## The Wider Impacts of Transport Infrastructure Investments

Agglomeration and Imperfect Competition in General Equilibrium

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Agglomeration and Imperfect Competition in General Equilibrium

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#### Abstract

The wider impacts of transport infrastructure investments are pecuniary externalities that are commonly reckoned to be positive. At least when inferred from partial equilibrium analysis, this can be viewed as reasonable. Though, for an economy with limited resources, it might be reasonable to presume that positive wider impacts are rather redistributive effects than self-contained benefits. If they are, what is the sign of the net wider impact? To study this sign and its determinants, I set up various general equilibrium models of spatial economies in which the costs of transportation are brought down by means of some tax-funded scheme. Applying two separate classes of models, I study the wider impacts both from imperfectly competitive markets in the wider economy and from the agglomeration of economic activity. The former contains a comprehensive base model, markets with exogenous mark-ups, industries with zero profits, monopolies, duopolies and monopolistically competitive industries, including one model with endogenous firm entry. The latter is a New Economic Geography model that is devised as to allow for a welfare analysis. The signs of the net wider impacts are found to be highly ambiguous.

JEL classification: D43, D50, D61, D62, F12, F15, R13, R42

**Keywords:** wider impacts, pecuniary externalities, agglomeration, center-periphery model, New Economic Geography, imperfect competition, monopoly power, general equilibrium, transport economics, cost-benefit analysis

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### Chapter 1

### Introduction

Two things are central to the study of economic geography. For one, there are the costs of covering some geographical distance in order to exchange information, ideas, services or physical goods. These costs inform many of our everyday decisions and actions, and they constitute the very reason for why geography is of any relevance in economics. Also, and partly as a consequence thereof, there are the various sources of the centripetal forces, e.g., economies of scale and spatial monopolies. They act toward spatial concentration and — as a matter of fact — are all market imperfections of some sort.<sup>1</sup> They counteract the centrifugal forces that result from the exogenous dispersion of resources, and all the forces ultimately even out to shape the spatial pattern of economic activity.<sup>2</sup>

When a network infrastructure investment leads to a reduction in some of the costs of covering some distance in space, the centripetal and centrifugal forces are both weakened. While the impact on the latter is quite straightforward, it is to be stated again that the cost reduction works through imperfect markets in weakening the former. So, when assessing the economic impact of such an investment, it would seem natural not to outright disregard these imperfections. Also, it would seem natural not to disregard that the spatial pattern might be altered as a consequence of the weakened spatial forces. The cost reduction might motivate some agents to change their decisions about where to locate, which might in turn exhibit some externality. Thus, when improving some network infrastructure, there are different potential effects on society that go beyond the immediate benefits to the users of the infrastructure, which are the so-called "wider impacts".

In general, such a network infrastructure could be that of a transport network, e.g., roads, railways or inland waterways. Yet, it could also be that of a telecommunication network, e.g., fiber optic cables, cell towers or satellites (Bröcker, 2013, 2012b, sec. 4.2, pp. 85–87). Just as with a reduction of informal barriers, the consideration of wider impacts is also applicable to a reduction of formal barriers like import tariffs, industry standards, harbor dues or road tolls, e.g., economic integration.

<sup>&</sup>lt;sup>1</sup> "Unvollkommene Märkte sind nicht störender Nebenaspekt der ökonomischen Geographie, sondern ihr Kern." (Bröcker, 2012b, p. 93; Imperfect markets are no disruptive minor point of economic geography, but its core.)

<sup>&</sup>lt;sup>2</sup> See Brakman et al. (2009, ch. 2, pp. 32–78), Bröcker (2012b) or Duranton (2008) on economic geography.

I choose to speak of a transport network, not just because it is convenient to think of the movement of physical goods rather than that of data or knowledge. More importantly, the externalities that exist in connection with transport have so far attracted considerable attention from scholars, planners and decision-makers. This is true of the technological externalities like environmental effects, accident risk and congestion, which are immediate consequences of transportation that cause an inefficient allocation of resources. Though, it is also true of the pecuniary externalities, i.e., the wider impacts, which are indirect consequences of a transport infrastructure investment that do not cause an inefficient allocation of resources. See Dodgson (1973, pp. 169–171) and Scitovsky (1954) for this distinction.

What I contemplate are the market imperfections and externalities in the wider economy. In the hypothetical case that these do not exist, i.e., if competition is perfect and externalities are nonexistent, a reduction in transport costs — in its entirety — feeds through into the wider economy. Primarily, it precipitates reductions in the prices of the goods and services that are produced using transportation as an input. Ultimately, it has effects on consumers that are not additional to the benefits to transport users, but equally large manifestations thereof (Dodgson, 1973). Regarding these effects on consumers as additional, even though they are not, would mean to double count. Though, it is safe to assume that externalities do exist and that competition is at least somewhat imperfect. So, when cost reductions feed through into the wider economy, they manifest themselves in effects on consumers that might very well not be of equal size to the transport user benefits. The difference between the two, i.e., the wider impacts, must be regarded as additional to the transport user benefits. Acknowledging that wider impacts do exist prompts two questions. First, what is their sign? Second, what is their scale, and does it justify their assessment?

In practice, different types of wider impacts are often claimed to be both positive and of a considerable scale, and are thus taken as justification for transport projects. Given what states spend on their transport infrastructure, it is obvious that such wider impacts, if relevant at all, could easily be of some significance in absolute terms. According to the OECD's International Transport Forum, ten member states of the EU15 spent between 0.6% and 1% of their respective gross domestic products on transport infrastructure investments during the year of 2014, as did many other industrialized countries.<sup>3,4</sup> In the case of Germany, this was 17.1bn, which was 0.6% of GDP. In the case of France, this was 20.5bn, which was 1% of GDP. The investments comprise the construction of new infrastructure as well as the improvement of existing infrastructure, though not its maintenance (OECD, 2016).

<sup>&</sup>lt;sup>3</sup> Those ten European countries are Austria, Denmark, Finland, France, Germany, Luxemburg, the Netherlands, Spain, Sweden and the United Kingdom. In the countries that have joined the European Union since 2004, the numbers are between 0.7% and 1.4% of GDP, except for the 2.1% in the case of Romania. Data on Malta and Cyprus are not available.

<sup>&</sup>lt;sup>4</sup> The other industrialized countries include Canada, Israel, Japan, South Korea, New Zealand, Switzerland, Turkey and the United States.

The terminology is such that the transport user benefits are sometimes referred to as the "direct impacts", which are the conventional measure of an investment's welfare effect, albeit that they might be passed on into the wider economy, at least in part. The "overall impacts", on the other hand, are all the impacts that result from the transport cost reduction being passed through into the wider economy — without any double counting, of course — and ultimately affecting social welfare, i.e., the welfare of not just the users of the infrastructure, but all households. The wider impacts are the difference between the overall impacts and the direct impacts. I.e., they measure by how much the conventional transport appraisal underestimates the actual impact on society. The wider impacts are also referred to in the literature as "wider economic benefits (WEBs)", yet this seems to be somewhat dated. I generally avoid to speak of wider *benefits*, so not to imply a positive sign. This is because, firstly, determining this sign is the actual aim of my resarch, and secondly, presupposing a positive sign — or even any sign — disagrees with my findings.

The models that I develop are all purely theoretical spatial models in which the costs of transportation are brought down through either state intervention or an exogenous shock. Including when I apply marginal analysis, I essentially evaluate the cost reduction's welfare effect by means of comparative statics, as is common practice in infrastructure appraisal (Mackie et al., 2011, p. 501). While the research on wider impacts has heavily relied on partial equilibrium analysis, I employ only general equilibrium frameworks. This incorporates the interactions between industries and/or regions so that it allows for a more accurate estimation of a transport scheme's net wider impact; see Vickerman (2009, sec. 5, pp. 54–56), Newbery (2002, sec. 1, pp. 1–3), Mas-Colell et al. (1995, sec. 15.E, pp. 538–540), Bröcker & Mercenier (2011), and Kanemoto & Mera (1985). Wider impacts might appear to be selfcontained and thus additional to the direct impacts. Yet, especially in developed regions, they might be redistributive impacts, and this can be assessed by the use of general equilibrium frameworks. A transport scheme's costs are the costs of employing the necessary labor, which is supplied by the households. In turn, the households pay a tax to finance the scheme. The net overall impact is measured as the impact on either the one representative household's or the multiple but identical households' welfare, where the overall impacts are ultimately conflated. There is no danger of double counting.

The main sources of wider impacts that are supposedly existent and relevant are changes in the outputs on imperfectly competitive markets, i.e., on markets where the prices do not equal the marginal social costs, changes in the economic geography, i.e., agglomeration, and imperfections of the labor market like, say, an income tax. See the guidelines by the United Kingdom's Department for Transport (DfT, 2014, sec. 2.2, pp. 2–3) for descriptions of these three types of wider impacts. See also Mackie et al. (2011, p. 513 et seqq.) and Holvad & Leleur (2015, sec. 2, pp. 260–265). I disregard the imperfections of labor markets (Venables, 2007; Zhu et al., 2009). Instead, I investigate the roles of imperfect competition and agglomeration, yet in separate classes of models. This thesis is structured as follows. Chapter 2 gives an overview of the relevant literature. The subsequent main body is divided into two parts, both of which start with a description of the methodology applied to model transportation and the possible infrastructure project, and a description of the methodology applied to assess the wider impact. Part I deals with imperfect competition. It starts with a broad model in chapter 3, followed by models in the three subsequent chapters that are more specific with regards to market structure. Part II deals with agglomeration. The model is introduced in chapter 7, and the two subsequent chapters consider one distinct case each. Chapter 10 concludes both parts of this thesis. Appendix A and appendix B belong to part I and part II, respectively, and they mostly append technical details, including the notation.

The notation that I use differs between the two parts, but is consistent within them. All the notation is listed in appendix A.1 and appendix B.1 for part I and part II, respectively. Variables and parameters are only explained where they are first introduced, and not with the model of every chapter. Differences in the use of subscripts and superscripts in the chapters of part I result from adaptations to the respective models at hand.

When there is a reference in the margin next to the heading of a part, chapter or section, it is to the corresponding place within the main body. In the case of a heading within the main body itself, it is to the corresponding place within the appendices.

### Chapter 2

### Literature

Some of the earliest works on the economic appraisal of transport infrastructure projects are by Tinbergen (1957) and by Bos & Koyck (1961). They develop examples of road construction projects and subject them to comparative–static assessments of their impacts on the national product. Yet, there are disadvantages to this method, as stated by Dodgson (1973, pp. 180–181), and it has not received as much attention as the conventional appraisal method based on the consumer surplus on the market for transportation (Bos & Koyck, 1961, pp. 19–20). Neither have other alternative methods. See Mohring (1976, ch. 9, pp. 105–113) for a comparison of these two methods.

The conventional method combines the decrease in transportation costs, which mostly results from time savings and lower vehicle operating costs, with the traffic volumes before and after the investment to evaluate its social benefits. These are then offset against its social costs. Some of the earliest such cost–benefit analyses (CBAs) of transport projects at the time were undertaken in the United Kingdom by Coburn et al. (1960) in the case of a motorway between London and Birmingham, and by Foster & Beesley (1963) as well as Beesley & Foster (1965) in the case of the Victoria Line, an underground railway in London. These applications to transport are based on the prior literature on government efficiency and welfare economics, e.g., Pigou (1920), as well as on research by engineers, like similar applications in fields such as water resource management.

The social benefits mentioned above, to be more precise, are the transport users' benefits from the transport cost reduction for the existing traffic and from the creation of additional traffic, i.e., the transport users' gains in consumer surplus. When calculating these benefits for a given link, this is typically done by multiplying the change in the unit cost of transportation by the average of the traffic volumes before and after the investment, which is an accurate measure of the change in the consumer surplus given a linear demand curve that does not shift due to the investment. This is the so-called "rule of a half", which was suggested by Neuburger (1971), though he was certainly not the first to mention or even use what is essentially this rule (Bos & Koyck, 1961, pp. 19–20). If demand is either nonlinear, shifted or simply unknown, or if the creation of traffic is moderate, the rule of a half is often taken to be a sufficient approximation (Small & Verhoef, 2007, p. 183).

The gray area in figure 2.1 delineates the change in consumer surplus as given by the rule of a half. Besides the one black, linear demand curve, there is a gray, strictly convex demand curve to exemplify a situation in which the effective gain in consumer surplus is overestimated by the rule of a half. See section 3.3.2 for an application of this rule.



Quantity transported

Figure 2.1 Transport user benefits and the rule of a half

The question of whether the benefits to transport users in any way reflect a transport infrastructure investment's effect on the welfare of society, is of course a fundamental one, and it has attracted considerable attention. Dodgson (1973) demonstrates that if there is perfect competition throughout the wider economy, and if externalities are absent, i.e., if all prices are equal to marginal social costs, this question can be answered in the affirmative. The transport user benefits are then a perfect reflection, and wider impacts do not exist. Mohring (1976, ch. 8, pp. 85–104) comes to the same conclusion using a more general framework in which demand is not necessarily linear, not applying the rule of a half.

Cost-benefit analysis still relies heavily on the estimation of transport user benefits. Yet, as it is acknowledged by Vickerman (2007a,b) as well as Small & Verhoef (2007, sec. 5.2, pp. 181–190) in their descriptions of the modern practice, an assumption of perfect competition might be the source of a considerable bias. Mackie & Nellthorp (2009) oppose this notion, at least for developed regions, and suppose little additionality of the wider impacts.<sup>5</sup>

There are various sources that serve as an introduction to the issue of wider impacts. Vickerman (2009), for instance, gives an introduction into the role of an economy's transport system with a focus on its desired efficiency/optimality. Vickerman (2008a) gives some theoretical as well as empirical background, and Vickerman (2008b) integrates the viewpoints of transport economics and urban economics. Laird et al. (2005) consider the network effects

<sup>&</sup>lt;sup>5</sup> See Small & Verhoef (2007, sec. 5.2.5, pp. 187–189), Mackie & Nellthorp (2009, sec. 4.2, pp. 166–169) and Vickerman (2007a, sec. 3, pp. 602–604) for the descriptions of the wider impacts. See Iacono & Levinson (2015) for another review of the current practice.

of transportation, a broader concept that includes impacts on the wider economy as well as the redistribution and creation of traffic, among other things. Mackie et al. (2011) as well as Holvad & Leleur (2015) give reviews of the (empirical) research on wider impacts in which they discuss the issue of additionality.

The European Commission's guidelines for the economic appraisal of investment projects suggest to exclude wider impacts because they are mostly redistributive effects, and because double counting should be avoided (EC, 2014, p. 25). Many countries do in fact exclude them (Mackie & Worsley, 2013). The United Kingdom, though, is an exception, and there has been quite some research on this issue during the past couple of decades. Venables & Gasiorek (1999) wrote a well-know and influential report to the Standing Advisory Committee on Trunk Road Assessment (SACTRA) with a number of numerical examples of a computable general equilibrium model, kicking off the development of appraisal techniques. The most recent version of the guidelines by the UK's Department for Transport (DfT, 2018a) follow that development which is documented in the report by Eddington (2006) as well as in other publications by the DfT (2005, 2008, 2012a,b, 2014).<sup>6</sup> A detailed overview of the development of guidelines for cost-benefit analysis in the UK is that by Worsley (2011, sec. 5, pp. 12–20).

There are currently three major transport projects in the UK for which an assessment of the wider impacts has been carried out. There is Crossrail, which is an urban railway line across London. Its construction project, which is one of Europe's largest, is currently under way and is planned to be complete by the end of 2019. According to Crossrail et al. (2011, pp. 9–12), this project will deliver wider impacts on welfare of between £6bn and £18bn, while costs are £5.6bn and direct impacts are between £11bn and £15.5bn. These are present values in 2002 prices. The wider impacts from imperfect competition have a value of £485m, which is less than a tenth of the impacts from agglomeration and 1–2 percent of the total overall impacts (Colin Buchanan, 2007, p. 21 et seqq.).<sup>7,8</sup> Worsley (2011, sec. 6, pp. 20–25) gives an overview of attempted quantifications of the wider impacts of Crossrail and some historical background.

Another project in the UK is HS2, a Y-shaped high-speed railway between London and a number of cities in central and northern England. Construction is planned to take till 2033. This transport scheme is expected to increase Britain's annual GDP by some £15bn in 2037 (in 2013 prices). This is the total productivity impact which consists of seven region–specific impacts that are all alleged to be positive, with either of two scenarios regarding the impact on business location (HS2, 2013a,b).<sup>9</sup>

<sup>&</sup>lt;sup>6</sup> For further documentation by the DfT, see https://www.gov.uk/guidance/transport-analysis-guidance-webtag (latest access: 2018-06-07).

<sup>&</sup>lt;sup>7</sup> See http://www.crossrail.co.uk/route/wider-economic-benefits (latest access: 2018-06-07).

<sup>&</sup>lt;sup>8</sup> See also Colin Buchanan (2007, ch. 5) for more details than in the summary report. See Jenkins et al. (2011, p. 104 et seq.) for a brief overview of the estimated impacts.

<sup>&</sup>lt;sup>9</sup> See https://www.hs2.org.uk (latest access: 2018-05-25).

The third project is an aviation project. There are three different schemes proposing to increase the capacity of the airport at either Gatwick or Heathrow. The main report by the Airports Commission (AC, 2015a,b) states that the agglomeration benefits with Heathrow are much larger than with Gatwick due to already existing business clusters. Also, Heathrow has an advantage in terms of the additional tax revenues created through the movement to more productive jobs. In total, the wider impacts amount to £11.5bn in the case of an expansion of the northwestern runway at Heathrow Airport. The estimates for the two alternative schemes are £10bn for the westerly extension of the runway at Heathrow Airport and £8.1bn for the new runway at Gatwick Airport. These are present values in 2014 prices.

#### 1 2.1 Imperfect competition

If a market in the wider economy is imperfectly competitive, the marginal willingness to pay of the consumers, i.e., the price, exceeds the marginal cost. This positive mark-up indicates that an increase in the quantity would give rise to a wider imipact beyond the gain in the consumer and producer surpluses on the transport market (DfT, 2018b, sec. 4, pp. 16–18). See also Mackie et al. (2011, p. 520), Jara-Díaz (1986) and Bröcker (1998, 2001).

If a transport investment makes the competition of a market become less imperfect, this might give rise to a wider impact. For instance, spatial monopolies exist because sufficiently high transport costs shield them from competition. When this shield is partly removed, other firms might enter the market, thereby altering its structure. See Vickerman (2009, p. 56) and DfT (2005, sec. 3.2, pp. 23–25).

The guidance by the DfT (2018b, sec. 4.3, pp. 17–18) suggests the simplified approach of adding a fixed share of 10% to the total business and freight user benefits. This estimate is the result of research into mark-ups and elasticities (DfT, 2005).

#### **II** 2.2 Agglomeration

Economic agents base their decisions about where to locate on various criteria that are informed by the costs of transportation to and from each possible location. For an individual, these can be the costs of commuting to their place of work. For a firm, these can be the costs of transporting its produce to customers. When a transport infrastructure investment reduces these costs, it might have an impact on the agents' choice of location. While this choice is made as to optimize the agent's own welfare, it might also impact that of others. Hence, there is an externality. Yet, since this externality is external to the market, it does not cause an inefficient allocation of resources. If agents do not relocate as a result of a transport scheme, there is no wider impact, as demonstrated by Newbery (2002). Though, if agents do relocate, a wider impact might exist. A positive wider impact from agglomeration is typically attributed to a net benefit from an increase in agglomeration at large. Agglomeration economies are in turn attributed to increases in productivity due to, say, an improved division of labor or knowledge spill-overs (Gibbons & Overman, 2009; Graham, 2005a,b, 2006, 2007a,b; Graham et al., 2009, 2010; Graham & van Dender, 2011). An increase in the agglomeration within, for instance, a city is due to an increase in either the geographical concentration within the city, the size of the city or the effective density of the city. The latter is brought about by a reduction in the costs of travel within the city, but not necessarily by the relocation of economic activity (DfT, 2018c). Comprehensive introductions to agglomeration economies are those by Eberts & McMillen (1999) and Rosenthal & Strange (2004). A review of work in the context of transport is that by Jenkins et al. (2011).

Venables (2007) develops a model in which the size of a monocentric city is increased through a reduction in the costs of commuting to the central business district, and the productivity of labor in that city is assumed to be increasing in the city's size. The productivity of labor in the periphery, though, is not affected by the expansion of the city. There is a positive wider impact because the shift of labor from the periphery to the city increases the productivity of all the workers in the city; yet, it does not decrease anyone's productivity. Meijers et al. (2012) investigate the distributive effects between center and periphery, with a real-life application to a tunnel that was constructed under the Westerschelde estuary. The model by Kanemoto (2013) is one with multiple cities, some of which might suffer a welfare loss as a consequence of a transportation improvement, while others enjoy a welfare gain. See also Krugman & Venables (1995). For the guidelines by the UK's Department for Transport on how to assess agglomeration's productivity impacts, see DfT (2018c).

The model that I develop in part II is a New Economic Geography (NEG) model that is an adaptation of the center-periphery model by Krugman (1991).<sup>10</sup> The adaptation is made as to allow for an assessment of the wider impact. Other such assessments are those by Baldwin et al. (2003), Charlot et al. (2006), Helpman (1998), Ottaviano & Thisse (2001, 2002), Robert-Nicoud (2006) and Tabuchi & Thisse (2002); see Pflüger & Südekum (2008b).

The one by Pflüger (2004) builds on the model by Forslid (1999). In that, it is like the Footloose–Capital Model (Baldwin et al., 2003, ch. 3, pp. 68–90), but capital owners are mobile in the long run and there is no repatriation of incomes. Also, the upper–tier utility function is quasi-linear rather than of the Cobb–Douglas type. The economy exhibits higher degrees of concentration than what is socially desirable; so, the wider impact is negative..<sup>11</sup>

<sup>&</sup>lt;sup>10</sup> For a background on the New Economic Geography, see Schmutzler (1999), Fujita & Krugman (2004), Bröcker (2012a) and Ottaviano & Thisse (2004). For the role of transport costs, see Alonso Villar (2007) and Lafourcade & Thisse (2011).

<sup>&</sup>lt;sup>11</sup> According to my supervisor, Johannes Bröcker, the sign of the wider impact is determined by introducing a social planner. Prices are set equal to marginal costs, a lump-sum tax is levied to compensate the capital owners (in terms of the numeraire good), and social welfare, which can simply be measured by summing up individual welfare levels due to the quasi-linear utility function, is maximized.

Pflüger & Südekum (2008b) develop an assessment based on that by Pflüger (2004) which also features over-agglomeration. They incorporate housing costs (Helpman, 1998) so that there is under-agglomeration and/or dispersion at low levels of the transport cost. Yet, the transition to agglomeration, which occurs at higher levels of the transport cost, is not socially desirable, at least not to the extent that is yielded by the market.

Ottaviano et al. (2002, sec. 5) set up another model with a quasi-linear utility function. There are zero marginal costs, the households have positive initial endowments with a numeraire good, and the transport costs are measured in units of that numeraire good. As opposed to the two models mentioned above, there is a catastrophic rahter than smooth transition from dispersion to concentration as a consequence of a transport cost reduction. At the transport cost threshold where the transition occurs, concentration yields a lower welfare level than dispersion. Hence, the wider impact is negative.

Tabuchi (1998) comes to the opposite conclusion. In his model, agglomeration is usually favorable compared to dispersion. So, he suggests to conduct policies that promote economic integration if that leads to more agglomeration, since the wider impact is likely to be positive.

Part I

Imperfect competition

А

This part investigates the sign of a transport cost reduction's wider impact resulting from imperfectly competitive industries in the wider economy. These industries are imperfectly competitive in that prices exceed marginal costs. When the transport cost reduction is fed through into the wider economy via the industries using the concerned means of transportation, this might trigger indirect effects on other industries that do or do not use these means of transportation. If any of these industries are imperfectly competitive, whether they are affected directly or indirectly, what is ultimately passed on to consumers might deviate from the original cost reduction. See DfT (2018b, sec. 4, pp. 16–18) and Mackie et al. (2011, p. 520). So, there might be a wider impact on the welfare of consumers in addition to the original cost reduction, and this wider impact might be either positive or negative. Which sign it is, and what the determinants are of this sign, is investigated in the models of the following chapters.

#### Transport costs and transport scheme

All the models developed through the course of not just this part, but the entire thesis, are built around the eventual necessity of having to overcome any spatial distance that might lie between economic agents whenever they engage in some sort of transaction. The corresponding costs that I consider are those of shipping at least one type of final good from the producer to the consumer. This is to say that these are the costs that are reduced, whether it is through some costly transport scheme or through an exogenous shock that is free of cost. I refer to either as a scheme. Other possible transport costs remain unaffected.

#### Transport costs

All marginal costs are constant in the quantities produced and/or transported. The marginal cost of production of firm j is denoted by the exogenous  $c_j > 0$ . The marginal cost of transportation unaffected by the scheme is denoted by  $\tau_j \ge 0$ , which is also exogenous. Prior to the scheme, the total marginal cost of transportation is  $\tau_j + t_j$ , where the exogenous  $t_j \ge 0$  denotes the fraction of this cost that is eliminated through the scheme. I occasionally amend the notation by a subscript other than j to distinguish between goods, firms, regions or the like.

#### **Transport scheme**

The scheme's progress is denoted by  $\varphi \in [0, 1]$ , and  $t_j$  identifies the effect of the full scheme. The total marginal cost of production and transportation combined is

$$m_j := c_j + \tau_j + (1 - \varphi) t_j .$$

This cost is modeled as linear in  $\varphi$ , i.e.,  $dm_j/d\varphi = -t_j$ , so the reduction thereof,  $\varphi t_j$ , is proportional to that of any other firm's cost. As the scheme progresses, it causes equable reductions of the costs of all the users of the concerned means of transportation.

The way that the scheme works is that households give up part of their income in form of a lump-sum tax which is redistributed to fund reductions in the costs for transport users. Through the reduced costs, the firms using transportation are led to lower their prices and increase their quantities, ceteris paribus, as partial equilibrium analysis suggests. Though in general equilibrium, if some quantities are increased, some quantities might be decreased, because the resources are limited.

The effect on the economy's production possibilities given the limited resources depends on how the cost reductions relate to the necessary costs of the scheme. If the scheme is free of cost, the production possibilities are expanded and the quantities of the final goods are increased at large. The higher the costs of the scheme are, the smaller are the changes in the quantities. If the costs of the scheme are equal to the aggregate cost reduction, e.g., if the scheme is a subsidy, it does not alter the production possibilities as social costs do not change. The marginal social cost of firm *j* would be  $\tilde{m}_j := m_j|_{\varphi=0}$  while  $m_j$  represents the marginal private cost. The difference,  $\varphi t_j$ , would be the cost external to the firm. So, if the quantity of one good is increased, the additional labor must be withdrawn from the production of some other good, thus decreasing this good's quantity.

#### Welfare assessment

I distinguish between the overall impact, *I*, the direct impact, *D*, and the wider impact, *W*. The overall impact is the impact on the households' welfare measured as the compensating variation. It is the net effect because the scheme's costs are taken into account by letting the households pay for them through the tax on their income. Therefore, I also calculate the direct impact as the net effect. The gross direct impact is the welfare effect on transport users measured as the change in the consumer surplus on the transport market, albeit — at least to some extent — fed through into the wider economy. The net direct impact follows by subtraction of the scheme's costs.

The wider impact is the extent by how much the direct impact underestimates the overall impact, i.e., the difference between the two. So, the scheme's costs cancel themselves out when *I* and *D* are offset against one another. There is no distinction to make with regards to the wider impact about whether *I* and *D* are inclusive of the scheme's costs. The scheme's costs only have an influence on the wider impact via their influence on the quantities of the various goods. In general equilibrium, the scheme absorbs some of the economy's available resources that would otherwise be employed in the production of these goods. The more resources the scheme absorbs, the smaller is the change in the production at large which follows as a consequence of the scheme, and the smaller is the wider impact.

## Chapter 3

### Base model

Assume a spatial world with an exogenous number of goods and one household consuming these goods. Each good is supplied by one industry, and one industry alone, which operates under either perfect or imperfect competition. The only factor of production that is employed by the industries is labor. The household supplies the labor fully inelastically. Moreover, labor is the numeraire. All industries' profits are transferred to the household.

When a transport scheme is conducted, reducing some of the costs of some of the industries, these reductions are fed through into the wider economy and are ultimately conflated into an impact on the household's welfare. Due to the imperfect competition in some industries, this impact might include an either positive or negative wider impact in addition to the original cost reduction.

#### 3.1 Industries

Beside the degrees of competition, there is one other fundamental difference between the industries, and this regards the degrees to which their costs are affected by the scheme. Firms of the same industry differ neither in regard to the transport cost nor in any other regard. Every firm of industry  $i \in \{1, ..., N\}$  has a marginal cost of production of  $c_i > 0 \forall i$  and a marginal cost of transportation following the scheme of  $\tau_i \ge 0 \forall i$ . It has a total marginal cost prior to the scheme of  $\tilde{m}_i := c_i + \tau_i + t_i$  and experiences a cost reduction of

$$t_i \ge 0 \ \forall i \quad \text{with} \quad t_i > 0 \ \exists i \ .$$
 (3.1a)

Hence, with  $\varphi \in [0, 1]$  as the scheme's progress, the total marginal cost of industry *i* is

$$m_i := c_i + \tau_i + (1 - \varphi) t_i$$
. (3.1b)

The price of any good *i*,

$$p_i = m_i + \mu_i , \qquad (3.2)$$

A.2

includes a mark-up atop the marginal cost of

$$\mu_i := p_i - m_i \ge 0 . \tag{3.3a}$$

If industry *i* is perfectly competitive, its mark-up is zero, and positive otherwise. In relative terms, the mark-up is

$$\mathbf{v}_i := \frac{p_i}{m_i} \ge 1 \ . \tag{3.3b}$$

The numbers of firms within the industries are assumed to be exogenous, and all firms of an industry are identical. Amounts are simply stated as aggregate values. The aggregate fixed input of industry *i* is denoted by  $F_i \ge 0 \forall i$ . The aggregate quantity is  $X_i$ , which is endogenous, so that the aggregate variable input is  $m_i X_i$ . Since the inputs are labor only, and since the wage rate is one, the aggregate profit is

$$\Pi_i := \mu_i X_i - F_i . \tag{3.4}$$

#### 3.2 General equilibrium

The household receives all industries' profits plus its earned income. The earned income equals *L*, which is the household's exogenous endowment with labor and the labor supply. The total labor demand comprises that by the industries and that by the scheme which is

$$S = S(\varphi) \tag{3.5}$$

with S(0) = 0 and  $S'(\varphi) := dS(\varphi)/d\varphi \ge 0$ . When the labor market clears, then

$$L = \sum_{i} (m_{i}X_{i} + F_{i}) + S.$$
(3.6)

The scheme is financed through a lump-sum tax that is imposed on the household. Therefore, the household's nominal disposable income is

$$y := L + \sum_{i} \Pi_{i} - S$$
 (3.7)

With (3.6), (3.4) and (3.3a), this yields the household's budget constraint:

$$y = \sum_{i} p_i X_i . aga{3.8}$$

#### 3.3 Welfare

#### 3.3.1 Overall impact

To assess the impact that the scheme has on the household's utility level, which is denoted by u, I work out what the compensating variation (CV) is and how it relates to  $\varphi$ . The CV reflects the value that, if *subtracted* from the household's income, would allow the household to attain some reference utility level denoted by  $\bar{u}$ . In other words, the CV is the difference between the income and the minimal expenditures necessary to attain  $\bar{u}$  at current prices. Other variables' or parameters' values in the reference situation are also denoted by a bar. With  $\boldsymbol{p} := (p_1, \ldots, p_N)$ , the indirect utility function,  $v(\boldsymbol{p}, \boldsymbol{y})$ , and the expenditure function,  $e(\boldsymbol{p}, u)$ , the definition of the overall impact, I, which is the CV, that I apply is

$$I := y - e(\boldsymbol{p}, \bar{u}) \iff v(\boldsymbol{p}, y - I) = \bar{u} .^{12}$$
(3.9)

When evaluating the marginal impact of a (further) increase in  $\varphi$ , the point of reference is just the point of evaluation, i.e.,  $\bar{\varphi} = \varphi$ . With examples of discrete increases in  $\varphi$ , the point of reference is  $\bar{\varphi} = 0$ . While  $I|_{\bar{\varphi}=\varphi} \equiv 0$ , the impact can have either sign as the result of an increase in  $\varphi$ . A positive *I* indicates a welfare gain as, even if the household were to give up some of its income, it would still be able to attain  $\bar{u}$ . A negative *I* indicates a welfare loss as, if the household were to be enabled to attain  $\bar{u}$ , it would need to be compensated for the changes triggered by the scheme.

Differentiating *I* (3.9) with respect to  $\varphi$  yields

$$I' := \left. \frac{\mathrm{d}I}{\mathrm{d}\varphi} \right|_{\bar{\varphi}=\varphi} = \frac{\mathrm{d}y}{\mathrm{d}\varphi} - \sum_{i} X_{i} \frac{\mathrm{d}p_{i}}{\mathrm{d}\varphi}$$
(3.10a)

by applying Shephard's lemma (Jehle & Reny, 2011, p. 37 et seqq.). The total differential of the budget constraint (3.8) is

$$dy = \sum_{i} p_i dX_i + \sum_{i} X_i dp_i .$$
(3.11)

The Marshallian demand functions,  $X_i = X_i(\mathbf{p}, y)$ , yield the changes in the quantities as

$$X'_{i} := \frac{\mathrm{d}X_{i}}{\mathrm{d}\varphi} = \sum_{j} \frac{\partial X_{i}}{\partial p_{j}} \frac{\mathrm{d}p_{j}}{\mathrm{d}\varphi} + \frac{\partial X_{i}}{\partial y} \frac{\mathrm{d}y}{\mathrm{d}\varphi} \,. \tag{3.12}$$

<sup>&</sup>lt;sup>12</sup> I apply the definition of the compensating variation (CV) by Mas-Colell et al. (1995, p. 82 et seqq.), Varian (1992, sec. 10.1, pp. 160–163) or Boadway & Bruce (1984, sec. 7.3, pp. 201–205), which is equivalent to that by Jehle & Reny (2011, p. 180 et seqq.) — except for the reversed sign.

