Robust stability analysis of synchronverters operating in parallel

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Suggested Citation
Abstract—Recent studies have shown how synchronization units of converters operating nearby may interact with each other, affecting the stability of the system. Synchronverters are able to self-synchronize to the grid without the need of a dedicated unit because they can reproduce the power synchronization mechanism of synchronous machines (SMs). Recently, the robust stability of a synchronverter has been investigated by means of structured singular values (SSV or \( \mu \)-analysis). In this paper, \( \mu \)-analysis is performed to investigate how the robust stability of a synchronverter is affected by the presence of another converter of the same type operating in parallel. It is demonstrated that the parallel operation of synchronverters reduces their robust stability and a possible solution is proposed, based on the implementation of virtual impedances in the control algorithm. An accurate state-space model of the system under study is developed by adopting the component connection method (CCM) and the robust stability analysis is validated against time-domain simulations in MATLAB/Simulink/PLECS and experimental results with a power-hardware in the loop (PHIL) test bench.

Index Terms—Parallel operation of synchronverters, synchronverter robust stability analysis, \( \mu \)-analysis, virtual impedance, structured singular values.

I. INTRODUCTION

The growth of power electronics-based generation has raised concerns among system operators about power system stability. New control strategies, which guarantee stable operation of the system even in scenarios with high penetration of inverters are requested. Virtual synchronous machines (VSMs) have been proposed as possible solutions and their capability to self-synchronize themselves to the main utility grid without the need of a dedicated unit is appealing [1]-[6]. In fact, recent studies have shown that the synchronization unit of a grid connected converter has a significant impact on the frequency behaviour of the converter itself, with consequences for its stability [7], [8]. Furthermore, interaction between synchronization units of converters operating nearby may occur, especially under weak grid conditions [9]. The synchronverter presented in [2] is among the most popular VSM implementations. Its stability and dynamic performance have been investigated in the literature [10]-[13] and modifications to the original control structure have been proposed [6], [14].

In recent years, the impedance-based stability criterion has been extensively adopted for stability analysis of grid connected converters [15], [16]. This approach has been used in [17] for stability analysis of a synchronverter-based wind farm. However, synchronverter stability and design have been mainly studied by means of eigenvalues analysis in the literature, where linearized models of the system have been adopted for the investigation [10]-[13]. Nevertheless, due to the multi-input multi-output (MIMO) nature of the studied system, multivariable analysis is required in order to provide an estimation of the stability margin of the system [18], [19]. In fact, it is well known that eigenvalues are a poor measure of gain for MIMO systems, since they provide only information about a specific system configuration and do not take into account interactions between inputs and outputs of different channels [18]. Recently, the robust stability of a synchronverter was investigated by means of structured-singular values (SSVs) (commonly called \( \mu \)-analysis) [20]. This approach is particularly suitable for introducing plant uncertainties in the analysis. In fact, due to the significant variations of characteristics and number of generation units and loads during the day, the common representation of the grid by means of a Thévenin equivalent at the connection point with a resistive-inductive impedance is often not sufficiently accurate [21].

Due to the intrinsic droop characteristic of synchronverters regulating power sharing among operating units, it is often taken for granted that synchronverters should not encounter particular problems when operating in parallel. However, this aspect has not yet been properly investigated in the literature. In [22], a non-linear model of a synchronverter-dominated microgrid is presented and the stability of the system is studied by means of bifurcation theory. [23] analyzes the behaviour of synchronverters operating in parallel by means of eigenvalue analysis and a linearized state-space model of multiple synchronverters operating in parallel is developed by means of the so-called component connection method (CCM) [24], so as linear control theory can be adopted for system stability assessment. The CCM technique allows connecting state-space representations of separated subsystems in a modular manner, such that the analysis can be easily extended to multiple operating units. The objective of this work is the investigation of the robust stability of synchronverters operating in parallel by means of \( \mu \)-analysis. Similarly to [10], the system is split into two parts, namely the control and the plant, whose model is obtained adopting the approach presented in [23]. The presence of one or more units connected in parallel affects the characteristics of the plant where the synchronverter is connected, with consequent effects on system stability.

This paper is structured as follows: the system under study is introduced in Section II, along with the CCM technique adopted to obtain the state-space representation of the system. In Section III, the concept of \( \mu \)-analysis is briefly introduced and then applied to the investigated case. In Section IV, experimental results performed with a power-hardware in the
loop (PHIL) test bench are reported and the developed model is validated against time-domain simulations and measurement. Section V is dedicated to the conclusions.

