

Monetary Policy with Imperfect Information and Bounded Rationality

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List of Acronyms

AR	Autoregressive
B	Beta distribution
C	Chartist
CB	Central Bank
DSGE	Dynamic Stochastic General Equilibrium
E5Y	Michigan Consumer Sentiment Index about expected economic conditions in the next five years
F	Fundamentalist
Fed	Federal Reserve Bank
FOMC	Federal Open Market Committee
FRB/US	Federal Reserve Board model for the economy of the United States
FRED	Federal Reserve Economic Data
GDP	Gross Domestic Product
HPDI	Highest Posterior Density Interval
IG	Inverse Gamma distribution
IS	Investment-Saving
LTV	Loan-To-Value
N	Normal distribution
NKM	New Keynesian Macroeconomics
NKPC	New Keynesian Phillips Curve
OLS	Ordinary Least Squares
Std.	Standard Deviation
VAR	Vector Autoregression
w.r.t.	with respect to

Chapter 1

Introduction

This dissertation consists of two papers that deal with monetary policy in the presence of imperfect expectation formation. The first paper deals with optimal monetary policy under imperfect and asymmetric information and potential welfare gains from easing these information frictions by using consumer confidence measures. The second paper develops a DSGE model with bounded rationality and dynamic interactions between endogenous central bank credibility, credit cycles and aggregate macroeconomic risk, and analyzes nonstandard conventional and unconventional monetary policy responses.

A lot of central banks have embraced price stability as their primary aim and seek to achieve this by pursuing inflation targeting. The theory behind this strategy stresses the overarching importance of the expectations channel for transmission and, thus, effectiveness of monetary policy tools such as setting key interest rates and open market operations. Macroeconomic models designed for the analysis of monetary policy typically rely on a New Keynesian setup with perfect rationality and information within the expectation formation mechanism of agents.

In such a framework expectations are fully model-consistent and homogeneous. However, evidence from surveys and laboratory experiments suggests that this is an unrealistically strong assumption and indicates substantial heterogeneity of output and inflation forecasts as Hommes and Lustenhouwer (2019) point out. This is important for monetary policy makers: bounded rationality restrains the central banks capabilities of *managing expectations*, imperfect information implies benefits from considering alternative informative

measures such as survey data, and asymmetries in forecasting and information affect the effectiveness of monetary policy tools.

In general there are two approaches to extend monetary policy models by augmenting the standard assumption of model-consistent expectations. First, one may incrementally extend the baseline rational expectations framework with constraints to information acquisition and processing. Alternatively, one may forgo rational expectations completely and replace it with a mechanism that relies on bounded rationality such as heuristic switching setups based on discrete choice learning. This dissertation represents both of these approaches by containing an exemplary paper for each of them.

Chapter 2 develops a framework in which the rational expectations of agents are subject to imperfect and asymmetric information. The approach is based on Barsky and Sims (2012) who assume that households cannot observe technology growth directly and forecast it instead by processing a noisy signal. I augment this channel by explicitly considering the information set of the central bank and applying this setup for analysis of (optimal) monetary policy. Crucially, I implement an endogenous consumer confidence measure and investigate if the monetary policy can improve aggregate welfare by paying attention to it. The analysis consists of two parts. First, the framework is integrated into a stylized New Keynesian model for which optimal monetary policy solutions under both discretion and commitment with asymmetric information are provided. Second, I integrate the information frictions into a microfounded empirical DSGE model with a monetary policy rule that depends on the degree of attention the central bank pays to confidence, and estimate it using a Bayesian Moment Matching methodology based on a quasi-likelihood function that reflects the distance between empirical and model-generated moments.

The main findings are the following. In both model versions paying attention to confidence implies quantitatively substantial welfare gains. The results stress the relevance of consumer confidence measures that are collected by public institutions with immense effort, but typically have no role in standard theoretical monetary policy settings. In the baseline New Keynesian model, optimal processing of confidence leads to a reduction of welfare loss caused by the information frictions of 63.7%. In the empirical DSGE model, the welfare effects are somewhat lower where the estimated gain is 21.8%. The analysis suggests that the Fed was able to attain 71.5% of the potential welfare gains between 1960 and 2018.

The chapter is based on the single-authored paper "Monetary Policy under Imperfect Information and Consumer Confidence".

Chapter 3 presents a microfounded DSGE model of New Keynesian type that is extended with three interactive transmission mechanisms. A dynamic heuristic switching mechanism based on discrete choice theory that involves boundedly rational learning along the lines of Brock and Hommes (1997) and gives rise to endogenous central bank credibility as pointed out by Hommes and Lustenhouwer (2019); a full-fledged microfounded credit market subject to default risk and collateral constraints akin to Becard and Gauthier (2022) that gives rise to endogenous credit cycles; and an aggregate macroeconomic risk measure similar to the one of Proaño and Lojak (2020) that depends on central bank credibility and affects credit market conditions.

It is shown that in this framework real disturbances are severely amplified by the interactive channels, and that risk shocks generate credit booms and busts that are a key driving force of economic fluctuations. The adverse effects of disturbances on stability and welfare are increased when accounting for a zero lower bound on the nominal interest rate which negatively affects central bank credibility when binding. This amplification mechanism is more severe in scenarios where the central bank faces relatively low levels of credibility, i.e. when agents are less inclined to trust in the policy makers capabilities to achieve its targets.

I propose and analyze multiple nonstandard conventional and unconventional monetary policy tools to ease the negative effects. The following results stand out. The relative efficiency of these tools crucially depends on the credibility environment. If credibility is relatively high, an optimal risk-augmented policy rule for the nominal interest rate outperforms all other tools by a wide margin. However, in scenarios associated with low credibility the central bank can attain higher welfare gains by engaging in direct credit intermediation. Applying active credit policy has also positive effects on economic stability, while the risk-augmented Taylor rule is very risky because parameter values slightly above the optimal ones can induce explosive behavior. Finally, when extending the model with an endogenous stock market central bank stock purchases provide positive welfare and stability effects. However, the efficiency is relatively muted in comparison to the other tools.

The chapter is based on the single-authored paper "Credit Risk, Endogenous Credibility, and Unconventional Monetary Policy".

Chapter 2

Monetary Policy under Imperfect Information and Consumer Confidence

Abstract

Although it is generally accepted that consumer confidence measures are informative signals about the state of the economy, theoretical macroeconomic models designed for the analysis of monetary policy typically do not provide a role for them. I develop a framework with asymmetric information in which the efficacy of monetary policy can be improved, when the imperfectly informed central bank includes confidence measures in its information set. The beneficial welfare effects are quantitatively substantial in both a stylized New Keynesian model with optimal monetary policy and an estimated medium-scale DSGE model.

JEL classification: D82, D83, D84, E52, E58, E71

Keywords: Consumer confidence, Monetary policy, Asymmetric information, Imperfect Information, New Keynesian macroeconomics, DSGE models

2.1 Introduction

In standard theoretical monetary policy settings there is no role for consumer confidence measures, although these are collected, processed, and discussed by public institutions with immense effort. I suggest an intuitive theoretical setup, in which central banks can explicitly improve the efficacy of monetary policy by considering confidence measures. I find that paying attention to confidence implies a quantitatively substantial positive welfare effect in both a stylized New Keynesian model under optimal monetary policy and an estimated medium-scale DSGE model. To the best of my knowledge, this study is the first that seeks to reconcile theoretical and perceived value of confidence measures for monetary policy making.

From a broader perspective, I contribute to the literature on imperfect information of monetary policy makers. As shown by Orphanides (2001), monetary policy recommendations differ substantially when based on real-time data instead of ex-post revised data. The seminal paper raises awareness to the great uncertainty about the underlying state of the economy that central banks face when making their decisions.¹ This paper does not only confirm the adverse effects of imperfect information of the central bank, but also proposes a way to mitigate them: by paying more attention to readily available signals like confidence measures when making monetary policy decisions.

The theoretical framework revolves around the approach of Barsky and Sims (2012), who assume incomplete information about technology growth. Instead of observing the growth rate directly, households forecast it by receiving and optimally processing a private noisy signal of it. I augment this channel by making explicit assumptions about the information set of monetary policy makers. More specifically, I assume that the central bank cannot observe the noisy signal of the private sector and has, thus, to forecast the private expectations in order to conduct monetary policy. In such a setup, the central bank can accommodate the welfare losses, caused by its informational disadvantage relative to the real sector, by taking into account an endogenous confidence measure. I argue that the link between confidence and macroeconomic key variables is of high importance for monetary policy when accounting for the policymakers' informational restrictions.

In a stylized New Keynesian model I analyze solutions for optimal monetary policy with consumer confidence for both discretion and commitment with asymmetric information. Under reasonable calibrations, I find

¹Based on this finding, a number of theoretical contributions highlight the features that the assumption of imperfect information provides to macroeconomic models. Neri and Ropele (2012) develop and estimate a model with imperfect knowledge about the state of the economy which has great explanatory power for the business cycle. Lippi and Neri (2007) analyze optimal monetary policy in a DSGE model with imperfect information. Collard et al. (2009) show that models with information constraints exhibit endogenous persistence which makes them less dependent on ad hoc assumptions like price indexation, habit formation or autoregressive shock processes.

that the effect of paying attention to confidence is large. Optimal processing of confidence signals leads to a reduction of 63.7% of the welfare loss induced by informational frictions.

The beneficial welfare effects persist in an enhanced medium-scale DSGE model. However, they are somewhat smaller: the estimated welfare gain of paying attention to confidence is 21.8%. The empirical assessment suggests that the central bank attained 71.5% of the potential welfare gain between 1960 and 2018.

Barsky and Sims (2012) find that confidence has powerful predictive implications for macroeconomic variables. They develop an estimated DSGE model, that reflects the empirical link between confidence and economic activity. One attractive feature of their approach is that it gives a role to two contradicting views, which assign a role to confidence in the macroeconomy: the *animal spirits* view and the *news* view.

On the one hand, the *animal spirits* view, which dates back to Keynes (1936), claims that autonomous fluctuations in beliefs have causal effects on economic activity. For example Blanchard (1993) explains the recession in 1990-1991 with an exogenous shift in pessimism, which had a causal effect on aggregate demand. On the other hand, supporters of the *news* view (Cochrane; 1994; Beaudry and Portier; 2006) argue that confidence merely contains fundamental information about the economy but has no causal role. Barsky and Sims (2012) find that animal spirits shocks are not an important source of the relation between confidence innovations and macroeconomic variables. Instead, news shocks explain the major part of the observed transmission from confidence to future activity.

A recent approach that implements confidence into a theoretical macroeconomic model in a different way is provided by Angeletos, Collard and Dellas (2018). In their model endogenous confidence emerges from the existence of higher-order uncertainty and frictions in expectation coordination. The confidence shock is a disturbance to the expected bias in the expectation of others. The shock shifts short-term expectations and is, thus, of very different nature than the news and animal spirits shocks of Barsky and Sims (2012) that shift medium- and long-term expectations. The authors argue that the stronger quantitative performance of their model can be explained by this difference. The model under investigation in this paper seeks to reconcile the nature of both approaches, as it introduces an additional *confidence measure shock* that affects the short-term forecasts of private agents, while also maintaining the two longer-term shocks of Barsky and Sims (2012).

Further influential contributions that discuss the relation of confidence about technology growth and real business cycles are Angeletos and La'O (2013) and Ilut and Schneider (2014). News and noise shocks in

macroeconomic frameworks are discussed in Jaimovich and Rebelo (2009) and Lorenzoni (2009). None of these contributions, however, makes explicit assumptions about restrictions to the information set of the central bank and could, thus, provide a role for a confidence measure as informative signal in the conduct of monetary policy.

This paper is also closely related to the literature that analyzes optimal monetary policy design when deviating from standard assumptions about expectations such as homogeneity, full information or rationality, e.g. Bask and Proaño (2016), Di Bartolomeo et al. (2016), Gasteiger (2014, 2021), and Lengnick and Wohltmann (2016). All of these contributions do however exclusively focus on heterogeneity among and constraints to expectation formation of *private agents*, while I explicitly include expectations of policymakers.

In Section 2, I investigate whether one can draw conclusions about the value monetary policy makers ascribe to confidence measures from reading central bank publications. Focusing on the Federal Open Market Committee (FOMC), I find that confidence is not explicitly mentioned in statements or publications on forecast methodologies. However, sifting through all minutes of FOMC meetings from 1959 to 2019 and counting mentions of the keywords confidence and sentiment reveals that monetary policy makers do indeed consider confidence measures as relevant source of information in their process of decision making.

In Section 3, I implement the imperfect information framework into an otherwise standard baseline New Keynesian model. I build heavily on the numerous influential monetary policy contributions to the New Keynesian literature, e.g. Clarida et al. (2000), Woodford (2003) and Galí (2015). The setup gives rise to an endogenous confidence measure. The information set of the central bank is explicitly restricted to not contain the private technology growth rate signal. In this asymmetric information setup, I provide the optimal commitment and discretion solutions, where I benefit from the general characterization of Svensson and Woodford (2004). Monte Carlo simulations of the calibrated model reveal that paying attention to confidence can mitigate the welfare losses caused by the information set restriction by almost two thirds.

To approximate the welfare effects of processing the confidence signal in a more empirically driven environment, Section 4 contains the analysis of an estimated New Keynesian DSGE model. The model is very similar to the one of Smets and Wouters (2007), but augmented with the same informational frictions as described above. The welfare effects are unsurprisingly lower than in the stylized model, but still statistically significant and quantitatively substantial.

Section 5 concludes.

2.2 Confidence and the FOMC

In standard models designed for the analysis of monetary policy, there is typically no role for confidence (or sentiment) of private market participants. In contrast, measures of these variables are frequently discussed in contributions, of both academics and media outlets, on the current and projected state of economic activity. In the light of this discrepancy, it is not obvious whether monetary policy makers regard confidence measures as a valuable source of information in their decision making process. To shed some light on this question, I broadly examine publications of the Federal Open Market Committee (FOMC), the principal monetary policy organ of the United States.

In its "Statement on Longer-Run Goals and Monetary Policy Strategy" (Federal Open Market Committee; 2019) the FOMC elaborates on its primary goals and communication principles without the slightest hint on consideration of market sentiment, let alone explicit mentioning.

However, confidence measures might be considered in the conduct of forecasts of macroeconomic responses to monetary policy. One prominent model, that is consulted by Federal Reserve Board members for this purpose, is the FRB/US model, a large-scale model of the U.S. economy with a neoclassical core and numerous rigidities. In a brief overview of the model structure, Brayton et al. (2014) acknowledge potential sentiment effects on firms' fixed investment, but solely as an explanation of why the latter is directly affected by current business output. The model does not envisage a role for market confidence besides confidence in the central bank's commitment to restore inflation to the target level. (Brayton et al.; 1997, p. 243)

The most straightforward way to gain insights into the relevance of confidence measures for FOMC decision making might be to sift through the minutes of its official meetings. Searching for the keywords *sentiment* and *confidence* in all minutes of FOMC meetings from January 6th 1959 and December 10th-11th 2019² reveals, that monetary policy makers indeed frequently cite household, business or finance (or sometimes simply *market*) sentiment to substantiate their assessment of the current state of economic activity and, thus, explain their monetary policy (in)action. After manually cleansing the data of all keyword references that refer either to confidence (about forecasts, estimates, assessments, capabilities etc.) or sentiment (for certain actions) of the FOMC meeting participants themselves, or confidence of private market participants in the capabilities or intentions of the Fed or public institutions, I observe 1252 mentions in 622 meetings, implying a mean of 2.01. However, the sample is fairly volatile: realizations fluctuate between 0 and 15, the

²For all meetings before 1993 I resort to the *Record of Policy Actions* as I find the format to be roughly equivalent to the *minutes* that are published to date.

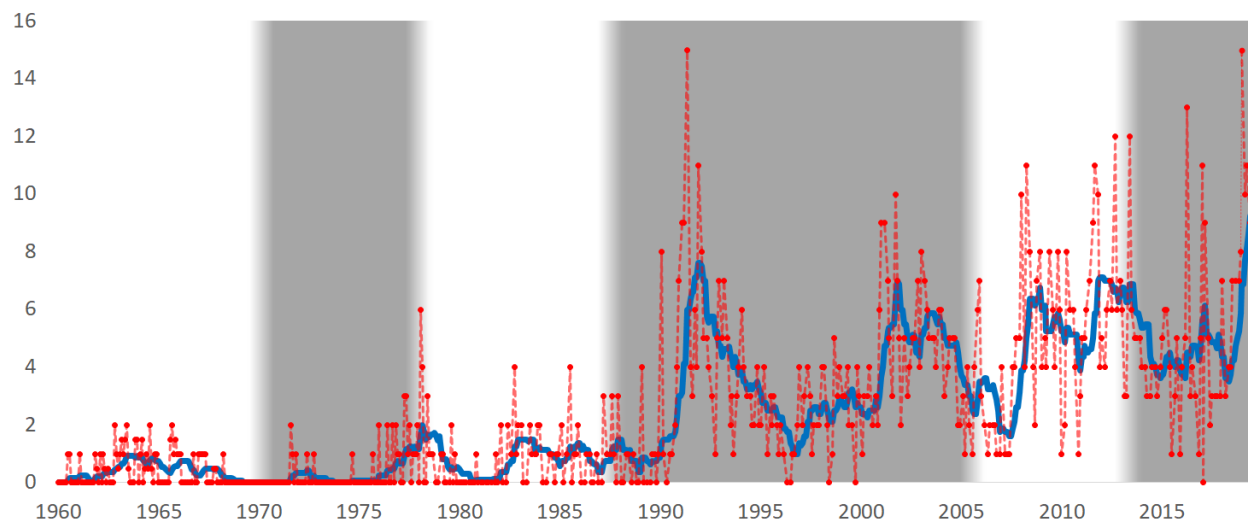


Figure 2.1: Mentions of the keywords 'sentiment' and 'confidence' in FOMC minutes from Jan. 1960 to Dec. 2019. Dashed (red) and solid (blue) line are mentions per month and its 12 months moving average, respectively. Alternating unshaded and shaded background indicate different chairs of the Board of Governors (William M. Martin, Arthur F. Burns, Paul Volcker, Alan Greenspan, Ben Bernanke, Janet Yellen), excl. the brief periods of G. William Miller (Mar. 1978 - Aug. 1979) and Jerome Powell (since Feb. 2018).

sample standard deviation is 2.67. In 260 minutes (41.8%) the keywords are not used at all. The time series and its 12-months moving average are shown in figure 2.1.

Of course, this experiment measures only imperfectly how closely the FOMC paid attention to confidence at each point in time. Nevertheless, it provides some intuition about the behavior of perceived importance of confidence for monetary policy. The following observations stand out to me. The perceived importance of confidence is positive but not constant. It fluctuates over time and exerts cyclical behavior. The perceived importance and its variance increase conspicuously in the early 1990s. While the moving average of mentions fluctuates between 0 and 2 before 1990, fluctuations thereafter occur roughly in the interval of 2 and 8.

At least some of these observations might be driven by individual or a group of FOMC members. To partially account for this, I highlight changes at the chair of the *Board of Governors* by using a shaded background for every other chair. One may notice that the keyword references are lower in frequency in the era of Burns (1970-1978) than under the chairmen Martin (-1970), Miller (1978-1979, not highlighted) and Volcker (1979-1987), while in turn the frequency is strikingly higher in the eras of Greenspan (1987-2006), Bernanke (2006-2014) and Yellen (2014-2018). However, there is no indication for changing FOMC personnel being a main driving force of perceived importance of confidence.

In summary, the results of this simple exercise are as follows. The FOMC considers confidence and sentiment measures as relevant for monetary policy making. However, the degree of attention it seems to dedicate to these variables fluctuates heavily. Does this imply that the impact of confidence on actual monetary policy decisions is positive and time-variant, too? This open question is dealt with in the empirical analysis of section 4.

2.3 Optimal Monetary Policy with Confidence and Asymmetric Information

A baseline macroeconomic model that enables analyzing the importance of considering confidence in the process of monetary policy making requires two features: nominal rigidities which provide effectiveness of monetary policy, and an endogenous confidence measure that contains information about the private sector which is otherwise not accessible for the central bank. The former is comfortably obtained by using the well established New Keynesian framework which is at the core of most modern theoretical monetary policy analyses. For the latter, I implement into this framework the unobservable growth rate and noisy signals approach from Barsky and Sims (2012) who also establish an endogenous confidence measure based on private agent expectations. By assuming imperfect central bank information about the growth rate signal one obtains a setup in which it is beneficial for monetary policy makers to explicitly add confidence to their information set.

2.3.1 The New Keynesian Model

The baseline New Keynesian model is extensively described in the seminal textbooks of Woodford (2003) and Galí (2015) who lay the foundation on which the following model is build upon. In the following, I briefly summarize its building blocks.

Households

A representative infinitely lived household maximizes lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\nu}}{1+\nu} \right) \quad (2.1)$$

subject to the period budget constraint

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t \quad (2.2)$$

where C_t is consumption, N_t denotes labor, P_t is the price level, W_t represents the nominal wage, B_t are (risk-free) government bond holdings purchased at price Q_t , and D_t denotes entrepreneurial profits.

Intermediate Goods Sector

A continuum of differentiated firms with production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \quad (2.3)$$

maximizes profit subject to monopolistic competition and Calvo (1983) price rigidity, i.e. only a fraction $1 - \theta$ of firms can reset prices in any period. Technology level, A_t , is defined further below. Firms face the household demand equations

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \quad (2.4)$$

where it is assumed that $Y_t = C_t$.

Price setting

The dynamics of aggregate gross rate of inflation are given by

$$\Pi_t^{1-\epsilon} = \theta + (1 - \theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \quad (2.5)$$

where reoptimizing firms choose the optimal price, P_t^* solving

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} \frac{C_{t+k}}{P_{t+k}} \left(P_t^* - \frac{\epsilon}{\epsilon-1} MC_{t+k|t} \right) \right\} = 0 \quad (2.6)$$

The marginal costs in period $t+k$ of a firm that last reoptimized in period t , $MC_{t+k|t}$, are given by nominal wage divided by marginal product of labor

$$MC_{t+k|t} = \frac{W_{t+k}}{(1-\alpha)A_t N_{t+k|t}^{-\alpha}} \quad (2.7)$$

Equilibrium

After log-linearizing around steady state values and rearranging the system, it can be expressed with two equations, the IS equation and the New Keynesian Phillips curve (NKPC). Writing all log-linearized variables with lowercase letters, respectively, the NKPC can be expressed as

$$\pi_t = \beta E_t [\pi_{t+1}] + \kappa x_t \quad (2.8)$$

where the output gap, $x_t \equiv y_t - y_t^*$, is defined as the percentage deviation of output from its *natural* level and $\kappa \equiv \frac{(1-\alpha)(1-\theta)(1-\beta\theta)}{(1-\alpha+\alpha\epsilon)\theta} \left(\sigma + \frac{\nu+\alpha}{1-\alpha} \right)$.

The IS equation describes the output gap dynamics and is defined as

$$x_t = E_t [x_{t+1}] - \frac{1}{\sigma} (r_t^n - E_t [\pi_{t+1}] - r_t^*) \quad (2.9)$$

with nominal interest rate r_t^n . Natural output, y_t^* , and natural real interest rate, r_t^* , are defined as realizations that would occur under flexible prices ($\theta = 0$). The dynamics of the latter are given by

$$r_t^* = \beta^{-1} + \sigma \psi_{ya} (E_t [a_{t+1}] - a_t) \quad (2.10)$$

where $\psi_{ya} \equiv \frac{1+\nu}{\sigma(1-\alpha)+\nu+\alpha}$.

2.3.2 Technology, News Shocks, and Animal Spirits

At this point, the so far standard New Keynesian approach is augmented with incomplete information and noisy signals about the technology growth rate. Following the assumptions of Barsky and Sims (2012), log technology follows a random walk with drift

$$a_t = a_{t-1} + g_{a,t-1} + \varepsilon_{a,t} \quad (2.11)$$

where the *unobservable* technology growth rate, $g_{a,t}$, follows an AR(1) process with news shock³ $\varepsilon_{g_{a,t}}$

$$g_{a,t} = (1 - \rho_{g_a})g_a^* + \rho_{g_a}g_{a,t-1} + \varepsilon_{g_{a,t}} \quad (2.12)$$

and unconditional mean g_a^* (which can be ignored in the log-linearized system of this section). All private agents receive a noisy signal of the growth rate, s_t , with animal spirits shock $\varepsilon_{s,t}$

$$s_t = g_{a,t} + \varepsilon_{s,t} \quad (2.13)$$

Whereas the news shock contains information about future technology dynamics, the animal spirits shock is pure noise.

Under the assumption of rational expectations, agents form optimal forecasts of the unobserved growth rate which can be computed via the *Kalman filter*.⁴ In each period t agents receive and process two informative signals to estimate the current technology growth rate: actual technology growth, $a_t - a_{t-1}$, and the additional noisy signal, s_t . The private information processing mechanism enters the New Keynesian system via the natural rate of real interest, r_t^* , which can now be expressed as

$$r_t^* = \beta^{-1} + \psi_{ya}\hat{g}_{a,t} \quad (2.14)$$

with the expected technology growth rate $\hat{g}_{a,t}$.

³Note that $\varepsilon_{g_{a,t}}$ affects technology at period $t+1$ such that it is in line with the definition of a news shock by e.g. Schmitt-Grohé and Uribe (2012) and Offick and Wohltmann (2013, 2016) in the special case of anticipation length $q = 1$.

⁴See appendix 2.7.1 for details.

2.3.3 Monetary Policy and Confidence

This section contains the core of the paper, as the New Keynesian framework with imperfect information provided by Barsky and Sims (2012) is now further augmented with explicit assumptions about the central bank's information set. The monetary policy makers are restricted regarding their ability to observe contemporary private signals of the technology growth rate. However, they may use an endogenous confidence measures to mitigate adverse effects of the informational frictions.

How can it be justified that the central bank has an informational disadvantage relative to the private sector? After all, central banks are, in general, large institutions that employ hundreds of experienced economists, statisticians and forecasters. However, the information set of the representative agent in the New Keynesian framework is the result of information aggregation. The above assumption implies by no means that individual consumers have superior information relative to monetary policy makers. Nevertheless, as Cochrane (1994) puts it, consumers have detailed information about their own prospects that correlate strongly with future aggregate income. Summing over consumers, one may obtain an aggregate private information set which contains idiosyncratic shocks that are unobservable for individual economists and consumers alike.

In the following, I analyze the impact of informational frictions on the efficacy of optimal monetary policy under both discretion and commitment. Typically, the performance of monetary policy regimes is evaluated using an approximated welfare loss function as initially proposed by Rotemberg and Woodford (1999). Following Galí (2015), the welfare loss function in the above New Keynesian setup is given by

$$W = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t (w_x x_t^2 + w_\pi \pi_t^2) \quad (2.15)$$

where $w_x \equiv \sigma + \frac{\nu+\alpha}{1-\alpha}$ and $w_\pi = \frac{\epsilon}{\kappa} w_x$. For simplification, I adopt a *timeless perspective* when optimizing monetary policy to neglect the starting point problem.

Optimal Monetary Policy with Asymmetric Information

To assign a role to confidence measures as informative tools for the central bank it is initially required to somewhat restrict its information set. Before considering this asymmetric information scenario, I briefly state the optimal monetary policy design of the benchmark symmetric scenario. As the set of all exogenous

disturbances $\{\varepsilon_{a,t}, \varepsilon_{g_{a,t}}, \varepsilon_{s,t}\}$ only affects the IS equation but not the NKPC, it is a well-known result that it is feasible for the monetary policy maker to completely stabilize output gap and inflation rate at all times.

Proposition 1 (Optimal monetary policy with symmetric information)

In the case of symmetric information, i.e. when the central bank's information set contains all private signals, optimal monetary policy from a timeless perspective under both discretion and commitment involves a period welfare loss of zero at all times. In both cases the monetary policy instrument, the nominal interest rate, is required to match the natural real rate of interest:

$$r_t^n = r_t^* = \beta^{-1} + \psi_{ya} \hat{g}_{a,t} \quad (2.16)$$

In the following, I assume that the central bank does not have complete information about private technology growth rate expectations. More precisely, the noisy signal, s_t , is not contained in the central bank's information set, I_t^{CB} , although it may contain other noisy signals that are informative in the absence of s_t . This specific assumption about the design of asymmetric information ensures that the central bank information set is completely nested in the private information set, a case for which Svensson and Woodford (2004) provide a general solution for optimal discretion and commitment. The following two propositions can be derived on its basis.

Proposition 2 (Optimal discretion with asymmetric information)

In the case of asymmetric information, under the assumption that the central bank's information set is fully nested in the private information set, optimal discretionary monetary policy implies that the nominal interest rate tracks the natural real rate of interest as closely as possible:

$$r_t^n = E[r_t^* | I_t^{CB}] = \beta^{-1} + \psi_{ya} E[\hat{g}_{a,t} | I_t^{CB}] \quad (2.17)$$

Proof: See appendix 2.7.3.

