





Real-Time Forecasting Using Mixed-Frequency VARs With Time-Varying Parameters

Markus Heinrich¹ | Magnus Reif² 📵

¹Stadtwerke Kiel, University of Kiel, Kiel, Germany | ²Deutsche Bundesbank, CESifo, Frankfurt, Germany

Correspondence: Magnus Reif (magnus.reif@bundesbank.de)

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ABSTRACT

This paper provides a detailed assessment of the real-time forecast accuracy of a wide range of vector autoregressive models that allow for both structural change and indicators sampled at different frequencies. We extend the literature by evaluating a mixed-frequency time-varying parameter vector autoregressive model with stochastic volatility. Monte Carlo simulation shows that the novel model is well-suited to estimate missing monthly observations in an environment that is subject to parameter instability. In a real-time forecast exercise, the model delivers accurate now- and forecasts and, on average, outperforms its competitors. Particularly, inflation and unemployment rate forecasts are more precise.

JEL Classification: C11, C53, C55, E32

1 | Introduction

Macroeconomists and, in particular, macroeconomic forecasters face two major challenges. First, there are structural changes within an economy. Second, in real time, forecasters need to process unbalanced datasets due to indicators sampled at different frequencies and indicator-specific publication lags. Concerning structural change, it is commonly found that particularly modeling time-varying volatilities enhances VAR-based inference and estimation, while fluctuations in the VAR coefficients are frequently considered to be less vital (for example, Chan and Eisenstat 2017; D'Agostino et al. 2013). Since the onset of the Great Recession, which probably caused important structural shifts, modeling time-varying links between variables has attracted anew interest. Concerning unbalanced datasets, literature stresses the merits of mixed-frequency approaches in computing precise now- and forecasts, and tracking the current state of the economy in real time (for instance, Kuzin et al. 2011; Schorfheide and Song 2015; Götz and Hauzenberger 2021).

However, evidence regarding the forecast performance of models allowing for structural shifts in a mixed-frequency setting is

rather sparse. This study aims at filling this gap by providing a detailed assessment of the real-time forecast accuracy of a bundle mixed-frequency models that allow for structural change. To this end, we estimate nonlinear and linear VARs with and without mixed-frequencies, including a fully-fledged model incorporating time-varying parameters, stochastic volatilities, and mixed-frequencies—a MF-TVP-SV-VAR. This analysis enables us to trace out the relative impact of the models' mixed-frequency part and the time-variation in the models' coefficients on the forecast accuracy. Our comparison relies on real-time out-of-sample now- and forecast accuracy of both point and density forecasts concerning four key US macroeconomic variables: GDP, industrial production, CPI, and the unemployment rate.

Overall, our forecast comparison provides two major findings. First, modeling structural change *and* intraquarterly dynamics is beneficial for point and density forecasts, notably for now and short-term forecasts. In particular, the accuracy of inflation and unemployment rate forecasts can be substantially increased. Second, the MF-TVP-SV-VAR delivers competitive point and density forecasts—on average over all variables it outperforms each competitor. Our results moreover suggest

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© 2025 The Author(s). Journal of Forecasting published by John Wiley & Sons Ltd. that the combination of mixed-frequencies, stochastic volatility, and time-varying parameters is particularly beneficial for inflation nowcasts computed with only little information about the respective quarters. In those cases, the MF-TVP-SV provides the largest gains in forecast accuracy. Inspecting the mixed-frequency models' forecasts during the Great Recession reveals that allowing for time-variation in the VAR coefficients and stochastic volatility is superior relative to only one of these specifications for inflation and the unemployment rate.

Estimation of the models' TVP-SV part follows Del Negro and Primiceri (2015). However, we estimate the hyperparameters that relate to the amount of time-variation in the parameters following Amir-Ahmadi et al. (2020). Estimation of the models' MF part is based on Mariano and Murasawa (2010) and Schorfheide and Song (2015).

2 | Data and Forecast Setup

We use a dataset consisting of four macroeconomic indicators, three of which are sampled at monthly frequency and one is observed quarterly. The quarterly series is US real GDP; the monthly series are industrial production (IP), the consumer price index (CPI), and the unemployment rate. GDP, IP, and CPI enter the models in log first differences times 100. The unemployment rate remains untransformed. For the VARs estimated on quarterly frequency, the monthly indicators enter the models as quarterly averages. We obtain real-time data from the Archival FRED (ALFRED) database. The sample runs from January 1960 until September 2017. The first 8 years are used to specify priors such that the estimation starts in January 1968.

We assess the predictions regarding the intraquarterly inflow of information using three information sets. We assume that the forecasts are generated around the middle of each month, when the current indicator releases are available. The first information set (I1) relates to the first month of each quarter such that the forecaster has information up to the middle of January, April, July, or October. In these months, the researcher has observations on IP growth, inflation, and unemployment until the end of the respective previous quarter and a first and preliminary estimate of GDP referring to the previous quarter. Concerning the second (third) information set I2 (I3), one (two) additional information for the monthly indicators and the first (second) GDP revision are available. As the quarterly VARs cannot cope with the monthly data flow, we estimate them in each recursion based on the balanced information set I1, which accounts for new information only in terms of data revisions.

We evaluate our forecasts for data vintages using an expanding window from January 1990 until September 2017. The predictions are evaluated based on quarterly averages. In the case of IP and CPI, this implies that we use the predicted monthly growth rates to reconstruct the monthly levels of the series, which in turn are used to compute the quarterly growth rates. For GDP growth, we use the aggregation rule given by (5) to obtain quarterly GDP growth. To abstract from benchmark revisions, we evaluate GDP growth forecasts based on the second available estimate, that is the forecast for period t+h is evaluated with the realization taken from the vintage published in t+h+2.

