C}{Y^C} \hat{C}_t + \frac{I}{Y^C} \hat{I}_t + \frac{G}{Y^C} \hat{G}_t \quad (6.65)$$

- Resource constraint:

$$\hat{Y}_t = \frac{wL}{Y} (\hat{w}_t + \hat{L}_t) + \frac{Nd}{Y} (\hat{N}_t + \hat{d}_t) + \frac{r^K K}{Y} (\hat{r}_t^K + \hat{K}_{t-1} + \hat{u}_t) \quad (6.66)$$

Shock processes

- Labor productivity:

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \varepsilon_t^z \quad (6.67)$$

- Entry costs:

$$\hat{f}_{E,t} = \rho_{f_E} \hat{f}_{E,t-1} + \varepsilon_t^{f_E} \quad (6.68)$$

- Investment-specific technology:

$$\hat{u}_t^I = \rho_I \hat{u}_{t-1}^I + \varepsilon_t^I \quad (6.69)$$

- Wage mark-up:

$$\hat{\mu}_t^w = \rho_\mu \hat{\mu}_{t-1}^w + \varepsilon_t^\mu + \nu \varepsilon_{t-1}^\mu \quad (6.70)$$

- Government spending:

$$\hat{G}_t = \rho_G \hat{G}_{t-1} + \varepsilon_t^G \quad (6.71)$$

6.B Appendix: Data

Table 6.8: Data source

Series ID	Description	Source
GDPC96	Real gross domestic product	BEA
PCND	Personal consumption expenditures: non-durable goods	BEA
PCESV	Personal consumption expenditures: services	BEA
PCDG	Personal consumption expenditures: durable goods	BEA
FPI	Fixed private investment	BEA
CBI	Change in private inventories	BEA
PRS85006033	Non-farm business hours worked index (2005=100)	BLS
PRS85006103	Non-farm business hourly compensation index (2005=100)	BLS
CES0500000008	Average hourly earnings of production	BLS
NBI	New business incorporations	SCB from BEA
CNP160V	Civilian noninstitutional population	BLS
GDPDEF	Gross domestic product: implicit price deflator	BEA
CPATAX	Corporate profits after tax with IVA and CCAdj	BEA

BEA: U.S. Bureau of Economic Analysis, BLS: U.S. Bureau of Labor Statistics, SCB: Survey of Current Businesses. Data series on profits are used in the sensitivity analysis.

Table 6.9: Construction of data series

Time Series	Construction	Description
$dl(GDP_t)$	$= dl \left(\frac{GDPC96_t}{CNP160V_t} \right)$	growth rate of real per capita GDP
$dl(CONS_t)$	$= dl \left(\frac{PCND_t + PCESV_t}{CNP160V_t \times GDPDEF_t} \right)$	growth rate of real per capita consumption
$dl(INV_t)$	$= dl \left(\frac{FPI_t + PCDG_t + CBI_t}{CNP160V_t \times GDPDEF_t} \right)$	growth rate of real per capita investment
$dl(WAGE1_t)$	$= dl \left(\frac{PRS85006103_t}{GDPDEF_t} \right)$	growth rate of first measure of real wage
$dl(WAGE2_t)$	$= dl \left(\frac{CES0500000008_t}{GDPDEF_t} \right)$	growth rate of second measure real wage
$l(HOURS_t)$	$= l \left(\frac{PRS85006033_t}{CNP160V_t} \right)$	logarithm of per capita hours worked
$dl(ENTRY_t)$	$= dl \left(\frac{NBI_t}{CNP160V_t} \right)$	growth rate of per capita new firms
$dl(PROFITS_t)$	$= dl \left(\frac{CPATAX_t}{CNP160V_t \times GDPDEF_t} \right)$	growth rate of real per capita profits
$dl(MARKUP_t)$	$= dl \left(\frac{GDPC96_t \times GDPDEF_t}{PRS85006103_t \times PRS85006033_t} \right)$	growth rate of inverse labor share

Note: The function l and dl stand for 100 times the demeaned logarithm and the demeaned log-difference, respectively. Data series on profits and mark-ups are used in the sensitivity analysis.

6.C Appendix: Robustness

Table 6.10: Results from the Bayesian estimation of the Translog model: Robustness checks

