

# Smooth Transition Regression Models in Finance

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# Chapter 1

## Overview

This thesis covers the further development of smooth transition regression models and their applications in finance. Smooth transition regression models are used to model nonlinearity of regime-switching type in empirical applications. Empirical application usually adopt the estimation framework suggested by Teräsvirta (1994). He proposed the empirical modelling cycle for smooth transition autoregressive (STAR) models consisting of specification, estimation and evaluation stages. Since then, the class of smooth transition regression models has been extended to cover a variety of specific properties of empirical applications like STR-GARCH models (Lundbergh and Teräsvirta, 1998), vector autoregressive (STR-VAR) models (Camacho, 2004), or panel regression (PSTR) models (González et al., 2005). Originally, smooth transition regression models are used with univariate transition functions, that is, a single transition variable is supposed to govern the transition between regimes. This commonly used technique was extended recently by Lof (2012) for smooth transition autoregressive models. He proposed the use of a multivariate transition function where the transition between regimes can be driven by more than one variable, each weighted by its estimated impact on the transition process. The weights of the transition variables sum up to unity which results in a new weighted transition variable which drives the regime switch.

In the first paper of my dissertation *The Relation Between Overreaction in Forecasts and Uncertainty: A Nonlinear Approach*, I transfer the use of multivariate transition functions to panel smooth transition regression models. Furthermore, in this single-authored paper I try to contribute to the debate whether forecasts are affected by uncertainty and if this is the case, whether uncertainty will tend to boost overreaction or underreaction. Overreaction with respect to the previous forecast is defined as overshooting the realized

value with a positive forecast change or undershooting the realized value with a negative forecast change. The paper uses oil price forecasts from the Survey of Professional Forecaster from the European Central Bank. The survey covers the expectations of oil price forecasters in the Euro area and their expectations on future oil price movements. In order to test for overreaction, I use the empirical framework from Amir and Ganzach (1998). I combine the framework with a panel smooth transition regression model which is supposed to capture possible regime-switches of forecasters. In a first step, I test with a univariate transition function whether uncertainty affects forecast changes. In a second step I enhance the transition function by additional potential transition variables. The multivariate transition functions is used to evaluate if the results change when I control for the presence of noisy signals and positive oil price returns in the transition function, two variables which are frequently found to affect forecast changes. The results indicate that oil price forecasters, at least in this data, are governed by overreaction. With increasing uncertainty, the overreaction is reduced leading to more cautious forecast changes. This finding is confirmed by the multivariate specification of the transition function in the panel smooth transition regression model.

The smooth transition regression framework can be generalized in further ways. Apart from multivariate transition functions a commonly found restriction is that the model is applied with homogeneous transitions. If a model with more than one regime-switching variable is used the transition between regimes can be either estimated with a single transition function for all regime-switching variables (homogeneous model) or with different transition functions (heterogeneous model) for each regime-switching variable. The model with a homogeneous transition function is often preferred because it limits the computational burden of an otherwise more flexible model. Applications with heterogeneous transition functions can be found for vector STAR models (de Dios Tena and Tremayne, 2009, Schleer and Semmler, 2013). The implementation and use of a panel smooth transition regression model with heterogeneous transition functions is the focus of the third chapter *The Role of a Changing Market Environment for Credit Default Swap Pricing*<sup>1</sup>. In this paper, Stefan Reitz and I estimate a heterogeneous panel smooth transition model with multivariate transition functions. Furthermore, we contribute to the literature concerned with time-varying influences of credit risks' driving

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<sup>1</sup>The paper is joint work with Stefan Reitz. My contribution consists of the programming and estimation of the models. We shared the writing of the paper.

forces. Credit default swaps (CDS) are a popular measure of credit risk for the specific entity. During the financial crisis and the following Euro area crisis credit risks show substantial swings and therefore became a major concern of monetary policy. For our study we use credit default swaps as a measure of credit risk. Our sample covers credit default swaps written on European firms listed in the EURO STOXX 50. We evaluate if the relation between CDS spreads and their standard pricing variables is nonlinear with respect to a changing market environment. The transition is modelled to be driven by a set of variables such as the ECB's systemic stress composite index, the Sentix index for current and future economic situation, and the VStoxx. As mentioned above, we allow the transition function of each pricing variable to be driven by a differently shaped transition function (heterogeneous model) with more than one transition variable (multivariate model). The results indicate the importance of nonlinearities in the pricing of risk derivatives during tranquil and turbulent times.

The previously introduced generalization of the smooth transition regression framework with heterogeneous transition functions increase the flexibility of the model. Using the heterogeneous smooth transition regression model instead of the homogeneous one is the cautionary choice, because estimating the homogeneous STR model when a heterogeneous specification is appropriate generally leads to biased parameter values. However, heterogeneous transition functions come at the price of convergence problems and increased estimation time, because for each regime-switching regressor a set of parameters specifying the nonlinear transition has to be estimated. A formal test might help to inform the research whether the parsimonious homogeneous STR is sufficient without sacrificing unbiased parameter estimates. In the third paper of my dissertation *Homogeneous vs. Heterogeneous Transition Functions in Smooth Transition Regressions - A LM-Type Test*<sup>2</sup> Stefan Reitz and I propose a test for the decision between homogeneous and heterogeneous transition functions. For the test we consider only the case where the transition variables are the same across all transition functions. This guarantees that the standard smooth transition regression model with homogeneous transition functions is nested within the heterogeneous model, a property we use to construct our LM test. We suggest to augment the specification step of the heterogeneous STR estimation procedure by a test for homogeneity against the heterogeneous STR

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<sup>2</sup>The paper is joint work with Stefan Reitz. My contribution consists of the programming and test development. We shared the writing of the paper.



alternative. We consider two versions of the test: a F-version of the test and a  $\chi^2$ -version. We use Monte Carlo simulations to show the empirical size and power of the test. We found the empirical size to be close to the nominal one and sufficient power against the alternative. The  $\chi^2$ -version, however, suffers from size distortion in small samples.