The changes in the prices (3.1b–2) are

$$\frac{\mathrm{d}p_i}{\mathrm{d}\varphi} = \frac{\mathrm{d}\mu_i}{\mathrm{d}\varphi} - t_i \;. \tag{3.13}$$

The changes in the mark-ups are discussed starting in section 3.3.4. Inserting (3.11) into (3.10a) yields

$$I' = \sum_{i} p_i X'_i . \tag{3.10b}$$

From the labor market's equilibrium condition (3.6) follows

$$dL = \sum_{i} m_i dX_i - \left(\sum_{i} t_i X_i - S'(\varphi)\right) d\varphi = 0.$$
(3.14)

Subtracting (3.14) from (3.10b) yields

$$I' = \sum_{i} \mu_{i} X'_{i} + \sum_{i} t_{i} X_{i} - S'(\varphi) .$$
(3.10c)

#### A.2.1 3.3.2 Direct impact

The discrete change in the costs of the scheme (3.5) is

$$\Delta S := S(\varphi) - S(\bar{\varphi}) . \tag{3.15}$$

Since the direct impact is measured as the change in the consumer surplus on the transport market, the gross and the net direct impact, respectively, with  $p = p(\varphi)$  and  $y = y(\varphi)$ , are

$$\Delta CS := \int_{\bar{\varphi}}^{\varphi} \left( \sum_{i} t_{i} X_{i}(\boldsymbol{p}(\phi), y(\phi)) \right) d\phi$$
(3.16a)

$$D := \Delta CS - \Delta S . \tag{3.16b}$$

These are zero at the point of reference, i.e.,  $D|_{\bar{\varphi}=\varphi} \equiv 0$ . Generally, the integration along the Marshallian demand functions is path-dependent (Johansson, 1991, sec. 4.2, pp. 42–47). Here, the path of integration arises out of the assumption of marginal costs being linear in  $\varphi$ . If one would use the Hicksian demand functions instead, thus calculating the compensating or equivalent variation rahter than the change in consumer surplus, the direct impact would not be path-dependent (Johansson, 1991, sec. 4.4, pp. 49–52). See also Boadway & Bruce (1984, p. 198 et seqq.).

The derivative of D (3.15–16) with respect to  $\varphi$ , by use of (3.14), is

$$D' := \left. \frac{\mathrm{d}D}{\mathrm{d}\varphi} \right|_{\bar{\varphi}=\varphi} = \sum_{i} t_i X_i - S'(\varphi) = \sum_{i} m_i X'_i \,. \tag{3.17}$$

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The same holds true if, instead of the change in consumer surplus, either the compensating or the equivalent variation is used in determining this marginal impact.

Using the rule of a half (Button, 2009, sec. 6, pp. 71–73) to approximate the change in consumer surplus (3.16a) gives

$$\Delta CS = (\varphi - \bar{\varphi}) \sum_{i} t_i \frac{X_i + \bar{X}_i}{2} . \qquad (3.18)$$

At  $\bar{\varphi} = \varphi$  and  $\bar{X}_i = X_i \ \forall i, D'$  (3.17) again follows as the marginal net direct impact; see appendix A.2.1.

#### 3.3.3 Wider impact

Since the wider impact is the extent by how much the direct impact underestimates the overall impact, (3.9) and (3.16b) yield

$$W := I - D = \sum_{i} \left( \Pi_{i} - \bar{\Pi}_{i} \right) - \left( e(\boldsymbol{p}, \bar{u}) - e(\bar{\boldsymbol{p}}, \bar{u}) \right) - \Delta CS .$$
(3.19)

See appendix A.2.2. At the point of reference, the wider impact is zero, i.e.,  $W|_{\bar{\varphi}=\varphi}\equiv 0$ .

The derivative of W (3.19) with respect to  $\varphi$  is the marginal wider impact and follows as the difference between either of (3.10b–c) and (3.17):

$$W' := \left. \frac{\mathrm{d}W}{\mathrm{d}\varphi} \right|_{\bar{\varphi}=\varphi} = l' - D' = \sum_{i} \mu_i X'_i \,. \tag{3.20a}$$

Figure 3.1 illustrates the impacts from a single industry with  $t_i > 0$ , a constant  $\mu_i > 0$ ,  $dX_i > 0$  and a linear demand curve, and for a discrete increase in  $\varphi$ . These impacts are denoted by  $l_i$ ,  $D_i$  and  $W_i$ , with  $W = \sum_i W_i$  etc.; cf. Mackie et al. (2011, Figure 21.3b, p. 521). The scheme's costs attributed to industry i are  $\Delta S_i := \bar{m}_i \bar{X}_i - m_i X_i$ . This is the amount of labor withdrawn from industry i, and  $\Delta S = \sum_i \Delta S_i$ ; see (3.6). Subtracting  $\Delta S_i$  from both the gross overall impact and the gross direct impact yields the respective net impacts,  $I_i$  and  $D_i$ , as delineated in figure 3.1.

The marginal wider impact exhibits a positive sign if activity is shifted toward industries with positive mark-ups and, if at all, away from industries with relatively low or even zero mark-ups. The distribution of the quantity shifts (3.12) depends on the price changes (3.13), which in turn depend on the cost reductions as well as the changes in the mark-ups. Also, the change in any normal good's quantity is the larger, the larger the change in the household's disposable income (3.7) is. Hence, the change in the production at large, and thus the marginal wider impact, is the larger, the lower the scheme's additional costs are.

A.2.2



**Figure 3.1** Welfare effect on the industry level ( $t_i > 0$ ,  $\mu_i > 0$ ,  $d\mu_i = 0$ ,  $dX_i > 0$ ,  $\bar{\varphi} = 0$ )

The mark-up,  $\mu_i$ , quantifies the wider impact from an additional unit of good *i* in terms of the additional surplus as it is the difference between the household's marginal willingness to pay and the marginal cost. It is also the profit contribution of an additional unit since  $\partial \Pi_i / \partial X_i = \mu_i$ ; see (3.4). From (3.19) with (3.13) and (3.16a) follows

$$W' = \sum_{i} \frac{\mathrm{d}\Pi_{i}}{\mathrm{d}\varphi} - \sum_{i} X_{i} \frac{\mathrm{d}\mu_{i}}{\mathrm{d}\varphi} . \qquad (3.20b)$$

Generally, the wider impact is channeled into the household's welfare via the changes in the profits and the mark-ups. The particular case of constant mark-ups follows in section 3.3.4, and that of constant profits follows in section 3.3.5.

#### 3.3.4 Exogenous mark-ups

If the mark-ups are exogenous, the cost reductions translate into equally large price reductions as  $d\mu_i = 0 \Rightarrow d\rho_i/d\varphi = -t_i$ ; see (3.13). Also, the changes in profits are only due to changes in the quantities as  $d\Pi_i = \mu_i dX_i$ , but not to changes in the mark-ups; see (3.4). It follows from (3.20a–b) that the marginal wider impact equals the marginal impact on the aggregate profit:  $d\mu_i = 0 \forall i \Rightarrow W' = \sum_i d\Pi_i/d\varphi$ .

#### A.2.3 3.3.5 Zero profits

For the purpose of this section, the mark-ups are endogenous. Presupposing that zero profits prevail, firms charge prices equal to average costs:  $p_i = a_i := m_i + F_i/X_i \Rightarrow \mu_i = F_i/X_i$ ; see (3.4). Thus,

$$\frac{\mathrm{d}\mu_i/\mu_i}{\mathrm{d}\varphi} = -\frac{X_i'}{X_i}; \qquad (3.21)$$

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see (3.13). From either (3.20a) with (3.21) for all N industries, or (3.20b) with  $d\Pi_i = 0 \forall i$ , follows

$$W' = -\sum_{i} X_{i} \frac{\mathrm{d}\mu_{i}}{\mathrm{d}\varphi} = -\sum_{i} X_{i} \frac{\partial a_{i}}{\partial X_{i}} X'_{i} . \qquad (3.20c)$$

Whether the marginal wider impact is positive, is a question of whether the quantity shifts bring down average costs at large. If they do, they bring down the mark-ups with them as they draw prices nearer to marginal costs. Since  $\partial a_i/\partial X_i = d\mu_i/dX_i = -F_i/X_i^2 \leq 0 \ \forall i$ , the average costs of any industry are only brought down if this industry exhibits economies of scale due to positive fixed costs and an increase in its quantity. If economic activity is shifted toward industries with economies of scale and, if at all, away from industries with relatively weak or even zero economies of scale, both average costs and mark-ups are brought down at large and the marginal wider impact is positive.

With a price equal to average costs, the elasticity of the average costs with regard to the quantity is equal to the inverse of the elasticity of the quantity with regard to the price. This inverse in turn gives the mark-up as a fraction of the price:

$$\epsilon_i := -\frac{\partial \ln(X_i)}{\partial \ln(p_i)} \tag{3.22a}$$

$$\Rightarrow \frac{1}{\epsilon_i} = -\frac{\partial \ln(a_i)}{\partial \ln(X_i)} = \frac{\mu_i}{p_i} .$$
 (3.22b)

The elasticity is assumed to be greater than one, so the output is increasing in the input. The inverse elasticity (3.22b) with the mark-up (3.3a) yields

$$\mu_i = \frac{1}{\epsilon_i - 1} m_i \tag{3.23a}$$

$$p_i = \frac{\epsilon_i}{\epsilon_i - 1} m_i . \tag{3.23b}$$

The higher the elasticity of the average costs (3.22b) is, the higher is the mark-up (3.23a). With zero profits, demand still needs to be shifted toward the industries with the relatively high mark-ups for the marginal wider impact to be positive. But now this is not because these industries are the ones with strong increases in their profits but with strong economies of scale and strong decreases in their mark-ups. The mark-up of good *i* is decreased if and only if  $(d\epsilon_i/\epsilon_i)/d\varphi > -t_i/p_i$ ; see appendix A.2.3.

Chapter 4 presents a fully specified model with free entry and exit of firms, and thus zero profits and endogenous mark-ups, though with constant scale elasticities instead of linear cost functions. It elaborates on the relevance and interrelation of the economies of scale, the mark-ups, the elasticities, the quantity shifts and the transport costs.

#### 3.3.6 Monopolies

If all N industries are monopolies, the mark-ups and the prices are as given by (3.23). The price elasticities of demand are as given by (3.22a); though, (3.22b) does not apply, except that  $\mu_i = p_i/\epsilon_i \ \forall i$ . The elasticities are assumed to be greater than one, so a solution to a firm's profit maximization exists. See Jehle & Reny (2011, p. 170 et seqq.), Varian (1999, ch. 24, pp. 414–432) or Mas-Colell et al. (1995, sec. 12.B, pp. 384–387). As opposed to section 3.3.5, the profits are not zero. Hence, the wider impact is channeled via changes in both profits and mark-ups; (A.1) applies.

#### 3.3.7 Zero direct impact A.2.4

With  $\alpha_i := p_i X_i / y$  as the expenditure share of good *i*, (3.20a) is

$$W' = \left[\sum_{i} \alpha_{i} \frac{\mu_{i}}{p_{i}} \frac{X'_{i}}{X_{i}}\right] y .^{13}$$
(3.22c)

Assume that  $S'(\varphi) = \sum_i t_i X_i$ , so there is no marginal direct impact (3.17) as the marginal costs of the scheme equal the marginal cost reduction for the current transport volume.<sup>14</sup> Due to the limitation regarding the available resources (3.14), (3.22c) is

$$W' = \left[\sum_{i} \alpha_{i} \frac{X'_{i}}{X_{i}}\right] y ; \qquad (3.22d)$$

see appendix A.2.4. The marginal wider impact is positive if the relative quantity shifts are positive on average when weighted by the expenditure shares.

Since D' = 0, it follows that W' = I'; see (3.20a). The price-to-marginal-cost ratios (3.3b) with W' (3.20a), (3.3a) and (3.14), cf. I' (3.10b-c), yield

$$W' = \sum_{i} \mathbf{v}_{i} m_{i} X'_{i} . \tag{3.22e}$$

Since  $m_i X'_i$  is the change in industry is employment of labor, the wider impact is these changes' sum weighted by the mark-ups, while  $\sum_i m_i X'_i = 0$ ; see (3.14). The impact is positive if resources are shifted away from the industries with the low  $v_i$ 's toward the industries with the high  $v_i$ 's. By use of an average mark-up,  $\hat{v}$ , and an average quantity shift,  $\hat{X}'$ , with

$$\hat{\mathbf{v}} := \sum_{i} \mathbf{v}_i \, \frac{m_i}{\sum_{i} m_i} = \frac{\sum_{i} p_i}{\sum_{i} m_i} \quad \text{and/or} \quad \hat{X}' := 0 , \qquad (3.24)$$

<sup>&</sup>lt;sup>13</sup> In section 3.3.5 and section 3.3.6,  $\mu_i/p_i = 1/\epsilon_i$ ; see (3.22b) and/or (3.23). <sup>14</sup> If the scheme is a subsidy, i.e., if  $S(\varphi) = \varphi \sum_i t_i X_i$ , then  $S'(0) = \sum_i t_i X_i$ . Compare section 4.5.2.
(3.22e) can be expressed as a correlation between mark-ups and resource shifts:

$$W' = \sum_{i} (v_{i} - \hat{v}) m_{i} \left( X'_{i} - \hat{X}' \right) . \qquad (3.22f)$$

For uniform mark-up factors, i.e.,  $v_i = v \forall i$ , (3.22e–f) with (3.14) and (3.24) yield W' = 0. There is no marginal welfare gain or loss, and thus no marginal wider impact, because the relative prices equal the relative marginal costs:  $p_i/p_j = (vm_i) / (vm_j) = m_i/m_j \forall i, j$ . Hence, the allocation of resources is socially optimal, even if the industries are imperfectly competitive in that v > 1.

#### 3.4 Two-industry economy

In the case of a two-industry economy with imperfect competition in industry 1 and perfect competition in industry 2, i.e.,  $\mu_1 > 0$  and  $\mu_2 = 0$ , the necessary and sufficient condition for the marginal wider impact (3.20a) to be positive is  $X'_1 > 0$ , irrespective of  $X'_2$ . If and only if economic activity is shifted toward the imperfectly competitive industry, the scheme has a positive marginal wider impact.

Assume that only the costs of industry 1 are affected by the scheme as  $t_1 > 0$  and  $t_2 = 0$ . Hence, the price of good 2 does not change while the price of good 1 decreases, assuming that the change in its mark-up is smaller than the decrease in the marginal cost; see (3.13). If demand for good 1 is decreasing in its own price, i.e.,  $\partial X_1/\partial p_1 < 0$ , and independent of income, i.e.,  $\partial X_1/\partial y = 0$ , then (3.12) yields that demand for good 1 does increase. Industry 1 is the only industry that benefits from the scheme so that its relative price decreases, shifting demand toward industry 1 and giving rise to a positive wider impact. Starting in chapter 5, I present models with two industries of such sort where I investigate the role of the market structure of the industry with imperfect competition.

## Chapter 4

## Monopolistic competition

The following model is a fully specified example. It builds on section 3.3.5 which considers zero profits within the previous framework. There is a spatial world with a single household consuming *N* heterogeneous goods. Each of the goods is produced by one industry, and one industry alone. All of these industries operate under monopolistic competition as developed by Dixit & Stiglitz (1977); see Fujita et al. (1999, ch. 4, pp. 45–59). Labor is the only factor of production as well as the numeraire. The household supplies it fully inelastically to the industries and the transport scheme.

Every firm of industry  $i \in \{1, ..., N\}$  is the sole supplier of one of  $n_i$  varieties of good i, which are heterogeneous to the household. Every variety's production features economies of scale due to fixed costs and a constant marginal cost. Hence, the prices include mark-ups. The relative mark-ups atop the total marginal costs are constant because demand for any single variety is isoelastic; so, the absolute mark-ups are nonincreasing in the scheme's progress. Every good i is a composite of its varieties. The numbers of varieties/firms are endogenous as firms can freely enter and exit the markets; so, they all earn a zero profit. All firms of an industry are assumed to be identical, and since all varieties of a good enter the household's utility function in the same way, the firm–level parameters and variables are uniform within the industries.

#### 4.1 Household

The household's utility is given by the Cobb-Douglas function

$$u := \prod_{i} X_i^{\alpha_i} \tag{4.1a}$$

with  $\alpha_i \in (0, 1) \ \forall i$  and  $\sum_i \alpha_i = 1$ , so  $\alpha_i$  is the income share spent on good *i*. Of every good *i* there are  $n_i$  varieties, and  $x_i$  units are consumed per variety. Each  $X_i$  is the quantity of the composite of the  $n_i$  varieties' quantities and given by a CES sub–utility function (A.2)

A.3

A.3.1

with  $\sigma_i > 1 \ \forall i$  as the constant elasticity of substitution between any two varieties:

$$X_i = U_i(x_i) = n_i \frac{\sigma_i}{\sigma_i - 1} x_i$$
 (4.1b)

From the utility function (4.1a) and the budget constraint,  $y = \sum_{i} P_i X_i$ , with  $P_i$  as the price index of the composite *i*, follow the Marshallian demand functions

$$X_i = X_i(P_i, y) := \frac{\alpha_i y}{P_i} . \tag{4.2a}$$

The Marshallian demand functions for the single varieties of good *i*, whose price is denoted by  $p_i$ , is

$$x_i = x_i(p_i, y) = p_i^{-\sigma_i} P_i^{\sigma_i - 1} \alpha_i y .$$
(4.2b)

Hence,  $\sigma_i$  is also the constant price elasticity of demand for any one variety of good *i*. The price index of good *i* is

$$P_{i} = n_{i}^{\frac{1}{1-\sigma_{i}}} p_{i} . (4.3)$$

See appendix A.3.1 for the derivations (Fujita et al., 1999, sec. 4.1, pp. 46–49).

#### 4.2 Industries

All firms of industry *i* are identical with regard to their cost structure. The fixed costs per firm are  $f_i > 0 \ \forall i$ . The marginal costs of production and transportation (following the scheme) are  $c_i > 0 \ \forall i$  and  $\tau_i \ge 0 \ \forall i$ , respectively. The cost reductions are  $t_i \ge 0 \ \forall i$  with  $t_i > 0 \ \exists i$ , and the progress of the scheme is  $\varphi \in [0, 1]$ . Thus, the total marginal cost is  $m_i := c_i + \tau_i + (1 - \varphi) \ t_i$ . Prior to the scheme, this is  $\tilde{m}_i := m_i|_{\varphi=0}$ . The mark-up of any variety of good *i* is denoted by  $\mu_i := p_i - m_i$ . A firm's profit is

$$\pi_i := \mu_i x_i - f_i . \tag{4.4}$$

Assuming a negligible effect of its single variety's price on the composite's price, a firm charges a price of

$$p_i = \frac{\sigma_i}{\sigma_i - 1} m_i , \qquad (4.5)$$

including a mark-up of

$$\mu_i = \frac{1}{\sigma_i - 1} \, m_i > 0 \, , \tag{4.6a}$$

with  $d\mu_i/d\varphi = -t_i/(\sigma_i - 1) \le 0$ . The relative mark-up is constant as

$$\mathbf{v}_i := \frac{p_i}{m_i} \equiv \frac{\sigma_i}{\sigma_i - 1} > 1 . \tag{4.6b}$$

#### 4.3 Long-run equilibrium

By solving the zero profit condition with (4.4) and (4.6a), I get the equilibrium quantity

$$x_i = \frac{(\sigma_i - 1) f_i}{m_i}$$
 (4.7a)

This is nondecreasing in  $\varphi$  because, due to the constant relative mark-up,  $v_i > 1$  (4.6b), if  $t_i$  is positive, the price decreases by more than the marginal cost (in absolute terms) when  $\varphi$  is increased. I.e., the absolute mark-up (4.6a) decreases. This means that the firm needs to sell a larger quantity in order to cover its fixed costs, which is what it merely does given free entry. Due to economies of scale, a variety's average costs are nonincreasing in  $\varphi$ .

The consumer spends a fixed share of their income on good *i* as  $P_iX_i = \alpha_i y$ ; see (4.2a). Besides, they purchase equally large amounts of the goods' varieties. So, the equilibrium amount consumed per variety of good *i* is

$$x_i = \frac{\alpha_i y}{n_i p_i} , \qquad (4.7b)$$

as follows from (4.2b–3). Setting (4.7a) equal to (4.7b), and solving for  $n_i$  by use of (4.5), gives

$$n_i = \frac{\alpha_i y}{\sigma_i f_i} \,. \tag{4.8}$$

From (4.7a) follows that a firm's total costs — and total revenues — are  $m_i x_i + f_i = \sigma_i f_i$ . Hence,  $n_i$  (4.8) is industry *i*'s aggregate revenues divided by a single firm's revenues. It is nonincreasing in  $\varphi$  because *y* is, since the costs of the scheme are nondecreasing in  $\varphi$ ; see section 4.4. Since the income is nonincreasing in  $\varphi$ , the expenditures on good *i*,  $P_i X_i = \alpha_i y$ , are so, too. Whether  $P_i$  and  $X_i$  are increasing or decreasing in  $\varphi$ , is ambiguous, unless *y* is constant or  $t_i = 0$ ; see (4.1b) and (4.3), respectively, and appendix A.3.2.

#### 4.4 General equilibrium

There are no profits to be transferred to the household. Hence, the household's nominal disposable income is earned income, *L*, which is its exogenous endowment with labor, minus the lump-sum tax,

$$S = S(\varphi) , \qquad (4.9)$$

with S(0) = 0 and  $S'(\varphi) := dS(\varphi)/d\varphi \ge 0$ . I.e.,

$$y := L - S \Rightarrow \delta := \frac{y}{L} = \frac{L - S}{L}$$
 (4.10)

So,  $\delta \in (0, 1]$  with  $\delta|_{\varphi=0} \equiv 1$  is nonincreasing in  $\varphi$ .

A.3.2

Transportation intensities are denoted by  $\chi_i := t_i/p_i \ge 0 \ \forall i$ . The average transportation intensity with the income shares for weights is

$$\hat{\chi} := \sum_{i} \alpha_i \chi_i > 0 , \qquad (4.11)$$

which is increasing in  $\varphi$ . With the numerical examples to follow, I distinguish between a transport scheme that is free of cost, i.e.,

$$S(\varphi) = 0 \Rightarrow \delta \equiv 1$$
, (4.12a)

and a scheme that is a subsidy, i.e.,

$$S(\varphi) = \varphi \sum_{i} t_{i} n_{i} x_{i} \Rightarrow \delta = \frac{1}{1 + \varphi \hat{\chi}} .^{15}$$
(4.12b)

Here,  $\delta$  follows from substituting  $S(\varphi)$  (4.12) into (4.10), with  $x_i$  (4.7b) if necessary. In the case of the subsidy,  $\delta$  is decreasing in  $\varphi$ .

#### A.3.3 4.5 Welfare

#### A.3.3.1 4.5.1 Overall impact

In order to construct a welfare measure in the way described in section 3.3.1, I set up the function w(m, y) (A.7a), which is an adaptation of the indirect utility function (A.6) for the long-run equilibrium. Since the varieties' prices are constant multiples of the respective marginal costs, I use the vector of the N goods' marginal costs,  $m := (m_1, \ldots, m_N)$ , as an independent variable. The corresponding expenditure function is denoted by f(m, u) (A.7b). See appendix A.3.3.1. Bars are used to denote an arbitrary reference situation.

Taking the difference between the income and the compensated income, as explained in appendix A.3.3.1, yields the overall impact as

$$I := y - f(\boldsymbol{m}, w(\bar{\boldsymbol{m}}, \bar{y})) = \left[\delta - \bar{\delta} \prod_{i} \left(\frac{m_{i}}{\bar{m}_{i}}\right)^{\alpha_{i}/\hat{v}}\right] L$$
(4.13)

with

$$\hat{\boldsymbol{\nu}} := \sum_{i} \alpha_i \boldsymbol{\nu}_i \tag{4.14}$$

as the average of the mark-up factors (4.6b), which is constant.

<sup>&</sup>lt;sup>15</sup> Note that  $S(\varphi) \neq \varphi \sum_{i} t_i X_i$  since  $X_i \neq n_i x_i$  (4.1b), as opposed to what footnote 14 on page 22 suggests for chapter 3. Also,  $\alpha_i = P_i X_i / y = n_i p_i x_i / y$ ; see (4.2a) and (4.7b), respectively. See also (4.1b) and (4.3). So,  $\alpha_i \neq p_i X_i / y$ , unlike in section 3.3.7.

4.5 Welfare

The scheme's marginal overall impact is the derivative of *I* (4.13) with respect to  $\varphi$ , see (A.8), at  $\bar{\delta} = \delta$  and  $\bar{m} = m$ :

$$I' := \left. \frac{\mathrm{d}I}{\mathrm{d}\varphi} \right|_{\bar{\varphi}=\varphi} = \frac{\delta}{\hat{\nu}} \left[ \sum_{i} \alpha_{i} \chi_{i} \nu_{i} + \hat{\nu} \, \frac{\mathrm{d}\delta/\delta}{\mathrm{d}\varphi} \right] L \,. \tag{4.15}$$

Figure 4.1 and figure 4.2 are numerical examples of the overall impact in the two-industry economy described in section 4.6. The welfare effects depicted in this chapter are calculated for discrete increases of  $\varphi$ , with  $\bar{\varphi} = 0$  as the reference situation. Since  $y|_{\varphi=0} \equiv L = 100$ , the welfare effects are stated in percent of the without-scheme income. The white lines are the contours at a zero effect. If the scheme is free (4.12a), the marginal overall impact is positive; see figure 4.1. If the scheme is a subsidy (4.12b), there can be a negative (marginal) overall impact instead, provided that the (marginal) costs of the scheme exceed the (marginal) gross overall impact on the household's welfare; see figure 4.2.

#### 4.5.2 Direct impact

The discrete change in the scheme's costs (4.9) is

$$\Delta S := S(\varphi) - S(\bar{\varphi}) . \tag{4.16}$$

The gross/net direct impacts, respectively, with  $n_i = n_i(\varphi)$ ,  $p_i = p_i(\varphi)$  and  $y = y(\varphi)$ , are

$$\Delta CS := \int_{\bar{\varphi}}^{\varphi} \left( \sum_{i} t_{i} n_{i}(\phi) x_{i}(p_{i}(\phi), y(\phi)) \right) d\phi$$
(4.17a)

$$D := \Delta CS - \Delta S . \tag{4.17b}$$

For the rule of a half as an approximation thereof, see appendix A.3.3.2.

The derivative of D (4.16–17) is the marginal direct impact:

$$D' := \left. \frac{\mathrm{d}D}{\mathrm{d}\varphi} \right|_{\bar{\varphi}=\varphi} = \delta \left[ \hat{\chi} + \frac{\mathrm{d}\delta/\delta}{\mathrm{d}\varphi} \right] L ; \qquad (4.18)$$

see appendix A.3.3.2. Figure 4.3 and figure 4.4 are numerical examples of the direct impact in the two-industry economy described in section 4.6.<sup>16</sup> If the scheme is free (4.12a), the marginal direct impact is positive; see figure 4.3. If the scheme is a subsidy (4.12b), the (marginal) direct impact is nonpositive because the (marginal) costs of the scheme are equal to or exceed the (marginal) aggregate transport cost reduction; see figure 4.4. A.3.3.2

<sup>&</sup>lt;sup>16</sup> The welfare effects depicted in this chapter are calculated with the accurate direct impact, i.e., using (4.17a), and not the rule of a half (A.10).



**Figure 4.1** Overall impact in a two-industry economy with a free scheme ( $\alpha_1 = \alpha_2 = 0.5$ ,  $\tilde{m}_1 = \tilde{m}_2 = 1$ ,  $v_1 = 1.6$ ,  $v_2 = 1.1$ ,  $t_2 = 0.2$ , L = 100,  $S(\varphi) = 0$  (4.12a),  $\bar{\varphi} = 0$ )



**Figure 4.2** Overall impact in a two-industry economy with a subsidy ( $\alpha_1 = \alpha_2 = 0.5$ ,  $\tilde{m}_1 = \tilde{m}_2 = 1$ ,  $v_1 = 1.6$ ,  $v_2 = 1.1$ ,  $t_2 = 0.2$ , L = 100,  $S(\varphi) = \varphi \sum_i t_i n_i x_i$  (4.12b),  $\bar{\varphi} = 0$ )



**Figure 4.3** Direct impact in a two-industry economy with a free scheme ( $\alpha_1 = \alpha_2 = 0.5$ ,  $\tilde{m}_1 = \tilde{m}_2 = 1$ ,  $v_1 = 1.6$ ,  $v_2 = 1.1$ ,  $t_2 = 0.2$ , L = 100,  $S(\varphi) = 0$  (4.12a),  $\bar{\varphi} = 0$ )



**Figure 4.4** Direct impact in a two-industry economy with a subsidy ( $\alpha_1 = \alpha_2 = 0.5$ ,  $\tilde{m}_1 = \tilde{m}_2 = 1$ ,  $v_1 = 1.6$ ,  $v_2 = 1.1$ ,  $t_2 = 0.2$ , L = 100,  $S(\varphi) = \varphi \sum_i t_i n_i x_i$  (4.12b),  $\bar{\varphi} = 0$ )

#### A.3.3.3 **4.5.3 Wider impact**

The wider impact is W := I - D; see (A.11). The marginal wider impact can be inferred from I' (4.15) and D' (4.18):

$$W' := \left. \frac{\mathrm{d}W}{\mathrm{d}\varphi} \right|_{\bar{\varphi}=\varphi} = I' - D' = \frac{\delta}{\hat{\nu}} \left[ \sum_{i} \alpha_i \chi_i \left( \nu_i - \hat{\nu} \right) \right] L \tag{4.19a}$$

$$= \frac{\delta}{\hat{\mathbf{v}}} \left[ \sum_{i} \alpha_{i} \left( \chi_{i} - \hat{\chi} \right) \left( \mathbf{v}_{i} - \hat{\mathbf{v}} \right) \right] L .$$
 (4.19b)

The sign of this expression is that of the sum within the square brackets, which is the correlation between the industries' mark-ups and transportation intensities. A single industry's summand is positive if the industry exhibits a mark-up and a transportation intensity that are both either above or below the respective economy-wide averages. Figure 4.5 and figure 4.6 are numerical examples of the wider impact in the two-industry economy described in section 4.6.

As I determined in section 3.3, economic activity needs to be shifted away from the industries with lower mark-ups toward the industries with higher mark-ups for the marginal wider impact to be positive. What was not determined at the time is how economic activity would shift, because demand was not specified. In this model, it is shifted toward the industries with both relatively high mark-ups and relatively high transportation intensities. It follows from (4.1b) with (4.7a) and (4.8), or from (A.4), that

$$\frac{X'_i}{X_i} := \frac{\mathrm{d}X_i/X_i}{\mathrm{d}\varphi} = \mathbf{v}_i \left(\chi_i + \frac{\mathrm{d}\delta/\delta}{\mathrm{d}\varphi}\right) . \tag{4.20}$$

Defining the relative quantity shift of a hypothetical average industry as

$$\dot{X}' := \hat{\nu} \left( \hat{\chi} + \frac{\mathrm{d}\delta/\delta}{\mathrm{d}\varphi} \right)$$

yields

$$W' = \frac{\delta}{\hat{\nu}} \left[ \sum_{i} \alpha_{i} \left( \frac{X'_{i}}{X_{i}} - \dot{X}' \right) \right] L .$$
(4.19c)

The marginal wider impact's sign is that of the average of the relative quantity shifts' deviations from the average quantity shift, with the income shares for weights.

With uniform transportation intensities, i.e.,  $\chi_i = \chi \ \forall i \Rightarrow \hat{\chi} = \chi$ , see (4.11), the marginal wider impact is zero; see (4.19b). The relative quantity shifts (4.20) are proportional to the  $v_i$ 's. If  $\chi = -(d\delta/\delta)/d\varphi$ , these shifts are all zero and there is no welfare effect at all, which is the case at  $\varphi = 0$  with (4.12b); see (A.9) with (4.11). Examples with two industries and  $\chi|_{\varphi=0} = 0.125$  are illustrated in the bottom–right diagrams of figure 4.7 and figure 4.8.



**Figure 4.5** Wider impact in a two-industry economy with a free scheme ( $\alpha_1 = \alpha_2 = 0.5$ ,  $\tilde{m}_1 = \tilde{m}_2 = 1$ ,  $v_1 = 1.6$ ,  $v_2 = 1.1$ ,  $t_2 = 0.2$ , L = 100,  $S(\varphi) = 0$  (4.12a),  $\bar{\varphi} = 0$ )



**Figure 4.6** Wider impact in a two-industry economy with a subsidy ( $\alpha_1 = \alpha_2 = 0.5$ ,  $\tilde{m}_1 = \tilde{m}_2 = 1$ ,  $v_1 = 1.6$ ,  $v_2 = 1.1$ ,  $t_2 = 0.2$ , L = 100,  $S(\varphi) = \varphi \sum_i t_i n_i x_i$  (4.12b),  $\bar{\varphi} = 0$ )

If all mark-ups that prevail in this economy are just the same, and thus equal to their average, i.e.,  $v_i = v \ \forall i \Rightarrow \hat{v} = v$ , see (4.14), the marginal wider impact (4.19) is zero. Examples with v = 1.6 are illustrated in the bottom–left diagrams of figure 4.7 and figure 4.8.

#### 4.6 Two-industry economy

Consider the case of two industries with  $\alpha_1 = \alpha_2 = 0.5$ . Then (4.19) is

$$W' = \frac{\delta}{2(v_1 + v_2)} (\chi_1 - \chi_2) (v_1 - v_2) L$$

If and only if industry 1 is the industry with the higher mark-up, i.e.,  $v_1 > v_2$ , then

$$\chi_1 > \chi_2 \Leftrightarrow \frac{t_1/m_1}{t_2/m_2} > \frac{\nu_1}{\nu_2} \Leftrightarrow W' > 0.$$

Since  $v_1/v_2 > 1$ ,  $t_1/m_1 > t_2/m_2$  does not suffice. The upper–left diagrams of figure 4.7 and figure 4.8 are examples with  $\chi_1|_{\varphi=0} = 5/16 > 2/11 = \chi_2|_{\varphi=0} \Rightarrow W'|_{\varphi=0} > 0$ , and the upper–right diagrams are examples with  $\chi_1|_{\varphi=0} = 1/8 < 5/11 = \chi_2|_{\varphi=0} \Rightarrow W'|_{\varphi=0} < 0$ .