The intuition of this result is straightforward. Without perfect information about the optimal level of the nominal interest rate, i.e. the natural real rate of interest, the optimal discretion policy involves setting the nominal interest rate to the expected optimal level at all times.

If commitment is feasible for the central bank, it can further reduce the welfare loss by committing to compensate for missing its target values (of zero inflation and output gap) in the future.

Proposition 3 (Optimal commitment with asymmetric information)

Under the same assumptions about asymmetric information as in Proposition 2, optimal monetary policy under commitment from a timeless perspective implies for the nominal interest rate:

$$r_t^n = E [r_t^* | I_t^{CB}] + \Phi \Xi_{t-1} \quad (2.18)$$

where Ξ_t contains the (backward-looking) shadow prices of (2.8) and (2.9), and follows the law of motion

$$\Xi_t = \begin{pmatrix} w_\pi & 0 \\ \kappa w_\pi & w_x \end{pmatrix} \begin{pmatrix} \pi_t \\ x_t \end{pmatrix} + \begin{pmatrix} 1 & \frac{1}{\beta\sigma} \\ \kappa & \frac{1}{\beta} + \frac{\kappa}{\beta\sigma} \end{pmatrix} \Xi_{t-1} \quad (2.19)$$

with initial value

$$\Xi_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Proof: See appendix 2.7.4.

Monetary Policy Implications of Confidence Signals

The conduct of optimal monetary policy under both commitment and discretion requires optimal forecasting of the natural real rate of interest which implies forecasting the perceived technology growth rate. By assuming that the central bank's information set is completely nested in the private sector's information set, this task is equivalent to forecasting the actual technology growth rate. Additionally, the assumption ensures that the central bank does not provide relevant information about the technology growth rate by setting its monetary policy instrument, the nominal interest rate.

To provide a role for a confidence measure, it is assumed that the central bank is subject to the following informational friction: when setting the nominal interest rate it has no knowledge about the private signal, s_t , which the private sector uses to form its expectations about the technology growth rate. However, the policymakers observe the realizations of all macroeconomic variables, e.g. x_t , π_t and a_t , *after* the nominal interest rate has been set. A fully rational central bank can, thus, deduce the private signal by reverse-engineering private sector expectations from observed behavior. More precisely, at each period t , s_{t-1} is included in the central bank's information set, I_t^{CB} , but s_t is not.

To mitigate the adverse welfare consequences of the informational friction, the central bank may process a confidence measure as additional signal. Following the assumption of Barsky and Sims (2012), confidence \tilde{C}_t is defined as the autoregressive process

$$\tilde{C}_t = (1 - \rho_c)\tilde{C}^* + \rho_c\tilde{C}_{t-1} + u_t \quad (2.20)$$

This specification is designed in order to capture the dynamics of a sentiment index which focuses on the long-term economic outlook of consumers. More specifically, Barsky and Sims (2012) developed this model structure in order to obtain a theoretical counterpart to the sentiment index $E5Y^5$ of the *Michigan Survey of Consumers*. The constant \tilde{C}^* is the unconditional confidence mean⁶ which is close to (but not necessarily equal to) the neutral position of 100 from which the index deviates by the fraction of favorable answers minus the fraction of unfavorable responses.

Confidence dynamics are driven by the confidence innovation, u_t , which is a linear combination of surprise increase in technology level and growth rate, respectively:

$$u_t = \zeta_1(a_t - a_{t-1} - \hat{g}_{a,t-1}) + \zeta_2(\hat{g}_{a,t} - \rho_{g_a}\hat{g}_{a,t-1}) + \varepsilon_{c,t} \quad (2.21)$$

The design of the endogenous confidence measure is adopted from Barsky and Sims (2012). Note that $\hat{g}_{a,t-1} \equiv E_{t-1}[g_{a,t-1}|I_{t-1}]$, i.e. how the private sector perceived technology growth rate one period earlier.

Figure 2.2 shows the perceived and actual technology growth rate responses to the four shocks for the parameterization $\sigma_a = 0.58$, $\rho_{g_a} = 0.73$, $\sigma_{g_a} = 0.17$, $\sigma_s = 0.13$, $\rho_c = 0.94$, $\zeta_1 = 1.01$, $\zeta_2 = 32.76$ and $\sigma_c = 3.78$, as estimated by Barsky and Sims (2012). In all cases, expectations of both real sector and central bank converge rapidly to actual growth rate, as the lines can barely be distinguished only four periods after the initial shock impulse.

The real sector expectations falsely ascribe technology level and animal spirits shocks partly to an actual growth rate increase, whereas perceived growth rate underestimates actual realizations in case of news shocks. Without an additional confidence signal, the central bank expectations deviate severely from their private sector counterparts on impact, as animal spirits and news shocks affect the received signals, $a_t - a_{t-1}$ and s_{t-1} , only with a delay of one period. Therefore, the central bank cannot react to these shocks on impact,

⁵E5Y is constructed from responses to the question "Turning to economic conditions in the country as a whole, do you expect that over the next five years we will have mostly good times, or periods of widespread unemployment and depression, or what?"

⁶Note that the constant is known to all agents in the model such that the calibration of \tilde{C}^* does not matter in any of the following analyses.

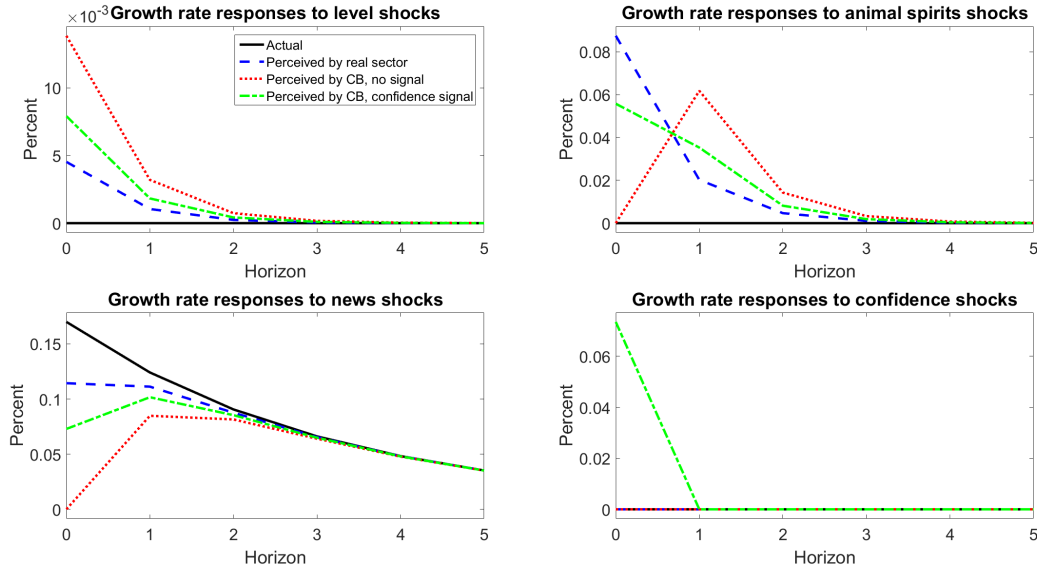


Figure 2.2: Technology growth rate responses to level, animal spirits, news and confidence shocks. For each shock, the figure compares the response of actual growth rate (solid, black), perceived growth rate by real sector (dashed, blue), and perceived growth rate by the central bank with (dashed-dotted, green) and without (dotted, red) an additional confidence signal.

whereas it overestimates the probability of increased growth rate in case of level shocks. When observing an additional confidence signal, the central bank can mitigate the distance to real sector expectations by almost two thirds in all of these cases. However, this improvement comes at a cost. The central bank exposes itself to a fourth stochastic shock, the confidence shock $\varepsilon_{c,t}$, that may lead to deviations from private sector expectations. Under fully rational information processing via the Kalman filter, perfect knowledge about the shock standard deviations implies, however, that average forecast errors must decrease with the additional signal.

As shown in appendix 2.7.5, under optimal discretion the realizations of inflation and output gap depend solely on the *contemporaneous* technology growth rate forecast error of the central bank:

$$\pi_t = -\kappa\psi_{ya} (E[\hat{g}_{a,t}|I_t^{CB}] - \hat{g}_{a,t}) \quad (2.22)$$

$$x_t = -\psi_{ya} (E[\hat{g}_{a,t}|I_t^{CB}] - \hat{g}_{a,t}) \quad (2.23)$$

The impulse responses of the system variables π_t , x_t and r_t^n are shown in figure 2.3. It is straightforward to see that, analogous to figure 2.2, using the confidence signal severely reduces output gap and inflation fluctuations in case of level, animal spirits and news shocks, at the cost of *one-off* deviations after confidence

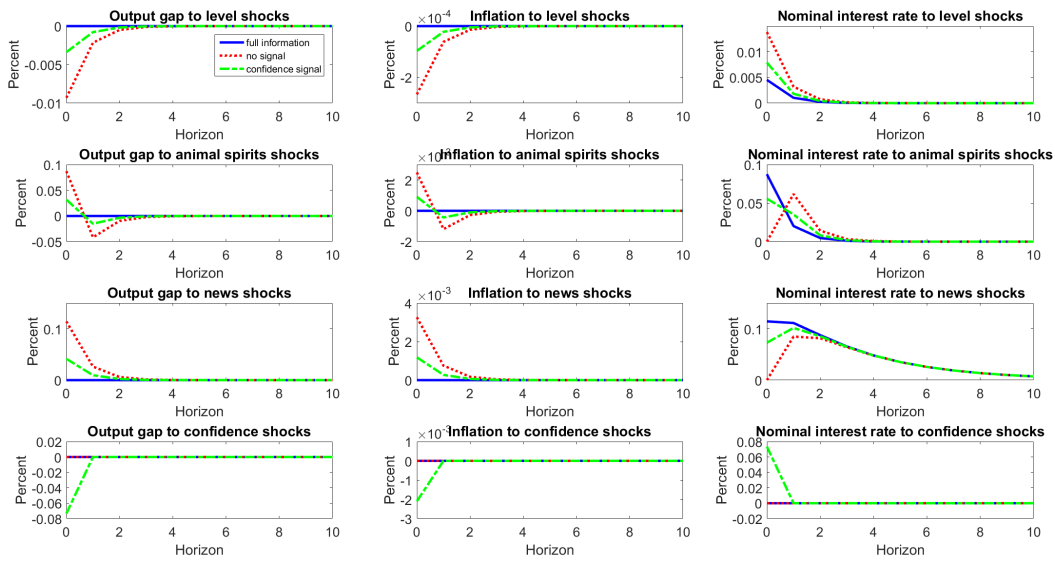


Figure 2.3: Output gap, inflation rate and nominal interest rate responses under optimal discretionary monetary policy to technology level, animal spirits, news and confidence shocks. For each shock, the figure compares the response under the informational regimes *full information* (solid, blue), *imperfect information without confidence signal* (dotted, red) and *imperfect information with confidence signal* (dashed, green).

shocks. A notable result is the observation of undershooting in case of animal spirits shocks, a behavior that is typically not observed in purely-forward looking New Keynesian models under discretion.

Under optimal commitment, inflation and output gap respond to disturbances by following more complex paths (see appendix 2.7.6). In this case the variables depend not only on the contemporaneous forecast error of the central bank, but on the complete history of forecast errors:

$$\begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = \sum_{i=0}^{\infty} \tilde{\Phi}_i (E[\hat{g}_{a,t-i} | I_{t-i}^{CB}] - \hat{g}_{a,t-i}) \quad (2.24)$$

where $\tilde{\Phi}_i$ is a vector of time-dependent weights. As illustrated by the impulse responses in figure 2.4, asymmetric information gives rise to non-monotonous dynamics. Output gap changes its sign twice in response to all shocks, while inflation dynamics show hump-shaped behavior. Once again, visual assessment suggests a fairly high contribution of confidence signal processing to stabilization.

To assess the welfare effects of the different informational regimes I estimate the average period loss function with Monte Carlo simulations. Under optimal discretion, the estimated loss under asymmetric information is 4.08% in the benchmark case. Processing the confidence signal helps to lower it to 1.48%. If it is feasible for

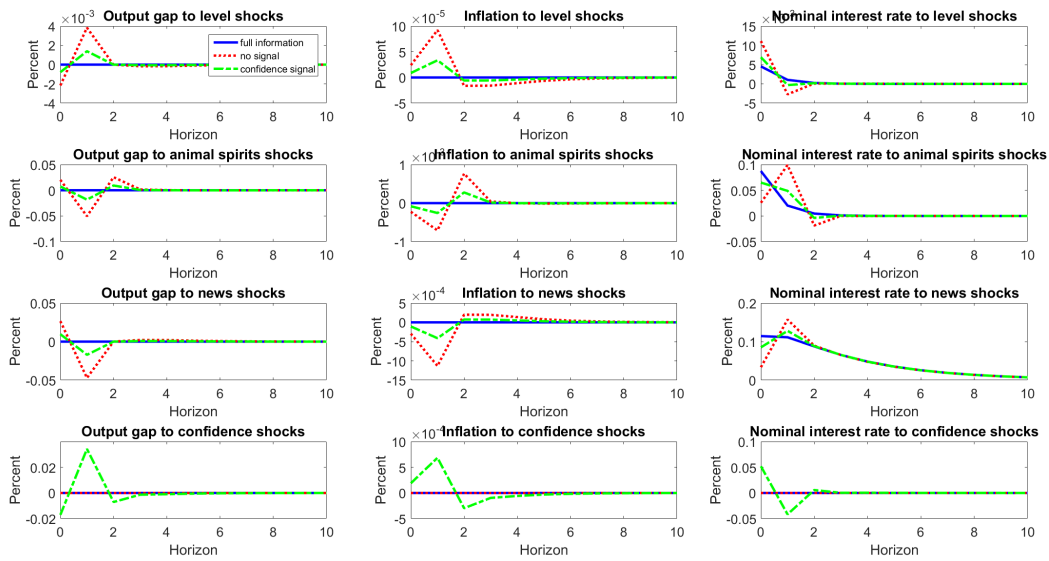


Figure 2.4: Output gap, inflation rate and nominal interest rate responses under optimal commitment to technology level, animal spirits, news and confidence shocks. For each shock, the figure compares the response under the informational regimes *full information* (solid, blue), *imperfect information without confidence signal* (dotted, red) and *imperfect information with confidence signal* (dashed, green).

policymakers to apply optimal commitment, the loss can be reduced to 0.98% and 0.36%, respectively. For both discretion and commitment, this implies a loss reduction of 63.7% caused by confidence signal processing. Independent of the underlying informational assumptions, monetary policy with optimal commitment provides a loss reduction of 75.9% in comparison to optimal discretion.

The impressive loss mitigation of almost two thirds when considering a confidence measure in the conduct of monetary policy has, of course, to be treated with caution as the underlying framework is very stylized. The exact value crucially depends on parameter calibration, most importantly on the standard deviation of confidence innovation, σ_c . Higher values decrease the confidence signal precision and, thus, make it less useful for welfare loss reduction. In the 95% confidence interval from the estimation of Barsky and Sims (2012) the estimated loss reduction is roughly between 80% and 45%, implying that the high degree of importance of the confidence signal is fairly robust to parameterization.⁷ Even for a value five times as high as the estimated value, confidence signal processing implies a loss reduction of more than 6%. Figure 2.5 shows the estimated welfare loss mitigation for different values of σ_c . Note that the *relative* loss mitigation does not depend on the monetary policy regime while the *absolute* welfare loss reduction does.

⁷Using the standard deviation that Barsky and Sims (2012) state for their estimated value of 3.81, one obtains the 95% confidence interval (2.22, 5.50).

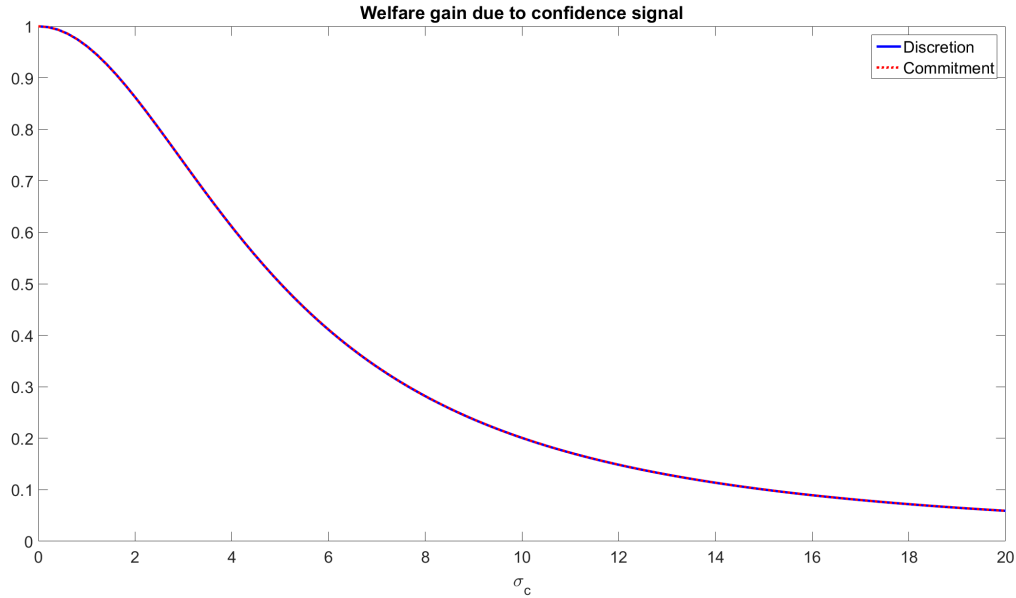


Figure 2.5: Estimated welfare loss mitigation due to confidence signal processing for different parameter calibrations of confidence innovation standard deviation, σ_c . Estimated average period loss is normalized by dividing by estimated loss without attention to confidence. Loss reduction is the same for optimal discretion (solid, blue) and commitment (dotted, red).

2.4 Empirical Analysis

The analysis in the previous section suggests that confidence information is a highly important tool for monetary policy makers to achieve welfare maximization. However, the model is very stylized and lacks a number of elements that provide modern DSGE models with more realistic dynamics. In this section I seek to answer the question how well this result translates to an estimated medium-scale DSGE model. The model is enhanced to include a measure of attention the central bank has allocated to confidence measures. I analyze whether its attention has shifted over time, and how much welfare was lost due to imperfect confidence signal processing.

2.4.1 Design of Monetary Policy and Informational Frictions

To describe monetary policy behavior I assume that nominal interest rates follow a rule of the type proposed by Taylor (1993):

$$r_t^n = \rho_r r_{t-1}^n + (1 - \rho_r) \left(\delta_\pi E[\pi_t | I_t^{CB}] + \delta_x E[x_t | I_t^{CB}] + \delta_{\Delta y} E[\Delta y_t | I_t^{CB}] \right) + \nu_t \quad (2.25)$$

where Δy_t denotes output growth. By including interest rate smoothing and both output gap and growth, the monetary policy includes all components from the original Taylor rule and the policy rule proposed by Barsky and Sims (2012). Monetary policy shock, ν_t , follows an AR(1) process.

Assuming the same informational frictions as in section 2.3, the central bank estimates of contemporaneous inflation and output crucially depend on its expectation about (perceived) technology growth rate, $E[\hat{g}_{a,t}|I_t^{CB}]$. To provide an intuitive measure on the degree of attention the central bank assigns to confidence, I assume that this expectation is a linear combination of the expected values that would be attained with and without confidence signal processing. Denoting the corresponding information sets as $I_t^{CB,C}$ and $I_t^{CB,0}$, respectively, central bank growth rate expectations are given by

$$E[\hat{g}_{a,t}|I_t^{CB}] = \omega_c E[\hat{g}_{a,t}|I_t^{CB,C}] + (1 - \omega_c) E[\hat{g}_{a,t}|I_t^{CB,0}] \quad (2.26)$$

with $0 \leq \omega_c \leq 1$. Confidence weight ω_c measures the importance the central bank assigns to confidence for monetary policy making.⁸

2.4.2 Empirical DSGE Model with Asymmetric Information and Confidence

Apart from the informational frictions the DSGE model used for the empirical analysis is relatively standard. It is a version of the one used in Smets and Wouters (2007) and, thus, a bit richer than the one of Barsky and Sims (2012) as it contains capital utilization and price indexation, as well as additional preference, investment demand and cost shocks. Appendix 2.7.2 contains the full description and derivation.

The solution under asymmetric information is obtained via fixed-point iteration as described in appendix 2.7.7, and is given by

$$X_t = \Gamma X_{t-1} + \tilde{\Gamma} X_{t-1}^f + \Gamma_r r_{t-1} + \Upsilon \eta_t + \Psi \hat{g}_{a,t} + \Psi^{CB} E[\hat{g}_{a,t}|I_t^{CB}] \quad (2.27)$$

$$X_t^f = \Gamma^f X_{t-1}^f + \Upsilon^f \eta_t + \Psi^f \hat{g}_{a,t} \quad (2.28)$$

$$y_t^f = \gamma_y^f X_{t-1}^f + \phi_y^f \eta_t + \psi_y^f \hat{g}_{a,t} \quad (2.29)$$

$$\eta_t = R \eta_{t-1} + J g_{a,t} + \epsilon_t \quad (2.30)$$

⁸The assumption of a constant confidence weight may seem to be contradicting the evidence in section 2.2. Therefore, I allow for a structural break in the attention to confidence later on to address potential time variance.

where X_t denotes the vector of all system variables of the sticky price model, $X_t^f \equiv (c_t^f, i_t^f, k_t^f)'$ contains all flex-price variables with backward-looking components, y_t^f is the flex-price output needed to compute the output gap in (2.25), and η_t denotes the vector of shock processes with white noise stochastic shocks ϵ_t .

2.4.3 Econometric Methodology

The parameters are estimated by matching the impulse response functions of empirical variables with the ones generated by model simulations. The moment matching approach used here is a Bayesian variant of the methodology in Barsky and Sims (2012). Pinning down parameter values by minimizing the distance between empirical and model-generated moments is frequently used in a variety of theoretical macroeconomic applications, e.g. Christiano, Eichenbaum and Evans (2005), Franke and Westerhoff (2012), and Altig et al. (2011). Following Kim (2002, 2014) and Christiano, Eichenbaum and Trabandt (2015), I approximate a quasi-likelihood function based on the moment distance. After reweighing with a prior density it is possible to explore the posterior parameter density as e.g. in Smets and Wouters (2007).

The data of selected key variables is used to estimate a VAR with four lags in companion form by OLS. The responses of the variables to orthogonalized innovations are computed involving a Cholesky factorization. The variables included in the VAR are:

$$V_t = \left(\Delta(y_t - n_t), \pi_t, c_t, y_t, i_t, \tilde{C}_t, r_t \right)' \quad (2.31)$$

Labor productivity, $y_t - n_t$ is measured by log of real output per hour (nonfarm business sector). Inflation, π_t , is measured by the growth rate of the personal consumption expenditures price index. Consumption, c_t , investment, i_t , and output y_t are measured by log of real personal consumption expenditures, private nonresidential fixed investment and GDP, respectively, each divided by civilian labor force level. Consumer confidence, \tilde{C}_t , is measured by the index E5Y which collects survey responses about expected economic conditions over the next five years. As mentioned above the definition of (2.20) was specifically developed by Barsky and Sims (2012) to provide a theoretical counterpart to E5Y. The real interest rate, r_t , is measured by the three months treasury bill rate net expected change of prices. All data is retrieved from FRED of the St. Louis Fed, except the confidence measure and inflation expectations which are provided by the Michigan Surveys of Consumers. The variables are measured in quarterly frequency. The sample encompasses the interval from 1960-Q1 to 2018-Q3 which implies a sample size of $T = 235$.

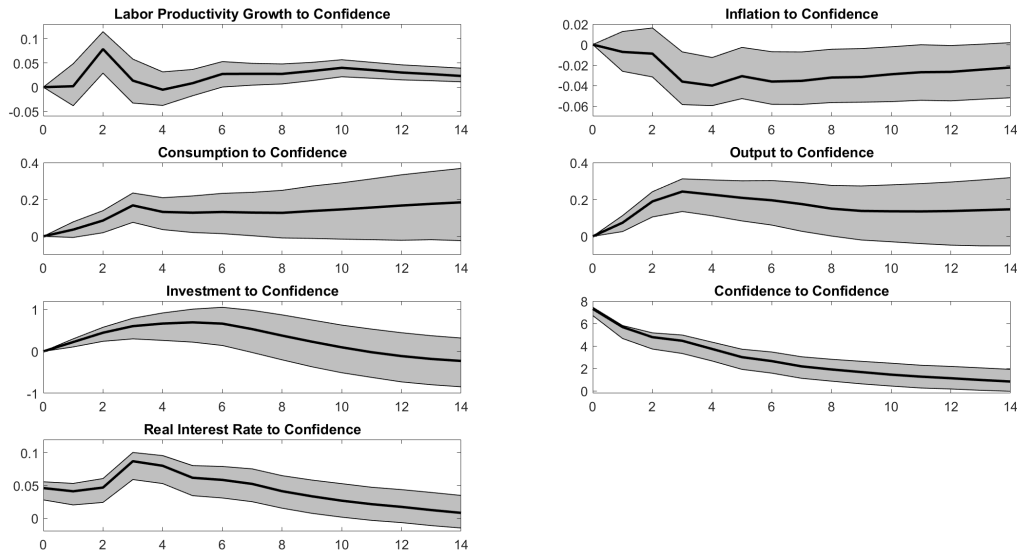


Figure 2.6: Empirical impulse responses to confidence innovations in an estimated VAR with three lags. The shaded area indicates 95% confidence intervals.

The ordering of variables is loosely based on Christiano, Eichenbaum and Trabandt (2015). As observed by Barsky and Sims (2012), only the real interest rate reacts contemporaneously to confidence innovations whereas output, consumption and inflation responses are slowly building over time. A similar dynamic behavior can be observed for investment and labor productivity. However, I follow the reasoning of Barsky and Sims (2012) that the VAR specification and structural interpretation is actually of second-order importance, as the capability of replicating empirical impulse responses is in itself a valuable feature for structural models.

The empirical responses of all variables to confidence innovations are shown in figure 2.6. As in Barsky and Sims (2012), consumption and output do not respond on impact to confidence innovations but increase permanently in the long run. The inflation rate responds temporarily negatively. The real interest rate increases on impact but the effect wears off in the long run. For the additionally included variables, labor productivity growth and investment, I observe temporary increases.

The empirical moments vector, $\hat{\mathbf{m}}$, contains the stacked responses of all variables to all orthogonalized innovations from impact to 20 lags. This provides a set of $n_m = 1008$ moments. Under regularity conditions, it holds for the asymptotic distribution of the empirical moments that

$$\sqrt{T}(\hat{\mathbf{m}} - \mathbf{m}(\theta)) \overset{a}{\sim} N(0, \mathbf{W}(\theta_0)) \quad (2.32)$$

where θ_0 denotes the *true* parameter values of the model, or equivalently

$$\hat{\mathbf{m}} \stackrel{a}{\sim} N(\mathbf{m}(\theta), \mathbf{V}) \quad (2.33)$$

with $\mathbf{V} \equiv T^{-1}\mathbf{W}(\theta_0)$.

Given these assumptions, one obtains the approximate likelihood function

$$f(\hat{\mathbf{m}}|\theta, V) = (2\pi)^{-N/2} |V|^{-1/2} \exp \left\{ -\frac{1}{2} (\hat{\mathbf{m}} - \mathbf{m}(\theta))' \mathbf{V} (\hat{\mathbf{m}} - \mathbf{m}(\theta)) \right\} \quad (2.34)$$

Denoting the prior parameter density by $p(\theta)$, the Bayesian posterior function can be expressed as

$$\varphi(\theta|\hat{\mathbf{m}}, \mathbf{V}) = \frac{f(\hat{\mathbf{m}}|\theta, V) p(\theta)}{\int f(\hat{\mathbf{m}}|\theta, V) p(\theta) d\theta} \quad (2.35)$$

The posterior function's properties are analyzed in an extensive numerical procedure. The logarithmic posterior with respect to a given parameter vector is provided by computing log-prior (which can be derived exactly) and estimating the log-likelihood function. For the latter, 2500 Monte Carlo simulations are conducted, each with simulation size equal to the empirical sample size T , to obtain average model-generated impulse responses that enter the vector of model-generated moments, $\mathbf{m}(\theta)$.