With its papers, this dissertation contributes to both the further development of the class of smooth transition regression models and its application in finance. The papers share the smooth transition regression model as common topic. In every paper, I try to contribute to the further development of the estimation framework. Furthermore, two of the three papers does not only focus on the estimation framework but also contribute to different problems in the finance literature.

## Chapter 2

# The Relation Between Overreaction in Forecasts and Uncertainty: A Nonlinear Approach

*Journal of Forecasting, 2016, 35(6): 564-572*

<http://dx.doi.org/10.1002/for.2402>

**Abstract** This paper examines overreaction of oil price forecasters. It takes into account impacts of uncertainty, measured by VSTOXX volatility; noisy signals, measured by oil price volatility; and oil price return on forecast changes. The panel smooth transition regression model is applied with different specifications of the transition function to account for nonlinear relations. Data on oil price expectations for different time horizons are taken from the European Central Bank Survey of Professional Forecasters. The results show that forecast changes are governed by overreaction. However, overreaction is markedly reduced when high levels of uncertainty prevail. On the other hand, noisy signals and positive oil price return tend to cause higher overreaction.

## Chapter 3

# The Role of a Changing Market Environment for Credit Default Swap Pricing

*with Stefan Reitz*<sup>1</sup>

*International Journal of Finance & Economics, 2016, 21(3): 209-223*

<http://dx.doi.org/10.1002/ijfe.1543>

**Abstract** This paper investigates the impact of a changing market environment on the pricing of credit default swaps (CDS) spreads written on debt from EURO STOXX 50 firms. A panel smooth transition regression reveals that parameter estimates of standard CDS-pricing variables are time varying depending on current values of a set of variables such as the European Central Bank's systemic stress composite index, the Sentix index for the current and future economic situation and the VStoxx. These variables describe the market's transition between different regimes, thereby reflecting the impact of substantial swings in agents' risk perception on CDS spreads. Overall, our results confirm the importance of nonlinearities in the pricing of risk derivatives during tranquil and turbulent times.

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<sup>1</sup>Note: This project has received funding from the European Union's Seventh Framework Programme for research, technological development and demonstration under grant agreement no. 612955.

## Chapter 4

# Homogenous vs. Heterogenous Transition Functions in Smooth Transition Regressions – A LM-Type Test

*with Stefan Reitz*

## 4.1 Introduction

Since its introduction, the class of smooth transition regression (STR) models has become increasingly popular in empirical research. The standard modelling framework for smooth transition models goes back to Teräsvirta (1994) originally suggesting a framework for a (homogeneous) smooth transition autoregressive model (STAR). Since then, this class of models has been extended to cover a variety of specific properties of empirical applications. Amongst others, smooth transition functions have been introduced in Vector Autoregressive (STR-VAR) models (Camacho, 2004), GARCH (STR-GARCH) models (Lundbergh and Teräsvirta, 1998), or panel regression (PSTR) models (González et al., 2005).

STR models with more than one regime-switching regressors can be build with either homogeneous or heterogeneous<sup>1</sup> transition functions. STR models with homogeneous transition functions use the same transition function for each regime-switching regressor. On the other hand, heterogeneous transition functions allow for different transition functions across regressors with either the same or a different set of transition variables. In our case heterogeneous transition functions across the model's regressors arise from differing parameters for a given set of transition variables. In the context of panel smooth transition models, Leppin and Reitz (2015) allow for heterogeneous transition functions. Albeit heterogeneous STR model are not common in panel smooth transition models, the idea is not new in the context of Vector STAR models. As mentioned in van Dijk et al. (2002), p.8 "It is straightforward to generalize the model to incorporate equation-specific transition functions ... and thereby allow for equation-specific regime-switching". Applications of Vector STAR models with equation specific transition functions can be found in de Dios Tena and Tremayne (2009) or Schleer and Semmler (2015). Teräsvirta and Yang (2014) outlines linearity testing for Vector Star models.

The standard STR model with common transition functions is nested within the heterogeneous STR model, a property we use to construct our LM test. Of course, the application of a more flexible heterogeneous STR model is the cautionary choice, because estimating the homogeneous STR model when a heterogeneous specification is appropriate generally leads to biased parame-

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<sup>1</sup>The use of heterogeneous nonlinearity in the context of STR model is not totally unambiguous. Another characterization of heterogeneous nonlinearity is used by Anderson and Vahid (1998). They derive a test against common nonlinear transition in multivariate regressions.

ter values. However, heterogeneous transition functions come at the price of convergence problems and increased estimation time, because for each regime-switching regressor a set of parameters specifying the nonlinear transition has to be estimated. While computation time is not a concern in small samples, it becomes an important factor with increasing sample size and increasing number of regime-switching parameters. Our LM test is constructed to inform the research whether the parsimonious homogeneous STR is sufficient without sacrificing unbiased parameter estimates. Thus, we suggest to augment the specification step of the heterogeneous STR estimation procedure by a test for homogeneity against the heterogeneous STR alternative.

