

Brownian molecular rotors: Theoretical design principles and predicted realizations

Jan Boyke Schönborn,¹ Rainer Herges,² and Bernd Hartke^{1,a)}

¹*Institut für Physikalische Chemie, Christian-Albrechts-Universität, Olshausenstraße 40, 24098 Kiel, Germany*

²*Otto-Diels-Institut für Organische Chemie, Christian-Albrechts-Universität, Olshausenstraße 40, 24098 Kiel, Germany*

(Received 5 March 2009; accepted 13 May 2009; published online 18 June 2009)

We propose simple design concepts for molecular rotors driven by Brownian motion and external photochemical switching. Unidirectionality and efficiency of the motion is measured by explicit simulations. Two different molecular scaffolds are shown to yield viable molecular rotors when decorated with suitable substituents. © 2009 American Institute of Physics.

[DOI: [10.1063/1.3148223](https://doi.org/10.1063/1.3148223)]

I. INTRODUCTION

Currently there is widespread interest in artificial molecular motors. Some of them are set up to mimic natural molecular motors (i.e., proteins); others are designed as effectors in artificial nanomachines. The topic has been extensively reviewed several times recently,^{1–3} therefore we briefly mention only a few subjectively selected systems here.

Koumura *et al.*^{4,5} and Klok *et al.*⁶ have proposed several generations of unidirectional molecular rotors. They essentially rely upon *cis-trans* isomerizations of sterically crowded alkenes, by a succession of thermal and photochemical steps. For a suitable potential of rotation about the C–C double bond with several minima, thermal activation transports the system to the next minimum that is energetically downhill. The following step would have to be energetically uphill in the electronic ground state. Such a movement is achieved by lifting the system to an electronically excited state where a suitable potential gradient leads to further forward rotation. Therefore, from a molecular design point of view, these systems are difficult to build since one has to ensure a correct form of the effective rotational potential in not one but two electronic states (plus an efficient and selective excitability of the desired upper state). Actual rotation of the system was proven by NMR and circular dichroism spectra of the sequential stages on a time scale of seconds to minutes.^{3,4} For the latest design variant,⁶ a possible rotational frequency on the order of a few MHz is estimated. To actually prove unidirectional rotation at such a frequency may turn out to be difficult, in spite of new detection ideas such as embedding the rotor molecules into chiral liquid crystals.⁷

Bermudez *et al.*⁸ and Leigh *et al.*⁹ have focused on intramolecular motion of catenanes, where one or two small rings move relatively to a larger ring that contains several docking stations, again induced by a suitable sequence of thermochemical and photochemical steps, on a timescale of

minutes to hours. Unidirectional motion could be achieved with the two-ring variant.⁹ As a sideline of interest for the present work, the authors point out that their rotor also shows signs of slippage, i.e., occasional excursions into the opposite rotational direction, driven by the random Brownian background noise.

Kelly *et al.*^{10,11} have synthesized prototypes of chemically driven rotors. Via a suitable reaction sequence, a chemical bond between rotator and stator part is formed temporarily, which helps to funnel thermal motion around the single bond axis into unidirectional rotation. So far, however, only a 120° turn was realized as proof of principle.

Hoki *et al.*^{12,13} and Fujimura *et al.*¹⁴ have presented variations in another basic concept, namely, to “kick-start” a rotation by suitable IR- and UV-laser pulse sequences. First, a spatially compact ground state wavepacket is prepared, which is then pumped to an excited state where the rotation is started (or promoted further) by the action of a suitable potential gradient. The concept hinges upon prior preparation of a pure enantiomer, which is in principle also possible using UV-laser pulses.^{15,16} Clearly, once started, this rotation will not continue indefinitely but it will ultimately turn into residual random motion after dissipation of energy into the environment or into other intramolecular degrees of freedom. Keeping the rotation alive requires further input of energy, e.g., by repeating pump-dump UV-laser pulses.¹⁴

The rotor design approach advocated in the present work differs in several respects from the approaches mentioned above. Most importantly, Brownian motion of the heat bath typically has a detrimental effect on the action of the rotors cited above, whereas we aim at the design of “Brownian rotors” for which random interaction with the heat bath is incorporated into the working principle and could even be considered the driving force. In contrast to the rotors by Koumura *et al.*^{4,5} and Fujimura *et al.*,¹⁴ we have to design a rotational potential of suitable form only in one electronic state (the ground state); as will be shown in the main part of this article, this task is already a difficult one. Similar to the rotors mentioned above, we also utilize a switching process,

^{a)}Electronic mail: hartke@phc.uni-kiel.de.

but the switching molecular unit is not directly associated with the rotation axis, again lending more flexibility to the molecular design.