The first step of the numerical procedure entail direct maximization of the logarithmic numerator the posterior parameter function (2.35) to obtain the estimated mode $\tilde{\theta}$. Afterwards, the posterior distribution is explored by running two simulations of the Metropolis-Hastings algorithm, which is a *Markov Chain Monte Carlo* method to sample indirectly from a probability distribution.⁹

In the estimation procedure the matrix V is replaced by its consistent estimator, $\bar{\mathbf{V}}$, resulting from the bias-corrected bootstrap of Kilian (1998). Following Christiano, Trabandt and Walentin (2010), I further

⁹At iteration step s , one draws a candidate parameter vector θ^c from a proposal distribution $N(\theta^{(s-1)}, c^2\tilde{\Sigma})$, where $\tilde{\Sigma}$ is the (estimated) inverse Hessian at $\tilde{\theta}$ and c is a tuning coefficient (to calibrate the acceptance ratio, typically to values between 25% and 40%). The change from $\theta^{(s-1)}$ to θ^c is accepted ($\theta^{(s)} = \theta^c$) with probability

$$r(\theta^{(s-1)}, \theta^c|\hat{\mathbf{m}}, \mathbf{V}) = \min \left\{ \frac{f(\hat{\mathbf{m}}|\theta^c, V) p(\theta^c)}{f(\hat{\mathbf{m}}|\theta^{(s-1)}, V) p(\theta^{(s-1)})}, 1 \right\} \quad (2.36)$$

and rejected otherwise ($\theta^{(s)} = \theta^{(s-1)}$). Repeating this for a large number of iteration steps provides an estimate of the posterior density function. Each instance of Metropolis-Hastings is conducted with a sample size of 30,000 and initialized at mode $\tilde{\theta}$. The algorithm is repeated to make use of the superior information the first run provided on $\tilde{\Sigma}$.

Parameter		Value
Discount factor	β	0.99
Capital depreciation rate	δ	0.03
Steady state income share of capital	α	0.36
Autocorrelation of government expenditure shocks	ρ_g	0.95
Standard deviation of government expenditure shocks	σ_g	0.25
Steady state income share of consumption	c_y	0.57
Steady state income share of government expenditures	g_y	0.20
Steady state income share of investment	i_y	0.23
Unconditional technology growth rate mean	g_a^*	0.33

Table 2.1: Calibrated parameter values

transform $\bar{\mathbf{V}}$ to account for its poor small-sample properties. As in Christiano, Eichenbaum and Evans (2005) and Christiano, Eichenbaum and Trabandt (2015), I apply the extreme case of this transformation to obtain the matrix $\hat{\bar{\mathbf{V}}}$ which contains only the diagonal elements of $\bar{\mathbf{V}}$.

2.4.4 Estimation Results

Nine parameters are calibrated as stated in table 2.1. These are the same parameter values that Barsky and Sims (2012) calibrate in their estimation and are, in general, fairly uncontroversial. The remaining 27 parameters are estimated with the procedure described above. Assumptions about prior distributions and estimated posteriors are summarized in table 2.2. The acceptance ratio of the two Metropolis-Hastings simulations are 32.7% and 29.2%, respectively.

The central bank's attention on confidence, ω_c , is estimated at 64.1%, indicating non-optimal information processing in the conduct of monetary policy making as the value is significantly lower than unity. However, confidence weight is significantly positive which suggests that consumer confidence is indeed considered when making policy decisions.

The interest rate smoothing coefficient of the monetary policy rule ($\rho_r = 0.517$) is relatively low in comparison to other estimates, but is well within the confidence bounds of Barsky and Sims (2012). Inflation ($\delta_\pi = 1.793$), output gap ($\delta_x = 0.143$) and output growth ($\delta_{\Delta y} = 0.244$) coefficients are fairly close to their counterparts in Smets and Wouters (2007) (2.03, 0.08 and 0.22, respectively), which suggests that monetary policy reacts strongly to inflation and that output growth prompts stronger responses than the output gap.

Parameter		Prior			Posterior	
		Shape	Mean	Std.	Median	90% HPDI
<i>Monetary policy</i>						
Attention to confidence	ω_c	B	0.50	0.10	0.641	[0.476, 0.777]
Interest rate smoothing	ρ_r	N	0.80	0.05	0.517	[0.455, 0.580]
Inflation coefficient	δ_π	N	1.50	0.125	1.793	[1.630, 1.979]
Output gap coefficient	δ_x	N	0.125	0.05	0.143	[0.104, 0.190]
Output growth coefficient	$\delta_{\Delta y}$	N	0.125	0.05	0.244	[0.182, 0.310]
Persistence of monetary shock	ρ_ν	B	0.50	0.20	0.635	[0.522, 0.739]
Std. of monetary shock	σ_ν	IG	0.50	0.20	0.220	[0.185, 0.256]
<i>Confidence</i>						
Persistence of confidence	ρ_c	B	0.95	0.10	0.876	[0.847, 0.901]
Perceived technology coefficient	ζ_1	N	1.00	0.25	0.796	[0.401, 1.188]
Perceived growth rate coefficient	ζ_2	N	32.0	0.80	32.36	[30.88, 33.73]
Std. of confidence shock	σ_c	IG	2.00	0.40	4.741	[3.556, 5.717]
<i>Technology and Noisy Signals</i>						
Std. of technology shock	σ_a	IG	0.50	0.20	0.483	[0.454, 0.512]
Persistence of growth rate shock	ρ_{g_a}	B	0.50	0.10	0.552	[0.491, 0.608]
Std. of news shock	σ_{g_a}	IG	0.50	0.20	0.172	[0.149, 0.193]
Std. of animal spirits shock	σ_s	IG	0.50	0.20	0.030	[0.015, 0.059]
<i>Household utility function</i>						
Consumption habit persistence	b	B	0.70	0.10	0.329	[0.265, 0.394]
Inv. Frisch labor supply elasticity	ν	N	0.50	0.05	0.423	[0.342, 0.501]
Persistence of preference shock	ρ_z	B	0.50	0.20	0.441	[0.312, 0.558]
Std. of preference shock	σ_z	IG	0.50	0.20	0.597	[0.521, 0.685]
<i>Capital and Investment</i>						
Investment adjustment cost	ϕ	N	4.00	1.00	4.277	[3.376, 5.311]
Capacity utilization adjustment cost	ψ	B	0.50	0.05	0.893	[0.863, 0.917]
Persistence of investment shock	ρ_i	B	0.50	0.20	0.399	[0.247, 0.546]
Std. of investment shock	σ_z	IG	0.50	0.20	1.887	[1.418, 2.459]
<i>Price setting</i>						
Price stickiness	θ	B	0.60	0.10	0.691	[0.646, 0.731]
Price indexation	ι_p	B	0.50	0.15	0.051	[0.021, 0.094]
Persistence of cost shock	ρ_p	B	0.50	0.20	0.892	[0.864, 0.913]
Std. of cost shock	σ_p	IG	0.50	0.20	1.137	[0.993, 1.300]

Table 2.2: Prior and (estimated) posterior distributions of the DSGE model parameters. Prior distributions of parameters have either Normal (N), Beta (B) or Inverse Gamma (IG) shape.

The coefficients that determine consumer confidence dynamics are similar to the original estimations of Barsky and Sims (2012). Confidence responds to the innovation in perceived technology ($\zeta_1 = 0.796$) considerably less than to innovations in perceived growth rate ($\zeta_2 = 32.36$). The standard deviation of confidence shocks ($\sigma_c = 4.741$) is larger than in the original estimation (3.78) but the standard errors on σ_c are large in both estimations. The persistence of confidence ($\rho_c = 0.876$) is somewhat lower than the original estimate of 0.94.

The standard deviations of technology level ($\sigma_a = 0.483$), news ($\sigma_{g_a} = 0.172$) and animal spirits shocks ($\sigma_s = 0.030$) are well within the confidence bounds of Barsky and Sims (2012), although the difference between the latter two is even more pronounced. The persistence of news shocks ($\rho_{g_a} = 0.552$) is fairly low in comparison to the previous result.

Inverse Frisch elasticity of labor supply ($\nu = 0.423$), elasticity of investment adjustment cost ($\phi = 4.277$) and Calvo price stickiness ($\theta = 0.691$) are all well within the region of typical estimates. The coefficient of consumption habit persistence ($b = 0.329$) is relatively low in comparison – e.g. Smets and Wouters (2007) estimate the parameter at 0.71 – but very close to the estimate of Barsky and Sims (2012), 0.31. The elasticity of capital utilization adjustment costs ($\psi = 0.893$) is notably higher than in other estimations, e.g. Smets and Wouters (2007) and Angeletos et al. (2018). However, many authors find a posterior that barely differs from the prior and has large HPDI¹⁰. The degree of price indexation ($\iota_p = 0.051$) is relatively low, the high persistence in inflation dynamics that is typically observed is instead provided by the autocorrelation coefficient of the cost shock process ($\rho_p = 0.892$).

Figure 2.7 shows the theoretical impulse responses of key variables to news, animal spirits and confidence shocks. Output reacts with a slowly building permanent increase to news shocks and increases temporarily after animal spirits shocks. The confidence shock induces a misjudgment by the central bank which has temporary negative effects on output. Inflation responds to animal spirits and confidence shocks very similar to output. In case of news shocks, inflation increases on impact, and proceeds on a non-monotonous dynamic path where inflation growth changes its sign twice. This pattern reveals an interesting interaction of the counteracting effects of increasing demand, decreasing marginal costs, and temporary forecast errors of the monetary policy maker. The real interest rate increases temporarily in response to all three shocks, with a non-monotonous path in response to news shocks similar to the one observed for inflation.

¹⁰ HPDI stands for *Highest Posterior Density Interval*. It describes the region of parameters values with the property that posterior density is higher at every point within the set than at any other point outside of it. It is often used as the Bayesian equivalent to a confidence region.

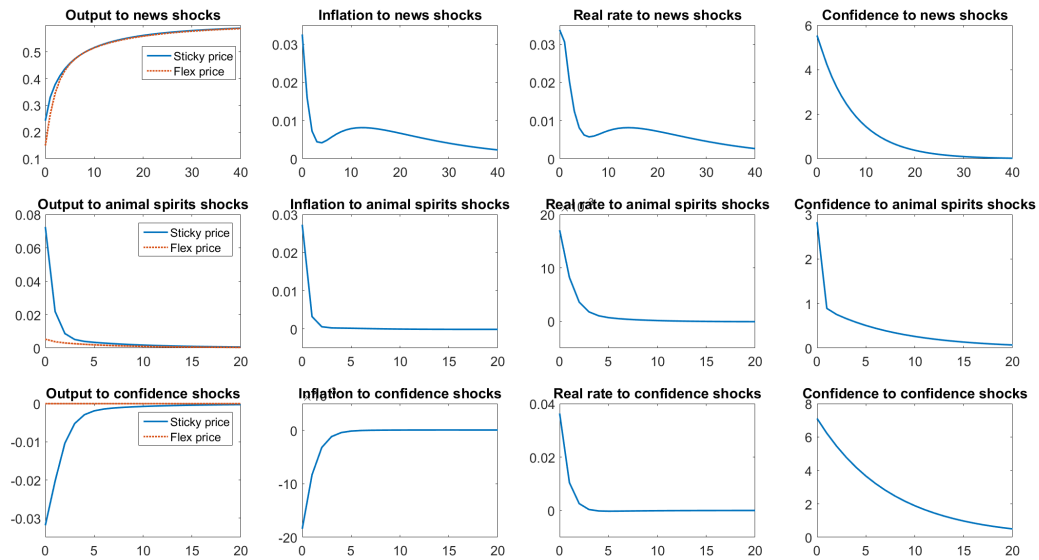


Figure 2.7: Theoretical impulse responses of output, inflation rate, real interest rate and consumer confidence to news, animal spirits and confidence shocks in the medium-scale DSGE model at the estimated parameter values

2.4.5 Beliefs about the Short-Term Economic Outlook

In the seminal approach of Angeletos, Collard and Dellas (2018), endogenous confidence emerges from the existence of higher-order uncertainty and frictions in expectation coordination. The confidence shock, a disturbance to the expected bias in the expectation of others, shifts *short-term* expectations about economic activity. As the authors argue, this detail accounts for the strong quantitative performance of their model, whereas they highlight that the news and animal spirits shocks of Barsky and Sims (2012) are of different nature because they shift medium- and long-term expectations.

Since the transmission mechanism of news and noise shocks has not fundamentally changed, this assertion is applicable to this modeling approach. The first two panels of figure 2.8 show that animal spirits and news shocks mainly shift long-term output level forecasts of private agents. However, as illustrated in the third panel, this does not hold for the newly introduced¹¹ confidence measure shock, which affects the short-term forecasts of private agents. Thus, the asymmetric confidence information mechanism is able to shift both short- and long-run prospects of private agents which allows to analyze confidence disturbances of different nature.

¹¹Barsky and Sims (2012) also consider a confidence measure shock, but without a transmission channel to any macroeconomic variable. The spillover effects to economic activity are rooted in the asymmetric confidence information approach introduced in this paper.

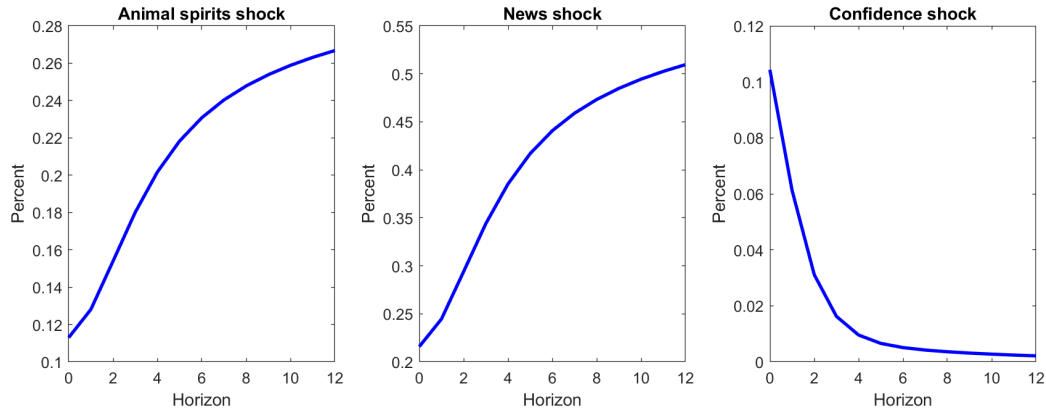


Figure 2.8: Private output level forecasts on impact of an animal spirits, a news, and a confidence measure shock

2.4.6 Welfare Analysis

In last section's empirical analysis, the parameter that governs central bank's attention to confidence, ω_c , is estimated at 0.641 which suggests positive yet not optimal information processing of confidence signals. In this section, I provide some insights into the welfare implications of this level of attention to confidence in the estimated DSGE model.

Analogous to section 2.3.3, I begin to compare the theoretical impulse responses to technology level, news, animal spirits and confidence shocks to output gap and inflation rate for three different confidence weights: no attention ($\omega_c = 0$), optimal confidence signal processing ($\omega_c = 1$), and estimated confidence weight. All other parameters are set to their posterior medians.

In figure 2.9, the impulse responses to technology level and news shocks are illustrated. The output and inflation responses to level shocks are virtually indistinguishable, implying that it is almost irrelevant to consider confidence measures when attempting to mitigate adverse welfare effects from this type of shock. On the contrary, monetary policy is markedly more successful in dampening business cycle fluctuations resulting from news shocks the higher central bank's attention to confidence is.

A similar observation is made for animal spirits shocks, as displayed in figure 2.10. The dynamic paths of output gap and inflation rate, differ however less strongly than in case of news shocks. Another marked effect of attention to confidence, albeit an adverse one, is observed for responses to confidence shocks. If the central bank does not process confidence signals, confidence shocks would not instigate dynamic responses

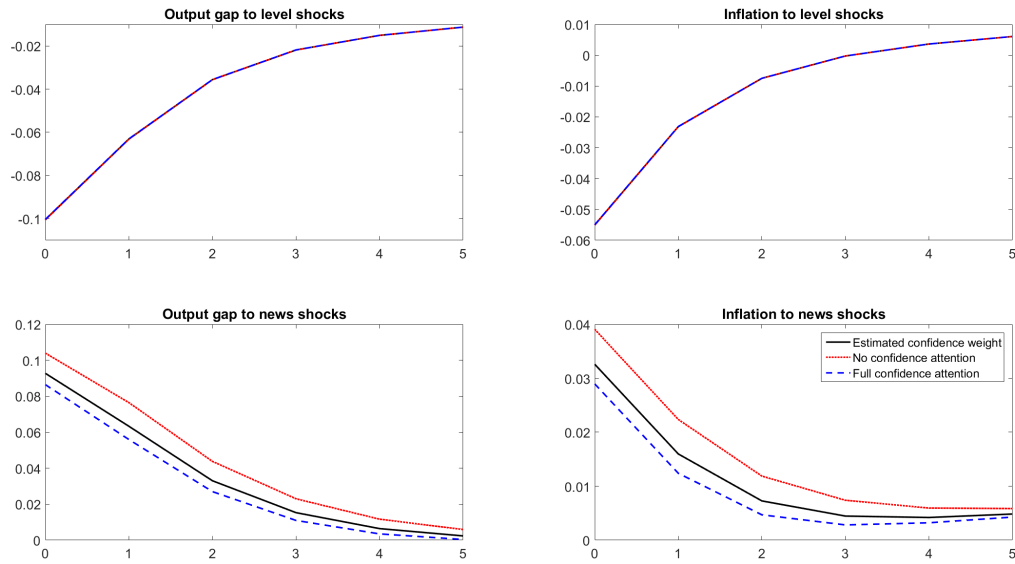


Figure 2.9: Theoretical impulse responses of output gap and inflation rate to technology level and news shocks in the medium-scale DSGE model for estimated central bank attention to confidence $\omega_c = 0.641$ (black solid line), no confidence attention $\omega_c = 0$ (red dotted line), and full confidence attention $\omega_c = 1$ (blue dashed line).

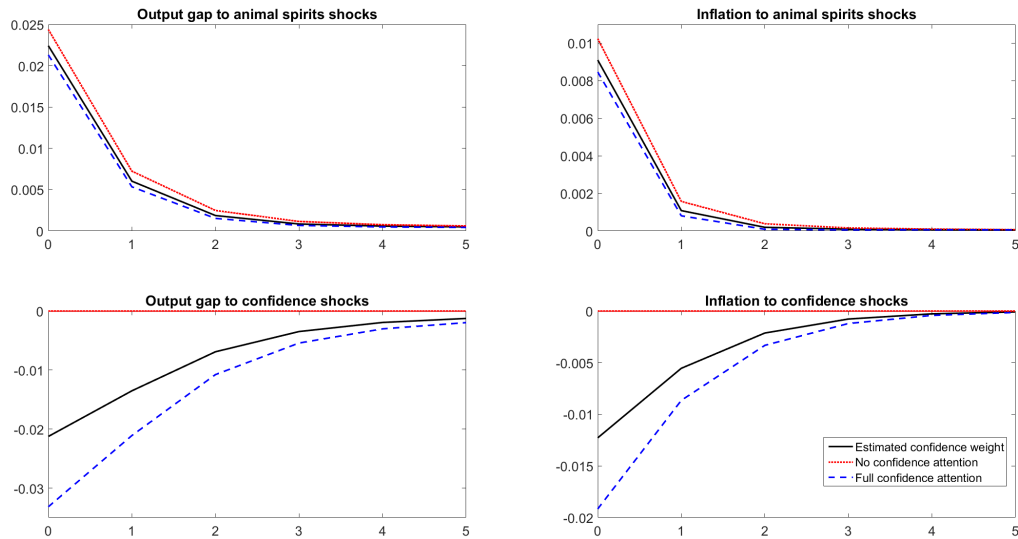


Figure 2.10: Theoretical impulse responses of output gap and inflation rate to animal spirits and confidence shocks in the medium-scale DSGE model for estimated central bank attention to confidence $\omega_c = 0.641$ (black solid line), no confidence attention $\omega_c = 0$ (red dotted line), and full confidence attention $\omega_c = 1$ (blue dashed line).

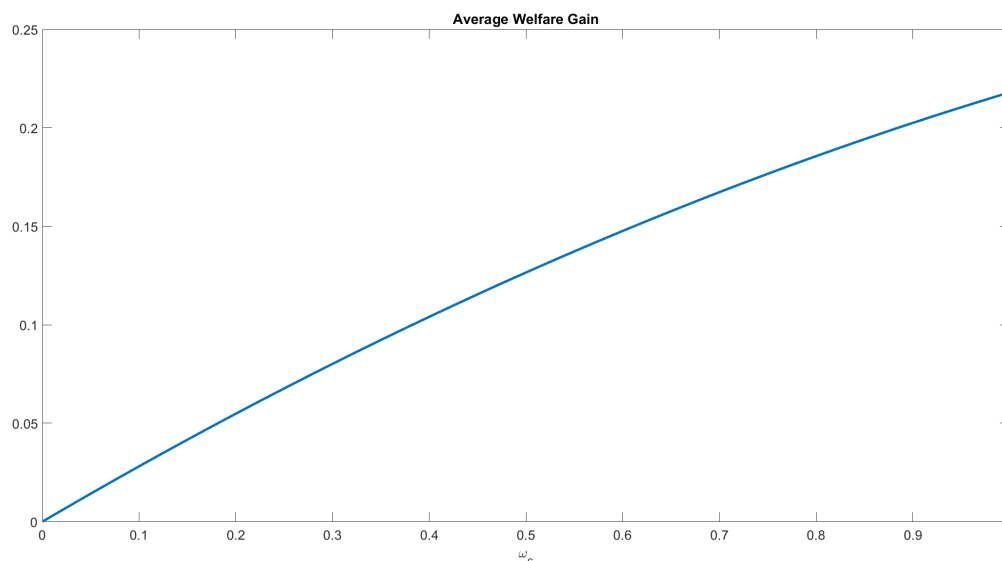


Figure 2.11: Monte Carlo estimated average welfare gain of paying attention to confidence in simulations of the DSGE model with technology level, news, animal spirits and confidence shock for varying confidence weights.

of any variables except confidence. The higher the confidence weight, the more strongly decline both output gap and inflation rate to positive confidence shocks.

The visual assessment of the impulse responses can, thus, once again (c.f. figures 2.3 and 2.4) not unambiguously clarify whether or not paying attention to confidence measures implies positive welfare effects. However, since information processing with the Kalman filter is optimal, central bank's forecast accuracy *must* improve with higher ω_c , such that negative welfare effects of attention to confidence would be immensely surprising.

For this purpose I conduct 10,000 Monte Carlo simulations of 1,000 periods and estimate the average of welfare loss, which is defined as the average percentage deviation of the period utility function from its flex-price counterpart. I repeat this exercise for all values of ω_c between zero and unity. By transforming the welfare loss distribution into its negative percentage deviation from the 'no attention to confidence' ($\omega_c = 0$) benchmark one obtains the welfare gain function shown in figure 2.11.

The maximum welfare gain, associated with optimal confidence information processing ($\omega_c = 1$), is 21.80%, which is slightly more than one third of the welfare gain obtained in the stylized New Keynesian model under optimal monetary policy. At the estimated value of ω_c , one obtains a welfare gain of 15.58% relative to the

Sample	Post. median of ω_c	90% HPDI	Welfare Gain	Share of max. Gain
1960-Q1 – 2018-Q3	0.641	[0.476, 0.777]	15.58%	71.48%
1960-Q1 – 1989-Q4	0.577	[0.417, 0.727]	14.29%	65.55%
1990-Q1 – 2018-Q3	0.669	[0.524, 0.791]	16.14%	74.03%

Table 2.3: Comparison of welfare gain associated with attention of confidence set to posterior median using (1) full sample, (2) subsample 1960-Q1 – 1989-Q4, and (3) subsample 1990-Q1 – 2018-Q3.

$\omega_c = 0$ case. This is 71.48% of the maximum welfare gain, suggesting that the central bank has, overall, done a good job in extracting relevant information from confidence measures.

2.4.7 Variation in Attention to Confidence

The assessment of FOMC minutes in section 2 suggests a potential breakpoint in attention to confidence. To investigate this suspicion I reestimate confidence weight ω_c with the same methodology as described in section 2.4 but divide the sample into the two subsamples 1960-Q1 – 1989-Q4 and 1990-Q1 – 2018-Q3. All other parameter values are set to their full sample estimates.

The results are summarized in table 2.3. In the first subsample, the estimated confidence weight, $\omega_c = 0.577$, is moderately lower, which is associated with a lower welfare gain (14.29%) that accounts for 65.55% of maximum welfare gain. In the reestimation procedure for the second subsample, one obtains a slightly higher degree of attention to confidence, $\omega_c = 0.669$. The welfare gain for this parameterization (16.14%) attains 74.03% of the welfare gain implied by perfect confidence signal processing.

The difference of the posterior medians is very modest and not statistically significant. In summary, there is no convincing evidence that the FOMC has increased its consideration of confidence measures over time, although the results suggest a small improvement.

2.5 Conclusion

Are consumer confidence measures informative about the state of the economy? Given its presence in the popular press, the expenses that public institutions are willing to make for collecting and processing

them, and the emergence of a strand of literature that assigns a role to confidence shocks in theoretical macroeconomic models, journalists, scientists and policymakers are likely to answer this question mostly unambiguously in the affirmative.

From this perspective, it appears puzzling that theoretical models designed for the analysis of monetary policy cannot provide a role for confidence measures as an informative signal for central banks. This paper seeks to fill this gap by providing an intuitive framework with informational frictions, in which monetary policy makers can improve their ability to achieve their targets when paying attention to confidence. The positive welfare effect is large in both a stylized New Keynesian model with optimal monetary policy and a more realistic estimated DSGE model.

2.6 References

- Altig, D., Christiano, L. J., Eichenbaum, M. and Linde, J. (2011). Firm-specific Capital, Nominal Rigidities and the Business Cycle, *Review of Economic Dynamics* **14**(2): 225–247.
- Angeletos, G.-M., Collard, F. and Dellas, H. (2018). Quantifying Confidence, *Econometrica* **86**(5): 1689–1726.
- Angeletos, G.-M. and La’O, J. (2013). Sentiments, *Econometrica* **81**(2): 739–779.
- Aoki, K. (2006). Optimal Commitment Policy under Noisy Information, *Journal of Economic Dynamics and Control* **30**(1): 81–109.
- Barsky, R. B. and Sims, E. R. (2012). Information, Animal Spirits, and the Meaning of Innovations in Consumer Confidence, *American Economic Review* **102**(4): 1343–77.
- Bask, M. and Proaño, C. R. (2016). Optimal monetary policy under learning and structural uncertainty in a new keynesian model with a cost channel and inflation inertia, *Journal of Economic Dynamics and Control* **69**: 112–126.
- Beaudry, P. and Portier, F. (2006). Stock Prices, News, and Economic Fluctuations, *American Economic Review* **96**(4): 1293–1307.
- Blanchard, O. (1993). Consumption and the Recession of 1990-1991, *American Economic Review* **83**(2): 270–274.

- Brayton, F., Laubach, T. and Reifschneider, D. L. (2014). The FRB/US Model: A Tool for Macroeconomic Policy Analysis, *Technical report*, Board of Governors of the Federal Reserve System (US).
- Brayton, F., Mauskopf, E., Reifschneider, D. and Tinsley, P. (1997). The Role of Expectations in the FRB/US Macroeconomic Model, *Federal Reserve Bulletin* **83**: 227.
- Calvo, G. A. (1983). Staggered Prices in a Utility-maximizing Framework, *Journal of Monetary Economics* **12**(3): 383–398.
- Christiano, L. J., Eichenbaum, M. and Evans, C. L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy, *Journal of Political Economy* **113**(1): 1–45.
- Christiano, L. J., Eichenbaum, M. S. and Trabandt, M. (2015). Understanding the Great Recession, *American Economic Journal: Macroeconomics* **7**(1): 110–67.
- Christiano, L. J., Trabandt, M. and Walentin, K. (2010). DSGE Models for Monetary Policy Analysis, *Handbook of Monetary Economics*, Vol. 3, Elsevier, pp. 285–367.
- Clarida, R., Gali, J. and Gertler, M. (2000). Monetary Policy Rules and Macroeconomic Stability: Evidence and some Theory, *Quarterly Journal of Economics* **115**(1): 147–180.
- Cochrane, J. H. (1994). Shocks, *Carnegie-Rochester Conference Series on Public Policy*, Vol. 41, Elsevier, pp. 295–364.
- Collard, F., Dellas, H. and Smets, F. (2009). Imperfect Information and the Business Cycle, *Journal of Monetary Economics* **56**: S38–S56.
- Di Bartolomeo, G., Di Pietro, M. and Giannini, B. (2016). Optimal monetary policy in a new keynesian model with heterogeneous expectations, *Journal of Economic Dynamics and Control* **73**: 373–387.
- Federal Open Market Committee (2019). Statement on Longer-Run Goals and Monetary Policy Strategy, *amended on January* **29**: 2019.
- Franke, R. and Westerhoff, F. (2012). Structural Stochastic Volatility in Asset Pricing Dynamics: Estimation and Model Contest, *Journal of Economic Dynamics and Control* **36**(8): 1193–1211.
- Gali, J. (2015). *Monetary Policy, Inflation, and the Business Cycle: an Introduction to the New Keynesian Framework and its Applications*, Princeton University Press.
- Gasteiger, E. (2014). Heterogeneous expectations, optimal monetary policy, and the merit of policy inertia, *Journal of Money, Credit and Banking* **46**(7): 1535–1554.