The model specification step of STR models generally consists of a set of linearity tests against the nonlinear alternative. The usual test is the Taylor expansion-based linearity test from Luukkonen et al. (1988). Monte Carlo-based tests for linearity are available as well, see for example Hansen (1996) and Hansen (1999). A comparison of the tests can be found in González and Teräsvirta (2006). The logic of these tests also allow for the post-estimation model evaluation. For instance, Eitrheim and Teräsvirta (1996) proposed a Taylor expansion-based test for no-remaining nonlinearity.<sup>2</sup>

Aside from being a useful tool in the specification step of the STR modelling cycle our test may also be applied in the final model evaluation stage as a misspecification test after the estimation of a homogeneous STR model. In this case, the test determines the adequateness of the estimated model specification from ex post. Since the test for multivariate transition functions requires the estimation of the model under the null of homogeneity it can be seen as well as a misspecification test after the estimation of a standard STR model with a common transition function.

During the Monte Carlo simulation we examine first the power and size of the test based on estimates from nonlinear least squares (NLS). The second part is dedicated to the local power curves of the test.

The remainder of the paper is as follows. Section 2 introduces the test against multivariate transition functions. Section 3 presents Monte Carlo simulations for the properties of the test. Section 4 concludes.

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<sup>2</sup>The authors also provide size and power simulations. Another evaluation of the test's properties can be found in Lundbergh and Teräsvirta (1998)

## 4.2 Test against Multivariate Transition Functions

The homogeneous STR model for time  $t = 1, \dots, T$  with  $k > 1$  exogenous regime-switching regressor variables  $x_{k,t}$  and a single transition function  $g_t$  is defined as

$$y_t = \sum_{k=1}^K \beta_{0,k} x_{k,t} + \sum_{k=1}^K \beta_{1,k} x_{k,t} g_{k,t}(q_t; \gamma, c) + u_t, \quad (4.1)$$

with a unique logistic transition function

$$g_{k,t}(q_t; \gamma, c) = (1 + \exp(-\gamma(q_t - c)))^{-1}, \quad (4.2)$$

which is used for all  $K$  regime-switching variables. The parameter  $c$  is a location parameter,  $\gamma$  is the speed of transition between regimes and  $q_t$  is the single transition variable. The heterogeneous STR differs from the previous model by allowing for regressor-specific transition functions

$$g_{k,t}(q_t; \gamma_k, c_k) = (1 + \exp(-\gamma_k(q_t - c_k)))^{-1}, \quad (4.3)$$

where the parameters  $\gamma_k$  and  $c_k$  are regressor-specific. The transition variable  $q_t$  is restricted to be the same across all transition functions.<sup>3</sup> The heterogeneous STR model boils down to the homogeneous alternative, if we restrict the parameters  $\gamma_k, c_k$  to be equal across all transition functions. In other words, the homogeneous STR model is nested in the heterogeneous STR model, which allows the construction of a formal test.

To derive the test assume that the data generating function of a variable  $y$  is driven by a smooth transition-type function with  $k > 1$  exogenous regressors  $x_{k,t}$  and  $K$  different transition functions  $g_{k,t}$  as specified in eq. (4.1) and eq. (4.3). All  $K$  transition functions depend on the same transition variable  $q_t$ . We further assume that the heterogeneous nature of the transition functions is ignored and a standard STR model with a single transition function  $g_t$  is estimated, i.e. the restriction is imposed that  $g = g_{1,t} = \dots = g_{K,t}$  which results

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<sup>3</sup>Otherwise no test for parameter equality is needed, because the transition functions would differ by definition.

in

$$y_t = \sum_{k=1}^K \beta_{0,k}^* x_{k,t} + \sum_{k=1}^K \beta_{1,k}^* x_{k,t} g_t(q_t; \gamma, c) + u_t. \quad (4.4)$$

Estimating equation (4.4) results in biased parameter estimates since  $\beta^* \neq \beta$ . If the error term in equation (4.4) is normally distributed a LM test can be constructed by evaluating the conditional log-likelihood function for observations  $t = 1, \dots, T$

$$l_t(\theta_{H_0}) = - (1/2) \log 2\pi - (1/2) \log \sigma^2 - (1/2\sigma^2) \left( y_t - \sum_{k=1}^K \beta_{0,k} x_{k,t} - \sum_{k=1}^K \beta_{1,k} x_{k,t} g_{k,t}(q_t; \gamma, c) \right)^2, \quad (4.5)$$

with  $\theta = \beta_{0,k}, \beta_{1,k}, \gamma, c$ , under the null hypothesis

$$H_0 : \gamma = \gamma_1 = \dots = \gamma_k, c = c_1 = \dots = c_k \quad (4.6)$$

The partial derivatives of the log-likelihood evaluated under the  $H_0$  with respect to the parameters  $\theta_{H_0}$  are

$$\frac{\partial \hat{l}_t(\theta_{H_0})}{\partial \beta_{0,k}} = (1/\sigma^2) x_{k,t} \quad (4.7)$$

$$\frac{\partial \hat{l}_t(\theta_{H_0})}{\partial \beta_{1,k}} = (1/\sigma^2) x_{k,t} \hat{g}_{k,t} \quad (4.8)$$

$$\frac{\partial \hat{l}_t(\theta_{H_0})}{\partial \gamma} = (1/\sigma^2) \beta_{1,k} x_{k,t} \left( (1+w)^{-2} w (q - \hat{c}) \right) \quad (4.9)$$

$$\frac{\partial \hat{l}_t(\theta_{H_0})}{\partial c} = (1/\sigma^2) \beta_{1,k} x_{k,t} \left( -(1+w)^{-2} w \hat{\gamma} \right), \quad (4.10)$$

with  $w = \exp(-\hat{\gamma}(q_t - \hat{c}))$ . Following Engle (1984) we reformulate the test of parameter equality to a test for omitted variables, where the omitted variables are simply the derivatives of the log likelihood with respect to the restricted parameters.