As the remaining variables are revised only rarely and slightly, we evaluate the forecast based on the latest vintage. The maximum forecast horizon h_{max} is set to 4 quarters. Thus, the mixed-frequency models generate forecasts for $h_m = 1, ..., 12$ months. Forecasts for horizons larger than one are obtained iteratively.

3 | Models

We evaluate the forecast performance of mixed-frequency, stochastic volatility, and time-varying parameter VARs, as well as the forecast performance of combinations of these features. For the stochastic volatility models, we use random walk stochastic volatility, which is a parsimonious and competitive specification (Clark and Ravazzolo 2015). Throughout the paper, $n = n_q + n_m$, where n, n_q , and n_m denote the number of total, quarterly, and monthly variables, respectively. Finally, p denotes the lag order.

3.1 | A VAR With Time-Varying Parameters and Stochastic Volatility

For the VAR with time-varying parameters and stochastic volatility (TVP-SV-VAR), we assume that the vector of variables (y_t) satisfies

$$y_t = B_{0,t} + \sum_{i=1}^{p} B_{i,t} y_{t-i} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \Omega_t)$$
 (1)

where $B_{0,t}$ is a $n\times 1$ vector of time-varying intercepts, $B_{i,t}$ are matrices of time-varying coefficients and Ω_t is the time-varying $n\times n$ variance-covariance matrix. Let $\Omega_t = A_t\Sigma_t A_t'$, where the diagonal elements of Σ_t are the stochastic volatilities and the lower-triangular elements of A_t are the contemporaneous relations between the variables. Moreover, let σ_t be the vector of the diagonal elements of Σ_t , a_t the vector of lower-triangular elements stacked by rows of A_t , and β_t the vector of stacked VAR coefficients. We assume that

$$\log \sigma_t = \log \sigma_{t-1} + e_t, \qquad e_t = (e_{1,t}, ..., e_{n,t})' \sim N(0, \Psi)$$
 (2)

$$a_t = a_{t-1} + v_t,$$
 $v_t = (v'_{1,t}, ..., v'_{n,t})' \sim N(0, \Phi)$ (3)

$$\beta_t = \beta_{t-1} + \chi_t, \qquad \chi_t \sim N(0, Q) \tag{4}$$

where
$$Q = diag\left(q_{\beta_1}^2, \dots, q_{\beta_{k_\beta}}^2\right)$$
. We obtain an SV-VAR by exclud-

ing time-variation in the VAR coefficients, and a TVP-VAR by excluding time-variation in the variance-covariance matrix.

3.2 | A Mixed-Frequency VAR

Estimation of the mixed-frequency VAR (MF-VAR) follows the Bayesian state-space approach of Schorfheide and Song (2015). Let $y_t = [y'_{q,t}, y'_{m,t}]'$, where $y_{m,t}$ collects the monthly variables and $y_{q,t}$ denotes the quarterly variables at monthly frequency. As the quarterly variables are observed only in the last month of each quarter, $y_{q,t}$ contains missing observations for the first and second month of each quarter. We construct the measurement

equation by assuming that quarterly GDP growth can be disaggregated into unobserved monthly GDP growth (Mariano and Murasawa 2003):

$$\Delta_{3} \log Y_{q,t} = y_{q,t} = \frac{1}{3} \tilde{y}_{q,t} + \frac{2}{3} \tilde{y}_{q,t-1} + \tilde{y}_{q,t-2} + \frac{2}{3} \tilde{y}_{q,t-3} + \frac{1}{3} \tilde{y}_{q,t-4}$$
(5)

Combining the unobserved with the observed monthly variables in $\tilde{y}_t = [\tilde{y}'_{q,t}, y'_{m,t}]'$, we define the state vector by $z_t = [\tilde{y}'_t, ..., \tilde{y}'_{t-p+1}]'$ and write the measurement equation as:

$$y_t = H_t z_t \tag{6}$$

Assuming that GDP growth is ordered first in the model, H_t is given by:

$$H_t = \begin{bmatrix} H'_{1,t} & H'_{2,t} \end{bmatrix}' \tag{7}$$

$$H_{1,t} = \begin{bmatrix} 1/3 & 0_{1\times n-1} & 2/3 & 0_{1\times n-1} & 1 & 0_{1\times n-1} & 2/3 & 0_{1\times n-1} & 1/3 & 0_{1\times n-1} & 0_{1\times (p-4)n} \end{bmatrix},$$

$$H_{2,t} = \begin{bmatrix} 0_{n-1 \times 1} & I_{n-1} & 0_{n-1 \times pn} \end{bmatrix}$$
 (9)

The missing observations in z_t are replaced by estimated states using simulation smoothing with a time-varying dimension of the state-space system (Durbin and Koopman 2001). The transition equation of the MF-VAR in state-space form is given by:

$$z_t = \mu + F z_{t-1} + v_t, \quad v_t \sim N(0, S)$$
 (10)

where μ and F contain the intercepts and AR-coefficients, respectively. S is a $pn \times pn$ variance-covariance matrix where the first $n \times n$ elements equal Ω and all remaining entries are zero. We obtain the MF-SV-VAR by setting the first $n \times n$ elements of S to Ω_t using the laws of motion in (2) and (3). The MF-TVP-VAR is obtained by allowing F to vary over time according to (4). Including both specifications leads to the MF-TVP-SV-VAR.

3.3 | Estimation Procedure and Prior Specification

We estimate the models using Bayesian estimation techniques. The mixed-frequency models are estimated with 4 lags; the quarterly models are estimated with 2 lags. 4 Prior specifications follow Clark (2011), Del Negro and Primiceri (2015), and Schorfheide and Song (2015). Additionally, for the hyperparameters that relate to the amount of time-variation in β_t , a_{it} , and $\log \sigma_{it}$, we follow Amir-Ahmadi et al. (2020) by implementing another layer of priors for those hyperparameters. A detailed description is provided in the supporting information.