- Gasteiger, E. (2021). Optimal constrained interest-rate rules under heterogeneous expectations, *Journal of Economic Behavior and Organization* **190**: 287–325.
- Ilut, C. L. and Schneider, M. (2014). Ambiguous Business Cycles, *American Economic Review* **104**(8): 2368–99.
- Jaimovich, N. and Rebelo, S. (2009). Can News about the Future drive the Business Cycle?, *American Economic Review* **99**(4): 1097–1118.
- Keynes, J. M. (1936). *The General Theory of Employment, Interest and Money*, Kessinger Publishing.
- Kilian, L. (1998). Small-sample Confidence Intervals for Impulse Response Functions, *Review of Economics and Statistics* **80**(2): 218–230.
- Kim, J.-Y. (2002). Limited Information Likelihood and Bayesian Analysis, *Journal of Econometrics* **107**(1-2): 175–193.
- Kim, J.-Y. (2014). An Alternative Quasi Likelihood Approach, Bayesian Analysis and Data-based Inference for Model Specification, *Journal of Econometrics* **178**: 132–145.
- Lengnick, M. and Wohltmann, H.-W. (2016). Optimal monetary policy in a new keynesian model with animal spirits and financial markets, *Journal of Economic Dynamics and Control* **64**: 148–165.
- Lippi, F. and Neri, S. (2007). Information Variables for Monetary Policy in an Estimated Structural Model of the Euro Area, *Journal of Monetary Economics* **54**(4): 1256–1270.
- Ljungqvist, L. and Sargent, T. J. (2018). *Recursive Macroeconomic Theory*, MIT press.
- Lorenzoni, G. (2009). A Theory of Demand Shocks, *American Economic Review* **99**(5): 2050–84.
- Neri, S. and Ropele, T. (2012). Imperfect Information, Real-Time Data and Monetary Policy in the Euro Area, *The Economic Journal* **122**(561): 651–674.
- Offick, S. and Wohltmann, H.-W. (2013). News shocks, nonfundamentalness and volatility, *Economics Letters* **119**(1): 17–19.
- Offick, S. and Wohltmann, H.-W. (2016). Volatility effects of news shocks in new keynesian models with optimal monetary policy, *Economics Letters* **147**: 78–82.
- Orphanides, A. (2001). Monetary Policy Rules based on Real-Time Data, *American Economic Review* **91**(4): 964–985.

- Rotemberg, J. J. and Woodford, M. (1999). Interest Rate Rules in an Estimated Sticky Price Model, *Monetary Policy Rules*, University of Chicago Press, pp. 57–126.
- Schmitt-Grohé, S. and Uribe, M. (2012). What’s news in business cycles, *Econometrica* **80**(6): 2733–2764.
- Smets, F. and Wouters, R. (2007). Shocks and Frictions in US Business Cycles: A Bayesian DSGE approach, *American Economic Review* **97**(3): 586–606.
- Svensson, L. E. and Woodford, M. (2004). Indicator Variables for Optimal Policy under Asymmetric Information, *Journal of Economic Dynamics and Control* **28**(4): 661–690.
- Taylor, J. B. (1993). Discretion versus Policy Rules in Practice, *Carnegie-Rochester Conference Series on Public Policy*, Vol. 39, Elsevier, pp. 195–214.
- Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

2.7 Appendix

2.7.1 Rational Growth Rate Forecasts via Kalman Filtering

Under the assumption of rational expectations, agents form optimal forecasts of the unobserved growth rate which can be computed via the *Kalman filter*, a procedure for instance described by Ljungqvist and Sargent (2018). In each period t agents receive and process two informative signals about the current technology growth rate: actual technology growth, $a_t - a_{t-1}$, and the additional noisy signal, s_t . Defining state vector $\chi_t \equiv (g_{a,t}, g_{a,t-1})'$ and signal vector $\xi_t \equiv (a_t - a_{t-1}, s_t)'$, the state space system is given by

$$\chi_{t+1} = \underbrace{\begin{pmatrix} \rho_{g_a} & 0 \\ 1 & 0 \end{pmatrix}}_{\equiv A} \chi_t + \underbrace{\begin{pmatrix} \sigma_{g_a} & 0 \\ 0 & 0 \end{pmatrix}}_{\equiv C} w_{t+1} \quad (2.37)$$

$$\xi_t = \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\equiv G} \chi_t + \underbrace{\begin{pmatrix} \varepsilon_{a,t} \\ \varepsilon_{s,t} \end{pmatrix}}_{\sim N(0,R)} \quad (2.38)$$

where $w_{t+1} \stackrel{i.i.d.}{\sim} N(0, I)$ and $R \equiv \begin{pmatrix} \sigma_a^2 & 0 \\ 0 & \sigma_s^2 \end{pmatrix}$.

Optimal signal processing involves iterating on the following recursion:

$$\hat{\chi}_{t+1} = A\hat{\chi}_t + K(\xi_t - G\hat{\chi}_t) \quad (2.39)$$

where the Kalman gain, K , is the solution of the system

$$K = A\Sigma G' (G\Sigma G' + R)^{-1} \quad (2.40)$$

$$\Sigma = CC' + K RK_t' + (A - KG)\Sigma(A - KG)' \quad (2.41)$$

2.7.2 Empirical DSGE Model

Households

As before, households maximize a lifetime utility function of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t) \quad (2.42)$$

where the period utility function now accounts for external consumption habit¹² and stochastic preference changes:

$$U(C_t, N_t; Z_t) = Z_t \left(\log(C_t - bC_{t-1}) - \frac{N_t^{1+\nu}}{1+\nu} \right) \quad (2.43)$$

The log of Z_t is the preference shock that follows an AR(1) process.

The budget constraint is

$$C_t + I_t + B_t + T_t = W_t N_t + \frac{R_{t-1}^n}{\Pi_t} B_{t-1} + R_t^k U_t K_{t-1} - \Psi(U_t) K_{t-1} + D_t \quad (2.44)$$

with investment I_t , lump-sum taxation T_t , gross nominal interest and inflation rate, R_t^n and Π_t , capital K_t , rental rate of capital R_t^k and capital utilization rate U_t . Here, W_t denotes the *real* wage. The cost function of changing capital utilization, $\Psi(\cdot)$ is scaled with parameter ψ according to $U \cdot \Psi''(U)/\Psi'(U) = \psi$ (where U denotes steady-state capital utilization).

Capital accumulates according to

$$K_t = (1 - \delta)K_{t-1} + \left(1 - \Phi \left(\frac{I_t}{I_{t-1}} \right) \right) I_t D_t^i \quad (2.45)$$

with depreciation rate δ . The log of D_t^i , the investment demand shock, follows an AR(1) process. Adjustment cost function $\Phi(\cdot)$ satisfies $\Phi'(\cdot) > 0$, $\Phi''(\cdot) > 0$, $\Phi(\gamma) = \Phi'(\gamma) = 0$ and $\Phi''(\gamma) = \varphi$ where γ is the constant steady-state growth rate.

¹²I experimented with internal habit assumptions but did not observe substantially different results.

One obtains the first-order conditions of utility maximization

$$\Lambda_t = \frac{Z_t}{C_t - bC_{t-1}} \quad (\text{I})$$

$$\Lambda_t W_t = Z_t N_t^\nu \quad (\text{II})$$

$$\Lambda_t = \beta E_t \left[\frac{R_t^n \Lambda_{t+1}}{\Pi_{t+1}} \right] \quad (\text{III})$$

$$\Lambda_t = \Lambda_t^k D_t^i \left(1 - \Phi \left(\frac{I_t}{I_{t-1}} \right) - \Phi' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) + \beta E_t \left[\Lambda_{t+1}^k D_{t+1}^i \Phi' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \quad (\text{IV})$$

$$R_t^k = \Psi'(U_t) \quad (\text{V})$$

$$\Lambda_t^k = \beta E_t \left[\Lambda_{t+1}^k (R_{t+1}^k U_{t+1} - \Psi(U_{t+1})) + (1 - \delta) \Lambda_{t+1}^k \right] \quad (\text{VI})$$

where Λ_t and Λ_t^k are the Lagrange multipliers of budget constraint and capital accumulation equation, respectively. Tobin's Q is defined as $Q_t \equiv \Lambda_t^k / \Lambda_t$.

Goods Producers and Price Setting

Intermediate goods production now requires physical capital as input:

$$Y_t(j) = A_t (K_t^s)^\alpha N_t^{1-\alpha} \quad (2.46)$$

K_t^s denotes the capital provided by households:

$$K_t^s = U_t K_{t-1} \quad (2.47)$$

First-order conditions of cost minimization are

$$W_t = (1 - \alpha) M C_t A_t (K_t^s)^\alpha N_t^{-\alpha} \quad (2.48)$$

$$R_t^k = \alpha M C_t A_t (K_t^s)^{\alpha-1} N_t^{1-\alpha} \quad (2.49)$$

which implies

$$K_t^s = \frac{\alpha}{1 - \alpha} \frac{W_t}{R_t^k} N_t \quad (2.50)$$

with marginal costs given by

$$MC_t = A_t^{-1} W_t (K_t^s)^{-\alpha} N_t^\alpha \quad (2.51)$$

Assumptions about price and demand aggregation as well as Calvo pricing remain as in the baseline NKM with the caveat that the latter is subject to partial indexation measured by parameter ι_p .

Government Expenditures and Resource Constraint

The log income share of government expenditures follows the AR(1) process

$$\log(G_t/Y_t) = (1 - \rho_g) \log(G/Y) + \rho_g \log(G_{t-1}/Y_{t-1}) + \varepsilon_{g,t} \quad (2.52)$$

Government expenditures are financed via lump-sum taxation

$$G_t + \frac{R_{t-1}^n}{\Pi_t} B_{t-1} = T_t + B_t \quad (2.53)$$

The aggregate resource constraint is given by

$$Y_t = C_t + I_t + G_t + \Psi(U_t) K_{t-1} \quad (2.54)$$

Log-linearized System

After detrending, log-linearizing the model equations around the steady state and denoting log-linearized variables by lowercase letters, one obtains the system of equations

$$c_t = \frac{1}{1+b} E_t [c_{t+1}] + \frac{b}{1+b} c_{t-1} - \frac{1-b}{1+b} (r_t^n - E_t [\pi_{t+1}]) + \frac{(1-\rho_z)(1-b)}{1+b} z_t \quad (2.55)$$

$$n_t = \frac{1}{\nu} w_t - \frac{1}{\nu(1-b)} (c_t - b c_{t-1}) \quad (2.56)$$

$$i_t = \frac{\beta}{1+\beta} E_t [i_{t+1}] + \frac{1}{1+\beta} i_{t-1} + \frac{1}{\varphi(1+\beta)} (g_t + d_t^i) \quad (2.57)$$

$$r_t^k = \frac{\psi}{1-\psi} u_t \quad (2.58)$$

$$q_t = \beta(1-\delta) E_t [q_{t+1}] + (1-\beta(1-\delta)) E_t [r_{t+1}^k] - (r_t^n - E_t [\pi_{t+1}]) \quad (2.59)$$

$$y_t = c_y c_t + i_y i_t + g_y (g_t + y_t) + \frac{1-\beta(1-\delta)}{\beta\delta} i_y u_t \quad (2.60)$$

$$y_t = \alpha k_t^s + (1-\alpha) n_t + a_t \quad (2.61)$$

$$k_t^s = u_t + k_{t-1} \quad (2.62)$$

$$k_t = (1-\delta) k_{t-1} + \delta (i_t + d_t^i) \quad (2.63)$$

$$k_t^s = n_t + w_t - r_t^k \quad (2.64)$$

$$m c_t = w_t + \alpha (n_t - k_t^s) - a_t \quad (2.65)$$

$$\pi_t = \frac{\beta}{1+\beta\iota_p} E_t [\pi_{t+1}] + \frac{\iota_p}{1+\beta\iota_p} \pi_{t-1} + \frac{\lambda}{1+\beta\iota_p} (m c_t + \varepsilon_{p,t}) \quad (2.66)$$

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t} \quad (2.67)$$

The system is closed with the monetary policy rule (2.25).

2.7.3 Optimal Discretion with Asymmetric Information - Proof of Proposition

2

Proof. Denoting $v_t \equiv (\chi_t, \hat{\chi}_t, \xi_t)'$, the equations (2.37), (2.38) and (2.39) can be rewritten as

$$\begin{pmatrix} I & 0 & 0 \\ 0 & I & 0 \\ -G & 0 & I \end{pmatrix} v_{t+1} = \begin{pmatrix} A & 0 & 0 \\ 0 & A - G & K \\ 0 & 0 & 0 \end{pmatrix} v_t + \begin{pmatrix} \varepsilon_{g_a,t+1} & 0 & 0 & 0 & \varepsilon_{a,t+1} & \varepsilon_{s,t+1} \end{pmatrix}' \quad (2.68)$$

$$\Leftrightarrow v_{t+1} = \tilde{A}_{11} v_t + \varepsilon_{v,t+1} \quad (2.69)$$

Together with equations (2.8) and (2.9), the system of equations is given by¹³

$$\begin{pmatrix} v_{t+1} \\ \tilde{E}E_t[\omega_{t+1}] \end{pmatrix} = \underbrace{\begin{pmatrix} \tilde{A}_{11} & 0 \\ \tilde{A}_{21} & \tilde{A}_{22} \end{pmatrix}}_{\equiv \tilde{A}} \begin{pmatrix} v_t \\ \omega_t \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ \tilde{B}_2 \end{pmatrix}}_{\equiv \tilde{B}} r_t^n + \begin{pmatrix} \varepsilon_{v,t+1} \\ 0 \end{pmatrix} \quad (2.70)$$

and the compact form of the period loss function, embedded in equation (2.15),

$$L_t = \omega_t' W \omega_t \quad (2.71)$$

where $\omega_t \equiv (\pi_t, x_t)'$. Using the more compact notation for central bank expectations $v_{t|t} = E[v_t | I_t^{CB}]$, the solution of optimal discretion has the general form

$$r_t^n = F^* v_{t|t} \quad (2.72)$$

$$\omega_{t|t} = G^* v_{t|t} \quad (2.73)$$

which is verified with the method of undetermined coefficients in the following.

From (2.72) and the upper block of (2.70), one obtains

$$\omega_{t+1|t} = G^* v_{t+1|t} = G^* \tilde{A}_{11} v_{t|t} \quad (2.74)$$

¹³Note that w.l.g. the constant β^{-1} in the natural real rate of interest equation can be neglected.

From the lower block of (2.70) one gets

$$\tilde{E}\omega_{t+1|t} = \tilde{A}_{21}v_{t|t} + \tilde{A}_{22}\omega_{t|t} + \tilde{B}_2r_t^n \quad (2.75)$$

Premultiplying the former with \tilde{E} , setting equal to the latter and solving for $w_{t|t}$ provides

$$\omega_{t|t} = \underbrace{\tilde{A}_{22}^{-1} \left(\tilde{E}G^* \tilde{A}_{11} - \tilde{A}_{21} \right)}_{\equiv A^*} v_{t|t} - \underbrace{\tilde{A}_{22}^{-1} \tilde{B}_2}_{\equiv B^*} r_t^n \quad (2.76)$$

With this result the expected period loss function can be rewritten as

$$L_{t|t} = \omega'_{t|t} W \omega_{t|t} = v'_{t|t} \underbrace{A^{*'} W A^*}_{\equiv Q^*} v_{t|t} + 2v'_{t|t} \underbrace{A^{*'} W B^*}_{\equiv U^*} r_t^n + r_t^{n'} \underbrace{B^{*'} W B^*}_{\equiv R} r_t^n \quad (2.77)$$

Under the assumption of (2.72), the optimal value to the problem has the quadratic form $v'_{t|t} V v_{t|t}$ such that the welfare minimization problem can be expressed by the following Bellman equation:

$$v'_{t|t} V v_{t|t} = \min_{r_t^n} \left\{ L_{t|t} + \beta E \left(v_{t+1|t+1} V v_{t+1|t+1} | I_t^{CB} \right) \right\} \quad (2.78)$$

with first-order condition

$$r_t^n = \underbrace{-R^{*-1} U^{*'}}_{\equiv F^*} v_{t|t} \quad (2.79)$$

and from (2.76)

$$\omega_{t|t} = \underbrace{A^* + B^* F^*}_{\equiv G^*} v_{t|t} \quad (2.80)$$

Therefore, it is verified that the optimal discretion solution has the form described in (2.72) and (2.73).

Noting that

$$F^* = -(B^{*-1} W B^*)^{-1} B^{*'} W A^*$$

and using

$$\tilde{A}_{22} \equiv \begin{pmatrix} 1 & -\kappa \\ 0 & 1 \end{pmatrix}, \quad \tilde{B}_2 \equiv \begin{pmatrix} 0 \\ \frac{1}{\sigma} \end{pmatrix}, \quad W \equiv \begin{pmatrix} w_\pi & 0 \\ 0 & w_x \end{pmatrix}$$

implies

$$G^* = (\kappa^2 w_\pi + w_x)^{-1} \begin{pmatrix} w_x & -\kappa w_x \\ -\kappa w_\pi & \kappa^2 w_\pi \end{pmatrix} A^*$$

We have

$$\tilde{A}_{21} \equiv \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\psi_{y_a} & 0 & 0 & 0 \end{pmatrix}, \quad \tilde{E} \equiv \begin{pmatrix} \beta & 0 \\ \frac{1}{\sigma} & 1 \end{pmatrix}$$

such that one may note that

$$(\kappa^2 w_\pi + w_x)^{-1} \begin{pmatrix} w_x & -\kappa w_x \\ -\kappa w_\pi & \kappa^2 w_\pi \end{pmatrix} \tilde{A}_{22}^{-1} \tilde{A}_{21} = \mathbf{0}_{2 \times 6}$$

It follows that

$$\begin{aligned} G^* &= (\kappa^2 w_\pi + w_x)^{-1} \begin{pmatrix} w_x & -\kappa w_x \\ -\kappa w_\pi & \kappa^2 w_\pi \end{pmatrix} \tilde{A}_{22}^{-1} \tilde{E} G^* \tilde{A}_{11} \\ &= \begin{pmatrix} \frac{w_\pi}{(\kappa^2 w_\pi + w_x)^{-1}} \\ \frac{-\kappa w_\pi}{(\kappa^2 w_\pi + w_x)^{-1}} \end{pmatrix} \begin{pmatrix} \rho_{g_a}(g_{11}^* + g_{16}^*) + g_{12}^* + g_{15}^* & 0 & \rho_{g_a} g_{13}^* & -g_{13}^* & k_{11} g_{13}^* + k_{21} g_{14}^* & k_{12} g_{13}^* + k_{22} g_{14}^* \end{pmatrix} \end{aligned}$$

where g_{ij}^* is the j th element of row i of matrix G^* , respectively. The unique solution for this equation is

$$G^* = \mathbf{0}_{2 \times 6}$$

Using this result gives

$$A^* = \tilde{A}_{22}^{-1} \tilde{A}_{21}$$

which implies

$$F^* = \begin{pmatrix} 0, & 0, & \sigma\psi_{ya}, & 0, & 0, & 0 \end{pmatrix}$$

Plugging this result into (2.79) confirms that the nominal interest rate tracks the natural real rate of interest under optimal discretion. \square

2.7.4 Optimal Commitment with Asymmetric Information - Proof of Proposition 3

Proof. Following (Svensson and Woodford; 2004, Ch. 3), the Lagrangian of the optimal commitment problem can be expressed in compact form as

$$\mathcal{L} = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[0.5\omega'_t W \omega_t + \tilde{\Xi}'_t \tilde{A}_{11} v_t + \Xi'_t \left(\tilde{A}_{21} v_t + \tilde{A}_{22} \omega_t + \tilde{B}_2 r_t^n \right) - \beta^{-1} \left(\tilde{\Xi}'_{t-1} v_t + \Xi'_{t-1} \tilde{E} \omega_t \right) \right] \right\} \quad (2.81)$$

where $\Xi'_t \equiv (\varphi'_{1,t}, \varphi'_{2,t})$ is the vector of Lagrangian multipliers. The first-order condition w.r.t. r_t^n is given by

$$B'_2 \Xi_{t|t} = 0 \quad (2.82)$$

which implies

$$\varphi_{2,t|t} = 0 \quad (2.83)$$

The first-order condition w.r.t. ω_t is

$$W \omega_t = -\tilde{A}'_{22} \Xi_t + \beta^{-1} \tilde{E}' \Xi_{t-1} \quad (2.84)$$

Note that the general setup is, thus, equivalent to the one in Aoki (2006) who contributes most of the following solution procedure.

The two rows of CB expectations of equation (2.84) are given by

$$\pi_{t|t} = \frac{1}{w_\pi} \varphi_{2,t|t} - \frac{1}{\beta \sigma w_\pi} \varphi_{1,t-1} - \frac{1}{w_\pi} \varphi_{2,t-1} \quad (2.85)$$

$$x_{t|t} = -\frac{\kappa}{w_x} \varphi_{2,t|t} - \frac{1}{\beta w_x} \varphi_{1,t-1} \quad (2.86)$$

Equation (2.85) implies

$$\pi_{t+1|t} = \frac{1}{w_\pi} \varphi_{2,t+1|t} - \frac{1}{w_\pi} \varphi_{2,t|t}$$

From (2.8) one has

$$\pi_{t+1|t} = \frac{1}{\beta} \pi_{t|t} - \frac{\kappa}{\beta} x_{t|t}$$

Plugging in (2.85) and (2.86), and setting the two expressions equal yields

$$\varphi_{2,t+1|t} - \left(1 + \beta^{-1} + \frac{\kappa^2 w_\pi}{\beta w_x}\right) \varphi_{2,t|t} + \left(\frac{1}{\beta^2 \sigma} - \frac{\kappa w_\pi}{\beta^2 w_x}\right) \varphi_{1,t-1} + \beta^{-1} \varphi_{2,t-1} = 0 \quad (2.87)$$

Iterating forward one period gives

$$\varphi_{2,t+2|t} - \left(1 + \beta^{-1} + \frac{\kappa^2 w_\pi}{\beta w_x}\right) \varphi_{2,t+1|t} + \beta^{-1} \varphi_{2,t|t} = 0 \quad (2.88)$$

The characteristic equation associated with (2.88) is

$$f(\lambda) = \lambda^2 - \left(1 + \beta^{-1} + \frac{\kappa^2 w_\pi}{\beta w_x}\right) \lambda + \beta^{-1} \quad (2.89)$$

Since it holds for the discriminant that

$$\mathcal{D} = \left(1 + \beta^{-1} + \frac{\kappa^2 w_\pi}{\beta w_x}\right)^2 - 4\beta^{-1} > 0$$

and $f(1) = -\beta^{-1} + \frac{\kappa^2 w_\pi}{\beta w_x} < 0$, $f(0) = \beta^{-1} > 0$, the roots of the characteristic equation, λ_1 and λ_2 , are real and satisfy

$$0 < \lambda_1 < 1 < \lambda_2$$

Thus, one obtains the unique stable solution

$$\varphi_{2,t+1|t} = \lambda_1 \varphi_{2,t|t} \quad (2.90)$$

Plugging this solution into (2.87) gives after a number of rearrangements

$$\varphi_{2,t|t} = \lambda_1 \left(\left(\frac{1}{\beta\sigma} + \frac{\kappa w_\pi}{\beta w_x} \right) \varphi_{1,t-1} + \varphi_{2,t-1} \right) \quad (2.91)$$

such that one can write

$$\Xi_{t|t} = \lambda_1 \begin{pmatrix} 0 & 0 \\ \frac{1}{\beta\sigma} + \frac{\kappa w_\pi}{\beta w_x} & 1 \end{pmatrix} \Xi_{t-1} \equiv \tilde{\Sigma} \Xi_{t-1} \quad (2.92)$$

Plugging the solution of $\Xi_{t|t}$ into (2.84) after taking expectations gives

$$\omega_t = W^{-1} \left(-\tilde{A}'_{22} \tilde{\Sigma} + \beta^{-1} \tilde{E}' \right) \Xi_{t-1} \equiv \tilde{\Gamma} \Xi_{t-1} \quad (2.93)$$

Taking expectations of (2.9) and solving for r_t^n gives

$$\begin{aligned} r_t^n &= E [r_t^* | I_t^{CB}] + \sigma \left(\left(\frac{1}{\sigma}, 1 \right) \omega_{t+1|t} - \left(0, 1 \right) \omega_{t|t} \right) \\ &= E [r_t^* | I_t^{CB}] + \sigma \left(\left(\frac{1}{\sigma}, 1 \right) \tilde{\Gamma} \Xi_{t|t} - \left(0, 1 \right) \tilde{\Gamma} \Xi_{t-1} \right) \\ &= E [r_t^* | I_t^{CB}] + \sigma \left(\left(\frac{1}{\sigma}, 1 \right) \tilde{\Gamma} \tilde{\Sigma} - \left(0, 1 \right) \tilde{\Gamma} \right) \Xi_{t-1} \\ &\equiv E [r_t^* | I_t^{CB}] + \Phi \Xi_{t-1} \end{aligned} \quad (2.94)$$

which completes the proof of proposition 3. □

2.7.5 Solution of the NKM under Discretion

Using the notation of section 2.7.3, one has under optimal discretion

$$\begin{aligned} \omega_t &= \tilde{A}_{22}^{-1} \tilde{E} E_t [\omega_{t+1}] + \tilde{A}_{22}^{-1} \tilde{B}_2 \sigma \psi_{ya} (\hat{g}_{a,t} - E [\hat{g}_{a,t} | I_t^{CB}]) \\ &= \sum_{i=0}^{\infty} \left(\tilde{A}_{22}^{-1} \tilde{E} \right)^i \tilde{A}_{22}^{-1} \tilde{B}_2 \sigma \psi_{ya} E_t [\hat{g}_{a,t+i} - E [\hat{g}_{a,t+i} | I_{t+i}^{CB}]] \end{aligned} \quad (2.95)$$

As private agents are aware that the central bank will have all current information in every future period, it holds that

$$E_t [E [\hat{g}_{a,t+i}|I_{t+i}^{CB}]] = E_t [\hat{g}_{a,t+i}] \quad \forall i > 0 \quad (2.96)$$

such that

$$\begin{aligned} \omega_t &= \tilde{A}_{22}^{-1} \tilde{B}_2 \sigma \psi_{ya} (\hat{g}_{a,t} - E [\hat{g}_{a,t}|I_t^{CB}]) \\ &= \begin{pmatrix} \kappa \\ 1 \end{pmatrix} \psi_{ya} (\hat{g}_{a,t} - E [\hat{g}_{a,t}|I_t^{CB}]) \end{aligned} \quad (2.97)$$

2.7.6 Solution of the NKM under Commitment

Together with (2.84), the system under optimal commitment is given by

$$\begin{pmatrix} \tilde{E} & \mathbf{0} \\ \mathbf{0} & \tilde{A}'_{22} \end{pmatrix} \begin{pmatrix} E_t \omega_{t+1} \\ \Xi_t \end{pmatrix} = \begin{pmatrix} \tilde{A}_{22} & \tilde{B}_2 \Phi \\ -W & \beta^{-1} \tilde{E}' \end{pmatrix} \begin{pmatrix} \omega_t \\ \Xi_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{B}_2 \sigma \psi_{ya} \\ \mathbf{0} \end{pmatrix} (E [\hat{g}_{a,t}|I_t^{CB}] - \hat{g}_{a,t}) \quad (2.98)$$

which implies

$$\begin{aligned} \begin{pmatrix} E_t \omega_{t+1} \\ \Xi_t \end{pmatrix} &= \underbrace{\begin{pmatrix} \tilde{E}^{-1} \tilde{A}_{22} & \tilde{E}^{-1} \tilde{B}_2 \Phi \\ -(\tilde{A}'_{22})^{-1} W & \beta^{-1} (\tilde{E} \tilde{A}_{22}^{-1})' \end{pmatrix}}_{\equiv \hat{A}} \begin{pmatrix} \omega_t \\ \Xi_{t-1} \end{pmatrix} \\ &\quad + \begin{pmatrix} \tilde{E}^{-1} \tilde{B}_2 \sigma \psi_{ya} \\ \mathbf{0} \end{pmatrix} (E [\hat{g}_{a,t}|I_t^{CB}] - \hat{g}_{a,t}) \end{aligned} \quad (2.99)$$

where it is assumed that saddlepath stability holds, i.e. that exactly two (of the four) eigenvalues of \hat{A} are larger than unity in modulus, respectively. Defining the matrix with the eigenvalues on its diagonal as

$$\Lambda \equiv \begin{pmatrix} \Lambda_1 & \mathbf{0} \\ \mathbf{0} & \Lambda_2 \end{pmatrix} \quad (2.100)$$

where Λ_1 contains all eigenvalues that are larger than unity in modulus, and the corresponding matrix of eigenvectors as

$$H \equiv \begin{pmatrix} H_1 & H_2 \end{pmatrix} \quad (2.101)$$

one can rewrite system (2.99) with the Jordan decomposition method as

$$\begin{pmatrix} E_t \omega_{t+1} \\ \Xi_t \end{pmatrix} = H \Lambda H^{-1} \begin{pmatrix} \omega_t \\ \Xi_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{E}^{-1} \tilde{B}_2 \sigma \psi_{ya} \\ \mathbf{0} \end{pmatrix} (E [\hat{g}_{a,t} | I_t^{CB}] - \hat{g}_{a,t}) \quad (2.102)$$

After defining auxiliary variables

$$\begin{pmatrix} E_t \tilde{\omega}_{t+1} \\ \tilde{\Xi}_t \end{pmatrix} \equiv H^{-1} \begin{pmatrix} E_t \omega_{t+1} \\ \Xi_t \end{pmatrix} \quad (2.103)$$

and

$$\hat{B} = \begin{pmatrix} \hat{B}_1 \\ \hat{B}_2 \end{pmatrix} \equiv H^{-1} \begin{pmatrix} \tilde{E}^{-1} \tilde{B}_2 \sigma \psi_{ya} \\ \mathbf{0} \end{pmatrix} \quad (2.104)$$

the system obtained after rearrangements

$$\begin{pmatrix} E_t \tilde{\omega}_{t+1} \\ \tilde{\Xi}_t \end{pmatrix} = \Lambda \begin{pmatrix} \tilde{\omega}_t \\ \tilde{\Xi}_{t-1} \end{pmatrix} + \hat{B} (E [\hat{g}_{a,t} | I_t^{CB}] - \hat{g}_{a,t}) \quad (2.105)$$

can be solved blockwise: the upper subsystem via forward iteration and the lower one via backward iteration.