This can be illustrated more directly with the use of Taylor Series approximations. Starting from equation (4.11)

$$y_t = \sum_{k=1}^K \beta_{0,k} x_{k,t} + \sum_{k=1}^K \beta_{1,k} x_{k,t} g_{k,t}(q_t; \gamma_k, c_k) + u_t, \quad (4.11)$$



it is possible to derive a reformulation of the test for the homogeneous smooth transition regression model against the alternative of a heterogeneous STR model. We expand equation (4.11) with the terms  $\beta_{1,k}x_{k,t}g_t(q_t; \gamma, c)$  to derive

$$y_t = \sum_{k=1}^K \beta_{0,k}x_{k,t} + \sum_{k=1}^K \beta_{1,k}x_{k,t}g_t(q_t; \gamma, c) + \sum_{k=1}^K \beta_{1,k}x_{k,t}[g_{k,t}(q_t; \gamma_k, c_k) - g_t(q_t; \gamma, c)] + u_t. \quad (4.12)$$

If the transition functions are in fact equal across the regressors, the  $\beta_{1,k}x_{k,t}[g_{k,t}(q_t; \gamma_k, c_k) - g_t(q_t; \gamma, c)]$  would cancel out ( $\gamma_k = \gamma, c_k = c$ ), which leaves us with the homogeneous STR model. In case of different transition functions the two terms  $\beta_{1,k}x_{k,t}g_t(q_t; \gamma, c)$  in equation (4.12) cancel out and we are back with equations (4.1). Thus, the rejection of  $H_0$  implies that at least one parameter is not equal across transition functions. With given estimates of parameters  $\gamma$  and  $c$  under the null we can reformulate equation (4.12) to test for the joint hypothesis of  $\gamma = \gamma_1 = \dots = \gamma_k$  and  $c = c_1 = \dots = c_k$  without estimating the model under the alternative. We approximate the terms  $g_{k,t}(q_t; \gamma_k, c_k) - g_t(q_t; \gamma, c)$  by Taylor series expansions with respect to  $c$  and  $\gamma$ . The approximation is located around the estimated values of  $c$  and  $\gamma$  under the  $H_0$ , denoted by  $c_{H0}$  and  $\gamma_{H0}$

$$y_t = \sum_{k=1}^K \beta_{0,k}x_{k,t} + \sum_{k=1}^K \beta_{1,k}x_{k,t}g_t(q_t; \gamma, c) + \sum_{k=1}^K \beta_{1,k}x_{k,t}[g_{k,t}(q_t; \gamma_k, c_k) - g_t(q_t; \gamma, c)] + u_t. \quad (4.13)$$

After a first-order Taylor series approximation we get

$$y_t = \sum_{k=1}^K \beta_{0,k}x_{k,t} + \sum_{k=1}^K \beta_{1,k}x_{k,t}g_t(q_t; \gamma, c) + \sum_{k=1}^K \beta_{1,k}x_{k,t} \left[ g(q; \gamma_{H0}; c_{H0}) + \frac{\partial g}{\partial \gamma_k} \Big|_{\substack{\gamma_{H0} \\ c_{H0}}} (\gamma_k - \gamma_{H0}) + \frac{\partial g}{\partial c_k} \Big|_{\substack{\gamma_{H0} \\ c_{H0}}} (c_k - c_{H0}) - g(q; \gamma_{H0}; c_{H0}) - \frac{\partial g}{\partial \gamma} \Big|_{\substack{\gamma_{H0} \\ c_{H0}}} (\gamma - \gamma_{H0}) - \frac{\partial g}{\partial c} \Big|_{\substack{\gamma_{H0} \\ c_{H0}}} (c - c_{H0}) \right]. \quad (4.14)$$

Since we approximate both  $g_{k,t}(q_t; \gamma_k, c_k)$  and  $g_t(q_t; \gamma, c)$  around the same values  $(\gamma_{H0}; c_{H0})$ , the terms  $g(q; \gamma_{H0}; c_{H0})$  of the Taylor approximations cancel out. We

fill in the required derivations of the transition function, which can be found in the appendix, and set  $w = \exp((-\gamma_{h0})(q_t - c_{h0}))$  to derive<sup>4</sup>

$$\begin{aligned} y_t &= \sum_{k=1}^K \beta_{0,k} x_{k,t} + \sum_{k=1}^K \beta_{1,k} x_{k,t} g_t(q_t; \gamma, c) \\ &+ \sum_{k=1}^K \beta_{1,k} x_{k,t} \left[ -1(1+w)^{-2} w(-q_t + c_{h0})(\gamma_k - \gamma_{h0}) - (1+w)^{-2} w \gamma_{h0} (c_k - c_{h0}) \right. \\ &\left. + (1+w)^{-2} w(-q_t + c_{h0})(\gamma - \gamma_{h0}) + (1+w)^{-2} w \gamma_{h0} (c - c_{h0}) \right] + u_t \end{aligned}$$

which can be further aggregated to

$$\begin{aligned} y_t &= \sum_{k=1}^K \beta_{0,k} x_{k,t} + \sum_{k=1}^K \beta_{1,k} x_{k,t} g_t(q_t; \gamma, c) + \sum_{k=1}^K \beta_{1,k} x_{k,t} \left\{ [(\gamma - \gamma_{h0}) + (\gamma_{h0} - \gamma_k)] \right. \\ &\left. (1+w)^{-2} w(-q_t + c_{h0}) + [(c - c_{h0}) + (c_{h0} - c_k)] (1+w)^{-2} w \gamma_{h0} \right\} + u_t. \end{aligned}$$