3.4 | Monte Carlo Evidence

In this section, we account for the novelty of the MF-TVP-SV-VAR and conduct a Monte Carlo simulation to assess the finite sample properties of the model. We focus on a VAR(4) without intercept term and consider four data generating processes (DGPs)—constant coefficients (DGP I), a single break in both the variance-covariance matrix and the coefficients (DGP II), drifting coefficients and variances (DGP III), and constant coefficients with outlier observations (DGP IV). We use the respective VAR process along with the disaggregation constraint (5) to mimic a dataset consisting of two monthly and one quarterly series. We assess accuracy of the MF-TVP-SV-VAR by computing the RMSEs of the estimated missing observations against the actual simulated values and compare the resulting figures with those derived from a (constant-coefficient) MF-VAR.

Table 1 shows that the MF-TVP-SV-VAR provides more accurate estimates for DGPs II and III. In particular, for the drifting-coefficient case (DGP III), the model strongly outperforms the MF-VAR. For DGP I, the MF-VAR provides, as expected, the best fit. Interestingly, the MF-TVP-SV-VAR seems

$$2/3 \quad 0_{1 \times n-1} \quad 1/3 \quad 0_{1 \times n-1} \quad 0_{1 \times (p-4)n}$$
, (8)

to be inadequate for DGP IV. Inspecting the estimated coefficients and variances reveals that the MF-TVP-SV-VAR interprets the outlier-induced hikes in volatility to be persistent, which leads to comparably poor estimates for the VAR's covariance matrix.6

In sum, these results suggest that the MF-TVP-SV-VAR is well-equipped to estimate missing monthly observation in our dataset, which is likely subject to some kind of parameter instability (see, for example, Chan and Eisenstat 2017, and the online Appendix).

4 | Results

For the point forecasts, which we evaluate in terms of root mean squared errors (RMSE), we first assess the models' nowcast accuracy. Afterwards, we evaluate the accuracy of the point forecasts and predictive densities with respect to the subsequent quarters. We evaluate the predictive densities using the continuous ranked probability score (CRPS).7 We provide results for the entire sample (1995Q1-2017Q4) and for a shorter sample period (2008Q1-2017Q4) to assess whether a possible structural break around the Great Recession affects the forecast performance.

TABLE 1 | RMSEs for estimated missing observations from Monte Carlo simulation.

	DGP I	DGP II	DGP III	DGP IV
MF-VAR	0.96	0.82	1.09	1.02
MF-TVP-SV- VAR	1.04	0.78	0.74	1.32

Note: DGP I: VAR with constant coefficients, DGP II: VAR with single break in coefficients and variances, DGP III: VAR with drifting coefficients and variances, DGP IV: VAR with outlier observations. RMSEs are based on 10 simulations per DGP. For each DGP, we set T = 500.

TABLE 2 | Real-time nowcast RMSEs.

Model	1990–2017			2008-2017		
	I1	12	I 3	I1	I2	I3
			GDP gro	owth		
MF-TVP-SV-VAR	0.90	0.86	0.88	0.82	0.80	0.83
MF-SV-VAR	0.89	0.82	0.81*	0.81	0.76	0.75
MF-TVP-VAR	0.85**	0.78	0.79*	0.78*	0.69*	0.72
MF-VAR	0.92	0.86	0.87	0.84	0.79	0.82
Q-TVP-SV-VAR	0.99	1.01	0.99	1.00	1.01	0.98
Q-TVP-VAR	0.98	0.99	0.99	0.97	0.98	0.98
Q-VAR	1.03	1.03*	1.03*	1.01	1.02	1.02
Q-SV-VAR	0.59	0.59	0.59	0.66	0.66	0.66
			IP grov	vth		
MF-TVP-SV-VAR	0.90	0.80	0.59**	0.84	0.68	0.52*
MF-SV-VAR	0.86	0.78	0.59**	0.82	0.70	0.52*
MF-TVP-VAR	0.86	0.78	0.58**	0.83	0.70	0.51*
MF-VAR	0.87	0.79	0.59**	0.84**	0.71	0.53*
Q-TVP-SV-VAR	0.99	0.99	0.99	0.93	0.94	0.94
Q-TVP-VAR	1.03	1.04	1.04	1.03	1.03	1.03
Q-VAR	1.06***	1.06***	1.07***	1.05**	1.06**	1.06**
Q-SV-VAR	1.13	1.10	1.09	1.34	1.34	1.34
			Inflati	ion		
MF-TVP-SV-VAR	0.77*	0.48**	0.28***	0.75 *	0.46**	0.26**
MF-SV-VAR	0.86	0.52**	0.29***	0.84	0.49**	0.27**
MF-TVP-VAR	0.82*	0.50**	0.28***	0.81	0.48**	0.27**
MF-VAR	0.90	0.53**	0.29***	0.87	0.50**	0.27**
Q-TVP-SV-VAR	0.91**	0.91**	0.91**	0.90**	0.90**	0.91**
Q-TVP-VAR	0.96***	0.96***	0.96***	0.96***	0.96***	0.96***
Q-VAR	1.04*	1.03*	1.04*	1.03	1.03	1.03
Q-SV-VAR	0.59	0.59	0.59	0.75	0.75	0.75
			Unemploym	ent rate		
MF-TVP-SV-VAR	0.80**	0.57***	0.37***	0.74**	0.52**	0.33***
MF-SV-VAR	0.80**	0.61***	0.38***	0.76**	0.55**	0.33***
MF-TVP-VAR	1.00	0.69***	0.37***	1.02	0.67**	0.33***
MF-VAR	0.79***	0.61***	0.38***	0.74**	0.56**	0.33***
Q-TVP-SV-VAR	0.94	0.93	0.95	0.92	0.92	0.93
Q-TVP-VAR	1.01	1.02	1.02	1.01	1.03	1.03
Q-VAR	1.03	1.03*	1.04*	1.02	1.03	1.03
Q-SV-VAR	0.27	0.26	0.25	0.30	0.30	0.30

Note: RMSEs are in absolute terms for the benchmark model (bottom row of each panel) and as ratios for the remaining models. Ratios below unity mark that the model outperforms the benchmark. Bold figures indicate the best result for the variable and information set. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively, according to the Diebold–Mariano-test with Newey–West standard errors.