The general principle of Brownian motors goes back to Smoluchowski¹⁷ and Feynman *et al.*¹⁸ They have shown that it is not possible to convert random Brownian heat bath movements into directed motion that can perform work. This would violate the second law of thermodynamics, which cannot be circumvented by any isolated machine, even if it is “asymmetrically” designed to seemingly favor motion into one direction, as in the famous ratchet-and-pawl mechanism.¹⁸ Nevertheless, slight variations in such an asymmetric setup, in particular addition of an external stimulus (e.g., periodic heating and cooling), can lead to directed motion. Many variations in this basic concept and predictions of directed motor action depending on design and parameter choices have been covered extensively by theoretical physicists.¹⁹ Actual realizations, however, have been comparatively rare.

The aim of this work is the design of molecular Brownian rotors, i.e., molecular rotors that operate continuously, with Brownian propulsion, and with directionality imposed by a suitable combination of molecular design and external stimuli. Our molecules consist of two parts: a rotator on an axis (e.g., a single bond) and stator fixing the axis in space and providing connection to a surface. We then imagine using laser pulses to periodically trigger switching between two different conformations of the whole setup, e.g., *cis-trans* isomerization of an azobenzene moiety in the rotator part. This leads to periodic switches between two different forms for the potential in the electronic ground state, as seen by the rotator part upon rotation relative to the stator part (in the terminology of Ref. 19, this is an “on-off Brownian rotor”). As demonstrated below, if these two potentials have suitable forms, the undirected Brownian motion of the heat bath surrounding the molecular rotor is channeled into unidirectional rotation. Since the larger portion of this unidirectional motion is driven by a potential gradient, it can do work. This whole process does not violate the second law of thermodynamics since the directionality of the motion (and/or the work done) is ultimately linked to the external switching stimulus and its energy entry into the molecular rotor system.

For this basic concept, we first show several idealized generic potential forms which we guessed in advance or abstracted from the real rotational potentials obtained in our calculations. We then demonstrate that these generic potential forms do lead to unidirectional rotation. Based on these concepts, we finally design actual, synthesizable molecules that approximate those idealized potential forms. We calculate their effective rotation potentials and demonstrate that also these realistic rotor potentials induce unidirectional rotor action. Hence, these molecules constitute promising proposals for molecular Brownian rotors.

The remainder of this paper is organized as follows. In Sec. II we present the essential details of the methods used in this study: one-dimensional (1D) canonical molecular dynamics (MD) simulations and quantum chemical calculations to generate effective 1D rotation potentials. In Sec. III we

present different functional forms that can be shown to induce unidirectional molecular rotation for our setup. Two different ideas for molecular rotor scaffolds are presented in Sec. IV. In both cases, actual molecules based on these scaffolds are given, for which our methods predict a net rotation. The article is concluded in Sec. V by a general discussion and an outlook to future work.

II. METHODS

As our central tool, we perform (1D-MD) in the canonical ensemble for a given pair of rotational potentials. We use this procedure as proof of concept, i.e., to show that certain rotational potential forms indeed lead to unidirectional rotation (cf. Sec. III), but also as proof for the rotational propensity of the real molecules proposed in Sec. IV.

This 1D-MD consists of a standard velocity Verlet algorithm, with a time step of 0.1 fs, coupled to an Andersen thermostat.²⁰ The bath collision frequency of the thermostat is adjusted to a best compromise between minimizing the influence on the dynamics and minimizing deviations from the target temperature, typically resulting in collision frequencies of about 1/(1.2 ps). For production, total propagation times are between 50 and 100 ns; final estimates of rotational action are taken from averages over 100 such trajectories.