II. MODELLING OF THE SYSTEM

A scheme of the system under study is shown in Fig. 1 (a). It is composed of two inverters with their respective controls and output filters, sharing the same point of common coupling (PCC). Similarly to [20], in order to perform the μ-analysis, the system is split into two parts, namely the control and the plant, whose inputs and outputs are shown in Fig. 1 (b). Differently from [20], the plant not only consists of the converter filter and the grid, but it also includes the control and the filter of the second synchronverter connected in parallel. Nevertheless, the small-signal model presented in [20] cannot be extended easily to the case of multiple synchronverters operating in parallel, since the state-space representation of the control and the plant, obtained through linearization of the respective equations, has been adapted to the specific case. The most efficient way for including other inverters in the analysis is to adopt the CCM technique [23], [24]. It consists of connecting state-space representations of several subsystems in a modular manner, such that the analysis can be extended easily to a more complex system. The state-space representation of the overall system is therefore obtained by connecting to each other independent subsystems by means of the so-called interconnection matrices. The approach presented in this paper is based on [23], where the CCM was used to obtain the linearised model of two synchronverters operating in parallel. However, differently from [23], here the CCM is used in order to obtain the model of the plant \( G \) indicated in Fig. 1 (b).

In Fig. 2, the block diagram representation of the overall system under study is shown, along with inputs and outputs of each subsystem. The dark red and dark blue arrows indicate inputs and outputs of the plant, labelled as \( u_{\text{plant}} \) and \( y_{\text{plant}} \), respectively. The state-space representation of the plant is given below:

\[
\begin{aligned}
\dot{x}_{\text{plant}} &= F_{\text{int}} x_{\text{plant}} + G_{\text{int}} u_{\text{plant}}, \\
y_{\text{plant}} &= H_{\text{int}} x_{\text{plant}} + J_{\text{int}} u_{\text{plant}},
\end{aligned}
\]

where \( F_{\text{int}}, G_{\text{int}}, H_{\text{int}} \) and \( J_{\text{int}} \) are defined as:

\[
\begin{aligned}
F_{\text{int}} &= A_d + B_d L_{11} (I - D_d L_{11})^{-1} C_d \\
G_{\text{int}} &= B_d L_{11} (I - D_d L_{11})^{-1} D_d L_{12} + B_d L_{12} \\
H_{\text{int}} &= B_d L_{21} (I - D_d L_{11})^{-1} C_d \\
J_{\text{int}} &= L_{21} (I - D_d L_{11})^{-1} D_d L_{12} + L_{22}
\end{aligned}
\]

\( A_d, B_d, C_d \) and \( D_d \) are sparse block diagonal matrices obtained from the state-space matrices of the single subsystems, namely:

\[
\begin{aligned}
A_d &= \text{diag} \{ A_{L1}, A_{L2}, A_{C2}, A_{PQ} \} \\
B_d &= \text{diag} \{ B_{L1}, B_{L2}, B_{C2}, B_{PQ} \} \\
C_d &= \text{diag} \{ C_{L1}, C_{L2}, C_{PQ} \} \\
D_d &= \text{diag} \{ D_{L1}, D_{L2}, D_{PQ} \}
\end{aligned}
\]

whereas \( L_{11}, L_{12}, L_{21} \) and \( L_{22} \) represent the aforementioned interconnection matrices, indicating how inputs and outputs of the sub-systems are connected to each other. Defining \( u_{\text{sub}} = [u_{L1} u_{L2} u_{C2} u_{PQ}]^T \) as the vector containing all the inputs of the subsystems and \( y_{\text{sub}} = [y_{L1} y_{L2} y_{PQ}]^T \) as the vector containing all the outputs, the following relation is valid:

\[
\begin{aligned}
u_{\text{plant}} &= L_{11} y_{\text{sub}} + L_{12} u_{\text{plant}}, \\
y_{\text{plant}} &= L_{21} y_{\text{sub}} + L_{22} u_{\text{plant}}
\end{aligned}
\]

A. Control

The description of the synchronverter control, along with its state-space representation are reported in [10]. The state-space matrices \( A_c, B_c, C_c \) and \( D_c \) of the control block in Fig. 2 are shown below:

\[
\begin{aligned}
A_c &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & -D_p & 0 \\ 0 & 1 & 0 \end{bmatrix}; \\
B_c &= \begin{bmatrix} 0 & 0 & \frac{1}{K} & -\frac{1}{K} - D_p \\ \frac{1}{\omega_0^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \\
C_c &= \begin{bmatrix} \omega_0 & M_j i_{f0} & 0 \\ 0 & 0 & 0 \end{bmatrix}; \\
D_c &= [0^{2 \times 5}].
\end{aligned}
\]

\( D_p, D_q, K \) and \( J \) are the control parameters shown in the control block diagram in Fig 1 (a), whereas \( \omega_0 \) and \( M_j i_{f0} \) are initial conditions of the system states [10].