4.1 | Nowcast Evaluation

Table 2 presents the results for the nowcast exercise taking into account the information sets I1 to I3. It provides three main takeaways. First, the mixed-frequency models outperform the quarterly models. On average, over all information sets and variables, the best nowcast performance is obtained by the MF-TVP-SV-VAR and the MF-SV-VAR, which improve on the benchmark (Q-SV-VAR) by roughly 30%. Second, most of the time, the nonlinear MF-models outperform the linear MF-VAR, indicating that—apart from using monthly information—parameter instability is beneficial also in a mixed-frequency setting. Third, the MF-models' relative performance improves with more information available, showing that the models are able to efficiently process the sequential data releases.

For GDP growth, only MF-models significantly outperform the benchmark. The best performance for both samples is obtained by the MF-TVP-VAR. This result suggests that, from a nowcasting perspective, it is more important to account for changes in output growth dynamics than to account for the decline in output growth volatility. Moreover, the mixedfrequency VARs' relative performance improves in the 2008-2017 sample, while the quarterly models performance remains rather unchanged. Thus, it appears that modeling the intraquarterly flow of information has gained importance. In the case of IP growth, accounting parameter instability does not yield more precise forecasts. In fact, the mixed-frequency models provide in both samples virtually identical RMSEs. Against the backdrop of the strong drop of IP growth volatility (see Appendix D of the supporting information), this results is rather surprising.

For inflation, the MF-TVP-SV-VAR delivers the best performance; it improves on the benchmark by, on average, 50%. Regarding both samples, the results indicate that notably with little information about the current quarter the MF-TVP-SV-VAR provides large gains in forecast accuracy relative to the competing models. The latter is particularly relevant because expert forecast are usually published in the second month of a quarter. With more information available, however, the differences towards the remaining MF-models vanish. For the unemployment rate, the MF-TVP-SV-VAR delivers the best nowcasts on average across information sets. However, it appears that non-linearity is not as important as for the remaining variables as the differences to the MF-VAR are negligible. The latter is maybe not surprising given that the fluctuations in volatility of the unemployment rate are less pronounced compared with the remaining variables (see Appendix D of the supporting information).

In sum, the nowcast exercise provides strong evidence in favor of non-linear models. For GDP and IP growth, allowing for time-varying parameters without stochastic volatility strongly improves forecast accuracy. For inflation and the unemployment rate the best forecast performance is obtained by models that account for both time-varying parameters and stochastic volatility.

4.2 | Forecast Evaluation

The results in Table 3 show that mixed-frequency VARs provide competitive forecasts even for higher horizons and for both samples. In the case of the unemployment rate, modeling within-quarter dynamics is particularly beneficial—at each horizon even the worst performing mixed-frequency VAR outperforms the best performing quarterly VAR. Moreover, the results reveal that the models' forecast performance substantially differs across variables. The best relative performance, over all variables and horizons, is delivered by the MF-SV-VAR and the MF-TVP-SV-VAR; the corresponding RMSEs are roughly 10% lower than those of the benchmark.

For GDP growth, the results provide three key findings. First, only the Q-TVP-SV-VAR significantly improves on the benchmark regarding both samples; for the shorter sample (right panel), the gains are even more pronounced. Second, the TVP models relative performance strongly improves in the shorter sample, suggesting that time-varying in the autoregressive coefficients has gained importance since the Great Recession. Third, also the mixed-frequency models performance strongly improves in the shorter sample (albeit in most cases insignificant) with the MF-TVP-SV-VAR providing the best performance. For IP growth, the benchmark is hard to beat (for both samples); no model significantly improves on the benchmark.

For inflation and the unemployment rate, the MF-TVP-SV-VAR delivers the best performance on average over all horizon, suggesting that time-variation in each coefficient is crucial for these variables. Moreover, for the unemployment rate, the MF-models consistently outperform the benchmark, while the quarterly models fail to do. Hence, the results provide evidence that both intraquarterly dynamics and time-variation in the VAR-coefficients are particularly important for these variables.

Thus, our results confirm the findings from previous studies based on quarterly models (see, among others, D'Agostino et al. 2013; Barnett et al. 2014) by use of mixed-frequency models. In sum, the results indicate that the gains in accuracy due to variations in the VAR-coefficients are smaller than the gains induced by stochastic volatility. However, using models with both features provides the most accurate forecasts on average over all variables. Finally, the results provide evidence that modeling within-quarter dynamics is beneficial also regarding short-term forecasts.

4.3 | Predictive Density Evaluation

The results for the CRPS are displayed in Table 4. Overall, the results point to the usefulness of within-quarter information in delivering well calibrated predictive densities; the mixed-frequency models provide better results on average over all variables and horizons than their quarterly counterparts. The MF-TVP-SV-VAR provides the best performance with a CRPS reduction of about 10% (on average over all variables and horizons). This emphasizes the importance of parameter instability for generating accurate predictive densities.

TABLE 3 | Real-time forecast RMSEs.