For realistic molecules, it would be desirable to show that their 1D rotational potential approximates one of the potential form prototypes recognized as rotorlike by the above 1D-MD, but then to demonstrate the actual rotor action in a less reductionistic simulation. This, however, is hardly possible for various reasons. Empirical force fields would make full-dimensional MD possible but simply are qualitatively wrong here since rotator-stator interaction is an intricate mix of steric hindrance, van der Waals interactions, π -stacking, hydrogen bonds and long-range Coulombic forces between partial charges. Recalibration of force fields is also not an option since by design the 1D rotational degree of freedom is strongly coupled to many other degrees of freedom, making the necessary potentials highly multidimensional. The interplay of many intramolecular interactions and the strong couplings also render an *a priori* guessed reduction in dimensionality infeasible. Full-dimensional *ab initio* MD would be theoretically ideal but is practically impossible since Brownian motors are by construction stochastic in the short time intervals that can be covered by *ab initio* MD; the directionality of the rotation becomes apparent only on the nanosecond timescale (cf. Sec. III).

We therefore resorted to the following strategy. For a proposed rotor molecule, we first performed a large series of quantum chemistry calculations according to a particular protocol, as described in the next paragraphs. From these calculations, we then extracted an *effective* 1D rotational potential that partially incorporates physically relevant excursions in the other degrees of freedom most strongly coupled to this 1D rotational motion. In this effective 1D rotational potential, we then performed the same 1D-MD rotational simulations described above, but with the correct moments of inertia for the specific molecule under study. This enabled

us to generate statistically relevant sets of trajectories on a timescale of 10–100 ns, for a set of several test molecules.

We tried several different protocols for extracting 1D rotational potentials from *ab initio* quantum chemistry calculations. For brevity, we describe only two of those for which we present results in the next section. The first one is a 50:50 average between a rigid scan (the desired rotational coordinate is incremented while all other coordinates are held fixed) and a fully relaxed scan (where all other coordinates are relaxed without constraints, at each point of the desired rotational coordinate). Clearly, a rigid scan corresponds to the assumption that the rotational motion is infinitely faster than all relaxational motions of the molecule perpendicular to it, and vice versa for the fully relaxed scan. According to our experience with direct *ab initio* MD for the present systems, neither of these assumptions is a good first approximation on its own. Therefore, we have used a 50:50 average of them. This avoids both unphysically close encounters of atoms and unrealistic molecular flexibility, but is obviously far too simple.

A better alternative is a protocol we dubbed *cooled MD scans*. It consists of incrementing the rotational angle, and at each increment performing first a very short sequence of geometry optimization steps and then a short sequence of MD steps. This particular combination of methods allows for performing a full rotation quickly (it may take a nanosecond when unforced, cf. next section, while forcing tends to give unchemical bond ruptures) while the optimization steps avoid spurious accumulation of energy in high-frequency modes and while MD steps ensure some physicality of the molecular motions.

For most of these protocols and for our target molecules, wave function-based *ab initio* methods with explicit electron correlation are too expensive. On the other hand (as already mentioned above), standard force fields and standard semi-empirical methods both turned out to be not even qualitatively correct. Hence, density functional theory (DFT) is the only viable alternative. Since nonbonded interactions are central to our systems, we have added the empirical dispersion correction (DFT-D) of Jurečka *et al.*,²¹ in conjunction with the BP86 functional, in the efficient parallel resolution of the identity (RI)-implementation in TURBOMOLE version 5.91.^{22,23} The standard triple zeta valence polarization (TZVP) bases of Schäfer *et al.*²⁴ are used unless mentioned otherwise. The nonstandard scan protocols discussed above were realized via shell scripts and additional helper programs of our own.