Parameters	Posterior distribution								
	Baseline Full sample		Baseline Sub sample		Baseline + Profit data Full sample		Baseline + Markup data Full sample		
	Mean	[5% , 95%]	Mean	[5% , 95%]	Mean	[5% , 95%]	Mean	[5% , 95%]	
<i>Structural parameters</i>									
Labor share in production	α	0.85	[0.81 , 0.90]	0.91	[0.86 , 0.96]	0.85	[0.81 , 0.90]	0.86	[0.81 , 0.90]
Labor utility	θ	3.64	[1.92 , 5.80]	4.14	[2.18 , 6.57]	3.62	[1.90 , 5.77]	3.60	[1.90 , 5.77]
Wealth elast. labor supply	γ	0.79	[0.63 , 0.93]	0.78	[0.59 , 0.93]	0.80	[0.63 , 0.93]	0.79	[0.63 , 0.93]
Consumption habit	b	0.76	[0.71 , 0.80]	0.77	[0.71 , 0.82]	0.75	[0.70 , 0.80]	0.75	[0.70 , 0.80]
Investment adj. cost	κ_I	3.82	[2.59 , 5.28]	3.81	[2.47 , 5.38]	3.83	[2.60 , 5.30]	3.93	[2.68 , 5.40]
Inv. elast. of capital util.	$\frac{\delta_2}{\delta_1}$	0.55	[0.33 , 0.89]	0.53	[0.32 , 0.84]	0.55	[0.33 , 0.90]	0.53	[0.33 , 0.85]
Price mark-up	μ^P	1.26	[1.16 , 1.36]	1.43	[1.31 , 1.55]	1.26	[1.16 , 1.36]	1.26	[1.17 , 1.36]
Entry adj. cost	κ_E	1.50	[1.09 , 1.99]	1.46	[1.01 , 2.00]	1.49	[1.08 , 1.98]	1.49	[1.07 , 1.97]
<i>Autocorrelation of shock processes</i>									
Labor productivity	ρ_z	0.96	[0.94 , 0.98]	0.98	[0.97 , 0.99]	0.96	[0.94 , 0.98]	0.96	[0.94 , 0.98]
Wage mark-up	ρ_μ	0.97	[0.95 , 0.98]	0.95	[0.93 , 0.97]	0.97	[0.95 , 0.98]	0.97	[0.95 , 0.98]
Invest. spec. tech.	ρ_I	0.24	[0.12 , 0.38]	0.19	[0.07 , 0.33]	0.25	[0.12 , 0.38]	0.24	[0.12 , 0.38]
Gov. spending	ρ_G	0.92	[0.88 , 0.94]	0.91	[0.87 , 0.95]	0.91	[0.88 , 0.94]	0.92	[0.88 , 0.95]
Entry cost	ρ_{fE}	0.95	[0.91 , 0.98]	0.98	[0.96 , 0.99]	0.95	[0.91 , 0.98]	0.95	[0.91 , 0.98]
<i>Standard deviation of innovations</i>									
Labor prod.	σ_{ε^z}	0.80	[0.70 , 0.90]	0.74	[0.65 , 0.85]	0.79	[0.70 , 0.90]	0.80	[0.70 , 0.90]
Wage mark-up	σ_{ε^μ}	4.30	[3.00 , 5.95]	4.98	[3.36 , 6.98]	4.28	[2.99 , 5.92]	4.28	[2.98 , 5.93]
Invest. spec. tech.	σ_{ε^I}	3.99	[2.39 , 6.54]	12.00	[4.91 , 26.14]	4.02	[2.39 , 6.69]	4.01	[2.42 , 6.50]
Preference	σ_{ε^X}	1.46	[1.07 , 1.91]	1.64	[1.17 , 2.20]	1.45	[1.07 , 1.90]	1.45	[1.07 , 1.88]
Gov. spending	σ_{ε^G}	1.87	[1.71 , 2.03]	1.78	[1.61 , 1.96]	1.87	[1.72 , 2.03]	1.87	[1.72 , 2.03]
Entry cost	$\sigma_{\varepsilon^{fE}}$	2.43	[1.96 , 2.97]	2.58	[2.12 , 3.08]	2.44	[1.97 , 2.99]	2.42	[1.95 , 2.97]
<i>Moving average parameter and loading coefficient</i>									
Wage mark-up shock	ν	0.41	[0.26 , 0.56]	0.27	[0.08 , 0.45]	0.41	[0.26 , 0.56]	0.41	[0.26 , 0.56]
Loading coefficient	λ	0.13	[0.06 , 0.19]	0.13	[0.04 , 0.22]	0.13	[0.06 , 0.19]	0.13	[0.06 , 0.19]

Note: Using a Random Walk Metropolis Hastings algorithm, we generate 2 chains of 2 Mio. parameter draws each. For each chain, we discard the first 1 Mio. draws and use the remaining draws to compute the posterior mean and percentiles. In our baseline estimation, the data sample ranges from 1964:Q1 to 2012:Q2 (full data sample) and the data set consists of data series on GDP, consumption, investment, hours worked, firm entry, and two wage measures. In the sub sample estimation, the data sample only ranges until 1998:Q3. In all estimations, we use the prior distributions of the baseline estimation given in table 6.4.