**Figure 4.7** Overall and wider impact in a two-industry economy with a free scheme  $(\alpha_1 = \alpha_2 = 0.5, \ \tilde{m}_1 = \tilde{m}_2 = 1, \ L = 100, \ S(\varphi) = 0 \ (4.12a), \ \bar{\varphi} = 0)$ 



**Figure 4.8** Overall and wider impact in a two-industry economy with a subsidy ( $\alpha_1 = \alpha_2 = 0.5$ ,  $\tilde{m}_1 = \tilde{m}_2 = 1$ , L = 100,  $S(\varphi) = \varphi \sum_i t_i n_i x_i$  (4.12b),  $\bar{\varphi} = 0$ )

If the ratios between the transport cost reductions and the total marginal costs are identical for the two industries, i.e.,  $t_1/m_1 = t_2/m_2$ , then — whatever the mark-ups — the marginal wider impact is nonpositive:

$$W' = -\frac{\delta (v_1 - v_2)^2}{2v_1 v_2 (v_1 + v_2)} \frac{t_1}{m_1} L \le 0$$

This shows up in figure 4.5 and figure 4.6, which are numerical examples with  $v_1 > v_2$  and  $\tilde{m}_1 = \tilde{m}_2$ , but with a varying  $t_1$ . If  $t_1 \le t_2 = 0.2$ , there is a negative marginal wider impact at  $\varphi = 0$ . If  $t_1$  is sufficiently high, there is a positive wider impact.

In order for a scheme to have a positive wider impact, one first needs to make sure that the industries differ with regard to their mark-ups. Second, one needs to make sure that they differ with regard to the transportation intensities. Third, the industries with the relatively high transportation intensities need to be the less competitive ones in that they exhibit higher mark-ups.

## Chapter 5

## Monopoly

In the style of what I considered in section 3.4, this model is that of an economy with two industries and two goods. The goods are called "good 1" and the "numeraire good". Good 1 is produced by a monopolist and with nondecreasing returns to scale. The numeraire good is produced under perfect competition with constant returns to scale. Both industries employ only labor, which is supplied by the one representative household. Both the wage rate and the marginal product of labor in the production of the numeraire good are one, so both labor and the numeraire good serve as numeraires. The household's preferences are quasi-linear, with the marginal utility of good 1 being decreasing and that of the numeraire good being constant.

Only the transport cost in the trade of good 1 between the monopolist and the consumer is reduced by the scheme. Since the mark-up charged by the monopolist is endogenous, a condition is stated for the price of good 1 to actually be decreased by the scheme. If it is, the marginal wider impact must be positive. This is because resources are shifted from the perfectly competitive industry toward the monopoly. Also due to the lack of an income effect in the consumption of good 1, this is a simple model to start assessing the wider impact on the level of a single industry.

In chapter 6, I consider two fully symmetric — though spatially distant — economies, each equivalent to the economy of this chapter. This adds the possibility of reciprocal dumping as the two firms of indusry 1 might export to each other's domestic market undercutting each other's monopoly price, thus creating two duopolies. Instead, they might attempt to ward off the foreign competition by means of predatory pricing. Either way, they are unable to sustain their monopoly power as a result of the transport scheme.

#### 5.1 Industry with imperfect competition

The variables and parameters of industry 1 are like those in the previous chapters, just without the subscript. The marginal cost is  $m := c + \tau + (1 - \varphi) t$  with c > 0 (production),  $\tau \ge 0$  and t > 0 (transportation),  $\varphi \in [0, 1]$  as the progress of the scheme and  $\tilde{m} := m|_{\varphi=0}$ . The price, the mark-up, the quantity and the fixed costs are p,  $\mu := p - m$ , X and  $f \ge 0$ .

A.4

The firm's profit is

$$\pi := \mu X - f . \tag{5.1}$$

Maximizing its profit, the firm charges a price of

$$p = \frac{\epsilon}{\epsilon - 1} \, m$$

with the price elasticity of demand as

$$\epsilon := -\frac{d\ln(X)}{d\ln(p)} > 1 ; \qquad (5.2)$$

see Mas-Colell et al. (1995, sec. 12.B, pp. 384–387), Varian (1999, ch. 24, pp. 414–432) or Jehle & Reny (2011, p. 170 et seqq.). The mark-up atop the marginal cost follows as

$$\mu = \frac{1}{\epsilon - 1} m > 0 . \tag{5.3}$$

#### 5.2 Household

The household has the quasi-linear utility function

$$u = u(X, z) := s(X) + z$$
 (5.4)

with the strictly concave sub-utility function s(X) and a marginal utility of good 1 that is greater than this good's price for sufficiently low quantities.<sup>17</sup> The quantity of the numeraire good is denoted by z.

Maximizing utility (5.4) subject to the budget constraint,

$$y = pX + z , (5.5)$$

yields the Marshallian demand function for good 1,

$$X = X(p) := s'^{-1}(p)$$
, (5.6a)

which is decreasing in *p*, and the demand function for the numeraire good,

$$z = z(p, y) := y - p X(p) .$$
<sup>18</sup> (5.6b)

Demand for good 1 is independent of the household's income given a sufficiently high income.

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<sup>&</sup>lt;sup>17</sup> The sub–utility function is such that s'(X) := ds(X)/dX > 0, s'(0) > p and  $s''(X) := d^2s(X)/dX^2 < 0$ . See Mas–Colell et al. (1995, p. 45) or Varian (1992, sec. 10.3, pp. 164–166) for the quasi–linear utility function.

<sup>&</sup>lt;sup>18</sup> The demand function is such that X'(p) := dX(p)/dp < 0; see footnote 17.

If and only if  $y \ge p X(p)$ , which is assumed to always be the case, the household will spend p X(p) on good 1 while spending the nonnegative residual of its income on the numeraire good, as given by (5.6).<sup>19</sup>

#### 5.3 General equilibrium

The costs of the transport scheme, and accordingly the lump-sum tax, are

$$S = S(\varphi) \tag{5.7}$$

with S(0) = 0 and  $S'(\varphi) := dS(\varphi)/d\varphi \ge 0$ . The household receives an earned income of L, which is its exogenous endowment with labor, as well as the monopolist's profit (5.1). Its nominal disposable income follows as

$$y := L + \pi - S .^{20}$$
(5.8)

With the numerical examples that follow, I distinguish between a transport scheme that is free of cost, i.e., an exogenous cost reduction with

$$S(\varphi) = 0 , \qquad (5.9a)$$

and a transport scheme that is equivalent to a subsidy as

$$S(\varphi) = \varphi t X . \tag{5.9b}$$

The equilibrium condition of the labor market, which follows from the household's income (5.8) with the budget constraint (5.5), is

$$L = mX + f + z + S . (5.10)$$

The first two summands to the right side of the equal sign are the variable and the fixed labor input in the production of good 1, respectively. The third summand, *z*, is the input in the production of the numeraire good. The fourth summand is the input in the provision of the transport scheme.

<sup>&</sup>lt;sup>19</sup> If it were that  $y \le p X(p)$ , then X = y/p and z = 0. Thus, the demand functions would need to be defined in a way other than that of (5.6).

<sup>&</sup>lt;sup>20</sup> It follows from (5.8) with (5.1) and (5.6a) that  $s'((L - f - S)/m) \le p \Leftrightarrow L - f - S \ge m X(p) \Leftrightarrow y \ge p X(p)$ ; cf. footnote 17.

#### 5.4 Welfare

#### A.4.1 5.4.1 Overall impact

Solving the labor market's equilibrium condition (5.10) for z, and substituting this into the utility function (5.4), along with the demand function for good 1 (5.6a), gives

$$u = w(p) := s(X(p)) - m X(p) + L - f - S.$$
(5.11)

This is an adaptation of the indirect utility function (A.12a) for the general equilibrium.

Since the household's utility function (5.4) is quasi-linear, the change in its utility is equal to the change in its consumer surplus as well as its compensating/equivalent variation (Varian, 1992, sec. 10.4, pp. 166–167). With bars denoting the parameters' and variables' values in an arbitrary reference situation, the scheme's additional costs are

$$\Delta S := S(\varphi) - S(\bar{\varphi}) ; \qquad (5.12)$$

see (5.7). The overall impact is

$$I := u - \bar{u} = s(X(p)) - s(X(\bar{p})) - (m X(p) - \bar{m} X(\bar{p})) - \Delta S .^{21}$$
(5.13)

It follows from the labor market's equilibrium condition (5.10) that

$$dL = m dX + dz - (tX - S'(\varphi)) d\varphi = 0.$$
(5.14)

For the marginal overall impact, one can differentiate (5.11) or (5.13), and then use (5.14):

$$I' := \left. \frac{\mathrm{d}I}{\mathrm{d}\varphi} \right|_{\bar{\varphi}=\varphi} = \left. \frac{\mathrm{d}u}{\mathrm{d}\varphi} \right|_{\bar{\varphi}=\varphi} = \mu \, X'(p) \, \frac{\mathrm{d}p}{\mathrm{d}\varphi} + t \, X(p) - S'(\varphi) = p \, X'(p) \, \frac{\mathrm{d}p}{\mathrm{d}\varphi} + \frac{\mathrm{d}z}{\mathrm{d}\varphi} \,. \tag{5.15}$$

This is the additional utility from a change in the quantity of good 1 plus the additional utility from the change in the quantity of the numeraire good.

#### A.4.2 **5.4.2 Direct impact**

Due to the quasi-linear preferences, there is no income effect with respect to good 1 so that the demand curve for good 1 does not shift, and the change in consumer surplus on the

<sup>&</sup>lt;sup>21</sup> Since *I* is independent of the values of *L* and *f*, only a minimum of  $\bar{u}$  (5.11) can be calculated to compare the welfare effects to. This minimum follows from  $L - f \ge m X(p) + S(\varphi)$ ; see footnote 20. With figure 5.1, where  $S(\varphi) = 0$  (5.9a), the condition is  $L - f \ge 0.09 \Rightarrow \bar{u} \ge 0.0975$  as m X(p) is maximal at  $\varphi = 0.8$ . With figure 5.2, where  $S(\varphi) = \varphi t X$  (5.9b), the condition is  $L - f \ge 0.175 \Rightarrow \bar{u} \ge 0.1825$  as  $m X(p) + S(\varphi) = \tilde{m} X(p)$  is maximal at  $\varphi = 1$ . The more costly the scheme is, the greater the abundance of resources beyond the fixed input of industry 1 needs to be.

transport market is equal to both the compensating and the equivalent variation (Mas-Colell et al., 1995, p. 83). The gross/net direct impacts, respectively, with  $p = p(\varphi)$ , are

$$\Delta CS := \int_{\tilde{\varphi}}^{\varphi} t \, X(p(\phi)) \, \mathrm{d}\phi \tag{5.16a}$$

$$D := \Delta CS - \Delta S . \tag{5.16b}$$

For the rule of a half, see appendix A.4.2.

The derivative of the direct impact (5.16b), with (5.14), is

$$D' := \left. \frac{\mathrm{d}D}{\mathrm{d}\varphi} \right|_{\bar{\varphi}=\varphi} = t \, X(p) - S'(\varphi) = m \, X'(p) \, \frac{\mathrm{d}p}{\mathrm{d}\varphi} + \frac{\mathrm{d}z}{\mathrm{d}\varphi} \,. \tag{5.17}$$

This is the additional input in the production of good 1 plus the additional input in the production of the numeraire good.

#### 5.4.3 Wider impact

The marginal wider impact, i.e., the derivative of W := I - D (A.13) at the reference values, with I' (5.15) and D' (5.17), is

$$W' := \left. \frac{\mathrm{d}W}{\mathrm{d}\varphi} \right|_{\bar{\varphi}=\varphi} = I' - D' = \mu X'(p) \, \frac{\mathrm{d}p}{\mathrm{d}\varphi} \,. \tag{5.18}$$

Since  $\mu > 0$  (5.3), there is a positive marginal wider impact if and only if the quantity produced by the monopolist is increased. Since X'(p) < 0, this is so if and only if  $dp/d\varphi < 0$ . If and only if demand is such that an increase in  $\varphi$  leads to a sufficiently moderate reduction, or even a rise, in the elasticity, i.e.,  $(d\epsilon/\epsilon)/d\varphi > -t/\mu$ , then the consequent change in the mark-up does not outweigh the simultaneous decrease in *m*, or it even complements it. See appendix A.4.3.

Figure 5.1 and figure 5.2 illustrate examples of the welfare effects given a linear demand for good 1 and a discrete increase in  $\varphi$ . The scheme is free of cost or a subsidy; see (5.9). Either way, the marginal wider impact is positive by choice of the demand function.

The welfare effects on the level of the monopoly,  $I_1$ ,  $D_1$  and W, which correspond to the previous figures, are illustrated in figure 5.3; cf. Mackie et al. (2011, Figure 21.3b, p. 521) and figure 3.1 on page 20. The scheme's costs attributed to industry 1 in deriving  $I_1$  and  $D_1$  are  $\Delta S_1 := \bar{m}\bar{X} - mX$ , which is the amount of labor withdrawn from the industry, and  $\Delta S = \Delta S_1 + \bar{z} - z$ ; see (5.10). See also section 3.3.3.

A.4.3



**Figure 5.1** Welfare effect with a free scheme ( $s(X) = 1.2 X - X^2$ ,  $c + \tau = 0.5$ , t = 0.5,  $S(\varphi) = 0$  (5.9a),  $\bar{\varphi} = 0$ )



**Figure 5.2** Welfare effect with a subsidy ( $s(X) = 1.2 X - X^2$ ,  $c + \tau = 0.5$ , t = 0.5,  $S(\varphi) = \varphi t X$  (5.9b),  $\bar{\varphi} = 0$ )



Figure 5.3 Welfare effect on the monopoly level ( $s(X) = 1.2X - X^2$ ,  $c + \tau = 0.5$ , t = 0.5,  $\bar{\varphi} = 0$ ,  $\varphi = 0.1$ )

## Chapter 6

## Reciprocal dumping

The framework below is based on the reciprocal dumping model originally presented by Brander (1981) and Brander & Krugman (1983), and it adds the possibility of firm entry. There are two fully identical, though spatially distant, economies of the type described in chapter 5. In each economy — or region for that matter — there are two industries and two goods. The one industry comprises a single firm which produces "good 1" with nondecreasing returns to scale. The other industry is perfectly competitive with constant returns to scale, producing the "numeraire good". Both industries of a region employ the labor supplied by the one local representative household. Labor and the numeraire good both serve as numeraires because the wage rate and the marginal product of labor in the production of the numeraire good are both one, and the numeraire good can freely be traded between the regions. The households are immobile between the regions. Their preferences are quasi-linear with a decreasing marginal utility of good 1 and a constant marginal utility of the numeraire good; see section 5.2.

Good 1 can also be traded between the regions. Yet, the transport cost of exporting to the foreign region is higher than that of supplying the domestic consumer. So, maybe there are no exports but two regions with one monopoly each, with each region being equivalent to the economy of chapter 5. As the full transport scheme eliminates the difference between the transport costs of industry 1, exports do eventually occur, unless the firms engage in entry deterrence; see section 6.3. The firms engage in reciprocal dumping into their respective foreign markets. They are able to undercut the monopoly price that is otherwise charged by the respective incumbents. Consequently, Cournot–type duopolies emerge. Instead, the firms might prevent each other's entry through predatory price setting; see section 6.4. Whether there is firm entry or just the threat thereof, 1 assess how this influences the scheme's wider impact in comparison to the sustained monopolies of chapter 5.

Since the economies are fully symmetric, variables and parameters simultaneously apply to both of them. For a background on the welfare implications of reciprocal dumping, see Greenhut et al. (1987, sec. 9.2.2, pp. 161–162) and Feenstra (2004, p. 241 et seqq.).

#### 6.1 Industry with imperfect competition

Industry 1's marginal cost of production is c > 0, and  $\tau \ge 0$  is its marginal cost of transportation unaffected by the scheme. The transport costs of the domestically consumed and the exported units, which are both eliminated by the scheme, are denoted by  $t_d$  and  $t_e$ , respectively, with  $t_e > t_d \ge 0$ . The total marginal costs are  $m_d := c + \tau + (1 - \varphi) t_d \le c + \tau + (1 - \varphi) t_e =: m_e \Rightarrow m_d|_{\varphi=1} = c + \tau = m_e|_{\varphi=1}$ . Prior to the scheme, the total marginal costs are  $\tilde{m}_d := m_d|_{\varphi=0} < m_e|_{\varphi=0} =: \tilde{m}_e$ . I use the subscripts d and e for other parameters and variables as well to distinguish between the firms' two separate operations of domestic supply and exporting.

With  $x_d$  denoting the domestically consumed quantity,  $x_e$  denoting the exported quantity and  $X := x_d + x_e$  denoting the aggregate quantity, each per firm/region, the export share is

$$\lambda := \frac{x_e}{X} \in [0, 0.5]$$

With *p* as the price and  $\mu_d := p - m_d \ge p - m_e =: \mu_e$  as the mark-ups, the average mark-up is  $\hat{\mu} := (1 - \lambda) \mu_d + \lambda \mu_e$ . With  $f \ge 0$  as the fixed costs per firm, a firm's profit is

$$\pi := \hat{\mu}X - f . \tag{6.1}$$

#### 6.2 General equilibrium

The lump-sum tax per household to cover the scheme's costs is

$$S = S(\varphi) \tag{6.2}$$

with S(0) = 0 and  $S'(\varphi) := dS(\varphi)/d\varphi \ge 0$ . As each household supplies *L* units of labor and receives the profit of one firm (6.1), each household's disposable income amounts to

$$y := L + \pi - S$$
. (6.3)

In the numerical examples given below, there is either a free scheme, i.e., an exogenous cost reduction with

$$S(\varphi) = 0 , \qquad (6.4a)$$

or a transport subsidy with the corresponding costs of

$$S(\varphi) = \varphi \hat{t} X . \tag{6.4b}$$

Here,  $\hat{t} := (1 - \lambda) t_d + \lambda t_e$  is the average transport cost reduction.

The average of the total marginal costs is  $\hat{m} := (1 - \lambda) m_d + \lambda m_e$ . Setting a household's income (6.3) equal to its expenditures (5.5), yields the equilibrium condition of either region's labor market:

$$L = \hat{m}X + f + z + S . (6.5)$$

#### 6.3 Simultaneous moves

Assume that the firms decide simultaneously about both their domestic and foreign supply. Maximizing profits, they charge a price on the domestic market and — if possible — on the foreign market of

$$p = p_d(\lambda) := \frac{\epsilon}{\epsilon + \lambda - 1} m_d > m_d \tag{6.6a}$$

$$p_e(\lambda) := \frac{\epsilon}{\epsilon - \lambda} m_e \ge m_e$$
 (6.6b)

The elasticity,  $\epsilon$ , is the price elasticity of *market* demand as defined by (5.2), with  $\epsilon > 1 - \lambda$ . See Varian (1999, sec. 27.8, pp. 481–482).

I need to distinguish between the case of no interregional trade (corner solution) and the case of interregional trade (interior solution). The relative mark-up that the firms are able to charge on their domestic markets for as long as they do not have to compete with imports is denoted by v. If

$$\frac{m_e}{m_d} \ge v := \frac{p_d(0)}{m_d}$$
, (6.7)

the firms are unable to compete on their foreign markets. Hence,

$$\lambda = x_e = 0$$
 and  $p = p_d(0) = vm_d \le m_e = p_e(0)$ 

If  $p_d(\lambda) \le p_e(\lambda)$  (6.6) at any level of  $\lambda$ , there is no interior but only this corner solution. See, for instance, the case of  $\varphi = 0$  in figure 6.1.

Even when there is a corner solution, a gradual (further) rise of  $\varphi$  will eventually make the marginal cost of exporting drop below the monopoly price. Figure 6.1 illustrates an example where the intraregional transport cost is affected by the scheme as  $t_d > 0$ . If instead  $t_d = 0$ , then  $p_d(\lambda)$  would not be shifted. The solution to  $p_d(\lambda) = p_e(\lambda)$  with (6.6) for  $m_e/m_d \leq v$  — is

$$p = \frac{\epsilon}{2\epsilon - 1} (m_d + m_e)$$
$$\lambda = \frac{\epsilon (m_d - m_e) + m_e}{m_d + m_e} .$$

This is the Nash equilibrium when each firm supplies the households of both regions.

A.5.1

The export threshold  $\check{v} := \{v | v = m_e/m_d\}$  is the solution to  $p_d(0) = p_e(0)$ , cf. (6.7), and hence the lower bound on the marginal–cost ratios for which there is no interregional trade. It defines the scheme's export threshold which needs to be exceeded for trade to occur as

$$\check{\varphi} := \begin{cases} \left\{ \varphi \middle| \varphi = \frac{\tilde{m}_e - \nu \tilde{m}_d}{t_e - \nu t_d} \right\} & \text{for } \tilde{m}_e / \tilde{m}_d \ge \nu|_{\varphi=0} \\ 0 & \text{for } \tilde{m}_e / \tilde{m}_d < \nu|_{\varphi=0} \end{cases}.$$

For there not to be any exports prior to the scheme, i.e.,  $\lambda = x_e = 0$  at  $\varphi = 0$ , it has to hold that  $\tilde{m}_e/\tilde{m}_d \ge v|_{\varphi=0}$ . Then  $\check{\varphi} \ge 0$ . Since  $m_e/m_d \ge 1$  is decreasing in  $\varphi$ , and since v > 1, it follows that  $m_e/m_d|_{\varphi=1} = 1 < v|_{\varphi=1}$ . So,  $\check{\varphi} < 1$ , and in conclusion,  $\check{\varphi} \in [0, 1)$ . If  $\tilde{m}_e/\tilde{m}_d < v|_{\varphi=0}$ , there is trade between the regions even prior to the scheme. If  $\check{\varphi} = 0$ , the first part of, say, (6.8a) is obviously superfluous. If  $t_d = 0$ , then  $m_d \equiv c + \tau$  and v is constant; if  $\check{v}$  exists, then  $\check{v} \equiv v$ .

The corner solutions and the interior solutions can be summarized by

$$p = \begin{cases} \frac{\epsilon}{\epsilon - 1} m_d & \text{for } \varphi < \check{\varphi} \\ \frac{\epsilon}{2\epsilon - 1} (m_d + m_e) & \text{for } \check{\varphi} \le \varphi \end{cases}$$

$$\lambda = \begin{cases} 0 & \text{for } \varphi < \check{\varphi} \\ \frac{\epsilon (m_d - m_e) + m_e}{m_d + m_e} & \text{for } \check{\varphi} \le \varphi \end{cases}$$
(6.8b)



Figure 6.1 Nash equilibrium with simultaneous moves ( $s(X) = 1.2 X - X^2$ ,  $c + \tau = 0.5$ ,  $t_d = 0.5$ ,  $t_e = 0.65$ ,  $\varphi = 0.3$ ,  $\check{\varphi} = 0.125$ ,  $\check{\nu} = 1.14$ )

#### 6.3.1 Overall impact

Solving (6.5) for z and substituting this into the utility function (5.4), along with the demand function for good 1 (5.6a), gives a household's utility in general equilibrium:

$$u = w(p) := s(X(p)) - \hat{m} X(p) + L - f - S.$$
(6.9)

This is an adaptation of the indirect utility function (A.12a) with equilibrium income (6.3).

As the scheme progresses, it causes additional costs of

$$\Delta S := S(\varphi) - S(\bar{\varphi});$$

see (6.2). The change in utility as given by w(p) (6.9), relative to some reference situation which is denoted by a bar, yields the overall impact as

$$I := u - \bar{u} = s(X(p)) - s(X(\bar{p})) - \left(\hat{m} X(p) - \bar{\tilde{m}} X(\bar{p})\right) - \Delta S .$$
 (6.10)

This is the change in consumer surplus as well as the compensating/equivalent variation (Varian, 1992, sec. 10.4, pp. 166–167).

From the labor market's equilibrium condition (6.5) follows

$$dL = \hat{m} dX + (1 - \varphi) (t_e - t_d) X d\lambda + dz - (\hat{t}X - S'(\varphi)) d\varphi = 0.$$
(6.11)

The marginal change in I (6.10),  $I' := dI/d\varphi|_{\bar{\varphi}=\varphi}$  with (6.11), is

$$l' = \begin{cases} \mu_d X'(p) \frac{dp}{d\varphi} + t_d x_d - S'(\varphi) & \text{for } \varphi < \check{\varphi} \\ \hat{\mu} X'(p) \frac{dp}{d\varphi} - (1 - \varphi) (t_e - t_d) X \frac{d\lambda}{d\varphi} + \hat{t} X - S'(\varphi) & \text{for } \check{\varphi} < \varphi \lor \check{\varphi} = 0 \end{cases}$$

$$= p X'(p) \frac{dp}{d\varphi} + \frac{dz}{d\varphi} .$$
(6.12a)
(6.12b)

As long as a firm can keep its monopoly on the domestic market, i.e., as  $\varphi < \check{\varphi} \Rightarrow \lambda = 0$ , the welfare effects are equivalent to what is described in section 5.4.

At the export threshold, i.e., at  $\varphi = \check{\varphi}$ , there is a leap in the marginal impact. While the export share,  $\lambda$ , is zero,  $d\lambda/d\varphi$  jumps from zero to some positive value. This reduces the marginal impact because an increasingly large share of what a household consumes of good 1 is imported from the other region, and imports are more costly than the domestically produced units. Besides, there is a jump in  $dp/d\varphi$  while  $\hat{\mu} = \mu_d > 0$ . If the increase in competition accelerates the decline in the price, this increases the marginal impact and thereby counteracts the effect of the increase in the export share. Finally, there might be some jump in  $S'(\varphi)$ . See appendix A.5.1.1.

A.5.1.1

#### A.5.1.2 **6.3.2 Direct impact**

Due to the quasi-linear preferences, good 1's demand curve does not shift because there is no income effect. Hence, the change in consumer surplus is equal to the compensating and equivalent variations (Mas-Colell et al., 1995, p. 83). The gross and net direct impacts, respectively, with  $\hat{t} = \hat{t}(\varphi)$  and  $p = p(\varphi)$ , are

$$\Delta CS := \int_{\bar{\varphi}}^{\varphi} \hat{t}(\phi) X(p(\phi)) \,\mathrm{d}\phi \tag{6.13a}$$

$$D := \Delta CS - \Delta S . \tag{6.13b}$$

For the rule of a half as an approximation of  $\Delta CS$ , see appendix A.5.1.2.

The marginal direct impact,  $D' := dD/d\varphi|_{\bar{\varphi}=\varphi}$ , follows from (6.13) with (6.11):

$$D' = \hat{t}X - S'(\varphi) \tag{6.14a}$$

$$= \begin{cases} m_d x_d \frac{\mathrm{d}\rho}{\mathrm{d}\varphi} + \frac{\mathrm{d}z}{\mathrm{d}\varphi} & \text{for } \varphi < \check{\varphi} \\ \hat{m} X \frac{\mathrm{d}\rho}{\mathrm{d}\varphi} + (1-\varphi) (t_e - t_d) X \frac{\mathrm{d}\lambda}{\mathrm{d}\varphi} + \frac{\mathrm{d}z}{\mathrm{d}\varphi} & \text{for } \check{\varphi} < \varphi \lor \check{\varphi} = 0 . \end{cases}$$
(6.14b)

#### A.5.1.3 **6.3.3 Wider impact**

From I' (6.12) and D' (6.14) follows the derivative of W := I - D (A.15):

$$W' := \left. \frac{\mathrm{d}W}{\mathrm{d}\varphi} \right|_{\bar{\varphi}=\varphi} = \begin{cases} \mu_d \, X'(p) \, \frac{\mathrm{d}p}{\mathrm{d}\varphi} & \text{for } \varphi < \check{\varphi} \\ \hat{\mu} \, X'(p) \, \frac{\mathrm{d}p}{\mathrm{d}\varphi} - (1-\varphi) \, (t_e - t_d) \, X \, \frac{\mathrm{d}\lambda}{\mathrm{d}\varphi} & \text{for } \check{\varphi} < \varphi \, \lor \, \check{\varphi} = 0 \; . \end{cases}$$
(6.15)

The lower  $dp/d\varphi$  and  $d\lambda/d\varphi$  are, the higher is the marginal wider impact. In turn, these are the lower, the higher  $d\epsilon/d\varphi$  is; see (A.14).

The next four figures are numerical examples of the welfare effects. The light-colored lines therein are the effects with sustained monopolies as in section 5.4. A comparison between the duopoly and the monopoly yields the following conclusion. At the export threshold, the jump to a positive  $d\lambda/d\varphi$  reduces W' because interregional trade consumes more resources than intraregional trade. If the entry of the foreign firm leads to a jump to a lower  $dp/d\varphi$ , e.g., if it accelerates the price's decline by posing additional competition, this increases W'. Figure 6.2 and figure 6.3, each with linear demand, exemplify leaps to lower levels of W' at  $\varphi = \check{\varphi}$ . Figure 6.4 and figure 6.5, each with isoelastic demand, exemplify leaps to higher levels of W'. Besides, the marginal wider impact beyond  $\check{\varphi}$  is decreasing in  $\lambda$  and increasing in p via  $\hat{\mu}$ , the average mark-up, if  $dp/d\varphi < 0$ .



**Figure 6.2** Welfare effect with simultaneous moves, linear demand and a free scheme  $(s(X) = 1.2 X - X^2, c + \tau = 0.5, t_d = 0.5, t_e = 0.65, S(\varphi) = 0$  (6.4a),  $\bar{\varphi} = 0$ )



**Figure 6.3** Welfare effect with simultaneous moves, linear demand and a subsidy  $(s(X) = 1.2 X - X^2, c + \tau = 0.5, t_d = 0.5, t_e = 0.65, S(\varphi) = \varphi \hat{t} X$  (6.4b),  $\bar{\varphi} = 0$ )



**Figure 6.4** Welfare effect with simultaneous moves, isoelastic demand and a free scheme  $(X(p) = 100 p^{-4}, c + \tau = 1, t_d = 0.4, t_e = 1, S(\varphi) = 0$  (6.4a),  $\bar{\varphi} = 0$ )



**Figure 6.5** Welfare effect with simultaneous moves, isoelastic demand and a subsidy  $(X(p) = 100 p^{-4}, c + \tau = 1, t_d = 0.4, t_e = 1, S(\varphi) = \varphi \hat{t} X$  (6.4b),  $\bar{\varphi} = 0$ )



**Figure 6.6** Welfare effect with entry deterrence, linear demand and a free scheme  $(s(X) = 1.2 X - X^2, c + \tau = 0.5, t_d = 0.5, t_e = 0.65, S(\varphi) = 0$  (6.4a),  $\bar{\varphi} = 0$ )



**Figure 6.7** Welfare effect with entry determence, linear demand and a subsidy ( $s(X) = 1.2 X - X^2$ ,  $c + \tau = 0.5$ ,  $t_d = 0.5$ ,  $t_e = 0.65$ ,  $S(\varphi) = \varphi \hat{t} X$  (6.4b),  $\bar{\varphi} = 0$ )



**Figure 6.8** Welfare effect with entry deterrence, isoelastic demand and a free scheme  $(X(p) = 100 p^{-4}, c + \tau = 1, t_d = 0.4, t_e = 1, S(\varphi) = 0$  (6.4a),  $\bar{\varphi} = 0$ )



**Figure 6.9** Welfare effect with entry determence, isoelastic demand and a subsidy  $(X(p) = 100 p^{-4}, c + \tau = 1, t_d = 0.4, t_e = 1, S(\varphi) = \varphi \hat{t} X$  (6.4b),  $\bar{\varphi} = 0$ )

#### 6.4 Strategic entry deterrence

So far, I assumed that when a firm exports to the foreign region, it makes its decision simultaneously to the incumbent firm. This section is a digression in that this decision is assumed to be sequential. The incumbent firm has the first-mover advantage. It anticipates the potential entrant's reaction (6.6b) and engages in strategic entry deterrence by means of predatory pricing. It sets its price equal to the total marginal cost of exports so that the potential entrant does not actually export:

$$p = \begin{cases} \frac{\epsilon}{\epsilon - 1} m_d & \text{for } \varphi < \check{\varphi} \\ m_e & \text{for } \check{\varphi} \le \varphi \\ \lambda = 0 \ . \end{cases}$$

The marginal wider impact follows from (6.15):

$$W' = \begin{cases} \mu_d X'(p) \frac{\mathrm{d}p}{\mathrm{d}\varphi} & \text{for } \varphi < \check{\varphi} \\ (1-\varphi) (t_e - t_d) (-t_e X'(p)) \ge 0 & \text{for } \check{\varphi} < \varphi \lor \check{\varphi} = 0 \end{cases}$$

The leap in this marginal wider impact at  $\check{\varphi}$  is bigger than with simultaneous moves or sustained monopolies since  $dp/d\varphi$  jumps to a lower level and  $d\lambda/d\varphi = 0$ . Since the full scheme,  $\varphi = 1$ , equalizes the total marginal costs and the price, it reduces the mark-ups and the marginal wider impact to zero.

The four figures above include the corresponding results for the numerical examples of section 6.3. They all exhibit positive leaps in W' at the export threshold, and — for the most part — the wider impacts beyond  $\check{\varphi}$  are higher than in the case of simultaneous moves. Figure 6.6 and figure 6.7, each with linear demand, exemplify wider impacts as a result of a full scheme,  $W|_{\varphi=1}$ , that are lower than in the monopoly case. Figure 6.8 and figure 6.9, each with isoelastic demand, exemplify full schemes that yield wider impacts that are higher than in the monopoly case.

Assume that  $t_d = 0$ ; so, W' = 0 for  $\varphi < \check{\varphi}$  since the transport scheme does not affect the costs of intraregional trade. Beyond the export threshold, the scheme does not create any interregional trade. Though, it does give rise to a wider impact by creating the threat of firm entry for the spatial monopolists. This threat could be brought about through a subsidy rather than an infrastructure investment, and this subsidy would be free of cost because it would not create any exports which would in turn induce subsidy payments.

END OF PART I

## Part II

# Agglomeration

В
In this part, I set up a core-periphery model that typically exhibits a catastrophic transition from dispersion to agglomeration when the transport cost declines and passes the threshold called the "break point". Given a transport cost right above the break point, it suffices to bring about an infinitesimal reduction in the transport cost, by means of some scheme, to give rise to this transition. This scheme would entail virtually no costs, and it would entail virtually no direct impact on welfare. Though, since it triggers a cumulative process of migration, it might cause a change in overall welfare. This impact would constitute a wider impact because there is virtually no direct impact. I show that the sign of this wider impact resulting from agglomeration is ambiguous within the applied model, and I investigate the sign's determinants.