Model	1990-2017			2008-2017			
	h=1	h=2	h=3	h=1	h=2	h=3	
			GDP gro	owth			
MF-TVP-SV-VAR	0.98	0.96	0.97	0.94	0.91**	0.89***	
MF-SV-VAR	0.99	1.00	0.98	0.99	0.99	0.97*	
MF-TVP-VAR	0.99	1.00	0.98	0.96	0.95	0.90**	
MF-VAR	1.05	1.05	0.99	1.02	1.00	0.95**	
Q-TVP-SV-VAR	0.98	0.93***	0.94**	0.94***	0.88	0.86***	
Q-TVP-VAR	0.99	0.93***	0.94**	0.98**	0.87***	0.86***	
Q-VAR	1.04***	1.00	1.00	1.01	0.98	1.00	
Q-SV-VAR	0.61	0.61	0.63	0.69	0.68	0.69	
			IP grov	wth			
MF-TVP-SV-VAR	1.04	1.05*	1.03	1.00	0.99	0.99	
MF-SV-VAR	1.00	1.02	0.99	1.01	1.02	0.99	
MF-TVP-VAR	1.05	1.04**	1.03	1.03	1.02	1.01	
MF-VAR	1.06	1.09***	1.04**	1.05	1.06*	1.03	
Q-TVP-SV-VAR	1.03	1.01	1.01	0.99	0.98	1.00	
Q-TVP-VAR	1.02	1.00	1.00	1.02	0.98	0.98	
Q-VAR	1.07***	1.05***	1.03***	1.05***	1.04**	1.03**	
Q-SV-VAR	1.23	1.26	1.30	1.46	1.48	1.52	
			Inflat	ion			
MF-TVP-SV-VAR	0.81***	0.83***	0.85***	0.81 ***	0.84***	0.89***	
MF-SV-VAR	0.96	1.00	1.01	0.96	0.98	1.00	
MF-TVP-VAR	0.91**	0.91***	0.89***	0.92*	0.94*	0.94**	
MF-VAR	1.06	1.16***	1.31***	1.03	1.05	1.11***	
Q-TVP-SV-VAR	0.89***	0.87***	0.87***	0.89***	0.88***	0.90***	
Q-TVP-VAR	0.93***	0.93***	0.91***	0.94***	0.95***	0.95***	
Q-VAR	1.07***	1.12***	1.18***	1.04***	1.07***	1.10***	
Q-SV-VAR	0.65	0.65	0.62	0.81	0.79	0.73	
			Unemployn	nent rate			
MF-TVP-SV-VAR	0.81***	0.88**	0.92	0.78***	0.84**	0.87**	
MF-SV-VAR	0.84***	0.91*	0.94	0.81**	0.88*	0.91	
MF-TVP-VAR	0.94	1.00	1.02	0.92	0.97	0.98	
MF-VAR	0.84***	0.91	0.95	0.80**	0.87**	0.91	
Q-TVP-SV-VAR	0.97	1.00	1.01	0.94*	0.97	0.98	
Q-TVP-VAR	1.03**	1.04***	1.04***	1.02**	1.03**	1.04***	
Q-VAR	1.03***	1.02***	1.02***	1.03**	1.02**	1.01	
Q-SV-VAR	0.45	0.65	0.85	0.54	0.80	1.06	

Note: See Table 2 for a description.

TABLE 4 | Real-time forecast CRPS.

Model	1990-2017			2008-2017		
	h=1	h=2	h=3	h=1	h=2	h=3
			GDP gro	owth		
MF-TVP-SV-VAR	0.99	0.97	0.99	0.93	0.91	0.90
MF-SV-VAR	0.99	1.01	0.99	0.99	1.01	0.98
MF-TVP-VAR	1.06*	1.12***	1.12***	1.00	1.06	1.04
MF-VAR	1.07**	1.08**	1.03	1.04	1.03	0.97
Q-TVP-SV-VAR	0.99	0.97	0.99	0.95	0.92	0.93
Q-TVP-VAR	1.04	1.02	1.02	1.01	0.93	0.93
Q-VAR	1.07***	1.05***	1.05***	1.03	1.02	1.03
Q-SV-VAR	0.33	0.33	0.34	0.36	0.36	0.37
			IP grov	wth		
MF-TVP-SV-VAR	1.03	1.06*	1.07*	0.95	0.98	1.03
MF-SV-VAR	1.00	1.03	1.02	1.01	1.03	1.02
MF-TVP-VAR	1.02	1.03	1.02	0.98	0.98	0.99
MF-VAR	1.09**	1.11***	1.07**	1.05	1.05	1.02
Q-TVP-SV-VAR	1.06***	1.03	1.03	1.00	1.00	1.01
Q-TVP-VAR	1.06***	1.04	1.04	1.04	1.00	0.99
Q-VAR	1.10***	1.06***	1.05***	1.05	1.02	1.02
Q-SV-VAR	0.64	0.66	0.69	0.73	0.76	0.78
			Inflati	ion		
MF-TVP-SV-VAR	0.81***	0.83***	0.81***	0.81***	0.85***	0.86***
MF-SV-VAR	0.97	1.06	1.08***	0.95	1.03	1.06
MF-TVP-VAR	0.88**	0.88***	0.81***	0.90*	0.91*	0.86***
MF-VAR	1.06	1.20***	1.32***	0.99	1.04	1.07
Q-TVP-SV-VAR	0.88***	0.85***	0.83***	0.89***	0.87***	0.86***
Q-TVP-VAR	0.93***	0.91***	0.87***	0.92***	0.91***	0.88***
Q-VAR	1.09***	1.12***	1.15***	1.04	1.05	1.04
Q-SV-VAR	0.32	0.34	0.34	0.42	0.41	0.40
			Unemployn	nent rate		
MF-TVP-SV-VAR	0.79***	0.85**	0.89*	0.75***	0.79***	0.83**
MF-SV-VAR	0.84***	0.89**	0.93*	0.80***	0.85**	0.89**
MF-TVP-VAR	1.02	1.11**	1.16**	1.02	1.09	1.14*
MF-VAR	0.84***	0.91**	0.94	0.80***	0.85**	0.88*
Q-TVP-SV-VAR	0.96	0.99	1.01	0.92**	0.95	0.97
Q-TVP-VAR	1.03*	1.04**	1.05**	1.02	1.03	1.04
Q-VAR	1.03**	1.01	1.00	1.01	1.00	0.98
Q-SV-VAR	0.23	0.33	0.43	0.27	0.40	0.54