B. LCL Filter

Choosing the current of the converter side inductor \( i_{L1} \), the current of the grid side inductor \( i_{L2} \) and the capacitor voltage
$v_c$ as state space variables of the subsystem, the equations of the LCL filter shown in Fig. 1 are reported below [10]:

$$\begin{align*}
L_f \frac{di_f}{dt} &= e - v_c - R_c(i_{L1} - i_{L2}) - R_{f1}i_{L1} \\
C_1 \frac{dv_c}{dt} &= i_{L1} - i_{L2} \\
L_f \frac{di_{L2}}{dt} &= v_c - v_{PCC} + R_c(i_{L1} - i_{L2}) - R_{f2}i_{L2}
\end{align*}$$

Writing (6) in $dq$ coordinates, the following state-space matrices $A_{LCL}$, $B_{LCL}$, $C_{LCL}$ and $D_{LCL}$ are obtained:

$$A_{LCL} = \begin{bmatrix}
-\frac{R_{f1} - R_c}{L_f} & -\frac{1}{L_f} & 0 & \frac{R_c}{L_f} & 0 \\
0 & -\frac{R_{f1} - R_c}{L_f} & 0 & \frac{R_c}{L_f} & 0 \\
0 & 0 & -\frac{1}{L_f} & 0 & 0 \\
0 & \frac{1}{L_f} & 0 & 0 & \frac{R_c}{L_f} \\
0 & 0 & \frac{1}{L_f} & 0 & 0
\end{bmatrix}$$

$$B_{LCL} = \begin{bmatrix}
\frac{\cos \omega_0}{L_f} - \frac{V_0 \sin \omega_0}{L_f} & 0 & 0 & 0 \\
\frac{-\sin \omega_0}{L_f} - \frac{V_0 \cos \omega_0}{L_f} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}$$

$$C_{LCL} = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \end{bmatrix} ; \quad D_{LCL} = [0^{2 \times 4}].$$

C. Grid

The grid has been modelled using its Thévenin equivalent at the PCC. In order to obtain the information about the voltage at the connection point, another state has been introduced in the grid model, namely a shunt capacitor at the input terminal as shown in Fig. 3. Choosing very high values for the shunt components $R_g$ and $C_g$, so that all the current $i_g$ flows into the grid, the PCC voltage is represented by the voltage $v_{cs}$ across the shunt capacitor. The corresponding grid equations are provided below:

$$\begin{align*}
L_g \frac{dv_{cs}}{dt} &= R_g i_g + v_{cs} + v_g - R_{Lg}i_L \\
C_g \frac{dv_{cs}}{dt} &= i_g - i_L
\end{align*}$$

which transformed in $dq$ coordinates result in the following state-space matrices of the grid subsystem:

$$A_g = \begin{bmatrix}
-\frac{R_{Lg} - R_g}{L_g} & -\frac{R_g}{L_g} & 0 & \frac{1}{L_g} \\
0 & -\frac{R_{Lg} - R_g}{L_g} & 0 & \frac{1}{L_g} \\
0 & 0 & -\frac{1}{C_g} & -\frac{\omega_0}{C_g} \\
0 & 0 & 0 & -\frac{1}{C_g}
\end{bmatrix} ; \quad B_g = \begin{bmatrix}
\frac{R_{Lg}}{L_g} & 0 & 0 & -1 \\
0 & \frac{R_g}{L_g} & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{1}{C_g} & 0 & 0 & 0
\end{bmatrix} ; \quad C_g = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \end{bmatrix} ; \quad D_g = [0^{2 \times 3}].$$

D. $P, Q$ and $V_{PCC}$ calculation

Calculation of active and reactive power of each converter, along with the amplitude of the PCC voltage involve algebraic equations. In order to obtain a state-space formulation out of these calculations, a first-order low pass filter with a very high cut-off frequency $\omega_c$ is introduced in the calculation of $\Delta P$, $\Delta Q$ and $\Delta V_{PCC}$ reported in [10]:
\[
\begin{align*}
\Delta P &= (\omega_c^2) \frac{1}{2} \left( I_{L1f2d0} \Delta V_{PCCd} + I_{L2f2q0} \Delta V_{PCCq} + V_{PCCd0} \Delta I_{L1f2d} + V_{PCCq0} \Delta I_{L2f2q} \right), \\
\Delta Q &= (\omega_c^2) \frac{1}{2} \left( -I_{L2f2d0} \Delta V_{PCCd} + I_{L1f2q0} \Delta V_{PCCq} + V_{PCCq0} \Delta I_{L2f2d} - V_{PCCd0} \Delta I_{L1f2q} \right), \\
\Delta V_{PCC} &= (\omega_c^2) \frac{V_{PCCd0} \Delta V_{PCCd} + V_{PCCq0} \Delta V_{PCCq}}{\sqrt{V_{PCCd0}^2 + V_{PCCq0}^2}}.
\end{align*}
\]

(10)

where the quantities with the subscript "0" indicate the values at the operating point and \( I_{L1f2dq} = \{ I_{L1f2d0}, I_{L1f2q0} \} \).