III. POTENTIAL FUNCTIONS

In this section we present several prototype potential forms that do lead to unidirectional rotation. Typically, conversion of unidirectional Brownian motion into directed motion is imagined to proceed via a saw-tooth potential. In a molecule that turns about an axis, the simplest way to mimic such a profile in the torsional potential appears to be the introduction of three substituents of different sizes close to this axis. This should result in the saw-toothlike potential depicted in Fig. 1. When switched to a suitable different

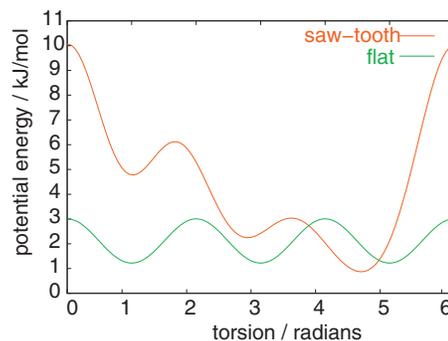


FIG. 1. (Color) Model potentials approximating a saw-tooth form (red) and a flat form (green), respectively, with a sum of three Gaussians. These are torsional potentials, shown for one full repetition period.

conformation that substantially diminishes the interaction between the rotator part and these three substituents, the torsional potential is expected to be much flatter (see also Fig. 1; it should be re-emphasized here that all of the potentials in this section are merely guessed, albeit with barriers of realistic height; they are not based on any calculations).

Periodic switching between these two conformations amounts to periodic switching between these two torsional potentials. In the calculations reported below, the saw-tooth potential was “on” for 900 fs, followed by the flat potential being in effect for 400 fs, and this cycle was repeated indefinitely. Using this setup in the simple 1D-MD with Andersen thermostat described in Sec. II, we obtained trajectories of random-walk character on short time scales, as shown in Fig. 2.

However, following up a representative set of 100 trajectories to much longer times reveals a systematic, directional drift on average, as shown in Fig. 3. The average behavior corresponds to 60 rotations per ns.

This proves that our basic concept does lead to unidirectional rotation (and it provides a first model prediction of its actual efficiency). We are currently performing optimization studies to arrive at optimal switching sequences and optimal potential forms. The results of these studies will be reported in the near future. The aim of the present contribution is to carry the connection to reality one step further by demonstrating that real molecules can be found with torsional potentials that also lead to directed rotation in these same tests. This provides a concrete basis for future realization of these concepts.

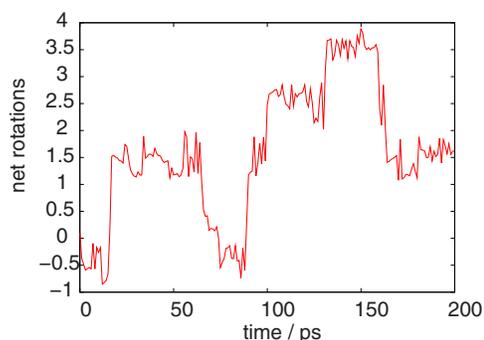


FIG. 2. (Color online) Short-time segment of a typical trajectory in the potentials of Fig. 1. One “net rotation” on the y-axis corresponds to the system covering the full width of the x-axis in Fig. 1 once.

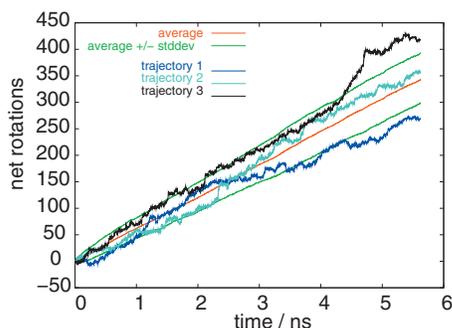


FIG. 3. (Color) Long-time behavior of 100 trajectories. Average overall trajectories (red), average plus/minus a standard deviation (green), and three randomly chosen actual trajectories.

In our study of possible rotor molecules, we discovered that the saw-tooth potential form is not a necessary requirement. Radically different forms also can lead to efficient rotor action. A prototype form of these is shown in Fig. 4; it is a maximally simplified version of a rotational potential we discovered during our calculations on realistic molecules (cf. Sec. IV).