Table 6.11: Standard deviations, relative volatilities and amplification indicators using estimated parameters from the subsample estimation

	Volatility			Amplification			<i>Corr</i> (<i>X</i> , <i>N</i>)
	Translog	CES-TrVE	CES-NoVE	Total	CE	VE	
<i>Std of GDP relative to ... shock</i>							$X = Y^r$
labor prod.	1.0382 [1.0066, 1.0556]	0.9402 [0.9065, 0.9580]	0.9304 [0.9058, 0.9483]	11.6% [9.3%, 13.2%]	10.4% [8.6%, 12.7%]	1.1 % [-0.5%, 1.8%]	0.95
wage mark-up	0.1628 [0.1067, 0.2308]	0.1470 [0.0957, 0.2096]	0.1456 [0.0956, 0.2088]	11.9% [9.4%, 13.4%]	10.8% [8.9%, 12.9%]	1.0 % [-0.5%, 1.8%]	0.89
inv. spec. tech.	0.1031 [0.0315, 0.1821]	0.1104 [0.0337, 0.1941]	0.1113 [0.0336, 0.1966]	-7.4% [-8.2%, -5.1%]	-6.7% [-7.6%, -5.1%]	-0.8 % [-1.4%, 0.5%]	-0.59
preference	0.0594 [0.0437, 0.0691]	0.0593 [0.0460, 0.0684]	0.0594 [0.0453, 0.0687]	-0.0% [-5.3%, 2.5%]	0.2% [-7.0%, 2.5%]	-0.2 % [-0.7%, 2.1%]	0.08
gov. spending	0.0648 [0.0533, 0.0781]	0.0678 [0.0572, 0.0812]	0.0681 [0.0574, 0.0812]	-4.8% [-8.2%, -2.7%]	-4.4% [-8.2%, -2.4%]	-0.5 % [-0.8%, 0.3%]	-0.22
<i>Absolute std of GDP</i>							
All five shocks	0.0541 [0.0442, 0.0786]	0.0495 [0.0403, 0.0715]	0.0491 [0.0402, 0.0714]	10.3% [8.7%, 12.1%]	9.3% [8.1%, 11.7%]	0.9 % [-0.5%, 1.6%]	0.52
<i>Std of consumption relative to ... shock</i>							$X = C^r$
labor prod.	1.3014 [1.1901, 1.4355]	1.0971 [0.9802, 1.2366]	1.0287 [0.9186, 1.1631]	26.5% [21.3%, 32.3%]	18.6% [14.7%, 23.3%]	6.7 % [5.5%, 7.8%]	0.99
wage mark-up	0.1766 [0.1164, 0.2545]	0.1416 [0.0895, 0.2056]	0.1322 [0.0835, 0.1935]	33.5% [26.3%, 42.8%]	24.7% [18.9%, 32.6%]	7.1 % [5.8%, 8.3%]	0.98
inv. spec. tech.	0.1359 [0.0394, 0.2378]	0.1469 [0.0431, 0.2554]	0.1501 [0.0436, 0.2625]	-9.5% [-12.5%, -6.5%]	-7.5% [-10.6%, -5.4%]	-2.2 % [-3.0%, -0.6%]	-0.59
preference	0.2893 [0.2497, 0.3227]	0.2961 [0.2575, 0.3299]	0.2949 [0.2560, 0.3293]	-1.9% [-2.8%, -1.3%]	-2.3% [-3.2%, -1.8%]	0.4 % [0.1%, 0.7%]	-0.14
gov. spending	0.1407 [0.1124, 0.1747]	0.1201 [0.0909, 0.1559]	0.1160 [0.0875, 0.1517]	21.3% [13.7%, 31.0%]	17.1% [10.9%, 25.6%]	3.6 % [2.4%, 4.5%]	0.96
<i>Absolute std of consumption</i>							
All five shocks	0.0651 [0.0517, 0.1043]	0.0553 [0.0428, 0.0897]	0.0524 [0.0405, 0.0846]	24.4% [20.8%, 30.6%]	17.7% [14.6%, 22.6%]	5.6 % [5.0%, 7.0%]	0.60
<i>Std of total investment relative to ... shock</i>							$X = TI^r$
labor prod.	1.5248 [1.0374, 1.8562]	1.7096 [1.2268, 2.0340]	1.6826 [1.2207, 1.9967]	-9.4% [-16.5%, -5.1%]	-10.8% [-16.5%, -7.5%]	1.6 % [-0.2%, 2.7%]	0.71
wage mark-up	0.3509 [0.2230, 0.4958]	0.3778 [0.2448, 0.5403]	0.3703 [0.2416, 0.5326]	-5.2% [-10.6%, -2.2%]	-7.1% [-11.2%, -4.9%]	2.0 % [0.5%, 3.1%]	0.58
inv. spec. tech.	0.5215 [0.1654, 0.9305]	0.5113 [0.1614, 0.9157]	0.5130 [0.1612, 0.9202]	1.6% [0.6%, 3.2%]	2.0% [1.2%, 3.1%]	-0.3 % [-0.7%, 0.2%]	-0.08
preference	0.5774 [0.4818, 0.6933]	0.5880 [0.4911, 0.7084]	0.5809 [0.4844, 0.7035]	-0.6% [-1.8%, 0.4%]	-1.8% [-2.6%, -1.1%]	1.2 % [0.7%, 1.7%]	0.28
gov. spending	0.2818 [0.1916, 0.3266]	0.2941 [0.2051, 0.3395]	0.2889 [0.2026, 0.3341]	-2.5% [-6.7%, -0.1%]	-4.2% [-7.5%, -2.4%]	1.8 % [0.7%, 2.5%]	0.45
<i>Absolute std of total investment</i>							
All five shocks	0.1140 [0.0950, 0.1240]	0.1208 [0.1026, 0.1330]	0.1193 [0.1018, 0.1314]	-4.5% [-10.1%, -2.6%]	-5.6% [-10.3%, -4.5%]	1.2 % [0.2%, 2.2%]	0.25