Note: Scores are in absolute terms for the benchmark (bottom row of each panel) and as ratios for the remaining models. Ratios below unity show that the model outperforms the benchmark. Bold figures mark the best result for the variable and horizon. *, **, and *** denote significance at the 10%, 5%, and 1% level, respectively, according to a t-test on the average difference in scores relative to the benchmark model with Newey–West standard errors.

For GDP growth density forecasts, no model significantly improves on the benchmark. Only the MF-TVP-SV-VAR and Q-TVP-SV-VAR provide an (insignificant) improvement. Thus, not only the mean of the predictive distribution is slightly more precise than those of the benchmark, but the entire distribution. As for point forecasts, IP growth density forecast from the benchmark are superior compared with the remaining models.

Regarding inflation, the results indicate two outcomes. First, the Q-TVP-SV-VAR and the MF-TVP-SV-VAR deliver the largest improvements on the benchmark. Hence, as for point forecasts, it is important to model time-variation in both the parameters and the residual variances to obtain precise predictive densities. Second, including time-variation in the parameters plays a vital role; both the MF-TVP-VAR and the Q-TVP-VAR strongly improve on the benchmark.

For the unemployment rate, the results support those from the point forecasts evaluation; the MF-TVP-SV-VAR delivers the best performance, improving on the benchmark by up to 20%. Imposing parameter instability only in the VAR coefficients seems to deteriorate the accuracy of the predictive densities. In fact, the MF-VAR and the Q-VAR outperform the TVP VARs. One reason for this results might be that the TVP VARs—particularly the MF-TVP-VAR—exaggerate the actual time-variation in the VAR coefficients. Moreover and in contrast to inflation, each mixed-frequency model improves both on the benchmark and on its quarterly counterpart. Thus, for generating precise predictive densities for the unemployment rate, it is crucial to include intraquarterly information and stochastic volatility.

In summary, the results of the predictive density evaluation support the findings from the point forecast evaluation. Using mixed-frequency models is beneficial over all variables and horizons. It significantly improves results for inflation and the unemployment rate. In addition, we confirm the importance of stochastic volatility in density forecasting by use of mixed-frequency VARs. We provide evidence that combining

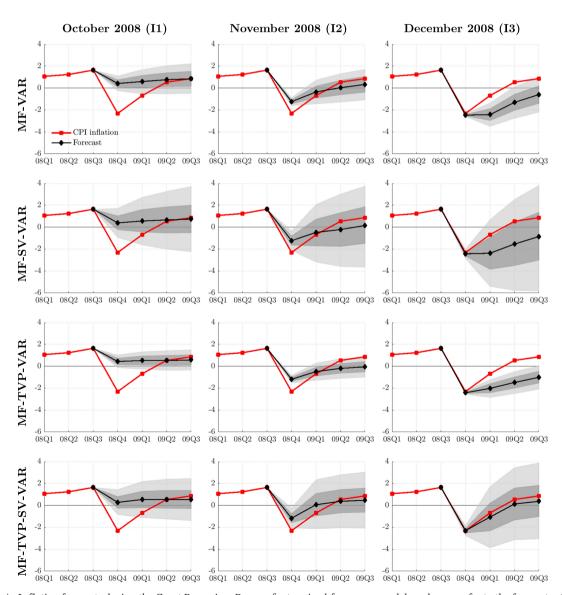


FIGURE 1 | Inflation forecasts during the Great Recession. Rows refer to mixed-frequency models; columns refer to the forecast origins. Actual values (red line) and forecasts: mean (black line) and 60% and 90% bands from the predictive distributions.

stochastic volatility, time-varying parameters, *and* mixed-frequencies significantly improves the accuracy of predictive densities.

4.4 | Forecasting During the Great Recession

So far we have demonstrated that modeling intraquarterly dynamics significantly improves forecast accuracy on average. Now we take a closer look at the MF-models' absolute performance during the Great Recession, which is of great interest, because many structural and nonstructural models failed to accurately forecast the steep contraction and the following upswing in 2008/2009. Figures 1 and 2 depict real-time quarter-on-quarter CPI inflation and the unemployment rate (red lines) along with both the means (black lines) and 60% as well as 90% error bands (shaded areas) from the predictive distributions, respectively. The figures' columns refer to the data vintages of October 2008 until December 2008.

First, for the inflation forecasts we consider the vintage of October 2008 (first column of Figure 1). Note that in this month the models do not have any information on the current quarter. In October 2008, the models' posterior means are similar for each horizon—for the nowcast, all of them lie at roughly 0.5%, which is about three percentage points too high compared with the realization. The MF-VAR (first row) and the MF-TVP-VAR (third row) deliver narrow intervals, assigning only a small fraction of probability mass to negative inflation rates. The MF-SV-VAR (second row) and the MF-TVP-SV-VAR (fourth row) generate much wider intervals, clearly including negative inflation rates. However, the realization is not included in any interval.