The corresponding state-space matrices are provided below:

\[
A_{PQ} = \omega_c \begin{bmatrix} f(5) \end{bmatrix}; \quad C_{PQ} = \begin{bmatrix} f(5) \end{bmatrix}; \quad D_{PQ} = \begin{bmatrix} 0^{5 \times 6} \end{bmatrix}
\]

\[
B_{PQ} = \omega_c \frac{3}{2} \begin{bmatrix}
I_{L1f2d0} & I_{L1f2q0} & V_{d0} & V_{q0} & 0 & 0 \\
-I_{L2f2d0} & I_{L2f2q0} & 0 & V_{q0} & -V_{d0} & 0 \\
I_{L1f2d1} & I_{L1f2q0} & 0 & 0 & V_{d0} & V_{q0} \\
-I_{L2f2d0} & I_{L2f2q0} & 0 & 0 & 0 & -V_{d0} \\
\end{bmatrix};
\]

(11)

where \( V_{d0} = V_{PCCd0} \) and \( \Delta = \sqrt{V_{PCCd0}^2 + V_{PCCq0}^2} \).

E. Interconnection matrices

\( L_{11}, L_{12}, L_{21}, \) and \( L_{22} \) in (4) are sparse matrices of zeros and ones accounting for the connection among inputs and outputs of the subsystems composing the overall interconnected system. These are reported in the Appendix of the paper and below only their sizes are shown:

\[
\begin{align*}
\begin{bmatrix} u_{\text{sub}} & y_{\text{sub}} \end{bmatrix} &= \begin{bmatrix} L_{11} & 22 \times 13 \end{bmatrix} + \begin{bmatrix} L_{12} & 22 \times 2 \end{bmatrix} u_{\text{plant}}, \\
\begin{bmatrix} y_{\text{plant}} & y_{\text{plant}} \end{bmatrix} &= \begin{bmatrix} L_{21} & 3 \times 13 \end{bmatrix} + \begin{bmatrix} L_{22} & 3 \times 2 \end{bmatrix} u_{\text{plant}}.
\end{align*}
\]

(12)

Although already defined at the beginning of this section, \( u_{\text{sub}}, u_{\text{plant}}, y_{\text{sub}} \) and \( y_{\text{plant}} \) are reported below for simplicity, where the notations of Fig. 2 for inputs and outputs of each subsystem are adopted:

\[
\begin{align*}
u_{\text{sub}} &= \begin{bmatrix} u_{c2} & u_{LCL1} & u_{LCL2} & u_{g} & u_{PQ} \end{bmatrix}^T, \\
y_{\text{sub}} &= \begin{bmatrix} y_{c2} & y_{LCL1} & y_{LCL2} & y_{g} & y_{PQ} \end{bmatrix}^T, \\
u_{\text{plant}} &= \begin{bmatrix} \Delta E_{p1} & \Delta Q_{1} & \Delta V_{PCC} \end{bmatrix}^T, \\
y_{\text{plant}} &= \begin{bmatrix} \Delta E_{p2} & \Delta Q_{1} & \Delta V_{PCC} \end{bmatrix}^T.
\end{align*}
\]

(13)

III. ROBUST STABILITY ANALYSIS

A. Introduction to \( \mu \)-analysis

Eigenvalue analysis is among the commonly adopted approaches for stability assessment of grid connected converters and generally for power systems studies [25]. By means of modal analysis, the states mainly contributing to the oscillatory modes of a large interconnected system can be determined [25], [27]. For instance, this approach has been adopted in [23] in order to investigate parallel operation of synchronverters. Although through eigenvalue analysis system stability can be efficiently assessed, it is well known that for MIMO systems, eigenvalues are a poor measure of robustness [18]. Indeed, eigenvalues provide only information about a specific configuration of the system when inputs and outputs are in the same direction, namely the direction of the eigenvectors. They do not take into account the possible interactions between different channels that typically occur in a MIMO system. For SISO analysis, gain and phase margin are identified as indicators of robustness. However, the definition of phase and gain margin of SISO systems cannot be easily extended to MIMO systems. For this reason, for accurate robustness assessment of MIMO systems, multivariable analysis is required and different methods have been developed, such as the structured singular values (SSV) or \( \mu \)-analysis [18]-[20]. To this extent, singular values provide better information about the gains of the plants and the robustness of the control is verified against a set of defined system uncertainties. This method allows to span a set of possible system configurations instead of verifying stability only for a specific condition.