Note: Volatilities and amplification measures are computed by simulating the model at the posterior mean reported in table 6.10. Figures in brackets give the corresponding fifth and ninety fifth percentiles. To compute these percentiles, we use the last 50,000 parameter draws (25,000 of each chain). For each parameter draw, we simulate the models and compute the volatility and amplification measures. The contemporaneous correlation in the last column is based on simulated data from the translog model at the posterior mean.

Table 6.12: Standard deviations, relative volatilities and amplification indicators using estimated parameters from the estimation with mark-up data

	Volatility			Amplification			<i>Corr</i> (<i>X</i> , <i>N</i>)
	Translog	CES-TrVE	CES-NoVE	Total	CE	VE	
<i>Std of GDP relative to ... shock</i>							$X = Y^r$
labor prod.	1.0019 [0.9509, 1.0304]	0.9334 [0.8930, 0.9614]	0.9175 [0.8786, 0.9451]	9.2% [6.7%, 10.9%]	7.3% [5.0%, 9.1%]	1.7 % [1.2%, 2.1%]	0.92
wage mark-up	0.1824 [0.1285, 0.2592]	0.1700 [0.1193, 0.2434]	0.1671 [0.1174, 0.2395]	9.1% [6.6%, 10.9%]	7.3% [4.9%, 9.1%]	1.7 % [1.2%, 2.1%]	0.93
inv. spec. tech.	0.2106 [0.1332, 0.3322]	0.2220 [0.1413, 0.3450]	0.2254 [0.1431, 0.3506]	-6.6% [-8.1%, -4.4%]	-5.1% [-6.7%, -3.1%]	-1.5 % [-1.8%, -1.1%]	-0.55
preference	0.0747 [0.0595, 0.0919]	0.0734 [0.0584, 0.0907]	0.0736 [0.0585, 0.0909]	1.6% [-0.1%, 3.5%]	1.9% [0.6%, 3.3%]	-0.3 % [-0.8%, 0.4%]	0.13
gov. spending	0.0632 [0.0526, 0.0751]	0.0645 [0.0541, 0.0764]	0.0649 [0.0543, 0.0768]	-2.7% [-5.1%, -0.3%]	-2.1% [-4.3%, -0.2%]	-0.6 % [-1.0%, -0.1%]	-0.17
<i>Absolute std of GDP</i>							
All five shocks	0.0519 [0.0437, 0.0679]	0.0486 [0.0408, 0.0641]	0.0478 [0.0402, 0.0630]	8.6% [6.3%, 10.1%]	6.8% [4.7%, 8.4%]	1.6 % [1.1%, 2.0%]	0.56
<i>Std of consumption relative to ... shock</i>							$X = C^r$
labor prod.	1.0768 [0.9221, 1.2289]	0.9320 [0.8005, 1.0771]	0.8872 [0.7642, 1.0270]	21.4% [14.6%, 26.5%]	15.5% [10.5%, 19.4%]	5.0 % [3.6%, 6.1%]	0.97
wage mark-up	0.2012 [0.1411, 0.2914]	0.1750 [0.1216, 0.2583]	0.1668 [0.1153, 0.2473]	20.6% [13.4%, 26.9%]	14.9% [9.4%, 19.8%]	5.0 % [3.5%, 6.1%]	0.97
inv. spec. tech.	0.2732 [0.1723, 0.4240]	0.2889 [0.1846, 0.4422]	0.2968 [0.1896, 0.4527]	-8.0% [-10.5%, -5.2%]	-5.4% [-7.6%, -3.2%]	-2.7 % [-3.4%, -1.9%]	-0.50
preference	0.2922 [0.2663, 0.3216]	0.2970 [0.2705, 0.3269]	0.2957 [0.2689, 0.3259]	-1.2% [-1.8%, -0.5%]	-1.6% [-2.2%, -1.0%]	0.5 % [0.3%, 0.7%]	-0.15
gov. spending	0.1448 [0.1215, 0.1717]	0.1312 [0.1087, 0.1594]	0.1281 [0.1060, 0.1563]	13.0% [7.7%, 17.6%]	10.4% [6.1%, 14.2%]	2.4 % [1.5%, 3.1%]	0.95
<i>Absolute std of consumption</i>							
All five shocks	0.0576 [0.0467, 0.0799]	0.0505 [0.0406, 0.0716]	0.0484 [0.0390, 0.0688]	18.9% [13.0%, 23.3%]	13.9% [9.3%, 17.2%]	4.4 % [3.3%, 5.4%]	0.55
<i>Std of total investment relative to ... shock</i>							$X = TI^r$
labor prod.	2.1944 [1.7275, 2.5336]	2.3274 [1.8844, 2.6394]	2.2571 [1.8347, 2.5570]	-2.8% [-7.2%, 0.8%]	-5.7% [-9.4%, -2.7%]	3.1 % [2.2%, 3.9%]	0.66
wage mark-up	0.3768 [0.2592, 0.5395]	0.4017 [0.2768, 0.5752]	0.3899 [0.2693, 0.5575]	-3.3% [-6.5%, -0.5%]	-6.2% [-8.7%, -3.8%]	3.0 % [2.3%, 3.8%]	0.68
inv. spec. tech.	1.2283 [0.7792, 1.9548]	1.2074 [0.7665, 1.9230]	1.2152 [0.7707, 1.9378]	1.1% [-0.0%, 2.2%]	1.7% [0.8%, 2.7%]	-0.7 % [-1.0%, -0.4%]	-0.11
preference	0.6002 [0.5326, 0.6911]	0.6115 [0.5415, 0.7046]	0.6025 [0.5325, 0.6955]	-0.4% [-1.6%, 0.9%]	-1.9% [-2.8%, -0.7%]	1.5 % [1.1%, 1.9%]	0.34
gov. spending	0.3123 [0.2444, 0.3601]	0.3219 [0.2540, 0.3687]	0.3143 [0.2481, 0.3608]	-0.6% [-2.9%, 1.5%]	-3.0% [-4.9%, -1.1%]	2.4 % [1.9%, 3.0%]	0.52
<i>Absolute std of total investment</i>							
All five shocks	0.1201 [0.1074, 0.1359]	0.1261 [0.1128, 0.1425]	0.1231 [0.1103, 0.1389]	-2.4% [-5.1%, -0.1%]	-4.7% [-6.9%, -2.9%]	2.4 % [1.8%, 3.1%]	0.43