The final equation is

$$y_t = \sum_{k=1}^K \beta_{0,k} x_{k,t} + \sum_{k=1}^K \beta_{1,k} x_{k,t} g_t(q_t; \gamma, c) + \sum_{k=1}^K \delta_k^a z_{k,t}^a + \sum_{k=1}^K \delta_k^b z_{k,t}^b + u_t \quad (4.15)$$

with

$$\begin{aligned} z_{k,t}^a &= (1+w)^{-2} w \gamma_{h0} x_{k,t} \\ z_{k,t}^b &= (1+w)^{-2} w (c_{h0} - q_t) x_{k,t} \\ \delta_k^a &= \beta_{1,k} (c - c_k) \\ \delta_k^b &= \beta_{1,k} (\gamma - \gamma_k). \end{aligned}$$

Testing for equality of parameters in the transition function can be carried out by testing the joint significance of the  $2(K-1)$  additional parameters  $\delta_k^a$  and  $\delta_k^b$  in equation (4.15). Therefore, the null hypothesis is reformulated as an omitted variable test. The associated LM statistic (in matrix notation) can be

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<sup>4</sup>Note that  $w$  is the same irrespectively if the transition function with common parameters  $(\gamma, c)$  or  $k$  individual parameters  $(\gamma_k, c_k)$  is considered. Both are evaluated at the estimated values under the  $H_0$ .

written as

$$LM = \hat{\sigma}^{-2} \hat{u}' Z (Z'Z - Z'V(V'V)^{-1}V'Z)^{-1} Z' \hat{u}, \quad (4.16)$$

where  $\hat{\sigma}^2 = \hat{u}'\hat{u}/T$  and  $\hat{u}$  is the  $(T \times 1)$  vector of residuals obtained from the model estimation under the null hypothesis. Furthermore,  $V = (X, D, A_1, A_2)$  with  $X = (x_1, \dots, x_K)$  where  $x_k$  are  $(T \times 1)$  vectors. Moreover,  $D = (x_1 g(q, \hat{\gamma}, \hat{c}), \dots, x_K g(q, \hat{\gamma}, \hat{c}))$  and  $A_1 = (\beta_{1,1} x_1 (\partial \hat{g} / \partial \gamma), \dots, \beta_{1,K} x_K (\partial \hat{g} / \partial \gamma))$ ,  $A_2 = (\beta_{1,1} x_1 (\partial \hat{g} / \partial c), \dots, \beta_{1,K} x_K (\partial \hat{g} / \partial c))$ . The matrix  $Z$  contains the omitted variables represented by the partial derivatives  $Z = (\beta_{1,1} x_1 (\partial \hat{g} / \partial \gamma_1), \dots, \beta_{1,K-1} x_{K-1} (\partial \hat{g} / \partial \gamma_{K-1}), \beta_{1,1} x_1 (\partial \hat{g} / \partial c_1), \dots, \beta_{1,K-1} x_{K-1} (\partial \hat{g} / \partial c_{K-1}))$ . Note that the derivations in matrix  $Z$  are now with respect to  $\gamma_k$  and  $c_k$  instead of  $\gamma$  and  $c$ . We only include up to  $2(K - 1)$  regressors since one restriction is redundant. To see this, note that we could easily reformulate  $H_0$  to  $\gamma_1 = \gamma_K, \gamma_2 = \gamma_K, \dots, \gamma_{K-1} = \gamma_K$  and  $c_1 = c_K, c_2 = c_K, \dots, c_{K-1} = c_K$ . Under the null hypothesis the LM statistic in (4.16) is asymptotically distributed as  $\chi^2(2(K - 1))$ . A F-version of the test is available, which is distributed as  $F(2(K - 1), T - 2k - 2 - (k - 1))$ . The F-version of the test has better size properties in small samples.

The test can be easily performed using the following steps

1. Estimate the model under the  $H_0$ , compute the residuals  $\hat{u}$  and the sum of squared residuals  $SSR_0$
2. Regress  $\hat{u}$  on  $X, D, A_1, A_2, Z$  as defined above and compute the sum of squared residuals  $SSR_1$
3. Compute the LM-test which is asymptotically  $\chi^2$ -distributed

$$LM = T(SSR_0 - SSR_1)/SSR_0$$

or the F-test

$$F = \frac{(SSR_0 - SSR_1)/(k - 1)}{SSR_1/(T - 2k - 2 - (k - 1))}.$$

### 4.3 Monte Carlo Simulation

In this section, we report the results from different Monte Carlo simulations.

### 4.3.1 Size and Power

We start with size and power simulations of the test against heterogeneous transition functions. We estimate the parameters  $\gamma$  and  $c$  under the  $H_0$  with nonlinear least squares<sup>5</sup> or maximum likelihood. Furthermore, we considered a version of the test where we estimate the parameters by a grid search. However, due to poor results we disregard this version in the paper.

The model investigated here has  $K = 2$  exogenous regressors  $x_k$ . The variables  $x_{k,t}$  are generated using a uniform(-10, 10) distribution. We set  $\beta_{0,k} = 1$  and  $\beta_{1,k} = -1$ .

$$y_t = x_{1,t} + x_{2,t} - x_{1,t}g_t(q_t; \gamma_1, c_1) - x_{2,t}g_t(q_t; \gamma_2, c_2) + u_t. \quad (4.17)$$

The values for the transition variable  $q_t$  are drawn from a normal (2,1) distribution. The number of observations are 50, 100, 200, or 1,000, and the error term  $u_t$  is either drawn from a  $N(0, 1^2)$  distribution, a  $N(0, 0.1^2)$  distribution or a  $N(0, 0.01^2)$  distribution. We carried out 5000 replications if the parameters under the  $H_0$  are estimated by NLS. The nominal significance level of the test is set to 5%. The following Table 4.1 contains the results for the power simulations using estimates of the parameters from NLS under the null hypothesis.