At high levels of the transport cost, dispersion typically exhibits a higher welfare level than agglomeration, and vice versa at low levels of the transport cost. The corresponding transport cost threshold of equal welfare levels is called the "utility switch" as agglomeration switches from being disadvantageous to being advantageous at the instant that the transport cost falls below it. So, if the break point is lower than the utility switch, there is a positive wider impact of the economy's transition to agglomeration as it brings about an increase in overall welfare, and vice versa.

Unlike part I, which comprehends models that are somewhat related, though quite distinct, this part only contains a single model. It is introduced in its most comprehensive form in chapter 7. Yet, this base model lacks analytical solvability for the most part and requires certain qualifications. By zooming in on one of three distinct cases with regard to the resources' substitutability in production, the model becomes more tractable. The two distinct cases that are relevant are described separately in the two subsequent chapters. Chapter 8 covers a substitutional case with a finite degree of substitutability. Chapter 9 covers the limitational case, i.e., that of no substitutability. These two versions do not just offer additional insights because they are relatively simple and tractable. They also allow for concise breakdowns of all parameter constellations and the corresponding sign of the wider impact. The case of an infinite substitutability is irrelevant because it exhibits only agglomeration, and thus no transition.

### Transport costs

I do not consider an actual transport scheme as it suffices to have an infinitesimal reduction in the transport cost. I rather just evaluate the effect of an exogenous reduction in the transport cost. In doing so, the point of reference is a sufficiently high level of the transport cost for dispersion to be stable which is as low as possible for an infinitesimal reduction thereof to give rise to the economy taking on an agglomerated state. In other words, the initial transport cost is right above the break point.

#### Iceberg-type transport costs

I model the transport costs as "iceberg costs" (Samuelson, 1954, 1983). To denote the units produced and shipped per unit arriving at its destination, I use  $\tau \ge 1$ . Within either of the two regions of this model, trade is generally free of cost, and there only is one type of good for which trade between the regions is not free. The transport costs are uniform over all producers as well as symmetric.

# Chapter 7

# Base model

In order to investigate a transport cost reduction's wider impact resulting from an alteration of the spatial pattern, I avail myself of the literature of the New Economic Geography (NEG). More precisely, I implement a center-periphery (CP) model much like that first presented by Krugman (1991), except for the one important adaptation motivated and presented below.<sup>22</sup> The CP model has a general equilibrium framework so that, beyond the impact on the center due to increased agglomeration therein, the impact on the economy's periphery due to emigration is also considered.

There are two types of industries, one agricultural and one manufacturing. The agricultural industry (called "A-sector") is perfectly competitive with constant returns to scale, and it produces one homogeneous good. The manufacturing industry (called "M-sector") features monopolistic competition as developed by Dixit & Stiglitz (1977) with increasing returns to scale, and it produces a large number of differentiated varieties.<sup>23</sup> Any agricultural or industrial firm's production facility is located in one of the two identical regions denoted by  $r, s \in \{1, 2\}$  with  $r \neq s$ .

Transporting the agricultural produce (called "A-good") is free of cost. The transportation of the manufactures (called "M-good") is only free within a region. Between the regions, whatever the direction of trade is, the transport cost per unit delivered is that of producing  $\tau - 1$  units of output. The households demand these goods, and they supply the firms with the factors of production.

With what is left of the general structure yet to be described, I depart from the model by Krugman (1991). The elements in question concern the allocation of resources and the technology with which the A-sector firms produce. Krugman assumes two industry–specific factors of production, namely skilled labor and unskilled labor, and every household supplies one unit of either the one or the other. Though, effectively having two types of households complicates any assessment of an effect on social welfare, especially since unskilled labor is immobile so that its welfare levels might differ between the regions. **B**.2

<sup>&</sup>lt;sup>22</sup> For Krugman's center-periphery model, see also Fujita et al. (1999, ch. 5), Brakman et al. (2009, ch. 3-4) or Baldwin et al. (2003, ch. 2).

<sup>&</sup>lt;sup>23</sup> For the Dixit-Stiglitz model of monopolistic competition, see also Fujita et al. (1999, ch. 4), Feenstra (2004, ch. 5) or Baldwin et al. (2003, ch. 2).

The way that I evade any such difficulties is by assuming every household to be of the same type as they all possess equal amounts of the two factors of production, which are land and labor. The total number of households in the economy is L := 2. Every household supplies one unit of labor to either of the industries in either of the regions. They can freely move, but they must always reside in the region of the firm that employs their labor.<sup>24</sup> Hence, the distribution of the total labor force, L, among the industries and regions is endogenous. The economy's endowment with the immobile land, however, is  $B_r := 1$  per region, and the households own equal shares of both of these units, irrespective of their place of residence. So, they all receive the same rent, which they then repatriate to wherever they reside. They then spend their income in that region on domestically produced as well as imported goods.

While the manufacturing industry only employs labor, the agricultural industry employs labor in combination with land. With the two factors both being utilized in the production of the A-good, they can be substitutable to some degree. This substitutability is limited in chapter 8. Chapter 9 considers the distinct case where the factors are not substitutable, making this most like the model by Krugman.

Though still, due to the deviations from it explained above, there are no sustainable equilibria with different welfare levels for different types of households, or even households of the same type who reside in different regions. Once a process of migration toward the region and/or industry promising the highest welfare has abated and all welfare levels have equalized, i.e., once such a sustainable equilibrium has been reached, there only is one welfare level as there only is one type of household. Social welfare thus simply parallels individual welfare. Evaluating a transition from one equilibrium to another, welfare, if it changes at all, either rises for all households or falls for all households.

### B.2.1 7.1 Preferences

The utility of any household in region *r* from consuming the homogeneous A-good and the composite M-good is given by a Cobb–Douglas function in combination with a CES function describing the M-good's composition of its differentiated varieties:

$$u_r := \left(X_r^M\right)^{\mu} \left(X_r^A\right)^{1-\mu} \tag{7.1a}$$

$$X_r^{\mathcal{M}} = U^{\mathcal{M}}(\mathbf{x}_r) := \left(\sum_i \left(x_r^i\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} .$$
(7.1b)

In units per household,  $X_r^A$  is the quantity of the A-good that is consumed in r, and  $X_r^M$  is the quantity of the M-good that is consumed in r. The latter is an amount measured in

<sup>&</sup>lt;sup>24</sup> With regard to the households' mobility, I make a distinction between the short run and the long run, as explained in section 7.4. Free movement between the regions only applies to the latter.

units of the composite, which is made up of  $x_r$  units of the various varieties. This is in turn a vector containing the amount of every variety *i*, each denoted by  $x_r^i$ . As is apparent from the demand function (7.2a),  $\mu \in (0, 1)$  is the income share spent on the M-good. It is one of the parameters that are pivotal in respect of the model's outcome. Another is  $\sigma \in (1, \infty)$ , which is the constant elasticity of substitution with regard to any couple of varieties of the M-good.

With  $y_r$  denoting the nominal income of a household in r, and with  $P^A$  and  $P_r^M$  denoting the goods' respective prices, the budget constraint is  $P^A X_r^A + P_r^M X_r^M = y_r$ . The upper–tier Marshallian demand functions follow as

$$X_r^M = \frac{\mu y_r}{P_r^M} \tag{7.2a}$$

$$X_r^A = \frac{(1-\mu) y_r}{P^A} = (1-\mu) y_r .$$
 (7.2b)

The price of the A-good is uniform over space due to the perfect competition, the constant returns to scale and the absence of transport costs. Besides, I select the A-good to be the numeraire, so  $P^A := 1$ . The price of the M-composite, though, is region–specific. The lower–tier Marshallian demand function for any variety *i* is

$$x_r^i = \left(p_r^i\right)^{-\sigma} \left(P_r^{\mathcal{M}}\right)^{\sigma-1} \mu y_r , \qquad (7.2c)$$

with  $p_r^i$  as the price of this variety when sold in region r, while not necessarily being produced in this very region. This demand function is isoelastic, and  $\sigma$  is the price elasticity of demand for any variety of the M-good.<sup>25</sup> The price of the industrial goods' composite is a price index of the varieties' prices:

$$P_r^{\mathcal{M}} = \left(\sum_i \left(p_r^i\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} .$$
(7.3)

See appendix B.2.1 (Fujita et al., 1999, sec. 4.1, pp. 46–49).

### 7.2 Industries

Section 7.2.1 sets up the part of the model which relates to the agricultural industry. It is stated in its most comprehensive form, i.e., for any degree of substitutability between land and labor. When considering the two distinct cases of the subsequent chapters, I offer corresponding versions of it. The part of the model relating to the manufacturing industry, which is described in section 7.2.2, remains unaffected thereof.

<sup>&</sup>lt;sup>25</sup> A single variety's price is assumed to have a negligible effect on  $P_r^M$  due to being one of — supposedly — very many varieties' prices.

#### 7.2.1 Agriculture

The production function is a linear-homogeneous CES function of  $l_r^A$  and  $b_r$ , which are the amount of labor and the amount of land, respectively. Due to the constant returns to scale and the perfect competition, I consider a region's A-sector to be one entity, and the inputs are thus to be understood as those of agriculture in region r as a whole. The amount of the agricultural product that is collectively produced in r is

$$\left[ \left( \eta \left( \frac{l_r^A}{\gamma} \right)^{\frac{\kappa-1}{\kappa}} + (1-\eta) b_r^{\frac{\kappa-1}{\kappa}} \right)^{\frac{\kappa}{\kappa-1}} \quad \text{for } \kappa > 0 \land \kappa \neq 1 \quad (7.4a) \right]$$

$$A_r := \begin{cases} \left(\frac{l_r^A}{\gamma}\right)^{\eta} b_r^{1-\eta} & \text{for } \kappa = 1 \end{cases}$$
(7.4b)

$$\left(\min\left\{l_r^A/\gamma, b_r\right\}\right) \qquad \text{for } \kappa = 0 \quad . \tag{7.4c}$$

Denoting the elasticity of substitution between the two factors of production is  $\kappa \in [0, \infty)$ . This represents what I have referred to as the "degree of substitutability". The parameters  $\eta \in (0, 1)$  and  $\gamma \in (0, \infty)$  jointly govern the relative input of land and labor. If  $\kappa = 0$ ,  $\eta$  disappears from the production function and  $\gamma$  needs to be used to adjust labor's units of measurement. If  $\kappa > 0$ , it suffices to set  $\gamma := 1$  and only use  $\eta$ . Anyway,  $\gamma$  is never left out from the description of this chapter, to guarantee for consistency.

Albeit that (7.4a) is not defined for an elasticity of substitution of either zero or one, the CES function's distinct forms of  $\kappa$  approaching either of these two values or infinity are well-defined. If  $\kappa \to 1$ , the production function is of the Cobb–Douglas type (7.4b). This is the one distinct case of this model, and it is covered in section 8.1. If  $\kappa \to 0+$ , the production function is of the Leontief type (7.4c). This case is covered in section 9.1. Both distinct forms follow from applying de l'Hôpital's rule to (7.4a); see Saito (2012).<sup>26</sup> If  $\kappa \to \infty$ , land and labor are perfect substitutes, which is apparent as all exponents in (7.4a) approach one, rendering the production function linear. This case is ignored because it makes the model exhibit full agglomeration for any level of the transport cost, and hence no transition.

Due to perfect competition, the agricultural industry generates a zero profit, and it supplies whatever it can produce given the available resources. The corresponding equilibrium condition of the A-good's market does not need to be stated explicitly, but it is fulfilled through Walras' law.<sup>27</sup> The conditions that constitute this model's system of equations (7.10–11) are developed hereafter. Section 7.4 summarizes and concludes the setup.

<sup>&</sup>lt;sup>26</sup> For the CES function, see Varian (1992, p. 19 et seq.) or Jehle & Reny (2011, p. 130 et seq.). See Abramowitz & Stegun (1965, p. 13) for de l'Hôpital's rule.

<sup>&</sup>lt;sup>27</sup> See Walras (1954, § 123, pp. 168–169), van Daal & Jolink (1993, pp. 36–39), Varian (1999, sec. 29.6, pp. 517–518) or Jehle & Reny (2011, p. 204 et seqq.) for Walras' law.

#### 7.2 INDUSTRIES

In order to determine the equilibrium conditions of the markets for agricultural labor and land, I first use the optimality condition for cost minimization that the marginal rate of technical substitution between inputs is to equal their relative price. The factor prices are denoted by  $w_r^A$  for the wage rate in agriculture and  $R_r$  for the rent:

$$MRTS_{lb} := -\frac{\mathrm{d}b_r}{\mathrm{d}l_r^A}\Big|_{\mathrm{d}A_r=0} = \frac{\partial A_r/\partial l_r^A}{\partial A_r/\partial b_r} = \frac{\eta}{\gamma(1-\eta)} \left(\frac{\gamma b_r}{l_r^A}\right)^{1/\kappa} \stackrel{!}{=} \frac{w_r^A}{R_r}$$
(7.5a)

$$\Rightarrow w_r^A = \frac{\eta}{\gamma (1 - \eta)} \left(\frac{\gamma b_r}{l_r^A}\right)^{1/\kappa} R_r .^{28}$$
(7.5b)

Either region's unit of land is supplied fully inelastically given a positive rent. For simplicity, I consider equilibrium supply and demand as always being  $b_r = B_r = 1$ ; see section 7.4. Since the agricultural product is the numeraire, it follows from the zero profit condition,  $w_r^A l_r^A + R_r b_r = P^A A_r$ , with (7.4a) and (7.5b) that the equilibrium rent is

$$R_r = (1 - \eta) \left( \eta \left( \frac{l_r^A}{\gamma} \right)^{\frac{\kappa - 1}{\kappa}} + (1 - \eta) \right)^{\frac{1}{\kappa - 1}} .$$
 (7.10a)

Substituting (7.10a) back into (7.5b) yields the equilibrium wage rate as

$$w_r^A = \frac{\eta}{\gamma} \left( \eta + (1 - \eta) \left( \frac{l_r^A}{\gamma} \right)^{\frac{1 - \kappa}{\kappa}} \right)^{\frac{1}{\kappa - 1}} .$$
 (7.10b)

These inverse demand functions declare the factor prices that ensure the demand of one unit of land plus  $l_r^A$  units of agricultural labor. Their distinct forms are (8.2) for  $\kappa = 1$  and (9.2) for  $\kappa = 0$ .

#### 7.2.2 Manufacturing

There are  $n_r$  manufacturing firms in region r, each producing one differentiated variety, and each variety is in turn only produced by one firm. Hence, the number of varieties produced in r is equal to  $n_r$ . The price that the manufacturers pay for both their m units of constant marginal input and their F units of fixed input per firm, the wage rate in manufacturing, is denoted by  $w_r^M$ .

Maximizing their profits subject to the demand function (7.2c), all firms in region r charge a mill price of

$$p_r = \frac{\sigma}{\sigma - 1} w_r^M m . aga{7.6}$$

B.2.2

<sup>&</sup>lt;sup>28</sup> If  $\kappa = 0$ , the production function (7.4c) is not differentiable at  $l_r^A/\gamma = b_r$ ; i.e.,  $MRTS_{lb}$  can not be determined and (7.5) does not apply. See section 9.1.

So, the consumer prices are  $p_r^i = p_r$  and  $p_s^i = \tau p_r$  if firm *i* is located in *r*.<sup>29</sup> Even though a firm is able to charge a mark-up on its marginal cost, it earns a zero profit due to the free entry and exit of firms. The quantity that allows it to cover its fixed costs is

$$q \equiv (\sigma - 1) \frac{F}{m} , \qquad (7.7)$$

irrespective of location or wage rate. To produce this quantity, a firm employs a total of  $mq + F = \sigma F$  workers. Hence, the equilibrium number of firms in region r is

$$n_r = \frac{l_r^M}{\sigma F} , \qquad (7.8)$$

with  $l_r^M$  being the number of workers in this region's M-sector.

Having derived the demand for any one variety as (7.2c) and the supply thereof as (7.7), the M-sector's wage equation is obtained from the equilibrium condition for any variety's market:

$$w_r^M = \left(Y_r \left(P_r^M\right)^{\sigma-1} + \phi Y_s \left(P_s^M\right)^{\sigma-1}\right)^{1/\sigma} . \tag{7.10c}$$

See appendix B.2.2. The variable  $Y_r$  denotes the total nominal income of all the households in region r and thus their total expenditure on both goods combined. As a measure of trade freeness, I use  $\phi := \tau^{1-\sigma} \in (0, 1]$ .

There are two mill prices, one per region that is. Consumers in region r need to pay  $p_r$  per unit of any domestically produced variety and  $\tau p_s$  per unit of consumption of any variety that is imported from region s. Hence, the price index of the composite manufacture (7.3) can be written as

$$P_r^M = \mu^{\frac{1}{\sigma-1}} \left( l_r^M \left( w_r^M \right)^{1-\sigma} + \phi l_s^M \left( w_s^M \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$
(7.10d)

by use of (7.6) and (7.8); see appendix B.2.2. I set  $m := (\sigma - 1) / \sigma$  and  $F := \mu / \sigma$  by choice of units, and without loss of generality, in deriving (7.10c–d).

### 7.3 Income

As all households own equal shares of land in both regions, the rents are distributed equally among them. In addition to the rents, every household supplies one unit of labor and earns the corresponding wage rate so that agricultural and industrial workers, respectively, receive

<sup>&</sup>lt;sup>29</sup> A single firm's decision is assumed to have only a negligible impact on  $P_r^M$  by simultaneously assuming a large  $n_r$ , as mentioned in footnote 25 with regard to the price elasticity of demand.

individual nominal incomes of

$$y_r^A := \frac{R_r + R_s}{2} + w_r^A$$
(7.9a)

$$y_r^M := \frac{R_r + R_s}{2} + w_r^M$$
. (7.9b)

A region's aggregate nominal income is

$$Y_r = l_r^A y_r^A + l_r^M y_r^M . aga{7.9c}$$

## 7.4 General equilibrium

The system of equations that describes the general equilibrium has already been set up for the most part. An equilibrium on the land market of r is brought about by a rent of

$$R_r = (1 - \eta) \left( \eta \left( \frac{l_r^A}{\gamma} \right)^{\frac{\kappa - 1}{\kappa}} + (1 - \eta) \right)^{\frac{1}{\kappa - 1}} .$$
 (7.10a)

An equilibrium on the agricultural labor market of r is brought about by a wage of

$$w_r^A = \frac{\eta}{\gamma} \left( \eta + (1 - \eta) \left( \frac{l_r^A}{\gamma} \right)^{\frac{1 - \kappa}{\kappa}} \right)^{\frac{1}{\kappa - 1}} .$$
 (7.10b)

A wage of

$$w_r^{\mathcal{M}} = \left(Y_r \left(P_r^{\mathcal{M}}\right)^{\sigma-1} + \phi Y_s \left(P_s^{\mathcal{M}}\right)^{\sigma-1}\right)^{1/\sigma}, \qquad (7.10c)$$

with

$$P_r^{\mathcal{M}} = \mu^{\frac{1}{\sigma-1}} \left( l_r^{\mathcal{M}} \left( w_r^{\mathcal{M}} \right)^{1-\sigma} + \phi l_s^{\mathcal{M}} \left( w_s^{\mathcal{M}} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} , \qquad (7.10d)$$

yields the clearing of the market for industrial products while (7.9) pose as the households' budget constraint. To complete the system of equations, an equilibrium condition needs to be added for the distribution of labor. Since I make a distinction between the short run and the long run with regard to the households' mobility, this equilibrium condition needs to be chosen accordingly.

If the distribution of labor across regions and industries was exogenous, the corresponding condition would be  $l_r^A + l_s^A + l_r^M + l_s^M = 2$ , which would need to be applied to (7.9a–10d). Yet, workers are more or less mobile, depending on the presumed time frame. I assume workers to always be mobile *within* regions — even in the short run — so that the intraregional distributions are left to the market. Only in the long run, though, they are also mobile *between* regions. So, in the short run, the distribution of labor between regions is exogenous.

B.2.3

It is expressed in terms of the shares of the economy's total labor force employed by the firms in either region,

$$\lambda_r := \frac{l_r^A + l_r^M}{2} , \qquad (7.10e)$$

with  $\lambda_r \in [0, 1]$  and

$$\lambda_r + \lambda_s = 1 . \tag{7.10f}$$

A condition needs to be included that constitutes whether an intraregional distribution is in equilibrium. Since workers are attracted by the industry that pays the higher wage, they gradually switch to this industry. Possibly, they thereby make the wages converge. Yet, diverging wages do not preclude a sustainable equilibrium. One must allow for corner solutions in the sense that the workers of a region concentrate in one industry. In order to encompass all possible distributions without intraregional incentives for migration, I use

$$\max\left\{\left(w_r^{\mathcal{M}} - w_r^{\mathcal{A}}\right) t_r^{\mathcal{A}}, \left(w_r^{\mathcal{A}} - w_r^{\mathcal{M}}\right) t_r^{\mathcal{M}}\right\} = 0.$$
(7.10g)

This complementarity makes sure that, if the wages diverge, there can only be workers in one of the two industries, and this industry must pay a wage at least as high as what the firms of the other industry were to pay if there were any. Equalized wages are of course a sufficient condition for any distribution to be in equilibrium.

After selecting a value for the exogenous  $\lambda_r$ , (7.9a–10g) yield the general equilibrium, including the households' intraregional distributions. Since all households in region r earn the same wage, it makes sense to define this wage and the corresponding individual income:

$$w_r := \max\left\{w_r^A, w_r^M\right\}$$
(7.10h)

$$y_r := \frac{R_r + R_s}{2} + w_r = \max\left\{y_r^A, y_r^M\right\} .$$
(7.10i)

The aggregate income,

$$Y_r = \lambda_r 2 y_r , \qquad (7.10j)$$

follows from (7.9c) by use of (7.10e) and (7.10g–i). This system of equations (7.10) describes the short-run equilibrium. It is summarized as (B.1) in appendix B.2.3.

For interregional comparisons, I define the welfare levels of either type of household,

$$\omega_r^A := y_r^A \left( P_r^M \right)^{-\mu} \tag{7.11a}$$

$$\omega_r^M := y_r^M \left( P_r^M \right)^{-\mu} \,. \tag{7.11b}$$

and the welfare level in a short-run equilibrium of any household in r,

$$\omega_r := y_r \left( P_r^M \right)^{-\mu} = \max \left\{ \omega_r^A, \, \omega_r^M \right\} \,. \tag{7.11c}$$

These represent the households' real incomes. They are expressed by a division through constants of the indirect utility function, which arises from (7.1a) with (7.2a–b).

To see how the regions' short-run welfare levels relate, and to infer from it the direction of interregional migration, the welfare difference can be computed for varying levels of  $\lambda_r$ . Figure 7.1 is a depiction for varying levels of  $\tau$ .<sup>30</sup> Figure 7.2 is a so-called "wiggle diagram", which is a depiction for a single level of the transport cost. The wiggle diagrams distinguish between the welfare difference between the agricultural industries,  $\omega_r^A - \omega_s^A$ , and the welfare difference between the manufacturing industries,  $\omega_r^M - \omega_s^M$ . The two can only diverge if either of the industries is nonexistent in either of the regions; see (7.10g) and note that  $w_r^A \ge w_r^M \Leftrightarrow \omega_r^A \ge \omega_r^M$  and  $w_r^A \le w_r^M \Leftrightarrow \omega_r^A \le \omega_r^M$ . In all chosen examples, this is the manufacturing industry in the periphery so that  $\omega_r - \omega_s = \omega_r^A - \omega_s^A$ . I generally ignore the possibility of a periphery with only manufacturing and no agriculture, as explained below. The vertical distance between the solid and the dashed line in a wiggle diagram is  $\omega_r^A - \omega_r^M$ on the left-hand side and  $\omega_s^M - \omega_s^A$  on the right-hand side.



**Figure 7.1** Short-run equilibria with a varying transport cost ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 8$ )

<sup>&</sup>lt;sup>30</sup> The levels of the transport cost denoted by  $\tau_b$ ,  $\tau_s$  and  $\tau_u$  are the break point (see page 74), the sustain point (see page 78), and the utility switch (see page 80), respectively.



**Figure 7.2** Short-run equilibria with no transport cost ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 8$ ,  $\tau = 1$ )

In the case of an uneven distribution of labor between the regions, I identify the center as region 1 and the periphery as region 2; i.e.,  $\lambda_1 > 0.5$ . As mentioned above, there is either an interior solution or a corner solution in terms of the labor distribution within the periphery. The former is an equilibrium with  $w_2^A = w_2^M$  and  $l_2^M \ge 0$ . The latter is an equilibrium with  $l_2^M = 0$  and  $w_2^A \ge w_2^M$ . The number of workers in the periphery's A-sector is positive, i.e.,  $l_2^A > 0$ , unless  $\lambda_2 = 0$ . The kinks in the solid line of figure 7.2 are the "transition points" between interior and corner solutions; so, they are characterized by  $l_2^M = 0$  and  $w_2^A = w_2^M$ . In figure 7.1, the transition points are depicted by white lines. The line along the crest in the front, where  $\lambda_r > 0.5$ , is visible. There is a corresponding line running along the rift in the back, which is hidden.

Everything up to this point considered the short run with only intraregional mobility. In the long run, though, I assume workers to also be mobile between regions. They wish to reside in a region with a welfare level that is at least as high as that of the other region, and they will gradually relocate if this is not the case. If  $\omega_r - \omega_s > 0$ , then  $\lambda_r$  will increase, and vice versa, until a long-run equilibrium is eventually attained. Any short-run equilibrium which satisfies  $\omega_r - \omega_s = 0$  is such a long-run equilibrium. So is any equilibrium with all households in the center which is sustainable as the center exhibits a welfare level that is at least as high as that in the periphery. To allow for either,

$$\max\left\{\left(\omega_{s}-\omega_{r}\right)\lambda_{r},\left(\omega_{r}-\omega_{s}\right)\lambda_{s}\right\}=0$$
(7.11d)

and (7.11c) need to be appended to the short-run system of equations (7.10) to produce the long-run system of equations (7.10–11);  $\lambda_r$  is now endogenous. See appendix B.2.3. The black lines in figure 7.1, whether thin or thick, indicate the long-run equilibria, which are also depicted in figure 7.3.

#### 7.4 GENERAL EQUILIBRIUM



**Figure 7.3** Long-run equilibria with a varying transport cost ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 8$ )

Apart from stability and instability, which is explained later, there are at least two types of long-run equilibria. In a fully symmetric two-region model like this, there always exists the symmetric one with perfect dispersion, as explained in section 7.4.1. All the other equilibria exhibit some degree of concentration. These concentrated long-run equilibria, especially the fully concentrated ones, which might or might not exist, are explained in section 7.4.2. "Full concentration" refers to the manufacturing industry only producing in the center so that, if anything, only agriculture takes place in the periphery. "Partial concentration" refers to agglomeration of some lesser degree.<sup>31</sup> Section 7.4.3 compares full concentration to dispersion. Whether the latter is stable, is explored on page 74. Whether the former, which is almost always stable if it exists, does actually exist, is explored on page 78.

An important qualification needs to be made in respect to the type of concentration that unfolds when there are so few — but some — workers in a region, typically the periphery, that they are all employed by the same industry. This requires a distinction regarding the elasticity of substitution in agriculture. If  $\kappa \in [1, \infty)$ , it can only ever be the manufacturing industry that does not operate in a region, because — if the supply of labor is sufficiently low — the agricultural firms pay a wage high enough to absorb this entire supply, but not vice versa; i.e.,  $\lambda_r > 0 \Leftrightarrow l_r^A > 0$  and  $\lambda_r = 0 \Leftrightarrow l_r^A = 0$ . This is because  $\lim_{l_r^A \to 0+} w_r^A = \infty$ while  $\lim_{l_r^M \to 0+} w_r^M$  is finite.<sup>32</sup> It follows that the periphery exhibits a higher welfare level than the center if  $\lambda_2$  is sufficiently low, as  $\lim_{\lambda_2 \to 0+} \omega_2 = \infty$ . Thus, a short-run equilibrium with, say,  $\lambda_2 = 0$  is not sustainable in the long run. Fully concentrated long-run equilibria are such that  $\lambda_2 \in (0, 0.5)$  since there is some agriculture in the periphery. Figure 7.3 is an example with  $\kappa = 1$ .

<sup>&</sup>lt;sup>31</sup> Some important remarks concerning the rare case of more than two types of concentrated long-run equilibria are made in section 7.4.3 and explained in more detail in appendix B.2.3.2.

<sup>&</sup>lt;sup>32</sup> For  $w_r^A$  with  $\kappa \in (1, \infty)$ , see (7.10b). For  $\kappa = 1$ , see (8.2b) on page 81.

If  $\kappa \in [0, 1)$ , then  $w_r^A \leq \lim_{\ell^A \to 0+} w_r^A = \eta^{\kappa/(\kappa-1)}/\gamma^{33}$  I.e., the agricultural firms of region r are unable to produce at an average cost of  $P^A = 1$  if the wage is too high. This indicates that the manufacturing firms might pay a wage high enough to absorb the entire labor supply while the agricultural firms are unable to compete, both on this region's labor market and the A-good's market. Nevertheless, I have not found a single numerical example of a shortrun equilibrium with  $l_r^A = 0$  but  $l_r^M > 0$ . This is why I suppose that equilibria with full intraregional concentration in manufacturing do not exist, with any  $\kappa \in [0, \infty)$ .<sup>34</sup> In a shortrun equilibrium with  $\lambda_2 = 0$ , the periphery might exhibit a lower welfare level than the center so that this equilibrium is sustainable. For an example of such a long-run equilibrium, see section 9.2.2.

For simplicity, I assume that the agricultural firms of region r always demand a quantity of land of  $b_r = B_r$ . The complementarity of the land market, as  $B_r = 1$ , is

$$\max\{(1-b_r)R_r, b_r-1\}=0$$
.

It would only be possible for the quantity demanded to be  $b_r < 1$  if  $R_r = 0$ , since the households would be indifferent regarding their supply. This requires a distinction regarding the elasticity of substitution in agriculture. If  $\kappa \in (1, \infty)$ ,  $R_r \ge R_r|_{l^A=0} = (1 - \eta)^{\kappa/(\kappa-1)} > 0$ as given by (7.10a). If  $\kappa \in (0, 1]$ , then  $\lim_{l^{A} \to 0^{+}} R_{r} = 0.35$  Also,  $\lim_{l^{A} \to 0^{+}} A_{r} = 0$ ; see (7.4a–b). The rent can only be zero if the A-sector produces no output as it employs no labor, whatever the amount of land it employs. Hence, the marginal product of land is zero. If  $\kappa = 0$ , then  $R_r = 0$  only if  $l_r^A/\gamma < 1$ ; see (9.2a). If so,  $A_r = l_r^A/\gamma$ ; see (7.4c). Hence, the marginal product of land beyond  $b_r = l_r^A / \gamma$  is zero. In conclusion, only if the marginal product of land is zero, the rent can also be zero. Even if the A-sector fimrs would demand less than a region's full unit of land, I can set  $b_r = 1$  because it makes no difference, neither with regards to costs nor with regards to output.

#### B.2.3.1 7.4.1 Dispersion

If the households are distributed evenly among the two regions, i.e., if  $\lambda_r = \lambda_s = 0.5$ , the economy takes on a fully symmetric state, which — to some extent — allows for an analytical investigation. I use variables without the subscripts, e.g.,  $R := R_r|_{\lambda_r=0.5} = R_s|_{\lambda_r=0.5}$ , to denote both of the two corresponding region-specific variables. As I will show, there are workers in both industries so that  $w = w^A = w^M$  and so forth.<sup>36</sup> Since the welfare levels are equal in the two regions, dispersion is generally a long-run equilibrium.

<sup>&</sup>lt;sup>33</sup> For  $w_r^A$  with  $\kappa \in (0, 1)$ , see (7.10b). For  $\kappa = 0$ , see (9.2b) on page 90.

<sup>&</sup>lt;sup>34</sup> If  $\kappa \to \infty$ , which I neglect, such equilibria do exist.

<sup>&</sup>lt;sup>35</sup> For  $R_r$  with  $\kappa \in (0, 1)$ , see (7.10a). For  $\kappa = 1$ , see (8.2a) on page 81. <sup>36</sup> If  $\kappa \to \infty$ , which I neglect, it is possible that  $l^A = 0$ ,  $l^M = 1$  and  $w = w^M \ge w^A$ ; cf. footnote 34.

#### 7.4 GENERAL EQUILIBRIUM

The short-run system of equations (7.10) can be conflated to

$$R = (1 - \eta) \left( \eta \left( l^{A} / \gamma \right)^{\frac{\kappa - 1}{\kappa}} + (1 - \eta) \right)^{\frac{1}{\kappa - 1}}$$
(7.12a)

$$w = \frac{\eta}{\gamma} \left( \eta + (1 - \eta) \left( l^{A} / \gamma \right)^{\frac{1 - \kappa}{\kappa}} \right)^{\frac{1 - \kappa}{\kappa}}$$
(7.12b)

$$w = \left(Y\left(P^{M}\right)^{\sigma-1}(1+\phi)\right)^{1/\sigma}$$
(7.12c)

$$y = Y = R + w \tag{7.12d}$$

$$P^{M} = \left(\frac{\mu}{1 - l^{A}} \frac{1}{1 + \phi}\right)^{\frac{1}{\sigma - 1}} w .$$
 (7.12e)

The long-run system of equations (7.11) can be conflated to

$$\omega = y \left( P^M \right)^{-\mu} \,. \tag{7.13}$$

First, I demonstrate that there typically is a solution to (7.12). Dividing (7.12a) by (7.12b) gives (7.14), which is equivalent to (7.5), and combining (7.12c-e) gives (7.15):

$$\frac{R}{w} = \frac{\gamma \left(1 - \eta\right)}{\eta} \left(l^{A} / \gamma\right)^{1/\kappa}$$
(7.14)

$$\frac{R}{w} = \frac{1 - \mu - l^A}{\mu} \ . \tag{7.15}$$

Equating the two yields an equation which needs to be solved numerically for  $l^A$ :

$$l^{A} + \mu \, \frac{\gamma \, (1 - \eta)}{\eta} \, \left( l^{A} / \gamma \right)^{1/\kappa} = 1 - \mu \,. \tag{7.16}$$

If  $\kappa \in (0, \infty)$ , a result exists with  $l^A$ ,  $l^M \in (0, 1)$ .<sup>37</sup> If  $\kappa = 0$ , then  $l^A = \min \{\gamma, 1 - \mu\}$ , though (7.12a–b), (7.14) and (7.16) do not apply at  $l^A = \gamma$ ; see section 9.2.1.