Remarkably, in this case, there is no inherent asymmetry in any of the two potentials themselves. This should make these torsional potentials much easier design targets. The asymmetry that finally drives the directed motion is only manifested in the (small) offset between the centers of the two peaks. In tests similar to the above, this pair of potentials exhibits 8 net rotations per ns.

IV. MOLECULAR DESIGN

A. Tripod rotors

The simplest realization of the saw-tooth prototype has the rotator as one substituent of an sp^3 -hybridized carbon atom (with the single bond constituting the rotor axis). The three other substituents and the C-atom itself constitute the stator part. A proper choice of these three stator substituents (in the simplest case just three substituents of different size) combined with a proper design of the switchable rotor part should lead to a saw-toothlike rotation potential. Figure 5 shows a generic setup, with three substituents of different sizes, and with an azobenzene moiety in the rotator part. The azobenzene can be switched between its *cis* and *trans* forms with suitable laser pulses. This should lead to stronger or weaker interaction between the rotator and stator parts and

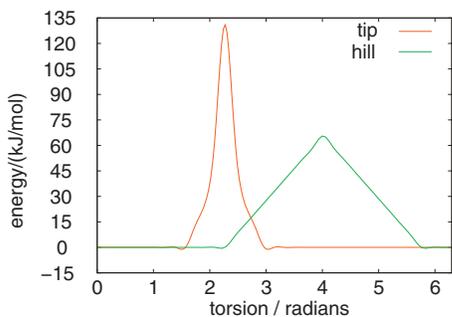


FIG. 4. (Color) The hill-tip paradigm. Two model potentials, each consisting only of a single symmetric cusp, but placed off-center relative to each other.

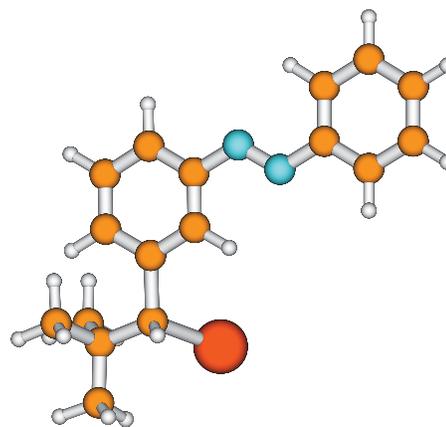


FIG. 5. (Color) Tripod rotor molecule in its simplest form.

hence could realize the potential forms depicted in Fig. 1. According to the tests reported in Sec. III we would then have a presumably working Brownian rotor.

However, first quantum chemical calculations of rotational potentials for rotors of this type revealed several defects of this simple setup. The switched rotator part (the azobenzene moiety) was too close to the stator part; for this reason, an acetylene group was put in between. Second, the stator part was too flexible; hence a bridge between two substituent positions was introduced, leading to norbornane derivatives. Last but not the least, contrary to initial expectations, the rotational potentials arise not primarily from steric hindrance, but rather a complicated interplay of additional interactions also adds in important contributions. Therefore, in most cases our calculated rotational potentials deviated rather strongly from what we expected, limiting the effectiveness of rational design and underlining the necessity to perform reliable quantum chemical calculations.

Nevertheless, after testing a variety of different substituent patterns for norbornane rotors (NRs), we were able to come up with successful variants. One of them is shown in Fig. 6. For both the *cis* and *trans* configuration of the azobenzene moiety, Fig. 7 shows the rotational potentials as 50:50 averages of a rigid and a fully relaxed scan in both cases.

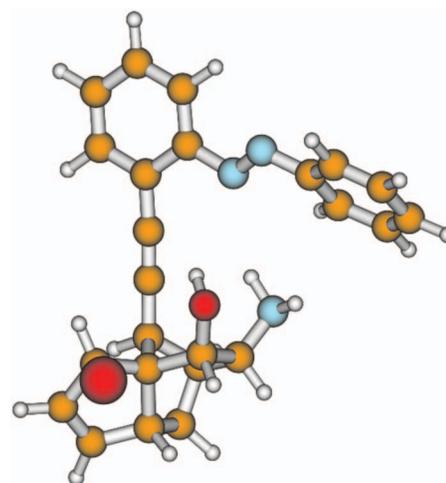


FIG. 6. (Color) Substituted NR.