In November 2008, the posterior means are still similar, but become much more pessimistic. The models correctly anticipate a negative inflation rate for 2008Q4 (approximately -1%). Moreover, the 90% intervals of the stochastic volatility models

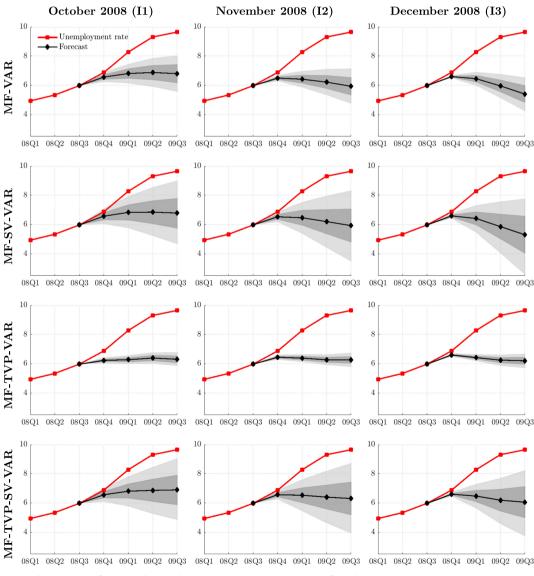


FIGURE 2 | Unemployment rate forecasts during the Great Recession. See Figure 1 for a description.

include the realization. Thus, as indicated in Section 4.1, the forecast errors become much smaller due to the additional monthly observations.

In December 2008, the models produce a forecast error of almost zero for 2008Q4 surrounded by a narrow forecast interval. The subsequent recovery, however, is best predicted by the MF-TVP-SV-model. Driven by the pessimistic nowcasts, each model except for the MF-TVP-SV-VAR forecasts negative inflation rates for the entire forecast horizon. The MF-TVP-SV-model, by contrast, provides a very accurate prediction of the recovery period. However, the large shock that hits the US economy in the fourth quarter of 2008 yields a strong increase of innovation volatility, which translates into wide predictive distributions of the stochastic volatility VARs.

For the unemployment rate (Figure 2), the MF-TVP-SV-VAR also provides the best performance. While the 90% intervals do not contain the realizations for 2009Q1 until 2009Q3, only the VARs with time-varying coefficients predicts a prolonged phase of increased unemployment, which is in turn is more pronounced according to the MF-TVP-SV-VAR.

In total, these results illustrate that the MF-VARs can translate intraquarterly information into more precise point and density forecasts. Furthermore, the findings from Sections 4.1 and 4.2 are supported; combining stochastic volatility with time-varying parameters is important for accurate now- and forecasts. Moreover, it is suggested that combining forecasts from different models would have increased forecast accuracy. In the supporting information, we detail on this using a forecast combination exercise.

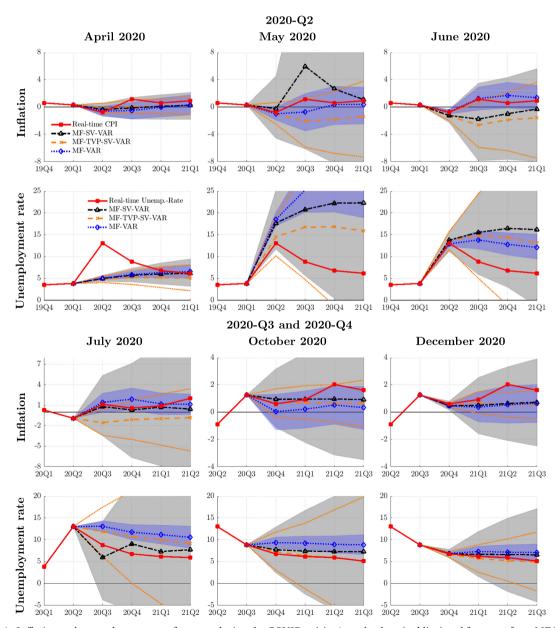


FIGURE 3 | Inflation and unemployment rate forecasts during the COVID-crisis. Actual values (red line) and forecasts from MF-VAR (dashed dotted line), MF-SV-VAR (dotted line), and MF-TVP-SV-VAR (dashed line). Shaded areas and dotted lines are 90% bands from the predictive distributions.

4.5 | Forecasting During the COVID-19 Pandemic

Since the onset of the COVID-19 pandemic, constant-coefficient VARs tend to exhibit implausible behavior (see, among other, Lenza and Primiceri 2022; Bobeica and Hartwig 2023; Schorfheide and Song 2024). Therefore, we examine whether the MF-TVP-SV-VAR is able to capture the economic developments during this period in a more meaningful way. To this end, we compare the predictions from this model with those from the MF-VAR and the MF-SV-VAR. The upper panel of Figure 3 presents the inflation and unemployment rate forecasts computed with the vintages from April-June 2020; the bottom panel presents the predictions from the July, October, and December vintages.

In the first half of 2020, the MF-VAR (dotted line) provides the most accurate inflation forecasts—particularly in June. By contrast, the MF-SV-VAR (dashed-dotted line) provides very volatile predictions that considerably overestimate the hike in inflation in May and underestimate it in June. The MF-TVP-SV-VAR (dashed line) generates less volatile forecasts but also fails to predict a prolonged increase in inflation in the first half of 2020. Moreover, exceedingly wide predictive distributions of the MF-SV-VAR indicate that the model overstates the true uncertainty. The MF-TVP-SV-VAR provides better calibrated predictive distributions, suggesting that changes in the VAR-coefficients have picked up parts of the COVID-shocks. The latter does not translate into more precise forecasts, though.

In the second half of 2020, the MF-SV-VAR outperforms the MF-VAR. Thus, it appears that the most recent observations, which are downweighted in the SV-models, distort the coefficient estimates of the MF-VAR. Except for July 2020, the MF-TVP-SV-VAR provides point forecasts, which are similar to the MF-SV-VAR but substantially better calibrated predictive densities. Hence, the results provide evidence that the combination of time-variation in both the VAR-coefficients and the residual variances is—as for the Great Recession—helpful for predicting the recovery phase in the second half of 2020.