Note: Volatilities and amplification measures are computed by simulating the model at the posterior mean reported in table 6.10. Figures in brackets give the corresponding fifth and ninety fifth percentiles. To compute these percentiles, we use the last 50,000 parameter draws (25,000 of each chain). For each parameter draw, we simulate the models and compute the volatility and amplification measures. The contemporaneous correlation in the last column is based on simulated data from the translog model at the posterior mean.

Table 6.13: Standard deviations, relative volatilities and amplification indicators using estimated parameters from the estimation with profit data

	Volatility			Amplification			<i>Corr</i> (<i>X</i> , <i>N</i>)
	Translog	CES-TrVE	CES-NoVE	Total	CE	VE	
<i>Std of GDP relative to ... shock</i>							$X = Y^r$
labor prod.	1.0056 [0.9570, 1.0335]	0.9341 [0.8962, 0.9618]	0.9183 [0.8823, 0.9454]	9.5% [6.8%, 11.1%]	7.7% [5.2%, 9.6%]	1.7 % [0.7%, 2.1%]	0.92
wage mark-up	0.1763 [0.1235, 0.2540]	0.1639 [0.1144, 0.2386]	0.1611 [0.1126, 0.2347]	9.4% [6.7%, 11.1%]	7.6% [5.0%, 9.7%]	1.7 % [0.6%, 2.1%]	0.93
inv. spec. tech.	0.2002 [0.0922, 0.3267]	0.2115 [0.0985, 0.3405]	0.2148 [0.0990, 0.3462]	-6.8% [-8.0%, -4.4%]	-5.4% [-6.8%, -3.1%]	-1.5 % [-1.8%, -0.5%]	-0.56
preference	0.0721 [0.0536, 0.0898]	0.0707 [0.0534, 0.0886]	0.0708 [0.0535, 0.0888]	1.8% [-1.1%, 4.0%]	2.0% [-0.9%, 3.6%]	-0.2 % [-0.7%, 0.6%]	0.14
gov. spending	0.0625 [0.0529, 0.0751]	0.0639 [0.0543, 0.0764]	0.0643 [0.0546, 0.0768]	-2.7% [-5.5%, -0.2%]	-2.1% [-4.9%, -0.2%]	-0.6 % [-0.9%, 0.0%]	-0.15
<i>Absolute std of GDP</i>							
All five shocks	0.0522 [0.0439, 0.0677]	0.0488 [0.0410, 0.0637]	0.0480 [0.0405, 0.0627]	8.8% [6.4%, 10.3%]	7.1% [4.8%, 9.0%]	1.6 % [0.6%, 2.0%]	0.56
<i>Std of consumption relative to ... shock</i>							$X = C^r$
labor prod.	1.0918 [0.9330, 1.2668]	0.9411 [0.8091, 1.0997]	0.8943 [0.7717, 1.0451]	22.1% [14.9%, 27.8%]	16.0% [10.7%, 20.5%]	5.2 % [3.7%, 6.3%]	0.97
wage mark-up	0.1954 [0.1347, 0.2881]	0.1692 [0.1153, 0.2539]	0.1609 [0.1091, 0.2435]	21.4% [13.6%, 29.6%]	15.5% [9.6%, 22.2%]	5.2 % [3.6%, 6.3%]	0.98