By use of an additional variable, I formulate (7.12–13) in such a way that it can readily be applied when calculating the dispersed equilibrium. The share of income that is earned income in the case of symmetry follows from (7.12d) with (7.14–15) as

$$\delta := \frac{w}{y} = \frac{\mu}{1 - l^A} = \frac{\eta}{\eta + \gamma (1 - \eta) \left( l^A / \gamma \right)^{1/\kappa}} .^{38}$$
(7.17)

<sup>&</sup>lt;sup>37</sup> The right-hand side of (7.16) is constant and positive; unless  $\kappa \to \infty$ , the left-hand side is zero at  $l^A = 0$ , yet increasing in  $l^A$ . At  $l^A = 1$ , the left-hand side is greater than the right-hand side. Hence, an  $l^A \in (0, 1)$  must exist that fulfills (7.16).

<sup>&</sup>lt;sup>38</sup> If  $\kappa = 0$ , then the last expression of (7.17) does not apply at  $l^A = \gamma$ , and  $\delta$  is given as (9.3). If  $\kappa \to \infty$ , then  $\delta \equiv \max \{\eta / (\eta + \gamma (1 - \eta)), \mu\}$ .

Rearranging (7.17) yields

$$l^{A} = \frac{\delta - \mu}{\delta} \tag{7.18a}$$

$$l^{\mathcal{M}} = \frac{\mu}{\delta} \tag{7.18b}$$

as  $l^{M} = 1 - l^{A}$  (7.10e). From (7.12–13) with (7.17) follow

$$R = (1 - \delta) Y \tag{7.19a}$$

$$w = \delta Y = \delta y \tag{7.19b}$$

$$w = \frac{1}{\gamma} \left( \eta^{\kappa} \frac{1-\mu}{\delta-\mu} \right)^{\frac{1}{\kappa-1}} \implies \lim_{\kappa \to 1} w = \frac{\eta}{\gamma} \left( \frac{\delta-\mu}{\gamma\delta} \right)^{\eta-1}$$
(7.19c)

$$P^{M} = \left(\frac{\delta}{1+\phi}\right)^{\frac{1}{\sigma-1}} w \tag{7.19d}$$

and

$$\omega = \frac{1}{\delta} \left( \frac{1+\phi}{\delta} \right)^{\frac{\mu}{\sigma-1}} w^{1-\mu} .$$
 (7.20)

The derivation of  $\lim_{\kappa \to 1} w$  is explained in appendix B.2.3.1.

While the dispersed equilibrium always is a long-run equilibrium, it can be either stable or unstable. A long-run equilibrium is stable if migration in one direction or the other gives rise to higher welfare in the region of origin than in the destination region. If it does, this creates an incentive for subsequent migration in the opposite direction leading back to the original equilibrium. If migration gives rise to higher welfare in the destination region, the long-run equilibrium is unstable because an incentive is created for further migration in the same direction triggering a cumulative process toward a different long-run equilibrium.

Differentiating the short-run system of equations (7.10) and the welfare function (7.11c) while using hats to denote the variables' relative changes, e.g.,  $\hat{\lambda} := d \log \lambda_r$ , and evaluating the derivatives at  $\lambda_r = \lambda_s = 0.5$ , so that  $d \log \lambda_s = -\hat{\lambda}$  etc., one obtains

$$\hat{\lambda} = l^A \hat{l}^A + l^M \hat{l}^M \tag{7.21a}$$

$$\hat{w} = -\frac{1}{\kappa} \frac{1-\eta}{\eta} \frac{\delta-\mu}{1-\mu} \left( l^A / \gamma \right)^{\frac{1-\kappa}{\kappa}} \hat{l}^A$$
(7.21b)

$$\hat{w} = \frac{Z}{\sigma} \left( \hat{Y} + (\sigma - 1) \hat{P}^M \right)$$
(7.21c)

$$\hat{Y} = \hat{\lambda} + \delta \hat{w} \tag{7.21d}$$

$$\hat{P}^{M} = Z\left(\frac{1}{1-\sigma}\hat{l}^{M} + \hat{w}\right)$$
(7.21e)

and

$$\hat{\omega} = \delta \hat{w} - \mu \hat{P}^M \,. \tag{7.22}$$

Integrating (7.21–22) yields the impact of immigration on local welfare as

$$\frac{\hat{\omega}}{\hat{\lambda}} = \frac{\left(\delta/\mu - \psi Z\right)\left(\mu - \delta Z\right)}{\sigma/Z - \delta - \psi Z\left(\sigma - 1\right)} + \frac{\delta Z}{\sigma - 1} , \qquad (7.23)$$

with  $Z := (1 - \phi) / (1 + \phi) \in [0, 1)$  as a measure of trade closedness and

$$\psi := 1 - \frac{\eta}{1 - \eta} \frac{1 - \mu}{\mu} \frac{\kappa}{\sigma - 1} \left(\frac{\delta - \mu}{\gamma \delta}\right)^{\frac{\kappa - 1}{\kappa}} . \tag{7.24}$$

See appendix B.2.3.1. If  $\hat{\omega}/\hat{\lambda} < 0$ , then symmetry is stable, and it is unstable if  $\hat{\omega}/\hat{\lambda} > 0$ , depending on, among other parameters, the transport cost represented by trade closedness, Z. If Z = 0, symmetry is neither stable nor unstable.

In the case of free trade, (7.23) is equal to zero, i.e.,  $\tau = 1 \Leftrightarrow Z = 0 \Rightarrow \hat{\omega}/\hat{\lambda} = 0$ , as one can infer from figure 7.2. Effectively, migration only takes place between the manufacturing industries, resulting in a proportional shift of firms. Yet, all the prices remain unaffected. As long as the supply of labor in the periphery does not fall below the demand by the local A-sector given a wage rate of *w*, migration from one region to the other has no impact on either region's welfare level, and thus does not create any incentives for migration.<sup>39</sup>



**Figure 7.4** Short-run equilibria with stable full concentration ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 8$ ,  $\tau = 1.35$ )

With a low but positive transport cost, i.e., a low  $\tau > 1$ , as in figure 7.4, dispersion is unstable because, unless  $\psi \to -\infty$ ,

$$\frac{\mathrm{d}}{\mathrm{d}Z}\frac{\hat{\omega}}{\hat{\lambda}}\Big|_{Z=0} = \frac{\delta\left(2\sigma - 1\right)}{\sigma\left(\sigma - 1\right)} > 0.$$
(7.25)

<sup>&</sup>lt;sup>39</sup> At Z = 0, according to (7.21c), (7.21e) and (7.22),  $\hat{\omega} = \hat{w} = \hat{P}^M = 0$ , and according to (7.21b),  $\hat{l}^A = 0$ . From (7.21a) and (7.21d) follows  $\hat{Y} = \hat{\lambda} = l^M \hat{l}^M$ .

In the wiggle diagrams, dispersion's (in)stability (7.23) is illustrated by a gray, dashed, straight line with the wiggle's slope at the dispersed equilibrium. Circles indicate unstable long-run equilibria and are hence to be found on upward sections of a wiggle; points indicate stable long-run equilibria and are hence to be found on downward sections.

With a prohibitive transport cost, i.e., with  $\tau \to \infty$ , (7.23) approaches a finite value:

$$\lim_{Z \to 1} \frac{\hat{\omega}}{\hat{\lambda}} = \frac{\left(\delta/\mu - \psi\right)\left(\mu - \delta\right)}{\sigma - \delta - \psi\left(\sigma - 1\right)} + \frac{\delta}{\sigma - 1} .$$
(7.26)

This value can be either positive or negative. If and only if it is negative, which is called the "no-black-hole condition", there is a level of the transport cost, denoted by  $\tau_b > 1$ , with  $\hat{\omega}/\hat{\lambda}|_{\tau=\tau_b} = 0$  so that  $\hat{\omega}/\hat{\lambda} < 0$  with any  $\tau > \tau_b$ . Otherwise, the dispersed equilibrium — at any level of  $\tau$  — is not stable because (7.23) is nonnegative. If (7.26) is in fact negative, setting (7.23) equal to zero yields the break point:

$$\tau_b := \left(\frac{\left(\psi\mu/\delta + \delta/\mu\right)\left(\sigma - 1\right) + \delta + \left(2\sigma - 1\right)}{\left(\psi\mu/\delta + \delta/\mu\right)\left(\sigma - 1\right) + \delta - \left(2\sigma - 1\right)}\right)^{\frac{1}{\sigma-1}} .$$
(7.27)

See appendix B.2.3.1. Figure 7.5 is a wiggle diagram similar to the previous ones, except that  $\tau = \tau_b$ .<sup>40</sup> If  $\kappa \to \infty \Rightarrow \psi \to -\infty$  and Z > 0, then  $\hat{\omega}/\hat{\lambda} = \mu/(\sigma - 1) > 0$ . Therefore, a break point does not exist and dispersion is generally not stable. This is why the case of perfect substitutability of land and labor is irrelevant with regard to the welfare assessment, because a transition from dispersion to applomeration does not occur.



**Figure 7.5** Short-run equilibria at the break point ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 8$ ,  $\tau = \tau_b \approx 1.5$ )

<sup>&</sup>lt;sup>40</sup> The break point is (usually) the maximum value of  $\tau$  for which the symmetric equilibrium is unstable, which is why this equilibrium is depicted as a circle in figure 7.5. This can be inferred from the welfare difference being nondecreasing in the labor share at and around symmetry because more than marginal immigration turns the welfare difference positive. Cf. figure B.2 in appendix B.2.3.2.

#### 7.4 GENERAL EQUILIBRIUM

Figure 7.6 depicts an example with  $\tau > \tau_b$ , and hence stable dispersion. Further increases in  $\tau$  are considered in section 7.4.2, with dispersion remaining stable. Graphical depictions of  $\hat{\omega}/\hat{\lambda}$  (7.23) as a function of  $\tau$  can be found in section 8.3.



**Figure 7.6** Short-run equilibria with stable dispersion and full concentration ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 8$ ,  $\tau = 1.6$ )

#### 7.4.2 Concentration

Figure 7.6 depicts partially concentrated as well as fully concentrated long-run equilibria. The former are of no particular interest due to their typical instability.<sup>41</sup> The latter feature manufacturing only in the center, and hence only agriculture in the periphery, if anything. They are virtually always stable — if they exist. Whether they do exist, or whether fully concentrated short-run equilibria are not actually sustainable in the long run, and whether they yield a higher or lower welfare than symmetry, is determined hereafter.

Consider long-run migration commencing at the transition point, i.e., at the fully concentrated short-run equilibrium with the highest possible number of workers in the periphery. If  $\omega_1 - \omega_2 > 0$  as welfare is lower in the periphery than in the center, there is migration toward the center. Either the welfare difference turns zero at some point, bringing migration to a halt, or the households all leave the periphery without the welfare difference ceasing to be positive. Either way, a fully concentrated long-run equilibrium is attained. I denote the corresponding values by an upside-down hat. So, either  $\lambda_2 \ge 0$  and  $\omega_1 = \omega_2$ , as in figure 7.6, or  $\lambda_2 = 0$  and  $\omega_1 \ge \omega_2$ , as in figure 9.6; see (7.11d). If  $\kappa = 0$ , there exists an analytical solution; see section 9.2.2. If  $\kappa \in (0, \infty)$ , a solution is derived numerically by solving (7.10–11) with  $\tilde{l}_2^M := 0$  and an otherwise endogenous labor distribution.

<sup>&</sup>lt;sup>41</sup> There can exist two types of partially concentrated long-run equilibria, one with weak concentration that is stable and one with strong (but partial) concentration that is unstable, if  $\tau < \tau_b$ . See appendix B.2.3.2.

I define  $\check{\omega} := \check{\omega}_1 \ge \check{\omega}_2$  as the welfare level of all households in the fully concentrated long-run equilibrium. To establish this equilibrium's sustainability, I define

$$\theta := \frac{\check{\omega}}{\check{\omega}_2^M} \tag{7.28}$$

with  $\theta|_{\tau=1} = 1$ . To be sustainable, it must hold that  $\theta \ge 1$ . Otherwise, there is an incentive for households to switch to the periphery's otherwise nonexistent manufacturing industry. If there is an analytical solution to the equilibrium, this test must be applied; see section 9.2.2. Though, a numerical solution can only yield a  $\theta \ge 1$  due to (7.10g).

Tracking the path of the transition point while continuing with the successive increases in the transport cost, as illustrated by the gray dashed lines in figure 7.7, one finds that households in the periphery are not necessarily worse off at the transition point than those in the center. So, full concentration is not necessarily sustainable. Since  $d\theta/d\tau|_{\tau=1} > 0$ , it is at low levels of the transport cost as  $\theta \ge 1$ . Though, there typically exists a level of the transport cost called the "sustain point",  $\tau_s > 1$ , with  $\theta|_{\tau=\tau_s} = 1$  and  $d\theta/d\tau|_{\tau=\tau_s} < 0$ . If it exists, then full concentration is unsustainable at at least some — but typically all levels of  $\tau > \tau_s$ ; see figure 7.8. There is no analytical solution to the sustain point, though it can be determined numerically by establishing that the transition point has to coincide with the fully concentrated long-run equilibrium. Calculating the positive transport cost,  $\tau > 1$ , and the labor distribution from which it follows that  $\check{\omega} = \check{\omega}_2^M \Leftrightarrow \theta = 1$ , yields the sustain point, if it exists; see figure 7.7.<sup>42</sup> Graphical depictions of  $\theta$  as a function of  $\tau$  can be found in section 8.3 and section 9.2.2. If  $\kappa = 0$ , this function is (9.12).



**Figure 7.7** Short-run equilibria at the sustain point ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 8$ ,  $\tau = \tau_s \approx 1.74$ )

<sup>&</sup>lt;sup>42</sup> At  $\tau = \tau_s$ , the fully concentrated long-run equilibrium is half stable and half unstable. The economy returns to this equilibrium if there is an exogenous migration shock toward the center. Yet, it transitions to symmetry if there is migration toward the periphery. This is why I use a circle in figure 7.7 for this very equilibrium.



**Figure 7.8** Short-run equilibria with stable dispersion ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 8$ ,  $\tau = 2$ )

If  $\kappa = 0$ , a sustain point does not necessarily exist. Though, if it does, there always is a transport cost threshold above the sustain point,  $\tau'_s > \tau_s$ , above which full concentration is sustainable as  $\theta \ge 1$ ; i.e.,  $\theta|_{\tau=\tau'_s} = 1$  and  $d\theta/d\tau|_{\tau=\tau'_s} > 0$ . See section 9.2.2.

#### 7.4.3 Dispersion vs. concentration

There is an important qualification in regards to the long-run equilibria that exist within some interval of the transport cost below the break point. The transition to agglomeration does not need to be subcritical, although it usually is. There can also be a supercritical transition into partial concentration with a subsequent leap from partial to full concentration as the former ceases to be sustainable. I disregard the possibility of a partially concentrated long-run equilibrium's stability for four reasons. First, such a case is unlikely to appear. Second, if it does appear anyway, the break point and the sustain point are usually very high so that the economy is agglomerated given a realistic level of the transport cost. Third, the welfare analysis explained hereafter does not apply. Fourth, the transport cost thresholds are sensitive to changes in the parameters. See appendix B.2.3.2.

Only considering subcritical transitions, any constellation of long-run equilibria essentially looks like that of figure 7.3. While all the households that decide during the transition to move to the center base their decisions on the potential gains in welfare which this brings for themselves, they do not consider the impact of their actions on the welfare of others, whether it is the people in the region of origin or the people in the destination region. Hence, the transition might or might not have a mutually beneficial outcome. This is to say that it remains to be determined whether a higher welfare for all can be brought about by reducing the transport cost to below the break point. B.2.3.2

I compare the respective welfare levels under full concentration and dispersion by using

$$\chi := \frac{\check{\omega}}{\omega} \tag{7.29}$$

with  $\chi|_{\tau=1} = 1$ . Since  $d\chi/d\tau|_{\tau=1} > 0$ , concentration exhibits a higher welfare level than dispersion at low values of the transport cost as  $\chi \ge 1$ . Yet, there typically exists a threshold of the transport cost called the "utility switch",  $\tau_u > 1$ , with  $\chi|_{\tau=\tau_u} = 1$  and  $d\chi/d\tau|_{\tau=\tau_u} < 0$ . If it exists, then full concentration exhibits a lower welfare level than dispersion at at least some — but typically all — levels of  $\tau > \tau_u$  as  $\chi < 1$ . There is no analytical solution to the utility switch, though it can be determined numerically. Calculating the positive transport cost,  $\tau > 1$ , and the labor distribution from which it follows that  $\check{\omega} = \omega \Leftrightarrow \chi = 1$ , yields the utility switch, if it exists. Graphical depictions of  $\chi$  as a function of  $\tau$  can be found in section 8.3 and section 9.2.2. If  $\kappa = 0$ , this function is (9.13).

If  $\kappa = 0$ , a utility switch does not necessarily exist. Though, if it does, there might be a transport cost threshold above the utility switch,  $\tau'_u > \tau_u$ , above which full concentration is advantageous as  $\chi \ge 1$ ; i.e.,  $\chi|_{\tau=\tau'_u} = 1$  and  $d\chi/d\tau|_{\tau=\tau'_u} > 0.^{43}$  See section 9.2.3.

### 7.5 Wider impact

Comparing the break point to the utility switch, the distinction goes as follows. If  $\tau_u < \tau_b$ , there is a negative wider impact because full concentration — which is what the economy transitions to — provides a lower welfare level at the break point than dispersion. If  $\tau_b < \tau_u$ , there is a positive wider impact, as in figure 7.1 and figure 7.3. When the transport cost decreases and eventually falls below the break point, the economy switches to concentration, thereby increasing the welfare of all households. Though, this is just an example, and I can just as easily find an example of a negative wider impact, e.g., that of figure 9.2. Therefore, I have demonstrated that, within the framework presented above, the sign of the wider impact is ambiguous.

Even within the distinct cases presented below, the sign of the wider impact is ambiguous. In section 8.3 and section 9.3, for  $\kappa = 1$  and  $\kappa = 0$ , respectively, I calculate parameter constellations that yield a zero wider impact. From these results, one can infer the sign of the wider impact at other parameter constellations. Besides, I calculate two examples each for given values of  $\sigma$  which comprehend the values of the break point and the signs of the wider impact at different constellations of the remaining parameters.

<sup>&</sup>lt;sup>43</sup> Since the fully concentrated long-run equilibrium does not exist if  $\tau \in (\tau_s, \tau'_s)$ , it follows for the utility switch and its reversal point that  $\tau_u, \tau'_u \notin (\tau_s, \tau'_s)$ .

# Chapter 8

# Cobb–Douglas case

This distinct case is not quite as tractable as that of chapter 9, but it requires far fewer considerations concerning the parameters and the types of solutions to the long-run equilibria. Compared to the base model, it has the advantage of an analytical solution to the symmetric equilibrium. Section 8.3 demonstrates the evaluation method of the welfare effect, which is also applied in section 9.3. The sign of the wider impact is shown to be ambiguous even as one confines the base model to this distinct case.

The technology of the A-sector is of the Cobb–Douglas type, with  $\eta \in (0, 1)$  being the constant cost share of labor so that  $1 - \eta$  is the cost share of land. As  $\eta$  suffices to adjust the input intensities, I set  $\gamma := 1$ . Land and labor are imperfect substitutes in agriculture, with an elasticity of substitution of one. Since the figures in chapter 7 are calculated for  $\kappa = 1$ , they specifically apply to the Cobb–Douglas case.

### 8.1 Agriculture

The A-sector of region r collectively produces the amount given by (7.4b):

$$A_r = \left(l_r^A\right)^{\eta} b_r^{1-\eta} . \tag{8.1}$$

Cost minimization yields constant cost shares as

$$MRTS_{lb} = \frac{\eta}{1-\eta} \frac{b_r}{l_r^A} \stackrel{!}{=} \frac{w_r^A}{R_r};$$

cf. (7.5a). In equilibrium, with  $b_r = 1$ ,  $P^A := 1$  and the production function (8.1), the factor prices follow from the zero profit condition, which is  $w_r^A l_r^A + R_r b_r = P^A A_r$ , as

$$R_r b_r = (1 - \eta) P^A A_r \Rightarrow R_r = (1 - \eta) \left( l_r^A \right)^\eta$$
(8.2a)

$$w_r^A l_r^A = \eta P^A A_r \Rightarrow w_r^A = \eta \left( l_r^A \right)^{\eta - 1}$$
 (8.2b)

**B**.3

### B.3.1 8.2 General equilibrium

The systems of equations (7.10–11) yield the short-run and long-run equilibria. Though, the rent and the agricultural wage rate (7.10a–b) need to be replaced by (8.2). Since this wage rate approaches infinity as the agricultural labor supply approaches zero, there always are workers in region r's agriculture if  $\lambda_r > 0$ , and  $w_r = w_r^A$ ; see page 71. Calculating the demand rather than inverse demand for agricultural labor (8.2b), and substituting this into the inverse demand for land (8.2a), yields

$$l_r^A = \left(\frac{\eta}{w_r}\right)^{\frac{1}{1-\eta}} \tag{8.3a}$$

$$R_r = (1 - \eta) \left(\frac{\eta}{w_r}\right)^{\frac{\eta}{1 - \eta}} .$$
(8.3b)

These can be used to conflate the short-run system of equations (7.10); see appendix B.3.1. As opposed to the dispersed equilibrium described hereafter, there is no analytical solution to the fully concentrated long-run equilibrium; section 7.4.2 and section 7.4.3 apply.

#### B.3.1.1 8.2.1 Dispersion

From (7.17) with (7.12c-e) and (8.3) follows

$$\delta \equiv 1 - (1 - \eta) (1 - \mu) , \qquad (8.4)$$

which is greater than both  $\eta$  and  $\mu$ , but less than unity. The equilibrium values follow from (7.18–20) with (8.4) and can be found in appendix B.3.1.1.

The differentials of the equations are given by (7.21–22), where (7.21b) is  $\hat{w} = (\eta - 1) \hat{l}^A$ , and (7.24) is

$$\psi \equiv 1 - \frac{\eta}{1 - \eta} \frac{1 - \mu}{\mu} \frac{1}{\sigma - 1} .$$
(8.5)

The impact of immigration on welfare under symmetry,  $\hat{\omega}/\hat{\lambda}$ , is (7.23) with (8.4–5).

With a prohibitive transport cost, the impact is (7.26) with (8.4–5):

$$\lim_{Z \to 1} \frac{\hat{\omega}}{\hat{\lambda}} = \frac{1}{\sigma - 1} \left( \delta - \frac{(\eta + \sigma (1 - \eta)) \eta^2 (1 - \mu)}{\eta + \mu (1 - \eta)^2} \right)$$

If and only if this is negative, the no-black-hole condition holds and dispersion is stable for a sufficiently high transport cost. I.e., the no-black-hole condition is

$$\sigma > \sigma_{nbh} := 1 + \mu \, \frac{\eta + \delta \, (1 - \eta)^2}{\eta^2 \, (1 - \delta)} \,, \tag{8.6}$$

with  $\sigma_{nbh} > 1$ . If this holds, a break point exists as given by (7.27) with (8.4–5). Figure 8.1

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is an example of a violated no-black-hole condition, and figure 8.2 etc. are examples of a no-black-hole condition that is fulfilled. The line representing immigration's impact on welfare as a function of the transport cost either intersects the zero level or forever continues above it.

#### 8.3 Wider impact

A numerical calculation for constant values of  $\eta$  and  $\mu$ , but varying values of  $\sigma$ , suffices to show that the sign of the wider impact is ambiguous and to explain the evaluation method. This includes the derivation of the order of the transport cost thresholds, first and foremost the break point,  $\tau_b$ , and the utility switch,  $\tau_u$ . Examples of the different types of results are listed in table 8.1. These include the corresponding values of  $\chi_b := \chi|_{\tau=\tau_b}$ , i.e., the welfare ratio between full concentration and dispersion at the break point. The results are illustrated by plotting  $\hat{\omega}/\hat{\lambda}$  (7.23),  $\theta$  (7.28) and  $\chi$  (7.29) over  $\tau$ .

η	μ	σ	Thresholds	χb	Wider impact	Figures		
0.7	0.4	4.3	$ au_u <  au_s$	n/a	n/a	8.1		
0.7	0.4	4.52	$ au_u <  au_s <  au_b$	n/a	n/a			
0.7	0.4	4.53	$ au_u <  au_b <  au_s$	n/a	n/a			B.1
0.7	0.4	5	$ au_u <  au_b <  au_s$	< 1	negative	8.2	B.4	
0.7	0.4	$\approx 5.41$	$ au_b =  au_u <  au_s$	= 1	zero	8.3	B.5	
0.7	0.4	8	$ au_b <  au_u <  au_s$	> 1	positive	8.4	B.6	7.3

**Table 8.1** Transport cost thresholds and the sign of the wider impact ( $\gamma = 1$ ,  $\kappa = 1$ ,  $\sigma_{nbh} \approx 4.51$ )



**Figure 8.1** Transport cost thresholds with no break point ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 4.3 < \sigma_{nbh} \approx 4.51$ )



**Figure 8.2** Transport cost thresholds with a negative wider impact ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 5$ )



**Figure 8.3** Transport cost thresholds with a zero wider impact ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = \sigma_{bu} \approx 5.41$ )



Figure 8.4 Transport cost thresholds with a positive wider impact ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 8$ )

According to (8.6), the no-black-hole condition is fulfilled if and only if  $\sigma > \sigma_{nbh} \approx 4.51$ , given that  $\eta = 0.7$  and  $\mu = 0.4$ . With the first example of table 8.1, this condition is violated and a break point does not exist, which is illustrated in figure 8.1. If  $\sigma$  is larger — but only slightly larger — than  $\sigma_{nbh}$ , as with the subsequent two examples, a break point does exist which is either higher or slightly lower than the sustain point. Hence, the transition to full concentration is not catastrophic, and the welfare assessment can not be applied; see appendix B.2.3.2. Given a sufficiently large  $\sigma$ , the break point is sufficiently lower than the sustain point for the transition to be catastrophic. Successive increases in  $\sigma$  decrease all transport cost thresholds, but they change their order in that they move the break point not just past the sustain point, but eventually past the utility switch.

Figure 8.2 illustrates the fourth example of table 8.1. It exhibits a break point above the utility switch; so,  $\chi_b < 1$  and the wider impact is negative. If  $\sigma$  is sufficiently large, as with the sixth example illustrated by figure 8.4, the break point is below the utility switch; so,  $\chi_b > 1$  and the wider impact is positive. Thus, there exists a threshold of  $\sigma$  at which the two transport cost thresholds are equal so that there is a zero wider impact, i.e.,  $\chi_b = 1$ . See figure 8.3, which illustrates the fifth example, with a  $\sigma = \sigma_{bu} := \{\sigma | \tau_b = \tau_u\} \approx 5.41$  and a  $\tau_{bu} := \{\tau_b | \tau_b = \tau_u\} \approx 2.33$ . See appendix B.3.2 for illustrations of these examples' welfare effects as a consequence of a transport cost reduction. This goes to show that the sign of the wider impact is ambiguous. Given the values of  $\eta$  and  $\mu$ , the wider impact is negative — if applicable — if  $\sigma \in (\sigma_{nbh}, \sigma_{bu})$ . It is positive if  $\sigma \in (\sigma_{bu}, \infty)$ .

An overview of the parameter constellations that yield a zero wider impact is given by figure 8.5. The solid lines delineate the combinations of  $\eta$  and  $\mu$  that yield no wider impact if  $\sigma$  is of a certain value. The dashed lines delineate the combinations of  $\eta$  and  $\mu$ , with  $\sigma = \sigma_{bu}$ , that yield a break point and a utility switch of a certain value. The sign of the wider impact at other parameter constellations can be inferred as follows. For instance, if  $\sigma = 8$ , there is a positive wider impact at the combinations of  $\eta$  and  $\mu$  below the line of  $\sigma_{bu} = 8$  as  $\sigma > \sigma_{bu}$ . The sixth example of table 8.1 ( $\eta = 0.7$ ,  $\mu = 0.4$ ) is one of these. At the combinations of  $\eta$  and  $\mu$  above this line, there is a negative wider impact if the welfare assessment is applicable. At, say,  $\eta = 0.7$  and  $\mu = 0.56$ , there is a negative wider impact. At, say,  $\eta = 0.7$  and  $\mu = 0.6$ , the assessment is not applicable since  $\sigma < \sigma_{nbh}$ . Cf. figure 8.7. Generally speaking,  $\tau_{bu}$  tends to be the lower, the higher  $\sigma_{bu}$  is. If  $\sigma > \sigma_{bu}$ , there is a positive wider impact at  $\tau_b < \tau_{bu}$ .

The values of  $\sigma_{bu}$  and  $\tau_{bu}$  in the upper area of figure 8.5 have not been computed because the no-black-hole condition yields very high values of  $\sigma_{nbh}$ , and  $\sigma_{bu}$  would be even higher. I defined rules for which values to compute. At any combination of  $\eta$  and  $\mu$ , I compute  $\sigma_{nbh}$ . If  $\sigma_{nbh} < 60$ , I compute  $\tau_b|_{\sigma=60}$ ; otherwise, the computation is aborted. This break point can be seen as a lower bound on  $\tau_{bu}$  because, if  $\sigma_{bu} < 60$ , then  $\tau_{bu} > \tau_b|_{\sigma=60}$ . If  $\tau_b|_{\sigma=60} > 5$ , the computation is aborted. After executing this computation of  $\sigma_{bu}$  and  $\tau_{bu}$  for multiple combinations of  $\eta$  and  $\mu$ , the lines are drawn as contours.



**Figure 8.5** Parameter constellations giving rise to a zero wider impact ( $\gamma = 1$ ,  $\kappa = 1$ )

Figure 8.6 and figure 8.7 assume given elasticities of substitution of  $\sigma = 5$  and  $\sigma = 8$ , respectively. The dotted lines outline the constellations of  $\eta$  and  $\mu$  below them that yield some break point as  $\sigma > \sigma_{nbh}$ . The solid lines delineate the parameter constellations that yield a break point of a certain value. The dashed lines delineate the constellations of no wider impact at  $\tau = \tau_b$ . Below these lines, the wider impact is positive.

The larger  $\sigma$  is, the smaller  $\mu$  is and the more medium  $\eta$  is, the more likely it is that the no-black-hole condition is fulfilled, the lower is the break point (if it exists), the more likely it is that the economy is spatially dispersed, and the more likely it is that the wider impact is positive.



**Figure 8.6** Sign of the wider impact with a low elasticity of substitution ( $\gamma = 1$ ,  $\kappa = 1$ ,  $\sigma = 5$ )



**Figure 8.7** Sign of the wider impact with a high elasticity of substitution ( $\gamma = 1$ ,  $\kappa = 1$ ,  $\sigma = 8$ )

# Chapter 9

# Limitational case

This distinct case is more tractable than that of chapter 8. There are analytical solutions, e.g., that to the fully concentrated long-run equilibrium, that otherwise do not exist. Though, they require considerations concerning the parameters and the types of solutions to the long-run equilibria. Section 9.3 applies the evaluation of the welfare effect demonstrated in section 8.3. Again, the sign of the wider impact is shown to be ambiguous.

The A-sector technology is of the Leontief type as the elasticity of substitution is  $\kappa = 0$ ; so, agricultural labor and land are perfect complements. In addition to one unit of land, the agricultural firms employ  $\gamma$  units of labor per unit of output. Only if  $\gamma \in (0, 1 - \mu)$ , the welfare assessment can be applicable; see section 9.2.1. The parameter  $\eta$  is of no relevance because it is not contained in the Leontief-type production function.

### 9.1 Agriculture

The whole of region r's agricultural industry produces the amount given by (7.4c):

$$A_r = \min\left\{l_r^A/\gamma, b_r\right\}$$
 .

Cost minimization yields a relative factor price of

$$\frac{w_r^A}{R_r} \stackrel{!}{=} MRTS_{lb} = \begin{cases} \infty & \text{for } l_r^A / \gamma < b_r \\ 0 & \text{for } l_r^A / \gamma > b_r \end{cases}$$
(9.1)

cf. (7.5a). The zero profit condition,  $R_r + \gamma w_r^A = P^A$ , with (9.1),  $b_r = 1$  and  $P^A := 1$  yields

$$R_{r} = \begin{cases} 0 & \text{for } l_{r}^{A} < \gamma \\ 1 & \text{for } l_{r}^{A} > \gamma \end{cases}$$
$$w_{r}^{A} = \begin{cases} 1/\gamma & \text{for } l_{r}^{A} < \gamma \\ 0 & \text{for } l_{r}^{A} > \gamma \end{cases}$$

At  $l_r^A/\gamma = b_r$ , the production function is nondifferentiable, and  $MRTS_{lb}$  is indeterminable. Though, the factor prices are determinable in spatial equilibrium.

#### B.4.1 9.2 General equilibrium

The short-run and long-run systems of equations which are to be implemented are (7.10–11) with the A-sector factor prices (7.10a–b) replaced by

$$R_r = 1 - \gamma w_r^A \tag{9.2a}$$

$$\begin{cases} w_r^A = 1/\gamma & \text{for } l_r^A < \gamma \\ w_r^A \in \left[\min\left\{w_r^M, 1/\gamma\right\}, 1/\gamma\right] & \text{for } l_r^A = \gamma \\ w_r^A = 0 & \text{for } l_r^A > \gamma . \end{cases}$$
(9.2b)

If  $l_r^A = \gamma$ , an intraregional distribution is in equilibrium if  $w_r^A = w_r^M$  while  $l_r^M \ge 0$  or if  $w_r^A \ge w_r^M$  while  $l_r^M = 0$ ; see (7.10g). The vertical sections in figure 9.1 exemplify the latter. The wage does not exceed  $1/\gamma$ , as  $R_r$  is nonnegative. The case of  $l_r^A > \gamma$  is irrelevant in general equilibrium since  $w_r^M$  is generally positive.