In [20], the robust stability of a synchronverter connected to the grid through an LCL filter has been investigated by means of \( \mu \)-analysis. The calculated \( \mu \)-factor provides an indication of system stability, namely the inverse of the highest peak of \( \mu \) over the investigated frequency range represents the stability margin of the system [18]-[20]. The results of the performed analysis are strongly related to the chosen plant uncertainty and, in [20], a multiplicative input uncertainty with specific characteristics has been defined. Among the results presented in [20], the fact that the robust stability of the synchronverter is augmented when the converter is connected to a weaker grid is probably the most interesting one. In fact, this represents an opposite trend compared to the behaviour shown by converters relying on dedicated synchronization units [9]. In the following, the \( \mu \)-analysis performed in [20] is extended to the case of two synchronverters operating in parallel. The presence of the second synchronverter modifies the characteristics of the plant with consequences on the results obtained in [20].

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverter rated power</td>
<td>( S_{in} = S_{p} )</td>
<td>330 kVA</td>
</tr>
<tr>
<td>Line-to-line voltage</td>
<td>( V_{LL} )</td>
<td>400 V (rms)</td>
</tr>
<tr>
<td>Rated grid frequency</td>
<td>( f_{g} )</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Grid inductance</td>
<td>( L_{g} )</td>
<td>0.05 pu</td>
</tr>
<tr>
<td>Inverter-side filter inductance</td>
<td>( L_{f1} = L_{f2} )</td>
<td>0.08 pu</td>
</tr>
<tr>
<td>Grid-side filter inductance</td>
<td>( L_{f3} = L_{f2} )</td>
<td>0.02 pu</td>
</tr>
<tr>
<td>Grid resistance</td>
<td>( R_{g} )</td>
<td>0.0005 pu</td>
</tr>
<tr>
<td>Inverter-side filter resistance</td>
<td>( R_{f1} = R_{f2} )</td>
<td>0.02 pu</td>
</tr>
<tr>
<td>Grid-side filter resistance</td>
<td>( R_{f3} = R_{f2} )</td>
<td>0.02 pu</td>
</tr>
<tr>
<td>Capacitor damping resistance</td>
<td>( R_{c} )</td>
<td>0.18 pu</td>
</tr>
<tr>
<td>Filter capacitor</td>
<td>( C_{f} )</td>
<td>0.05 pu</td>
</tr>
<tr>
<td>Virtual inertia</td>
<td>( J_{l} = J_{g} )</td>
<td>0.6687</td>
</tr>
<tr>
<td>K factor</td>
<td>( K_{f} = K_{g} )</td>
<td>37459</td>
</tr>
<tr>
<td>P-Droop coefficient</td>
<td>( D_{p1} = D_{p2} )</td>
<td>60.8 (5%)</td>
</tr>
<tr>
<td>Q-Droop coefficient</td>
<td>( D_{q1} = D_{q2} )</td>
<td>18371 (5%)</td>
</tr>
</tbody>
</table>
B. Application to the system under study

The same operating conditions of the converter considered in [20] are assumed for the analysis. Filter and control parameters of the two synchronverters are reported in Table I, whereas the considered frequency dependent uncertainty function is shown in Fig. 4. It presents an amplitude of 50% at low frequency, increasing till 500% at very high frequencies. This accounts for low frequency uncertainties due to parametric uncertainty in the model as well as high frequency neglected dynamic effects or resonances in the grid due to the presence of other converters operating nearby.

![High-frequency uncertainty](image)

**Figure 4.** Multiplicative input uncertainty used for the μ-analysis.

In Fig. 5, the μ-factor over the investigated frequency range related to the defined set of plant uncertainties is shown. The blue curve indicates the results when the converter operates alone, while the red curve correspond to the parallel operation. The results clearly show how the highest peak of μ of approximately 0.61 at a frequency of 277 rad/s is shifted to a value of μ ≈ 0.65 at the same frequency for parallel operation, corresponding to a reduction of the stability margin.