Using (2.96), one obtains as the solution of the former

$$\begin{aligned} \tilde{\omega}_t &= \Lambda_1^{-1} E_t \tilde{\omega}_{t+1} - \Lambda_1^{-1} \hat{B}_1 (E [\hat{g}_{a,t} | I_t^{CB}] - \hat{g}_{a,t}) \\ &= - \sum_{i=0}^{\infty} \Lambda_1^{-i-1} \hat{B}_1 E_t [(E [\hat{g}_{a,t+i} | I_{t+i}^{CB}] - \hat{g}_{a,t+i})] \\ &= -\Lambda_1^{-1} \hat{B}_1 (E [\hat{g}_{a,t} | I_t^{CB}] - \hat{g}_{a,t}) \end{aligned} \quad (2.106)$$

The solution of the latter is given by

$$\begin{aligned}\tilde{\Xi}_t &= \Lambda_2 + \hat{B}_2 (E [\hat{g}_{a,t}|I_t^{CB}] - \hat{g}_{a,t}) \\ &= \sum_{i=0}^{\infty} \Lambda_2^i \hat{B}_2 (E [\hat{g}_{a,t-i}|I_{t-i}^{CB}] - \hat{g}_{a,t-i})\end{aligned}\quad (2.107)$$

Using (2.103), the solution for ω_t is

$$\omega_t = \sum_{i=0}^{\infty} \tilde{\Phi}_i (E [\hat{g}_{a,t-i}|I_{t-i}^{CB}] - \hat{g}_{a,t-i})\quad (2.108)$$

where

$$\tilde{\Phi}_0 \equiv -\Lambda_1^{-1} \hat{B}_1\quad (2.109)$$

and

$$\tilde{\Phi}_i \equiv \Lambda_2^i \hat{B}_2 \quad \forall i > 0\quad (2.110)$$

2.7.7 Solution of the DSGE model via Fixed-Point Iteration

The sticky-price model described in section 2.4 has the form

$$AE_t X_{t+1} + BX_t + CX_{t-1} + D\eta_t + Fr_t^n = 0\quad (2.111)$$

where X_t is the vector collecting all endogenous variables and η_t denotes the vector containing all exogenous shock processes for which it holds

$$\eta_t = R\eta_{t-1} + Jg_{a,t} + \varepsilon_t\quad (2.112)$$

The monetary policy rule has the general form

$$r_t^n = \rho_r r_{t-1}^n + PE [X_t|I_t^{CB}] + P_l X_{t-1} + P^f E [y_t^f|I_t^{CB}] + Q\eta_t\quad (2.113)$$

where y_t^f denotes output that would be obtained in a hypothetical flex-price economy.

The solution for all variables of the flex-price economy with backward-looking elements, X_t^f , is given by¹⁴

$$X_t^f = \Gamma^f X_{t-1}^f + \Upsilon^f \eta_t + \Psi^f \hat{g}_{a,t} \quad (2.114)$$

The flex-price output (and all other variables without backward-looking elements) can be expressed as

$$y_t^f = \gamma_y^f X_{t-1}^f + \phi_y^f \eta_t + \psi_y^f \hat{g}_{a,t} \quad (2.115)$$

It will be shown in the following via *guess-and-verify* that the solution has the form

$$X_t = \Gamma X_{t-1} + \tilde{\Gamma} X_{t-1}^f + \Gamma_r r_{t-1}^n + \Upsilon \eta_t + \Psi \hat{g}_{a,t} + \Psi^{CB} E [\hat{g}_{a,t} | I_t^{CB}] \quad (2.116)$$

which implies for central bank expectations

$$E [X_t | I_t^{CB}] = X_t + \Psi (E [\hat{g}_{a,t} | I_t^{CB}] - \hat{g}_{a,t}) \quad (2.117)$$

Plugging (2.115) and (2.117) into the monetary policy rule (2.113) yields

$$\begin{aligned} r_t^n &= P X_t + P_l X_{t-1} + P^f \gamma_y^f X_{t-1}^f + \rho_r r_{t-1}^n + (P^f \phi_y^f + Q) \eta_t - P \Psi \hat{g}_{a,t} \\ &\quad + (P \Psi + P^f \psi_y^f) E [\hat{g}_{a,t} | I_t^{CB}] \end{aligned} \quad (2.118)$$

Using (2.112), (2.116) and $E_t [\hat{g}_{a,t+1}] = E_t [E [\hat{g}_{a,t+1} | I_{t+1}^{CB}]] = \rho_{g_a} \hat{g}_{a,t}$ one obtains for the real sector expectations

$$E_t [X_{t+1}] = \Gamma X_t + \tilde{\Gamma} X_t^f + \Gamma_r r_t^n + \Upsilon R \eta_t + (\Upsilon J + (\Psi + \Psi^{CB}) \rho_{g_a}) \hat{g}_{a,t} \quad (2.119)$$

¹⁴The solution is fairly simply obtained with the same fixed-point iteration methodology described here and is, thus, left out.

Finally, plugging (2.118) and (2.119) into (2.111) yields

$$\begin{aligned}
-(A\Gamma + (A\Gamma_r + F)P + B)X_t &= ((A\Gamma_r + F)P_l + C)X_{t-1} \\
&+ \left((A\Gamma_r + F)P^f\gamma_y^f + A\tilde{\Gamma}\Gamma^f \right) X_{t-1}^f \\
&+ (A\Gamma_r + F)\rho_r r_{t-1}^n \\
&+ \left((A\Gamma_r + F)(P^f\phi_y^f + Q) + A\tilde{\Gamma}\Upsilon^f + A\Upsilon R + D \right) \eta_t \\
&+ \left(-(A\Gamma_r + F)P\Psi + A\tilde{\Gamma}\Psi^f + A(\Upsilon J + (\Psi + \Psi^{CB})\rho_{g_a}) \right) \hat{g}_{a,t} \\
&+ (A\Gamma_r + F)(P\Psi + P^f\psi_y^f) E[\hat{g}_{a,t} | I_t^{CB}]
\end{aligned} \tag{2.120}$$

which is indeed of the same form as guess (2.116) with coefficient matrices

$$\begin{aligned}
\Gamma &= -(A\Gamma + (A\Gamma_r + F)P + B)^{-1}((A\Gamma_r + F)P_l + C) \\
\tilde{\Gamma} &= -(A\Gamma + (A\Gamma_r + F)P + B)^{-1}\left((A\Gamma_r + F)P^f\gamma_y^f + A\tilde{\Gamma}\Gamma^f\right) \\
\Gamma_r &= -(A\Gamma + (A\Gamma_r + F)P + B)^{-1}(A\Gamma_r + F)\rho_r \\
\Upsilon &= -(A\Gamma + (A\Gamma_r + F)P + B)^{-1}\left((A\Gamma_r + F)(P^f\phi_y^f + Q) + A\tilde{\Gamma}\Upsilon^f + A\Upsilon R + D\right) \\
\Psi &= -(A\Gamma + (A\Gamma_r + F)P + B)^{-1}\left(-(A\Gamma_r + F)P\Psi + A\tilde{\Gamma}\Psi^f + A(\Upsilon J + (\Psi + \Psi^{CB})\rho_{g_a})\right) \\
\Psi^{CB} &= -(A\Gamma + (A\Gamma_r + F)P + B)^{-1}(A\Gamma_r + F)(P\Psi + P^f\psi_y^f)
\end{aligned}$$

The above coefficient equations are solved numerically.

Chapter 3

Credit Risk, Endogenous Credibility, and Unconventional Monetary Policy

Abstract

A microfounded medium-scale DSGE model with endogenous credibility, bounded rationality, credit cycles and aggregate risk is presented. The key mechanisms interact with each other in a feedback loop that amplifies real disturbances and risk-related shocks. The adverse effects on welfare and stability are larger if a zero lower bound is considered and if the central bank faces an unfavorable credibility environment. I propose a number of nonstandard conventional and unconventional monetary policy tools, namely a risk-augmented Taylor rule, active credit policy and central bank stock purchases, and analyze their efficiency.

JEL classification: D84, E44, E52, E58, E71

Keywords: Monetary policy, Bounded Rationality, Animal Spirits, Credit cycles, New Keynesian macroeconomics, DSGE models

3.1 Introduction

In this paper I investigate efficiency of nonstandard conventional and unconventional monetary policy measures at the zero lower bound subject to imperfect central bank credibility, credit cycles and risk-related disturbances. In a microfounded macroeconomic model of New Keynesian type with boundedly rational expectations, shocks to aggregate risk, collateral constraints, and credit default risk generate credit booms and busts that are a key driving force of economic fluctuations. The adverse effects of credit crunches on economic stability and welfare are amplified when accounting for a binding zero lower bound. The amplifier is more severe in scenarios where the central bank faces low levels of credibility. I propose three nonstandard monetary policy measures to ease the negative effects: an aggregate risk augmented policy rule, direct credit intermediation, and central bank stock purchases. The efficiency of these tools crucially depends on the underlying credibility environment.

At the heart of the model there are two main mechanisms. First, akin to Hommes and Lustenhouwer (2019) endogenous central bank credibility is determined based on discrete choice theory as proposed by Brock and Hommes (1997). When forming their expectations, heterogeneous agents choose between *believing that the central bank will achieve its target value* and an alternative (backward-looking) heuristic. The fraction of agents that anchors their expectations to the central bank target, a natural measure for credibility, depends on the central bank's track record of achieving its goals in the past but may also be reduced in the presence of a binding effective lower bound on the nominal interest rate. Second, I adopt the aggregate macroeconomic risk measure of Proaño and Lojak (2020) that depends negatively on credibility and determines the risk-induced interest rate spread.

Unlike in Proaño and Lojak (2020), where the measure is linked to the risk premium on government bonds in an ad-hoc manner, aggregate risk is transmitted to the economy through the credit market via two links: it is positively correlated with the default risk of indebted entrepreneurs and the loan restrictions imposed by banks. Based on the structure of Becard and Gauthier (2022), the credit market consists of banks that intermediate deposits from households as loans to entrepreneurs subject to idiosyncratic default risk. The intermediaries mitigate this risk by requiring collateral subject to an endogenous loan-to-value (LTV) ratio that is seized in case of credit default.

This setting is integrated into a medium-scale DSGE model with capital accumulation, nominal pricing rigidities of Calvo (1983) type, active fiscal policy and endogenous government debt accumulation. Fluctuations

in credit risk and collateral value cause the emergence of endogenous credit cycles that are transmitted to the real economy and lower aggregate welfare. The real disturbances lower central bank credibility which, in turn, increases macroeconomic risk which further aggravates the adverse effects of the credit market shocks. The negative welfare consequences are substantially amplified when taking into account a zero-lower bound which restrains the central bank's capacity to apply *conventional* expansionary monetary policy measures, further undermining its credibility, and gives rise to deflationary spirals that endanger economic stability.

To salvage its incapability to respond to credit crunches in proximity to the lower bound the central bank is provided with a number of both conventional and *unconventional* tools which are examined regarding their effectiveness in reducing aggregate welfare loss. The following results stand out. First, the relative effectiveness of the measures depends crucially on the credibility environment. If agents are, in general, fairly inclined to believe in the central bank's capabilities, i.e. credibility is high, an optimal risk-augmented policy rule outperforms all other tools under consideration by a wide margin. However, in low-credibility scenarios it is more efficient for the central bank to engage in direct credit intermediation and facilitate lending. Second, while active credit policy has positive effects on economic stability it is more risky to tie the policy rule to aggregate risk as parameter values only modestly higher than the optimal ones (in terms of welfare maximization) can cause explosive simulation paths. Third, when extending the model with an endogenous stock market along the lines of Lengnick and Wohltmann (2016) where shares of entrepreneurs are traded, the central bank can reduce the detrimental welfare effects of the feedback loop between the financial and the real sector by engaging in stock purchases. However, the effect is relatively muted compared to the other tools under consideration.

This work is closely related to that of De Grauwe and Macchiarelli (2015) and De Grauwe and Gerba (2018) who investigated the feedback loop between credit cycles and the real sector in heuristic switching models. I contribute to this work by incorporating a more comprehensive credit market with frictions that are endogenously tied to aggregate risk and central bank credibility, suggesting an extension with an explicitly modeled stock market and by providing a thorough analysis on (optimal) unconventional policy tools. I also build heavily on the influential literature on heuristic switching models in macroeconomic settings which includes Branch and McGough (2010), De Grauwe (2011), Massaro (2013) and De Grauwe and Ji (2019).

The paper is structured as follows. In section 2, the medium-scale DSGE model with capital accumulation, credit market, and active fiscal policy subject to government debt accumulation is presented. It also contains specifications of expectation formation, central bank credibility and aggregate macroeconomic risk. Section

3 sheds some light on model dynamics. The role of key mechanisms in the transmission of real disturbances is analyzed. It also discusses the impact of credibility and the zero lower bound on aggregate welfare, stability, and severity of recessions caused by risk shocks. Section 4 contains the specification and analysis of optimal conventional and unconventional policy measures. In section 5, the extended version that includes an endogenous stock market is given. It also contains investigations on the effectiveness of central bank stock purchases. Section 6 concludes.

3.2 A behavioral macroeconomic framework with credit, credibility, and risk shocks

The model equations are solved as log-linear approximations around the steady state. The microfoundation and the complete derivation of the model is given in appendix 3.8.1.

3.2.1 Households

Households maximize intertemporal utility subject to consumption and working hours. Aggregate consumption c_t is given by the Euler equation

$$c_t = \tilde{E}_t c_{t+1} - \sigma_c^{-1} \left(r_t^n - \tilde{E}_t \pi_{t+1} \right) + \varepsilon_{c,t} \quad (3.1)$$

with nominal (risk-free) interest rate¹ r_t^n , inflation rate π_t , and consumption demand shock $\varepsilon_{c,t}$. \tilde{E} is the aggregate expectation operator to be defined below. Labor supply is described with the equation

$$w_t = \sigma_c c_t + \sigma_l l_t \quad (3.2)$$

with real wage w_t and working hours l_t .

¹The underlying assumption is that households acquire one-period bonds which are either bank deposits or government debt. Both savings vehicles are perfect substitutes and pay out the same interest rate.

3.2.2 Goods producers

Goods are produced with a standard technology

$$y_t = \alpha k_{t-1} + (1 - \alpha)l_t + a_t \quad (3.3)$$

using capital k_{t-1} and labor as inputs, where y_t and a_t denote aggregate output and a technology shock, respectively. Firms compensate households for working hours with real wage w_t , and rent capital from entrepreneurs at rate r_t^k . Cost minimization implies

$$r_t^k - w_t = l_t - k_{t-1} \quad (3.4)$$

Goods producers are subject to standard Calvo (1983) pricing frictions such that inflation dynamics are given by the New Keynesian Phillips curve

$$\pi_t = \beta \tilde{E}_t \pi_{t+1} + \lambda \psi_t + \varepsilon_{\pi,t} \quad (3.5)$$

with cost-push shock $\varepsilon_{\pi,t}$ and discount factor β . Marginal costs ψ_t are given by

$$\psi_t = \alpha r_t^k + (1 - \alpha)w_t \quad (3.6)$$

3.2.3 Capital producers

Capital producers produce capital and sell it to entrepreneurs. After being used in goods production, depreciated capital is repurchased. Capital producers adjust capital stock using goods as investment subject to adjustment costs. Thus, the capital accumulation function is given by

$$k_t = (1 - \delta)k_{t-1} + \delta i_t \quad (3.7)$$

with investment i_t and depreciation rate δ . Profit maximization implies investment Euler equation

$$i_t = \frac{\beta}{1 + \beta} \tilde{E}_t i_{t+1} + \frac{1}{1 + \beta} i_{t-1} + \frac{1}{\varphi_i(1 + \beta)} q_t + \varepsilon_{i,t} \quad (3.8)$$

with capital price q_t , investment demand shock $\varepsilon_{i,t}$ and investment adjust cost elasticity φ_i .

3.2.4 Credit market: entrepreneurs and banks

The credit market, essentially a version of the one in Becard and Gauthier (2022), is characterized by banks extending loans to entrepreneurs. Entrepreneurs combine the obtained loans with their equity to finance the purchase of capital that it rents out to goods producers. However, there is a positive probability p_t^d that they face an idiosyncratic shock that reduces the value of its assets and forces them to default. To mitigate credit risk a bank requires entrepreneurs to back their debt with collateral. In case of default it seizes collateralized assets and sells them in the open market. The fraction of entrepreneurs' capital that banks demand as security is given by the endogenous loan-to-value (LTV) ratio ϕ_t .

A defaulting entrepreneur terminates its business and transfers all remaining assets to households as one-time dividend payment. To maintain stability, an equal number of new entrepreneurs enters the market and is endowed by households with a fraction of the received transfers. Following from these assumptions, one can derive the log-linearized net worth of entrepreneurs n_t accumulation equation

$$n_t = (1 - n_1)n_{t-1} + n_1(q_{t-1} + k_{t-1}) + n_2R_t^k - n_3r_t^e - n_4\hat{p}_{t-1}^d - n_5\hat{\phi}_{t-1} \quad (3.9)$$

Intuitively, entrepreneur net worth depends positively on its real capital stock and return to capital R_t^k , but negatively on credit rates r_t^e , entrepreneur default risk \hat{p}_{t-1}^d and LTV ratio $\hat{\phi}_{t-1}$ (where the hat denotes log differences from steady state, respectively). Net worth exerts a high amount of persistence as it depends on its preceding realization with coefficient $1 - n_1$. The log-linear version of the equation for return to capital R_t^k is given by

$$R_t^k = q_1q_t + (1 - q_1)r_t^k - q_{t-1} \quad (3.10)$$

Entrepreneurs operate under full competition and maximize expected (pre-dividend) net worth with respect to their leverage ratio $\hat{l}_t = q_t + k_t - n_t$. The first-order condition is given by the following equation that captures the dynamics of capital prices:

$$q_t = q_1q_{t-1} + (1 - q_1)r_{t-1}^k - \left(r_t^e - \tilde{E}_t\pi_{t+1}\right) + q_2\hat{p}_t^d - q_3\hat{\phi}_t \quad (3.11)$$

The capital price is given by a backward-looking process depending on the credit market conditions from the perspective of the entrepreneur. It increases when the real credit rate decreases or the rental rate of capital rises, both of which increases capital demand of entrepreneurs. Somewhat unintuitively, q_t responds

positively when default probability increases and the loan conditions tighten. However, it makes sense when considering that the real credit rate responds overproportionally (see below) to these credit market distortions such that the total response of capital prices is negative.

Since banks do not have information on the idiosyncratic shock in advance, every entrepreneur receives the same debt contract. In this case, the bank participation constraint is binding which implies the credit spread equation

$$r_t^e - r_t^n = b_1 \hat{l}_t + b_2 \hat{\rho}_t^d - b_3 \hat{\phi}_t \quad (3.12)$$

Banks demand higher credit rates (relative to the riskless rate) when entrepreneurs are highly leveraged, face high default risk and when the LTV ratio is low. Since both b_2 and b_3 exceed unity, worsening credit market conditions induce rising capital prices in equation (3.11).

3.2.5 Fiscal policy, (conventional) monetary policy, market clearing

As in Proaño and Lojak (2020), government expenditures g_t are assumed to follow the simple fiscal policy rule

$$g_t = -\delta_{gy} y_{t-1} - \delta_{gb} b_{t-1}^g \quad (3.13)$$

where b_t^g is government debt. The government finances its budget with both lump-sum taxes and the issuance of bonds where the latter accumulates according to

$$b_t^g = g_1 \left(b_{t-1}^g + r_t^n - \tilde{E}_t \pi_{t+1} \right) + (1 - g_1) g_t \quad (3.14)$$

Government debt is issued in form of riskless one-period bonds which are perfect substitutes of bank deposits. Accordingly, both instruments pay out the same nominal interest rate r_t^n . In *normal times*, r_t^n is under perfect control of the central bank that sets it to a target value r_t^T according to a monetary policy rule of Taylor type

$$r_t^T = \rho_r r_{t-1}^T + (1 - \rho_r) [\delta_\pi (\pi_{t-1} - \pi^*) + \delta_y (y_{t-1} - y^*)] + \varepsilon_{r,t} \quad (3.15)$$

subject to interest rate smoothing and a monetary policy disturbance $\varepsilon_{r,t}$. π^* and y^* denote the central bank's inflation rate and output target, respectively. Under the assumption that a (zero) lower bound on the nominal interest rate exists, it holds that $r_t^n \geq 0$.² In summary, the central bank can either set the nominal interest rate according to the Taylor rule $r_t^T = r_t^n \geq 0$ or is restricted by a binding zero lower bound such that $r_t^T < r_t^n = 0$.

Goods market clearing implies the resource constraint for aggregate demand

$$y_t = y_c c_t + y_i i_t + y_g g_t \quad (3.16)$$

where $y_c = C/Y$, $y_i = I/Y$ and $y_g = G/Y$ are steady state income shares of consumption, investment and government expenditures, respectively.³

3.2.6 Expectations and credibility

Agents form their expectations by choosing between two different rules-of-thumb: a *target* and a *static* heuristic. *Targeters* believe output and inflation rate to be equal to the central bank target values y^* and π^* , while followers of the static expectational rule predict the variables to be equal to the last observed realizations y_{t-1} and π_{t-1} . This implies for aggregate expectations

$$\tilde{E}_t y_{t+1} = \omega_t^{CB} y^* + (1 - \omega_t^{CB}) y_{t-1} \quad (3.17)$$

$$\tilde{E}_t \pi_{t+1} = \omega_t^{CB} \pi^* + (1 - \omega_t^{CB}) \pi_{t-1} \quad (3.18)$$

Since central bank targets and steady state values coincide, it holds that $y^* = \pi^* = 0$. ω_t^{CB} is the endogenous fraction of agents that *trust* in the central bank's capabilities and follow the target heuristic. It is determined with a discrete choice approach according to

$$\omega_t^{CB} = \frac{\exp\{\mu(U_t^y + U_t^\pi + U_t^r) + \bar{c}\}}{1 + \exp\{\mu(U_t^y + U_t^\pi + U_t^r) + \bar{c}\}} \quad (3.19)$$

²In practice the effective lower bound on e.g. deposit rates is negative due to tools such as penalty rates. For simplicity these instruments are disregarded.

³While y_g is a parameter that can be freely chosen (see parameterization section below), y_c and y_i are given in the steady state solution (see appendix 3.8.3).

with intensity of choice μ . Following Hommes and Lustenhouwer (2019) the endogenous measure for central bank credibility is defined as

$$m_t^{CB} = \omega_t^{CB} - (1 - \omega_t^{CB}) = 2\omega_t^{CB} - 1 \quad (3.20)$$

There are three endogenous components that influence credibility. The first two capture the central bank's track record of achieving its targets of output and inflation stabilization:

$$U_t^y = \xi U_{t-1}^y - (1 - \xi)(y_{t-1} - y^*)^2 \quad (3.21)$$

$$U_t^\pi = \xi U_{t-1}^\pi - (1 - \xi)(\pi_{t-1} - \pi^*)^2 \quad (3.22)$$

with memory coefficient ξ . The third component captures measures how large the induced deviation from the monetary policy rule is if the zero-lower bound is binding:

$$U_t^r = r_t^n - r_t^T \quad (3.23)$$

The constant \bar{c} is set to calibrate steady state credibility m^{CB} and captures the agents' fundamental pre-disposition for trusting the central bank. For $\bar{c} = \infty$ one obtains $\omega_t^{CB} = m_t^{CB} = 1$ which implies that all agents fully trust in the central banks' ability to stabilize the economy at all times. If, however, $\bar{c} = -\infty$, the credibility environment is extremely unfavorable and all agents follow the backward-looking heuristic instead: $\omega_t^{CB} = 0$ and $m_t^{CB} = -1$.

Regarding expectations on the *components* of aggregate output, I follow the argument of Proaño and Lojak (2020) and assume that consumption expectations are linked linearly to output expectations⁴ :

$$\tilde{E}_t c_{t+1} = \alpha_c \tilde{E}_t y_{t+1} \quad (3.24)$$

In contrast to consumption expectations that are formed by the households, investment is forecasted by profit maximizing capital producers who are likely to consider not only news on aggregate output but also

⁴When plugging aggregate consumption expectations into Euler equation (3.1) one obtains an equation that is reminiscent of the fundamental psychological law of consumption by Keynes as it links consumption to aggregate output (albeit with a lag of one period) multiplied with a factor between zero and one. However, the specification of Proaño and Lojak (2020) is not motivated by behavioral assumptions about the spending behavior of households. Instead it describes how agents heuristically forecast the individual output components by exploiting their high correlation with aggregate output to save cognitive costs.

on recent credit market developments such as entrepreneur default risk and lending requirements. Thus, investment expectations are modified to be slightly more sophisticated and given by

$$\tilde{E}_t i_{t+1} = \alpha_i \tilde{E}_t y_{t+1} - k_{i,p^d} \hat{p}_t^d + k_{i,\phi} \hat{\phi}_t \quad (3.25)$$

Capital producers correctly anticipate that a high number of defaulting entrepreneurs and more excessive collateral requirements will increase the price of capital and squelch future investment.

3.2.7 Risk

In this section, a measure for *perceived* aggregate macroeconomic risk, ζ_t , is defined along the lines of Proaño and Lojak (2020). It is developed with the intention to heuristically capture the sentiment of the agents within the economy towards contemporary aggregate economic conditions.

$$\zeta_t = -\tau_m (m_t^{CB} - m^{CB}) - \tau_y y_{t-1} + \tau_b b_{t-1}^g + \tau_\pi |\tilde{E}_t \pi_{t+1} - \pi^*| + \varepsilon_{\zeta,t} \quad (3.26)$$

It can be interpreted as a measure of perceived uncertainty within the macroeconomic system. The first component ties aggregate risk negatively to the deviation of central bank credibility from its steady state: a lower level of trust in the capabilities of policymakers to stabilize the economy induces a higher level of uncertainty. The other components are included with the reasoning that agents will also consider recent economic developments when evaluating contemporary macroeconomic uncertainty. Thus, ζ depends negatively on aggregate output, but positively on government debt and both inflationary and deflationary expectations, i.e. inflation forecasts above or below the central bank target inflation. Finally, $\varepsilon_{\zeta,t}$ is an aggregate risk shock.

While Proaño and Lojak (2020) use this measure as an ad-hoc risk premium that determines the spread between bond rate and policy rate, in this setup it exerts influence on interest rate spreads indirectly by being transmitted to the economy via two channels. First, it increases the default risk of entrepreneurs according to

$$\hat{p}_t^d = \gamma_d \zeta_t + \varepsilon_{d,t} \quad (3.27)$$

with entrepreneur default shock $\varepsilon_{d,t}$. Second, it affects the collateral constraint as banks tighten loan restrictions in the face of risk disturbances:

$$\hat{\phi}_t = -\gamma_\phi \zeta_t + \varepsilon_{\phi,t} \quad (3.28)$$

where $\varepsilon_{\phi,t}$ is an LTV shock.