**Figure 9.1** Short-run equilibria with stable full concentration ( $\gamma = 0.45$ ,  $\kappa = 0$ ,  $\mu = 0.5$ ,  $\sigma = 6$ ,  $\tau = 1.4 < \tau_R \approx 1.47 < \tau_b \approx 1.68$ )

In figure 9.1, the long-run equilibria — both dispersed and fully concentrated — exhibit amounts of agricultural labor of  $l_r^A = l_s^A = \gamma$ . Figure 9.2 depicts such long-run equilibria for varying levels of the transport cost. Yet, both types of long-run equilibria can instead exhibit an amount of agricultural labor below  $\gamma$  in one region and/or the other so that land is abundant and free. Either way, there are algebraic solutions to both types of long-run equilibria, and hence to  $\theta$  and  $\chi$ . So, all transport cost thresholds can readily be derived, including two thresholds that are unique to the limitational case.



**Figure 9.2** Long-run equilibria with a varying transport cost ( $\gamma = 0.3$ ,  $\kappa = 0$ ,  $\mu = 0.4$ ,  $\sigma = 6$ ,  $\tau_R \approx 13.33$ ,  $\tau_\lambda \approx 43.66$ )

#### 9.2.1 Dispersion

From (7.17) with (7.12c-e) and (9.2) follows

$$\delta \equiv \begin{cases} \frac{\mu}{1-\gamma} & \text{for } \gamma + \mu \le 1 \quad D_1 \\ 1 & \text{for } \gamma + \mu \ge 1 \quad D_2 \end{cases} .$$
(9.3a)

Either  $\gamma + \mu \leq 1 \Rightarrow \delta \equiv \mu/(1 - \gamma) \leq 1$  so that  $l^A = \gamma$  and  $R \geq 0$ , which is the solution to the dispersed equilibrium denoted by D<sub>1</sub>, or  $\gamma + \mu \geq 1 \Rightarrow \delta \equiv 1$  so that  $l^A = 1 - \mu \leq \gamma$ and R = 0, which is D<sub>2</sub>. If and only if the income share spent on the A-good,  $1 - \mu$ , is greater than  $\gamma$ , the labor share employed in agriculture is  $\gamma$  and the rent is positive. The equilibrium values that follow from (7.18–19) with (9.3) can be found in appendix B.4.1.1.

The respective welfare levels follow from (7.20), (7.19c) and (9.3). In D1, it is

$$\omega = \left(\frac{1}{1-\mu}\right)^{1-\mu} \left(\left(\frac{1-\gamma}{\mu}\right)^{\sigma} (1+\phi)\right)^{\frac{\mu}{\sigma-1}} .$$
(9.4a)

In D2, it is

$$\omega = \left(\frac{1}{\gamma}\right)^{1-\mu} (1+\phi)^{\frac{\mu}{\sigma-1}} . \tag{9.4b}$$

The derivatives at symmetry of the system of equations are (7.21a), (7.21c–e) and (7.22) with  $\hat{l}^A = 0$  for D1 and  $\hat{w} = 0$  for D2, and (7.24) is

$$\begin{cases} \psi \equiv 1 & \text{for } \gamma + \mu \le 1 & \text{D1} \end{cases}$$
(9.5a)

$$\begin{cases}
\psi \to -\infty & \text{for } \gamma + \mu > 1 \quad D_2 .
\end{cases}$$
(9.5b)

The impact of immigration on welfare under symmetry,  $\hat{\omega}/\hat{\lambda}$ , is (7.23) with (9.3) and (9.5).

B.4.1.1

If  $\gamma + \mu > 1$ , the impact is  $\hat{\omega}/\hat{\lambda} = \mu/(\sigma - 1) > 0$  with any Z > 0. Hence, a break point does not exist because dispersion is not stable at any level of the transport cost, and the welfare assessment is not applicable with D<sub>2</sub>, irrespective of  $\sigma$ . With D<sub>1</sub>, though, a break point might exist. Given a prohibitive transport cost, the impact is

$$\lim_{Z \to 1} \frac{\hat{\omega}}{\hat{\lambda}} = \delta \left( \frac{1}{\sigma - 1} - \frac{\gamma^2}{1 - \gamma - \mu} \right) ;$$

cf. (7.26). This is negative, i.e., the no-black-hole condition is fulfilled, if and only if

$$\sigma > \sigma_{nbh} := 1 + \frac{1 - \gamma - \mu}{\gamma^2} , \qquad (9.6)$$

with  $\sigma_{nbh} \ge 1$ . If this holds true, a break point exists, which is

$$\tau_{b} = \left(\frac{(1+(1-\gamma)(3-\gamma))(\sigma-1)+1-\gamma+\mu}{\gamma^{2}(\sigma-1)-(1-\gamma)+\mu}\right)^{\frac{1}{\sigma-1}};$$
(9.7)

cf. (7.27). See appendix B.4.1.1.

#### B.4.1.2 9.2.2 Concentration

It exists an analytical solution to the fully concentrated long-run equilibrium. From (7.10c–j) and (9.2) with  $\check{l}_2^M := 0$  follow  $\check{l}_1^A + \check{l}_2^A + \check{l}_1^M = 2$  and

$$\check{y}_1 = 1 + \frac{(2 - \gamma)\check{w}_1 - \gamma\check{w}_2}{2}$$
(9.8a)

$$\check{y}_2 = 1 + \frac{(2 - \gamma)\check{w}_2 - \gamma\check{w}_1}{2}$$
(9.8b)

$$\check{Y}_1 = \left(\check{l}_1^A + \check{l}_1^M\right)\check{y}_1 \tag{9.8c}$$

$$\check{Y}_2 = \check{l}_2^A \check{y}_2 \tag{9.8d}$$

$$\check{P}_{1}^{M} = \left(\frac{\mu}{\check{l}_{1}^{M}}\right)^{\frac{1}{\sigma-1}}\check{w}_{1}$$
(9.8e)

$$\check{P}_{2}^{M} = \left(\frac{\mu}{\check{l}_{1}^{M}}\right)^{\frac{1}{\sigma-1}} \tau \check{w}_{1} = \tau \check{P}_{1}^{M}$$
(9.8f)

$$\begin{cases} \check{w}_1 = 1/\gamma & \text{for } \check{l}_1^A < \gamma \\ \check{w}_1 \in (0, 1/\gamma] & \text{for } \check{l}_1^A = \gamma \end{cases}$$
(9.8g)

$$\begin{cases} \check{w}_2 = 1/\gamma & \text{for } \check{l}_2^A < \gamma \\ \check{w}_2 \in (0, 1/\gamma] & \text{for } \check{l}_2^A = \gamma \end{cases}$$
(9.8h)

$$\check{w}_1 = \left(\check{P}_1^M\right)^{\frac{\sigma-1}{\sigma}} \left(\check{Y}_1 + \check{Y}_2\right)^{1/\sigma} . \tag{9.8i}$$

The wage that manufacturing firms in the periphery would pay, if there were any, follows from (7.10c) with (9.8f):

$$\check{w}_2^{\mathcal{M}} = \left(\check{P}_1^{\mathcal{M}}\right)^{\frac{\sigma-1}{\sigma}} \left(\phi\check{Y}_1 + \check{Y}_2/\phi\right)^{1/\sigma} .$$
(9.8j)

Welfare in the center must equal that in the periphery if  $\check{\lambda}_2 > 0 \Leftrightarrow \check{l}_2^A > 0$ . From (7.11c) with (9.8f) follows  $\check{\omega}_1 = \check{\omega}_2 \Leftrightarrow \tau^{\mu}\check{y}_1 = \check{y}_2$ . If  $\check{\lambda}_2 = 0 \Leftrightarrow \check{l}_2^A = 0$ , then  $\check{\omega}_1 \ge \check{\omega}_2$ . See (7.11d).

Adding up the regions' aggregate incomes (9.8c-d) yields

$$\check{Y}_1 + \check{Y}_2 = 2 + \left(\check{l}_1^A + \check{l}_1^M - \gamma\right)\check{w}_1 - \left(\gamma - \check{l}_2^A\right)\check{w}_2$$

Substituting this and (9.8e) into the equation for the wage in the center (9.8i) yields

$$\check{w}_{1} = \frac{2 - \left(\gamma - \check{l}_{2}^{A}\right)\check{w}_{2}}{\gamma - \check{l}_{1}^{A} + \left((1 - \mu)/\mu\right)\check{l}_{1}^{M}}.$$
(9.9a)

The rent in the center follows from (9.2a):

$$\check{R}_{1} = \frac{\gamma - \check{l}_{1}^{A} + ((1 - \mu) / \mu) \check{l}_{1}^{M} - \gamma \left(2 - \left(\gamma - \check{l}_{2}^{A}\right) \check{w}_{2}\right)}{\gamma - \check{l}_{1}^{A} + ((1 - \mu) / \mu) \check{l}_{1}^{M}} .$$
(9.9b)

If  $\check{\omega}_2 = \check{\omega}_1 \Leftrightarrow \check{y}_2 = \tau^{\mu}\check{y}_1$ , then (9.8a–b) yield

$$\check{w}_2 = \frac{2(\tau^{\mu} - 1) + (2\tau^{\mu} - \gamma(\tau^{\mu} - 1))\check{w}_1}{2 + \gamma(\tau^{\mu} - 1)} .$$
(9.9c)

The rent in the periphery then follows from (9.2a):

$$\check{R}_{2} = \frac{2 - \gamma \left(\tau^{\mu} - 1 + (2\tau^{\mu} - \gamma \left(\tau^{\mu} - 1\right)\right)\check{w}_{1})}{2 + \gamma \left(\tau^{\mu} - 1\right)} .$$
(9.9d)

There are four types of solutions to (9.8–9) to distinguish between; see appendix B.4.1.2 for the equilibrium values. If  $\gamma/2 + \mu \leq 1$ , there are three possible solutions, depending on how the transport cost relates to the thresholds

$$\tau_R := \left(1 + \frac{2}{\gamma} \frac{1 - \gamma - \mu}{1 - \gamma + \mu}\right)^{1/\mu} \tag{9.10a}$$

$$\tau_{\lambda} := \left(1 + \frac{2}{\gamma} \frac{2(1-\mu) - \gamma}{2-\gamma}\right)^{1/\mu} , \qquad (9.10b)$$

with  $\gamma + \mu \leq 1 \Rightarrow \tau_R \geq 1$ ,  $\gamma/2 + \mu \leq 1 \Rightarrow \tau_\lambda \geq 1$  and  $\tau_R < \tau_\lambda$ . If  $\gamma + \mu \leq 1$  and  $\tau \in [1, \tau_R]$ , the equilibrium exhibits  $\check{l}_1^A = \check{l}_2^A = \gamma$  and  $\check{R}_1$ ,  $\check{R}_2 \geq 0$ , which is the solution C1; see figure 9.1. If  $\tau \in [\tau_R, \tau_\lambda]$ , or  $\tau \in [1, \tau_\lambda]$  if  $\gamma + \mu \geq 1$ , the equilibrium exhibits  $\check{l}_1^A = \gamma$ ,  $\check{l}_2^A \leq \gamma$ ,  $\check{R}_1 \geq 0$  and

 $\check{R}_2 = 0$ , which is C2; see figure 9.3 and figure 9.5. If  $\tau \in [\tau_\lambda, \infty)$ , the equilibrium exhibits  $\check{l}_1^A = \gamma$ ,  $\check{l}_2^A = 0$ ,  $\check{R}_1 \ge 0$  and  $\check{R}_2 = 0$ , which is C3; see figure 9.6. Finally, if  $\gamma/2 + \mu \ge 1$ , the equilibrium exhibits  $\check{l}_1^A \le \gamma$ ,  $\check{l}_2^A = 0$  and  $\check{R}_1 = \check{R}_2 = 0$  irrespective of  $\tau$ , which is C4.

The respective welfare levels follow from (7.11b-c). In C1, they are

$$\check{\omega} = \frac{1}{2 + \gamma (\tau^{\mu} - 1)} \left(\frac{2}{1 - \mu}\right)^{1 - \mu} \left(\frac{2(1 - \gamma)}{\mu}\right)^{\frac{\mu \sigma}{\sigma - 1}}$$
(9.11a)  
$$\check{\omega}_{0}^{\mathcal{M}} = \frac{\tau^{-\mu}}{1 - \mu} \left(\frac{2}{1 - \mu}\right)^{1 - \mu} \left(\frac{2(1 - \gamma)}{\mu}\right)^{\frac{\mu \sigma}{\sigma - 1}}$$

$$\omega_{2}^{m} = \frac{1}{2 + \gamma (\tau^{\mu} - 1)} \left( \frac{1}{1 - \mu} \right) \left( \frac{1}{\mu} \right) \left( \frac{1}{\mu} \right) \left( \frac{1}{1 - \mu} \left( \frac{(2 - \gamma) \tau^{1 - \sigma} + \gamma \tau^{\mu + \sigma - 1}}{2 + \gamma (\tau^{\mu} - 1)} \right)^{1/\sigma} \right) \right].$$
(9.11b)

In C<sub>2</sub>, they are

$$\begin{split} \check{\omega} &= \frac{2}{2 - \gamma \left(\tau^{\mu} - 1\right)} \left( \frac{2 - \gamma \left(\tau^{\mu} - 1\right)}{2\tau^{\mu} - \gamma \left(\tau^{\mu} - 1\right)} \frac{1}{\gamma} \right)^{1 - \mu} \left( 2 \frac{2 + (2 - \gamma) \left(\tau^{\mu} - 1\right)}{2 + (2\mu - \gamma) \left(\tau^{\mu} - 1\right)} \right)^{\frac{\mu}{\sigma - 1}} \quad (9.11c) \\ \check{\omega}_{2}^{M} &= \frac{2\tau^{-\mu}}{2 - \gamma \left(\tau^{\mu} - 1\right)} \left( \frac{2 - \gamma \left(\tau^{\mu} - 1\right)}{2\tau^{\mu} - \gamma \left(\tau^{\mu} - 1\right)} \frac{1}{\gamma} \right)^{1 - \mu} \left( 2 \frac{2 + (2 - \gamma) \left(\tau^{\mu} - 1\right)}{2 + (2\mu - \gamma) \left(\tau^{\mu} - 1\right)} \right)^{\frac{\mu}{\sigma - 1}} \\ &\quad \cdot \frac{1}{2} \left[ \left[ \frac{(2 - \gamma \left(\tau^{\mu} - 1\right)\right)^{\sigma - 1}}{2\tau^{\mu} - \gamma \left(\tau^{\mu} - 1\right)} \left( \tau^{1 - \sigma} \left( 2 \left(2\mu + \gamma\right) + \left(4\mu - \gamma^{2}\right) \left(\tau^{\mu} - 1\right) \right) \right) \right]^{1 / \sigma} \right] \quad (9.11d) \\ &\quad + \tau^{\mu + \sigma - 1} \left( 4 \left(1 - \mu\right) - \gamma \left(2 + \left(2 - \gamma\right) \left(\tau^{\mu} - 1\right) \right) \right) \right]^{1 / \sigma} + \gamma \left(\tau^{\mu} - 1\right) \right] . \end{split}$$

In C<sub>3</sub>, they are

$$\check{\omega} = \frac{1}{2} \left( \frac{1}{1-\mu} \right)^{1-\mu} \left( \frac{2-\gamma}{\mu} \right)^{\frac{\mu\sigma}{\sigma-1}}$$
(9.11e)

$$\check{\omega}_{2} = \frac{4(1-\mu)-\gamma^{2}}{2\gamma(2-\gamma)\tau^{\mu}} \left(\frac{1}{1-\mu}\right)^{1-\mu} \left(\frac{2-\gamma}{\mu}\right)^{\frac{\mu\sigma}{\sigma-1}}$$
(9.11f)

$$\check{\omega}_{2}^{M} = \frac{2 - \gamma - 2\mu \left(1 - \phi^{1/\sigma}\right)}{2 \left(2 - \gamma\right) \tau^{\mu}} \left(\frac{1}{1 - \mu}\right)^{1 - \mu} \left(\frac{2 - \gamma}{\mu}\right)^{\frac{\mu\sigma}{\sigma - 1}}$$
(9.11g)

with  $\check{\omega} \geq \check{\omega}_2$ . In C4, also with  $\check{\omega} \geq \check{\omega}_2$ , they are

$$\check{\omega} = \left(\frac{1}{\gamma}\right)^{1-\mu} 2^{\frac{\mu}{\sigma-1}} \tag{9.11h}$$

$$\check{\omega}_2 = \tau^{-\mu} \left(\frac{1}{\gamma}\right)^{1-\mu} 2^{\frac{\mu}{\sigma-1}} \tag{9.11i}$$

$$\check{\omega}_{2}^{M} = \tau^{-\mu - \frac{\sigma - 1}{\sigma}} \left(\frac{1}{\gamma}\right)^{1 - \mu} 2^{\frac{\mu}{\sigma - 1}} . \tag{9.11j}$$


**Figure 9.3** Short-run equilibria with stable dispersion and full concentration I ( $\gamma = 0.45$ ,  $\kappa = 0$ ,  $\mu = 0.5$ ,  $\sigma = 6$ ,  $\tau_R \approx 1.47 < \tau_b \approx 1.68 < \tau = 2 < \tau_s \approx 2.55 < \tau_\lambda \approx 6.64$ )



**Figure 9.4** Short-run equilibria with stable dispersion ( $\gamma = 0.45$ ,  $\kappa = 0$ ,  $\mu = 0.5$ ,  $\sigma = 6$ ,  $\tau_R \approx 1.47 < \tau_s \approx 2.55 < \tau = 3 < \tau'_s \approx 4.18 < \tau_\lambda \approx 6.64$ )

A solution's sustainability is verified by use of (7.28). For C1, (9.11a-b) yield

$$\theta = \tau^{\mu} \left[ 1 - \mu \frac{2 + \gamma (\tau^{\mu} - 1)}{2 (1 - \gamma)} \left( 1 - \left( \frac{(2 - \gamma) \tau^{1 - \sigma} + \gamma \tau^{\mu + \sigma - 1}}{2 + \gamma (\tau^{\mu} - 1)} \right)^{1/\sigma} \right) \right]^{-1} .$$
(9.12a)

For C<sub>2</sub>, (9.11c-d) yield

$$\theta = 2\tau^{\mu} \left[ \left[ \frac{(2 - \gamma (\tau^{\mu} - 1))^{\sigma - 1}}{2\tau^{\mu} - \gamma (\tau^{\mu} - 1)} \left( \tau^{1 - \sigma} \left( 2 (2\mu + \gamma) + \left( 4\mu - \gamma^{2} \right) (\tau^{\mu} - 1) \right) + \tau^{\mu + \sigma - 1} \left( 4 (1 - \mu) - \gamma (2 + (2 - \gamma) (\tau^{\mu} - 1)) \right) \right) \right]^{1/\sigma} + \gamma (\tau^{\mu} - 1) \right]^{-1}.$$
 (9.12b)

For C<sub>3</sub>, (9.11e) and (9.11g) yield

$$\theta = \frac{(2 - \gamma) \tau^{\mu}}{2 - \gamma - 2\mu \left(1 - \phi^{1/\sigma}\right)} \ge 1 .$$
 (9.12c)

For C<sub>4</sub>, (9.11h) and (9.11j) yield

$$\theta = \tau^{\mu + \frac{\vartheta - 1}{\vartheta}} \ge 1 . \tag{9.12d}$$

Figure 9.4 is the one example where there is no sustainable fully concentrated equilibrium. The kinks in the wiggle diagrams' solid lines other than the transition points are due to the initial emigration from the periphery's A-sector in consequence of increased concentration.



**Figure 9.5** Short-run equilibria with stable dispersion and full concentration II ( $\gamma = 0.45$ ,  $\kappa = 0$ ,  $\mu = 0.5$ ,  $\sigma = 6$ ,  $\tau_R \approx 1.47 < \tau_s \approx 2.55 < \tau'_s \approx 4.18 < \tau = 5.5 < \tau_\lambda \approx 6.64$ )



**Figure 9.6** Short-run equilibria with stable dispersion and full concentration III ( $\gamma = 0.45$ ,  $\kappa = 0$ ,  $\mu = 0.5$ ,  $\sigma = 6$ ,  $\tau'_s \approx 4.18 < \tau_\lambda \approx 6.64 < \tau = 7$ )

Since  $\theta|_{\tau=1} = 1$  and  $d\theta/d\tau|_{\tau=1} > 0$ , full concentration is sustainable at low values of  $\tau$  as  $\theta \ge 1$ ; see appendix B.4.1.2. Since  $\lim_{\tau\to\infty} \theta = \infty$ , full concentration is also sustainable at high levels of  $\tau$ . Only maybe, it is unsustainable at medium levels of  $\tau$ . If and only if a sustain point,  $\tau_s$ , exists, there also exists its reversal point,  $\tau'_s$ , and there exists no fully concentrated long-run equilibrium at  $\tau \in (\tau_s, \tau'_s)$  since (9.12) yields a  $\theta < 1$ . Figure 9.7 is an example where these thresholds exist, which belongs to the wiggle diagrams above.<sup>44</sup> In figure 9.8, these thresholds do not exist.



**Figure 9.7** Transport cost thresholds ( $\gamma = 0.45$ ,  $\kappa = 0$ ,  $\mu = 0.5$ ,  $\sigma = 6$ ,  $\tau_R \approx 1.47 < \tau_u \approx 1.72 < \tau_s \approx 2.55 < \tau'_s \approx 4.18 < \tau_\lambda \approx 6.64$ )



**Figure 9.8** Transport cost thresholds with no sustain point ( $\gamma = 0.6$ ,  $\kappa = 0$ ,  $\mu = 0.5$ ,  $\sigma = 6.85$ ,  $\tau_u \approx 2.98 < \tau'_u \approx 3.64 < \tau_\lambda \approx 3.81$ )

<sup>&</sup>lt;sup>44</sup> As opposed to the figures of section 8.3 for the Cobb–Douglas case, figure 9.7 displays  $\theta$  and  $\chi$  even where a fully concentrated long-run equilibrium does not actually exist as  $\theta < 1$ . This is just because, in the Leontief case,  $\theta$  and  $\chi$  at  $\tau \in (\tau_s, \tau'_s)$  can actually be determined.

#### B.4.1.3 9.2.3 Dispersion vs. concentration

Of the two solutions to the dispersed equilibrium and the four solutions to the fully concentrated long-run equilibrium, there are six possible combinations, as summarized in table 9.1. The welfare ratio,  $\chi$ , is (7.29) with  $\omega$  (9.4) and  $\check{\omega}$  (9.11).

Dispersion		Full concentration					
$\gamma + \mu$		$\gamma/2 + \mu$	$ au_R$	$ au_\lambda$	τ		X
≤ 1	D1		≥ 1	> 1	$\in$ [1, $\tau_R$ ]	C1	9.13a
					$\in$ [ $\tau_R$ , $\tau_\lambda$ ]	C2	9.13b
					$\in$ [ $ au_{\lambda}$ , $\infty$ )	C <sub>3</sub>	9.13c
≥ 1	D2	<u>≤</u> 1		≥ 1	$\in$ [1, $\tau_{\lambda}$ ]	C2	9.13d
					$\in$ [ $ au_{\lambda}$ , $\infty$ )	C <sub>3</sub>	9.13e
		≥ 1			∈ [1, ∞)	C4	9.13f

**Table 9.1** Co-occurrences of dispersed and fully concentrated long-run equilibria ( $\kappa = 0$ )

If  $\gamma + \mu \leq 1$ , the dispersed equilibrium is D1. Also,  $\tau_R \geq 1$ , and there is one of three types of fully concentrated long-run equilibria. If  $\tau \in [1, \tau_R]$ , it is C1, and (9.4a) and (9.11a) yield

$$\chi = \frac{2}{2 + \gamma (\tau^{\mu} - 1)} \left(\frac{2}{1 + \phi}\right)^{\frac{\mu}{\sigma - 1}} .$$
 (9.13a)

If  $\tau \in [\tau_R, \tau_\lambda]$ , it is C2, and (9.4a) and (9.11c) yield

$$\chi = \frac{2}{2 - \gamma (\tau^{\mu} - 1)} \left( \frac{2 - \gamma (\tau^{\mu} - 1)}{2\tau^{\mu} - \gamma (\tau^{\mu} - 1)} \frac{1 - \mu}{\gamma} \right)^{1 - \mu} \cdot \left( \left( \frac{2 + (2 - \gamma) (\tau^{\mu} - 1)}{2 + (2\mu - \gamma) (\tau^{\mu} - 1)} \right) \left( \frac{\mu}{1 - \gamma} \right)^{\sigma} \frac{2}{1 + \phi} \right)^{\frac{\mu}{\sigma - 1}} .$$
 (9.13b)

If  $\tau \in [\tau_{\lambda}, \infty)$ , it is C<sub>3</sub>, and (9.4a) and (9.11e) yield

$$\chi = \left(\frac{1}{2}\right)^{1-\mu} \left( \left(\frac{2-\gamma}{2(1-\gamma)}\right)^{\sigma} \frac{2}{1+\phi} \right)^{\frac{\mu}{\sigma-1}} .$$
(9.13c)

If  $\gamma + \mu \ge 1$ , the dispersed equilibrium is D2. Since this equilibrium is generally unstable, a break point does not exist, and the assessment of the wider impact is not applicable. However, if  $\gamma/2 + \mu \le 1$ , then  $\tau_{\lambda} \ge 1$ , and there is one of two types of fully concentrated long-run equilibria. If  $\tau \in [1, \tau_{\lambda}]$ , it is C2, and (9.4b) and (9.11c) yield

$$\chi = \frac{2}{2 - \gamma (\tau^{\mu} - 1)} \left( \frac{2 - \gamma (\tau^{\mu} - 1)}{2\tau^{\mu} - \gamma (\tau^{\mu} - 1)} \right)^{1 - \mu} \left( \frac{2 + (2 - \gamma) (\tau^{\mu} - 1)}{2 + (2\mu - \gamma) (\tau^{\mu} - 1)} \frac{2}{1 + \phi} \right)^{\frac{\mu}{\sigma - 1}} .$$
(9.13d)

#### 9.3 WIDER IMPACT

If  $\tau \in [\tau_{\lambda}, \infty)$ , it is C<sub>3</sub>, and (9.4b) and (9.11e) yield

$$\chi = \left(\frac{\gamma}{2(1-\mu)}\right)^{1-\mu} \left(\left(\frac{2-\gamma}{2\mu}\right)^{\sigma} \frac{2}{1+\phi}\right)^{\frac{\mu}{\sigma-1}} .$$
(9.13e)

If  $\gamma/2 + \mu \ge 1$ , the fully concentrated long-run equilibrium is C4, irrespective of the level of the transport cost, and (9.4b) and (9.11h) yield

$$\chi = \left(\frac{2}{1+\phi}\right)^{\frac{\mu}{\sigma-1}} \ge 1 . \tag{9.13f}$$

Since  $\chi|_{\tau=1} = 1$  and  $d\chi/d\tau|_{\tau=1} > 0$ , full concentration exhibits a higher welfare level than dispersion at low values of  $\tau$  as  $\chi \ge 1$ . If  $\lim_{\tau\to\infty} \chi \le 1$ , a utility switch,  $\tau_u$ , exists, yet not its reversal point,  $\tau'_u$ , and full concentration exhibits a lower welfare level than dispersion at  $\tau \in (\tau_u, \infty)$  since (9.13) yields a  $\chi < 1$ ; see figure 9.7. If  $\lim_{\tau\to\infty} \chi > 1$ , a utility switch might exist. If and only if it does,  $\tau'_u$  also exists, and  $\chi < 1$  at  $\tau \in (\tau_u, \tau'_u)$ . In figure 9.8, these thresholds exist. See appendix B.4.1.3.

#### 9.3 Wider impact

Like in section 8.3, it suffices to do a numerical calculation for constant values of  $\gamma$  and  $\mu$ , but varying values of  $\sigma$ , to show that the sign of the wider impact is ambiguous. Examples of the different types of results are listed in table 9.2. Provided that  $\gamma = 0.3$  and  $\mu = 0.4$ , a break point exists if and only if  $\sigma > \sigma_{nbh} = 13/3$ ; see (9.6). The transport cost thresholds follow from  $\tau_b$  (9.7),  $\theta$  (9.12) and  $\chi$  (9.13).

γ	μ	σ	Thresholds	χb	Wider impact	Figures
0.3	0.4	1.5		n/a	n/a	
0.3	0.4	2.5	$ au_u$	n/a	n/a	
0.3	0.4	4	$ au_u <  au_s$	n/a	n/a	
0.3	0.4	5	$ au_u <  au_s <  au_b$	n/a	n/a	
0.3	0.4	6	$ au_u <  au_b <  au_s$	< 1	negative	9.2
0.3	0.4	pprox 8.48	$ au_b =  au_u <  au_s$	= 1	zero	
0.3	0.4	10	$ au_b <  au_u <  au_s$	> 1	positive	

**Table 9.2** Transport cost thresholds and the sign of the wider impact ( $\kappa = 0$ ,  $\sigma_{nbh} = 13/3$ )

An overview of the parameter constellations that yield a zero wider impact is depicted in figure 9.9. The straight dotted line outlines the combinations of  $\gamma$  and  $\mu$  above it that yield the dispersed equilibrium D<sub>2</sub> which generally has no break point, whatever the value of  $\sigma$ . The solid lines delineate the combinations of  $\gamma$  and  $\mu$  that yield no wider impact if  $\sigma$  is of a certain value. The dashed lines delineate the combinations of  $\gamma$  and  $\mu$ , with  $\sigma = \sigma_{bu}$ , that

yield a break point and a utility switch of a certain value. It turns out that these thresholds can only be equal with the fully concentrated equilibrium C<sub>1</sub>; i.e.,  $\tau_{bu} < \tau_R$ . The reason is that, as opposed to the Cobb–Douglas case, a utility switch does not necessarily exist. The curved dotted line outlines the combinations of  $\gamma$  and  $\mu$  above it that yield D<sub>1</sub> but no  $\sigma_{bu}$  and  $\tau_{bu}$ . If, say,  $\sigma = 10$ , the wider impact is positive at the combinations of  $\gamma$  and  $\mu$ above the line of  $\sigma_{bu} = 10$  since  $\sigma > \sigma_{bu}$ ; see the seventh example of table 9.2 ( $\gamma = 0.3$ ,  $\mu = 0.4$ ) and compare figure 9.11. With the fifth example, there is a negative wider impact since  $\sigma = 6 < \sigma_{bu}$ . Like with the Cobb–Douglas case,  $\tau_{bu}$  tends to be the lower, the higher  $\sigma_{bu}$  is. If  $\sigma > \sigma_{bu}$ , there is a positive wider impact at  $\tau_b < \tau_{bu}$ .



**Figure 9.9** Parameter constellations giving rise to a zero wider impact ( $\kappa = 0$ )

Figure 9.10 and figure 9.11 assume given elasticities of  $\sigma = 8$  and  $\sigma = 10$ , respectively. The straight dotted lines outline the constellations of  $\gamma$  and  $\mu$  above them that yield D2 which generally has no break point. The curved dotted lines in the bottom left-hand corners outline the constellations to their right that yield some break point. The solid lines delineate the constellations that yield a break point of a certain value. The black dashed lines outline the constellations to their left that yield no fully concentrated long-run equilibrium at the break point as  $\theta_b := \theta|_{\tau=\tau_b} < 1$ . The gray dashed lines outline the constellations above them that yield a positive wider impact at  $\tau = \tau_b$ . The gray dotted lines delineate the constellations of  $\tau_b = \tau_R$ .



**Figure 9.10** Sign of the wider impact with a low elasticity of substitution ( $\kappa = 0$ ,  $\sigma = 8$ )



**Figure 9.11** Sign of the wider impact with a high elasticity of substitution ( $\kappa = 0$ ,  $\sigma = 10$ )

The larger  $\sigma$  and  $\mu$  are, the more likely it is that the no-black-hole condition is fulfilled, the lower is the break point (if it exists), the more likely it is that the economy is dispersed, and the more likely it is that the wider impact is positive. The larger  $\gamma$  is, the more likely it is that the no-black-hole condition is fulfilled, and the lower is the break point. With regards to  $\mu$ , this is the other way round from the Cobb-Douglas case; see section 8.3.

End of part II  $\Box$ 

## Chapter 10

## Conclusion

I have demonstrated, as others have before me, that there exist wider impacts as a consequence of a transport infrastructure investment that result from imperfect competition in the wider economy or from an agglomeration externality. Whether the wider impacts are of a considerable scale, is something I do not attempt to assess. What I can say is that the signs of the wider impacts are ambiguous. Strictly speaking, I would even be very cautious to conjecture which sign is by and large the more probable one. Presupposing a positive sign, and maybe even a considerable scale, whether for imperfect competition or agglomeration, would constitute a considerable bias in transport appraisal.

I agree with what seems to be consensus anyway, that there probably could not be established any simple rules of thumb for the evaluation of transport projects' wider impacts (Vickerman, 2008a, p. 80). It rather appears necessary to conduct project-based assessments, maybe as addenda to conventional cost-benefit analyses. In line with my reluctance to predict the wider impacts' potential scale, I am just as reluctant to predict whether the wider impacts are significant enough to justify their ex ante assessment.

#### **10.1** Imperfect competition

If an imperfectly competitive market's equilibrium quantity is affected by a transport scheme — whether directly or indirectly — this induces a wider impact. As the mark-up is positive, i.e., as the consumers' marginal willingness to pay is above the marginal cost, the equilibrium quantity is below the socially optimal level at which the marginal willingness to pay equals the marginal cost. If the quantity is increased, the market induces positive welfare effects, and if the quantity is decreased, it induces negative welfare effects. The price quantifies the marginal overall impact as it represents the consumers' willingness to pay for an additional unit of the good. The marginal cost quantifies the marginal direct impact as it represents the resources reallocated to this industry — possibly from transportation. The mark-up thus quantifies the marginal wider impact. For a market whose marginal cost of transportation is directly affected by the scheme, partial equilibrium analysis suggests that the quantity is increased and that the welfare effects are thus positive. L

In a general equilibrium framework with multiple industries, the outcome of a transport scheme is typically such that some quantities are increased while others are decreased. For the net wider impact to be positive, the quantities of goods with positive mark-ups need to be increased, while the decreases need to be of the quantities of goods with relatively low or even zero mark-ups. An increase in the demand for any normal good can be due to a decrease in the good's price, which is in turn due to a reduction in the mark-up and/or the marginal cost of transportation. Though, the latter comes at the cost of running the scheme, and the scheme is financed by taxing the households. The higher the scheme's costs are, the lower is the households' disposable income, and the lower is the households' demand for any normal good. In other words, running the scheme absorbs some of the economy's limited resources. Yet, it does so in order to reduce the costs of transportation. The more efficient it is at doing this, the more resources remain available for the production of the various goods. This entire reallocation of the resources, which is triggered by the scheme, is in accordance with the quantity shifts described above.