The  $\chi^2$ -test and the F-test have no problems to identify correctly multivariate transition functions when the transition functions for  $k = 1$  and  $k = 2$  differ with respect to both parameters  $c$  and  $\gamma$ . This holds for any combination of  $T$  given the error term variance is either  $0.1^2$  or  $0.01^2$ . In the case of an error term with variance  $\sigma^2 = 1$ , the test has problem regarding the rejection of the false null hypothesis. The reason is the relatively small difference between the two transition functions. As we will show in the section on local power curves, higher differences are required for an error term variance of 1. The specification where differences occur only with respect to the location parameter  $c$  yields similar results. If the transition functions differ with respect to the speed of transition  $\gamma$ , the test has less power. For the specification where the error term follows a  $N(0, 0.1^2)$  distribution, the power is markedly reduced in small samples. The results indicate that over 200 observations are required to reliably reject the false null hypothesis in this case. For a  $N(0, 1^2)$  distribution and a difference of 0.1 between  $\gamma_1$  and  $\gamma_2$ , the test shows nearly no power against the alternative.

The empirical size of the test when we use nonlinear least squares to estimate

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<sup>5</sup>We implemented the estimator in Stata and C++. We used components of the C++ libraries ‘‘Armadillo’’ by Sanderson (2010) and ‘‘GNU Scientific Library’’ by Galassi et al. (2009).

Table 4.1: Empirical power:  $H_0$  estimated with STR model

	T=50		T=100		T=200		T=1000	
	$\chi^2$	F	$\chi^2$	F	$\chi^2$	F	$\chi^2$	F
$\gamma_1 = 2.1, \gamma_2 = 2, c_1 = 2.1, c_2 = 2$								
$\sigma^2 = 0.01$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\sigma^2 = 0.1$	0.960	0.946	1.000	1.000	1.000	1.000	1.000	1.000
$\sigma^2 = 1$	0.081	0.058	0.100	0.087	0.129	0.122	0.458	0.456
$\gamma_1 = \gamma_2 = 2, c_1 = 2.1, c_2 = 2$								
$\sigma^2 = 0.01$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\sigma^2 = 0.1$	0.939	0.917	1.000	1.000	1.000	1.000	1.000	1.000
$\sigma^2 = 1$	0.083	0.063	0.087	0.078	0.113	0.106	0.417	0.415
$\gamma_1 = 2.1, \gamma_2 = 2, c_1 = c_2 = 2$								
$\sigma^2 = 0.01$	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$\sigma^2 = 0.1$	0.159	0.123	0.263	0.243	0.477	0.463	0.996	0.996
$\sigma^2 = 1$	0.063	0.047	0.055	0.048	0.061	0.058	0.077	0.075

Note: Rejection frequencies of the  $H_0$ .

Table 4.2: Empirical size

	T=50		T=100		T=200		T=1000	
	$\chi^2$	F	$\chi^2$	F	$\chi^2$	F	$\chi^2$	F
<i><math>H_0</math> estimated with STR model</i>								
$\sigma^2 = 0.01$	0.070	0.051	0.058	0.050	0.052	0.048	0.052	0.050
$\sigma^2 = 0.1$	0.067	0.049	0.059	0.050	0.057	0.054	0.049	0.049
$\sigma^2 = 1$	0.067	0.051	0.062	0.054	0.050	0.047	0.049	0.048

Note: Rejection frequencies of the  $H_0$ .

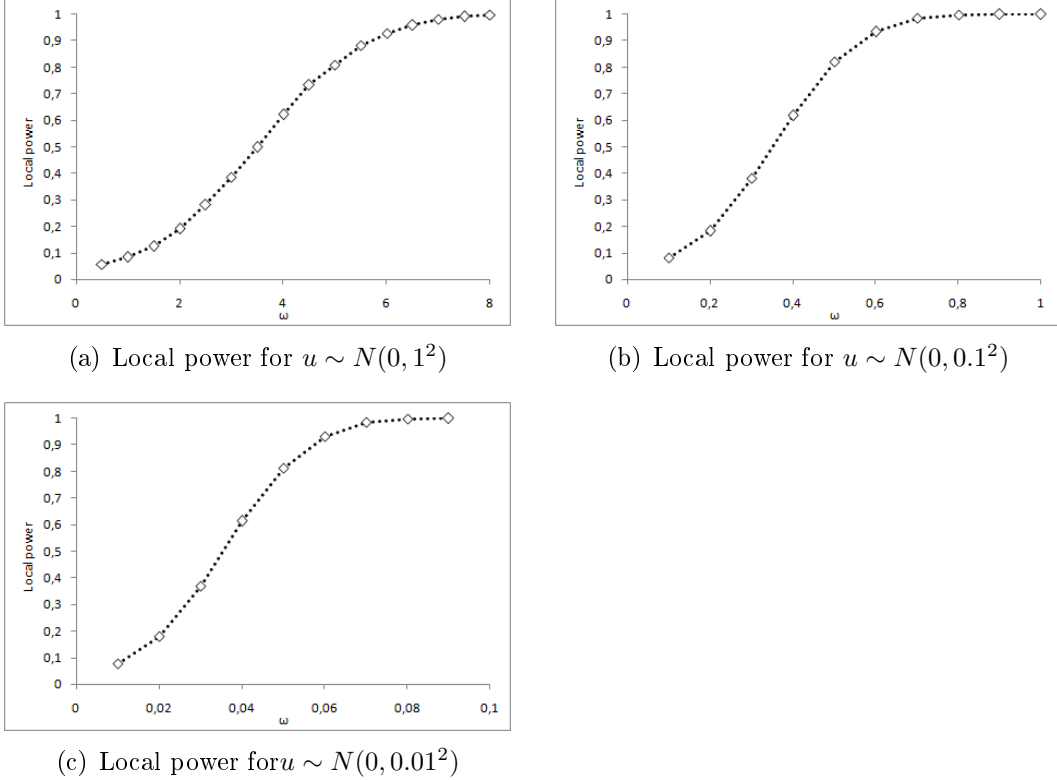
the model under the null hypothesis is shown in Table 4.2. For low numbers of observations, the  $\chi^2$ -test is slightly oversized. This well-known property of the  $\chi^2$ -test in small samples is overcome by using the F-version of the test. For the F-test, the empirical size is always close to the nominal one.