3.3 Model dynamics with credibility and risk at the zero lower bound

3.3.1 Parameterization

The parameterization of the above system is fully in line with the microfounded structure provided in the appendix 3.8.1. Deep parameters are set by making use of the extensive empirical New Keynesian literature or by defaulting to widely accepted values. For the household coefficients, I assign the default value 0.99 to the discount factor β , and set inverse Frisch labor supply elasticity to unity which implies $\sigma_l = 1$. The intertemporal rate of substitution σ_c is 1.4 and the investment adjustment cost coefficient φ_i is 5.48, which are the posterior modes of Smets and Wouters (2007). Following Altig et al. (2011) I use a steady state income share of capital α of 0.36. In line with Gertler and Karadi (2011), the capital depreciation rate is set to $\delta = 0.025$, elasticity of substitution to $\varepsilon = 4.167$, and Calvo pricing coefficient to $\theta = 0.779$.

The monetary policy rule is parameterized with the typical values $\delta_\pi = 1.5$ and $\delta_y = 0.5$, akin to the original proposal of Taylor (1993), and smoothing coefficient $\rho_r = 0.8$. While I follow Proaño and Lojak (2020) in setting the output coefficient in the fiscal policy rule to $\delta_{gy} = 0.5$, I choose a slightly lower value for the government debt coefficient $\delta_{gb} = 0.05$ for stability concerns. Steady state income share of government expenditures is set to $y_g = 0.2$ as in Brenneisen (2020). Government debt issuance is calibrated to obtain a steady state debt to GDP ratio of $y_{bg} = 1.2$ which is roughly in line with recent estimates for the United States.

The parameters in the aggregate risk equation, $\tau_m = 0.1$, $\tau_y = 0.05$, $\tau_b = 0.015$, and $\tau_\pi = 0.05$, are equal to the setting of Proaño and Lojak (2020). The aggregate risk transmission coefficients to entrepreneur default risk, γ_{p^d} , and LTV ratio, γ_ϕ , are both set to unity. The intensity of choice is set to $\mu = 10$ as e.g. in Lengnick

Goods market		Policy		Risk transmission		
Households	Production	Monetary	Fiscal	Expectations	Risk	Credit
$\beta = 0.99$	$\delta = 0.025$	$\rho_r = 0.8$	$\delta_{gy} = 0.5$	$\mu = 10$	$\tau_m = 0.1$	$p^d = 0.02$
$\sigma_c = 1.4$	$\alpha = 0.36$	$\delta_\pi = 1.5$	$\delta_{gb} = 0.05$	$\xi = 0.8$	$\tau_y = 0.05$	$w^d = 0.5$
$\sigma_l = 1$	$\varphi_i = 5.48$	$\delta_y = 0.5$	$y_g = 0.2$	$\alpha_c = \alpha_i = 0.9$	$\tau_b = 0.015$	$\phi = 0.6$
	$\epsilon = 4.167$		$y_{bg} = 1.2$	$k_{i,p^d} = k_{i,\phi} = 1$	$\tau_\pi = 0.05$	$\kappa = 0.03$
	$\theta = 0.779$				$\gamma_d = \gamma_\phi = 1$	

Table 3.1: Baseline parameterization

and Wohltmann (2016).⁵ A memory coefficient of $\xi = 0.8$ is roughly in line with a broad interpretation of Coibion and Gorodnichenko (2012), who find that the half-life of forecast errors is less than a year.⁶ Predicted output coefficients in consumption and investment expectations are $\alpha_c = \alpha_i = 0.9$ as in Proaño and Lojak (2020). Entrepreneur default and LTV ratio coefficients, k_{i,p^d} and $k_{i,\phi}$ in investment expectations are set to unity, respectively. Steady state entrepreneur default risk p^d is 0.02 which is similar to the posterior mode of Becard and Gauthier (2022). The remaining credit market parameters are set as follows: quantity of entrepreneur capital is $w^d = 0.5$, steady state LTV ratio is $\phi = 0.6$, and the proportional transfer to entering entrepreneurs is $\kappa = 0.03$. All parameter values are summarized in table 3.1. For simplicity, all stochastic shocks follow an AR(1) process with persistence of 0.8 and standard deviation of 0.005.

3.3.2 Amplification of real disturbances

In order to get an intuition on the model dynamics, the impulse responses of the deterministic system to a negative consumption demand shock $\varepsilon_{c,t}$ are illustrated below in figures 3.1 and 3.2. The negative impulse to aggregate demand induces output and inflation reductions and a persistent deflationary recession. In response the nominal interest rate r_t^n sharply decreases according to the monetary policy rule (3.15) and $r_t^T = r_t^n$ for $r_t^n \geq 0$. If the initial impulse is large enough this can lead to a prolonged period of a binding zero lower bound such that $r_t^T < r_t^n = 0$. In such a scenario it is possible that the *real* interest rate *increases* (not included in the figures) which further deepens the ongoing economic crisis.

⁵The calibration of the intensity of choice coefficient is not straightforward because empirical evidence on the heuristic switching parameters cannot be interpreted detached from the underlying structure. Therefore, I adopt the value of $\mu = 10$ from Proaño and Lojak (2020) and Lengnick and Wohltmann (2016), two models that are closely related to the one in this paper, instead of using results of e.g. Cornea-Madeira et al. (2019) who estimate the intensity of choice in a behavioral inflation dynamics model at 5.04.

⁶Angeletos et al. (2018) interpret this survey evidence in a similar manner.

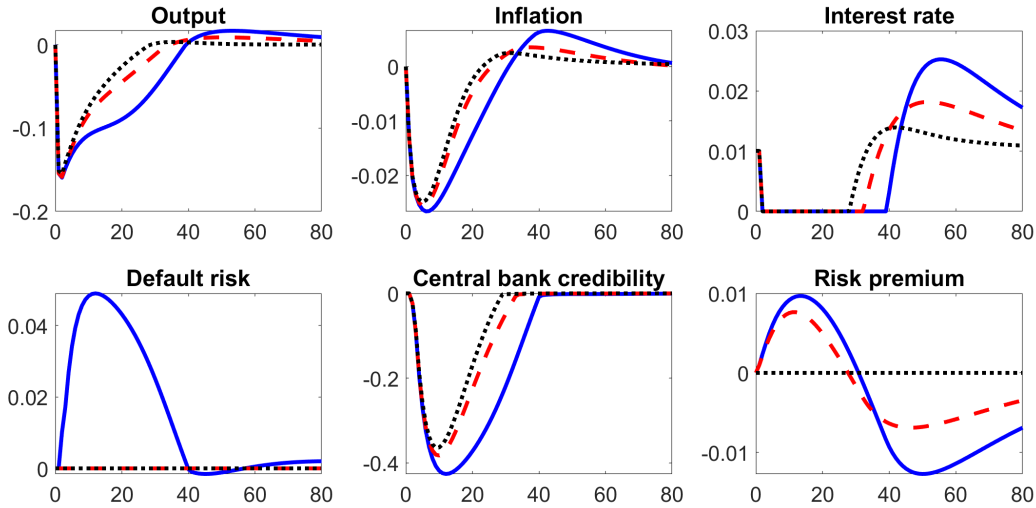


Figure 3.1: Impulse responses of the deterministic system to a negative demand shock $\varepsilon_{c,t} < 0$ without credit frictions (dotted black line), with credit frictions but without aggregate risk transmission (dashed red line), and the full model with credit frictions and risk transmission (solid blue line).

There are four major transmission mechanisms present in the model that *amplify* the adverse effects of real disturbances.⁷ The discrete choice switching in expectations that causes negative credibility effects in response to economic turmoil. The credit market frictions that give rise to endogenous credit cycles. The aggregate risk channel that reinforces said credit market fluctuations by directly affecting entrepreneur default risk and tightness of loan restrictions. And finally the transmission of credit market disturbances to investment expectations.

Credit frictions and risk

The solid black line in figure 3.1 shows the impulse responses of the model without credit market imperfections. This implies a zero interest rate spread such that the risk premium does not respond to the impulse. The negative output and inflation responses induce a lower nominal interest rate and a credibility drop. Since the zero lower bound restriction is binding, trust in the central bank's capabilities deteriorates further which in turn reinforces the persistence in the economy since the share of backward-looking agents rises. Notably, output, inflation and interest rate slightly overshoot their steady state values in the later stages of the dynamic adjustment path.

⁷The mechanisms also amplify the effects of other real disturbances such as cost shocks. While a stagflationary shock $\varepsilon_{\pi,t} > 0$ does never induce an initial reduction of the nominal interest rate with the given parameterization, a negative shock $\varepsilon_{\pi,t} < 0$ can induce a binding zero lower bound if the deflationary effects are large enough. In this case it is even possible that inflation expectations decrease more strongly than the nominal interest rate such that the real interest rate decreases and aggregate output declines.

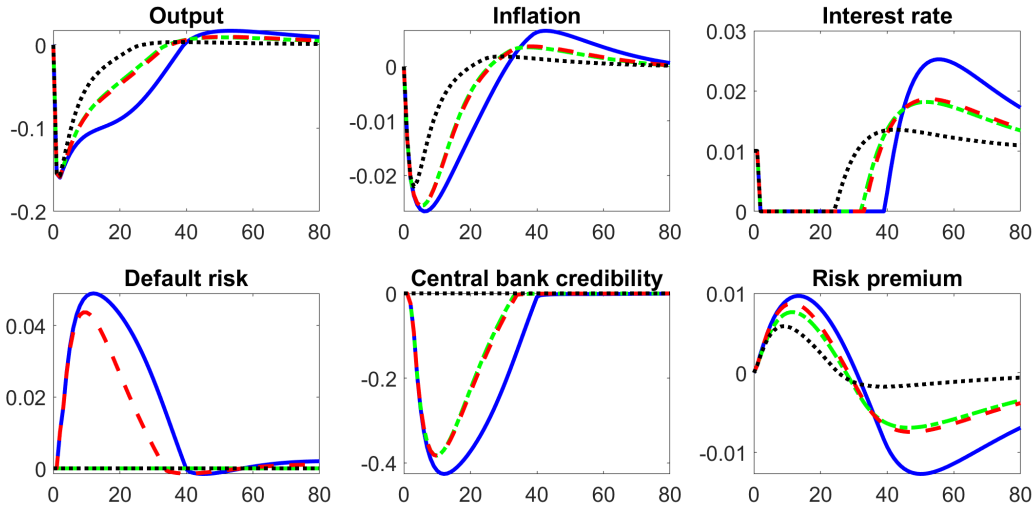


Figure 3.2: Impulse responses of the deterministic system to a negative demand shock $\varepsilon_{c,t} < 0$ without dynamic switching and aggregate risk transmission (dotted black line), with dynamic switching but without aggregate risk transmission (dash-dotted green line), with dynamic switching and aggregate risk transmission but without credit cycle transmission to investment expectations (dashed red line), and the full model with dynamic switching, aggregate risk transmission, and credit cycle transmission to investment expectations (solid blue line).

The implementation of credit frictions, in the form of *constant* entrepreneur default risk and collateral requirements ($\gamma_{p^d} = \gamma_\phi = 0$), introduces credit cycles to the model (dashed red line in figure 3.1): the interest rate *spread* (risk premium) responds to the demand shock by rising above the steady state initially but subsequently cyclically converging back to it. The newly added frictions amplify the initial size and persistence of the negative output and inflation responses, such that the economy undergoes a prolonged period of binding zero lower bound and a deeper credibility crunch.

The solid blue line in figure 3.1 illustrates the effect of implementing the aggregate risk measure and its transmission to default risk and LTV ratio ($\gamma_{p^d} > 0$, $\gamma_\phi > 0$). The reduction in credibility and the worsening of the macroeconomic situation causes aggregate risk to increase which translates to rising entrepreneur default risk and tightening credit conditions. In turn, the amplitude of credit cycles, the response of output and inflation, the length of the zero lower bound episode, and reduction of central bank credibility are intensified.

Expectations and risk

Figure 3.2 illustrates the role of the switching mechanism and its interaction with the aggregate risk measure. The dotted black line shows the responses of the model without switching ($\mu = 0$) and risk transmission to the credit market ($\gamma_{p^d} = \gamma_\phi = 0$). Thus, both credibility and entrepreneur default risk do not respond to the disturbance. Notably, the interest rate spread does not undershoot its steady state after the initial increase. It becomes evident that for the emergence of credit cycles the expectations mechanism with discrete choice learning is required *in conjunction* with credit frictions.

When implementing dynamic switching with $\mu > 0$ (dash-dotted green line in figure 3.2) credibility dynamically adjusts and decreases in response to the shock. Credit cycles emerge and output, inflation and interest rate responses are substantially amplified. Adding the transmission of aggregate risk into the credit market ($\gamma_{p^d} > 0$, $\gamma_\phi > 0$) has a much lower impact (dashed red line) and only slightly intensifies the model dynamics when not yet integrating the link between expectations and credit market conditions ($k_{i,p^d} = k_{i,\phi} = 0$). It is the link of default risk and LTV ratio to investment expectations that completes the feedback loop between dynamic heuristic switching, aggregate risk and credit cycles, which substantially increases size and persistence of impulse responses (solid blue line in figure 3.2).

3.3.3 Aggregate risk shocks and credibility

Shocks to aggregate macroeconomic risk induce credit cycles that are transmitted to the goods sector over investment demand. A positive shock $\varepsilon_{\zeta,t} > 0$ increases risk which raises entrepreneur default risk and tightens loan requirements. The worsening of credit market conditions causes an increase of the interest rate spread which lowers investment incentives, an effect that is reinforced with the direct link of credit market disturbances to investment expectations. Consequently, output and inflation are reduced which is accompanied by a drop in credibility and a decrease of the nominal interest rate via the monetary policy rule.

Figure 3.3 illustrates these model dynamics with and without consideration of the zero lower bound for credibility parameter $\bar{c} = 0$. If the central bank is not restricted it has much leeway to react with a massive drop of the nominal interest rate which keeps the output and inflation responses fairly moderate such that its credibility only takes a minor dip. However, if the zero lower bound is implemented and binding two

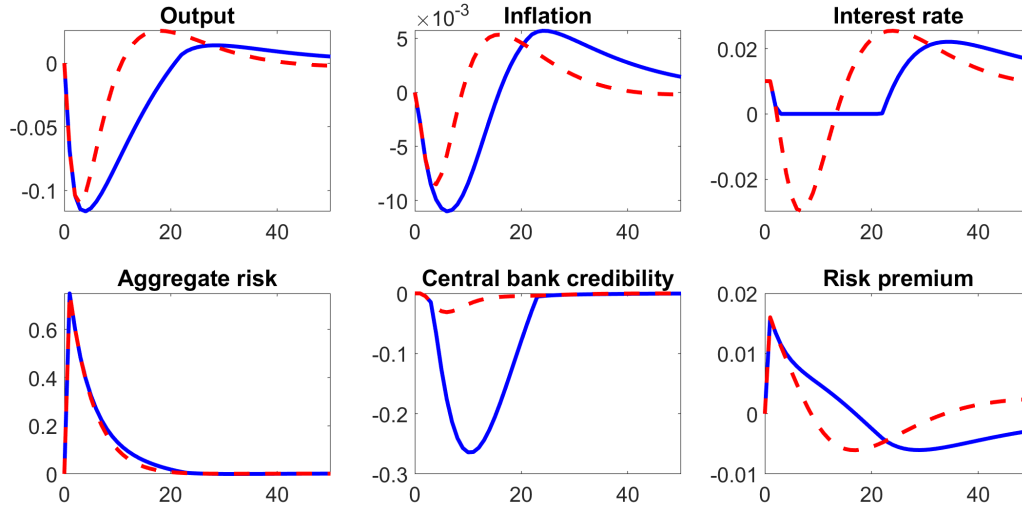


Figure 3.3: Impulse responses of the deterministic system to a positive aggregate risk shock $\varepsilon_{\zeta,t} > 0$ with (solid blue line) and without (dashed red line) the zero lower bound for *intermediate* credibility parameter $\bar{c} = 0$.

effects increase the economic fluctuations in conjunction. First, the central bank cannot set the nominal interest rate as low as the monetary policy rule suggests such that it can offset less of the adverse effects of the risk shock. Second, a binding zero lower bound directly lowers credibility which increases the share of backward-looking expectations such that the persistence within the system increases and the effects on output and inflation are longer lasting.

Overall, the dynamic responses of the endogenous variables are much more pronounced when considering the lower bound on the nominal interest rate but the system returns to steady state over time. This is not the case when analyzing a scenario in which agents are, in general, less inclined to trust the central bank ($\bar{c} = -1$). As shown in figure 3.4, when facing the same shock as in the previous example the dynamic system now involves explosive behavior.

The opposite can be said for a scenario that is much more favorable for the central bank ($\bar{c} = 1$) as illustrated in figure 3.5. If monetary policy makers can build on a higher level of credibility to begin with, the economic fluctuations in response to the risk shock are much lower. Importantly, the difference between the scenarios with and without zero lower bound have all but vanished. In the former case, the monetary policy restriction is only binding for a fairly short period. Output and inflation dynamics exhibit persistence of much lower degree, and the size of credibility reduction is substantially lower both in absolute terms and especially relative to its larger steady state value.

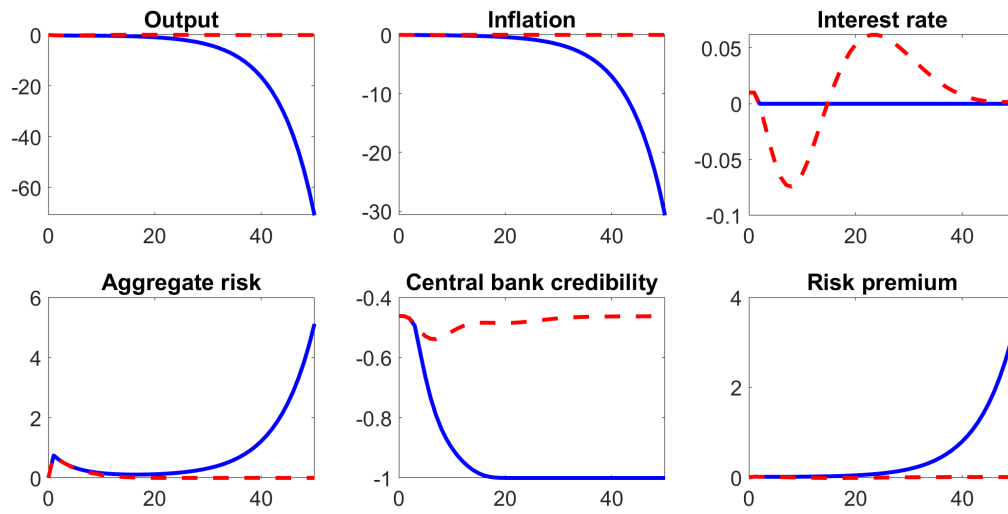


Figure 3.4: Impulse responses of the deterministic system to a positive aggregate risk shock $\varepsilon_{\zeta,t} > 0$ with (solid blue line) and without (dashed red line) the zero lower bound for *low* credibility parameter $\bar{c} = -1$.

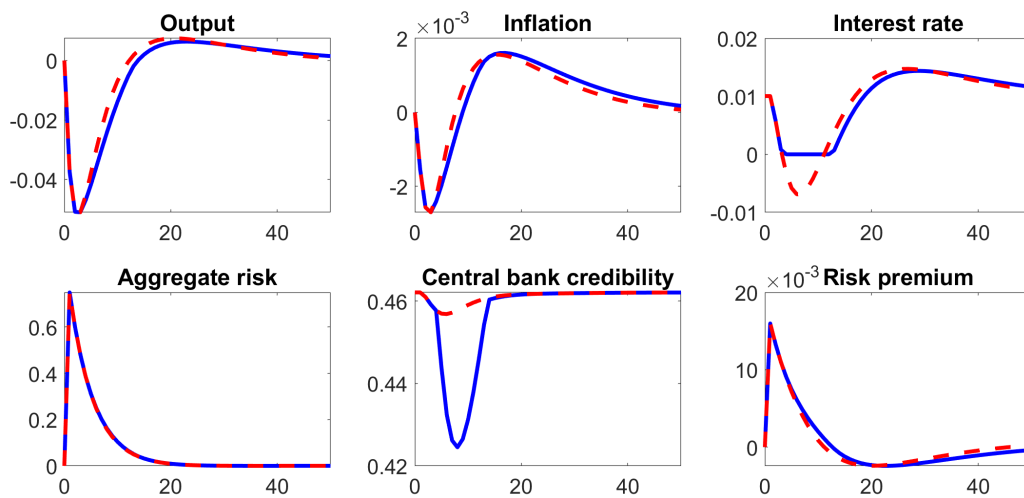


Figure 3.5: Impulse responses of the deterministic system to a positive aggregate risk shock $\varepsilon_{\zeta,t} > 0$ with (solid blue line) and without (dashed red line) the zero lower bound for *high* credibility parameter $\bar{c} = 1$.

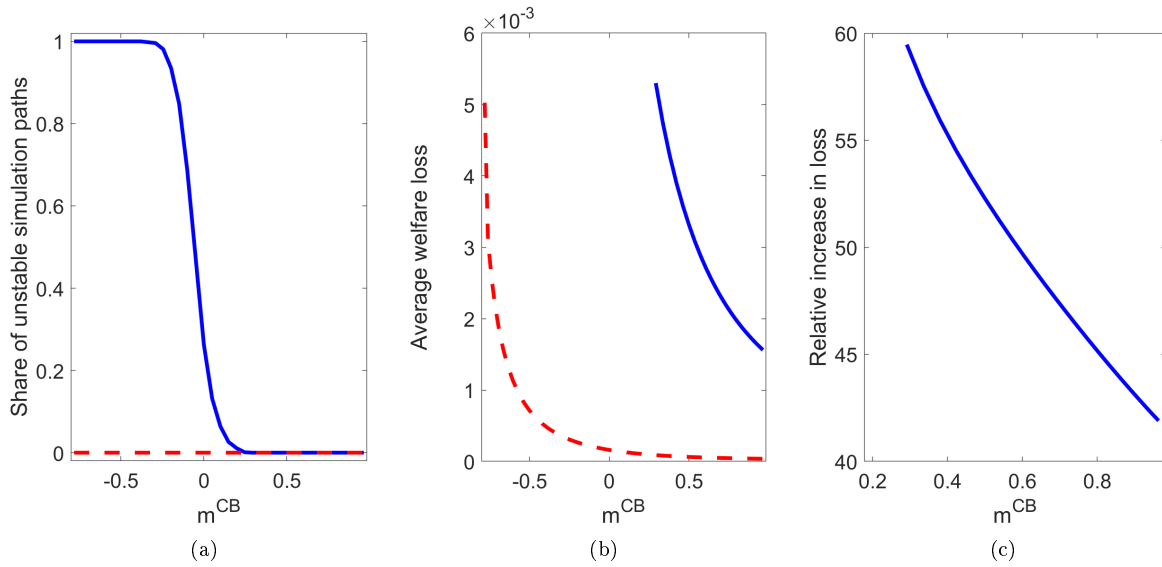


Figure 3.6: (a) Share of explosive simulation paths with (solid blue line) and without (dashed red line) the zero lower bound, (b) Average welfare loss with (solid blue line) and without (dashed red line) the zero lower bound, and (c) Relative increase in welfare loss due to the zero lower bound, for different values of steady state credibility m^{CB} .

3.3.4 The role of credibility for stability and welfare

Both the zero lower bound and the credibility environment are of major importance for stability and magnitude of economic fluctuations as shown in figure 3.6. For a plethora of values of \bar{c} , which imply different steady state credibility values m^{CB} , stochastic simulations of the full model are conducted for 1000 different random seeds and simulation size of 1000 periods.

Panel (a) illustrates the share of explosive simulation runs in this Monte Carlo experiment. While the model is always stable if the zero lower bound is not considered (dashed red line), the share of stable simulations with a zero lower bound (solid blue line) depends heavily on the credibility environment. For low values of m^{CB} the dynamic system exhibits explosive behavior in every single random draw. The share of unstable simulations rapidly decreases for higher values of credibility. Full stability of all stochastic simulations is reached for steady state credibility of roughly $m^{CB} = 0.3$.

Panel (b) shows average welfare loss measured as deviation of average household utility from its hypothetical counterpart without frictions and disturbances. While the relationship between welfare loss and credibility is monotonously negative, it is evident that the adverse consequences of the zero lower bound are of significant quantity: a scenario with lower bound on the nominal interest rate and steady state credibility of 0.3 is

associated with a similar welfare loss as a scenario without monetary policy restrictions and m^{CB} of -0.8. Both the absolute and relative (as shown in panel (c)) difference in welfare loss with and without the lower bound decrease in m^{CB} .

3.4 Conventional and unconventional monetary policy

A binding zero lower bound severely constrains the central bank. The adverse consequences are further enhanced by the interactive credibility, credit cycles and risk channels such that the significance of finding effective policy tools that provide economic stability and mitigate welfare losses is even higher than in standard applications. In the following sections, a number of nonstandard monetary policy approaches are proposed, optimized and examined regarding their performance.

3.4.1 Risk-augmented Taylor rule

Akin to Proaño and Lojak (2020), the first proposal is a risk-augmented monetary policy rule:

$$r_t^T = \rho_r r_{t-1}^T + (1 - \rho_r) [\delta_\pi \pi_{t-1} + \delta_y y_{t-1} - \delta_\zeta \zeta_{t-1}] + \varepsilon_{r,t} \quad (3.29)$$

A binding zero lower bound lowers the component U_t^r that in turn decreases central bank credibility m_t^{CB} , such that the risk measure ζ_t rises. Since risk affects the policy rate with a lag of one period, zero lower bound periods induce monetary policy to be more loose than usual even after the constraint is not binding anymore. Additionally to this mechanism that is also present in Proaño and Lojak (2020), the presence of the fitness measures U_t^y and U_t^π , as in Hommes and Lustenhouwer (2019), gives rise to another stabilizing effect. When the central bank misses its targets of output and inflation stabilization, endogenous credibility decreases which lowers monetary policy effectiveness and increases the probability of hitting the zero lower bound in the future. By responding to credibility induced surge in aggregate risk, the central bank is able to *avoid* zero lower bound periods in certain scenarios.

As shown in figure 3.7, augmenting the monetary policy rule with an aggregate risk component can partially offset the negative welfare consequences of the zero lower bound. In panel (a), the average welfare loss associated with a risk-augmented Taylor rule with welfare-maximizing risk coefficient δ_ζ (dotted cyan line) is

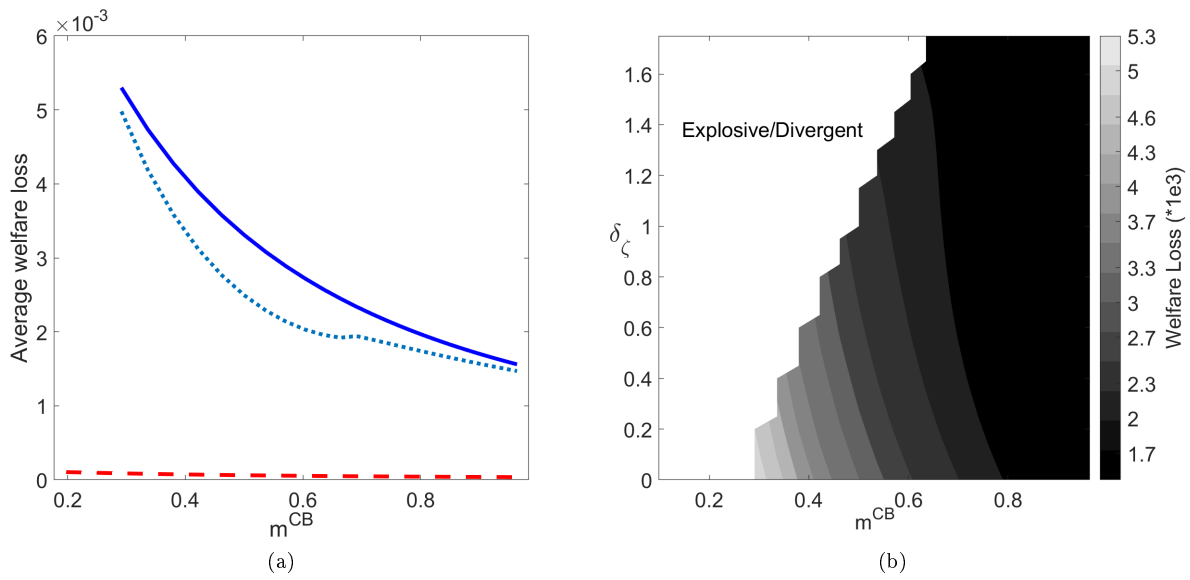


Figure 3.7: (a) Average welfare loss with an optimal risk-augmented Taylor rule and the zero lower bound (dotted cyan line) for different values of steady state credibility m^{CB} , in comparison to the baseline model with (solid blue line) and without (dashed red line) the zero lower bound. (b) Average welfare loss for different values of risk coefficient δ_ζ and steady state credibility m^{CB} where darker areas indicate lower levels of loss.

compared to the baseline cases of figure 3.6 (b). The policy measure reduces welfare loss substantially with highest (absolute and relative) effects for steady state credibility around 0.5. However, the policy measure cannot increase stability even with the optimal parameter value in the sense that it is not able to stabilize any of the explosive simulation paths.