inv. spec. tech.	0.2589 [0.1227, 0.4158]	0.2744 [0.1322, 0.4347]	0.2820 [0.1351, 0.4453]	-8.2% [-11.1%, -5.5%]	-5.7% [-8.3%, -3.4%]	-2.7 % [-3.5%, -1.9%]	-0.50
preference	0.2901 [0.2613, 0.3195]	0.2950 [0.2666, 0.3245]	0.2935 [0.2650, 0.3233]	-1.2% [-2.1%, -0.5%]	-1.6% [-2.5%, -1.0%]	0.5 % [0.3%, 0.7%]	-0.15
gov. spending	0.1457 [0.1218, 0.1704]	0.1313 [0.1082, 0.1569]	0.1281 [0.1053, 0.1536]	13.7% [8.0%, 19.6%]	10.9% [6.3%, 16.0%]	2.5 % [1.6%, 3.2%]	0.95
<i>Absolute std of consumption</i>							
All five shocks	0.0583 [0.0473, 0.0802]	0.0510 [0.0411, 0.0717]	0.0488 [0.0393, 0.0687]	19.5% [13.1%, 25.2%]	14.3% [9.4%, 18.9%]	4.6 % [3.3%, 5.6%]	0.55
<i>Std of total investment relative to ... shock</i>							$X = TI^r$
labor prod.	2.1574 [1.5919, 2.5244]	2.2925 [1.7766, 2.6176]	2.2257 [1.7411, 2.5355]	-3.1% [-10.2%, 0.9%]	-5.9% [-11.5%, -2.6%]	3.0 % [1.5%, 3.9%]	0.66
wage mark-up	0.3603 [0.2492, 0.5185]	0.3846 [0.2682, 0.5539]	0.3736 [0.2615, 0.5382]	-3.6% [-8.2%, -0.3%]	-6.3% [-9.8%, -3.7%]	2.9 % [1.6%, 3.8%]	0.67
inv. spec. tech.	1.1486 [0.5220, 1.9399]	1.1295 [0.5081, 1.9112]	1.1368 [0.5092, 1.9255]	1.0% [-0.1%, 2.4%]	1.7% [0.7%, 2.8%]	-0.7 % [-1.0%, -0.3%]	-0.12
preference	0.5978 [0.5321, 0.6903]	0.6084 [0.5405, 0.7032]	0.5995 [0.5315, 0.6949]	-0.3% [-1.5%, 1.0%]	-1.8% [-2.7%, -0.7%]	1.5 % [1.0%, 1.9%]	0.33
gov. spending	0.3141 [0.2426, 0.3660]	0.3237 [0.2535, 0.3737]	0.3162 [0.2483, 0.3657]	-0.7% [-4.0%, 1.6%]	-3.0% [-5.6%, -0.9%]	2.4 % [1.6%, 2.9%]	0.51
<i>Absolute std of total investment</i>							
All five shocks	0.1200 [0.1066, 0.1345]	0.1259 [0.1128, 0.1412]	0.1231 [0.1105, 0.1379]	-2.5% [-6.8%, 0.0%]	-4.8% [-7.9%, -2.8%]	2.3 % [1.2%, 3.1%]	0.43

Note: Volatilities and amplification measures are computed by simulating the model at the posterior mean reported in table 6.10. Figures in brackets give the corresponding fifth and ninety fifth percentiles. To compute these percentiles, we use the last 50,000 parameter draws (25,000 of each chain). For each parameter draw, we simulate the models and compute the volatility and amplification measures. The contemporaneous correlation in the last column is based on simulated data from the translog model at the posterior mean.