In Fig. 6 (a) and (b), the μ-factor for a sweep of the grid SCR from 20 to 2 is shown for both examined cases. When the synchronverter operates alone, the increase of the

![μ factor - Sweep SCR one Synchronverter](image)

![μ factor - Sweep SCR two Synchronverters](image)

**Figure 6.** μ factor, sweep SCR from 20 to 2; (a) one synchronverter, (b) two synchronverters.

impedance between the converter and the grid enhances the robust stability of the converter against high frequency uncertainties in the plant. This has been justified in [20] by the fact that a synchronverter reproduces the behaviour of a real SM and basically behaves as a voltage source behind an impedance. The increase of the impedance between the voltage source and the grid produces the effects of a stability margin improvement [6]. Fig. 6 (b) clearly shows how the increase of the grid impedance does not produce the same effects when the synchronverter is operating in parallel with a second synchronverter. In fact, the other synchronverter nearby also behaves as a grid forming unit [28], reducing the benefits provided by the higher impedance between the converter and the grid.

C. Possible countermeasure

The analysis performed in the previous section has highlighted that the stability margin of a synchronverter is reduced when operating in parallel to a converter of the same type. The issue related to parallel operation is not only confined to synchronverters, but this is a general challenge for power electronics-based converters. Interactions among current controls and filters have been extensively investigated in the literature [15], [26], [27], along with the negative effects of synchronization units on the converter stability which has been addressed in several works [7], [8], [16]. Since synchronverters do not require a dedicated unit for their synchronization to the grid, the issues related to interactions among synchronization units of converters operating in parallel observed in [9] and [29] can be avoided. Although the control loops of a synchronverter are different from those of standard grid connected converters using dedicated synchronization units, the results of the analysis performed in this work show that the parallel operation among these types of converters has also effects on...
their stability. According to the considerations reported in the previous subsections and in [20], the impedance between the synchronverter and any other equivalent voltage source operating nearby has to be increased in order to enhance its robust stability. This can be interpreted as the need for an increase of the electrical distance between the two voltage sources. It is however important to point out that the synchronverter is not the only control structure proposed in the literature based on the power synchronization mechanism of a SM. In [30], a generalized formulation for power synchronization-based control algorithms has been presented and the results shown in this paper can be extended to such generalized structure.

Typical values of stator impedance of real SMs are generally much higher than the output impedance of standard converter filters. Normally, synchronous reactances of SMs are in the range of 1.5 to 2 pu [25], whereas the output impedance of a converter filter is generally significantly lower (in the range of 0.1-0.2 pu) [6]. Hence, the effects of an increase of filter parameters on the the µ-factor are reported in Fig. 7. The blue and red curves already shown in Fig. 5 are compared to the yellow curve accounting for the results of the µ-analysis obtained when the converter filter impedances of both synchronverters have been increased by 20%. The results show clearly that the maximum peak of the µ-factor is reduced to a value of µ ≈ 0.53, corresponding to an increase of the stability margin. Fig. 8 shows the variation of the µ-factor for a sweep of the grid SCR from 20 to 2 when the two converters are connected in parallel and whose filter parameters, namely \( L_{f1}, L_{f2}, R_{f1}, \) and \( R_{f2}, \) have been increased of 20%. Effects of increased filter impedance on the dynamic performance of the control are shown in the section dedicated to the experimental results.

Due to the fact that converter filters are typically designed so as to optimize the trade-off between power quality and size of the filter components, a simple and efficient way for increasing the output impedance of the converter without necessarily oversizing the hardware components is represented by the emulation of a virtual impedance through the control. This represents a well-established technique in micro-grid applications and several possible solutions can be found in the literature [31]-[35]. Different techniques have been proposed for a variety of purposes, e.g. proper sharing of active and reactive power among units [32], current limitations [33] or harmonics suppression [34], [35]. Assuming that the filter impedance can be arbitrarily modified, for example by means of a virtual impedance implementation technique, in the next section, an indication about optimal parameter choice for design purposes is provided investigating the effects of converter output filter parameter variations on the µ-factor.