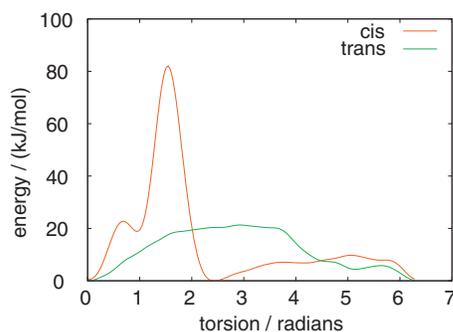


FIG. 7. (Color) Rotational potentials for the NR rotor of Fig. 6; 50:50 averages of a rigid and a fully relaxed scan, calculated with RI-DFT-D BP86/TZVP.

Obviously, the potentials of Fig. 7 correspond closely to the new hill-tip paradigm (Sec. III) rather than to the saw-tooth form. Using these potentials together with the corresponding moments of inertia in our 1D-MD tests indicates a preference for directed rotation, albeit with only 0.8 rotations per ns. Also, attempts to better take into account relaxations in the other degrees of freedom by using more refined scan modes (e.g., the cooled MD scan described in Sec. II) lead to different potentials, which in turn do not show a convincing rotation propensity anymore. Therefore, we also studied other rotor systems, as described in the next section.

B. Platform rotors

Here, the rotator part (again an azobenzene moiety) is placed on the central carbon atom of a triazatriangulenium ion “platform” as stator, with different substituents on the platform periphery. In the following, we refer to this design as TATA rotors. The platform offers the practical advantage of well-defined monolayer assembly on surfaces.²⁵ Also, attaching substituents of different sizes on the platform periphery makes it possible to tune the lateral distance between the switching molecules in the monolayer. In this way, close contacts that could hinder the photochemical switching and/or the rotor action can be avoided.

Again, several different substituent patterns were tried, without convincing success. Finally, we attempted to link two adjacent substituents by a ramplike bridge (cf. Fig. 8), to enforce a saw-toothlike potential pattern. However, cooled MD scans with DFT-D revealed that the actual torsional 1D-

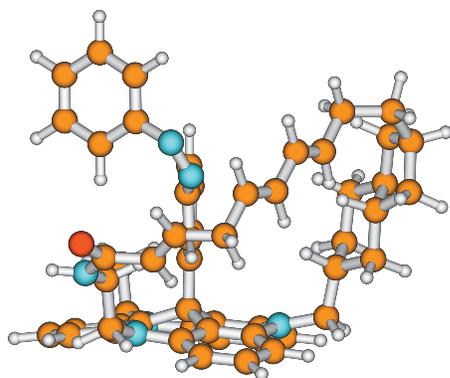


FIG. 8. (Color) TATA rotor with two substituent positions linked by a ramp.

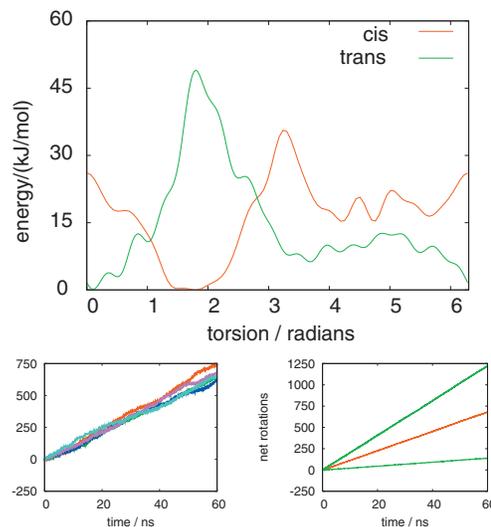


FIG. 9. (Color) Partially relaxed rotational potentials for the TATA rotor of Fig. 8 (top); a few representative trajectories (bottom left), and average trajectory as well as average \pm standard deviation (bottom right) for a set of 100 trajectories.

potentials are somewhere in-between a pure saw-tooth and our new hill-tip form (Fig. 9, top). Closer inspection of these DFT-D scans reveals that π -stacking between the rotator and stator parts leads to this marked deviation from the saw-tooth form. Our 1D-MD tests show that this rotor actually works rather efficiently, coming close to the efficiency of the artificial *ad hoc* potentials of Sec. III. Depending on the exact details of how we do the rotational potential scan, we obtained unidirectional rotation with 1.6–11.3 rotations per ns (Fig. 9, bottom).