Table 6.14: Results from the Bayesian estimation of the Translog and the CES models

Parameters	Prior distribution				Posterior distribution					
	Type	Mean	STD	Translog		CES-TrVE		CES-NoVE		
				Mean	[5% , 95%]	Mean	[5% , 95%]	Mean	[5% , 95%]	
<i>Structural parameters</i>										
Labor share in production	α	Beta	0.7	0.2	0.85	[0.81 , 0.90]	0.85	[0.83 , 0.88]	0.85	[0.83 , 0.88]
Labor utility	θ	Gamma	2.0	1.0	3.64	[1.92 , 5.80]	3.83	[2.11 , 5.95]	3.91	[2.17 , 6.06]
Wealth elast. labor supply	γ	Beta	0.5	0.2	0.79	[0.63 , 0.93]	0.79	[0.63 , 0.93]	0.79	[0.62 , 0.93]
Consumption habit	b	Beta	0.5	0.2	0.76	[0.71 , 0.80]	0.75	[0.70 , 0.80]	0.74	[0.69 , 0.79]
Investment adj. cost	κ_I	Gamma	4.0	1.0	3.82	[2.59 , 5.28]	3.70	[2.53 , 5.09]	3.61	[2.44 , 4.99]
Inv. elast. of capital util.	$\frac{\delta_2}{\delta_1}$	Igamma	1.0	1.0	0.55	[0.33 , 0.89]	0.55	[0.34 , 0.89]	0.56	[0.34 , 0.90]
Price mark-up	μ^P	Gamma	1.3	0.2	1.26	[1.16 , 1.36]	–	–	–	–
Entry adj. cost	κ_E	Gamma	4.0	1.0	1.50	[1.09 , 1.99]	1.56	[1.13 , 2.06]	1.52	[1.09 , 2.02]
<i>Autocorrelation of shock processes</i>										
Labor productivity	ρ_z	Beta	0.5	0.2	0.96	[0.94 , 0.98]	0.96	[0.94 , 0.98]	0.96	[0.94 , 0.98]
Wage mark-up	ρ_μ	Beta	0.5	0.2	0.97	[0.95 , 0.98]	0.97	[0.95 , 0.98]	0.97	[0.95 , 0.98]
Invest. spec. tech.	ρ_I	Beta	0.5	0.2	0.24	[0.12 , 0.38]	0.23	[0.11 , 0.36]	0.22	[0.10 , 0.35]
Gov. spending	ρ_G	Beta	0.5	0.2	0.92	[0.88 , 0.94]	0.92	[0.88 , 0.95]	0.92	[0.89 , 0.95]
Entry cost	ρ_{fE}	Beta	0.5	0.2	0.95	[0.91 , 0.98]	0.94	[0.89 , 0.98]	0.94	[0.89 , 0.98]
<i>Standard deviation of innovations</i>										
Labor prod.	$\sigma_{\varepsilon z}$	Igamma	2.0	1	0.80	[0.70 , 0.90]	0.79	[0.71 , 0.89]	0.80	[0.72 , 0.89]
Wage mark-up	$\sigma_{\varepsilon \mu}$	Igamma	2.0	1	4.30	[3.00 , 5.95]	4.39	[3.12 , 5.96]	4.43	[3.14 , 6.01]
Invest. spec. tech.	$\sigma_{\varepsilon I}$	Igamma	2.0	1	3.99	[2.39 , 6.54]	3.91	[3.21 , 4.69]	3.94	[3.25 , 4.71]
Preference	$\sigma_{\varepsilon \chi}$	Igamma	2.0	1	1.46	[1.07 , 1.91]	1.42	[1.06 , 1.85]	1.36	[1.01 , 1.75]
Gov. spending	$\sigma_{\varepsilon G}$	Igamma	2.0	1	1.87	[1.71 , 2.03]	1.87	[1.72 , 2.03]	1.87	[1.72 , 2.04]
Entry cost	$\sigma_{\varepsilon fE}$	Igamma	2.0	1	2.43	[1.96 , 2.97]	2.04	[1.57 , 2.55]	1.98	[1.53 , 2.49]
<i>Moving average parameter and loading coefficient</i>										
Wage mark-up shock	ν	Normal	0.0	0.2	0.41	[0.26 , 0.56]	0.42	[0.27 , 0.56]	0.42	[0.27 , 0.56]
Loading coefficient	λ	Normal	1.0	2.0	0.13	[0.06 , 0.19]	0.13	[0.06 , 0.19]	0.13	[0.06 , 0.19]

Note: Using a Random Walk Metropolis Hastings algorithm, we generate 2 chains of 2 Mio. parameter draws each. For each chain, we discard the first 1 Mio. draws and use the remaining draws to compute the posterior mean and percentiles. To estimate the CES models, we calibrate the steady state price mark-up at the posterior mean of the translog estimation. The data set ranges from 1964:Q1 to 2012:Q2 and consists of data series on GDP, consumption, investment, hours worked, firm entry, and two wage measures.