The total impact on the industries' profits is the combined impact via changes in the mark-ups and the quantities. The latter by itself is equal to the wider impact. Provided that the mark-ups are exogenous, the wider impact is equal to the total impact on profits. Provided that the profits are zero, the wider impact is equal to the impact on the profits via changes in the mark-ups, but with a reversed sign. These changes in the mark-ups then represent the impact on average costs, and the wider impact represents the benefits from economies of scale.

If the costs of the scheme are such that there is no direct impact, then the wider impact is equal to the overall impact. The marginal overall impact is positive if there is a positive correlation between the mark-up factors atop the marginal costs and the resource shifts. I.e., it is positive if resources are shifted from industries with below–average mark-up factors toward industries with above–average mark-up factors.

An economy with monopolistically competitive industries yields demand/resource shifts that are positively correlated with the mark-up factors and the transportation intensities. A transportation intensity is a good's marginal cost reduction as a fraction of the price. The marginal wider impact is positive if there is a positive correlation between the mark-up factors and the transportation intensities. I.e., it is positive if the industries with the above– average mark-up factors are the ones with the above–average transportation intensities.

With the potential suppliers of an industry differing with regards to their marginal costs of transportation, a scheme can make the costs converge, leading to an increase in the number of suppliers. Firm entry has two effects on the marginal wider impact. There is a positive effect if the decline of the price is accelerated through enhanced competition. There is a negative effect since the entrant has higher transport costs than the incumbent firms. If the incumbents deter other firms from entering, e.g., by means of predatory pricing, the marginal wider impact is both positive and higher than without entry deterrence.

### 10.2 Agglomeration

The agglomeration of economic activity can yield benefits due to, say, economies of scale, enhanced competition or avoidance of transport costs. Yet, it can also yield costs due to, say, scarce resources. Moreover, agglomeration in one region must go along with deglomeration in another region if there is a fixed number of agents, as in a general equilibrium framework. When a transport cost reduction brings about a redistribution of economic activity, there maybe is a net benefit for one region and a net loss for another region. There is generally either a net benefit or a net loss for the economy as a whole.

This ambiguity of the sign of the wider impact is what I have demonstrated within a single New Economic Geography model. While the approach by Tabuchi (1998) also yields an ambiguous sign, the ones by Pflüger & Südekum (2008b) and Ottaviano et al. (2002) yield negative signs.

In my model, the sign of the wider impact is negatively correlated with the households' love for variety regarding the manufactures, of which the elasticity of substitution of the households' preferences regarding the varieties of the manufactures is an inverse measure. The stronger the love for variety is, the more likely it is that the wider impact is negative. But the stronger the love for variety is, the stronger are the economy's centripetal forces. Given the level of the transport cost, the economy is already concentrated given a sufficiently strong love for variety. The economy is dispersed given a sufficiently weak love for variety; so, a successive transport cost reduction eventually triggers a transition to concentration. The weaker the love for variety is, the more likely it is that the wider impact is positive. Ш

# Appendices

## Appendix A

## **Imperfect competition**

### A.1 Notation

A bar with any parameter or variable, e.g.,  $\bar{u}$ , denotes the corresponding value in an arbitrary reference situation for the purpose of the welfare assessment.

Symbol	Meaning
ai	Average costs of industry <i>i</i>
С	Marginal cost of production of industry 1
Ci	Marginal cost of production of industry <i>i</i>
cj	Marginal cost of production of firm <i>j</i>
CS	Consumer surplus on the transport market
d	Subscript for the domestic firm of industry 1
<i>D</i> , <i>D</i> <sub><i>i</i></sub>	Direct impact (from industry $i$ ) on the welfare of the transport users
е	Subscript for the foreign firm of industry 1
$e(\cdot)$	Expenditure function
f	Fixed costs per firm of industry 1
$f(\cdot)$	Adaptation of the expenditure function
$f_i$	Fixed costs per firm of industry <i>i</i>
$F_i$	Aggregate fixed costs of industry <i>i</i>
i	Subscript for the industries/goods
I, I <sub>i</sub>	Overall impact (from industry $i$ ) on the welfare of the household
j	Subscript for a firm or an industry/good other than <i>i</i>
L	Endowment with labor and labor supply
т	Total marginal cost of industry 1's monopolist
m <sub>d</sub>	Total marginal cost of industry 1's domestic firm
m <sub>e</sub>	Total marginal cost of industry 1's foreign firm
m <sub>i</sub>	Total marginal cost of industry <i>i</i>
m <sub>j</sub>	Total marginal cost of firm <i>j</i>
ŵ	Average of total marginal costs of industry 1's duopolists
ñ	Total marginal cost of industry 1's monopolist prior to the scheme

I

Symbol	Meaning
<i>m̃</i> <sub>d</sub>	Total marginal cost of industry 1's domestic firm prior to the scheme
<i>т</i> <sub>е</sub>	Total marginal cost of industry 1's foreign firm prior to the scheme
<i>m</i> <sub>i</sub>	Total marginal cost of industry <i>i</i> prior to the scheme
${ ilde m}_j$	Total marginal cost of firm <i>j</i> prior to the scheme
m	Vector of all industries' total marginal costs
$n_i, n_i(\cdot)$	Number of firms/varieties of industry/good <i>i</i>
N	Number of industries/goods
$p, p(\cdot)$	Consumer price of good 1
$p_d(\cdot)$	Function of the consumer price of good 1 on the domestic market
$p_e(\cdot)$	Function of the consumer price of good 1 on the export market
$p_i, p_i(\cdot)$	Consumer price of (any variety of) good <i>i</i>
<b>ρ</b> , <b>ρ</b> (·)	Vector of all goods' consumer prices
$P_i$	Consumer price index of good <i>i</i>
Р	Vector of all goods' consumer price indices
$s(\cdot)$	Sub-utility function regarding good 1 with quasi-linear preferences
$S, S(\cdot), S_i$	Lump-sum tax and costs of the scheme (attributed to industry <i>i</i> )
t	Reduction in the marginal transport cost of industry 1's monopolist
$t_d$	Reduction in the marginal transport cost of industry 1's domestic firm
t <sub>e</sub>	Reduction in the marginal transport cost of industry 1's foreign firm
ti	Reduction in the marginal transport cost of industry <i>i</i>
tj	Reduction in the marginal transport cost of firm <i>j</i>
$\hat{t}, \hat{t}(\cdot)$	Average reduction in the marginal transport costs of industry 1's duopolists
и	Utility of the household
$u(\cdot)$	Utility function
$U_i(\cdot)$	Sub–utility function regarding good <i>i</i> with two–tier preferences
$v(\cdot)$	Indirect utility function
$w(\cdot)$	Adaptation of the indirect utility function
W, W <sub>i</sub>	Wider impact (from industry <i>i</i> ) on the welfare of the household
Xd	Quantity of good 1 by domestic firm
x <sub>e</sub>	Quantity of good 1 by foreign firm
x <sub>i</sub>	Quantity of any variety of good <i>i</i>
$x_i(\cdot)$	Marshallian demand function of any variety of good <i>i</i>
<i>x</i> <sub>i</sub>	Quantity of any variety per unit of good <i>i</i>
X	Quantity of good 1
$X(\cdot)$	Demand function of good 1
$X_i$	Quantity of good <i>i</i>
$X_i(\cdot)$	Marshallian demand function of good <i>i</i>
$X'_i$	Impact on the quantity of good <i>i</i>

#### A.1 NOTATION

Symbol	Meaning
$X^h_i(\cdot)$	Hicksian demand function of good <i>i</i>
$\hat{X}'$	Average impact on the quantities
Χ́′	Relative impact on the quantity of an average goood
$y, y(\cdot)$	Nominal disposable income of the household
Ζ	Quantity of the numeraire good
$z(\cdot)$	Marshallian demand function of the numeraire good
$\alpha_i$	Expenditure share of good <i>i</i>
γί	Constant with the demand for good <i>i</i> in long-run equilibrium
$\delta$	Disposable income's share of earned income
$\epsilon$	Price elasticity of (market) demand for good 1
$\epsilon_{i}$	Price elasticity of demand for good <i>i</i>
η	Constant with the expenditure function and the indirect utility function
λ	Export share of industry 1's duopolists
μ	Absolute mark-up of industry 1's monopolist
$\mu_d$	Absolute mark-up of industry 1's domestic firm
$\mu_e$	Absolute mark-up of industry 1's foreign firm
$\mu_i$	Absolute mark-up of industry <i>i</i>
μ	Average mark-up of industry 1's duopolists
V	Relative mark-up on the total marginal cost of industry 1's monopolist or uniform relative mark-up
v <sub>i</sub>	Relative mark-up on the total marginal cost of industry <i>i</i>
Ŷ	Average of the relative mark-ups on the total marginal costs
ř	Export threshold of the total-marginal-costs ratio of industry 1's duopolists
π	Profit per firm of industry 1
$\pi_i$	Profit per firm of industry <i>i</i>
$\Pi_i$	Aggregate profit of industry <i>i</i>
$\sigma_i$	Elasticity of substitution between any two varieties of good <i>i</i>
τ	Marginal transport cost of industry 1 unaffected by the scheme
$ au_i$	Marginal transport cost of industry $i$ unaffected by the scheme
$ au_j$	Marginal transport cost of firm <i>j</i> unaffected by the scheme
arphi	Progress of the scheme
$\check{arphi}$	Export threshold of the progress of the scheme
$\phi$	Variable of integration for the progress of the scheme
X	Uniform cost-reduction-to-price ratio
Xi	Cost–reduction–to–price ratio of industry <i>i</i>
Ŷ	Average cost-reduction-to-price ratio

Table A.1 Notation

#### 3 A.2 Base model

#### 3.3.2 A.2.1 Direct impact

The derivative of (3.16b), with (3.18) for  $\Delta CS$  and (3.15), with respect to  $\varphi$  is

$$\frac{\mathrm{d}D}{\mathrm{d}\varphi} = \frac{1}{2} \sum_{i} t_i \left( X_i + \bar{X}_i + (\varphi - \bar{\varphi}) X_i' \right) - S'(\varphi) \; .$$

For the reference values, i.e., for  $\bar{\varphi} = \varphi$  and  $\bar{X}_i = X_i \forall i$ , follows D' (3.17).

#### 3.3.3 A.2.2 Wider impact

With I (3.9), y (3.7), S (3.5), D (3.16b),  $\Delta S$  (3.15) and  $y = e(\mathbf{p}, u)$ , W (3.19) follows as

$$W = I - D = \underbrace{L + \sum_{i} \Pi_{i} - S(\varphi) - e(\mathbf{p}, \bar{u}) - \Delta CS + \underbrace{S(\varphi) - S(\bar{\varphi})}_{=\Delta S}}_{=y}$$
$$= L + \underbrace{\sum_{i} \Pi_{i} - e(\mathbf{p}, \bar{u}) - \Delta CS - S(\bar{\varphi})}_{=\sum_{i} \left(\Pi_{i} - \bar{\Pi}_{i}\right) - \left(e(\mathbf{p}, \bar{u}) - e(\bar{\mathbf{p}}, \bar{u})\right) - \Delta CS}.$$

The wider impact is not fully independent of S as the  $\Pi_i$ 's and  $\Delta CS$  depend on the  $X_i$ 's, which in turn depend on y (3.7).

#### 3.3.5 A.2.3 Zero profits

It follows from (3.23) that

$$\frac{\mathrm{d}\epsilon_i/\epsilon_i}{\mathrm{d}\varphi} > -\frac{t_i}{p_i} \Rightarrow \frac{\mathrm{d}\mu_i/\mu_i}{\mathrm{d}\varphi} = -\frac{1}{m_i} \left( t_i + p_i \frac{\mathrm{d}\epsilon_i/\epsilon_i}{\mathrm{d}\varphi} \right) < 0 \tag{A.1a}$$

$$\frac{\mathrm{d}\epsilon_i/\epsilon_i}{\mathrm{d}\varphi} > -\frac{t_i}{\mu_i} \Rightarrow \frac{\mathrm{d}p_i/p_i}{\mathrm{d}\varphi} = -\frac{1}{m_i} \left( t_i + \mu_i \frac{\mathrm{d}\epsilon_i/\epsilon_i}{\mathrm{d}\varphi} \right) < 0 .$$
 (A.1b)

#### 3.3.7 A.2.4 Zero direct impact

Combining (3.14) with  $\alpha_i = p_i X_i / y$  and (3.3a) yields

$$\sum_{i} \alpha_{i} \frac{p_{i} - \mu_{i}}{p_{i}} \frac{X_{i}'}{X_{i}} = 0 \implies \sum_{i} \alpha_{i} \frac{\mu_{i}}{p_{i}} \frac{X_{i}'}{X_{i}} = \sum_{i} \alpha_{i} \frac{X_{i}'}{X_{i}} .$$

Hence the step from (3.22c) to (3.22d). Cf. I' (3.10b) with  $p_i = \alpha_i y / X_i$ .

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#### A.3 Monopolistic competition

#### A.3.1 Household

Because the household consumes equal amounts of all the  $n_i$  varieties of any good i, the CES sub-utility functions can be reduced to (4.1b):

$$X_{i} = U_{i}(x_{i}) := \left(\sum_{1}^{n_{i}} x_{i}^{\frac{\sigma_{i}-1}{\sigma_{i}}}\right)^{\frac{\sigma_{i}}{\sigma_{i}-1}} = \left(n_{i}x_{i}^{\frac{\sigma_{i}-1}{\sigma_{i}}}\right)^{\frac{\sigma_{i}}{\sigma_{i}-1}} = n_{i}^{\frac{\sigma_{i}}{\sigma_{i}-1}}x_{i} .$$
(A.2)

Minimizing the expenditures  $P_i := n_i p_i \tilde{x}_i$  subject to the constraint that  $U_i(\tilde{x}_i) \stackrel{!}{=} 1$  with (4.1b) yields

$$\tilde{x}_i = \left(\frac{p_i}{P_i}\right)^{-\sigma_i} . \tag{A.3}$$

This is the amount that the household consumes per variety of good *i* in order to consume one unit of the composite *i* at minimal expenditures. The Marshallian demand functions for single varieties (4.2b) are obtained by multiplying  $\tilde{x}_i$  (A.3) by  $X_i$  (4.2a). Substituting (A.3) back into the objective function, and solving for  $P_i$ , yields (4.3). In deriving the demand for a single variety, and its price elasticity,  $p_i$  is assumed to have a negligible influence on  $P_i$ due to a — supposedly — large  $n_i$ . See Fujita et al. (1999, sec. 4.1, pp. 46–49).

#### A.3.2 Long-run equilibrium

From substituting  $n_i$  (4.8) together with  $x_i$  (4.7a) into the sub–utility function (4.1b) follows

$$X_i = \gamma_i \, \frac{(\alpha_i y)^{\frac{\sigma_i}{\sigma_i - 1}}}{m_i} \tag{A.4}$$

with

$$\gamma_i := \left(\frac{\sigma_i^{\sigma_i}}{(\sigma_i - 1)^{\sigma_i - 1}} f_i\right)^{\frac{1}{1 - \sigma_i}}$$

as a collection of constants. From substituting  $n_i$  (4.8) and  $p_i$  (4.5) into (4.3) follows

$$P_i = \frac{1}{\gamma_i} \left( \alpha_i y \right)^{\frac{1}{1 - \sigma_i}} m_i .$$
(A.5)

Because  $m_i$  and y (4.10) are both nonincreasing in  $\varphi$ , the signs of the dependencies of  $X_i$ and  $P_i$  on  $\varphi$  are ambiguous. If  $t_i = 0$  so that  $m_i$  is constant,  $X_i$  is nonincreasing while  $P_i$ is nondecreasing in  $\varphi$ . If y is constant,  $X_i$  is nondecreasing while  $P_i$  is nonincreasing in  $\varphi$ ; see section 4.4. 4 4.1

4.3

#### 4.5 A.3.3 Welfare

#### 4.5.1 A.3.3.1 Overall impact

Substituting the composite quantities (4.2a) into the utility function (4.1a) gives the indirect utility function

$$u = v(\boldsymbol{P}, \boldsymbol{y}) := \left(\prod_{i} \alpha_{i}^{\alpha_{i}}\right) \frac{\boldsymbol{y}}{\prod_{i} P_{i}^{\alpha_{i}}}$$
(A.6)

with  $P := (P_1, \ldots, P_N)$ . Substituting the equilibrium price indices (A.5) into (A.6) yields

$$u = w(\boldsymbol{m}, \boldsymbol{y}) := \eta \, \frac{\boldsymbol{y}^{\hat{\boldsymbol{\nu}}}}{\prod_{i} m_{i}^{\alpha_{i}}} \tag{A.7a}$$

with  $\hat{v}$  as (4.14) and

$$\eta:=\prod_i \left( {lpha_i}^{m{v}_i}m{\gamma}_i 
ight)^{lpha_i}$$
 ,

which is a constant. Rearranging (A.7a) for y gives the corresponding adaptation of the expenditure function:

$$f(\boldsymbol{m}, \boldsymbol{u}) := \left(\frac{\boldsymbol{u}}{\eta} \prod_{i} m_{i}^{\alpha_{i}}\right)^{1/\nu} .$$
(A.7b)

I take the difference between the equilibrium income (4.10) and the minimal expenditures as given by the expenditure function (A.7b), for the current marginal costs and the reference utility level which is in turn given by the indirect utility function (A.7a):

$$I = y - f(\boldsymbol{m}, \bar{u}) = y - \left(\frac{\bar{u}}{\eta} \prod_{i} m_{i}^{\alpha_{i}}\right)^{1/\hat{v}}$$
  
$$= y - f(\boldsymbol{m}, w(\bar{\boldsymbol{m}}, \bar{y})) = y - \left(\frac{1}{\eta} \left(\eta \frac{\bar{y}^{\hat{v}}}{\prod_{i} \bar{m}_{i}^{\alpha_{i}}}\right) \prod_{i} m_{i}^{\alpha_{i}}\right)^{1/\hat{v}}$$
  
$$= \delta L - f(\boldsymbol{m}, w(\bar{\boldsymbol{m}}, \bar{\delta}L)) = \left[\delta - \bar{\delta} \prod_{i} \left(\frac{m_{i}}{\bar{m}_{i}}\right)^{\alpha_{i}/\hat{v}}\right] L.$$

The marginal overall impact with any reference situation, see *I* (4.13), is

$$\frac{\mathrm{d}I}{\mathrm{d}\varphi} = \frac{\delta}{\hat{\nu}} \left[ \sum_{i} \alpha_{i} \chi_{i} \nu_{i} \left( \frac{\bar{\delta}}{\delta} \prod_{j} \left( \frac{m_{j}}{\bar{m}_{j}} \right)^{\alpha_{j}/\hat{\nu}} \right) + \hat{\nu} \frac{\mathrm{d}\delta/\delta}{\mathrm{d}\varphi} \right] L \,. \tag{A.8}$$

For  $\bar{\delta} = \delta$  and  $\bar{m} = m$  follows l' (4.15).

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#### A.3.3.2 Direct impact

In the case of a free scheme (4.12a),  $\Delta S = 0$ , and in the case of a subsidy (4.12b),

$$\Delta S = \sum_{i} t_{i} \left( \varphi n_{i} x_{i} - \bar{\varphi} \bar{n}_{i} \bar{x}_{i} \right) = \left[ \sum_{i} \alpha_{i} \chi_{i} \left( \varphi \delta - \bar{\varphi} \bar{\delta} \frac{m_{i}}{\bar{m}_{i}} \right) \right] L.$$

See (4.16).

The derivative of (4.16) and (4.9) is

$$\frac{\mathrm{d}\Delta S}{\mathrm{d}\varphi} = S'(\varphi) = \frac{\mathrm{d}S(\varphi)}{\mathrm{d}\varphi} = -\frac{\mathrm{d}\delta}{\mathrm{d}\varphi} L$$

since  $S = L - y = (1 - \delta) L$ ; see (4.10). With (4.12a),  $d\delta = 0$ , and with (4.12b),

$$\frac{\mathrm{d}\delta/\delta}{\mathrm{d}\varphi} = -\delta \sum_{i} \alpha_{i} \chi_{i} \, \frac{\tilde{m}_{i}}{m_{i}} \,. \tag{A.9}$$

The derivative of (4.17a) is

$$\frac{\mathrm{d}\Delta CS}{\mathrm{d}\varphi} = \sum_{i} t_{i} n_{i} x_{i} = \delta \hat{\chi} L \,.$$

In the case of (4.12a), (4.18) is  $D' = \delta \hat{\chi} L$ . In the case of (4.12b), the marginal direct impact at  $\varphi = 0$  is zero as  $S'(0) = \hat{\chi} L = d\Delta CS/d\varphi|_{\varphi=0}$ . From (4.18) follows

$$D' = \delta \left[ \hat{\chi} - \delta \sum_{i} \alpha_{i} \chi_{i} \frac{\tilde{m}_{i}}{m_{i}} \right] L \Rightarrow D' \big|_{\varphi=0} = 0$$

Compare section 3.3.7.

Using the rule of a half (Button, 2009, sec. 6, pp. 71–73) instead of (4.17a) yields

$$\Delta CS = (\varphi - \bar{\varphi}) \sum_{i} t_{i} \frac{n_{i} x_{i} + \bar{n}_{i} \bar{x}_{i}}{2} = \frac{\varphi - \bar{\varphi}}{2} \left[ \sum_{i} \alpha_{i} \chi_{i} \left( \delta + \bar{\delta} \frac{m_{i}}{\bar{m}_{i}} \right) \right] L .$$
(A.10)

Differentiating *D*, (4.17b) with (A.10), with respect to  $\varphi$  gives

$$\frac{\mathrm{d}D}{\mathrm{d}\varphi} = \delta \left[ \hat{\chi} + \frac{\mathrm{d}\delta/\delta}{\mathrm{d}\varphi} - \frac{1}{2} \sum_{i} \alpha_{i} \chi_{i} \left( 1 - \frac{\bar{\delta}}{\delta} \frac{m_{i}}{\bar{m}_{i}} - (\varphi - \bar{\varphi}) \left( \mathbf{v}_{i} \chi_{i} + \frac{\mathrm{d}\delta/\delta}{\mathrm{d}\varphi} \right) \right) \right] L.$$

For any reference situation, i.e., for  $\bar{\varphi} = \varphi$ ,  $\bar{\delta} = \delta$  and  $\bar{m}_i = m_i \forall i$ , follows D' (4.18).

4.5.2

#### 4.5.3 A.3.3.3 Wider impact

The wider impact with I (4.13),  $y = L - S(\varphi) = f(\mathbf{m}, u)$  (A.7b), D (4.17b) and  $\Delta S$  (4.16) is

$$W = I - D = \underbrace{L - S(\varphi)}_{=y} - f(\boldsymbol{m}, \bar{u}) - \Delta CS + \underbrace{S(\varphi) - S(\bar{\varphi})}_{=\Delta S}$$

$$= L - f(\boldsymbol{m}, \bar{u}) - \Delta CS - S(\bar{\varphi})$$

$$= - (f(\boldsymbol{m}, \bar{u}) - f(\bar{\boldsymbol{m}}, \bar{u})) - \Delta CS .$$
(A.11)

The marginal wider impact with any reference situation follows from  $dI/d\varphi$  (A.8) and  $dD/d\varphi$  (4.18), or from W (A.11), i.e., if (4.17a) is applied, as

$$\frac{\mathrm{d}W}{\mathrm{d}\varphi} = \frac{\delta}{\hat{\nu}} \left[ \sum_{i} \alpha_{i} \chi_{i} \left( \nu_{i} \frac{\bar{\delta}}{\delta} \prod_{j} \left( \frac{m_{j}}{\bar{m}_{j}} \right)^{\alpha_{j}/\hat{\nu}} - \hat{\nu} \right) \right] L.$$

For  $\bar{\delta} = \delta$  and  $\bar{m} = m$  follows W' (4.19).

#### 5 A.4 Monopoly

#### 5.4.1 A.4.1 Overall impact

The indirect utility function follows from substituting the demand functions (5.6) into the utility function (5.4):

$$u = v(p, y) := u(X(p), z(p, y)) = s(X(p)) - p X(p) + y .$$
(A.12a)

Substituting y (5.8) into this yields w(p) (5.11). The corresponding expenditure function is

$$e(p, u) := u - s(X(p)) + p X(p)$$
. (A.12b)

#### 5.4.2 A.4.2 Direct impact

The approximation of (5.16a) by use of the rule of a half (Button, 2009, sec. 6, pp. 71–73) is

$$\Delta CS = (\varphi - \bar{\varphi}) t \frac{X + \bar{X}}{2} .$$

The corresponding marginal direct impact, see (5.16b), is

$$\frac{\mathrm{d}D}{\mathrm{d}\varphi} = \frac{t}{2} \left( X(p) + X(\bar{p}) + (\varphi - \bar{\varphi}) X'(p) \frac{\mathrm{d}p}{\mathrm{d}\varphi} \right) - S'(\varphi) \; .$$

For the reference values, i.e., for  $\bar{\varphi} = \varphi$  and  $\bar{p} = p$ , follows D' (5.17).

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#### A.4.3 Wider impact

The wider impact with *I* (5.13), *D* (5.16b), e(p, u) (A.12b) and  $\pi$  (5.1) is

$$W = I - D = u - \bar{u} - \Delta CS + \Delta S$$
  
=  $s(X(p)) - s(X(\bar{p})) - (m X(p) - \bar{m} X(\bar{p})) - \Delta CS$  (A.13)  
=  $\pi - \bar{\pi} - (e(p, \bar{u}) - e(\bar{p}, \bar{u})) - \Delta CS$ .

The price is decreasing in the progress of the scheme if there is a sufficiently moderate decrease, or even increase, in the elasticity as

$$\frac{\mathrm{d}\epsilon/\epsilon}{\mathrm{d}\varphi} > -\frac{t}{p} \implies \frac{\mathrm{d}\mu/\mu}{\mathrm{d}\varphi} = -\frac{1}{m} \left( t + p \, \frac{\mathrm{d}\epsilon/\epsilon}{\mathrm{d}\varphi} \right) < 0$$
$$\frac{\mathrm{d}\epsilon/\epsilon}{\mathrm{d}\varphi} > -\frac{t}{\mu} \implies \frac{\mathrm{d}p/p}{\mathrm{d}\varphi} = -\frac{1}{m} \left( t + \mu \, \frac{\mathrm{d}\epsilon/\epsilon}{\mathrm{d}\varphi} \right) < 0 \; .$$

#### A.5 Reciprocal dumping

#### A.5.1 Simultaneous moves

#### A.5.1.1 Overall impact

The marginal impacts on p (6.8a) and  $\lambda$  (6.8b) are

$$\frac{\mathrm{d}p/p}{\mathrm{d}\varphi} = \begin{cases} -\frac{1}{m_d} \left( t_d + \mu_d \, \frac{\mathrm{d}\epsilon/\epsilon}{\mathrm{d}\varphi} \right) & \text{for } \varphi < \check{\varphi} \\ -\frac{1}{m_d + m_e} \left( t_d + t_e + \frac{p}{\epsilon} \, \frac{\mathrm{d}\epsilon/\epsilon}{\mathrm{d}\varphi} \right) & \text{for } \check{\varphi} < \varphi \lor \check{\varphi} = 0 \end{cases}$$

$$\frac{\mathrm{d}\lambda}{\mathrm{d}\varphi} = \begin{cases} 0 & \text{for } \varphi < \check{\varphi} \\ \frac{t_e - t_d}{m_d + m_e} \left( \frac{\epsilon}{p} \, (c + \tau) - (1 - \varphi) \, \frac{\mathrm{d}\epsilon}{\mathrm{d}\varphi} \right) & \text{for } \check{\varphi} < \varphi \lor \check{\varphi} = 0 \end{cases}$$
(A.14a)
$$(A.14b)$$

For instance, if demand is isoelastic, there is a decline in the price that is accelerated at the export threshold. If and only if  $(d\epsilon/\epsilon)/d\varphi < ((1 - \varphi) p/(c + \tau))^{-1}$ , the export share is increasing at a progress of the scheme beyond the export threshold. With either linear or isoelastic demand, i.e., with an elasticity that is either decreasing in  $\varphi$  or constant, this inequality holds as the right-hand side is positive.

5.4.3

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6.3 6.3.1

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#### 6.3.2 A.5.1.2 Direct impact

The approximate change in the consumer surplus on the market for transportation (6.13a), when applying the rule of a half (Button, 2009, sec. 6, pp. 71–73), is

$$\Delta CS = (\varphi - \bar{\varphi}) \, \frac{\hat{t}X + \bar{t}\bar{X}}{2} \; .$$

The corresponding marginal impact on D (6.13b) is

$$\frac{\mathrm{d}D}{\mathrm{d}\varphi} = \frac{1}{2} \left[ \hat{t}X + \bar{\hat{t}}\bar{X} + (\varphi - \bar{\varphi}) \left( \hat{t}X'(p)\frac{\mathrm{d}p}{\mathrm{d}\varphi} + (t_e - t_d)X\frac{\mathrm{d}\lambda}{\mathrm{d}\varphi} \right) \right] - S'(\varphi) \; .$$

At the reference values, i.e., at  $\bar{\varphi} = \varphi$ ,  $\bar{\hat{t}} = \hat{t}$  and  $\bar{X} = X$ , this is D' (6.14).

#### 6.3.3 A.5.1.3 Wider impact

The wider impact with *I* (6.10), *D* (6.13b), e(p, u) (A.12b) and  $\pi$  (6.1) is

$$W = I - D = u - \bar{u} - \Delta CS + \Delta S$$
  
=  $s(X(p)) - s(X(\bar{p})) - (\hat{m} X(p) - \bar{\hat{m}} X(\bar{p})) - \Delta CS$  (A.15)  
=  $\pi - \bar{\pi} - (e(p, \bar{u}) - e(\bar{p}, \bar{u})) - \Delta CS$ .

## Appendix B

## Agglomeration

### B.1 Notation

Symbol	Meaning
A	Quantity of the agricultural good produced per region under symmetry
$A_r$	Quantity of the agricultural good produced in region r
b <sub>r</sub>	Quantity of land employed by agriculture in region r
Br	Endowment with land and land supply of region <i>r</i>
F	Quantity of labor employed as fixed input by any industrial firm
i	Superscript for industrial firms/varieties
$l^A$	Quantity of agricultural labor per region under symmetry
$l^{\mathcal{M}}$	Quantity of industrial labor per region under symmetry
lА	Relative change in $l_r^A$ under symmetry
$\hat{l}^{\mathcal{M}}$	Relative change in $l_r^M$ under symmetry
$l_r^A$	Quantity of agricultural labor in region r
$l_r^M$	Quantity of industrial labor in region <i>r</i>
$\check{l}_r^A$	Quantity of agricultural labor in region r under full concentration
$\check{l}^{\mathcal{M}}_{r}$	Quantity of industrial labor in region r under full concentration
L	Endowment with labor and labor supply of the economy
т	Quantity of labor employed as marginal input by industrial firms
$MRTS_{lb}$	Marginal rate of technical substitution of agricultural labor for land
п	Number of industrial firms/varieties per region under symmetry
n <sub>r</sub>	Number of industrial firms in region $r$ and number of varieties produced in $r$
p <sub>r</sub>	Mill price of any industrial variety produced in region r
$p_r^i$	Consumer price of industrial variety <i>i</i> in region <i>r</i>
$P^A$	Price of the agricultural good
$P^M$	Consumer price of the industrial composite under symmetry
$\hat{P}^M$	Relative change in $P_r^M$ under symmetry

||

Symbol	Meaning
$P_r^M$	Consumer price of the industrial composite in region <i>r</i>
$\check{P}_r^M$	Consumer price of the industrial composite in $r$ under full concentration
q	Quantity produced per industrial firm/variety
r	Subscript for regions
R	Rent under symmetry
R <sub>r</sub>	Rent in region <i>r</i>
Ř <sub>r</sub>	Rent in region r under full concentration
5	Subscript for the region other than <i>r</i>
u <sub>r</sub>	Utility of a household in region <i>r</i>
$U^{M}(\cdot)$	Lower-tier utility function for the industrial composite
W	Wage rate under symmetry
ŵ	Relative change in $w_r$ under symmetry
Wr	Wage rate in region <i>r</i>
Ŵr	Wage rate in region <i>r</i> under full concentration
w <sup>A</sup>	Agricultural wage rate under symmetry
$w^{\mathcal{M}}$	Industrial wage rate under symmetry
$w_r^A$	Agricultural wage rate in region <i>r</i>
$w_r^M$	Industrial wage rate in region r
$\check{w}_r^M$	Industrial wage rate in region r under full concentration
<i>x</i> <sub>r</sub>	Vector of all varieties' quantities consumed by a household in region <i>r</i>
<i>x</i> <sub>r</sub>	Vector of all varieties' quantities per unit of the composite consumed in $r$
$x_r^i$	Quantity of variety <i>i</i> consumed by a household in region <i>r</i>
$\tilde{x}_r^i$	Quantity of variety $i$ per unit of the composite consumed in region $r$
$X_r^A$	Quantity of the agricultural good consumed by a household in region $r$
$X_r^M$	Quantity of the industrial composite consumed by a household in region $r$
y	Nominal individual income under symmetry
y <sub>r</sub>	Nominal individual income in region <i>r</i>
ў,	Nominal individual income in region <i>r</i> under full concentration
$y_r^A$	Nominal individual income in agriculture of region r
y <sup>M</sup>	Nominal individual income in manufacturing of region r
ў <sup>м</sup>	Nominal individual income in manufacturing of $r$ under full concentration
Y	Aggregate nominal income per region under symmetry
Ŷ	Relative change in $Y_r$ under symmetry
Y <sub>r</sub>	Aggregate nominal income of region <i>r</i>
Ý <sub>r</sub>	Aggregate nominal income of region <i>r</i> under full concentration
Z	Trade closedness

#### B.1 NOTATION

Symbol	Meaning
$Z_b$	Value of $Z$ at the break point
γ	Agricultural labor per unit of land/output with Leontief technology
$\delta$	Earned income's share of total nominal income under symmetry
η	Cost share of labor in agriculture with Cobb–Douglas technology
θ	Welfare under full concentration relative to industrial welfare in periphery
$ heta_b$	Value of $ heta$ at the break point
K	Elasticity of substitution between agricultural labor and land
λ	Relative change in $\lambda_r$ under symmetry
$\lambda_r$	Labor share in region <i>r</i>
$\check{\lambda}_r$	Labor share in region <i>r</i> under full concentration
μ	Expenditure share for manufactures
σ	Elasticity of substitution of the households between any industrial varieties
$\sigma_{bu}$	Threshold of $\sigma$ that yields a zero wider impact
$\sigma_{nbh}$	Maximum value of $\sigma$ for the no–black–hole condition not to be fulfilled
τ	Iceberg parameter for interregional trade in manufactures
$ au_b$	Value of $ au$ at the break point
$ au_{bu}$	Threshold of $ au$ that yields a zero wider impact
$ au_R$	Minimum of $ au$ for a zero rent in the periphery under full concentration
$ au_s$ , $ au_s'$	Value of $ au$ at the sustain point and at its reversal point
$ au_u$ , $ au_u'$	Value of $ au$ at the utility switch and at its reversal point
$ au_\lambda$	Minimum of $ au$ for a zero labor share in the periphery under full concentration
$\phi$	Trade freeness
$\phi_b$	Value of $\phi$ at the break point
X	Welfare under full concentration relative to welfare under symmetry
Хb	Value of $\chi$ at the break point
$\psi$	Collection of constants
ω	Welfare under symmetry
ŵ	Relative change in $\omega_r$ under symmetry
ŭ	Welfare under full concentration
ω <sub>r</sub>	Welfare in region r
<i>ω</i> <sub>r</sub>	Welfare in region <i>r</i> under full concentration
$\omega_r^A$	Agricultural welfare in region <i>r</i>
$\omega_r^{\mathcal{M}}$	Industrial welfare in region r
$\check{\omega}^{\mathcal{A}}_{r}$	Agricultural welfare in region r under full concentration
$\check{\omega}^{\mathcal{M}}_r$	Industrial welfare in region r under full concentration

Table B.1 Notation

#### 7 B.2 Base model

#### 7.1 B.2.1 Preferences

The lower-tier Marshallian demand functions (7.2c) follow from minimizing  $P_r^M := \sum_i p_r^i \tilde{x}_r^i$  subject to  $U^M(\tilde{x}_r) = 1$  as given by (7.1b), with  $\tilde{x}_r$  being the vector of all  $\tilde{x}_r^{i\prime}$ s, and these in turn being the amounts of the single varieties that are consumed per unit of the composite so that the expenditures per unit of the composite are minimal. First, this yields  $\tilde{x}_r^i = (p_r^i/P_r^M)^{-\sigma}$ . The lower-tier demand functions are calculated as  $x_r^i = \tilde{x}_r^i X_r^M$  with  $X_r^M$  as given by (7.2a). The expression for the manufactures' price index as a CES function of the varieties' prices (7.3) follows from rearranging  $P_r^M = \sum_i p_r^i \tilde{x}_r^i = \sum_i (p_r^i)^{1-\sigma} (P_r^M)^{\sigma}$  for  $P_r^M$ . See Fujita et al. (1999, sec. 4.1, pp. 46–49).