Panel (b) illustrates welfare loss for different values of δ_ζ and m^{CB} where darker areas indicate lower loss while white areas mark observations of explosive behavior. Unsurprisingly, higher values of steady state credibility are unambiguously associated with lower loss. The same can be said about the risk coefficient but only when disregarding the fact that too high parameter values lead to unstable simulation runs even for credibility settings that were completely stable with a standard monetary policy rule. Thus, the implementation of a risk-augmented Taylor rule is very risky: if the central bank fails to correctly determine the optimal parameterization and sets the coefficient only slightly too high it may destabilize the system.

3.4.2 Credit policy

As an alternative, I consider an unconventional monetary policy tool that involves active credit policy akin to Gertler and Karadi (2011). The central bank engages in the credit market by directly extending loans to entrepreneurs. This policy tool eases the adverse effects of credit frictions since the central bank has a significant advantage over private lenders: it can collect lump-sum taxes from households, the owners of entrepreneurs, to offset default loss and, thus, does not have to resort to seizing collateralized assets.

The degree of public involvement in the credit market is given by ς_t which denotes the fraction of total loans the central bank is willing to fund. The credit spread equation (3.12) becomes

$$r_t^e - r_t^n = b_1 \hat{l}_t + b_2 \hat{p}_t^d - b_3 \hat{\phi}_t - b_4 \varsigma_t \quad (3.30)$$

Accordingly, the active credit policy can improve loan conditions for entrepreneurs by lowering the risk premium. However, in line with the assumption of (partially) bond-financed government budget, it ties the public debt to the growth of public credits to entrepreneurs. The government debt equation (3.14) becomes

$$b_t^g = g_1 \left(b_{t-1}^g + r_t - \tilde{E}_t \pi_{t+1} \right) + (1 - g_1) g_t + g_2 (n_t - n_{t-1}) + g_3 (\hat{l}_t - \hat{l}_{t-1}) + g_4 (\varsigma_t - \varsigma_{t-1}) \quad (3.31)$$

The first specification of a credit policy rule ties central bank credit injections to aggregate risk:

$$\varsigma_t = \delta_{\varsigma, \zeta} \zeta_{t-1} \quad (3.32)$$

As shown in figure 3.8, the welfare effects of an optimal risk-dependent credit policy rule are overall fairly low in comparison to the risk-augmented Taylor rule. However, the effects are larger than the previous policy measure for relatively low credibility. The risk-dependent credit policy has also some positive effects on stability as illustrated in panel (b): there exists a range of values for $\delta_{\varsigma, \zeta}$ that ensures stability of previously explosive simulation runs.

Alternatively, one may argue that it is superior to specify the credit policy rule as a function of the interest rate spread since it is a more direct and complete measure of current credit market conditions than aggregate

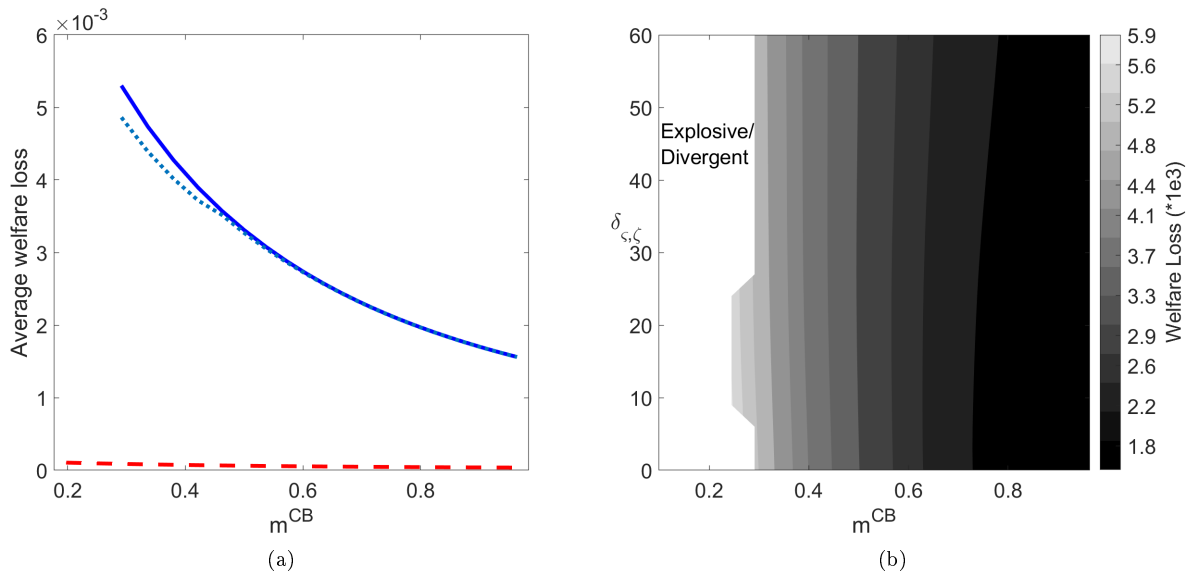


Figure 3.8: (a) Average welfare loss with an optimal risk-dependent credit policy rule and the zero lower bound (dotted cyan line) for different values of steady state credibility m^{CB} , in comparison to the baseline model with (solid blue line) and without (dashed red line) the zero lower bound. (b) Average welfare loss for different values of risk coefficient $\delta_{s,\zeta}$ and steady state credibility m^{CB} where darker areas indicate lower levels of loss.

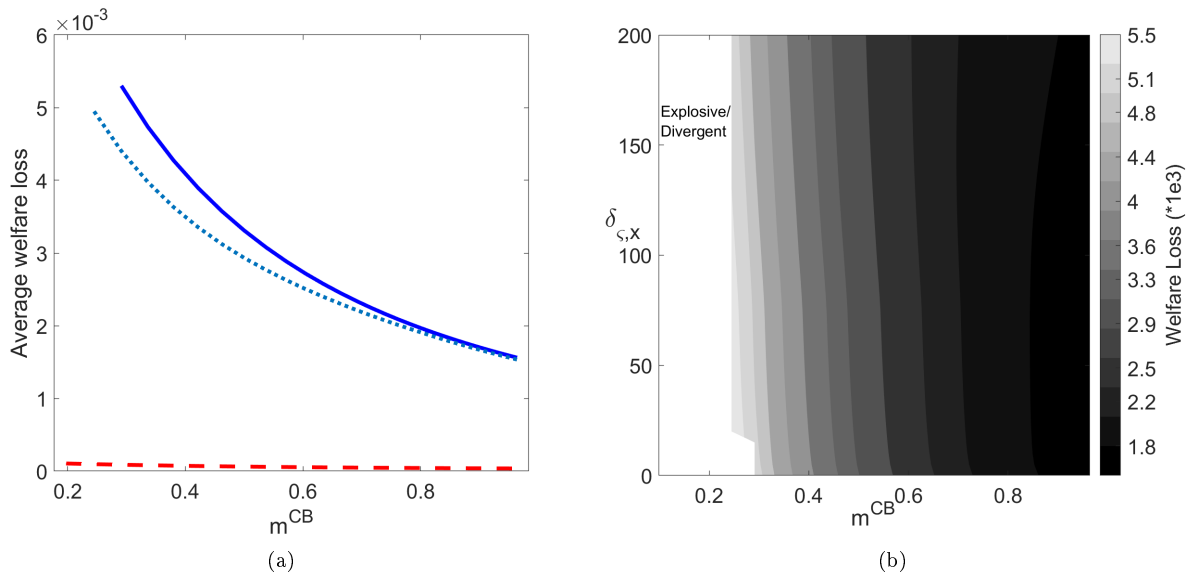


Figure 3.9: (a) Average welfare loss with an optimal spread-dependent credit policy rule and the zero lower bound (dotted cyan line) for different values of steady state credibility m^{CB} , in comparison to the baseline model with (solid blue line) and without (dashed red line) the zero lower bound. (b) Average welfare loss for different values of spread coefficient $\delta_{s,r}$ and steady state credibility m^{CB} where darker areas indicate lower levels of loss.

risk, which is only transmitted to the credit market over default risk and LTV ratio. A spread-dependent credit policy rule is given by

$$\varsigma_t = \delta_{\varsigma,r} (r_{t-1}^e - r_{t-1}) \quad (3.33)$$

A look at figure 3.9 reveals that the spread-dependent credit policy rule has quantitatively the same implications as its risk-dependent counterpart but the effects are more pronounced: welfare is strongly increased for low levels of credibility and the rule raises stability.

In summary, a risk-augmented Taylor rule provides the larger welfare effects if steady state credibility is sufficiently large while credit policy performs better for relatively low credibility. Regarding stability it is very risky to use the enhanced interest rate rule since it involves the danger of raising the probability of observing explosive dynamics. In contrast, active credit policy has exclusively positive stability effects. A spread-dependent credit policy rule strictly dominates its risk-dependent counterpart independent of the credibility environment.

3.5 Extension: Dynamic stock market trading and central bank stock purchases

In this section the model is extended by assuming that shares of entrepreneurs are traded in a dynamic stock market which allows the implementation of another type of unconventional monetary policy: stock purchases by the central bank. The specification of the stock market is very similar to the structure of Lengnick and Wohltmann (2016). Adopting the authors *stock in the utility* assumption provides the following microfounded household stock demand function:

$$d_t^s = \frac{\sigma_c}{\sigma_d} c_t - \frac{1}{\sigma_d} (\bar{s}_t - p_t) - \frac{\beta}{\sigma_d(1-\beta)} \left(r_t^n - \tilde{E}_t [\Delta \bar{s}_{t+1}] \right) \quad (3.34)$$

with stock elasticity of substitution σ_d , stock demand d_t^s and \bar{s}_t denoting (average) stock price in quarter t . The demand for stocks depends negatively on real stock price $\bar{s}_t - p_t$ and the difference between nominal interest rate and expected stock price growth $r_t^n - \tilde{E}_t [\Delta \bar{s}_{t+1}]$ where the latter captures the opportunity costs of stock purchases.⁸

⁸As in Lengnick and Wohltmann (2016), quarterly stock price expectations are determined with the same heuristic switching approach as the other macroeconomic expectations of variables π_t and y_t .

Following the assumption of Lengnick and Wohltmann (2016), the stock market operates at a daily frequency in contrast to the quarterly frequency of the macroeconomy. Since a quarter has on average roughly 64 trading days, the stock price in quarter t is given by

$$\bar{s}_t = \frac{1}{64} \sum_{j=64(t-1)+1}^{64t} s_j \quad (3.35)$$

where stock price on *day* j , s_j , is determined by the agent-based financial market model with learning given below.

3.5.1 A stock market model with boundedly-rational learning of daily frequency

Similar to the expectation formation within the macroeconomic model, expectations in the stock market model are provided by an heuristic switching setup. Stock traders can apply three different strategies to determine their stock demand. The chartist (C) rule involves expecting that the recent stock price trend continues:

$$E_j^C s_{j+1} = s_j + k^C (s_j - s_{j-1}) \quad (3.36)$$

Observing a stock price increase of one percentage point implies that chartists expect the stock price to increase by k^C .

In contrast, the fundamentalist (F) rule determines stock price forecasts by assuming a reversal to the *fundamental* stock price s_j^f :

$$E_j^F s_{j+1} = s_j + k^F (s_j^f - s_j) \quad (3.37)$$

The strategy anchors stock price expectations to a fundamental value and expects contemporary deviations to diminish over time with rate k^F .

The excess stock demand of both of these groups is given by a linear function depending on their respective stock price growth forecasts:

$$D_j^C = \ell (E_j^C s_{j+1} - s_j) \quad (3.38)$$

$$D_j^F = \ell (E_j^F s_{j+1} - s_j) \quad (3.39)$$

The third applicable strategy is *no trading* ($D_j^0 = 0$) which is a viable choice in case the other two strategies fail to reliably generate profits. The market fraction of a strategy $h \in \{C, F, 0\}$ is given by the discrete choice equation

$$W_j^h = \frac{\exp \{eA_j^h\}}{\exp \{eA_j^C\} + \exp \{eA_j^F\} + \exp \{eA_j^0\}} \quad (3.40)$$

where attractivity of strategy h , A_j^h is a weighted average of past profits,

$$A_j^h = mA_{j-1}^h + (\exp \{s_j\} - \exp \{s_{j-1}\}) D_{j-2}^h \quad (3.41)$$

with memory coefficient m . In contrast to the macroeconomic expectation formation mechanism, it is possible to use a more direct measure for the success of heuristics, namely accumulated profits, rather than the more abstract fitness measures discussed above.

Recent macroeconomic developments enter the stock market through two channels. First, the fundamental stock price is determined by the value of expected entrepreneur dividend payments. As shown in appendix 3.8.2, it is approximately given by

$$s_j^f = s_1 \left(R_t^k + n_t + \hat{l}_t - \frac{\phi}{1-\phi} \hat{\phi}_{t-1} \right) + s_2 \hat{p}_t^d + s_3 \hat{p}_{t-1}^d \quad (3.42)$$

While increasing entrepreneur default risk (due to more frequent dividend payments) and decreasing LTV ratios (due to lower collateral payments) have a direct positive impact on the fundamental price, the total effect is negative since s_t^f depends positively on entrepreneur net worth and return to capital, which react negatively to these distortions either directly or via capital price transmission.

Stock market		
Structure	Heuristics	Learning
$\ell = 1$	$k^C = 0.04$	$e = 300$
$a = 1$	$k^F = 0.04$	$m = 0.975$
$k = 0.2$		

Table 3.2: Stock market parameterization

Here, $t = \text{floor}\left(\frac{j-1}{64}\right)$ denotes the quarter of the most recently published macroeconomic data at day j .

Second, household stock demand growth is incorporated in the stock price impact function:

$$s_{j+1} = s_j + a \left(W_j^C D_j^C + W_j^F D_j^F + \frac{k}{64} \Delta d_t^s \right) \quad (3.43)$$

The stock price growth depends linearly on the aggregate stock demand, given by the weighted sum of net purchases of chartist traders, fundamentalist traders and households. Since the latter variable is computed on a quarterly basis from the macroeconomic DSGE model, an assumption about the timing of household purchases is required: following Lengnick and Wohltmann (2016) are assumed to spread their purchases evenly among the 64 trading days of each quarter.

The stock market parameterization is the same as in Lengnick and Wohltmann (2016) and summarized in table 3.2.⁹

3.5.2 Unconventional monetary policy: stock purchases

In this section the central bank is provided with another unconventional policy tool: the option to directly intervene in the stock market with stock purchases $d_t^{s,CB}$ that affects stock prices over the new impact function

$$s_{j+1} = s_j + a \left(W_j^C D_j^C + W_j^F D_j^F + \frac{k}{64} \left(\Delta d_t^s + \Delta d_t^{s,CB} \right) \right) \quad (3.44)$$

For simplicity, it is assumed that the central bank does not optimize the timing of its purchases with respect to trading days, but spreads them uniformly across the quarter just as the households.

⁹Stock elasticity of substitution is set to $\sigma_d = \frac{\beta}{1-\beta}$ to match the calibration $c_4 = 1$ in the household stock demand equation of Lengnick and Wohltmann (2016).

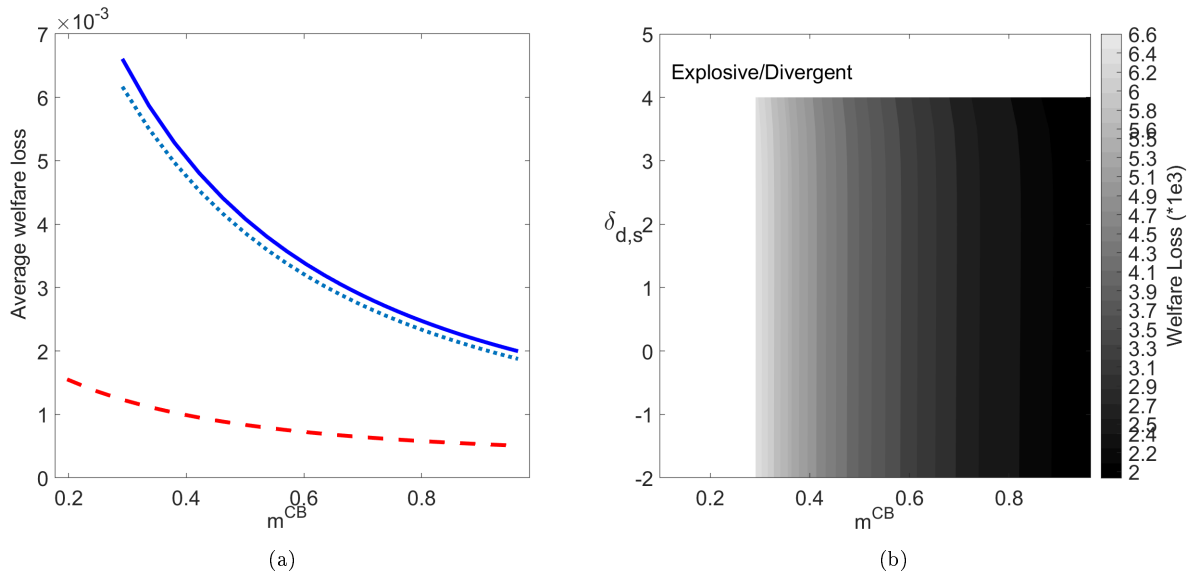


Figure 3.10: (a) Average welfare loss with zero lower bound and an optimal stock purchase rule depending on stock price gap (dotted cyan line) for different values of steady state credibility m^{CB} , in comparison to the baseline model with (solid blue line) and without (dashed red line) the zero lower bound. (b) Average welfare loss for different values of stock price gap coefficient $\delta_{d,s}$ and steady state credibility m^{CB} where darker areas indicate lower levels of loss.

In this setup, stock market developments are transmitted to the government debt accumulation equation which is now given by

$$\begin{aligned}
 b_t^g = & g_1 \left(b_{t-1}^g + r_t - \tilde{E}_t \pi_{t+1} \right) + (1 - g_1) g_t + g_2 (n_t - n_{t-1}) + g_3 (\hat{l}_t - \hat{l}_{t-1}) + g_4 (\varsigma_t - \varsigma_{t-1}) \\
 & + g_5 \left(\Delta \bar{s}_t + \Delta d_t^{s,CB} \right)
 \end{aligned} \tag{3.45}$$

where the latter component is now added and describes that government debt increases with the cost of net stock purchases conducted by the central bank.

The first specification of a stock purchases rule depends on the stock price deviation from its fundamental value:

$$\Delta d_t^{s,CB} = \delta_{d,s} \left(\bar{s}_t - s_t^f \right) \tag{3.46}$$

which is the unconventional monetary policy tool proposed by Lengnick and Wohltmann (2016). As shown in figure 3.10, the central bank can use it to modestly increase welfare. However, if parameter $\delta_{d,s}$ is set higher than a threshold of roughly 4, the model becomes explosive for every value of m^{CB} . Critically, the

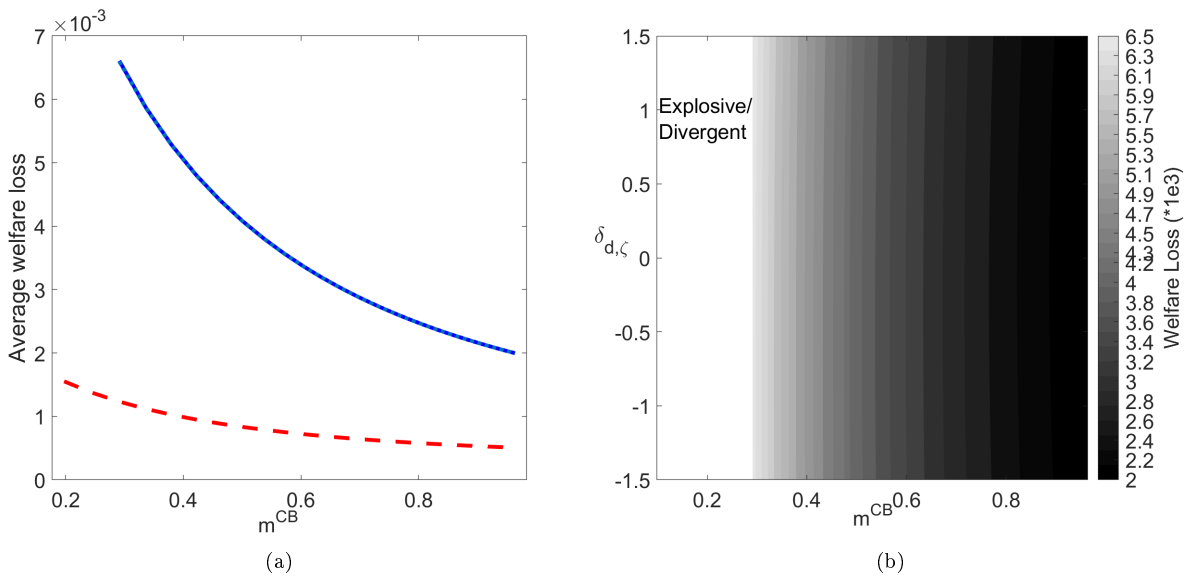


Figure 3.11: (a) Average welfare loss with zero lower bound and an optimal risk-dependent stock purchase rule (dotted cyan line) for different values of steady state credibility m^{CB} , in comparison to the baseline model with (solid blue line) and without (dashed red line) the zero lower bound. (b) Average welfare loss for different values of risk coefficient $\delta_{d, \zeta}$ and steady state credibility m^{CB} where darker areas indicate lower levels of loss.

optimal values are located at the fringe of this threshold such that applying this tool is extremely risky - a result reminiscent of the danger involved with a risk-dependent Taylor rule.

Another option is to tie central bank stock purchases to aggregate risk:

$$\Delta d_t^{s, CB} = \delta_{d, \zeta} \zeta_t \quad (3.47)$$

The results, illustrated in figure 3.11, are disappointing: the policy rule can not provide relevant welfare gains.

Finally, a spread-dependent rule is given by

$$\Delta d_t^{s, CB} = \delta_{d, r} (r_t^e - r_t^n) \quad (3.48)$$

which performs slightly better as can be seen in figure 3.12. While it is still outperformed by the rule based on stock price deviation, it is a safer option that is capable of providing positive welfare effects. However, the unconventional policy measures involve, in general, lower welfare effects than active credit policy and

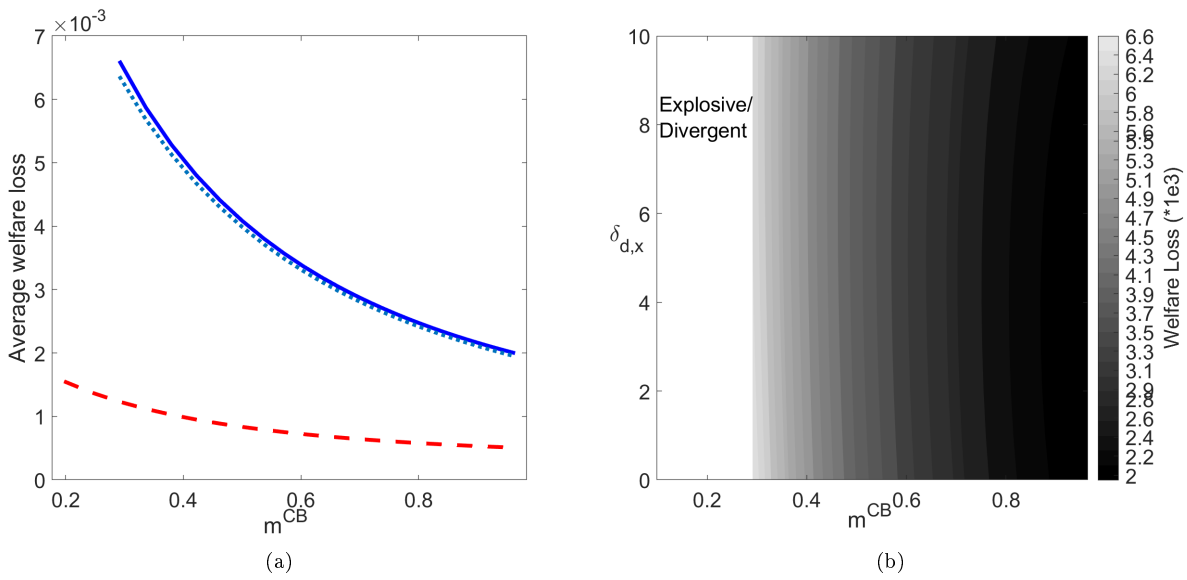


Figure 3.12: (a) Average welfare loss with zero lower bound and an optimal spread-dependent stock purchase rule (dotted cyan line) for different values of steady state credibility m^{CB} , in comparison to the baseline model with (solid blue line) and without (dashed red line) the zero lower bound. (b) Average welfare loss for different values of spread coefficient $\delta_{d,r}$ and steady state credibility m^{CB} where darker areas indicate lower levels of loss.

the risk-augmented Taylor rule. Therefore, they can at best be seen as a useful supplement rather than a substitute to these tools.

3.6 Conclusion

This paper presents a microfounded DSGE model that involves three key mechanisms that interact with each other in a feedback loop: endogenous credibility that is based on boundedly-rational discrete choice learning akin to Hommes and Lustenhouwer (2019), credit cycles that arise in a credit market specification based on Becard and Gauthier (2022) subject to default risk and collateral requirements, and aggregate macroeconomic risk as in Proaño and Lojak (2020). The feedback loop acts as an amplifying mechanism for real disturbances and risk shocks. Stability and welfare loss of the full model crucially depend on both the existence of a zero lower bound on the nominal interest rate and the credibility environment for the central bank.

Three policy tools are proposed to ease the adverse effects: a nonstandard conventional policy tool with the risk-augmented Taylor rule, and two unconventional measures namely active credit policy and central bank

stock purchases. The welfare and stability effects of the instruments depend on the predisposition of agents to *trust* the central bank. The following results stand out.

The risk-augmented Taylor rule clearly outperforms all others tools regarding welfare maximization if credibility is sufficiently high. However it is very risky: slightly too large parameter settings involve the emergence of explosive simulation paths. In contrast, active credit policy is a much safer option as it has only beneficial effects on stability. For sufficiently low credibility it also outperforms the augmented interest rate rule regarding welfare. Central bank stock purchases provides positive but relatively low welfare gains and does not have noticeable effects on stability. Both unconventional monetary policy measures perform better with spread-dependent rules than risk-dependent ones.

While the efficiency of the proposed unconventional tools may appear relatively low in comparison to the massive adverse effects of the zero lower bound, it should be noted that the results in this paper provide more of a lower threshold than an accurate estimate. Following the approach of Proaño and Lojak (2020), a currently binding zero lower bound has devastating effects on credibility which amplifies economic disturbances. Is this setup still reasonable if the central bank has effective unconventional tools at its disposal that do not face any constraints? If agents observe the successful implementation of these tools it is conceivable that it reduces the credibility penalty at the lower bound (U_r) which in turn further increases the efficiency of said measures. The implementation of such a mechanism is, however, beyond the scope of this paper and is left for future research.

3.7 References

- Altig, D., Christiano, L. J., Eichenbaum, M. and Linde, J. (2011). Firm-specific capital, nominal rigidities and the business cycle, *Review of Economic dynamics* **14**(2): 225–247.
- Angeletos, G.-M., Collard, F. and Dellas, H. (2018). Quantifying confidence, *Econometrica* **86**(5): 1689–1726.
- Becard, Y. and Gauthier, D. (2022). Collateral shocks, *American Economic Journal: Macroeconomics* **14**(1): 83–103.
- Branch, W. A. and McGough, B. (2010). Dynamic predictor selection in a New Keynesian model with heterogeneous expectations, *Journal of Economic Dynamics and Control* **34**(8): 1492–1508.
- Brenneisen, J.-N. (2020). Monetary policy under imperfect information and consumer confidence, *Economics Working Paper, University of Kiel*.
- Brock, W. A. and Hommes, C. H. (1997). A rational route to randomness, *Econometrica* pp. 1059–1095.
- Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework, *Journal of Monetary Economics* **12**(3): 383–398.
- Coibion, O. and Gorodnichenko, Y. (2012). What can survey forecasts tell us about information rigidities?, *Journal of Political Economy* **120**(1): 116–159.
- Cornea-Madeira, A., Hommes, C. and Massaro, D. (2019). Behavioral heterogeneity in US inflation dynamics, *Journal of Business & Economic Statistics* **37**(2): 288–300.
- De Grauwe, P. (2011). Animal spirits and monetary policy, *Economic Theory* **47**(2): 423–457.
- De Grauwe, P. and Gerba, E. (2018). The role of cognitive limitations and heterogeneous expectations for aggregate production and credit cycle, *Journal of Economic Dynamics and Control* **91**: 206–236.
- De Grauwe, P. and Ji, Y. (2019). Inflation targets and the zero lower bound in a behavioural macroeconomic model, *Economica* **86**(342): 262–299.
- De Grauwe, P. and Macchiarelli, C. (2015). Animal spirits and credit cycles, *Journal of Economic Dynamics and Control* **59**: 95–117.
- Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the New Keynesian framework and its applications*, Princeton University Press.