### 4.3.2 Local Power Curves

In this section we simulate local power curves by using Monte Carlo simulation. In order to investigate for which values of  $c_k$  and  $\gamma_k$  the test for homogeneous transition functions has power against the alternative, we generated a model set-up with  $K = 2$  exogenous regressors for differences in  $c$

$$y_t = x_{1,t} + x_{2,t} - x_{1,t}g_{1,t}(q_t; \gamma_1 = 2, c_1 = 2 + \omega/\sqrt{T}) - x_{2,t}g_{2,t}(q_t; \gamma_2 = 2, c_2 = 2) + u_t \quad (4.18)$$

Figure 4.1: Local power against heterogeneous transition functions ( $c$ )



Note: Local power curves for  $c$ . With  $\gamma = 2$ ,  $\beta_{0,k} = 1$ ,  $\beta_{1,k} = -1$  and  $T = 1000$ .

and differences in  $\gamma$

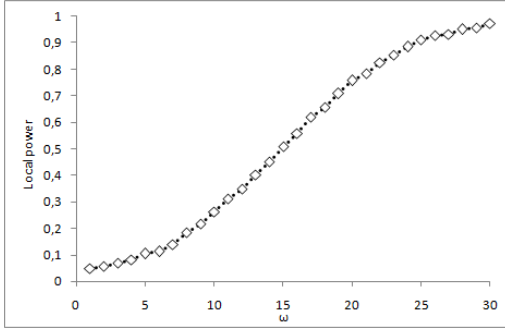
$$y_t = x_{1,t} + x_{2,t} - x_{1,t}g_{1,t}(qt; \gamma_1 = 2 + \omega/\sqrt{T}, c_1 = 2) - x_{2,t}g_{2,t}(qt; \gamma_2 = 2, c_2 = 2) + u_t \quad (4.19)$$

We consider three different error terms:  $u \sim N(0, 1^2)$ ;  $u \sim N(0, 0.1^2)$ ;  $u \sim N(0, 0.01^2)$ . For each scenario, we run 5000 repetitions.

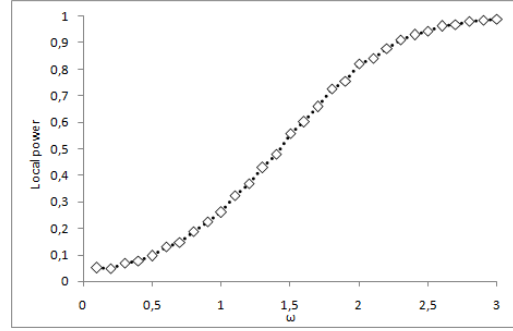
Figure 4.1 shows local power curves for differences in the parameter  $c$ , denoted by  $\omega$ , between the two transition functions. Not surprisingly, higher error variances require higher parameter differences as the comparison of Figures 4.1(a)-(c) reveals. For an error variance of  $\sigma = 1$ , the test requires that the differences between  $c_1$  and  $c_2$  is greater than 0.126 ( $4/\sqrt{1000}$ ) in order to have a power of more than 0.6. For an error variance of  $\sigma = 0.1$ , the required difference for power of more than 0.6 shrinks to 0.0126.

Figure 4.2 shows local power curves for differences in the parameter  $\gamma$ . As for the case of different  $c$ , higher error variances requires greater differences of the parameters in order to yield the same power. Furthermore, we can observe that differences in  $\gamma$  are harder to detect than differences in  $c$ . When we compare Figure 4.1(a) with 4.2(a), we can observe that a difference of 0.126 ( $4/\sqrt{1000}$ ) between  $c_1$  and  $c_2$  is

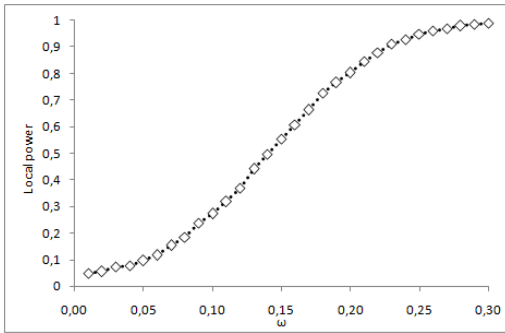
Figure 4.2: Local power against heterogeneous transition functions ( $\gamma$ )



(a) Local power for  $u \sim N(0, 1^2)$



(b) Local power for  $u \sim N(0, 0.1^2)$



(c) Local power for  $u \sim N(0, 0.01^2)$

Note: Local power curves for  $\gamma$ . With  $c = 2$ ,  $\beta_{0,k} = 1$ ,  $\beta_{1,k} = -1$  and  $T = 1000$ .

sufficient for a power of 0.6 while in the case of  $\gamma$ , a difference of 0.538 ( $17/\sqrt{1000}$ ) between  $\gamma_1$  and  $\gamma_2$  is required.