### D. Tuning of the virtual impedance

The curves shown in Fig. 9 have been obtained varying the magnitude of the resistive and inductive components of the filter parameters shown in Table I. The blue line represents the µ-factor for the case when the two synchronverters are simply connected in parallel and their respective filters have
the nominal values shown in Table I, which coincides with the red curve of Fig. 5 and Fig. 7. Indicating with \( L_f \) the sum of \( L_{f1} \) and \( L_{f2} \), the red curve of Fig. 9 shows the \( \mu \)-factor for an increase of \( L_f \) of 100%. Although this parameter choice causes a reduction of the amplitude of the curve in almost the whole investigated frequency range, a resonance peak around the fundamental frequency can be observed. On the contrary, the same magnitude of variation applied to the filter resistance \( R_f = R_{f1} + R_{f2} \) provides significant damping to the aforementioned resonance peak, whereas it produces an increase of the \( \mu \)-factor in the range \( \omega = [10; 100] \) rad/s, as indicated by the yellow line in the figure. Finally, the violet curve represents the case when the filter parameters \( L_{f1}, L_{f2}, R_{f1}, \) and \( R_{f2} \) are increased of 50%. Compared to the blue curve, this parameter choice ensures a reduction of the \( \mu \)-factor in the whole investigated frequency range and provides satisfactory damping around the fundamental frequency. As it will be shown in the next section, the increase of filter impedance causes a detriment of the dynamic response of the system. Fig. 9 shows how the increase of the filter impedance enhances significantly the robust stability of the synchronverter for parallel operation with other converters of the same type. According to these considerations, it is recommended to choose the parameters of the virtual impedance such as the corresponding total filter resistance \( R_f = [0.07; 0.15] \) \( pu \) and the total filter inductance \( L_f = [0.15; 0.25] \) \( pu \), in order to obtain a good compromise between robustness and dynamic performances.

IV. EXPERIMENTAL RESULTS

Experimental tests in a laboratory environment have been carried out so as to validate the results of the robust stability analysis performed in this paper. The laboratory set-up used for the tests is shown in Fig. 10 (a) and (b), where a schematic drawing and a picture of the laboratory environment are shown, respectively. Two converters Danfoss Series FC-302, 4 kW rated power, operating with a switching frequency of 10 kHz and equipped with output filters have been used. Each converter is additionally equipped with a transformer in order to provide galvanic isolation. The two converters are connected to a 4-quadrant linear power amplifier PAS 15000 from Spitzenerger-Spies (single phase rated power 15 kVA, total three phase rated power 45 kVA). The control algorithms of the two converters are implemented in a dSPACE control Desk DS1202 MicroLabBox, whereas a grid model is simulated in real-time by means of an RTDS simulator. The set-up enables reproducing the simulated voltages at the output terminals of the power amplifier instantaneously for easy testing of the converters under different grid conditions.

The developed linearized model of the two synchronverters connected in parallel is validated against electromagnetic transient (EMT) time-domain simulations in MATLAB/Simulink/PLECS (modelling the converter as a voltage-source) and against measurements performed with the described test set-up. Set-up parameters along with control parameters of the two synchronverters are reported in Table II. Results are shown in Fig. 11, where steps of 0.25 pu of active and reactive power setpoints of the synchronverter 1 indicated as \( P_{set1} \) and \( Q_{set1} \), respectively, are performed and the dynamic behaviour of active and reactive power output of the two converters \( P_1, P_2, Q_1 \) and \( Q_2 \) are observed. The results show a good match between simulations and measurements for quantities within the same converter, namely: \( \frac{\Delta P_1}{\Delta P_{set1}} \approx \frac{\Delta Q_1}{\Delta Q_{set1}} \) as well as for the cross-coupling effects between active and reactive power: \( \frac{\Delta P_2}{\Delta Q_{set1}} \approx \frac{\Delta Q_2}{\Delta P_{set1}} \). A satisfactory match between measurements and simulations can be also observed for the cross-coupling effects between the two converters: \( \frac{\Delta P_1}{\Delta P_{set2}}, \frac{\Delta P_2}{\Delta P_{set2}}, \frac{\Delta Q_1}{\Delta Q_{set2}}, \frac{\Delta Q_2}{\Delta Q_{set2}} \).

A. Virtual impedance implementation

In order to prove the results of the analysis performed in this paper, experimental tests are reported in the following,
showing the effects of the virtual impedance implementation on the robust stability of the converter. The virtual impedance implementation proposed in [6] is adopted in this work, whose principle is briefly explained below. It represents an intuitive and efficient solution for virtually increasing the filter impedance of the converter by a factor \( n \). According to the considerations about the virtual impedance implementation, Figure 12 (a), which shows the equivalent single phase electrical circuit of the VSM, one could imagine splitting the total impedance into a physical part, represented by the filter impedance, and a virtual additional component to be implemented in the control, whose amplitude is \( (n-1) \) times the magnitude of the filter impedance. Calculating the quantity \( e_a' \) from the equivalent scheme shown in Fig. 12(a):

\[
\Delta P_1 / \Delta P_{\text{set1}}
\]

\[
\Delta Q_1 / \Delta Q_{\text{set1}}
\]

\[
\Delta P_1 / \Delta P_{\text{set2}}
\]

\[
\Delta Q_1 / \Delta Q_{\text{set2}}
\]