- Gertler, M. and Karadi, P. (2011). A model of unconventional monetary policy, *Journal of Monetary Economics* **58**(1): 17–34.
- Hommel, C. and Lustenhouwer, J. (2019). Inflation targeting and liquidity traps under endogenous credibility, *Journal of Monetary Economics* **107**: 48–62.
- Lengnick, M. and Wohltmann, H.-W. (2016). Optimal monetary policy in a New Keynesian model with animal spirits and financial markets, *Journal of Economic Dynamics and Control* **64**: 148–165.
- Massaro, D. (2013). Heterogeneous expectations in monetary DSGE models, *Journal of Economic Dynamics and Control* **37**(3): 680–692.
- Proaño, C. R. and Lojak, B. (2020). Animal spirits, risk premia and monetary policy at the zero lower bound, *Journal of Economic Behavior & Organization* **171**: 221–233.
- Smets, F. and Wouters, R. (2007). Shocks and frictions in US business cycles: A Bayesian DSGE approach, *American Economic Review* **97**(3): 586–606.
- Taylor, J. B. (1993). Discretion versus policy rules in practice, *Carnegie-Rochester conference series on public policy*, Vol. 39, Elsevier, pp. 195–214.
- Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

3.8 Appendix

3.8.1 A microfounded New Keynesian Model with Credit and Collateral Shocks

The general approach is based on the seminal approach of De Grauwe (2011) who integrates bounded rationality in form of a discrete choice learning mechanism into an otherwise fully microfounded macroeconomic model. In line with this strand of literature I set up the equations of a medium-scale DSGE model in the following sections, and afterwards replace the mathematical expectations operator E_t with the boundedly rational expectations operator \tilde{E}_t (described above) in the log-linearized approximation of the system that is given in the main text.

Households

A representative household maximizes the intertemporal utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t Z_t \left[\frac{C_t^{1-\sigma_c}}{1-\sigma_c} - \frac{L_t^{1+\sigma_l}}{1+\sigma_l} \right] \quad (3.49)$$

subject to the budget constraint

$$C_t + D_t \leq W_t L_t + R_t D_{t-1} + \Delta_t + T_t \quad (3.50)$$

where D_t is the total short term debt acquired by households that consists of riskless one-period bonds which are either bank deposits or government debt. Both savings vehicles are perfect substitutes and pay out the same gross real interest rate R_t .¹⁰ One obtains the first-order conditions

$$W_t = L_t^{\sigma_l} C_t^{\sigma_c} \quad (3.51)$$

$$C_t^{-\sigma_c} = \beta^{-1} E_t \left\{ C_{t+1}^{-\sigma_c} R_{t+1} \frac{Z_{t+1}}{Z_t} \right\} \quad (3.52)$$

which are represented by the log-linear approximations (3.1) and (3.2).¹¹

¹⁰Following Woodford (2003) I consider the limit of the cashless economy.

¹¹Note that in the log-linear specification the real interest rate has been replaced with the standard Fisher equation.

Goods producers

A competitive final goods firm uses the technology

$$Y_t = \left(\int_0^1 Y_{j,t}^{1-1/\epsilon} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad (3.53)$$

with demand elasticity ϵ . Intermediate goods $Y_{j,t}$ are produced by a continuum of differentiated intermediate goods producers that combines labor of households and capital rented from entrepreneurs according to the technology function

$$Y_{j,t} = A_t K_{j,t-1}^\alpha L_{j,t}^{1-\alpha} \quad (3.54)$$

with technology shock A_t . Cost minimization implies for the ratio of real wages and rental rate of capital:

$$\frac{r_t^k}{W_t} = \frac{\alpha}{1-\alpha} \frac{L_{j,t}}{K_{j,t-1}} \quad (3.55)$$

Intermediate goods producers are subject to standard Calvo pricing frictions. In every period only a fraction of $1 - \theta$ firms can adjust their prices, while the remaining fraction θ keeps prices unchanged. This implies the condition for optimal price P_t^*

$$\sum_{t=0}^{\infty} (\beta\theta)^k E_t \left\{ \Lambda_{t+k,t} Y_{t+k|t} \left(\frac{P_t^*}{P_{t+k}} - \frac{\epsilon}{\epsilon-1} \Psi_{t+k|t} \right) \right\} \quad (3.56)$$

with stochastic discount factor $\Lambda_{t+k,t}$ and where $Y_{t+k|t}$ and $\Psi_{t+k|t}$ denote output and (real) marginal costs in period $t+k$ of a firm that could adjust the price in period t for the last time. Aggregate inflation dynamics are given by

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \quad (3.57)$$

Along the lines of e.g. Galí (2015), after some manipulations inflation dynamics in log-linear form can be expressed as the New Keynesian Phillips Curve (3.5), where $\lambda = \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta(1-\alpha+\alpha\epsilon)}$. Aggregate marginal costs are given by

$$\Psi_t = A_t^{-1} \left(\frac{r_t^k}{\alpha} \right)^\alpha \left(\frac{W_t}{1-\alpha} \right)^{1-\alpha} \quad (3.58)$$

Capital producers

A representative competitive capital producer builds capital according to

$$K_t = (1 - \delta)K_{t-1} + \left(1 - S\left(\zeta_{i,t} \frac{I_t}{I_{t-1}}\right)\right) I_t \quad (3.59)$$

which it sells to entrepreneurs for price Q_t . After the capital has been used in goods production, it repurchases depreciated capital at price Q_{t+1} and adjusts the capital stock using final goods as investment subject to adjustment cost function $S(\cdot)$. Profit maximization implies the first-order condition

$$Q_t \cdot [1 - S(\chi_t) - \chi_t S'(\chi_t)] = 1 - \beta E_t \left\{ Q_{t+1} \Lambda_{t+1,t} \chi_{t+1} \frac{I_{t+1}}{I_t} S'(\chi_{t+1}) \right\} \quad (3.60)$$

with $\chi_t \equiv \zeta_{i,t} \frac{I_t}{I_{t-1}}$. Under the assumption of an adjustment cost function with properties $S(1) = S'(1) = 0$ and $S''(1) = \varphi_i$, the first-order condition in log-linear form is reduced to the Euler investment equation (3.8).

Entrepreneurs

A continuum of competitive entrepreneurs obtains loans $B_{j,t}$ to finance capital purchases $Q_t K_{j,t}$. Each entrepreneur faces the budget constraint

$$Q_t K_{j,t} = N_{j,t} + B_{j,t} \quad (3.61)$$

where $N_{j,t}$ denotes the net worth of the entrepreneur. It rents out its capital to goods producers at rental rate r_t^k . At the beginning of period $t + 1$, the entrepreneur faces an idiosyncratic shock $w_{j,t+1}$ that reduces the value of its assets to $w^d Q_t K_t$ ($0 < w^d < 1$) with probability p_t^d and keeps it unchanged else. The net worth evolves according to

$$N_{j,t+1} = R_{t+1}^k \cdot w_{j,t+1} Q_t K_{j,t} - R_{t+1}^e B_{j,t} \quad (3.62)$$

with loan rate, R_{t+1}^e , and return to capital

$$R_{t+1}^k = (r_{t+1}^k + (1 - \delta)Q_{t+1}) / Q_t \quad (3.63)$$

It is assumed that an entrepreneur who faces a negative realization of the idiosyncratic shock $w_{j,t+1} = w^d$ always defaults. Financial intermediaries enforce a loan-to-value constraint to mitigate the losses in case of default

$$B_{j,t} \leq \phi_t K_{j,t} \quad (3.64)$$

where $\phi_t K_{j,t}$ is the fraction of assets that are seized by creditors. Thus, net value of defaulting entrepreneurs in period $t+1$ is $p_t^d w^d (1 - \phi_t) R_{t+1}^k Q_t K_{j,t}$ which is transferred to households as lump-sum dividend payment. An equal number of new entrepreneurs enters the system and receives a fraction of κ/p_t^d as initial endowment. Thus, the aggregate net value of entrepreneurs evolves according to

$$N_{t+1} = (1 - p_t^d) [(R_{t+1}^k - R_{t+1}^e) Q_t K_t + R_{t+1}^e N_t] + \kappa w^d (1 - \phi_t) R_{t+1}^k Q_t K_t \quad (3.65)$$

The compound parameters in its log-linear approximation (3.9) are given by $n_1 \equiv 1 - (1 - p^d) R^e$, $n_2 \equiv n_1 + (1 - p^d) \hat{L} R^e$, $n_3 \equiv (1 - n_1) (\hat{L} - 1)$, $n_4 \equiv p^d ((R^k - R^e) \hat{L} + R^e)$, and $n_5 \equiv \kappa w^d \phi R^k \hat{L}$, with steady state return to capital R^k , steady state credit rate R^e , steady state leverage ratio \hat{L} , steady state entrepreneur default risk p^d , and steady state LTV ratio ϕ .¹²

Entrepreneurs choose leverage ratio $\hat{L}_{j,t} \equiv Q_t K_{j,t} / N_{j,t}$ to maximize expected pre-dividend net worth:

$$\max_{\hat{L}_t} E_t \left\{ (1 - p_t^d) \left[(R_{t+1}^k - R_{t+1}^e) \hat{L}_{j,t} + R_{t+1}^e \right] + p_t^d w^d (1 - \phi_t) R_{t+1}^k \hat{L}_{j,t} \right\} \quad (3.66)$$

which implies the first-order condition

$$(1 - p_t^d + p_t^d w^d (1 - \phi_t)) E_t R_{t+1}^k = (1 - p_t^d) R_{t+1}^e \quad (3.67)$$

One obtains the log-linear approximation (3.11) with compound parameters $q_1 \equiv \frac{1-\delta}{R^k}$, $q_2 \equiv \frac{R^e - R^k}{(1-p^d)R^e}$, and $q_3 \equiv \frac{p^d w^d \phi R^k}{(1-p^d)R^e}$.¹³

¹²The real credit rate is replaced in the log-linear specification with the standard Fisher equation.

¹³To avoid further complication, it is assumed that $\tilde{E}_t r_{t+1}^k = r_{t-1}^k$ and $\tilde{E}_t q_{t+1} = q_{t-1}$ when replacing mathematical expectations with bounded rationality operator \tilde{E}_t .

Credit policy

Akin to Gertler and Karadi (2011), the central bank is facilitating lending by intermediating loans to entrepreneurs in addition to private intermediation. Total loans to entrepreneurs B_t are the sum of privately intermediated loans B_t^p and loans provided with government assistance B_t^{ga} :

$$B_t = B_t^p + B_t^{ga} \quad (3.68)$$

In comparison to private lending (specified below), the government has a strategic advantage when dealing with defaulting entrepreneurs. While private lenders have to resort to seizing collateralized assets to mitigate credit risk, the government can collect lump-sum taxes from households, who are the recipients of the defaulting entrepreneur's remaining assets.

Assuming that the central bank is willing to fund the fraction ς_t of total loans, one obtains

$$B_t^p = (1 - \varsigma_t)B_t \quad (3.69)$$

Financial intermediaries

A representative bank intermediates deposits of households to loans of entrepreneurs. Since they do not have information about the idiosyncratic shock in advance, every entrepreneur receives the same standard debt contract. The following bank participation constraint must hold

$$(1 - p_t^d)R_{t+1}^e B_{j,t}^p + p_t^d E_t R_{t+1}^k w^d \phi_t Q_t K_{j,t} \geq R_{t+1} B_{j,t}^p \quad (3.70)$$

Together with the first-order condition of entrepreneurs (3.67) it implies the log-linear approximation for the credit spread (3.30) with compound parameters $b_1 \equiv [1 - (1 - p^d)\beta R^e] / (\hat{L} - 1)$, $b_2 \equiv \frac{p^d}{1-p^d}\beta R^e - [1 - (1 - p^d)\beta R^e] (1 - q_2)$, $b_3 \equiv [1 - (1 - p^d)\beta R^e] (1 + q_3)$, and $b_4 \equiv 1 - (1 - p^d)\beta R^e$. If the assumption of central bank intermediation is dropped, equation (3.30) can be replaced with equation (3.12).

The LTV ratio is set endogenously to minimize the costs of redeploying seized assets, which are captured by the quadratic function

$$\Theta(\phi_t, \Xi_t) = \frac{\phi_t^2}{2} \cdot \frac{\Xi_t^{\gamma_\phi}}{\exp\{\varepsilon_{\phi,t}\}} \quad (3.71)$$

with fundamentals-based risk measure Ξ_t and stochastic LTV shock $\varepsilon_{\phi,t}$. Cost minimization implies

$$\phi_t = \frac{\exp\{\varepsilon_{\phi,t}\}}{\Xi_t^{\gamma_\phi}} \quad (3.72)$$

Government budget constraint and market clearing

Government expenditures G_t are financed with lump-sum taxes on households T_t and government debt B_t^g issued to a representative institutional investor that operates outside the remaining financial system (to avoid further complication).¹⁴ The government budget constraint is given by

$$G_t + R_t B_{t-1}^g + \varsigma_t B_t = T_t + B_t^g + (1 - p_t^d) R_t^e \varsigma_{t-1} B_{t-1} \quad (3.73)$$

It is assumed that the government sets lump-sum taxes T_t to collect the assets it lost due to entrepreneur default, to finance a fraction τ of government expenditures and interest payments on government debt. For simplification it transfers all revenues from loan intermediation to households, $(R_t^e - 1) \varsigma_{t-1} B_{t-1}$. This implies

$$T_t = (1 - (1 - p_t^d) R_t^e) \varsigma_{t-1} B_{t-1} + \tau (G_t + R_t B_{t-1}^g) \quad (3.74)$$

One obtains the government debt accumulation equation

$$B_t^g = (1 - \tau) G_t + (1 - \tau) R_t B_{t-1}^g + \varsigma_t B_t - \varsigma_{t-1} B_{t-1} \quad (3.75)$$

Together with the fiscal policy rule (3.13), this implies the log-linear equation (3.31) with $g_1 \equiv (1 - \tau)\beta^{-1}$, $g_2 \equiv \varsigma B / B^g$, $g_3 \equiv g_2 \cdot \hat{L} / (\hat{L} - 1)$, and $g_4 \equiv B / B^g - g_2$.

¹⁴Note that lump-sum dividend payments to households were imposed to offset any direct effects of government debt on aggregate demand.

The resource constraint is given by

$$Y_t = C_t + \left(1 + S \left(\zeta_{i,t} \frac{I_t}{I_{t-1}}\right)\right) I_t + G_t \quad (3.76)$$

3.8.2 Extension with stock market

Household utility maximization with stock demand in the utility

The representative household maximizes a utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t Z_t \left[\frac{C_t^{1-\sigma_c}}{1-\sigma_c} + \alpha_d \frac{(D_t^s)^{1-\sigma_d}}{1-\sigma_d} - \frac{L_t^{1+\sigma_l}}{1+\sigma_l} \right] \quad (3.77)$$

with household stock demand D_t^s , an approach that Lengnick and Wohltmann (2016) refer to as *stock in the utility*. The budget constraint in real terms is

$$C_t + D_t + S_t D_t^s \leq W_t L_t + R_t D_{t-1} + S_t D_{t-1}^s + \Delta_t + T_t \quad (3.78)$$

with real stock price S_t . This gives rise to the same first-order conditions as in the baseline model plus the following equation that determines stock demand

$$\alpha_d (D_t^s)^{-\sigma_d} = C_t^{-\sigma_c} \left(S_t - \frac{S_{t+1}}{R_{t+1}} \right) \quad (3.79)$$

In log-linearized form one obtains equation (3.34).

Unconventional monetary policy: stock purchases

If the central bank engages in stock purchases the government budget constraint becomes

$$G_t + R_t B_{t-1}^g + \varsigma_t B_t + S_t D_t^{s,CB} = T_t + B_t^g + (1 - p_t^d) R_t^e \varsigma_{t-1} B_{t-1} + \frac{S_{t-1}}{\Pi_t} D_{t-1}^{s,CB} \quad (3.80)$$

such that one obtains the log-linearized version (3.45) with $g_5 = \frac{S D^{s,CB}}{B^g}$.

Approximation of fundamental stock price

The fundamental stock price is determined as the value of expected dividend payments. Thus, we have for the fundamental stock price of entrepreneurs

$$S_t^f = \tilde{E}_t^f \left\{ \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j,t} \left(\prod_{k=1}^j (1 - p_{t+k-2}^d) \right) p_{t+j-1}^d D_{t+j} \right\} \quad (3.81)$$

$$\begin{aligned} &= p_{t-1}^d D_t + \beta(1 - p_{t-1}^d) p_t^d \tilde{E}_t^f (\Lambda_{t+1,t} D_{t+1}) + \beta^2(1 - p_{t-1}^d)(1 - p_t^d) \tilde{E}_t^f (p_{t+1}^d \Lambda_{t+2,t} D_{t+2}) \\ &\quad + \beta^3(1 - p_{t-1}^d)(1 - p_t^d) \tilde{E}_t^f ((1 - p_{t+1}^d) p_{t+2}^d \Lambda_{t+3,t} D_{t+3}) + \dots \end{aligned} \quad (3.82)$$

where D_t is the value of assets a defaulting entrepreneur transfers to the household, i.e.

$$D_t = w^d(1 - \phi_{t-1}) R_t^k Q_{t-1} K_{t-1} \quad (3.83)$$

Under the reasonable assumption that macroeconomic expectations of *fundamentalist* stock traders for all future quarters are equal to the steady state values, it holds for all $j \geq 1$ that

$$\tilde{E}_t^f \Lambda_{t+j,t} = 1 \quad (3.84)$$

$$\tilde{E}_t^f p_{t+j}^d = p^d \quad (3.85)$$

$$\tilde{E}_t^f D_{t+j} = D = w^d(1 - \phi) R^k K \quad (3.86)$$

such that

$$S_t^f = p_{t-1}^d D_t + \beta D(1 - p_{t-1}^d) p_t^d + \frac{\beta^2 p^d D}{1 - \beta(1 - p^d)} (1 - p_{t-1}^d)(1 - p_t^d) \quad (3.87)$$

The log-linear approximation (3.42) contains the compound parameters $s_1 = 1 - \beta(1 - p^d)$, $s_2 = \beta p^d s_1 - \beta^2(1 - p^d) p^d$ and $s_3 = (1 - \beta p^d) s_1 - \beta^2(1 - p^d) p^d$.

3.8.3 Steady state analysis

The zero inflation ($\Pi = 1$) steady state of the system given by (3.51), (3.52), (3.54), (3.55), (3.58), (3.59), (3.60), (3.61), (3.63), (3.65), leverage ratio definition, (3.67), (3.69), (3.70), (3.75), and (3.76) implies the following:

$$W = L^{\sigma_l} C^{\sigma_c} \quad (3.88)$$

$$R = \beta^{-1} \quad (3.89)$$

$$Y = K^\alpha L^{1-\alpha} \quad (3.90)$$

$$\frac{r^k}{W} = \frac{\alpha}{1-\alpha} \frac{L}{K} \quad (3.91)$$

$$\Psi = \left(\frac{r^k}{\alpha}\right)^\alpha \left(\frac{W}{1-\alpha}\right)^{1-\alpha} = \frac{\epsilon - 1}{\epsilon} \quad (3.92)$$

$$I = \delta K \quad (3.93)$$

$$Q = 1 \quad (3.94)$$

$$QK = N + B \quad (3.95)$$

$$R^k = r^k/Q + 1 - \delta \quad (3.96)$$

$$N = (1 - p^d) [(R^k - R^e)QK + R^e N] + \kappa w^d (1 - \phi) R^k QK \quad (3.97)$$

$$\hat{L} = QK/N \quad (3.98)$$

$$(1 - p^d)R^e = (1 - p^d + p^d w^d (1 - \phi)) R^k \quad (3.99)$$

$$B^p = (1 - \varsigma)B \quad (3.100)$$

$$RB^p = (1 - p^d)R^e B^p + p^d R^k w^d \phi QK \quad (3.101)$$

$$B^g = (1 - \tau)G + (1 - \tau)RB^g \quad (3.102)$$

$$Y = C + I + G \quad (3.103)$$

A slightly reduced system is

$$W = L^{\sigma_l} C^{\sigma_c} \quad (3.104)$$

$$Y = K^\alpha L^{1-\alpha} \quad (3.105)$$

$$\frac{r^k}{W} = \frac{\alpha}{1-\alpha} \frac{L}{K} \quad (3.106)$$

$$\Psi = \left(\frac{r^k}{\alpha}\right)^\alpha \left(\frac{W}{1-\alpha}\right)^{1-\alpha} = \frac{\epsilon - 1}{\epsilon} \quad (3.107)$$

$$K = N + B \quad (3.108)$$

$$R^k = r^k + 1 - \delta \quad (3.109)$$

$$N = (1 - p^d) [(R^k - R^e)K + R^e N] + \kappa w^d (1 - \phi) R^k K \quad (3.110)$$

$$\hat{L} = K/N \quad (3.111)$$

$$(1 - p^d)R^e = (1 - p^d + p^d w^d (1 - \phi)) R^k \quad (3.112)$$

$$B = \beta(1 - p^d)R^e B + \beta p^d R^k w^d \phi (1 - \varsigma)^{-1} K \quad (3.113)$$

$$B^g = \frac{1 - \tau}{1 - (1 - \tau)\beta^{-1}} G \quad (3.114)$$

$$Y = C + G + \delta K \quad (3.115)$$

From definition of leverage constraint and entrepreneur budget constraint, it follows that

$$B = (\hat{L} - 1)N \quad (3.116)$$

such that one obtains from the bank participation constraint

$$\hat{L} = \frac{1 - \beta(1 - p^d)R^e}{1 - \beta((1 - p^d)R^e + p^d R^k w^d \phi (1 - \varsigma)^{-1})} \quad (3.117)$$

Inserting the entrepreneur FOC yields

$$\hat{L} = \frac{1 - \beta(1 - p^d)R^e}{1 - \beta(1 - p^d)R^e \cdot \left(1 + \frac{p^d w^d \phi}{(1 - p^d + p^d w^d (1 - \phi))(1 - \varsigma)}\right)} \quad (3.118)$$

while the entrepreneur net worth equation implies

$$\begin{aligned}
\hat{L} &= \frac{1 - (1 - p^d)R^e}{(1 - p^d)(R^k - R^e) + \kappa w^d(1 - \phi)R^k} \\
&= \frac{1 - (1 - p^d)R^e}{(1 - p^d)R^e \cdot \left(\frac{1 - p^d + \kappa w^d(1 - \phi)}{1 - p^d + p^d w^d(1 - \phi)} - 1 \right)} \\
&= \frac{(1 - (1 - p^d)R^e)(1 - p^d + p^d w^d(1 - \phi))}{(1 - p^d)w^d(1 - \phi)R^e \cdot (\kappa - p^d)}
\end{aligned} \tag{3.119}$$

The two leverage ratio equations imply the quadratic equation in R^e

$$p_0 - p_1 R^e + p_2 (R^e)^2 = 0 \tag{3.120}$$

with

$$\begin{aligned}
p_0 &= 1 - p^d + p^d w^d(1 - \phi) \\
p_1 &= (1 - p^d)(1 - p^d + \kappa w^d(1 - \phi) + \beta(1 - p^d + p^d w^d(1 - \phi) + p^d w^d \phi(1 - \varsigma)^{-1})) \\
p_2 &= \beta(1 - p^d)^2(1 - p^d + \kappa w^d(1 - \phi) + p^d w^d \phi(1 - \varsigma)^{-1})
\end{aligned}$$

The equation has two real solutions for R^e if $(p_1)^2 \geq 4p_0 p_2$ where for realistic parameterization only the solution

$$R^e = \frac{p_1}{2p_2} - \sqrt{\left(\frac{p_1}{2p_2}\right)^2 - p_0/p_2} \tag{3.121}$$

coincides with a steady state leverage ratio above unity, $\hat{L} > 1$, and is, thus, economically interpretable. We calibrate $G/Y = y_g$. Steady state real wage is determined as

$$W = (1 - \alpha) \left(\frac{\epsilon - 1}{\epsilon} \right)^{\frac{1}{1-\alpha}} \left(\frac{r^k}{\alpha} \right)^{-\frac{\alpha}{1-\alpha}} \tag{3.122}$$

It follows that

$$L = \frac{1 - \alpha}{\alpha} \frac{r^k}{W} K = \left(\frac{\epsilon r^k}{(\epsilon - 1)\alpha} \right)^{\frac{1}{1-\alpha}} K \tag{3.123}$$

such that

$$Y = \frac{\epsilon r^k}{(\epsilon - 1)\alpha} K \tag{3.124}$$

With

$$C^{\sigma_c} = \frac{W}{L^{\sigma_l}} \quad (3.125)$$

$$= \frac{W^{1+\sigma_l}}{[(1-\alpha)/\alpha \cdot r^k K]^{\sigma_l}} \quad (3.126)$$

$$= (1-\alpha) \left(\frac{\epsilon-1}{\epsilon} \right)^{\frac{1+\sigma_l}{1-\alpha}} \left(\frac{r^k}{\alpha} \right)^{-\frac{\alpha+\sigma_l}{1-\alpha}} K^{-\sigma_l} \quad (3.127)$$

Thus,

$$Y - G = C + I \quad (3.128)$$

$$(1-y_g)Y = \left((1-\alpha) \left(\frac{\epsilon-1}{\epsilon} \right)^{\frac{1+\sigma_l}{1-\alpha}} \left(\frac{r^k}{\alpha} \right)^{-\frac{\alpha+\sigma_l}{1-\alpha}} K^{-\sigma_l} \right)^{1/\sigma_c} + \delta K \quad (3.129)$$

which implies

$$\left((1-\alpha) \left(\frac{\epsilon-1}{\epsilon} \right)^{\frac{1+\sigma_l}{1-\alpha}} \left(\frac{r^k}{\alpha} \right)^{-\frac{\alpha+\sigma_l}{1-\alpha}} \right)^{1/\sigma_c} K^{-\frac{\sigma_l+\sigma_c}{\sigma_c}} = \frac{(1-y_g)\epsilon r^k}{(\epsilon-1)\alpha} - \delta \quad (3.130)$$

Steady state with stock market extension

All steady state equations of the baseline model apply to the extended model as well. Additionally, we have

$$\alpha_d (D^s)^{-\sigma_d} = C^{-\sigma_c} (1-\beta) S \quad (3.131)$$

with

$$S = S^f = \frac{p^d w^d (1-\phi) R^k K}{1-\beta(1-p^d)} \quad (3.132)$$

The steady state stock purchases of the central bank are assumed to be a fraction of household stock demand

$$D^{s,CB} = \varsigma_s D^s \quad (3.133)$$

Chapter 4

Outlook

Effective monetary policy requires the policymaker to take into account the specifics of the expectations channel which is an important mechanism to obtain objectives such as price stability. Typically models designed for monetary policy analysis rely on both perfect rationality and information of agents which involves completely homogeneous and model-consistent expectations. However, many studies have shown that these assumptions are unrealistically strong. In general, there are two approaches to accommodate these findings: by incrementally extending the rational expectations framework with constraints to information acquisition and processing, or by foregoing the assumption of full rationality and replacing it with an alternative mechanism such as boundedly rational heuristic switching. It is an important task for future research on monetary policy theory to bring the two strands of literature together, each represented by a paper in this dissertation.

Chapter 2 analyzes a monetary policy model that builds on the rational expectations assumption by implementing limited and asymmetric information. However, in this setup the available information is perfectly processed such that all agents act completely rational with respect to their informational limitations. Future research should focus on relaxing this assumption by integrating bounded rationality. Additionally, one could enhance the news shock mechanism with a variable adjustment length to align it with the literature.

Chapter 3 is based on a heuristic switching framework where expectations are the result of boundedly rational discrete choice learning. The setup is used in conjunction with modern components from the monetary policy literature such as a credit market subject to default risk and collateral requirements, endogenous macroeconomic risk, a dynamic stock market, and central bank credibility. Extensions to this work should implement a thorough empirical analysis. A particularly interesting route for future research on dynamic switching models constitutes the inclusion of rational expectations in the set of available forecast strategies.

Eidesstattliche Erklärung (Affidavit)

Ich erkläre hiermit, dass ich meine Doktorarbeit „*Monetary Policy with Imperfect Information and Bounded Rationality*“ selbstständig und ohne fremde Hilfe angefertigt habe und dass ich alle von anderen Autoren wörtlich übernommenen Stellen, wie auch die sich an die Gedanken anderer Autoren eng anlehenden Ausführungen meiner Arbeit, besonders gekennzeichnet und die Quellen nach den mir angegebenen Richtlinien zitiert habe.

Datum

Unterschrift