Table 6.15: Standard deviations, relative volatilities and amplification indicators using estimated translog and CES models

	Volatility			Amplification			<i>Corr</i>
	Translog	CES-TrVE	CES-NoVE	Total	CE	VE	(<i>X, N</i>)
<i>Std of GDP relative to ... shock</i>							$X = Y^r$
labor prod.	1.0006	0.9365	0.9202	8.7%	6.8%	1.8 %	0.92
wage mark-up	0.1811	0.1628	0.1599	13.2%	11.3%	1.8 %	0.93
inv. spec. tech.	0.2166	0.2078	0.2111	2.6%	4.3%	-1.6 %	-0.55
preference	0.0749	0.0766	0.0768	-2.4%	-2.3%	-0.2 %	0.14
gov. spending	0.0627	0.0608	0.0611	2.6%	3.2%	-0.6 %	-0.16
<i>Std of consumption relative to ... shock</i>							$X = C^r$
labor prod.	1.0705	0.9396	0.8939	19.8%	13.9%	5.1 %	0.97
wage mark-up	0.1995	0.1731	0.1649	21.0%	15.2%	5.0 %	0.97
inv. spec. tech.	0.2797	0.2695	0.2769	1.0%	3.8%	-2.7 %	-0.49
preference	0.2915	0.3023	0.3010	-3.2%	-3.6%	0.4 %	-0.15
gov. spending	0.1461	0.1384	0.1352	8.1%	5.6%	2.4 %	0.95
<i>Std of total investment relative to ... shock</i>							$X = TI^r$
labor prod.	2.2082	2.2929	2.2256	-0.8%	-3.7%	3.0 %	0.67
wage mark-up	0.3743	0.3580	0.3480	7.6%	4.6%	2.9 %	0.68
inv. spec. tech.	1.2574	1.1249	1.1317	11.1%	11.8%	-0.6 %	-0.12
preference	0.5985	0.6090	0.6005	-0.3%	-1.7%	1.4 %	0.34
gov. spending	0.3138	0.3152	0.3080	1.9%	-0.4%	2.3 %	0.52

Note: Volatilities and amplification measures are computed at the respective posterior mean given in table 6.14. Note that we are not able to give the probability bands for our amplification measures in the above specification. This would require to know the joint posterior distribution under the translog and the CES models. The contemporaneous correlation in the last column is based on simulated data from the translog model at the posterior mean.

Chapter 7

Summary and outlook

The main contribution of this thesis to the existing theoretical and empirical literature on New Keynesian Macroeconomics are as follows: First, we link the lag polynomial associated with news shocks to the class of cyclotomic polynomials. By doing so, we are able to analytically show that the roots of the lag polynomial are all equal in modulus and are independent of the length of the anticipation horizon. Hence, the destabilizing effects of news shocks are not related to the problem of nonfundamentalness. Second, we study the volatility implications of news shocks under optimal monetary policy and boundedly rational expectations. We find that the (volatility) effects of news shocks on the economy and, thus, their importance for business cycle fluctuations depend on the assumption of forward-looking rational expectations. Under boundedly rational expectations, news shocks may generate less volatility than unanticipated shocks of the same form. Third, we introduce partially anticipated monetary policy shocks for which the public has imperfect information about the shock process. We find that this type of shock may lead to a lower volatility than fully anticipated shocks of the same form. Fourth, we integrate a money and credit market into a baseline New Keynesian model based on a money-and-credit-in-the-utility approach. This framework enables us to study changes in the monetary base and in the refinancing rate. It gives implicitly rise to a credit channel as an additional transmission mechanism of monetary shocks in which the (current and future) bond and loan rate directly affect current goods demand. Finally, we estimate a medium-scale real business cycle model with endogenous firm entry for the U.S. economy in order to quantify the amplification mechanism associated with firm entry. The amplification mechanism works through the competition and the variety effect. Both effects are statistically significant and substantially amplify the volatility in output and consumption.

This thesis forms the basis for several possible directions of future research. The following two avenues seem particularly promising and are already in working process: First, over the last years central banks around the globe have made an effort to explain and communicate their future policy intentions. Since interest rates are at the zero lower

bound, central banks are forced to use unconventional instruments to stimulate the economy. In light of this, it would be interesting to study the dynamic effects of anticipated disturbances in the monetary base. Based on this thesis, a possible way would be to use a dynamic version of the New Keynesian model with money and credit markets developed in chapter 5 and extend it with news shocks. Second, another research idea in the context of anticipated monetary policy is to extend the analysis of chapter 4 and study the optimal communication strategy of the central bank. In particular, what is the optimal anticipation length of monetary policy shocks as response to anticipated and unanticipated disturbances?

Eidesstattliche Erklärung

Hiermit erkläre ich an Eides statt, dass ich meine Doktorarbeit „*News Shocks, Monetary Policy, and Amplification Effects in New Keynesian Macroeconomics*“ selbstständig und ohne fremde Hilfe angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehrenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen nach den mir angegebenen Richtlinien zitiert habe.

Kiel, 19. Oktober 2015

Sven Offick