Regarding the unemployment rate, the models predict that the COVID-shocks exhibit persistent effects in the first half of 2022—for the June 2020 vintage, the unemployment rate forecasts for 2021-Q1 range from 12% to 17% while the realization is about 6%. The strong rise in unemployment is best captured by the MF-TVP-SV-VAR. In fact, the MF-VAR exhibits explosive behavior in May 2020 with forecasts approaching 30%. 10 Compared with the rapid decline of the unemployment rate, the forecasts are excessively pessimistic and reflect that the COVID-shock entails markedly different effects than previous shocks. While the MF-VAR and the MF-SV-VAR continue to forecast persistently elevated levels of the unemployment rate throughout the second half of 2022, the MF-TVP-SV-VAR provides an accurate prediction of unemployment rate path starting in October 2020. The latter is particularly interesting given the finding of Schorfheide and Song (2024) that a linear MF-VAR, which includes considerably more information than our MF-TVP-SV-VAR, fails to provide precise unemployment rate forecasts in October 2020. Thus, these results suggest that time-variation in the VAR-coefficients helps the model to react to the extraordinary nature of the COVID-shock and translates into more precise forecasts.

However, the predictive distribution of both the MF-SV-VAR and the MF-TVP-SV-VAR are apparently too wide since the onset of COVID-crisis. The latter stems from the fact that we implement stochastic volatility via random walks for the log-variances. This implies that the SV-models cannot differentiate between transitory and persistent volatility hikes but expect permanent volatility shifts. As highlighted by Lenza and Primiceri (2022) and Carriero et al. (2024), modeling outliers in the volatility process, which do not implicate persistent shifts, is a possible solution to this problem. Another solution might be to resort to non-parametric VARs as proposed by Huber et al. (2023).

In total, our results suggest that the combination of mixed-frequencies, stochastic volatility, and time-varying parameters, would have been, to some extent, useful for policy-makers during the COVID-19 pandemic. In particular, the rapid decline of the unemployment rate predicted by the MF-TVP-SV-VAR stands out. However, given the large forecast errors and excessively wide predictive distributions, the benefits would have been clearly limited.

5 | Conclusion

Several studies show that modeling structural change improves forecast accuracy. We contribute to this discussion by investigating whether allowing for structural change in a mixed-frequency VAR setup further improves performance.

We conduct a rigorous real-time out-of-sample forecast experiment and generate predictions for GDP growth, IP growth, CPI inflation, and the unemployment rate. Our findings show that modeling monthly dynamics substantially improves forecast accuracy. Nowcasts and short-term forecasts especially benefit from within-quarter information, while for longer horizons, the advantages vanish in most cases. The MF-TVP-SV-VAR provides, on average, the best point and density forecast performance. Both inflation and unemployment rate forecasts benefit considerably from modeling both monthly dynamics and structural change. Furthermore, we assess the forecast performance during the Great Recession and the COVID-crisis. For the Great Recession, we show that the combination of time-varying parameters and stochastic volatility yields overall the best performance for the downturn and subsequent recovery. For the COVID-crisis, we provide evidence that allowing for parameter flexibility would have been somewhat helpful to capture the extraordinary nature of the shock.

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Data Availability Statement

The data that support the findings of this study are openly available in the ALFRED database of the St. Louis Fed (https://alfred.stlouisfed.org/).

Endnotes

- ¹ Amir-Ahmadi et al. (2020) show that the magnitude of the hyperparameters changes significantly when estimated on monthly data compared with quarterly data, which affects the time-variation in the model's coefficients.
- ² While the random walk has the undesired property that it is unbounded, this property is innocuous if the process operates for a finite period of time and exhibits a small variance (which is what we obtain). Moreover, the random walk is more robust to misspecification and allows for permanent shifts. Finally, the random walk has the desired property that the predictive distribution widens with the forecast horizon, reflecting the rising uncertainty about the future path of the variable under consideration.
- 3 The diagonal assumption is made to simplify inference. We also estimated the respective models with a non-diagonal Q matrix and obtained off-diagonal elements close to zero. Thus, in line with, for example, Eisenstat et al. (2016), the results for models using both a diagonal and non-diagonal innovation covariance matrix are fairly similar.
- ⁴ We set p=2 for the quarterly model to be consistent with the literature on US data (see, e.g., Primiceri 2005; D'Agostino et al. 2013; Clark and Ravazzolo 2015). The monthly models have 4 lags to keep them computationally feasible and comparable to the quarterly models.
- ⁵ Details on the Monte Carlo design are provided in the supporting information.
- ⁶ In Section 4.5, we elaborate further on this issue.
- ⁷ We prefer the CRPS over the frequently applied log-scores, because it is better able to evaluate forecasts close but not equal to the realization and less sensitive towards extreme outcomes (Gneiting and Raftery 2007). Moreover, we abstract from evaluating the nowcasts with respect to predictive densities. Depending on the information sets, the nowcasts of the mixed-frequency models consist of quarterly averages over draws from the simulation smoother and realizations. Therefore, the nowcast densities of the mixed-frequency models are very narrow compared with the quarterly models and thus hardly comparable.
- 8 As the marginal impact of an additional month of information becomes less important for forecasts at higher horizons, the RMSEs for higher horizons become similar across the information sets. For the forecast evaluation, we therefore compute total RMSEs by averaging over the entire forecast sample. Appendix C of the supporting information provides plots for the relative RMSE for each information set.
- ⁹ To circumvent this issue, Schorfheide and Song (2024) propose to ignore the observations from January 2020 onward for the model estimation.
- ¹⁰ These results resemble those of Schorfheide and Song (2024) and Carriero et al. (2024), who use medium-sized MF-VARs. We have trimmed the figure to 25% to facilitate readability.

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Supporting Information

Additional supporting information can be found online in the Supporting Information section.