Figure 11. Comparison experimental results (blue), EMT simulations (red), linearized model (yellow). (a) \( \Delta P_1 / \Delta P_{\text{set1}} \), (b) \( \Delta Q_1 / \Delta Q_{\text{set1}} \), (c) \( \Delta P_1 / \Delta P_{\text{set2}} \), (d) \( \Delta Q_1 / \Delta Q_{\text{set2}} \), (e) \( \Delta P_1 / \Delta P_{\text{set1}} \), (f) \( \Delta Q_1 / \Delta Q_{\text{set1}} \), (g) \( \Delta Q_1 / \Delta Q_{\text{set2}} \), (h) \( \Delta Q_1 / \Delta Q_{\text{set2}} \).

\[
\Delta P_1 / \Delta P_{\text{set2}}
\]

\[
\Delta Q_1 / \Delta Q_{\text{set2}}
\]

The effects of the additional impedance are reproduced by imposing the calculated \( e_a' \) for the generation of the converter pulses in Fig. 11. In Fig. 13, measurements results are compared to EMT simulations. According to the considerations about the optimal filter tuning presented in section III.D, the approach

where \( e_a' \) represents the phase \( a \) component of the synchron-
previously described has been implemented in the synchronverter control in order to increase the filter impedance of twice its nominal value. Measurements are compared to simulations, showing the reference behaviour of the converter when the filter impedance is physically increased by the corresponding factor. The step responses $\Delta P_{1/s}$, $\Delta Q_{1/s}$, $\Delta P_{2/s}$, and $\Delta Q_{2/s}$ are depicted, showing a good match between measurements and the reference dynamic behaviour.

![Comparison experimental results (blue), EMT simulations (red), linearized model (yellow). (a) $\Delta P_{1/s}$ without virtual impedance, (b) $\Delta P_{1/s}$ with virtual impedance, (c) $\Delta Q_{1/s}$ without virtual impedance, (d) $\Delta Q_{1/s}$ with virtual impedance.](image)

Figure 14. Comparison experimental results (blue), EMT simulations (red), linearized model (yellow). (a) $\Delta P_{1/s}$ without virtual impedance, (b) $\Delta P_{1/s}$ with virtual impedance, (c) $\Delta Q_{1/s}$ without virtual impedance, (d) $\Delta Q_{1/s}$ with virtual impedance.

In [10] and [20], the effects of the parameter $K$ on the dynamic performance and on the stability of the synchronverter have been shown. Although higher values of $K$ generally increase the damping of the reactive power loop, the robust stability analysis performed in [20] shows that an increase of this control parameter over a certain limit simply worsen the dynamic performance without improving robustness. When the synchronverter is operating alone, the lowest value of $K$ causing the instability of the converter is $K_{lim} \approx 50$. However, when the second synchronverter is connected in parallel, the critical $K$ causing instability increases to $K_{lim} \approx 60$. Fig. 13 (a) and (c) show the step responses of $\Delta P_{1/s}$ and $\Delta Q_{1/s}$ of the synchronverter 1 when operating in parallel to synchronverter 2, both having $K_1 = K_2 = 55$ and when the virtual impedance is not implemented in the control. Fig. 13 (b) and (d) show the behaviour of the converter when operating in parallel to the second converter, both having $K_1 = K_2 = 55$ and when the virtual impedance is instead implemented in the control of synchronverter 1. This demonstrates the benefit of the virtual impedance implementation on the converter stability, since the new critical $K$ causing the instability of both synchronverters for parallel operation is reduced to $K_{lim} \approx 20$ when it is implemented in both controls.

**V. CONCLUSION**

In this paper, the robust stability of two synchronverters operating in parallel has been investigated by means of $\mu$-analysis. In order to obtain the state-space representation of the system under study, the CCM method has been adopted. This enables easy modelling a complex system modularly, by splitting it into smaller interconnected subsystems. The results of the performed analysis show a decrease of the stability margin of the synchronverter when another operating unit of the same type is operating electrically close to it. In order to improve the stability margin of the converter, the virtual increase of the filter impedance has been considered and implemented. Time-domain simulations as well as experimental results in a laboratory environment with a PHIL test bench confirm the validity of the presented analysis.

**APPENDIX**

Interconnection matrices $L_{11}$, $L_{12}$, $L_{21}$ and $L_{22}$ of (12) are reported below:

$$L_{11} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$  \hspace{1cm} (16)

$$L_{12} = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$  \hspace{1cm} (17)

$$L_{21} = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$  \hspace{1cm} (18)

$$L_{22} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$  \hspace{1cm} (19)

**REFERENCES**


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