V. SUMMARY AND OUTLOOK

In this article, we have reported on the basic idea of a single-molecule rotor, driven by stochastic Brownian motion of the surrounding heat bath but provided with directionality through external energy input in the form of periodic switches between two different conformers. We have shown that switching the rotational potential between the well-known asymmetric saw-tooth form and a (symmetric) flat form leads to directional rotation. Additionally, however, we have discovered that quite different potential forms are also effective for directional rotation, including cases where both potentials are symmetric, e.g., the hill-tip form. This is particularly interesting since molecules with such potentials should prove to be easier targets for molecular design and synthesis.

We have presented two basic molecular scaffolds that can be used to build up real molecular rotors: tripod rotors (with their norbornane derivatives), and TATA rotors. For each of these scaffolds, it is possible to arrange substituents around the rotational axis in such a way that the desired rotational potentials are achieved.

We have also shown that, however, actually arriving at the desired rotational potential in this way is less straightforward than one may have thought. By design, the rotational potential is the result a complicated superposition of many weak long-range force contributions of different parts of the

molecule. Hence, it is not possible to rely on just one contribution (e.g., steric hindrance) to dominate over all others, which would lead to a much more transparent design process. Additionally, the true rotational potential is a complicated effective potential which also contains deformations and excursions in many other degrees of freedom coupled strongly to the actual rotation. In this work, we have used several simple recipes to generate first approximations to such effective rotational potentials. They will be improved in future work.

With these preliminary recipes, we have found suitable substituent patterns for directed rotation, for two of our molecular scaffolds: for the NRs and for the TATA rotors. Following up on the theoretical design phase presented in this article, we will now undertake the actual synthesis of the proposed rotor molecules. This should be possible since all our setups were designed with synthesizability in mind.

The photochemical *cis-trans*-isomerization of azobenzene, which is required for our NR and TATA rotors, should be experimentally possible. Controlled reversible *cis-trans*-isomerization of azobenzene by laser pulses has been demonstrated in various settings, including surface-mounted cases^{26,27} as envisaged here for the TATA rotors. However, addressing a single molecule in a controlled fashion clearly is a challenge. Nonetheless, it should be pointed out that the present scheme is inherently rather robust in this respect, since the rotational motion is driven stochastically anyway (on the femtosecond-picosecond-timescale). Therefore, it does not matter much if a single rotor molecule does not see every pulse in the imagined pulse train that periodically switches between the two potentials (free/saw-tooth, or hill/tip). This is in contrast to, e.g., simple variants of coherent control where it is implicitly assumed that each molecule perfectly interacts with the whole, well-designed laser pulse train. Nevertheless, in ongoing work we are investigating how robust our rotors are when the switching is less than perfect overall and combined with a stochastic element.

Experimental proof of rotor action, however, still is an open problem. One could possibly use Förster resonance energy transfer (FRET) effects between suitably modified rotor and stator parts or between the rotor and its surrounding, and detect periodic oscillations in the FRET signal. Clearly even more difficult is an experimental proof of unidirectionality of the rotation on the nanosecond timescale (while being stochastic on the picosecond timescale). Probably it will be necessary to include a rotor molecule as motor in a nanomachine, i.e., to couple it to a mechanism that performs work which in turn can be detected on the macroscale. Such plans are the topic of future work in our laboratories.

ACKNOWLEDGMENTS

We would like to thank Petr Jurečka of the Hobza group and also Stefan Grimme for providing us with their DFT-D implementations. Part of the calculations were performed on the machines of the computer center of the Christian-Albrechts University Kiel. This work was inspired by collaboration between the authors within the special research area SFB 677 "Function by Switching."

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