## 4.4 Conclusion

This paper suggests a Lagrange multiplier test for homogeneous smooth transition regressions against a heterogeneous model alternative. In the Monte Carlo simulation, the LM-test shows sufficient power against the alternative and its empirical size is close to the nominal one. While the latter favourable results occurs using the F-version of the test, the  $\chi^2$ -version suffers from size distortion in small samples. The local power curves show that the test can detect differences in the location parameter easier than differences in the speed of transition. Using the test against heterogeneous transition functions based on parameters from a grid search is in general not advisable. Besides providing a useful tool in the specification step of the STR estimation cycle our test against heterogeneous smooth transition regression functions is also a useful in the model evaluation because the test for multivariate transition functions requires the estimation of the model under the null of homogeneity.

## 4.5 Appendix

Derivatives of the (logistic) transition function

$$\begin{aligned}\frac{\partial g}{\partial \gamma} &= -1[1 + \exp\{-\gamma(q - c)\}]^{-2} \exp\{-\gamma(q - c)\}(-q + c) \\ &= (1 + w)^{-2} w(q - c)\end{aligned}$$

$$\begin{aligned}\frac{\partial g}{\partial c} &= -1[1 + \exp\{-\gamma(q - c)\}]^{-2} \exp\{-\gamma(q - c)\} \gamma \\ &= -(1 + w)^{-2} w \gamma\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 g}{\partial \gamma^2} &= 2(1 + w)^{-3} w^2 (q - c)^2 - (1 + w)^{-2} w (q - c)^2 \\ &= (1 + w)^{-2} w (q - c)^2 (2(1 + w)^{-1} w - 1)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 g}{\partial c^2} &= 2(1 + w)^{-3} w^2 \gamma^2 - (1 + w)^{-2} w \gamma^2 \\ &= (1 + w)^{-2} w \gamma^2 (2(1 + w)^{-1} w - 1)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 g}{\partial c \partial \gamma} &= \frac{\partial^2 g}{\partial \gamma \partial c} = -2(1 + w)^{-3} w^2 (q - c) \gamma + (1 + w)^{-2} w (q - c) \gamma - (1 + w)^{-2} w \\ &= (1 + w)^{-2} w \left( -2(1 + w)^{-1} w (q - c) \gamma + (q - c) \gamma - 1 \right)\end{aligned}$$

$$\begin{aligned}\frac{\partial^3 g}{\partial \gamma^3} &= 6(1 + w)^{-4} w^3 (q - c)^3 - 6(1 + w)^{-3} w^2 (q - c)^3 + (1 + w)^{-2} w (q - c)^3 \\ &= (1 + w)^{-2} w (q - c)^3 (6(1 + w)^{-2} w^2 - 6(1 + w)^{-1} w + 1)\end{aligned}$$

$$\begin{aligned}\frac{\partial^3 g}{\partial c^3} &= -6(1 + w)^{-4} w^3 \gamma^3 + 6(1 + w)^{-3} w^2 \gamma^3 - (1 + w)^{-2} w \gamma^3 \\ &= (1 + w)^{-2} w \gamma^3 \left( -6(1 + w)^{-2} w^2 + 6(1 + w)^{-1} w - 1 \right)\end{aligned}$$



$$\begin{aligned}
\frac{\partial^3 g}{\partial \gamma^2 \partial c} &= \frac{\partial^3 g}{\partial \gamma \partial c \partial \gamma} = \frac{\partial^3 g}{\partial c \partial \gamma^2} = -6(1+w)^{-4} w^3 (q-c)^2 \gamma + 6(1+w)^{-3} w^2 (q-c)^2 \gamma \\
&\quad - 4(1+w)^{-3} w^2 (q-c) - (1+w)^{-2} w (q-c)^2 \gamma + 2(1+w)^{-2} w (q-c) \\
&= (1+w)^{-2} w (q-c) \left( -6(1+w)^{-2} w^2 (q-c) \gamma + 6(1+w)^{-1} w (q-c) \gamma \right. \\
&\quad \left. - 4(1+w)^{-1} w - (q-c) \gamma + 2 \right)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^3 g}{\partial \gamma \partial c^2} &= \frac{\partial^3 g}{\partial c \gamma \partial c} = \frac{\partial^3 g}{\partial c^2 \partial \gamma} = 6(1+w)^{-4} w^3 (q-c) \gamma^2 - 6(1+w)^{-3} w^2 (q-c) \gamma^2 \\
&\quad + (1+w)^{-2} w (q-c) \gamma^2 - 2(1+w)^{-2} w \gamma + 4(1+w)^{-3} w^2 \gamma \\
&= (1+w)^{-2} w \gamma \left( 6(1+w)^{-2} w^2 (q-c) \gamma - 6(1+w)^{-1} w (q-c) \gamma \right. \\
&\quad \left. + 4(1+w)^{-1} w + (q-c) \gamma - 2 \right)
\end{aligned}$$

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## Eidesstattliche Erklärung

Ich erkläre hiermit, dass ich meine Doktorarbeit "*Smooth Transition Regression Models in Finance*" selbstständig und ohne fremde Hilfe angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehrenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen nach den mir angegebenen Richtlinien zitiert habe.

Hamburg, den 30.12.16

Julian Sebastian Leppin