#### 7.2.2 B.2.2 Manufacturing

The wage equation for the manufacturing industry (7.10c) is derived by setting supply equal to demand for a single variety of the manufactures. In order to cope with the iceberg cost, this is done in value terms. The supply by any one firm in region r is  $p_rq$ , and the demand by a ssingle household for any variety i is given by (7.2c) multiplied by  $p_r^i$ . While the variety i is produced in only one region, say, in region r, it is consumed in both r and s. Since consumers in r are faced with the mill price  $p_r$  and consumers in s with the price  $\tau p_r$ , and since the consumers' preferences are homothetic, total demand for this variety from r is  $(p_r/P_r^M)^{1-\sigma} \mu Y_r + (\tau p_r/P_s^M)^{1-\sigma} \mu Y_s$ . Individual income is simply replaced by total income due to the homotheticity, and  $\mu Y_r$  represents a region's total expenditure on manufactures, as one can infer from (7.2a). Setting supply equal to demand, and substituting into it the mill price (7.6) as well as the quantity produced (7.7), this equation can be rearranged for the wage rate to yield

$$w_r^M = \frac{\sigma - 1}{\sigma m} \left( \frac{\mu m}{(\sigma - 1) F} \right)^{1/\sigma} \left( Y_r \left( P_r^M \right)^{\sigma - 1} + \phi Y_s \left( P_s^M \right)^{\sigma - 1} \right)^{1/\sigma} .$$

By choice of units, i.e.,  $m := (\sigma - 1) / \sigma$  and  $F := \mu / \sigma$ , this expression is reduced to (7.10c) as it loses its constant factor.

Taking the reverse perspective, i.e., that of a consumer in region r having to pay  $p_r$  for any domestically produced variety and  $\tau p_s$  for any imported variety, the M–composite's price given by (7.3) can be written as

$$P_r^M = \left(n_r p_r^{1-\sigma} + n_s (\tau p_s)^{1-\sigma}\right)^{\frac{1}{1-\sigma}} .$$

Substituting the numbers of firms (7.8) and the mill prices (7.6) into this, and factorizing

B.2 BASE MODEL

the constants, one gets

$$P_r^M = \frac{\sigma m}{\sigma - 1} \left( \sigma F \right)^{\frac{1}{\sigma - 1}} \left( l_r^M \left( w_r^M \right)^{1 - \sigma} + \phi l_s^M \left( w_s^M \right)^{1 - \sigma} \right)^{\frac{1}{1 - \sigma}} .$$

By choice of units, as stated above, this becomes (7.10d).

#### B.2.3 General equilibrium

Section 7.4 outlines the systems of equations that are summarized as follows. To solve for a short-run equilibrium, i.e., a general equilibrium with intraregional mobility but interregional immobility, one selects values for  $\lambda_r$  and the parameters. The numerical solution follows from (7.10), which is

$$R_r = (1 - \eta) \left( \eta \left( l_r^A / \gamma \right)^{\frac{\kappa - 1}{\kappa}} + (1 - \eta) \right)^{\frac{1}{\kappa - 1}}$$
(B.1a)

$$w_r^A = \frac{\eta}{\gamma} \left( \eta + (1 - \eta) \left( l_r^A / \gamma \right)^{\frac{1 - \kappa}{\kappa}} \right)^{\frac{1 - \kappa}{\kappa}}$$
(B.1b)

$$w_r^{\mathcal{M}} = \left(Y_r \left(P_r^{\mathcal{M}}\right)^{\sigma-1} + \phi Y_s \left(P_s^{\mathcal{M}}\right)^{\sigma-1}\right)^{1/\sigma}$$
(B.1c)

$$w_r = \max\left\{w_r^A, w_r^M\right\}$$
(B.1d)

$$y_r = \frac{R_r + R_s}{2} + w_r \tag{B.1e}$$

$$Y_r = \lambda_r 2 y_r \tag{B.1f}$$

$$P_r^M = \mu^{\frac{1}{\sigma-1}} \left( l_r^M \left( w_r^M \right)^{1-\sigma} + \phi l_s^M \left( w_s^M \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$
(B.1g)

plus

$$\lambda_r + \lambda_s = 1 \tag{B.1h}$$

$$\lambda_r = \frac{l_r^A + l_r^M}{2} \tag{B.1i}$$

$$\max\left\{\left(w_r^{\mathcal{M}} - w_r^{\mathcal{A}}\right)l_r^{\mathcal{A}}, \left(w_r^{\mathcal{A}} - w_r^{\mathcal{M}}\right)l_r^{\mathcal{M}}\right\} = 0.$$
(B.1j)

All but (B.1h) are to be included for both  $r \in \{1, 2\}$ .

To solve for a long-run equilibrium, i.e., a general equilibrium with full mobility, I let the labor distribution be endogenous and append

$$\omega_r = y_r \left( P_r^M \right)^{-\mu} \tag{B.2a}$$

$$\max\left\{\left(\omega_{s}-\omega_{r}\right)\lambda_{r},\left(\omega_{r}-\omega_{s}\right)\lambda_{s}\right\}=0\tag{B.2b}$$

7.4

to (B.1) in order to produce the long-run system of equations (B.1–2). When attempting to find a numerical solution, one should be aware that there always is at least one long-run equilibrium because dispersion always is a long-run equilibrium.

#### 7.4.1 B.2.3.1 Dispersion

Applying de l'Hôpital's rule (Abramowitz & Stegun, 1965, p. 13) to the A-sector wage (7.19c) yields

$$\lim_{\kappa \to 1} \ln(w) = \lim_{\kappa \to 1} \left( \ln \left( \frac{\eta}{\gamma} \right) - \frac{\mathrm{d}\delta/\mathrm{d}\kappa}{\delta - \mu} \right) \; .$$

From the total differentials of (7.17–18a) follows

$$\lim_{\kappa \to 1} \frac{\mathrm{d}\delta}{\mathrm{d}\kappa} = \frac{(1-\delta)\left(\delta-\mu\right)}{1-\mu} \cdot \ln\left(\frac{\delta-\mu}{\gamma\delta}\right) \; .$$

With  $\delta$  (8.4), these yield the wage for  $\kappa = 1$  as given by (7.19c). Merging (7.21c–e) results in

$$(\sigma/Z - \delta - Z (\sigma - 1)) \hat{w} = \hat{\lambda} - Z \hat{l}^{M}$$

With (7.21a–b) and (7.18), and  $\psi$  (7.24), this yields

$$\frac{\hat{w}}{\hat{\lambda}} = \frac{1 - \delta Z/\mu}{\sigma/Z - \delta - \psi Z (\sigma - 1)} . \tag{B.3}$$

From (7.22) with (7.21e), (7.21a-b), (7.18), (7.24) and (B.3) follows (7.23).

Using (7.23) to solve  $\hat{\omega}/\hat{\lambda} \stackrel{!}{=} 0$  for trade closedness, one solution is  $Z = 0 \Leftrightarrow \tau = 1$ . The other is the trade closedness at the break point:

$$Z = Z_b := \frac{2\sigma - 1}{\left(\frac{\psi\mu}{\delta + \delta/\mu}\right)(\sigma - 1) + \delta} . \tag{B.4}$$

The corresponding trade freeness is

$$\phi_b := \frac{(\psi\mu/\delta + \delta/\mu)(\sigma - 1) + \delta - (2\sigma - 1)}{(\psi\mu/\delta + \delta/\mu)(\sigma - 1) + \delta + (2\sigma - 1)}$$
(B.5)

as  $\phi = (1 - Z) / (1 + Z)$ . The break point in terms of the iceberg factor (7.27) is derived from (B.5) with  $\tau = \phi^{1/(1-\sigma)}$ . The no-black-hole condition, which is the condition for (B.4–5) and (7.27) to be valid solutions, i.e., for  $Z_b > 0 \Leftrightarrow \phi_b < 1 \Leftrightarrow \tau_b > 1$ , is that (7.26) must be negative; see (8.6) and (9.6), respectively, for corresponding minimum values of  $\sigma$  for either of the two distinct cases.

#### B.2.3.2 Subcritical vs. supercritical pitchfork bifurcation

The transition from dispersion to full concentration is either subcritical, like in figure 7.3, or supercritical, like in figure B.1; see Pflüger (2004), Pflüger & Südekum (2008a, 2011), Fujita et al. (1999, appx. to ch. 3) and Grandmont (1988). It is subcritical if the curve at and around  $\tau = \tau_b$  and  $\lambda_r = 0.5$  depicting the partially concentrated long-run equilibria is bent to the right. When the transport cost falls below  $\tau_b$ , dispersion turns unstable and the economy leaps from dispersion to full concentration. If the curve is bent to the left, the economy gradually transitions into partial concentration before leaping to full concentration, i.e., the transition is supercritical.



**Figure B.1** Long-run equilibria with a supercritical transition ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 4.53$ )

Figure B.2 illustrates the short-run equilibria at  $\tau = \tau_b$ . This indicates the supercritical transition as the welfare difference is nonincreasing in the labor share at and around the symmetric equilibrium, which is thus stable. I.e., the third derivative of the welfare level with respect to the labor share at  $\lambda_r = 0.5$  and  $\tau = \tau_b$  is negative. The first derivative (7.23) is — by definition — zero at the break point. The second derivative is zero because symmetry is a saddle point. The third derivative is negative. If the third derivative is positive, as in figure 7.5, the transition is subcritical, and the welfare difference is nondecreasing at and around the symmetric equilibrium, which is thus unstable.

If the transition is supercritical, there exists a stable long-run equilibrium with only partial concentration within some interval of the transport cost below the break point, as in figure B.3. If the transport cost is below the sustain point, there also exists an unstable long-run equilibrium with partial (but stronger) concentration and a stable fully concentrated long-run equilibrium. In case of a subcritical transition, the fully concentrated long-run equilibrium is the only stable one at a transport cost below the break point; see figure 7.4.

7.4.3



**Figure B.2** Short-run equilibria at the break point with stable dispersion ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 4.53$ ,  $\tau = \tau_b \approx 7.86$ )



**Figure B.3** Short-run equilibria with stable partial concentration ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 4.53$ ,  $\tau = 5.75$ )

The transition is supercritical if  $\sigma$  is only slightly larger than  $\sigma_{nbh}$ , i.e., if  $\tau_b$  is either higher than or slightly lower than  $\tau_s$ . Since the transport cost thresholds are decreasing in  $\sigma$ , they are quite high when the transition is supercritical. Thus, they are likely to be above  $\tau$  so that the economy is agglomerated and a transition does not occur, rendering the analysis of the wider impact moot. Also, the thresholds — and thus the very occurrence of a supercritical transition — are sensitive to changes in the parameters. The assessment of the wider impact is based on the comparison of the welfare levels at the break point under dispersion and full concentration. If the transition from dispersion to full concentration is not subcritical, this assessment is not applicable. This is the justification for my disregard of a supercritical transition given in section 7.4.3.

### B.3 Cobb–Douglas case

#### B.3.1 General equilibrium

Integrating (7.10) or (B.1) with (8.3) yields

$$l_r^M = \lambda_r 2 - \left(\frac{\eta}{w_r}\right)^{\frac{1}{1-\eta}}$$
$$w_r^M = \left(Y_r \left(P_r^M\right)^{\sigma-1} + \phi Y_s \left(P_s^M\right)^{\sigma-1}\right)^{1/\sigma}$$
$$y_r = \frac{1-\eta}{2} \left(\left(\frac{\eta}{w_r}\right)^{\frac{\eta}{1-\eta}} + \left(\frac{\eta}{w_s}\right)^{\frac{\eta}{1-\eta}}\right) + w_r$$
$$Y_r = \lambda_r 2y_r$$
$$P_r^M = \mu^{\frac{1}{\sigma-1}} \left(l_r^M \left(w_r^M\right)^{1-\sigma} + \phi l_s^M \left(w_s^M\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}$$

and

$$\lambda_r + \lambda_s = 1$$

$$\max\left\{w_r^{\mathcal{M}} - w_r, \left(w_r - w_r^{\mathcal{M}}\right)l_r^{\mathcal{M}}\right\} = 0.$$

#### B.3.1.1 Dispersion

The equilibrium values follow from substituting  $\delta$  (8.4) into (7.18–20):

$$\begin{split} l^{A} &= \frac{\eta \left(1-\mu\right)}{\mu+\eta \left(1-\mu\right)} \\ l^{M} &= \frac{\mu}{\mu+\eta \left(1-\mu\right)} \\ R &= \left(1-\eta\right) \left(\frac{\eta \left(1-\mu\right)}{\mu+\eta \left(1-\mu\right)}\right)^{\eta} \\ w &= \eta \left(\frac{\eta \left(1-\mu\right)}{\mu+\eta \left(1-\mu\right)}\right)^{\eta-1} \\ y &= Y = \frac{1}{1-\mu} \left(\frac{\eta \left(1-\mu\right)}{\mu+\eta \left(1-\mu\right)}\right)^{\eta} \\ P^{M} &= \eta \left(\frac{\eta \left(1-\mu\right)}{\mu+\eta \left(1-\mu\right)}\right)^{\eta-1} \left(\frac{\mu+\eta \left(1-\mu\right)}{1+\phi}\right)^{\frac{1}{\sigma-1}} \\ \omega &= \frac{\eta^{-\mu}}{1-\mu} \left(\frac{\eta \left(1-\mu\right)}{\mu+\eta \left(1-\mu\right)}\right)^{\mu+\eta \left(1-\mu\right)} \left(\frac{1+\phi}{\mu+\eta \left(1-\mu\right)}\right)^{\frac{\mu}{\sigma-1}} . \end{split}$$

8.2

8

8.2.1

#### 8.3 **B.3.2 Wider impact**

A transport cost reduction has an impact on welfare that possibly includes a wider impact at the instant of the cost passing the break point. Table 8.1 lists the following examples. Figure B.4 depicts a negative wider impact, figure B.5 depicts a zero wider impact, and figure B.6 depicts a positive wider impact. The black dashed lines are the welfare levels under symmetry, and the gray solid lines are the welfare levels under full concentration. The black solid lines mark the welfare levels in the states that the economy is in, provided that there is a decreasing transport cost. One can see that, as the transport cost decreases, it eventually passes the break point,  $\tau_b$ , making the economy transition from dispersion to full concentration. As it does, welfare might leap to a higher or lower level. A drop represents a negative wider impact, and a rise represents a positive wider impact.



Figure B.4 Welfare development with a negative wider impact ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 5$ )



**Figure B.5** Welfare development with a zero wider impact ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = \sigma_{bu} \approx 5.41$ )



Figure B.6 Welfare development with a positive wider impact ( $\gamma = 1$ ,  $\eta = 0.7$ ,  $\kappa = 1$ ,  $\mu = 0.4$ ,  $\sigma = 8$ )

### B.4 Limitational case

9

9.2

9.2.1

#### B.4.1 General equilibrium

#### **B.4.1.1 Dispersion**

If the equilibrium is D1, i.e., if  $\gamma + \mu \leq 1 \Rightarrow \delta \equiv \mu/(1 - \gamma)$  (9.3a), (7.18–19) yield

$$l^{A} = \gamma$$

$$l^{M} = 1 - \gamma$$

$$R = \frac{1 - \gamma - \mu}{(1 - \gamma)(1 - \mu)}$$

$$w = \frac{\mu}{(1 - \gamma)(1 - \mu)}$$

$$y = Y = \frac{1}{1 - \mu}$$

$$P^{M} = \frac{1}{1 - \mu} \left( \left(\frac{\mu}{1 - \gamma}\right)^{\sigma} \frac{1}{1 + \phi} \right)^{\frac{1}{\sigma - 1}}$$

If it is D2, i.e., if  $\gamma + \mu \geq 1 \Rightarrow \delta \equiv 1$  (9.3b), the equilibrium values are

$$l^{A} = 1 - \mu$$

$$l^{M} = \mu$$

$$R = 0$$

$$w = y = Y = \frac{1}{\gamma}$$

$$P^{M} = \frac{1}{\gamma} \left(\frac{1}{1+\phi}\right)^{\frac{1}{\sigma-1}}$$

With D1, i.e., with (9.3a) and (9.5a), (B.3) and (7.23) are

$$\begin{aligned} \frac{\hat{w}}{\hat{\lambda}} &= \frac{1 - Z/(1 - \gamma)}{\sigma/Z - \delta - Z(\sigma - 1)} \\ \frac{\hat{w}}{\hat{\lambda}} &= \frac{(\delta - \mu Z)(1 - Z/(1 - \gamma))}{\sigma/Z - \delta - Z(\sigma - 1)} + \frac{\delta Z}{\sigma - 1} \end{aligned}$$

The intermediate steps of calculating  $au_b$  (9.7) from  $\hat{\omega}/\hat{\lambda}=0$  are

$$Z_{b} = \frac{2\sigma - 1}{(1 - \gamma + 1/(1 - \gamma))(\sigma - 1) + \delta}$$
  
$$\phi_{b} = \frac{\gamma^{2}(\sigma - 1) - (1 - \gamma - \mu)}{(1 + (1 - \gamma)(3 - \gamma))(\sigma - 1) + 1 - \gamma + \mu};$$

cf. (B.4–5).

#### 9.2.2 B.4.1.2 Concentration

The equilibrium values of the fully concentrated long-run equilibrium follow from (9.8–9) with (7.9b), (7.10e) and (7.11b–d). In C<sub>1</sub>, i.e., if  $\gamma + \mu \leq 1$  and  $\tau \in [1, \tau_R]$  (9.10a), they are

$$\begin{split} \check{\lambda}_{1} &= \frac{2 - \gamma}{2} \\ \check{\lambda}_{2} &= \frac{\gamma}{2} \\ \check{l}_{1}^{A} &= \check{l}_{2}^{A} = \gamma \\ \check{l}_{1}^{M} &= 2(1 - \gamma) \\ \check{R}_{1} &= \frac{1 - \gamma - \mu}{(1 - \gamma)(1 - \mu)} \\ \check{R}_{2} &= \frac{2 - \gamma \left((1 - \gamma + \mu) / (1 - \gamma - \mu)\right)(\tau^{\mu} - 1)}{2 + \gamma (\tau^{\mu} - 1)} \frac{1 - \gamma - \mu}{(1 - \gamma)(1 - \mu)} \\ \check{w}_{1} &= \frac{\mu}{(1 - \gamma)(1 - \mu)} \\ \check{w}_{2} &= \frac{2 + (\gamma + 2(1 - \gamma) / \mu)(\tau^{\mu} - 1)}{2 + \gamma (\tau^{\mu} - 1)} \frac{\mu}{(1 - \gamma)(1 - \mu)} \\ \check{w}_{2}^{M} &= \left(\frac{(2 - \gamma)\tau^{1 - \sigma} + \gamma\tau^{\mu + \sigma - 1}}{2 + \gamma (\tau^{\mu} - 1)}\right)^{1/\sigma} \frac{\mu}{(1 - \gamma)(1 - \mu)} \\ \check{y}_{1} &= \frac{2}{2 + \gamma (\tau^{\mu} - 1)} \frac{1}{1 - \mu} \\ \check{y}_{2} &= \frac{2\tau^{\mu}}{2 + \gamma (\tau^{\mu} - 1)} \frac{1}{1 - \mu} \end{split}$$
B.4 Limitational case

$$\begin{split} \check{y}_{2}^{M} &= \frac{2}{2 + \gamma \left(\tau^{\mu} - 1\right)} \frac{1}{1 - \mu} \\ & \cdot \left[ 1 - \mu \frac{2 + \gamma \left(\tau^{\mu} - 1\right)}{2 \left(1 - \gamma\right)} \left( 1 - \left(\frac{\left(2 - \gamma\right)\tau^{1 - \sigma} + \gamma \tau^{\mu + \sigma - 1}}{2 + \gamma \left(\tau^{\mu} - 1\right)}\right)^{1/\sigma} \right) \right] \\ \check{Y}_{1} &= \frac{2 - \gamma}{2 + \gamma \left(\tau^{\mu} - 1\right)} \frac{2}{1 - \mu} \\ \check{Y}_{2} &= \frac{\gamma \tau^{\mu}}{2 + \gamma \left(\tau^{\mu} - 1\right)} \frac{2}{1 - \mu} \\ \check{P}_{1}^{M} &= \frac{2}{1 - \mu} \left(\frac{\mu}{2 \left(1 - \gamma\right)}\right)^{\frac{\sigma}{\sigma - 1}} \\ \check{P}_{2}^{M} &= \frac{2\tau}{1 - \mu} \left(\frac{\mu}{2 \left(1 - \gamma\right)}\right)^{\frac{\sigma}{\sigma - 1}} . \end{split}$$

In C2, i.e., if  $\gamma/2 + \mu \leq 1$  and  $\tau \in [\tau_R, \tau_\lambda]$  (9.10), or  $\tau \in [1, \tau_\lambda]$  if also  $\gamma + \mu \geq 1$ , they are

$$\begin{split} \check{\lambda}_{1} &= \left( 2\mu \frac{2 + (2 - \gamma)(\tau^{\mu} - 1)}{2 + (2\mu - \gamma)(\tau^{\mu} - 1)} + \gamma \right) \frac{1}{2} \\ \check{\lambda}_{2} &= \frac{4(1 - \mu) - \gamma(2 + (2 - \gamma)(\tau^{\mu} - 1))}{2 + (2\mu - \gamma)(\tau^{\mu} - 1)} \frac{1}{2} \\ \check{l}_{1}^{A} &= \gamma \\ \check{l}_{2}^{A} &= \frac{4(1 - \mu) - \gamma(2 + (2 - \gamma)(\tau^{\mu} - 1))}{2 + (2\mu - \gamma)(\tau^{\mu} - 1)} \\ \check{l}_{1}^{M} &= 2\mu \frac{2 + (2 - \gamma)(\tau^{\mu} - 1)}{2 + (2\mu - \gamma)(\tau^{\mu} - 1)} \\ \check{R}_{1} &= \frac{2(\tau^{\mu} - 1)}{2\tau^{\mu} - \gamma(\tau^{\mu} - 1)} \frac{1}{\gamma} \\ \check{R}_{2} &= 0 \\ \check{w}_{1} &= \frac{2 - \gamma(\tau^{\mu} - 1)}{2\tau^{\mu} - \gamma(\tau^{\mu} - 1)} \frac{1}{\gamma} \\ \check{w}_{2}^{M} &= \left[ \frac{(2 - \gamma(\tau^{\mu} - 1))^{\sigma - 1}}{(2\tau^{\mu} - \gamma(\tau^{\mu} - 1))^{\sigma - 1}} \left( \tau^{1 - \sigma} \left( 2(2\mu + \gamma) + \left( 4\mu - \gamma^{2} \right)(\tau^{\mu} - 1) \right) \right) \right. \right. \\ &+ \tau^{\mu + \sigma - 1} \left( 4(1 - \mu) - \gamma(2 + (2 - \gamma)(\tau^{\mu} - 1)) \right) \right) \right]^{1/\sigma} \frac{1}{\gamma} \\ \check{y}_{1} &= \frac{2}{2\tau^{\mu} - \gamma(\tau^{\mu} - 1)} \frac{1}{\gamma} \\ \check{y}_{2} &= \frac{2\tau^{\mu}}{2\tau^{\mu} - \gamma(\tau^{\mu} - 1)} \frac{1}{\gamma} \end{split}$$

$$\begin{split} \check{y}_{2}^{M} &= \frac{2}{2\tau^{\mu} - \gamma(\tau^{\mu} - 1)} \frac{1}{\gamma} \\ &\quad \cdot \frac{1}{2} \left[ \left[ \frac{(2 - \gamma(\tau^{\mu} - 1))^{\sigma - 1}}{2\tau^{\mu} - \gamma(\tau^{\mu} - 1)} \left( \tau^{1 - \sigma} \left( 2(2\mu + \gamma) + \left( 4\mu - \gamma^{2} \right)(\tau^{\mu} - 1) \right) \right) \right. \\ &\quad + \tau^{\mu + \sigma - 1} \left( 4\left( 1 - \mu \right) - \gamma\left( 2 + \left( 2 - \gamma \right)(\tau^{\mu} - 1) \right) \right) \right]^{1/\sigma} + \gamma\left(\tau^{\mu} - 1\right) \right] \\ \check{Y}_{1} &= \frac{2(2\mu + \gamma) + \left( 4\mu - \gamma^{2} \right)(\tau^{\mu} - 1)}{2 + (2\mu - \gamma)(\tau^{\mu} - 1)} \frac{2}{2\tau^{\mu} - \gamma(\tau^{\mu} - 1)} \frac{1}{\gamma} \\ \check{Y}_{2} &= \frac{4(1 - \mu) - \gamma\left( 2 + \left( 2 - \gamma \right)(\tau^{\mu} - 1) \right)}{2 + (2\mu - \gamma)(\tau^{\mu} - 1)} \frac{2\tau^{\mu}}{2\tau^{\mu} - \gamma(\tau^{\mu} - 1)} \frac{1}{\gamma} \\ \check{P}_{1}^{M} &= \frac{2}{\gamma} \frac{2 - \gamma(\tau^{\mu} - 1)}{2 + (2\mu - \gamma)(\tau^{\mu} - 1)} \left( \frac{1}{2} \frac{2 + (2\mu - \gamma)(\tau^{\mu} - 1)}{2 + (2 - \gamma)(\tau^{\mu} - 1)} \right)^{\frac{\sigma}{\sigma - 1}} \\ \check{P}_{2}^{M} &= \frac{2\tau}{\gamma} \frac{2 - \gamma(\tau^{\mu} - 1)}{2 + (2\mu - \gamma)(\tau^{\mu} - 1)} \left( \frac{1}{2} \frac{2 + (2\mu - \gamma)(\tau^{\mu} - 1)}{2 + (2 - \gamma)(\tau^{\mu} - 1)} \right)^{\frac{\sigma}{\sigma - 1}} . \end{split}$$

In C3, i.e., if  $\gamma/2 + \mu \leq 1$  and  $\tau \in [\tau_{\lambda}, \infty)$  (9.10b), they are

$$\begin{split} \check{\lambda}_{1} &= 1 \\ \check{\lambda}_{2} &= 0 \\ \check{l}_{1}^{A} &= \gamma \\ \check{l}_{2}^{A} &= 0 \\ \check{l}_{1}^{M} &= 2 - \gamma \\ \check{R}_{1} &= \frac{2(1-\mu)-\gamma}{(2-\gamma)(1-\mu)} \\ \check{R}_{2} &= 0 \\ \check{W}_{1} &= \frac{\mu}{(2-\gamma)(1-\mu)} \\ \check{W}_{2} &= \frac{1}{\gamma} \\ \check{W}_{2}^{M} &= \frac{\mu\phi^{1/\sigma}}{(2-\gamma)(1-\mu)} \\ \check{Y}_{1} &= \frac{1}{2(1-\mu)} \\ \check{Y}_{2} &= \frac{4(1-\mu)-\gamma^{2}}{2\gamma(2-\gamma)(1-\mu)} \\ \check{Y}_{2}^{M} &= \frac{2-\gamma-2\mu(1-\phi^{1/\sigma})}{2(2-\gamma)(1-\mu)} \\ \check{Y}_{1} &= \frac{1}{1-\mu} \\ \check{Y}_{2} &= 0 \end{split}$$

B.4 Limitational case

$$\begin{split} \check{P}_1^M &= \frac{1}{1-\mu} \left(\frac{\mu}{2-\gamma}\right)^{\frac{\sigma}{\sigma-1}} \\ \check{P}_2^M &= \frac{\tau}{1-\mu} \left(\frac{\mu}{2-\gamma}\right)^{\frac{\sigma}{\sigma-1}} \,. \end{split}$$

In C4, i.e., if  $\gamma/2 + \mu \ge 1$ , they are

$$\begin{split} \breve{\lambda}_{1} &= 1 \\ \breve{\lambda}_{2} &= 0 \\ \breve{l}_{1}^{A} &= 2 (1 - \mu) \\ \breve{l}_{2}^{A} &= 0 \\ \breve{l}_{1}^{M} &= 2\mu \\ \breve{R}_{1} &= \breve{R}_{2} &= 0 \\ \breve{w}_{1} &= \breve{w}_{2} &= \breve{y}_{1} \\ \breve{w}_{2}^{M} &= \breve{y}_{2}^{M} &= \frac{1}{\gamma} \\ \breve{w}_{2}^{M} &= \breve{y}_{2}^{M} &= \frac{1}{\gamma} \phi^{1/\sigma} \\ \breve{w}_{2}^{M} &= \breve{y}_{2}^{M} &= \frac{1}{\gamma} \phi^{1/\sigma} \\ \breve{Y}_{1} &= \frac{2}{\gamma} \\ \breve{Y}_{2} &= 0 \\ \breve{P}_{1}^{M} &= \frac{1}{\gamma} 2^{\frac{1}{1 - \sigma}} \\ \breve{P}_{2}^{M} &= \frac{\tau}{\gamma} 2^{\frac{1}{1 - \sigma}} . \end{split}$$

It follows from (9.12) that

$$\frac{\mathrm{d}\theta}{\mathrm{d}\tau}\Big|_{\tau=1} = \begin{cases} \frac{\mu(2\sigma-1)}{\sigma} & \text{for } \gamma+\mu \le 1 \\ \frac{\mu(2\sigma-1)-(1-\gamma-\mu)(\sigma-1)}{\sigma} & \text{for } \gamma+\mu \ge 1 \land \gamma/2+\mu \le 1 \\ \mu+\frac{\sigma-1}{\sigma} & \text{for } \gamma/2+\mu \ge 1 \\ \end{cases}$$

which is positive.

## 9.2.3 B.4.1.3 Dispersion vs. concentration

It follows from (9.13) that

$$\frac{\mathrm{d}\chi}{\mathrm{d}\tau}\Big|_{\tau=1} = \begin{cases} \frac{\mu(1-\gamma)}{2} & \text{for } \gamma+\mu<1 & \text{D1 C1} \\ \mu\left(\mu\left(1+\frac{1-\mu}{\sigma-1}\right)-\frac{1-\gamma}{2}\right) & \text{for } \gamma+\mu\geq1\wedge\gamma/2+\mu<1 & \text{D2 C2} \\ \frac{\mu}{2} & \text{for } \gamma/2+\mu\geq1 & \text{D2 C4} \end{cases}$$

which is positive. Besides,

$$\lim_{\tau \to \infty} \chi = \begin{cases} \frac{1}{2} \left( \frac{2 - \gamma}{1 - \gamma} \right)^{\frac{\mu \sigma}{\sigma - 1}} & \text{for } \gamma + \mu \le 1 & \text{D1 C}_3 \\ \\ \frac{1}{2} \left( \frac{\gamma}{1 - \mu} \right)^{1 - \mu} \left( \frac{2 - \gamma}{\mu} \right)^{\frac{\mu \sigma}{\sigma - 1}} & \text{for } \gamma + \mu \ge 1 \land \gamma/2 + \mu \le 1 & \text{D2 C}_3 \\ \\ \frac{2^{\frac{\mu}{\sigma - 1}} > 1}{2^{\frac{\mu}{\sigma - 1}} > 1} & \text{for } \gamma/2 + \mu \ge 1 & \text{D2 C}_4 \end{cases}$$

which can be either less than one or greater than one if  $\gamma/2 + \mu \leq 1$ .

End of Appendices  $\Box$ 

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## Erklärung zum selbstständigen Verfassen der Arbeit

Ich erkläre hiermit, dass ich meine Doktorarbeit

"The Wider Impacts of Transport Infrastructure Investments: Agglomeration and Imperfect Competition in General Equilibrium"

selbstständig und ohne fremde Hilfe angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehnenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen nach den mir angegebenen Richtlinien zitiert habe.

Kiel, den 13. Juli 2018

Michael Holtkamp